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A hybrid approach to cardinality constraint portfolio selection problem based on nonlinear neural network and genetic algorithm

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ABSTRACT

Portfolio Optimization (PO) is an extremely popular method that selects the best portfolio for an investor. When the classic methods fail to find an exact solution, heuristic techniques are designed to find an approximate solution. In the literature, such techniques have often been used in the portfolio selection problem. However, hardly any of these methods utilize a neural network to apportion the extent of stocks. The basic objective of portfolio optimization is to minimize the risk of the portfolio while maximizing the expected return of the portfolio. In reality, the Standard Portfolio Optimization Model does not fulfill stock market requirements in a money related world. Indeed, this problem cannot limit the amount of stock in the portfolio. On the other hand, the precise number of stocks are taken into account by the Cardinality Constraint Portfolio Optimization model, which is the Mixed-Integer Quadratic Programming problem. While minimizing the risk of the portfolio, limiting the expected return and the number of stocks is the main subject of this study. In this study, a hybrid approach is proposed, based on the Nonlinear Neural Network and the Genetic Algorithm to solve the Cardinality Constraint Portfolio Optimization Model. To investigate the effectiveness of the proposed hybrid approach, the Istanbul Stock Exchange (ISE-30[1]) data is used. The ISE-30 data[1] consists of daily prices, from May 2015 to May 2017. The ISE-30 data[1] from May 2017 to July 2018 is used as out-of-sample. To clarify the effectiveness proposed, the method was applied to publicly data sets which are used for numerous different portfolio selection strategies in many articles. The proposed hybrid approach to the cardinality constraint portfolio optimization problem has more viable outcomes than current strategies.

1. Introduction

The portfolio optimization problem is a conspicuous issue in the world of economics. Until the 1950s, the conventional portfolio approach was managed by simply minimizing risk. Harry Markowitz (1952) and Markowitz (1959) proposed modern portfolio theory which is a pioneering work in modern portfolio theory. The main objective is that the expected (mean) return minimizes, while at the same time, the risk (variance) of the portfolio minimizes. Markowitz's portfolio optimization problem is known as the Mean-Variance Model. Indeed, the proportion of stocks that investors will buy and hold, are estimated by Markowitz's Mean-Variance PO. On the other hand, the Mean-Variance Optimization Method does not meet the expectations of today's financial world. Solving a large-scale quadratic programming problem is the most important issue. Because it is an NP-hard (non-deterministic polynomial-time hardness) problem, computing time is longer so Konno & Yamazaki (1991) replace the risk function in the Markowitz Mean-Variance Model with the absolute deviation of the risk function and formulate a Mean Absolute Deviation Portfolio Optimization Model. After this, many models have been generated based on the Markowitz Mean-Variance Model. One of these models is the Mean-Variance Model proposed by Jorion (1996). Simaan compares the Mean-Variance Model with the Mean Absolute Deviation Model (Simaan, 1997). Moreover, Rockafellar and Uryasev (2000) studied the Mean-Cvar Model. Yan and Li (2009) propose the Multi-Period Semi-Variance Model. Junhui et al. (2009) present a Nonlinear Futures Hedging Model based on skewness risk and kurtosis risk. These models need exact estimation; mean of return, variance, and covariance due to probability theory. Chang et al. (2000), suggest a lower and upper proportion of a particular asset constrained, which means the size of holding assets is limited and quantity is constrained. In the literature, limiting the number of assets has been studied in extensive reviews. Bruni et al. (2015) propose a risk-

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return approach to the Enhanced Indexation Problem. There have been many studies on this topic, such as Canakgoz and Beasley (2009), and Guastaroba and Speranza (2012). In these studies, the fitness function is a linear programming problem that maximizes excess return regardless of the underperformance of risk. On the other hand, the Markowitz Mean-Variance Model with cardinality constraint is called the MVCCPO Problem (Mean-Variance Cardinality Constraint Portfolio Optimization Problem) and is a type of mixed-integer quadratic optimization problem (MIQP). In this problem, while the risk is minimized, the target return should be satisfied. The cardinality constraint is the number of assets to be invested.

Up to now, many exact and heuristic algorithms have been designed to solve the Mean-Variance Cardinality Constraint Portfolio Optimization Problem. A number of the exact algorithms are branch-and-cut algorithms, different to the convex algorithm (Bienstock,1996). Konno and Wijayanayake (2001) propose a branch and bound algorithm in operational research to calculate the optimal solution of a portfolio selection problem under concave transaction costs and minimal transaction unit constraints. Recently, Xue, Di, Zhao, and Zhang (2019) use uncertain portfolio selection with mental accounts and realistic constraints. Furthermore, Babazadeh and Esfahanipour (2019) suggest the multi-period mean-VaR Portfolio Optimization Model, considering practical constraints and transaction costs.

Since exact algorithms have a high computing time and they only find an optimal solution, heuristic algorithms have aroused interest in the optimization world. Heuristic algorithms endeavor to locate ideal arrangements in a sensible time; however, they do not ensure achieving ideal solutions. Therefore, heuristic algorithms are used to solve MVCCPO problems, such as Differential Evolution, Evolutionary Algorithm, Tabu Search, Particle Swarm Optimization, Simulated Annealing, and others. For the portfolio selection algorithm, Lai uses a double-stage genetic (Lai, Yu, Wang, & Zhou, 2006). Charpentier and Oulidi (2009) suggest a simple method to estimate optimal allocation, based on a Value-at-Risk minimization constraint, as well as getting empirical-confidence intervals. Juan (2012) considered a portfolio optimization algorithm for multi-objective planning. The presented method forms a genetic algorithm (GA) with a multi-objective optimization portfolio planning system.

Classical optimization methods, such as the Simplex Method, the Interior Point Method, the Active Set Method (ASM), and the Gradient Projection Method are commonly-preferred to solve quadratic problems. These methods could not meet today demands, and furthermore, they need much computational time and cost. Therefore, in 1986, Tank and Hopfield (who initiated neural network models), suggested a recurrent neural network based on a gradient method. The major advantage of this method is its implementation using analog electronic circuits. This method uses analog electronic circuits for the implementation which is its main advantage. To minimize computational time, these circuits operate in parallel. Kennedy and Chua (1988) proposed a method based on the Karush-Kuhn-Tucker conditions, which guarantees convergence. However, this method only gives an approximated optimal solution. Maa and Schanblatt (1992) developed a two-phase model that provides an exact solution. Because of complex models, system parameter selection needs more attention. Zhang and Constantinides (1992) created a neural network based on lagrangian programming that suggests new variables to deal with inequality constraints. Having high dimension resulting variables leads to more computational time. On the other hand, to solve linear programs Wang (1994) added a time-variant variable to the neural network. Xia (1996) suggests a method that solves primal and dual problems simultaneously. These methods overcome all of the problems. In 2000, a recurrent neural network model was presented (Nyguen, 2000) which has Xia's model advantages. In addition, the recurrent neural network has a more rapid convergence rate and a more intuitive economic interpretation. In 2014, QP problems could be solved thanks to an improved model (Yan, 2014).

The Portfolio optimization model is an important theme in finance theory Nonlinear Neural Networks (NNN) are used to sketch graph efficient frontiers associated with the portfolio selection problem (Bohra & Bhatia, 2012). Recently, the neural network has been frequently used in financial expectations. In that particular study, Bohra classified assets with a membership of expected returns. Markowitz's Mean-Variance Portfolio Optimization with cardinality constraint is not only quadratic optimization adding cardinality constraint to the problem, it becomes a Mixed Integer Quadratic Programming Problem (MIQP). In this study, to solve this Mixed-Integer Quadratic Programming Optimization Problem, a hybrid algorithm is suggested as a solution for the MVCCPO Problem. To solve the binary integer part, a genetic algorithm is used, meanwhile, portfolio optimization is solved by a new neural network. The proposed neural network is based on solving primal and dual problems simultaneously. In this respect, Istanbul Stock Exchange-30 data is used to solve the nonlinear neural network which is adapted to solve portfolio optimization. The paper is organized as follows. In Section 2, portfolio selection is presented. In Section 3, the Nonlinear Neural Network is described. In Section 4, the genetic algorithm is briefly given. In Section 5, we propose a hybrid algorithm based on a Nonlinear Neural Network and Genetic Algorithms (NNNGA). In Section 6, we illustrate the hybrid algorithm with ISE-30 data[1] by computations on both ISE-30 and other stock exchanges. In Section 7, we finish with a discussion.

2. Portfolio selection

The aim of the investment is to have maximum returns, and to take a little risk. Because of this, the tradeoff between risk and return is a vital issue for the investor. Standard portfolio optimization was presented in 1959 by Markowitz. The main purpose of this problem is to minimize risk while ensuring that the portfolio has an expected return of R^*, and that the proportion of assets totals one. Standard Portfolio Optimization (Markowitz, 1959) determines asset proportions (x_i, $i = 1, 2, \cdots, N$), where N indicates the number of assets that investors could select. The Standard Portfolio Optimization Problem is a nonlinear programming model that minimizes the fitness function,

$$Min \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij},$$

Subject to

$$\sum_{i=1}^{N} x_i = 1,$$

$$\sum_{i=1}^{N} x_i \mu_i = R^*,$$

$$0 \leq x_i \leq 1, i = 1, \cdots, N \tag{1}$$

The expected return of the portfolio $E(R_p) = \sum_{i=1}^{N} x_i \mu_i$, risk of the portfolio $\sigma_p = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \sigma_{ij}}$, where $N$is the number of assets in the portfolio, and $\mu_i$presents the expected return of the i^{th} asset. σ_{ij} is the correlation between the i^{th} and j^{th} assets and x_i; i^{th} assets' proportion of being in the portfolio and R^* is the expected return in the portfolio. The Sharpe Ratio (S_p), Sortino Ratio (SR), Information Ratio(IR), and Average Return (AR) denote respectively;

$$S_p = (E(R_p) - R_f)/\sigma_p,$$

$$SR = E\left[R^{out} - R_f\right]/\sigma\left(minR^{out} - R_f, 0\right),$$

$$IR = E[R^{out} - R_{benchmark}]/\sigma_p(R^{out} - R_{benchmark}),$$

$$AR = E[R^{out}]. \tag{2}$$

The Sharpe Ratio (S_p) is the ratio of expected return $E(R_p)$ to the risk of the portfolio (σ_p). We expect the portfolio we have chosen to have the highest Sharpe Ratio. R_f is the risk-free rate of return. The Sortino Ratio (SR) is the average of $R^{out} - R_f$ and its downside deviation. Like the Sharpe Ratio, the larger Sortino Ratio is preferable. The Information Ratio (IR) is the difference between the portfolio return of the out sample and the benchmark index. The average return is the expected return of the out-sample of a portfolio (Bruni et al., 2017).

The main goal is to get a proportion of the assets in the selected portfolio (x_i). However, Markowitz's Mean-Variance has no constraint that limits the proportion of assets in the portfolio. In order to limit the number of assets that are selected the Markowitz Mean-Variance Model with cardinality constraint is suggested. The parameter of $z_i$is limited with to proportion of assets being in the portfolio. Portfolio selection under the cardinality constraint model is as follows:

$$Min\sum_{i=1}^{N}\sum_{j=1}^{N}x_iz_i\sigma_{ij}z_jx_j,$$

$$\sum_{i=1}^{N}x_i = 1,$$

$$\sum_{i=1}^{N}z_i = K,$$

$$\sum_{i=1}^{N}x_i\mu_i = R^*,$$

$$\varepsilon_iz_i \leq x_i \leq \delta_iz_i, z_i\in\{0,1\}.i = 1,\cdots,N. \tag{3}$$

In addition to the Markovitz model, there are cardinality and bounding constraints. One of the cardinality constraints, K, defines the number of different assets in the portfolio. A second constraint, z_i, determines the existence of assets in the portfolio, which means if z_i is 1, the i^{th} asset is included in the portfolio, otherwise z_i is 0 which means the asset is not in the portfolio. The bounding constraints are ε_i and δ_i respectively; the lower and upper bounds for the proportion of capital to be invested in asset i at the range $0 \leq \varepsilon_i \leq \delta_i \leq 1$. Markowitz's Mean-Variance Problem has become a MIQP model with the addition of cardinality and bounding constraints that efficient algorithms do not have.

3. Genetic algorithm

The main purpose of the traditional and heuristic algorithms is to achieve the best possible solution in a reasonable period. Indeed, classical methods cannot solve complex problems because of insufficient time. Heuristic algorithms cannot guarantee an exact solution, only an approximate solution in a reasonable time period. Therefore, heuristic techniques were developed to solve these complex optimization problems, such as Particle Swarm Optimization, Ant Colony Optimization, the Genetic Algorithm, and others. Holland (1992) proposed the Genetic Algorithm (GA) in 1975, which is a heuristic search inspired by Charles Darwin's theory of natural selection. Moreover, the GA is an adaptive search method, which is composed of selection, inheritance, mutation, and certain population dynamics. In addition, the GA can solve both constrained and unconstrained optimization problems. Salvatore, Arnone, Loraschi, and Tettamanzi (1993) suggest the GA for the Unconstrained Portfolio Optimization Problem, with the risk associated with the portfolio being measured by a downside risk. Shoaf and Foster (1998) solved Markowitz's Mean-Variance Model by the Genetic Algorithm. The GA used to support portfolio optimization for index fund management.

In the Genetic Algorithm, firstly, initialize a population which contains a randomly generated constant number of chromosomes. Related to portfolio optimization problems, each chromosome represents the proportion of assets in the portfolio. To get possible asset weights, the fitness function is optimized.

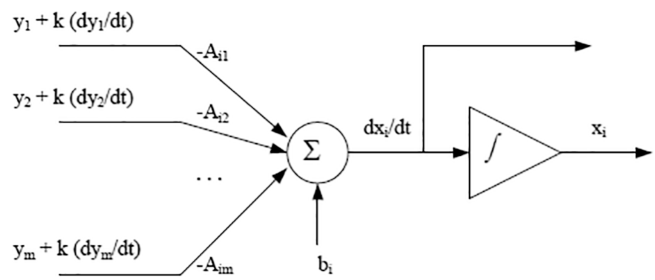

Fig. 1. Structure of the primal neuron.

The chromosomes represent solutions that are evaluated by the evaluation function. Moreover, there are strategies to converge good fit chromosomes; the crossover, mutation values, and natural selection. The chromosome with good fitness results is the best result. Furthermore, the best chromosome gives the selected proportion of assets in the portfolio.

4. The steps of the genetic algorithm

Step1: Initialize population which is randomly generated.
Step2: Evaluate the fitness of the population.
Step3: Apply elitist selection: carry on the best individuals to the next generation from reproduction, crossover, and mutation.
Step4: Replace the current population by the new population.
Step5: If the termination criterion is satisfied then stop, or else go to Step 2.

In the reproduction process, the children's chromosomes are generated by parental chromosomes. The reproduction process terminated when all of the children's chromosomes have generated. The best solution is the proportion of assets that have minimum risk.

5. Nonlinear neural network for portfolio optimization problem

Nguyen presents a Nonlinear Neural Network to solve linear programming models. Nguyen's network solves the primal problem and dual problem simultaneously. The Nonlinear Neural Network consists of two layers. One is a primal neuron and the other is a dual neuron. Inputs of primal neurons are outputs of dual neurons and derivatives. Similarly, inputs of dual neurons are outputs of the primal neuron and their derivatives. In contrast to Tank and Hopfield's (2019) proposed algorithm, Nyguyen's neural networks' neurons are symmetrical. Fig. 1 also demonstrates the configuration of a primal neuron (1988).

The improvement in this method is that not only are dual neurons taken as an input of the model, but also their derivatives. This structure ensures nonlinearity in the system. The dual neurons have a similar configuration. Nguyen's neural network is extended by Yan (2014) to solve quadratic programming problems. Background information is given in the following articles: Maa and Schanblatt (1992), Nyguen (2000), Tank and Hopfield (1986), Wang (1994), Xia (1996), Kennedy and Chua (1988), Zhang and Constantinides (1992), and Yan (2014). Consider a QP problem given as:

$$\frac{1}{2}x^TQx + e^Tx,$$

$$Dx = b,$$

$$Ax \geq c,$$

$$x \geq 0 \tag{4}$$

where x and e are n-dimensional vectors, Q is an $n \times n$ symmetric pos-

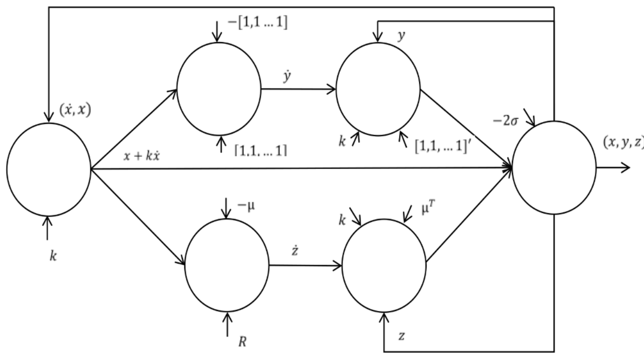

Fig. 2. The topology of the nonlinear neural network for solving portfolio optimization.

itive definite matrix, $D \in R^{p \times n}$, $A \in R^{m \times n}$,$b \in R^{p \times 1}$, and $c \in R^{m \times 1}$. The Lagrangian Function of this minimization problem can be written as:

$$\mathscr{L}(x, y, z) = (1/2)x^T Qx + e^T x - y^T (Dx - b) - z^T (Ax - c) \tag{5}$$

where $z \in R^p_+ = \{z \in R^p | z \geq 0\}, y \in R^m$ are Lagrangian multipliers. According to Karush-Kuhn-Tucker (KKT) conditions, $x^*$is a solution of the Lagrangian Function given by Eq. (5) if $y^* \in R^m$and$z^* \in R^p_+$exist, so that (x^*, y^*, z^*) satisfies the conditions. Yan (1992) proposes the recurrent neural network for solving the quadratic programming problem given as:

$$\dot{x} = -Q(x + k\dot{x}) - e + D^T (y + k\dot{y}) + A^T (z + k\dot{z}), x \geq 0$$

$$\dot{y} = b - D(x + k\dot{x}),$$

$$\dot{z} = -A(x + k\dot{x}) + c, z \geq 0. \tag{6}$$

The Mean-Variance Portfolio Optimization Problem (MVPO) is a quadratic programming problem. The main purpose of the MVPO problem is to minimize risk while being sure that R is the expected return of the portfolio and the total proportion of assets is equal to one. The Nonlinear Neural Network can solve quadratic programming problems, which are types of nonlinear programming problems.

In the Nonlinear Neural Network Problem, both primal and dual problems are solved simultaneously. Outputs of dual neurons and their derivatives are the inputs of primal neurons. Similarly, the primal neuron's outputs and derivatives are the inputs of dual neurons and their derivatives. In this paper, a neural network is proposed to solve the portfolio optimization problem given as:

$$\dot{x} = -2\sigma(x + k\dot{x}) + I^T (y + k\dot{y}) + \mu^T (z + k\dot{z}), x > 0, \tag{7a}$$

$$\dot{y} = [1, 1, \cdots, 1] - [1, 1, \cdots, 1](x + k\dot{x}), \tag{7b}$$

$$\dot{z} = -\mu(x + k\dot{x}) + R, z > 0 \tag{7c}$$

The topology of this neural network is given by Fig. 2.

Nonlinear Neural Network Algorithm for PO

Step 1: Initialize the $x = 0, y = \{0, 1, 2, 3\}, z = -1, 0, 1, 2$and dx, dy, and dz as a zero's vectors.

Step 2: Assign mean and variance of assets to the corresponding variables μ, σ.

Step 3: Calculate the first node to get$(x + k\dot{x})$.

Step 4a: Take the first node's output as an input to calculate $\dot{y}$ as output using Eq. (7b). (I is the$[1, 1, \cdots, 1]$vector.)

Step 4b: Take first node i.e. $(x + k\dot{x})$ to calculate output $\dot{z}$ using Eq. (7c).

Step 5a: Take $\dot{y}$and$y$as an input to calculate node i.e. $(y + k\dot{y})$.

Step 5b: Take $\dot{z}$ and z as an input to calculate node i.e. $\mu^T (z + k\dot{z})$.

Step 6: Calculate $\dot{x}$ using former node outputs as an input of the last node using Eq. (7a).

Step 7: If termination criteria are satisfied, go to Step 8 and if not satisfied go to Step 3.

Step 8: Calculate expected return, risk, and Sharpe ratio of the portfolio with selected proportions of x_i using Eq. (2).

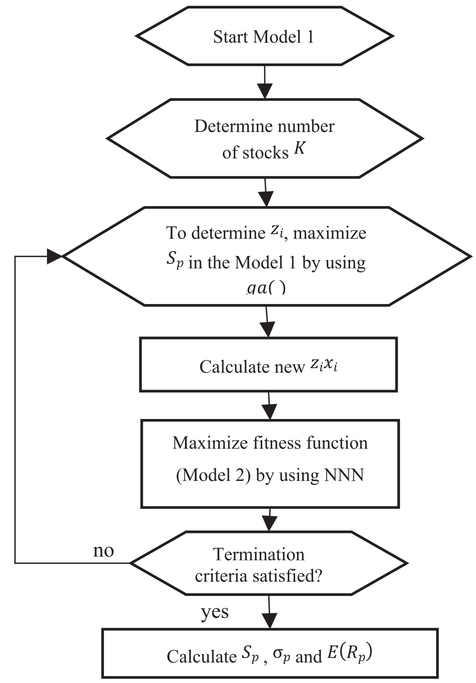

Fig. 3. Flow chart of the proposed hybrid algorithm.

6. A Hybrid Approach to Cardinality Constraint Portfolio Selection Based on the Nonlinear Neural Network and Genetic Algorithm

In this section, we discuss Markowitz's Mean-Variance Model, which added the cardinality constraint. While adding the cardinality constraint, this optimization problem becomes a Mixed-Integer Quadratic Programming Problem with z_i taking binary variables {0, 1}. The suggested hybrid method involves two optimization algorithms to solve the cardinality constraint PO problem. The cardinality part of the algorithm is solved by the genetic algorithm using the GA function '$ga(\cdots)$' in MATLAB. In the first part of the algorithm, the Sharpe Ratio, which is also given in Eq. (8), maximizes the expected return-risk ratio of the portfolio which the cardinality constraint z_i has to satisfy. After this, the second part of the algorithm has minimized the risk of the portfolio using the nonlinear neural network. Therefore, the MIQP is solved by a hybrid algorithm which is the Genetic Algorithm and the Nonlinear Neural Network.

Model 1

$$S_p = \frac{E(R_p)}{\sigma_p}$$

$$\sum_{i=1}^{N} z_i = K$$

$$z_i = 0, 1$$

$$i = 1, 2, \cdots, N. \tag{8}$$

Model 2

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_i z_i \sigma_{ij} z_j x_j$$

$$\sum_{i=1}^{N} x_i = 1$$

$$\sum_{j=1}^{N} x_i \mu_i = R^*$$

$$0 \le x_i \le 1, i = 1, 2, \cdots, N. \tag{9}$$

6.1. Proposed hybrid algorithm for cardinality constraint PO

The steps of the algorithm are given by the flow chart in Fig. 3 as follows:

Step 1: Number of stocks (K) are taken into the portfolio determined by the first model.

Step 2: While the Sharpe Ratio (S_p) is maximized the genetic algorithm function is created to enable decision variable z_i to be taken as binary integer values.

Step 3: A new x_i vector is created for decision variables (assets) to be taken into the portfolio according to z_i.

Step 4: The x_i vector for stock rates is determined by a nonlinear neural network based on the portfolio optimization algorithm.

Step 5: The Sharpe Ratio, risk and expected return of the portfolio, are calculated by the determinate vector for stock rates x_i.

6.2. Computational experiments

In this study, we hybridize the Nonlinear Neural Network for portfolio optimization problems and genetic algorithms to solve the MVCCPO problem. The Istanbul Stock Exchange-30, Turkey, (ISE 30) data shows the daily prices between June 2015 and May 2017, composed of 480 days in total. This data is obtained from https://tr.investing.com/. Heuristic and traditional methods are compared to clarify the advantages and effectiveness of the proposed algorithm. The active set method, the conventional method, tackles the Quadratic Optimization Problem and also solves a sequence of equality-constrained quadratic problems. This strategy comprises two stages. In the first stage, the objective is ignored while a feasible point is found for the constraints. In the second stage, the objective is minimized while feasibility is maintained. In the Portfolio Optimization Problem, the active set method is frequently used. In this study, to compare the suggested method, firstly, the Active Set Method is used by the function 'optimoptions', which is used in MATLAB2018b. Secondly, NNN for the PO is compared with the suggested method, which is a hybrid method based on the NNN and GA for the MVCCPO. To examine algorithms, MATLAB2018b is used. The covariance matrix $\sigma_{30\times30}$ and the mean vector $\mu_{30\times1}$ of ISE-30[1] assets are calculated as follows:

$$\mu = \begin{bmatrix} 0.000665 \\ 0.001305 \\ 0.002515 \\ 0.000546 \\ 0.000650 \\ 0.001745 \\ 0.000466 \\ \vdots \\ 0.000778 \\ 0.000943 \\ 0.000366 \end{bmatrix} \tag{10}$$

$$\sigma = \begin{bmatrix} 0.00034056 & 0.0001635 & 0.00013038 \cdots & 0.0002866 \\ \vdots & & \ddots & \vdots \\ 0.00028666 & 0.0001615 & 0.00013677 \cdots & 0.00035471 \end{bmatrix} \tag{11}$$

According to the principle of superiority, the low risky investment will be selected where investments have the same variance. Moreover, an increasing expected return leads to an increase in the risk in the portfolio. The expected return, the risk of the portfolio, and the fitness function ($f(x)$) for the different initial values of y and z are given in Table 1.

Table 1
The selected proportion by the proposed method.

i	Assets	z_i	x_i	i	Assets	z_i	x_i
1	AKBNK	0	0	16	OTKAR	1	0.1250
2	ARCLK	0	0	17	PGSUS	0	0
3	ASELS	1	0.3870	18	PETKM	1	0.3870
4	BIMAS	0	0	19	SISE	0	0
5	DOHOL	0	0	20	TAVHL	0	0
6	ECILC	0	0	21	TKFEN	0	0
7	EKGYO	0	0	22	THYAO	0	0
8	ENKAI	1	0.0460	23	TOASO	0	0
9	EREGL	0	0	24	TUPRS	1	0.0550
10	GARAN	0	0	25	TTKOM	0	0
11	SAHOL	0	0	26	TCELL	0	0
12	KRDMD	0	0	27	HALKB	0	0
13	KCHOL	0	0	28	ISCTR	0	0
14	KOZAL	0	0	29	VAKBN	0	0
15	KOZAA	0	0	30	YKBNK	0	0

Table 2
Comparison of Methods.

Name of assets	ASM	NNN	NNNGA
ARCLK	0.0135	0	0
ASELS	0.0600	0	0.3870
BIMAS	0.1869	0.3500	0
DOHOL	0.0368	0.0700	0
ECILC	0	0.0200	0
ENKAI	0.1514	0.0100	0.0460
GARAN	0	0	0
KCHOL	0	0	0
OTKAR	0.1065	0.0800	0.1250
PETKM	0.1267	0.3400	0.3870
TKFEN	0.0210	0.1300	0
TUPRS	0.1019	0	0.0550
TTKOM	0.0780	0	0
TCELL	0.1173	0	0
Expected Return%	0.0940	**0.2000**	**0.2000**
Risk	0.0010	**0.0123**	0.0125
Information Ratio	−0.0056	0.0143	**0.1014**
Sortinio Ratio	0.1542	0.1946	**0.2975**
Sharpe Ratio	0.0416	**0.1625**	0.1592
$f(x)$*10^{-1}	0.0005	**0.0015**	**0.0015**
Run time (seconds)	2.53	**0.92**	23.34

6.3. An application of the hybrid approach to cardinality constraint portfolio selection

When the cardinality constraint is added to the Markowitz Mean-Variance Model, the problem with the ISE-30 data[1] is transformed into the models given in Eq. (11). The cardinality parameter of $z_i$takes the value1or 0.Therefore, it is decided whether or not the related assets will be included in the portfolio. Therefore, the investor will choose Kassets. In this study, the Genetic Algorithm is used to select the binary integer parameter z_i in the model. The constants are ($K = 5, R = 0.002$) decided. The names of selected assets and ratios obtained from the solution of the models are given in Table 1.

Therefore, the assets and rates are determined as 39% ASELS, 4% ENKAI, 13% OTKAR, 39% PETKM, and 5% TUPRS. The expected return, risk, and the Sharpe Ratio of the selected portfolio are, 0.0020, 0.0125, and 0.592, respectively. Concerning all of the forecasting comparisons reported in this work, we examine the following three months, which is ISE-30 data[1] between May 14, 2017 to August 16, 2017. The distribution

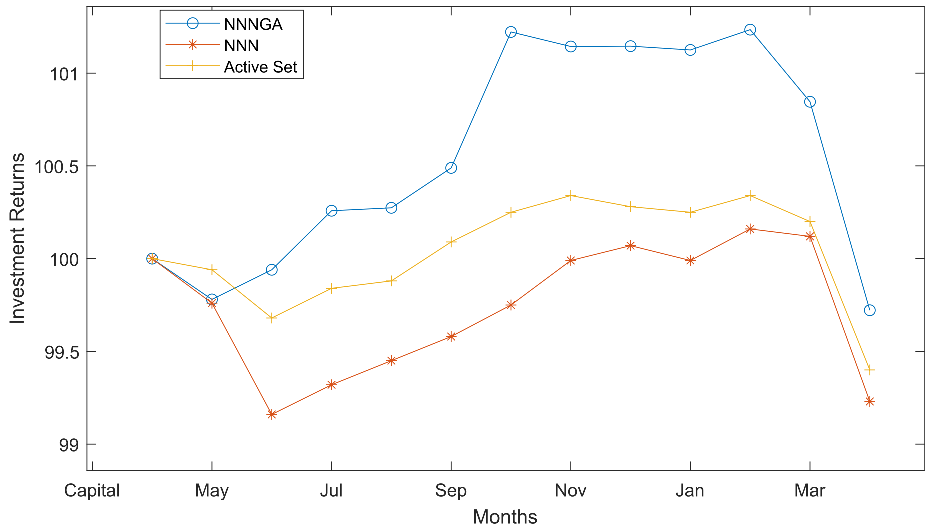

Fig. 4. Comparison of methods in following 12-months investment.

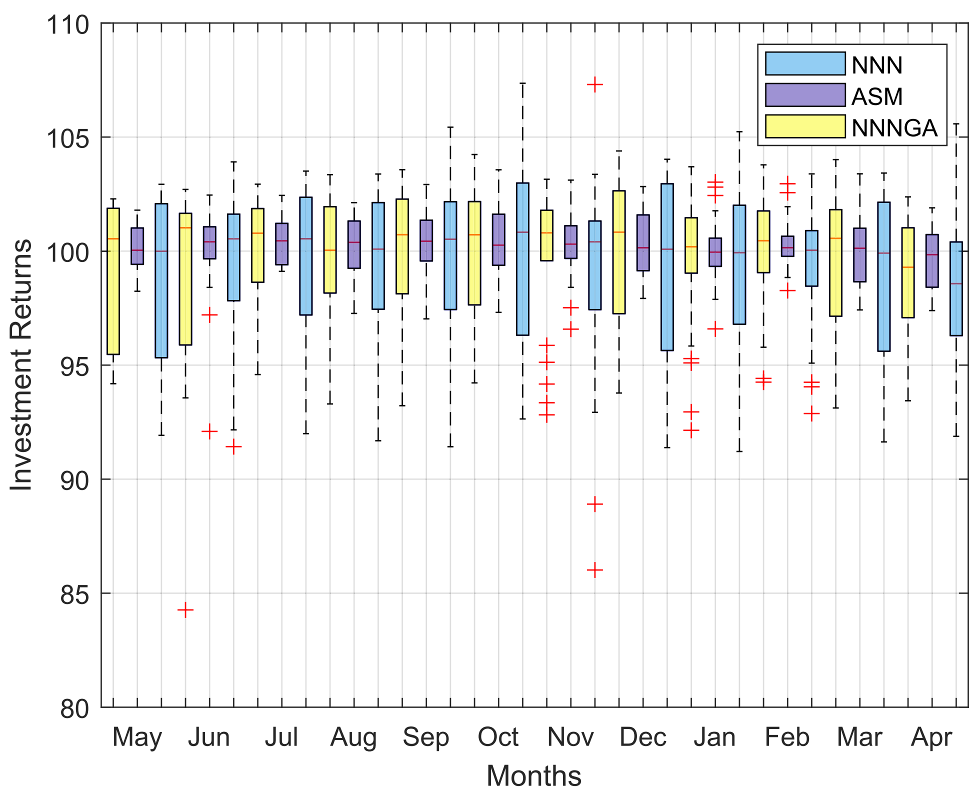

Fig. 5. Boxplots of methods in following 12-months investment.

of shares according to the methods is given in Table 2. The proposed method limited the number of assets with $K = 5$.

At first glance, the simple Nonlinear Neural Network for portfolio optimization seems to give efficient results. To derive real results, we invest 100 units of the capital in the selected portfolios. The investor invests 100-unit money to the selected portfolios by twelve methods. In Fig. 4, it can be seen that for the selected portfolio by hybrid algorithm, the proposed nonlinear neural network with the genetic algorithm, earns more money than other portfolios in the following twelve months. For the detailed information, boxplots of each method can be seen in Fig. 5. These methods tested by ANOVA to determine the difference of means for the 12 months data. In fact, there is a significant difference between the means of each method ($p = 0.000 < 0.05$).

6.4. *Application for public data sets*

Publicly data sets used to get the efficiency of the proposed method. These public datasets also have a larger temporal horizon. Portfolio selection strategies can be tested on publicly available datasets to compare their efficiencies. Bruni et al. (2016) obtains portfolios by several strategies, mainly concerning stochastic dominance In light of the information in that article data sets; Dow Jones Industrial Average (Dow Jones), National Association of Securities Dealers Automated Quotation (NASDAQ100), Financial Time Stock Exchange (FTSE100), and Fama & French 49 Industrial portfolios (FF49) are chosen for comparison with the proposed method. These datasets can be obtained from Bruni et al. (2016) in the Appendix. Detailed information regarding

Table 3
Information about major stock market datasets.

Name of Datasets	Time interval	Number of assets	Observations	Country
Dow Jones	Feb 1990–April 2016	28	1363	USA
Nasdaq 100	Nov.2004–April 2016	82	596	USA
FTSE100	July 2002–April 2016	83	717	UK
FF49	July 1969–July 2015	49	2325	USA

Table 4
Comparison of Methods with NNNGA.

	Methods	Sharpe	Sortino	Information ratio	Average return*10^{-2}
Dow Jones	CZεSD	0.09	0.14	**0.09**	0.23
	L-SSD	0.08	0.11	0.01	0.17
	KP-SSD	0.12	**0.17**	0.08	0.36
	NNNGA	**0.14**	**0.17**	0.06	**0.38**
Nasdaq100	CZεSD	0.12	0.17	0.14	0.40
	L-SSD	0.13	0.20	0.07	0.34
	KP-SSD	0.12	0.16	0.09	0.55
	NNNGA	**0,16**	**0.28**	**0.16**	**0.59**
FTSE 100	CZεSD	0,09	0,12	0,18	0.23
	L-SSD	0.13	0.17	0.14	0.28
	KP-SSD	0.14	0.20	0.14	**0.50**
	NNNGA	**0.39**	**0.39**	**0.23**	0.49
FF49	CZεSD	0.20	0.25	0.15	0.50
	L-SSD	0,22	**0.28**	−0,01	0.42
	KP-SSD	0.20	0,26	**0,08**	**0.60**
	NNNGA	**0.24**	0.13	−0.03	0.45

the major stock market datasets sets is given in Table 3.

We apply the proposed method to these public data sets. We divide the following data sets into 80% in-sample data, and 20% out-sample data. In Table 4, there are other techniques for portfolio selection which are frequently used in articles. Bruni et al. (2017) compares many other methods for the portfolio selection problem. In Table 4, the best results of portfolio selection strategies for each dataset are marked in bold.

7. Discussion

In this paper, we proposed a hybrid algorithm based on nonlinear neural network and genetic algorithm to solve the mean–variance cardinality constraint portfolio optimization problem. The proposed algorithm has been run to hold five assets i.e. $K = 5$. Results are compared with a classic method (Active Set Method) and an unlimited number of assets' portfolio problems which are given in Table 2. The ISE-30 data[1] is used for the comparison of the nonlinear neural network, a hybrid approach for GA and NNN, and the ASM that is frequently used for solving quadratic optimization problems such as portfolio optimization problem. The nonlinear neural network and hybrid approach have the same expected return which shows both satisfied the expected return constraint. On the other hand, the risk of the hybrid approach is much lower than NNN. Because of this reason, the hybrid approach based on NNN, and the genetic algorithm has a smaller Sharpe Ratio then the NNN for portfolio optimization. Sharpe Ratio is the ratio of expected return to risk. We expected the portfolio which has a higher Sharpe Ratio. Nonlinear neural network for portfolio optimization has no cardinality constraint so that it selects 7 assets which are: 35% BIMAS, 7% DOHOL, 2% ECILC, 1% ENKAI, 8% OTKAR, 34% PETKM, 13% TKFEN. The proposed method has cardinality constraint K = 5, selected five assets 38% ASELS, 5% ENKAI, 12% OTKAR, 39% PETKM, 6% TUPRAS. As a result of these experiments proposed hybrid algorithm based on the nonlinear neural network to solve the cardinality constraint portfolio optimization problem can assume as a powerful model. To show the performance of the proposed method different datasets that Dow Jones, Nasdaq100, FTSE100, and FF49 are analyzed. The results compare with the other methods proposed in the literature. It seems that from numerical application the proposed method is applicable for the portfolio selection problem. For further studies, performance indicators that are not given in this article can be used for comparison. Besides, the outsample ratio can be chosen differently from 20% which is used in this article. Moreover, portfolio selection performance for the proposed method (NNNGA) will be improved if fuzzy logic will be included in future studies.

CRediT authorship contribution statement

Ilgım Yaman: Investigation, Data curation, Resources, Writing - original draft. **Türkan Erbay Dalkiliç:** Validation, Conceptualization, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.eswa.2020.114517.

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Further reading

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