

Tabu Search Algorithm for Network Synthesis

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Abstract—Large scale combinatorial problems such as the network expansion problem present an amazingly high number of alternative configurations with practically the same investment, but with substantially different structures (configurations obtained with different sets of circuit/transformer additions). The proposed parallel tabu search algorithm has shown to be effective in exploring this type of optimization landscape. The algorithm is a third generation tabu search procedure with several advanced features. This is the most comprehensive combinatorial optimization technique available for treating difficult problems such as the transmission expansion planning. The method includes features of a variety of other approaches such as heuristic search, simulated annealing and genetic algorithms. In all test cases studied there are new generation, load sites which can be connected to an existing main network: such connections may require more than one line, transformer addition, which makes the problem harder in the sense that more combinations have to be considered.

Index Terms—tabu search, network synthesis, combinatorial optimization.

I. INTRODUCTION

THE allocation of transmission costs in a competitive environment requires the determination of optimal transmission network expansion plans [1]. However, most of the conventional optimization algorithms used in practice are unable to generate optimal, or even near-optimal, solutions for larger, complex transmission network expansion problems. On one hand, the number of options to be analyzed increases exponentially with the size of the network (large scale combinatorial problem). On the other hand, the expansion problem presents a multimodal landscape in which the chances of being trapped in a local minimum solution increases with the size of the problem.

The purpose of this paper is to report research in the adaptation of recent developments in combinatorial optimization to transmission expansion planning. The paper describes a parallel Tabu Search algorithm for solving the static transmission network expansion problem (single stage expansion) Tabu search belongs to a family of methods which also includes Simulated Annealing and Genetic Algorithms [6], [7], [10]. Tabu search evolved in three distinct phases. In the first phase TS was presented basically as a search algorithm with short-term memory,

a tabu list (movements in the list were temporarily forbidden), and an aspiration criterion by means of which very attractive tabu movements are allowed. This basic TS algorithm was applied to some combinatorial power system problems such as the ones described in Refs. [8], [9]. In the second phase, TS included the mechanisms of diversification, intensification and long-term memory. Finally, in the third phase, path relinking, elite configurations, intelligent initial configuration selection, strategic oscillations, neighborhood reduction, and hybrid versions adding features of other combinatorial methods such as Genetic Algorithms and Simulated Annealing [7], [10] were included [6]. The transmission expansion planning method of this paper uses a third generation TS algorithm extended to handle families of configurations (parallel TS).

The paper is organized as follows: first, the initialization process is described; second, the configuration space is characterized; third the concept of reduced neighborhood is presented; next, the proposed parallel tabu search algorithm is detailed; finally, test results are presented and the relevant conclusions are summarized. The paper also includes an appendix dealing with the mathematical formulation of the problem.

II. SET OF INITIAL CONFIGURATIONS

Rather than using a single configuration, as in more conventional tabu search algorithms, a family of concurrent configurations is kept throughout the optimization process. This allows for a more comprehensive search on the space of configurations. Although the process could be actually mapped on a parallel machine, this is not necessarily so, and serial machines can be used as well.

Initially, a family of configurations is obtained with a modified Garver's algorithm: the idea is to obtain a first configuration using the algorithm as is, and then turning tabu active (i.e., forbidden) some essential attributes (e.g., line additions) of this configuration leading to other configuration belonging to different regions of the search space. Garver's algorithm uses a transportation model (Linear Programming), i.e. a model in which only Kirchhoff's current law is taken into account. When this is done the algorithm, although approximate, is able to pinpoint the main transmission trunks of the expanded network. These configurations normally contain the most interesting attributes which will eventually be found in optimal solutions which makes Garver's approach an extremely handy initializer for the optimal search process. Although the proposed approach, as well as SA and GA algorithms in general, can be randomly initialized, Garver's initialization has proved to be more effective both regarding computational speed and final solution quality.

Manuscript received November 3, 1998; revised June 16, 1999. This work was supported in part by FAPESP, Fundação de Amparo à Pesquisa do Estado de São Paulo, and by CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico.

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Publisher Item Identifier S 0885-8950(00)03777-9.

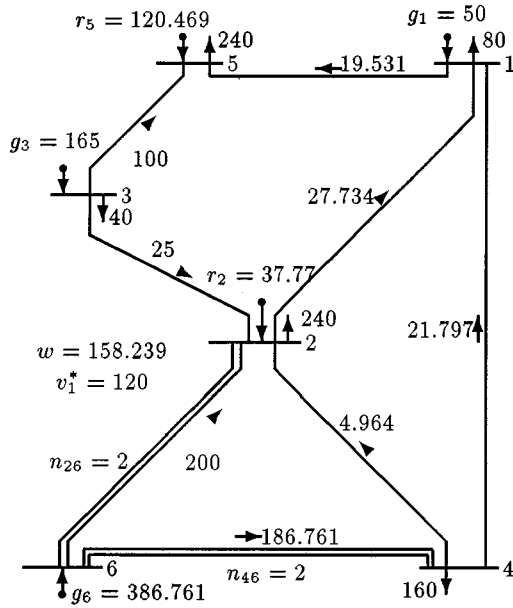


Fig. 1. Example transmission expansion plan for Garver's network with the addition of two circuits in branch 2-6 and two circuits in branch 4-6.

III. CONFIGURATION SPACE

Let n_{ij} be the number of circuits added in branch $i - j$ (right-of-way). A configuration is an investment alternative for which the integer values of n_{ij} are specified for all branches. A feasible configuration provides a solution of the network equations without loss of load (see LP problem formulation in the Appendix). Fig. 1 shows a typical example transmission expansion plan for Garver's 6-bus system. In this example, the basic configuration has a single circuit in each of the following branches: 1-2, 1-4, 1-5, 2-3, 2-4, and 3-5. The basic configuration, which corresponds to a network with no circuit additions, is then represented as:

$$n = \begin{array}{cccccccccccccccc} 1-2 & 1-3 & 1-4 & \dots & & & & & & & & & & & & 5-6 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

In the configuration of Fig. 1, four circuits have been added to the basic configuration, i.e., two circuits in branch 2-6 and two circuits in branch 4-6. These additions can be represented as:

$$n = \begin{array}{cccccccccccccccc} & & & & & & & & & & 2-6 & & & & & 4-6 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 \end{array}$$

with objective function $v = 911.19$ for $\alpha = 5$ (penalty factor described in the Appendix). This configuration is not feasible and leads to the loss of load indicated in Fig. 1. The optimal expansion plan for this system is given by:

$$n = \begin{array}{cccccccccccccccc} & & & & & & & & & & 2-6 & 3-5 & & & & 4-6 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 1 & 0 & 0 & 2 & 0 \end{array}$$

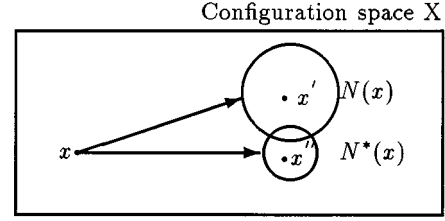


Fig. 2. Neighborhood $N(x)$ and reduced neighborhood $N^*(x)$ of a generic configuration x .

and an example of feasible, nonoptimal configuration is as follows:

$$n = \begin{array}{cccccccccccccccc} & & & & & & 2-3 & & 2-6 & & 3-5 & & & & 4-6 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 4 & 0 & 1 & 0 & 0 & 3 & 0 \end{array}$$

IV. REDUCED NEIGHBORHOOD

A critical issue in tabu search is the characterization of the neighborhood $N^*(x)$ of a given configuration x , as illustration is shown in Fig. 2. Working with full neighborhoods is practical only for relatively small problems, since for larger systems the exploration of a neighborhood makes use of large numbers of LP routine calls. Thus larger systems require a careful definition of neighborhoods. A neighborhood is considered then as a reduced list of circuit additions or swaps. This list is obtained using the best alternatives found with three different methods: (a) approximate methods [3]–[5] (Garver's method based on a transportation network [3], least effort criterion based on a DC power flow [4], and a minimum loss of load criterion based on a specialized LP algorithm [5]); (b) circuits or sets of circuits adjacent to buses with significant unserved load, generation (buses with large loss of load); and (c) a set of additions/swaps randomly chosen. The list of neighbors found by these methods is a compromise between greedy algorithms (which are good for local searches) and random search (which are less prone to be trapped into local optimal points). For situations of so far unconnected load/generation sites, the list may contain elements with multiple simultaneous additions (say, a connected set of circuits and transformers or *paths* as defined in [5]).

V. PARALLEL TABU SEARCH

The general search strategy is illustrated in Fig. 3. The search initiates with a family of k configurations ($i = 1, 2, \dots, k$) shown in level-a of Fig. 3. To each member of the family it is applied a Limited Tabu Search (ITS) algorithm. This yields a parallel thread for each member of the family. As the process evolves, communication is established between threads, as indicated in Fig. 3 (see discussion below).

A. Limited Tabu Search—LTS

The LTS algorithm comprises: a tabu list with an aspiration criterion, short-term memory, reduced neighborhood, strategic oscillation, intensification for local search, and limited diversification (the resulting family of configurations is shown in level-b of Fig. 3). A *tabu list* contains temporarily forbidden movements

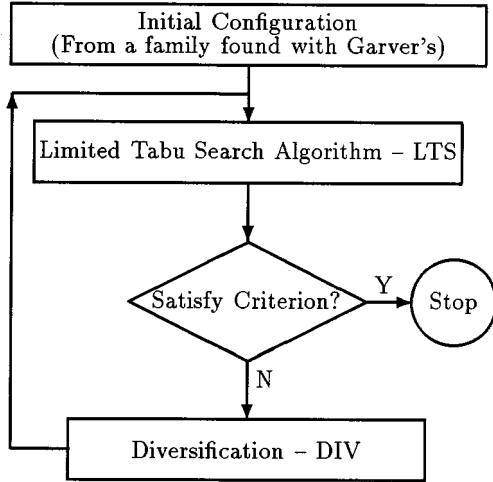


Fig. 5. Sequential tabu search algorithm used in each thread of the parallel algorithm of Fig. 3.

configuration. Path relinking has been adopted for producing part of the configurations of the next generation (the other configurations are generated by guided crossover, a feature taken from Genetic Algorithms). As a result of path relinking, we can get either a new superior solution in the neighborhood of the current solution (intensification) or a solution which will be far apart from the original one, thus allowing the exploration of unvisited regions of the search space (diversification). Although both effects are always present, in the network transmission expansion planning problem, it has been observed that the diversification effect is more pronounced than the intensification one, as illustrated in Fig. 6.

VI. TEST RESULTS

In this section a practical evaluation of the parallel tabu search algorithm (Fig. 3) is presented. Computing requirements are indirectly measured by the number of LP (Linear Programming) routine calls (this routine is used to find feasible network solutions; unfeasibilities are managed by loss of load).

A. Garver's 6-Bus System

AU the relevant data for this test case can be found in [3]. In this case the family of configurations was initialized randomly; initialization by Garver's would turn the search trivial. Table I shows a set of ten representative situations; in these cases the generation levels are considered constant (no rescheduling allowed) which makes the problem more difficult and convergence harder. ND is the number of diversifications performed. $LP's(stop)$ is the number of LP routine calls (network solutions) until convergence. $LP's(v_{op})$ is the number of LP routine calls until the optimal solution is found for the first time. The optimal configuration for this case is: investment cost $v = 200$; and additions $n_{26} = 4$ circuits, $n_{35} = 1$ circuit and $n_{46} = 2$ circuits.

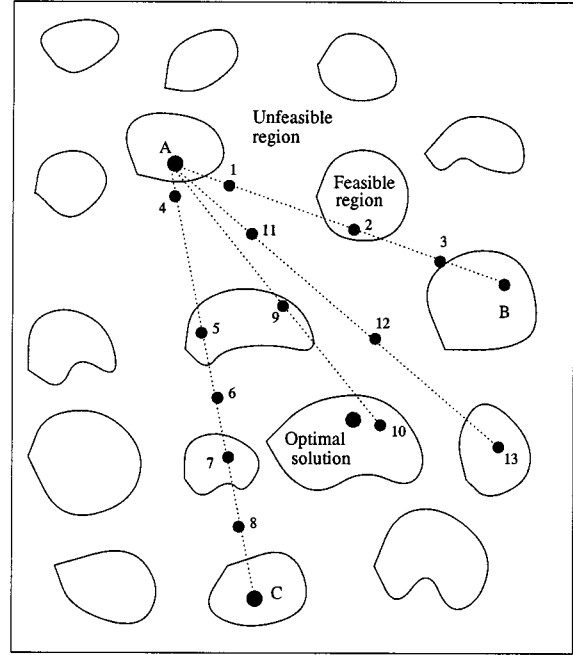


Fig. 6. Path relinking.

TABLE I
RESULTS FOR GARVER'S 6-BUS EXAMPLE SYSTEM

ND	LP's (stop)	LP's (v_{op})
11	562	80
9	707	62
9	649	171
8	608	67
8	748	72
9	698	77
9	638	140
8	737	72
8	600	99
8	645	134
Average	660	97

B. 46-Bus Southern System

AR the relevant data for this test case can be found in [4]. If redispatch is allowed (the less demanding situation) the best configuration found by the modified Garver's initializer is practically the optimal solution, with minor differences. Thus this situation will not be considered in the tests. If no redispatch is allowed (fixed generation/load levels) a larger number of additions is required which makes the problem more difficult for the algorithm, Table II shows a set of ten representative cases with two types of initialization: random and Garver's. The optimal configuration for this case is: investment cost of US\$154, 420, 000.00; and additions $n_{20-21} = 1$, $n_{42-43} = 2$, $n_{46-06} = 1$, $n_{19-25} = 1$, $n_{31-32} = 1$, $n_{28-30} = 1$, $n_{26-29} = 3$, $n_{24-25} = 2$, $n_{29-30} = 2$ and $n_{05-06} = 2$.

TABLE II
RESULTS FOR THE 46-BUS SYSTEM

Random		Garver	
LP's (stop)	LP's (v_{op})	LP's (stop)	LP's (v_{op})
6313	3131	3061	356
6422	1040	2477	69
5877	434	2344	345
6166	498	2439	424
5837	2148	3213	270
5641	152	3286	875
5878	922	2366	439
5548	153	3493	657
5492	330	2343	80
6044	1263	2662	653
5922	997	2768	417

C. North-Northeastern System Without Rescheduling

This test system represents the part of the Brazilian network corresponding to the North and Northeastern regions; the system has 87 buses and 183 candidate branches (The complete data set for this test system is available from the authors). Total demand is 29 754 MW and there is no known optimal solution for this system. This is a very hard, complex practical system which presents a great variety of local optimal solutions. All the tests reported here were made using Garver's initialization. On average, 155 000 LP routine calls are required until convergence. As an illustration two of the best solutions found are summarized below. Although both solutions present practically the same investment level, the actual configurations differ substantially from each other. The best configuration requires investments of US\$ 2 574 745 000.00 and proposes the following additions:

$$\begin{aligned}
&n_{01-02} = 1, n_{02-04} = 1, n_{04-05} = 4, n_{04-81} = 3, n_{05-58} = 4, \\
&n_{12-15} = 1, n_{13-15} = 4, n_{14-45} = 1, n_{15-16} = 4, n_{15-46} = 1, \\
&n_{16-44} = 6, n_{16-61} = 2, n_{18-50} = 11, n_{18-74} = 6, n_{19-22} = 1, \\
&n_{20-21} = 3, n_{20-38} = 2, n_{22-37} = 1, n_{22-58} = 2, n_{24-43} = 1, \\
&n_{25-55} = 3, n_{26-54} = 1, n_{27-53} = 1, n_{30-31} = 2, n_{30-63} = 2, \\
&n_{35-51} = 2, n_{36-39} = 1, n_{36-46} = 3, n_{40-45} = 2, n_{41-64} = 2, \\
&n_{43-55} = 2, n_{43-58} = 2, n_{48-49} = 1, n_{49-50} = 4, n_{52-59} = 1, \\
&n_{54-58} = 1, n_{54-63} = 1, n_{61-64} = 1, n_{61-85} = 3, n_{67-69} = 2, \\
&n_{67-71} = 3, n_{68-69} = 1, n_{69-87} = 1, n_{71-72} = 1, n_{72-73} = 1, \\
&n_{73-74} = 2, n_{73-75} = 1, n_{75-81} = 1.
\end{aligned}$$

The second best configuration, which presents negligible-loss of load (3 MW or 0.01 % of total demand) requires investments of US\$ 2 573 941 000.00 and proposes the following line additions:

$$\begin{aligned}
&n_{01-02} = 1, n_{02-87} = 1, n_{04-05} = 4, n_{04-81} = 2, n_{05-56} = 1, \\
&n_{05-58} = 3, n_{06-37} = 1, n_{12-15} = 1, n_{13-15} = 4, n_{14-59} = 1, \\
&n_{15-16} = 4, n_{15-46} = 1, n_{16-44} = 6, n_{16-61} = 1, n_{18-50} = 11, \\
&n_{18-74} = 6, n_{21-57} = 2, n_{22-37} = 1, n_{24-43} = 2, n_{25-55} = 4, \\
&n_{30-31} = 1, n_{30-63} = 2, n_{35-51} = 1, n_{36-46} = 1, n_{39-42} = 1, \\
&n_{39-86} = 3, n_{40-45} = 1, n_{40-46} = 2, n_{41-64} = 2, n_{42-44} = 2, \\
&n_{43-55} = 3, n_{43-58} = 3, n_{48-49} = 1, n_{49-50} = 4, n_{52-59} = 1, \\
&n_{53-86} = 1, n_{54-55} = 1, n_{54-63} = 1, n_{56-57} = 1, n_{61-64} = 1, \\
&n_{61-85} = 2, n_{67-69} = 2, n_{67-71} = 3, n_{68-83} = 1, n_{69-87} = 1, \\
&n_{71-72} = 1, n_{72-73} = 2, n_{73-83} = 1, n_{73-74} = 2, n_{81-83} = 1.
\end{aligned}$$

The main differences between the two configurations are: the second configuration utilizes 16 right-of-ways not utilized by the first one, and in 12 other right-of-ways the number of circuits added are different from the first configuration.

At this point it is worth illustrating the explosive nature of the problem. For instance, if 11 circuit additions are allowed in each right-of-way, then the total number of possible configurations is 12^{183} . Thus the TS algorithm only visits a small fraction of the entire search space. Although the selection of good initial configuration may help, its importance becomes less critical as the size of the problem increases.

VII. CONCLUSIONS

A parallel tabu search, TS, algorithm for transmission network expansion planning is proposed. This is a third generation TS algorithm which includes advanced features such as path re-linking, elite configurations, intelligent initialization, strategic oscillations, neighborhood reduction, and hybrid features taken from other combinatorial methods such as Genetic Algorithms and Simulated Annealing. The new method is more flexible and more comprehensive than other combinatorial approaches. For small and medium size cases optimal solutions were found in shorter times than the ones obtained with other combinatorial methods. Also, for these systems the proposed initialization procedure has shown excellent performance improvements. For a difficult large scale system a number of attractive, near-optimal solutions were found. In this case there are an amazingly high number of alternative configurations within a 1 % investment range, but which may present substantial differences in configuration (different sets of circuit/transformer additions). In all test cases there are parts of the network which are initially unconnected (new generation, load sites). In the case of the larger system the connection of these new sites to the main network may require the addition of multiple circuits/transformers, which makes the combinatorial problem even more difficult).

APPENDIX

This appendix describes the mathematical formulation of the synthesis problem. The static (one stage) transmission expansion

sion planning problem can be formulated a mixed integer non-linear programming problem:

$$\begin{aligned} \min \quad & v = \sum_{ij} c_{ij} n_{ij} + \sum_i \alpha_i r_i \\ & B(x + \gamma^o)\theta + g + r = d \\ & (x_{ij} + \gamma_{ij}^o)|\theta_i - \theta_j| \leq (x_{ij} + \gamma_{ij}^o)\bar{\phi}_{ij} \\ & 0 \leq g \leq \bar{g} \\ & 0 \leq r \leq d \\ & 0 \leq n_{ij} \leq \bar{n}_{ij} \end{aligned} \quad (1)$$

where:

- c_{ij} Cost of the addition of a circuit in branch i - j .
- x_{ij} Total susceptance in branch i - j .
- $B(\cdot)$ Susceptance matrix.
- θ Vector of nodal voltage angles.
- γ^o Vector of initial susceptances, whose elements are γ_{ij}^o , i.e. the summation of the susceptances in branch i - j at the beginning of the optimization.
- n_{ij} Number of circuits added in branch i - j : $n_{ij} = x_{ij}/\gamma_{ij}$; where γ_{ij} is the susceptance of the new circuits.
- $\bar{\phi}_{ij}$ Defined as the ratio: $\bar{\phi}_{ij} = \bar{f}_{ij}/\gamma_{ij}$; where \bar{f}_{ij} is the maximum flow in a circuit i - j .
- d Demand vector.
- g Generation vector.
- \bar{g} Vector of maximum generation capacity.
- r Vector of artificial generations.
- α Penalty parameter associated with loss of load caused by lack of transmission capacity.

For a given set of decisions variables x_{ij} , Problem (1) becomes a linear programming problem:

$$\begin{aligned} \min \quad & w = \sum_i \alpha_i r_i \\ & B(x^k + \gamma^o)\theta + g + r = d \\ & (x_{ij}^k + \gamma_{ij}^o)|\theta_i - \theta_j| \leq (x_{ij}^k + \gamma_{ij}^o)\bar{\phi}_{ij} \\ & 0 \leq g \leq \bar{g} \\ & 0 \leq r \leq d \end{aligned} \quad (2)$$

which is solved for testing the adequacy of a candidate solution; adequacy is indicated by zero loss of load. Notice that Problem (1) is always feasible due to the presence of the loss of load factor $\sum_i \alpha_i r_i$ in the objective function; thus whenever a tentative solution set x_{ij} is inadequate, feasibility is achieved by the use of artificial generators (loss of load). It has been observed that this procedure facilitates the search process to come out from local minima by temporarily moving through regions

of inadequate solutions, but still keeping the feasibility of the mathematical problem.

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