Homework 5_1 [order of postorder traversal]

postorder (n11)
postorder (n11 → left: n5)
postorder (n5 → left: n3)
postorder (n3 → left: n1)
postorder (n1 → left: null)
postorder (n1 → right: null)
print (n1)
postorder (n3 → right: n2)
postorder (n2 → left: null)
postorder (n2 → right: null)
print (n2)
print (n3)
postorder (n5 → right: n4)
postorder (n4 → left: null)
postorder (n4 → right: null)
print (n4)
print (n5)
postorder (n11 → right: n10)
postorder (n10 → left: n6)
post order (n6 → left: null)
post order (n6 → right: null)
print (n6)
postorder (n10 → right: n9)
postorder (n9 → left: n7)
postorder (n7 → left: null)
postorder (n7 → right: null)
print (n7)
postorder (n9 → right: n8)
postorder (n8 → left: null)
postorder (n8 → right: null)
print (n8)
print (n9)
print (n10)
print (n11)

Homework 5_2 [Tree_successor.cpp]

Variable analysis

TreeNode	
type	**name**
int	data
TreeNode*	left
TreeNode*	right
TreeNode*	parent

- data : a value that node has
- left : address of left child node
- right : address of right child node
- parent : address of parent node

Function analysis

1) Tree_successor

: in order to find successor in inorder traversal, the existence of right subtree is important. Because the sequence of inorder traversal is left ->vertex ->right, which means the next node of current(vertex) is one of the nodes in right subtree. This Algorithm is divided into two cases on the basis of the right subtree of the input node being empty or not

1. If right subtree of *node* is not *NULL*, then succ lies in right subtree. So, starting from top of right subtree, return leftmost node.
2. If right subtree of *node* is NULL, then *succ* is one of the ancestors.
   Travel up using the parent pointer until you see a node which is left child of its parent. The parent of such a node is the *succ*. The reason why we allow only left child relation is that the parent node has already been passed if it has right child relation

```
TreeNode* tree_successor(TreeNode* x)

{         //case 1: x's right subtree is not null
          if (x->right != NULL)
          {
                    x = x->right; //set right sub tree
                    while (x->left != NULL) { //leftmost of right subtree
                              x = x->left; //move to left child
                    }
                    return x;

          }
```

```
                  //case 2: x's right subtree is null
         TreeNode* y = x->parent;
                  while (y != NULL and x == y->right) { // continue searching until the relation between child and parent is left.
                           x = y;
                           y = y->parent;
                  }
         return y; //return parent
}
```

2) main

```
void main()
{
         TreeNode* exp = &n7;
         TreeNode *q = exp;
         n1.parent = &n3;
         n2.parent = &n3;
         n3.parent = &n7;
         n4.parent = &n6;
         n5.parent = &n6;
         n6.parent = &n7;
         n7.parent = NULL;
         while (q->left) q = q->left; // Go to the leftmost node
         do
         {
                  printf("%c\n", q->data); // Output data
                  // Call the successor
                  q = tree_successor(q);
         } while (q); // If not null
}
```

: To start from first node in inorder traversal, move to leftmost node. And ouput data and move to successor until q is null. By doing this, we can traverse all nodes in tree.

[result]

```
A
C
B
G
D
F
E
```

Homework 5_3 [Tree_predecessor.cpp]

Variable analysis

Same as Homework 5_2 [Tree_successor.cpp]

Function analysis

1) Tree_predecessor

Pseudo code

```
TreeNode* tree_predecessor(TreeNode* p) {

    if x->left != NULL //x's left subtree is not null
        return the rightmost node of left subtree

    //x's left subtree is null
    y= x->parent
        while (y != NULL and x == y->left) {
            x = y;
            y = y->parent;
        }
    return y;

}
```

: in order to find predecessor in inorder traversal, the existence of left subtree is important. Because the sequence of inorder traversal is left ->vertex ->right, which means the previous node of current(vertex) is one of the nodes in left subtree. Algorithm implemented is divided into two cases on the basis of the left subtree of the input node being empty or not

1. If left subtree of *node* is not *NULL*, then pred lies in left subtree. So, starting from top of left subtree, return rightmost node.
2. If left subtree of *node* is NULL, then pred is one of the ancestors.
   Travel up using the parent pointer until you see a node which is right child of its parent. The parent of such a node is the pred. The reason why we allow only right child relation is that the parent node is one of the next nodes if it has left child relation

```
TreeNode* tree_predecessor(TreeNode* x) {

         if (x->left != NULL) //x's left subtree is not null
                  {
                           x = x->left; //set left sub tree
                           while (x->right != NULL) { //rightmost of left subtree
                                    x = x->right; //move to right child
                           }
                           return x;
                  }
         //x's left subtree is null
         TreeNode* y = x->parent;
                  while (y != NULL && x == y->left) {// continue searching until the
relation between child and parent is right.
                           x = y;
                           y = y->parent;
```

```
                }
        return y;
}
```

2) main

```
void main() {
        TreeNode* q = exp;
        n1.parent = &n3;
        n2.parent = &n3;
        n3.parent = &n7;
        n4.parent = &n6;
        n5.parent = &n6;
        n6.parent = &n7;
        n7.parent = NULL;

        while (q->right) q = q->right; // Go to the rightmost node
                do {
                        printf("%c\n", q->data); // Output data
                                // Call the predecessor
                        q = tree_predecessor(q);
                } while (q); // If not null

}
```

: To start from last node in inorder traversal, move to rightmost node. And ouput data and move to predecessor until q is null. By doing this, we can traverse all node in tree reversely.

[result]

```
E
F
D
G
B
C
A
```

Homework 5_4 [bst_insertion_deletion.cpp]

Variable analysis

TreeNode	
type	**name**
int	data
TreeNode*	left

TreeNode*	right

- data : a value that node has

- left : address of left child node

- right : address of right child node

Function analysis

1) delete_node

When we delete a node, three possibilities arise.

Case 1) Node to be deleted is the leaf: Simply remove from the tree.

```
              50                            50
           /     \         delete(20)      /   \
          30      70       --------->    30     70
         /  \    /  \                     \    /  \
       20   40  60   80                   40  60   80
```

Case 2) Node to be deleted has only one child: Copy the child to the node and delete the child

```
              50                            50
           /     \         delete(30)      /   \
          30      70       --------->    40     70
            \    /  \                          /  \
            40  60   80                       60   80
```

Case 3) Node to be deleted has two children: Find inorder successor of the node. Copy contents of the inorder successor to the node and delete the inorder successor. Note that inorder predecessor can also be used. *We can use predecessor in this time.*

```
        50                        60
     /      \        delete(50)     /    \
   40        70      --------->   40      70
           /   \                            \
         60    80                            80
```

However, Unlike above two examples, we has no parent information in struct TreeNode this time. But, we need information about parent of given key.

```
void delete_node(TreeNode **root, int key)
```

: So to find parent of key, we set variable root as parameter. The reason why we use double pointer(**) is the root can be changed while implementing this function.

```
TreeNode *p, *child, *pred, *pred_p, *t;
        // search node t with key, p: t's parent
        p = NULL;
        t = *root;
        while (t != NULL && t->key != key) {
                p = t;
                t = (key < t->key) ? t->left : t->right;
        }
```

: after this routine, t points to node with key and p points to t's parent.

```
// Case 1: deletion of leaf node
        if ((t->left == NULL) && (t->right == NULL)) { // no both subtree = leaf node
                // If the parent node is not NULL,
                if (p != NULL) {
                        if (p->left == t)
                                p->left = NULL;
                        else p->right = NULL;
                }
                // If the parent node is NULL, the node to be deleted is the root
                else
                        *root = NULL;
```

: There are two possibilities in case 1.

1. the parent node is not NULL : depending on relation between t and t's parent (left child or right child), disconnect that connection.

2. the parent node is NULL : the node to be deleted is the root, which is only one left node in tree. So delete root and make root pointer point to nothing.

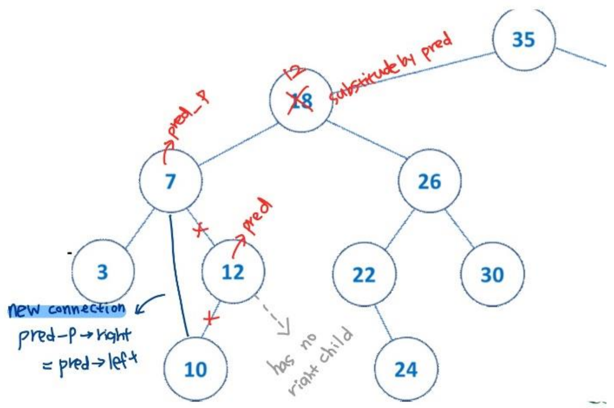
```
// Case 2 : deletion of node with single sub tree
        else if ((t->left == NULL) || (t->right == NULL)) {
                child = (t->left != NULL) ? t->left : t->right;
                if (p != NULL) {
```

```
				if (p->left == t)
					p->left = child;
				else p->right = child;
			}
			// If the parent node is NULL, the node to be deleted is the root
			else
				*root = child;
		}
```
: firstly, find child depending on which place it has child at. Then there are two possibilities.

1. the parent node is not NULL : connect parent with child depending on what relation parent and t(deleted node) has.

2. the parent node is NULL : the node to be deleted is the root. So just change root to t's child.

```
// Case 3 : deletion of node with both sub tree
		else {
			// Find the predecessor at left subtree
			pred_p = t;
			pred = t->left;
			// Keep moving to the right and find the predecessor
			while (pred->right != NULL) {
				pred_p = pred;
				pred = pred->right;
			}
			// new connection between pred_p and pred 'child
			if (pred_p->left == pred)
				pred_p->left = pred->left;
			else
				pred_p->right = pred->left;
			t->key = pred->key; //copy contents of pred
			t = pred; //set node that have to be free().
		}
```
: If the deleted node has both subtree, the problem becomes more complex. We have to find substitution of the deleted in this tree that doesn't break the rules. But we can simply solve this problem by finding predecessor of the deleted. Because Predecessor is the largest value among nodes smaller than the deleted in subtree. A predecessor in inorder traversal can be found as the rightmost node of left subtree. (same logic as Homework 5_3 [Tree_predecessor.cpp]).



After find predecessor, we just copy contents of the inorder predecessor to the deleted position and make new connection between parent of pred and pred's child. The reason why using not `child` but `pred ->right` is pred has no right subtree since it is rightmost node.

[result]

1) key =18

```
Binary tree
3       7       10      12      18      22      24      26      30      35      68      99

Binary tree
3       7       10      12      22      24      26      30      35      68      99
```

2) key =35

```
Binary tree
3       7       10      12      18      22      24      26      30      35      68      99

Binary tree
3       7       10      12      18      22      24      26      30      68      99
```

3) key =7

```
Binary tree
3       7       10      12      18      22      24      26      30      35      68      99

Binary tree
3       10      12      18      22      24      26      30      35      68      99
```