<u>Matlab Assignment #1</u>

Math 547, Fall 2013

General Instructions: For this assignment, you will answer problems on the material in sections 5.1-5.4 of the textbook using Matlab. Complete all work in Matlab on the m-file for this assignment which can be downloaded from the course website by right-clicking on the link and selected "save link as". This m-file contains data for parts I and III. Use the "%" symbol to provide comments on your m-file when necessary to indicate what you are doing in your m-file. In addition to the m-file, complete a written assignment with answers to each of the questions. Explanations should be kept brief. Embed any plots into your written assignment as well. All plots should have x and y axes labeled (with units when applicable). Both your m-file and written assignment should be printed out and turned in.

<u>Part I: Fundamentals of Vector Projections onto Subspaces of $\mathbb{R}^n$</u>

Consider the vectors:

$$\vec{v_1} = \begin{bmatrix} 1 \\ 5 \\ 7 \\ 9 \end{bmatrix} \quad \vec{v_2} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 0 \end{bmatrix} \quad \vec{v_3} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 2 \end{bmatrix}$$

1. Let A be the matrix with $\vec{v_1}$ as its first column, $\vec{v_2}$ as its second column, and $\vec{v_3}$ as its third column. Use Matlab to calculate some property of A in order to confirm that the vectors $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ are linearly independent. In your written assignment, provide the result you calculated and explain why this means the vectors are linearly independent.

Matlab tip: you can make a matrix out of existing column vectors in the following way: "A= [v1,v2,v3]".

2. Let V be the subspace given by image(A). Since $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ are linearly independent, they form a basis of V. Using Matlab, verify that $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ are *not* an orthonormal basis of V by calculating the length of each vector and the dot product of each pair of vectors. Provide your answers in the written assignment and make sure to explain why each calculation shows that the vectors are not orthonormal.

Matlab tip: you can calculate the length of any vector in Matlab using the function: norm(x). To take the dot product of two vectors *x* and *y*, you can either multiply the two vectors (but make sure the first vector is a row vector and the second vector is a column vector), or use the function: dot(x,y).

3. Now use the Gram-Schmidt process to construct three new vectors ($\overrightarrow{u_1}$, $\overrightarrow{u_2}$, and $\overrightarrow{u_3}$) that form an orthonormal basis of *V*. Do this in Matlab in two ways: First, use the algorithm given in Theorem 5.2.1 in the textbook to calculate the new vectors in Matlab. Second, use the built-in Matlab command [Q,R]=qr(A,0). This command will calculate the QR factorization of your matrix A, and return the matrix Q whose columns are the vectors $\overrightarrow{u_1}$, $\overrightarrow{u_2}$, and $\overrightarrow{u_3}$ (the "0" is needed in the Matlab function otherwise Matlab will return Q with an extra column). In your written assignment, provide the results from both methods and verify they are the same.

4. Now let $T(\vec{x}) = P\vec{x}$ be the linear transformation that represents the orthogonal projection of a vector $\vec{x}$ in $\mathbb{R}^4$ onto the subspace *V* (defined above as the image of matrix *A*). Using your results from #3, use Matlab to calculate the matrix *P*. In your written assignment, give the result you calculated and briefly explain why *P* can be calculated in this way.

5. Now consider the two vectors:

$$\overrightarrow{b_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{ and } \overrightarrow{b_2} = \begin{bmatrix} 0 \\ 3 \\ -2 \\ 26 \end{bmatrix}$$

In Matlab, use the matrix *P* you found in #4 to calculate $T(\overrightarrow{b_1})$ and $T(\overrightarrow{b_2})$. In your written assignment, provide your answers. Describe what you notice about $T(\overrightarrow{b_2})$. Given that $\overrightarrow{b_2} = 2\overrightarrow{v_1} - 3\overrightarrow{v_2} + 4\overrightarrow{v_3}$, explain why your answer for $T(\overrightarrow{b_2})$ makes sense.

Part II: Fitting a Parabola to the Height of a Projectile

The height (*y*) of a projectile vs. time (*t*) can be described by the following equation (assuming no air resistance):

$$y = -\frac{1}{2}at^2 + bt + c \qquad \text{(eq. 1)}$$

where *a* is the object's acceleration in the vertical direction, *b* is the object's initial velocity, and *c* is the object's initial height. For this problem, we will let *a*, *b*, and *c* be free parameters that we will solve for.

6. I conducted an experiment where I launched a small ball in the air. Images taken during that experiment at different times can be found in a separate file on the website. Use this image data to make a vector of the times the images were taken and a vector of the ball height that corresponds with those times. Next to the vectors in your m-file, make sure to comment the units of each vector.

7. Consider the linear system of equations that would allow you to solve for a parabola (as described by eq. 1) that would fit all the data perfectly. Put this system of equations in matrix form and input the augmented matrix of that system in Matlab. Use Matlab to show that this system has no solutions. Make sure to describe in your written assignment what you calculated in Matlab and why it shows the system has no solutions. What does this mean about the data?

Matlab tip: For the system $A\vec{x} = \vec{b}$, if you first input the coefficient matrix A and the vector $\vec{b}$ in Matlab, you can form the augmented matrix using the command: "Aaug = [A,b]"

8. Use Matlab to find the least-squares solution for the system you set up above in vector form. Briefly describe what this least-squares solution represents geometrically. Give the equation for the parabola that best fits the data given your least-squares solution. Given the equation for your parabola, calculate the maximum height the ball is predicted to have reached.

9. Compare the values of the parameters a and c in your least-squares solution to what these values *should* be given your data and what these parameters represent. Use Matlab to plot the best-fit parabola you found and plot the data (as points) on top in some other color. Embed your plot in your written assignment.

Matlab tip: To plot a function in Matlab, make a vector t that spans the range of time you want to plot over. You can use the Matlab command: "t=linspace(x1,x2,n)" to make a vector t that has n equally spaced points such that the first entry is x1 and the last entry is x2. Making n a large number will allow your function to appear more smooth when plotted. Then calculate the corresponding vector y you want to plot using the equation of your function. You should plot functions as lines and data as points. See the "Matlab Help" section of the course website for more help on plotting.

Part III: Fitting an Exponential Curve to Global Human Population Growth

On your Matlab m-file, I have provided you with historical global human population estimates vs. time (Data from Population Reference Bureau; http://en.wikipedia.org/wiki/World_population_estimates). The simplest model of population growth assumes an exponential dependence of population (y) on time (t) according to the following equation:

$$y = ae^{rt} \qquad \text{(eq. 2)}$$

where a is the population at time 0 and r is a growth rate parameter. For this problem, we will let a, and r be free parameters that we will solve for.

10. In its present form, we cannot fit a function like eq. 2 to our data using the linear least-squares method since one of the parameters is in the exponent. Transform eq. 2 by taking the natural logarithm of both sides to get a new equation of the form:

$$y^* = c_1 + c_2 t \qquad \text{(eq. 3)}$$

What does the new dependent variable y^* equal? Based on this, using the original data given to you, make a new vector y^* in your m-file. How do the new parameters c_1 and c_2 in your transformed equation relate to the original parameters in eq. 2?

11. Using the transformed equation, use Matlab to calculate the least-squares solution that best fits the data provided. For your best fit equation, what are the values of the parameters a and r as given in eq. 2?

12. Plot the best fit exponential curve of population vs. year and plot the data points on top in a different color. Embed your plot in your written assignment.

13. Use Matlab to calculate a vector that gives the squared error for each data point, where the squared error is defined as the square of the difference between the actual value and the value predicted by your model. Plot the values of squared error you calculated vs. year and embed your plot in your written assignment. For which data points is error lowest and for which data points is error highest?