Matlab Assignment #2

Math 547, Fall 2013

General Instructions: For this assignment, you will answer the problems below concerning the concepts presented in sections 7.1-7.6 of the textbook. You will use Matlab, completing all your work in Matlab on two m-files (one for Part I and one for Part II). The two m-files for this assignment can be downloaded from the course website by right-clicking on the link and selected "save link as". Use the "%" symbol to provide comments on your m-file when necessary to indicate what you are doing in your m-file. In addition to the m-files, complete a written assignment with answers to each of the questions. Explanations should be kept brief. Embed any plots into your written assignment as well. All plots (besides images) should have x and y axes labeled (with units when applicable). Both your m-files and written assignment should be printed out and turned in.

Part I: Dyamical Systems – Fish Population in a Lake System

Consider the dynamical system below.
Five connected lakes (Lake Artichoke, Lake Banana, Lake Cabbage, Lake Dragonfruit, and Lake Eggplant) are home to a species of fish. After a big storm, all the fish in Lake Banana, Lake Cabbage and Lake Eggplant are wiped out and only 200 fish remain in Lake Artichoke and 100 fish remain in Lake Dragonfruit. The population of fish in each lake, *t* weeks after the storm, can be described by the vector:

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{bmatrix} \begin{array}{l} \text{population of fish in Lake Artichoke} \\ \text{population of fish in Lake Banana} \\ \text{population of fish in Lake Cabbage} \\ \text{population of fish in Lake Dragonfruit} \\ \text{population of fish in Lake Eggplant} \end{array}$$

Suppose that:

$$\vec{x}(t+1) = \begin{bmatrix} 0.7 & 0 & 0 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0 & 0.2 & 0.1 \\ 0 & 0.2 & 0.8 & 0 & 0 \\ 0.1 & 0.2 & 0.2 & 0.6 & 0.1 \\ 0 & 0.1 & 0.1 & 0 & 0.9 \end{bmatrix} \vec{x}(t)$$

1. Explain the significance of the entries of the transformation matrix. Sum the columns of the transformation matrix and explain what this means for the population of fish in each lake. What is $\vec{x}(0)$?

2. Using Matlab, directly calculate $\vec{x}(t)$ for $t = 0$ to 20 weeks using the recursive formula with the transformation matrix given above. Then plot on one figure the fish population in each of the lakes (using a different color or symbol for each lake) vs. time. Also include on the same plot the total population of the fish in all the lakes combined.

Matlab tip: In Matlab, it is easiest to represent $\vec{x}(t)$ as a matrix X where the first column is $\vec{x}(0)$, the second column is $\vec{x}(1)$, the third column is $\vec{x}(2)$, etc. You can access the ijth entry of a matrix X using the notation "X(i,j)". You can also access an entire row or column of a matrix X using the notation "X(i,:)" (for the ith row) or the notation "X(:,j)" (for the jth column). You can also write what are called *for loops* in Matlab to do some task a set number of times. For example the code below will repeat 10 times (for i from 1 to 10) and for each i, it will make the entry in the ith row and 1st column of X equal to i.

```
for i=1:10
    X(i,1)=i;
end
```

3. Calculate the eigenvalues and eigenvectors of the transformation matrix of the dynamical system using Matlab. On your written assignment, give the eigenvectors and eigenvalues you calculated. Derive a closed formula for $\vec{x}(t)$ using these eigenvectors and eigenvalues you found and show your work on your written assignment. Use this closed formula to calculate $\vec{x}(20)$ in Matlab. In your written assignment, compare your calculation from your closed formula with your calculation of $\vec{x}(20)$ that you calculated in problem 2.

Matlab tip: Use the notation "[V,D]=eig(A)" to calculate the eigenvectors and eigenvalues of a matrix A in Matlab. V will return you a matrix where each column is an eigenvector. D will return you a diagonal matrix where the entry in the ith row and ith column is the ith eigenvalue (corresponding to the eigenvector in the ith column of V).

4. Looking at the eigenvectors of A and your closed formula for $\vec{x}(t)$, describe the behavior of the dynamical system as $t \to \infty$ in your written

assignment. Based on the eigenvectors and eigenvalues, which lake will have the least fish as $t \to \infty$? If after some long time the lake with the least number of fish has p fish, how many fish can you predict the all of the other lakes will have? Plot in Matlab the trajectory of the system on the $x_3 - x_4$ phase plane (that is, with the population of fish in Lake Cabbage on the x-axis and the population of fish in Lake Dragonfruit on the y-axis). On the same figure, also plot the dominant eigenvector (See Figure 7 in Section 7.1 for an example of a trajectory on a phase-plane).

5. This problem represents a discrete dynamical system. Given what the values in the transition matrix represent, explain why this situation may be better represented by a continuous dynamical system.

Part II: Eigenfaces and Face Recognition

In this part, you will use principal component analysis on a set of images of faces from students in the class. You will test how well this method can be used to recognize faces of known individuals. To run the m-file provided for this section, first download and unzip the Class Face Database folder (on Sakai) that contains Matlab image files of students in the class. To run the m-file, you will need to have this Class Face Database folder chosen as the Current Folder in the Matlab Command Window. This folder contains 70 images that make up the Class Face Database (2 for each student in the class) which are labeled "face_001" to "face_070". The folder also contains 43 test images (labeled "test_face_001" to "test_face_043") including 35 images of known faces (one for each student in the class), 5 images of unknown faces (courtesy of AT&T Laboratories Cambridge, Database of Faces), and 3 images of non-faces (my dog, a statue, and a plant). All of the images in the Class Face Database and all of the test images can be viewed by downloading pdf files on Sakai that display the images for your reference.

6. In the m-file provided, matrix $A = [\Phi_1, \Phi_2, \Phi_3, \ldots \Phi_m]$ where Φ_i represents the difference between the i^{th} image in the face database (X_i) and the mean of all the images in the face database (MF): $\Phi_i = X_i - MF$ where Φ_i is then reshaped to be a vector. What is the size of matrix A and what do each of these dimensions represent in terms of our Face Database? We learned in class that principal component analysis is used to reduce the number of variables in a set of data. In our Face Database, what are the

“variables” and how many do we have? You may find it helpful to use the paper *Face Recognition Using Eigenface Approach* (Slavković and Jevtić, 2012) which can be downloaded on Sakai.
Use Matlab to make a figure displaying the mean face (MF) of the class and embed in your written assignment.

Matlab tip: To display an image of a matrix A, use the following commands:
figure
imagesc(A)
colormap(‘gray’)
axis(‘image’)

7. How is the covariance matrix of A calculated? For our problem, how large is the covariance matrix? Explain how we can use the eigenvalues and eigenvectors of $A^T A$ to find the eigenvalues and eigenvectors of the covariance matrix. Again, you might want to use Slavković and Jevtić, (2012) as a reference.

8. In the m-file, “eigenval” gives the eigenvalues of $A^T A$ in order from largest to smallest. Use Matlab to calculate the fraction of the total variance that each eigenvalue represents by dividing the eigenvalues by the sum of all the eigenvalues. Plot the variance fraction of each eigenvalue (y-axis) vs. eigenvalue number (on the x-axis) and embed the plot in your written assignment.

Matlab tip: sum(x) will sum all the entries in a vector x.

9. Make a figure displaying the first eigenface (that is the one corresponding to the largest eigenvalue). In the m-file, the jth eigenface can be accessed by typing “E(:,:,j)”. Embed the figure in your written assignment and explain what this eigenface represents.

10. Now run the eigenface program using the first 10 eigenfaces (that is the eigenfaces corresponding to the 10 largest eigenvalues). How much total variance do these 10 eigenfaces account for?
Then using this reduced-dimension face space, you will reconstruct an image of yourself from the Class Face Database. Set the image you want to reconstruct on line 55 of the m-file by setting j equal to the image number from the Class Face Database that corresponds to a picture of you (or someone else if you are not included in the Class Face Database).

The program reconstructs a face from the database by projecting its difference from the mean face onto the space spanned by the first K eigenfaces (where K=10 in this case).
Given that the eigenfaces form an orthonormal basis for this space, use what you know about orthonormal projections to write a formula for the reconstructed face in your written assignment. For notation, you can use X to represent the oringal face, *MF* to represent the mean face, and *E1*, *E2*, … *EK* to represent the first through K eigenfaces.

Make a figure displaying your reconstructed face using the first 10 eigenfaces and embed the figure in your written assignment. (The reconstructed face is given as "reconst_face" in the m-file). How come it doesn't look like the original image?

Matlab tip: Line 38 of the mfile: "ind=[1:10]" tells the program which eigenfaces to use. It is initially set to use the first 10 eigenfaces. To use a different number of eigenfaces, change the 10 to the maximum eigenvalue number you want to use in the face reconstruction. For example, ind=[1:25]; will tell the program to use the first 25 eigenfaces.

11. Now run the eigenface program using the first 35 eigenfaces (that is the eigenfaces corresponding to the 35 largest eigenvalues – see *Matlab tip* from previous problem). How much total variance do these 35 eigenfaces account for?
Reconstruct the same image that you reconstructed in problem 10 but now using these 35 eigenfaces. Make a figure displaying your reconstructed face and embed your figure in your written assignment. Does it look more or less like the original image than the reconstructed image you made with 10 eigenfaces? Why?

12. Now we will perform face recognition with our program using a set of images that do not appear in the Class Face Database. For this problem and all following problems concerning face recognition, use the first 35 eigenfaces as you did in problem 11.
First perform face recognition on the test_face image of you. On line 65 of the m-file, input the name of the test_face file that corresponds to you (or someone else if there is no picture of you). Plot the Euclidean distance (given as the vector "dist" in the m-file) between the test face and each of the images in the Class Face Database (see Slavković and Jevtić, 2012). Embed your plot in your written assignment. For which image in the Class

Face Database is this Euclidean distance minimized? Did the program recognize you as you?

13. Perform face recognition for a few more test_faces from people in the class (test_face_001 to test_face_035) such that you find at least one face where the student was recognized correctly and one face where the student was not recognized correctly. On your written assignment, state the test_face attempted, whether it was successfully recognized or not, and what the minimum Euclidean distance was.
Perform face recognition for one of the test_faces of unknown people (test_face_036 to test_face_040) and give the minimum Euclidean distance on your written assignment.
Given these trials, on your written assignment suggest a threshold Euclidean distance such that if the calculated minimum distance for a face is below the threshold the program defines that face to be recognized as a known person and if the calculated minimum distance for a face is above the threshold the program defines that face as an unknown person. What happens if you set the threshold too low? What happens if you set the threshold too high?

14. Lastly, perform face recognition on one of the test_faces that is a non-face (test_face_041 to test_face_043) and give the minimum Euclidean distance in your written assignment. Suggest a different threshold such that above this threshold the program defines the image as not a face.
Make a figure displaying the reconstructed face of this non-face and embed in your written assignment. Why does this reconstructed non-face still look like a human face?