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Dependence Modeling

Benefits of Diversification and Assessing Dependencies in Risk Pools

Overview: Analysts and managers faces three discrete stages when considering dependence among insurance risks: (i) detecting dependencies, (ii) representing dependencies using mathematical models, and (iii) establishing dependence as an important feature in risk management. This essay focuses on the third stage and seeks to provide a framework for thinking about the importance of dependence in insurance risk pools. To do this, we also give some insights into selected approaches for introducing tools to represent dependencies in insurance risk pools.

Framework for Managing Uncertainty in Risk Pools

To understand the importance of dependence, we first introduce a framework for managing uncertainty in risk pools.

1.1 What are Risk Pools?

Let us use the term "risk pool" to simply mean a collection of insurance contracts. These contracts may be underwritten by (i) a government agency, (ii) a private company that is managed by policyholders for their mutual benefit, or (iii) a private company that is owned by individuals and other corporations. On the one hand, insurance is just one example of a risk pool, gambling systems and investment funds being other examples, and so insights that we draw from studies of insurance may lead to insights into other areas. On the other hand, insurance is an important sector of the global economy and is worthy of being studied in its own right.

In contrast to other financial institutions, insurers exist to spread risks. Specifically, insurers:

- Promote efficient use and economies of scale concerning their expertise,
- Spread risk, and
- Provide the ability to charge premiums in advance and pay compensation afterwards.

1.2 Risk Sources

What are the sources of uncertainty that an insurer faces? One common framework is based on organizing risk sources by (i) underwriting, (ii) financial, and (iii) non-financial attributes, c.f., Doff (2011). Financial uncertainty is important and can be further decomposed into market, credit, and liquidity components:

- "Market risk" is uncertainty due to changes in financial market variables, such as interest rates, stock prices, exchange rates, real estate prices, and so forth.
- "Credit risk" is uncertainty due to changes in the ability of counter-parties to fulfill their obligations.

- "Liquidity risk" is uncertainty due to unexpectedly high depletion of assets when complying with obligations involving a loss.

Non-financial risks are sources of uncertainty faced by all organizations, financial as well as non-financial. They are typically decomposed into "operational risks" (due to shortcomings in internal processes, people, systems or external events) and \business risks" (due to changes in the competitive environment or internal flexibility).

Financial and non-financial risks are important and worthy of study. In our work, we seek to provide a foundation so that a Wisconsin governmental agency, described in Section 3, can make informed risk decisions. In line with this goal, our focus is on the underwriting source of uncertainty.

1.3 Risk Controls

Let us use the term "risk control" for an action or a process that an insurer may use to manage the distributions of its risks.

Risk control begins with "underwriting," the process whereby an insurer classifies a prospective policyholder into one of several established groups known as a "rating class," thereby establishing a price for agreed upon insurance coverage. The uncertainty associated with this process is that premiums will be inadequate to coverage insurance claims, both initially and over the life of the agreement. Further, there is a risk that several policies may submit claims simultaneously due to a large-scale calamity.

"Reinsurance," which is insurance for insurers, is another important risk management tool for insurance companies. As described in Brockett et al. (1991), reinsurance can be used to provide additional capacity for insurers to write additional business, provide needed stability in managing cash flows, mitigate the impact of catastrophic events, and reduce barriers to market entry and exit through risk transfers and sharing of specialized market knowledge.

Managing the distribution of an individual risk can be done at the initial acceptance of a risk as part of the underwriting stage by modifying contracts by changing the exposure through deductibles, coverage limits, and coinsurance. Ratemaking is also an important part of the underwriting process whereby the insurer determines appropriate pricing mechanisms at initial acquisition and renewal stages. The distribution of each risk can also be managed by reinsurance agreements. For example, "quota share reinsurance" is a form of proportional reinsurance which specifies that a fixed percentage of each policy written will be transferred to the reinsurer, although allowances are made due to sales and other administrative expenses of the primary, or "ceding," insurer. Table 1 summarizes risk controls that influence the distribution of each risk.

Table 1: Risk Controls for Managing Individual Risks

Change the exposure through deductibles, coverage limits, and coinsurance
Establish pricing mechanisms at initial acquisition and renewal stages
Mitigate risks through reinsurance arrangements

Naturally, these risk controls are interrelated. For example, a policy coverage limit and a reinsurance treaty may both influence the amount of risk retained by the insurer; the coverage limit dictates the amount of risk retained by the policyholder and a reinsurance agreement specifies the risk amount ceded to a reinsurer. They serve as complimentary vehicles for mitigating an insurer's risk exposure.

2 Benefits of Diversification

Insurance systems are predicated on the pooling of contracts. For a risk pool, or portfolio, a key element of risk control is the diversification of risks. Risk controls for managing the distribution of individual policies are important but it is the distribution of the portfolio that is critical to an insurer's solvency and profitability.

2.1 Portfolio Risk Controls

Table 2 summarizes several tools that the insurer has at its disposal for managing a risk portfolio. Although these control mechanisms affect the distribution of each individual risk, their more important impact is on the portfolio distribution.

Table 2: Risk Controls for Managing Portfolio Risks

Risk Control	*Example*
Elect to not insure (or limit the coverage selected types of perils (a "peril" is a cause of loss)	Earthquakes in homeowners insurance
Elect to not insure (or limit the coverage selected types of coverages	Not offer comprehensive automobile coverage and write only third party liability in motor insurance
Elect to not underwrite selected groups of policies, either at initial application or at renewal	High risk automobile drivers or policy holders from a specific geographical area
Mitigate risks through reinsurance arrangements	"Excess of loss" is a type of non-proportional agreement at the portfolio level where the primary insurer is responsible for all losses for a portfolio up to a specified amount (the retention limit) and the reinsurer is responsible for the excess

As with the risk controls that are more geared to managing individual risk distributions, portfolio risk controls are interrelated. For example, by limiting exposure in selected geographical areas, an insurer can

effectively eliminate exposure to earthquake risk. As another example, an insurer can manage the amount of exposure to specified coverages through the pricing mechanisms, either by making prices low to attract business or high to reduce risk exposure.

2.2 Diversification as a Key Element of Risk Control

Insurers pool risks in order to enjoy the benefits of diversification. To illustrate, consider an extreme scenario where two risks are perfectly negatively related in the sense that one risk decreases as the other increases. When viewing the sum of risks, their random tendencies perfectly offset one another so that the risks provide the insurer with a natural hedge. For another classic example, consider n independent risks with the same distribution and use the standard deviation to measure the uncertainty of the portfolio sum of risks. Easy calculations show that the portfolio standard deviation is proportional to $\sqrt{n}$. This can be interpreted to mean that the uncertainty becomes smaller relative to the portfolio mean, that is proportional to n, as the number of risks n increases.

The Section 2.2 portfolio risk controls represent actionable strategies that insurers can use to establish the composition of a portfolio. Selection of risks can be based on coverages, perils, or policyholders, or indirectly through ratemaking. Reinsurance treaties influence the insurer's retained risk, thus helping to determine the portfolio composition.

This then raises the question as to how can an insurer measure the benefits of diversification achieved through a risk control strategy. Intuitively, it seems plausible that the individual and portfolio risk controls tools should be helpful in managing a portfolio. But, how does one quantify the effectiveness of these tools? Moreover, can one provide some quantitative guidelines for choosing an appropriate risk control tool when confronted with a given portfolio of risks?

As is common in financial risk modeling, our assertion is that measures of the portfolio's distribution of potential outcomes can provide a basis for making decisions about a firm's potential insolvency, capital adequacy and profitability. To measure this uncertainty, it is also our assertion that analysts now understand the distribution of each risk in most situations.

Rather it is the relationship, or dependence, among risks where additional research is needed; quantifying dependencies is critical for understanding the uncertainty of the portfolio. Sections 4 and 5 of this essay introduce our approaches to modeling dependencies.

For simple examples, we have already described the situation of two perfectly negative risks and n independent, identical risks. As another example, suppose that one has n identical risks that perfectly depend on one another. In this case, both the mean and the standard deviation are proportional to n so that no diversification benefits accrue through pooling.

Naturally, reality can be far from these token examples. Risks are not identical nor are they perfectly dependent nor independent. Moreover, more comprehensive risk measures than standard deviation are needed for practical analyses. Nonetheless, through these simple examples, one can interpret our research strategy:

- Develop a dependence structure that adequately captures relationships among risks.
- Identify risk measures that summarize the portfolio uncertainty.
- With a summary measure of the uncertainty that depends on risk control mechanisms, risk managers will be able to observe changes in the risk summary under different risk control settings, and thereby make informed decisions.

3 Local Government Property Insurance Fund

Although this work concerns developing general risk pool relationships, we are honing our thought processes in the context of a specific risk pool, the Local Government Property Insurance Fund (LGPIF), that is administered by the Wisconsin Office of the Insurance Commissioner (OCI). The LGPIF was established to provide property insurance for local government entities that include counties, cities, towns, villages, school districts, and library boards. The fund insures local government property such as government buildings, schools, libraries, and motor vehicles. The fund covers all property losses except those resulting from flood,

earthquake, wear and tear, extremes in temperature, mold, war, nuclear reactions, and embezzlement or theft by an employee.

The fund covers over a thousand local government entities who pay approximately $25 million in premiums each year and receive insurance coverage of about $75 billion. State government buildings are not covered; the LGPIF is for local government entities that have separate budgetary responsibilities and who need insurance to moderate the budget effects of uncertain insurable events. Claims for state government buildings are charged to another state fund that essentially self-insures its properties.

The fund offers three major groups of insurance coverage: building and contents, inland marine (construction equipment), and motor vehicles.

In effect, the LGPIF acts as a stand-alone insurance company, charging premiums to each local government entity (policyholder) and paying claims when appropriate. Although the LGPIF is not permitted to deny coverage for local government entities, these entities may go onto the open market to secure coverage. Thus, the LGPIF acts as a "residual" market to a certain extent, meaning that other sources of market data may not reflect its experience.