Physics 1154
General Physics Lab I

Second Edition

(Revision 1)

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Contents

Preface

This text is second edition of the revised lab manual for PHYS-1154 a one-semester laboratory course for students enrolled in PHYS 1110 or 2110. This manual covers experiments in mechanics, pressure, thermodynamics and sound. I have endeavored to write this book so that it is accessible to students with only knowledge in algebra while including several calculus concepts so that we can actually talk about the physics that we are studying. The first revised edition of this manual was released in August 2014 and sought to update the manual written by Dr. Glenn Sowell in 2003. This mainly consisted of moving the text into an unified format across the book as well as updating the procedures to reference instructions in PASCO's Capstone software.

Changes in this edition include many, many small fixes across the manuscript that both my colleagues and my students pointed out to improve the manuscript. Also, some larger structural changes to the course: The section on vector arithmetic has been split from kinetic friction in the fourth chapter and moved to the first chapter where it is now paired with a section on general measurements and uncertainty. This will hopefully adjust the timing of the course to match closer with the lecture course and help reinforce the concepts learned in both courses. To compensate for the loss of the vectors section, the friction chapter now includes a section on static friction. Archimedes' principle has been moved to the sixth chapter since it utilizes primarily forces to determine the density of our objects. Also, the monster chapter of torque and angular momentum has finally been split! This is something that I have wanted to do ever since I took the class myself. Torque now has its own dedicated chapter and now includes a section on dynamic torque where students will explore the relation between torque, moment of inertia, and angular acceleration. I have left angular momentum largely unchanged as I felt that the section and experiment was more than enough all by itself. The harmonics and calorimetry chapters have swapped places so that they follow the progression found in most lecture textbooks. Finally, the standing waves chapter that used to end our semester has been dropped from the course. None of the lecture courses ever got to the chapters on standing waves and with the splitting of torque and angular momentum an extra week was needed so unfortunately standing waves lost to scheduling.

Benjamin Floyd

Chapter 1

Vectors & Measurements

Goal

For our first meeting we will discuss some of the basic preliminaries that we will need for this course. Namely, the accuracy of our measurements and the reliability of our equipment. We also will discuss some basic vector arithmetic as many of the quantities that we use in physics are a certain mathematical structure called vectors.

Equipment

- Four Meter sticks
- Block of wood
- Masses
- Triple-beam balance

Theory

Measurements

Whenever we take a measurement or write a number down in physics it is extremely important that we some how indicate what this number is. We typically do this by using units. The standard system units used in all of science are known as the International System of Units or SI (abbreviated from the French name, *Le Système International d'Unités*). SI was developed to replace the old metric system and was first published in 1960. It is comprised of seven base units: metre, kilogram, second, ampere, kelvin, candela, and mole which define length, mass, time, electric current, temperature, luminous intensity, and amount of substance respectively. We will be using these units and units that can be derived from the seven base units throughout the semester.

Particular to experimentation is the need to be careful of the uncertainties and errors in our measurements. In our class we will generally use percent discrepancy which allows us to compare experimental values with standard or otherwise pre-agreed upon values and is defined below,

$$\text{Percent discrepancy} = \frac{|\text{experimental value} - \text{standard value}|}{\text{standard value}} \times 100\%.$$

Sometimes though, there is no "standard value" to compare against. In these situations we will want to make many measurements in order to obtain an average value. With an average value we can then calculate a relative discrepancy. The formula for relative discrepancy is the same as that for percent discrepancy except that in place of the "standard value" one uses the *average* of the various experimental results.

$$\text{Relative discrepancy} = \frac{|\text{experimental value} - \text{average value}|}{\text{average value}} \times 100\%$$

We will also need to be aware of both systematic and random errors in our experiments. Systematic errors are those which arise mainly from deficiencies in the measuring device and they almost always tend to make *all* of the readings too high or all of the readings too low. Random errors can generally be attributed to the physical limitations of the observer and measuring device. Given a set of data we show that the best, most accurate value is simply the arithmetic mean,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n},$$

where $\bar{x}$ is the mean of n numbers. The uncertainty of our data points in our set is generally measured by what is called the standard deviation, σ, given by,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

where $\bar{x} - x_i$ are the deviations of the individual readings and n is the total number of readings.

n.b., This is in no way an exhaustive discussion about measurements and uncertainties. For a much more extensive discussion and examples, please refer to Appendix A

Vector Arithmetic

Vector arithmetic can sometimes be strange to use as it follows different rules as the regular arithmetic of real numbers we are typically used to. A vector is typically thought of as an arrow in space that has two parts: magnitude or the

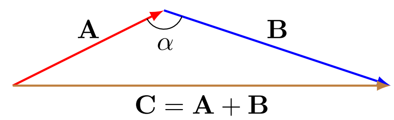


Figure 1.1

length of the arrow and direction or, in a flat, two-dimensional plane, the angle the arrow makes with an axis. When we add or subtract two vectors we need to take both parts into consideration. For example consider the vector addition in Figure 1.1. In order to find the magnitude of vector $\mathbf{C}$ we would need to know the magnitudes $A = |\mathbf{A}|$ and $B = |\mathbf{B}|$ and the angle between them, α. We remember from trigonometry that to find the length of the third side of any triangle, we use the law of cosines,

$$C = \sqrt{A^2 + B^2 - 2AB\cos\alpha}. \tag{1.1}$$

Using Equation (1.1) we can find C but it can sometimes lead to confusion as to how the angle α is defined in relation to the angles θ_A and θ_B that define the orientation of vectors $\mathbf{A}$ and $\mathbf{B}$ respectively. Also, while Equation (1.1) is simple now, with the addition of more than two vectors our job can become much more difficult. So instead of using the law of cosines, let us develop another method that is more procedural and easier to follow.

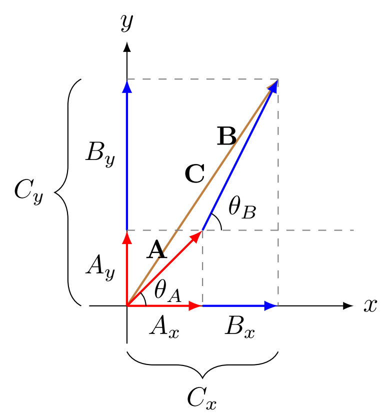
Consider the vectors in Figure 1.2a, We want to compute both the magnitude of vector $\mathbf{C}$ and its angle θ_C (as measured from the x-axis). To do this we must first break the vectors $\mathbf{A}$ and $\mathbf{B}$ into their x and y components. The advantage of this is that instead of dealing with an obtuse triangle we can focus on one right triangle per vector, which is much easier to handle and can easily be expanded to as many vectors as we need.

To determine the components of our vectors we use the basic trigonometric identities,

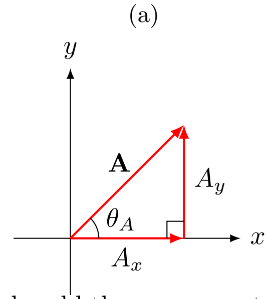
$$\cos\theta_A = \frac{A_x}{A} \text{ or } A_x = A\cos\theta_A, \tag{1.2}$$

$$\sin\theta_A = \frac{A_y}{A} \text{ or } A_y = A\sin\theta_A. \tag{1.3}$$

Once we have reduced all our vectors to components along the axes, we can then simply add the components that are



(a)



(b)

Figure 1.2

in the same direction to get the components of our resultant vector, **C**. When we have our resultant components, we can use the Pythagorean theorem to find the magnitude of our resultant,

$$\begin{aligned} C &= \sqrt{C_x^2 + C_y^2} \\ &= \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}. \end{aligned} \tag{1.4}$$

To find the the direction, we use the identity,

$$\tan\theta_C = \frac{C_y}{C_x} \quad \text{or} \quad \theta_C = \arctan\left(\frac{C_y}{C_x}\right). \tag{1.5}$$

Now that we understand how to add vectors, let's talk about multiplication. Vector multiplication is perhaps the most different from the multiplication of real numbers that we are used to. The most stark difference is that for vectors there is not one but *two* distinct types of multiplication. The first is called the *scalar product* or *dot product* which produces a scalar quantity. The other type of multiplication is called the *vector product* or *cross product* which produces a vector quantity.

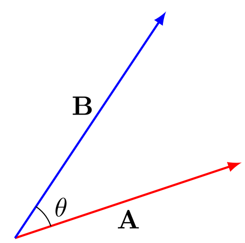


Figure 1.3

Scalar Product

The scalar product of two vectors **A** and **B** is denoted as $\mathbf{A}\cdot\mathbf{B}$ and thus is often also called the "dot product." This multiplication inputs two vectors and outputs a scalar. We will use this later in Chapter 7 when we study work. To compute magnitude of the scalar product $\mathbf{A}\cdot\mathbf{B}$ we want to project our second vector **B** onto our first vector **A**. We can do this by choosing the component of **B** in the direction of $\mathbf{A}, B\cos\theta$, where θ is the angle between the two vectors as seen in Figure 1.3. So we have,

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta. \tag{1.6}$$

It is also worth noting that the scalar product is commutative. i.e., $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$.

Vector Product

The vector product of two vectors **A** and **B** is denoted as $\mathbf{A}\times\mathbf{B}$ and thus is also called the "cross product." This multiplication inputs two vectors and outputs a new vector. This product will come up in Chapter 10 when we study torque. To compute the magnitude of the vector product $\mathbf{A}\times\mathbf{B}$ we want choose the component of our second vector **B** that is perpendicular to **A**, $B\sin\theta$ where θ is the angle between **A** and **B** *measured from* **A** *towards* **B**. This directionality is important as we will see later. Thus we have,

$$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}|\sin\theta. \tag{1.7}$$

In contrast to the scalar product, the vector product is *not* commutative. i.e., $\mathbf{A}\times\mathbf{B} \neq \mathbf{B}\times\mathbf{A}$. This is at first a confusing concept to many students because the multiplication that we are most familiar with is the multiplication of real numbers which is commutative. Here though, the order of the vectors in our multiplication does matter. One of the defining properties of the vector product is that the resultant vector will always be *orthogonal* or mutually perpendicular to both its parent vectors. Thus we can determine the direction of our resultant by what is known as the "right-hand rule." This simple mnemonic is true for all vector products and is setup as follows: The index finger of the right hand is pointed in the direction of the first vector of the product (**A**), the middle finger is pointed in the direction of the second vector (**B**), then the thumb will be pointing in the correct direction of the resultant vector.

Note that this is not a complete discussion on the two vector multiplications. There are methods to compute the products when we are given the vectors **A** and **B** in component form. This is especially important when considering the cross product as it outputs a new vector. However, in this class we will typically handle our vectors as magnitudes and find the direction afterwards.

Finally, a note: While we may often use degrees to measure angles out of habit or convenience in physics it should be noted that the standard unit for an angle is a radian, where 1 rad = 57.3°.

Setup I: Measurements

1. Each member of the group should measure the length of the block of wood using the meter sticks at the table. Then once everyone has measured the block calculate the average measured length and the standard deviation of the readings.

2. Using the triple-beam balance provided, measure the mass of several of the disks with the same printed mass. Calculate the average measured mass and standard deviation of the readings.

Setup II: Vector Arithmetic

1. Use the three vectors assigned by the instructor to solve for the resultant vector $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$ by using the component method described in the theory section. Record the values on the data sheet. Show sample calculations.

2. Use the additional pair of vectors assigned by the instructor to calculate the resultant vector $\mathbf{C} = \mathbf{A} - \mathbf{B}$ by using the component method. Hint: $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$ where $-\mathbf{B}$ has the same magnitude as $\mathbf{B}$ but is in the opposite direction, e.g., if for $\mathbf{B}$ the components are $B_x = 2$ m and $B_y = 5$ m then $-\mathbf{B}$ has components $B_x = -2$ m and $B_y = -5$ m.

3. Another way to visualize the cross product is as the area of a parallelogram with the length of the sides equal to the magnitudes of the vectors. Verify this by using the final pair of vectors given by the instructor to

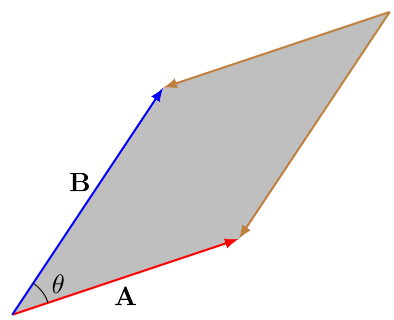


Figure 1.4

calculate the resultant vector $\mathbf{C} = \mathbf{A} \times \mathbf{B}$. Show that both the area of the parallelogram and the magnitude of the resultant vector $\mathbf{C}$ are the same. Also, determine which direction the vector $\mathbf{C}$ is pointing i.e., in or out of the paper.

Chapter 1: **Vectors & Measurements**
Fill out the accompanying report template. The points are distributed as follows:

1. **(5)** Purpose — What was the point of these experiments and problems? What information can we gain from them?

2. **(10)** Theory — Describe in one or two paragraphs how to find the sum, difference, and both scalar and vector products. Include both complete sentences and equations.

3. **(20)** Data Sheets

4. **(10)** Show work for all calculations. (May be written on data sheets.)

5. **(5)** Conclusion — Close with a short summary of the experiments and any final thoughts about our experiments and problems.

Vectors & Measurements Assignment

Name:______________________________

Purpose:

Theory:

Data Sheets

Measurements

Length of wooden block

Lengths x_i	Deviations, $x_i - \bar{x}$	$(x_i - \bar{x})^2$

Average length ($\bar{x}$)= ______________

Standard deviation = ______________

Measured masses

Masses x_i	Deviations, $x_i - \bar{x}$	$(x_i - \bar{x})^2$

Average mass ($\bar{x}$)= ______________

Standard deviation = ______________

Vector Arithmetic

$\mathbf{A} =$ ______________ at ______________

$\mathbf{B} =$ ______________ at ______________

$\mathbf{C} =$ ______________ at ______________

Vector	x Component	y Component
Resultant ($\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C}$)		

Magnitude= ______________ $\tan\theta =$ ______________ $\theta =$ ________

$\mathbf{A} =$ ______________ at ______________

$\mathbf{B} =$ ______________ at ______________

Vector	x Component	y Component
Resultant ($\mathbf{C} = \mathbf{A} - \mathbf{B}$)		

Magnitude= ______________ $\tan\theta =$ ______________ $\theta =$ ________

$\mathbf{A} =$ ______________ at ______________

$\mathbf{B} =$ ______________ at ______________

Area of parallelogram = ______________

Magnitude $|\mathbf{C}| = |\mathbf{A}\times\mathbf{B}| =$ ______________ Direction of $\mathbf{C}$=______________

Conclusion:

Chapter 2

Displacement & Velocity

Goal

These experiments will explore the concepts of displacement and velocity by examining position vs. time and displacement vs. time plots. We will use a motion sensor to detect the motion of objects moving towards and away from the sensor. Also, we will look at the difference between average and instantaneous velocity.

Equipment

- 850 Universal Interface
- PASCO Capstone Software
- Motion Sensor
- Two Photogates
- Low-friction dynamics track and cart
- Cart-mountable picket fence
- Masonite targets

Theory

Displacement describes the motion of an object from one location in space to another. We can represent this mathematically by,

$$\Delta x = x - x_0,$$

where x describes our current position, x_0 describes our previous position, and Δx represents the change in x. If we were to look at how our displacement changes over a certain time interval, we would know another piece of information about our object called *velocity.* We can define an *average* velocity as,

$$v_{ave} = \frac{x - x_0}{t - t_0} = \frac{\Delta x}{\Delta t},$$

where t is the time when the object is at position x and t_0 is the time at position x_0. While an average velocity works very well when the velocity is constant, if our object's velocity is changing we could be missing out on information by averaging over a large time span. To compensate for this, we can shorten our time interval to make our average more accurately describe our velocity at a single instant in time. If we keep shortening our time interval until our starting and ending time are infinitesimally close we can get what is called an *instantaneous* velocity. Those with some calculus background will recognize this as a limiting process. Thus, we can write our instantaneous velocity as,

$$v = \lim_{t \to t_0} \frac{x(t) - x(t_0)}{t - t_0} = \frac{dx}{dt}.$$

Setup I: Motion Graph Matching.

1. Turn the 850 Universal Interface (850UI) on.

2. Attach the Motion Sensor into digital inputs 1 and 2. Use channel 1 for the yellow plug and channel 2 for the black one. (Any two adjacent channels can be used but the yellow plug must always be in the one on the right.) Set the Motion Sensor to the "people" (wide angle) setting.

3. Open Capstone from the desktop. In the upper left-hand side click on the button labeled "Hardware Setup." There should be a picture of the 850UI shown. If not, ask the instructor for help.

4. For this experiment we want to load a saved workbook from the computer. Open the file titled "Motion Graph Matching" by clicking . The file should be located in ..\Documents\Phys Lab Files\Phys1154.

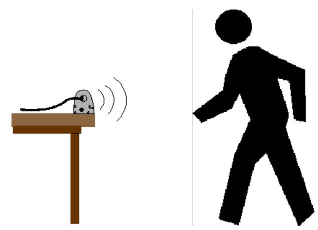
5. Place the Motion Sensor on the edge of a table at a place where there is at least 2.5 m of empty room in front of the sensor.

6. Aim the Motion Sensor at the Masonite board while standing in front of the sensor.

7. Position the computer monitor so you can see the screen while you move away from the Motion Sensor. The Motion Sensor puts out 10 pulses each second for 10 seconds. The Index on the horizontal axis is the number of pulses, so an Index of 10 corresponds to 1 second.

Procedure

1. Each group member should do the following for one Position graph and one Velocity graph.
2. Stand an appropriate distance (see start position on Position 1–3)(50 cm for Velocity plots) in front of the Motion Sensor.
3. After you click RECORD, there is a three-second countdown before data recording begins. Watch the Ready clock on the bottom of the screen and be ready to move when it reaches zero.



4. The Score display will show how closely you match the graph. The closer to 100, the better.
5. Click RECORD. The Motion Sensor will make a faint clicking noise to tell you it is on and the green LED will flash.
6. Try to match the graph by moving forward or backward. The recording will stop automatically after 10 sec.
7. Repeat the data recording process as many times as you need (time permitting) to get your best match. Use the white triangle by the Delete Last Run icon at the bottom of the page to delete unwanted runs (too many recorded runs can cause problems with the program). If you want to examine a previous run, use the black triangle by the Run Select icon in the graph toolbar above the graph. It is fairly easy to get a score above 95 on the position plots. The velocity plots are harder and scores above 80 are good.
8. Repeat the process for Velocity 1–4.

Position plots

Question 1. What does a horizontal line on a position graph mean?

Question 2. What is the difference between the parts of the plot with positive slope and the parts with negative slope?

Question 3. On the Position 3 plot, what is happening between 5 and 10 seconds (50 to 100 index)?

Velocity plots

Question 4. What does a horizontal line on a velocity graph mean?

Question 5. What is the difference between the parts of the plot with positive slope and the parts with negative slope?

Question 6. Consider the Velocity 2 plot. What is the difference between places where the slope is large and places where the slope is near zero?

Setup II: Looking at Position & Velocity graphs together.

1. We now want to clear all our data and start a new experiment. We can do this by clicking on the “New Experiment” button  or going to “File” >“New Experiment.”
2. Before we move on we need to tell Capstone that we are using a Motion Sensor again. To do this click on the “Hardware Setup” button and on the picture of the 850UI click the port that the Motion Sensor is plugged into. (For sensors like the Motion Sensor, always click on the port that the yellow plug is in.) Simply type in “Motion Sensor” and click on “Motion Sensor II.” If we have done this correctly, there should be a small icon of the Motion Sensor in the picture with two green lines indicating which ports the sensor is plugged into. Click the “Hardware Setup” again to hide the window.
3. We want to create a graph for our experiment. To do this, double-click on the button labeled “Graph” in the toolbar on the right.
4. To tell Capstone what data to plot, click on the “<Select Measurement>” button on the vertical axis and select “Position.”
5. Now we want to also show a velocity plot on the same graph. To do this in the toolbar above the graph, click on this button  This will add a second graph below our current one.
6. Repeat the process from step 4, but this time select “Velocity.”

Procedure

1. One member of the group should be the target and do the following: Collect data as the target walks away from the motion detector at a constant speed, pauses for a second, and then walks toward the motion detector at a constant speed. This may take several tries. Pick the best attempt and hide the rest. Rescale the data with this button  to fit the data.

2. Notice that both graphs are drawn together. Both should have 3 regions corresponding to the 3 movements in Step 1.

3. While the Position graph is selected, click on the Coordinates/Delta Tool  and drag it to a point in the first region.

4. To get access to the "Delta Tool," right-click the Coordinates tool in the graph and select "Tool Properties." Check "Show Delta" and change the "Delta Tool Style" to be "$\Delta y/\Delta x$ with slope.." Click "Okay" to close the window.

5. Drag the new tool to another data point along the straight-line portion of our graph. The graph should show a box with $\Delta x/\Delta t =$ value, where the value is the *slope* of the straight line connecting the two points. This is the *average velocity* of our motion between the two times we have selected. Record the midpoint of the time interval and the slope shown in the Delta Tool in the data table.

6. Now look at the Velocity versus time graph. In the data table, record the same time as you recorded above and the velocity at this time as given on this graph. (Use the Coordinate Tool to display the point.)

7. Repeat Steps 3–6 above for the other straight regions on your graph. **Print** out a copy of the graph displaying the data in each region (use multiple Coordinate Tools) for all group members.

Question 7. Taking the velocity from the velocity-vs.-time graph as the standard, calculate the percent discrepancy with the average value of the velocity from the position-versus-time graph for the three cases.

$$\%\ \text{discrepancy} = \frac{|\text{experimental} - \text{standard}|}{\text{standard}} \times 100\%$$

Question 8. Why are the two values of the velocity from the two graphs not the same? Or, if they are the same, should they be?

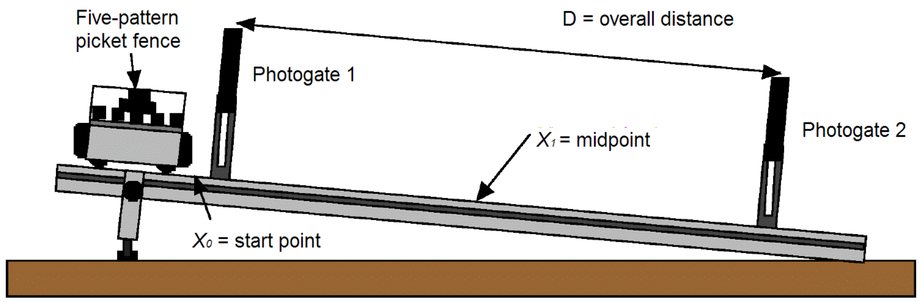
Question 9. What can you say about the position-versus-time graph when the velocity is negative as shown on the velocity-versus-time graph?

Setup III: Instantaneous Velocity.

1. Disconnect the (real) motion sensor from 850UI and wrap the cord up. Place it aside; out of the way.

2. Open the file called "DispVel3." It should be located in ..\Documents\Phys Lab Files\Phys1154.

3. Place the Photogate connected to Digital Channel 1 ("Gate 1") at the top end of the track 40 cm away from the X_1 point. Place the Photogate connected to Digital Channel 2 ("Gate 2") at the bottom end of the track 40 cm away from the X_1 point.



4. Place the "five-pattern picket fence" into the accessory tray on the top of the dynamics cart. Place the picket fence so that *one of the solid bands* will block the Photogate beam as the cart moves down the track. Put the cart on track. Adjust the height of both Photogates so that the Photogate beams are blocked when the cart and picket fence move down the track. Before recording any data for later analysis, you should experiment with the Photogate, cart, and picket fence. Put the cart at the starting point on the track and release it. Do the lights on both Photogates come on? Adjust them as necessary.

Procedure

1. The goal of this section is to determine the instantaneous velocity of the cart at X_1, the midpoint between the two Photogates. You will do this by measuring the *average* velocity of the cart as we progressively reduce the distance between the Photogates.

2. Record the location of the X_1 as indicated by the scale on the track. Check to see that the Photogates are equidistant from this point. Record the initial position of the cart. You will release the cart from this position for every run.

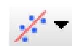
3. Click on the "Preview" button to begin the experiment.

4. Release the cart so it moves down the track. Data recording begins when the Photogate beam is first blocked. If the cart successfully passes through both gates (watch for the lights!), enter the distance between the gates in the table and press the "Keep Sample" button. Make sure **not** to click the "Stop" button as it will stop your current run.

5. Move the two Photogates closer to the midpoint X1 by the same amount of distance—say, 5 cm. Adjust the heights of the Photogates above the track so that the cart's picket fence will still block them.

6. Perform another run at this new position: Release the cart from the same initial position as before. If it passes successfully through both Photogates, enter the new distance in the table and click "Keep Sample."

7. Repeat Steps 5–6 until the Photogates are as close as possible to X_1.

8. After you have collected the last data, then click on the "Stop" button. This will end the data run. Make a *Linear* fit  of the data in the graph. **Print** out a copy of this graph for each of the group members.

Question 10. Which of the average speeds that we measured gives the closest approximation to the instantaneous speed of the cart as it moved through the midpoint X_1? Why?

Question 11. What is the relationship between the "y-intercept" on the graph and the instantaneous speed of the cart as it moved through the midpoint X_1?

Question 12. What factors (accuracy of timing, release of object, accuracy of position measurements, etc.) influence the results? Do errors in these factors produce large changes in your results or small ones? Are the errors random or systematic?

Question 13. How does a Photogate work?

Chapter 2: **Displacement & Velocity**

1. **(1)** Title Page — Title, name, name of lab partners, date of lab.

2. **(4)** Purpose — What was the goal of Displacement & Velocity III?

3. **(10)** Theory — Discuss what information can be gathered from a position vs. time graph and a velocity vs. time graph. Explain how this information is found. Make sure to include the equations for *both* average and instantaneous velocity.

4. **(4)** All graphs.

5. **(3)** Data Table.

6. **(24)** Answers to all questions.

7. **(4)** Conclusion — What were the findings from Displacement & Velocity III? How good were they?

Data Table

Setup II

Region	Midpoint (s)	v_{ave} (m/s)	v_{inst} (m/s)	% discrepancy
Left				
Center				
Right				

Chapter 3

Acceleration

Goal

This week we will preform three different activities to learn about the relationship of position, velocity, and acceleration. We will start with looking at two different ways to calculate the acceleration due to gravity, then finish with an experiment using a motor to accelerate the cart.

Equipment

- 850 Universal Interface (850UI)
- PASCO Capstone
- Motion Sensor, Photogate
- Low-friction track, regular cart, fan cart, and support rod
- Large picket fence
- Meter stick

Theory

We can describe the position of an object, given a constant acceleration, a, by the following equation,

$$x = x_0 + v_0 t + \frac{1}{2} a t^2, \tag{3.1}$$

where x_0 and v_0 are the initial position and velocity respectively. We can identify this equation as quadratic equation.

If we were to take the derivative of Equation (3.1) with respect to time we would get,

$$\frac{d}{dt}x = v = v_0 + at, \tag{3.2}$$

which is another of the kinematic equations that describes the velocity of an object under acceleration.

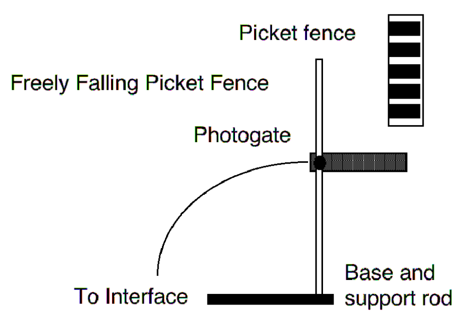
Finally, if we take the derivative of Equation (3.2) or equivalently, the second derivative of Equation (3.1) we have,

$$\frac{dv}{dt} = \frac{d^2x}{dt^2} = a. \tag{3.3}$$

For those without any prior exposure to calculus, don't panic! All we need to know is that a derivative is an equation that defines the slope of a curve at a single point. Thus, the second derivative is simply the "slope of the slope" and so on.

For a freely-falling object the acceleration experienced will be due to the gravitational force. We give this acceleration a special symbol,g, as it is the same value everywhere near the surface of Earth, $g = 9.81$ m/s^2. As acceleration is a vector, and since the vertical direction going upward is normally taken as positive, we often use a negative sign with g to indicate that the acceleration is downward.

Setup I: Acceleration on a freely-falling body



1. Attach the Photogate to digital channel 1. Setup the Photogate at the edge of the table so that the regular picket fence can pass through as shown.

2. Open Capstone.

3. Under "Hardware Setup" in the toolbar on the left, click on digital channel 1 on the picture of the 850UI and add a Photogate. You can do this by scrolling through the list and clicking or by typing "Photogate" into the search bar. Note: We want *"Photogate"* not "Photogate with Pulley."

4. There will now be a new button in the toolbar on the left called “Timer Setup.” We need to tell Capstone how the Photogate will be used.

5. Click on “Timer Setup” and follow the on-screen instructions to setup a Pre-Configured Timer for Photogate, Ch 1. On step 3 of the timer, choose “Picket Fence.” Make sure that “Speed,” “Acceleration,” and “Position” are all checked. Finally, make sure the Flag Spacing is set to 0.05 m. Click “Finish” to complete the timer setup. Click on “Timer Setup” to hide the window.

6. We want to create a graph for our experiment. To do this, double-click on the button labeled “Graph” in the toolbar on the right.

7. To tell Capstone what data to plot, click on the “<Select Measurement>” button on the vertical axis and select “Position.”

8. We want to also show velocity and acceleration on the same graph. To do this in the toolbar above the graph, click on this button  twice. This will add two graphs below our current one.

9. Repeat the process from Step 7 to select “Speed” and “Acceleration.”

Procedure

1. Click on the Record button at the bottom. Capstone will begin data collection when the Photogate is first blocked.

2. Do not let the picket fence hit the floor. Put a book bag, coat or other soft object underneath the Photogate so that the picket fence doesn’t break.

3. Drop the regular picket fence through the Photogate. Click Stop in the toolbar. If the picket fence hits the Photogate, then do it again. When you have a good run hide any “bad” data runs, so that you only see the good one. Press the Scale to fit button . n.b., how you rescaled graph looks may depend on which graph panel (there are 3 here) is active at a time. Experiment with clicking on one panel (say, Position vs. Time) and then clicking on the Scale to Fit button. Repeat with each panel. Which should you choose? Choose the one that shows all of the data in all 3 panels.

Analysis

1. Click on the Position versus Time graph. Fit the data  with a Quadratic Fit. You can move the fit info box around so that it does not obscure the data. The A value is the quadratic coefficient, B the linear coefficient, and C the constant term. Comparison with Equation (3.1) will show you what each of these coefficients correspond to.

2. Click on the Speed versus Time graph. Fit the data with a Linear Fit.
3. Click on the Acceleration versus Time graph. Calculate the mean and standard deviation of the data by selecting the Statistics button,  in the graph toolbar and choosing those options. Uncheck Maximum and Minimum options. Click the Σ to make the results show on the graph.
4. **Print** a copy of this graph for each member of the group.
5. Using the results from all three graphs, compare the experimental values to the standard value of g using a percent discrepancy.

Question 1. From the step above which graph gives the best value of g? Which is the worst? Why?

Question 2. The experiment indicates that the acceleration due to gravity is a positive number, not a negative. What is the significance of this? Does it matter?

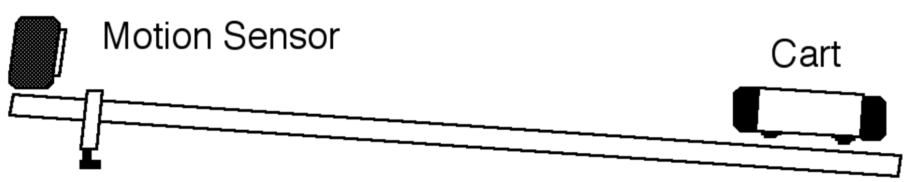
Setup II: Acceleration on an inclined plane.

In this experiment, we will again be determining g. However, instead of just dropping an object, let's roll a cart down a track. In this situation, the acceleration of the cart will be actually slower than if it were just falling. The acceleration of the cart will be,

$$a = g \sin\theta, \tag{3.4}$$

where θ is the angle the track makes with the table.

1. Remove any extraneous equipment from earlier activities, wrap the cords and put aside.



2. Position the Motion sensor on the dynamics track as shown. Plug the Motion sensor into the 850UI.
3. Choose "New Activity" from the File menu in Capstone. Answer "Discard" when asked to save the previous activity.
4. Under "Hardware Setup" add the Motion Sensor to the appropriate ports.
5. Double-click the Graph icon in the toolbar on the right.

6. Create a three-paneled graph as we did in Setup I for Position, Velocity, and Acceleration.

Procedure

1. Calculate the angle of the inclined track. We can do this by measuring the height of the track and the length of the track and using the following trigonometric definition to find the angle,

$$\theta = \arcsin\left(\frac{Height}{Length}\right).$$

   To get an accurate value of the angle, make sure to measure the height of the bottom surface of the track, not the top surface.

2. Position the Motion sensor at the high end of the track. The cart will start at the low end of the track and be pushed up toward the motion sensor.
3. Before recording any data for later analysis, you should experiment with the motion sensor to make sure it is aligned and can "see" the cart as it moves.
4. Place the cart on the low end of the track (i.e., the end opposite to the motion sensor).
5. Click the Record button to begin recording data.
6. Give the cart a firm push up the track so the cart will move up the inclined plane toward and then away from the motion sensor. Don't push the cart so firmly that it gets closer than 15 cm to the sensor. Continue collecting data until the cart has returned to the bottom of the track. Click the Stop button to end data recording. (If the data points do not appear on the graph or do not form a smooth graph, check the alignment of the motion sensor, remove the bad data, and try again.)
7. Click the Scale To Fit button to automatically rescale the Graph.

Analysis

1. Click the mouse in the "Velocity vs. Time" panel of the graph.
2. Choose a linear fit to the graph. The slope of this line is the average acceleration.
3. In the plot of velocity, use the Selection Tool,  to select the points of the plot that shows the cart's motion after the push and before it stopped at the bottom of the track. This will force Capstone to only use the points within the box in calculating the fit of the line.

4. Click the mouse in the “Acceleration vs. Time” panel of the graph.

5. In the plot of acceleration, use the Selection Tool to include only the region of the plot that corresponds to the cart’s motion after the push and before it stopped at the bottom of the track. Record the mean of the acceleration.

6. Click on the triangle to the right of the Show Selected Statistics button . Check the “Mean” and “Standard Deviation” choices and uncheck everything else. Finally click the Σ to actually display the information.

7. **Print** a copy of this graph for each member of your lab table.

8. Calculate the theoretical value for the acceleration of the cart using Equation (3.4) and $g = 9.81\ \mathrm{m/s^2}$.

Question 3. Describe the position versus time plot of the Graph display. Why does the distance begin at a maximum and decrease as the cart moves up the inclined plane?

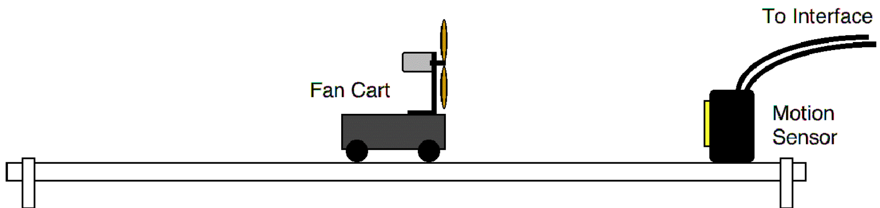
Question 4. By averaging the results of both the Velocity vs. Time and the Acceleration vs. Time plots, calculate the percent discrepancy using the theoretical value from Analysis step 8.

Question 5. If the mass of the cart is doubled, how should the results be affected? Explain.

Setup III: Acceleration from a fan.

For this last activity we will NOT be using the acceleration due to gravity, but instead the acceleration provided by a Fan Cart. We shall compare the position, velocity and acceleration data of the fan cart with Equations (3.1), (3.2), and (3.3).

Procedure



Position, Velocity, and Acceleration of a Fan Cart

1. Arrange the Fan Cart, Motion Sensor, and track as shown. Tie a string to the fan cart so that we can hold the fan cart before releasing it without interfering with the motion sensor.
2. We will use the same graph configuration as we did in Setup II. However, make sure that all of the previous data runs have been deleted.
3. Hold the string so that the fan cart remains stationary about 40 cm in front of the Motion Sensor. Also, make sure that the fan cart is pulling away from the motion sensor!
4. Click on "Record" to begin data collection.
5. Release the fan cart.
6. When the cart has traveled the length of the available track, click "Stop" to finish data collection.

Analysis

1. Scale your data to best fit your graph window.
2. Click on the Selection Tool, . Drag and resize the box to only include a relatively small region of the Position vs. Time plot that shows a smooth parabolic curve. Try to include about five or six data points in the box.
3. Press Scale to fit, . This will rescale all three graphs so that they display the same time interval. If you are not satisfied with your selection, right-click in the selection box, and click "Delete Highlighter," press the Scale to fit button again, then repeat step 2.
4. Click on the Position vs. Time graph. Fit the data with a Quadratic Fit. You can move the fit info box around so that it does not obscure the data. The A value is the quadratic coefficient, B the linear coefficient, and C the constant term.
5. Click on the Velocity versus Time graph. Fit the data with a Linear Fit.
6. Click on the Acceleration versus Time graph. Calculate the mean and standard deviation of the data by selecting the Statistics triangle  in the graph toolbar and choosing those options.
7. **Print** this graph for all group members.
8. Identify the value of a from each of the three graphs using the appropriate results from the curve fits.

Question 6. Calculate the percent discrepancy of each of the two other accelerations using the mean from the Acceleration vs. Time graph as the standard.

Chapter 3: **Acceleration**

1. **(1)** Title page — title, your name, name of lab partners, date of lab.
2. **(5)** Purpose — What was the goal of Acceleration II?
3. **(12)** Theory — Show how Equations (3.1), (3.2), and (3.3) are related. If you are in Algebra-based physics, talk about slopes. If you are in Calculus-based physics, use derivatives. What shape should the position, velocity, and acceleration graphs have for freely falling objects?
4. **(3)** Data Table
5. **(6)** All graphs. There are 3 of them.
6. **(6)** Calculations — Show calculations e.g., percent discrepancies, theoretical values
7. **(12)** Answer all questions.
8. **(5)** Conclusion — What were your findings from Acceleration II? Explain.

Data Table

Setup II

Item	Value
Angle of track	degrees
Acceleration (slope)	m/s^2
Acceleration (mean)	m/s^2
Acceleration (theoretical)	m/s^2

Chapter 4

Force

Goal

This week, we will look into how the dynamics of motion of an object can change with different amounts of net force. We will also examine the relationship between the applied force, acceleration, and the mass of an object.

Equipment

- 850 Universal Interface (850UI)
- PASCO Capstone
- Dynamics track and cart
- Masses and string
- Photogate with Pulley, Motion Sensor, Force Sensor
- Triple-Beam Balance

Theory

Isaac Newton described the relationship of the net force applied to an object and the acceleration it experiences in the following way: the acceleration (a) of an object is directly proportional to and in the same direction as the net force (F_{net}), and inversely proportional to the mass (m) of the object:

$$a = \frac{F_{net}}{m} \tag{4.1}$$

Consider the system illustrated below. A cart of mass M on a horizontal track is attached via a "frictionless, massless" pulley to a vertically hanging mass m. We can show that for this specific situation, that the acceleration of the cart rolling along the track will be,

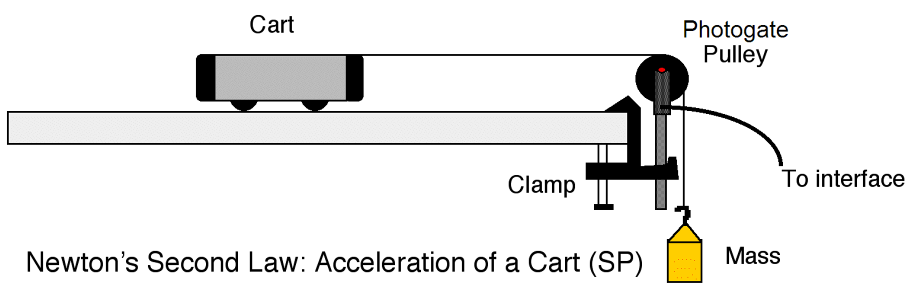
$$a = \frac{mg}{M+m}. \tag{4.2}$$

(This can be found using free body diagrams, and will be discussed in lab.) Using Newton's Second Law, we can then say that the tension on the string, and thus the net force on the cart is,

$$T = Ma = \frac{Mmg}{M+m} \tag{4.3}$$

Setup I: Acceleration and Force of a Cart.

1. Open Capstone
2. Attach the Photogate to the 850UI and tell Capstone what port the sensor is plugged into. Make sure to select the *"Photogate with Pulley"* in Capstone. This will automatically load the proper settings for the Photogate when we are using it with a pulley.
3. Hide the "Hardware Setup" and double-click on "Graph" in the toolbar to the right.
4. For the vertical axis choose "Linear Speed (m/s)."
5. Arrange the dynamics track, cart and pulley as shown. Level the track so that the cart will not roll one way or the other on its own.



Newton's Second Law: Acceleration of a Cart (SP)

6. Attach a string to the dynamics cart. Make the string long enough so that when the cart is next to the Pulley and the string is over the pulley, the string reaches the ground. Attach a mass hanger to the other end of the string. Put the string that connects the cart and the mass hanger over the Pulley. Adjust the Pulley so that the string from the cart is parallel to the level track or the top of the table.

7. Place about 20 grams of mass on the 50 gram mass hanger so that the total initial hanging mass is 70 grams. Add about 200 grams of mass to the top of the cart.

Procedure

1. Measure and record the total mass of the cart (M) in the data table provided.
2. Measure and record the total mass of the mass hanger and masses (m).
3. When we are ready to collect data, pull the cart away from the Photogate Pulley until the mass hanger almost touches the pulley. Turn the pulley so that the photogate beam of the Smart Pulley is unblocked (the light-emitting diode (LED) on the photogate is off).
4. Click the Record button to begin data recording. Release the cart so it can be pulled by the falling mass hanger. Data recording will begin when the Pulley photogate is first blocked.
5. Stop the data recording just before the mass hanger reaches the floor by clicking "Stop." n.b., Do not let the cart hit the Pulley as it could damage it.
6. In the graph window, rescale the data by using the Scale to Fit button . Fit a linear line to the data. The slope of the line is the acceleration of the cart. Record this value in the data table provided.

   Question 1. Why is the the slope of this line the acceleration of the cart?

7. Change the applied force by moving masses from the cart to the hanger. This changes the force without changing the total mass. Measure and record the new values for M (total mass of cart) and m (mass hanger and masses).
8. Repeat steps 3–6 for a total of at least five times. Each time, move some mass from the cart to the mass hanger. Measure and record the values for M (cart) and m (mass hanger and masses). Record the value of the slope for each trial. **Print** the graph of the last trial for each group member.

Analysis

1. Calculate the theoretical acceleration using Equation (4.2). Record the theoretical acceleration in the data table.

2. Calculate the net force acting on the cart for each trial. The net force on the cart is the tension in the string minus the friction forces. If friction is neglected, the net force is,

$$F_{net} = M_{cart}a,$$

where a is the theoretical acceleration from step 1.

3. Find an experimental net force on the cart using M (mass of cart) and the experimental acceleration.

4. Calculate the percent discrepancy between the experimental and theoretical forces.

5. Also calculate the total mass $(M + m)$ that is accelerated in each trial.

Question 2. What is the relationship between force and acceleration?

Setup II: Measuring Force on a cart directly.

Here we will be using the same experimental setup as before but with one addition. We will now mount a force sensor to the top of the cart and attach the string to the sensor. The force sensor is a device that we can use to measure how much force is being applied on the hook. The hook is connected to a variable resistor so that when the hook is pulled or pushed the circuit reads the change in voltage and sends that information to Capstone. Capstone then assigns certain voltage values to specific force values. While Capstone does use a default setting, we will often calibrate the force sensor manually ourselves.

1. Use the same setup as in part one. Delete any data from that part. The Graph window should be displaying “Linear Speed versus Time.”

2. Attach a Force Sensor to the Analogue channel “A” on the 850UI.

3. In Capstone, under “Hardware Setup” click on the port that the force sensor is plugged into and add the sensor. There are a few versions of the Force Sensor in Capstone, the one that we will always use is simply called “Force Sensor.”

4. Create a table for the Force data by double-clicking the “Table” button in the toolbar on the right. This will create a table in the same window as our graph. For the first column choose “Force (N)” to be the measurement, “Time (s)” for the second column.

5. We need to calibrate the Force Sensor.

   (a) Mount the Force Sensor on a horizontal rod so that the sensor’s hook is pointing downwards. Do *not* put an object on the hook yet.

(b) In the toolbar on the left, there is a button labeled "Calibration." Click on this. We will follow the on-screen instructions. We will be calibrating Force.

(c) In step 3, we will be using the "Two Standards (2 point)" option.

(d) For the first point we will set our "zero point." Press the tare button on the side of the physical force sensor. In Capstone set the standard value to be 0 N and press the "Set Current Value to Standard Value" button, then click "Next."

(e) For the second point, hang an object of known mass, e.g., 1 kg or 0.5 kg works best.

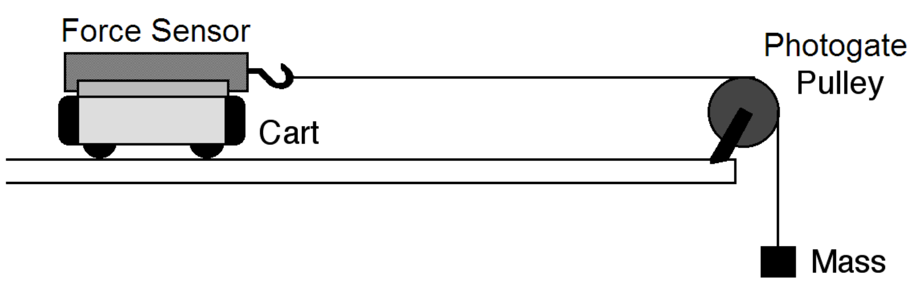
(f) Type in the object's weight in Newtons for the Standard Value. For this experiment, we will want to enter this value as negative to indicate that the force is pulling away from the sensor. Press the "Set Current Value to Standard Value" button, then click "Next."

(g) We should be presented with a summary of the calibration. Press "Finish" to complete the calibration then click the "Calibration" button on the left to hide the window.

6. Carefully measure and record the mass of the cart and the Force Sensor M. The cart and sensor are too massive for the balance to measure together so we will need to measure them separately.

7. Use a # 0 Phillips head screwdriver to mount the Force Sensor onto the accessory tray of the cart.

8. Use a string that is 10 cm longer than the length needed to reach the floor when the cart is next to the pulley. Attach one end to the Force Sensor's hook.



9. Add 20 or 30 grams to the mass hanger. Carefully measure and record the total mass m.

10. Attach the mass hanger to the other end of the string and put the string in the pulley's groove. Adjust the height of the pulley so that the string is parallel to the track.

Procedure

1. Pull the cart toward the left end of the track, and don't let the mass hanger bump into the pulley.

2. Click the Record button to begin data collection and then release the cart.

3. Click Stop to end recording just before the cart reaches the pulley. Stop the cart before it collides with the pulley.

4. In the *Graph* rescale the data by clicking the Scale To Fit button. Fit a linear line to the data. The slope of the straight line is the acceleration.

5. In the *Table* click on the small triangle to the right of the Show Selected Statistics button . Check the Mean and Standard Deviation choice and uncheck everything else. Click on the Σ to actually show the results.

6. **Print** a copy of the Graph and Table for all group members.

Analysis

1. In the Graph of the velocity of the cart, use the Selection tool  to create a rectangle around the region of the plot that shows the movement of the cart. Record the value of the acceleration a_{exp} (slope) for this region in the data table provided.

2. Note the beginning and ending times of your selected rectangle.

3. In the Table of the force on the cart, click on the Data Highlighting Tool  and use the cursor to select the data for the same times. Record the mean of the force in the data table.

4. Calculate and record the mean force found on the Force-Time table exerted on the cart / Force Sensor, T_{meas}.

5. We can calculate two values of the net force on the cart. One is $T_{theo} = Ma_{th}$ the other $T_{calc} = Ma_{exp}$. Calculate and record both of these. Note that a_{th} is calculated based on Equation (4.2) using $g = 9.81$ m/s^2 and the measured values of M and m.

6. Calculate and record the percent discrepancies of T_{meas} and T_{calc} with respect to T_{theo}.

Question 3. What is the percent discrepancy of a_{exp} with respect to a_{th}?

Question 4. What is the percent discrepancy of T_{meas} with respect to T_{theo}?

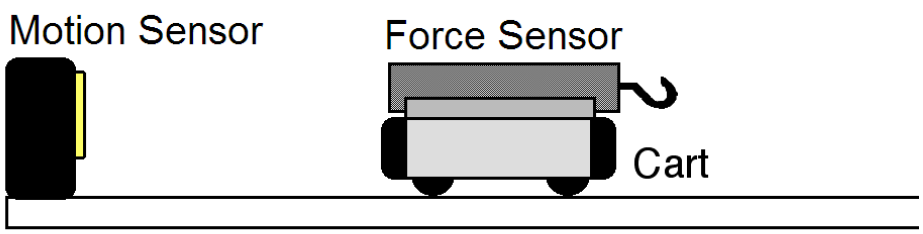
Question 5. What is the percent discrepancy of T_{calc} with respect to T_{theo}?

Question 6. Should we expect T_{theo} or T_{calc} to be closer to T_{meas}? Why? Is this borne out by our results?

Question 7. Here we ignored friction. If friction were a factor, would the tension in the string be more or less than the current theoretical value T_{theo}? Why? Do our results indicate that friction was a significant factor?

Setup III: Finding the mass of the cart.

For this activity, we will push and pull a dynamics cart back-and-forth on a level track. The Motion Sensor will measure the motion of a cart, and the force sensor will measure the force you exert on the cart. Capstone will calculate the acceleration of the cart as it moves. The graph of force versus acceleration reveals the mass of the moving object.



Pushing and Pulling a Cart

1. Remove the Photogate Pulley, string, and hanging weights. Leave the dynamics track, cart and Force probe. The Force probe should remain attached to the cart. Add the Motion sensor to the end of the track.

2. Attach the Motion Sensor to the 850UI.

3. In “Hardware Setup” remove the Photogate by clicking on the icon and pressing Delete. Add the Motion Sensor to the appropriate port.

4. Open a new page by clicking the “Add Page” button . Double-click “Graph” from the toolbar on the right.

5. We will want a Force vs. Acceleration graph for this experiment. For the vertical axis choose “Force (N)” and for the horizontal axis choose “Acceleration (m/s^2).”

Procedure

1. Make sure the track is level. We can check this by placing a cart on the track. If the cart rolls one way or the other, use the adjustable feet at one end of the track to raise or lower that end until the track is level and the cart does not roll one way or the other.

2. The minimum distance the Motion Sensor can read is about 15 cm. Make a mark on the track at 20 cm so that we don't get too close to the sensor.

3. Measure and record the mass of the cart plus the Force Sensor in the data table.

4. Before recording any data, make sure that Motion Sensor is correctly aligned and has a clear line-of-sight with the cart.

5. Position the cart an press the tare button on the side of the Force Sensor.

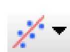
6. Click Record to begin data collection.

7. Firmly grasp the hook of the Force Sensor and push and pull the Force Sensor to make the cart move back and forth in a smooth and even motion. Make sure the cart does not get too close to the Motion Sensor.

8. Click Stop to end the data recording.

Analysis

1. Rescale the data in the Graph window.

2. Use a Proportional Fit  on the data. The constant of proportionality, A, should be the mass M of the cart and Force Sensor together. Record this value in the data table.

3. **Print** a copy of this graph for all group members.

Question 8. Why should the constant of proportionality A of the Force vs. Acceleration graph be the object's mass?

Question 9. What is the percent discrepancy between the actual and measured mass?

Chapter 4: **Force**

1. **(1)** Title Page
2. **(13)** Theory for Setup II only. Include a free body diagram.
3. **(6)** Graphs — One from each setup.
4. **(6)** Data Tables — One from each setup.
5. **(6)** Show sample calculations for Setup II.
6. **(18)** Answers to all questions.

Data Tables

Setup I

Trial	M (kg)	m (kg)	Total Mass (kg)	a_{th} (m/s^2)	a_{exp} (m/s^2)	F_{th} (N)	F_{exp} (N)	% Discrepancy

Setup II

Item	Value
Mass of cart & sensor M	kg
Mass of hanger & masses m	kg
Acceleration (slope) a_{exp}	m/s^2
Acceleration (calculated) a_{th}	m/s^2
Force (mean) T_{exp}	N
Force (calculated) T_1	N
Force (calculated) T_2	N

Setup III

Item	Value
Mass of cart & sensor (measured) M_{th}	kg
Mass of cart & sensor (A) M_{exp}	kg

Chapter 5

Static & Kinetic Friction

Goal

Last chapter, we took steps to ensure that friction was minimized by using the dynamics track and cart. This time we want to study the influence of frictional forces on our system. We will do this by substituting the dynamics track and cart for a plank of wood and a wooden block respectively.

Equipment

- PASCO Capstone Software
- 850 Universal Interface (850UI)
- Wooden board, wood block with felt attached
- Support rod
- Meter stick
- Mass hanger and masses
- Photogate Pulley
- String
- Triple-beam balance

Theory

Static Friction

Imagine we are trying to move a box across the floor. When we initially apply a force to the box it may not move because the *static frictional force* that the floor applies to the box is equal and opposite to our applied force. In order to move the box, we will need to overcome this static frictional force. As we increase the applied force on the box, the static frictional force, $\boldsymbol{f}_s$, will increase roughly linearly as seen in Figure 5.1. The static frictional force is defined by the relation,

$$f_s \leq \mu_s N, \tag{5.1}$$

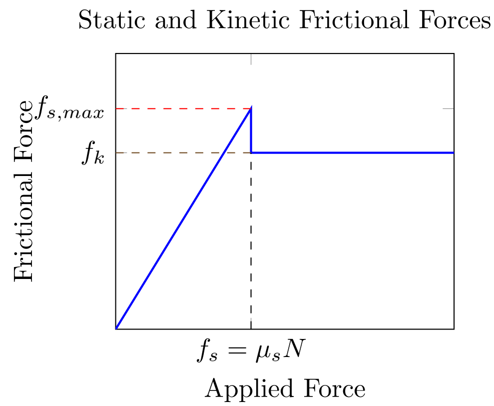


Figure 5.1

where f_s and N are the magnitudes of the static frictional force and the normal force and μ_s is called the *coefficient of static friction*.

In our experiment we will experimentally determine μ_s of our two materials on our object. Here we will use an inclined plane as shown in Figure 5.2. To determine μ_s we want to set θ so that our object is on the verge of moving. At this point, $f_s = f_{s,max} = \mu_s N$. If we analyze our setup we can write the system of equations,

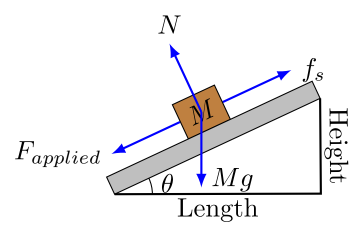


Figure 5.2

$$\sum F_x = F_{app} - f_s = 0 \tag{5.2a}$$

$$\sum F_y = N - mg\cos\theta = 0. \tag{5.2b}$$

With some careful manipulation of the above system of equations we find that the coefficient of static friction can be found as,

$$\mu_s = \tan\theta = \frac{Height}{Length}, \tag{5.3}$$

where *Height* and *Length* are as defined on Figure 5.2.

Kinetic Friction

In our kinetic friction experiment we will use the same setup as in Chapter 4, but with one important difference. Whereas in Chapter 4 we used the dynamics track and cart to minimize friction, this time we will use a block of wood against a plank of wood to generate a noticeable frictional force.

The kinetic frictional force $\boldsymbol{f}_k$ describes a force that opposes the velocity of the body relative to its frictional substrate. In common cases, the relative velocity $\mathbf{v}_{rel}$ is generated by an applied force, so $F_{applied}$ and $\mathbf{v}_{rel}$ point in the same direction, as in Figure 5.3. The kinetic frictional force can be experimentally found to be be approximately proportional to the *magnitude* of of the normal force by the relation,

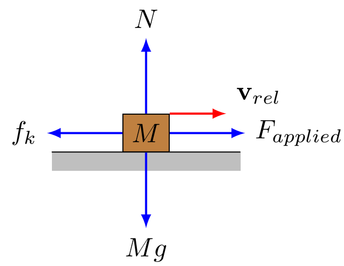


Figure 5.3

$$f_k = \mu_k N, \tag{5.4}$$

where μ_k is a constant called the *coefficient of kinetic friction*. In reality Equation (5.4) is only an approximation because μ_k might depend on the velocity of the object relative to the surface. For the purposes of our experiment however we will consider that μ_k *is dependent primarily on the materials involved* and possibly on a few other variables that we will check in today's experiment.

As in Chapter 4, we consider a mass M moving on a horizontal surface and pulled into motion by the weight of a vertically hanging mass m. If we set up our force diagram as we did last time we can get the system of equations,

$$M : \sum F_x = T - f_k = Ma \tag{5.5a}$$

$$\sum F_y = N - Mg = 0 \tag{5.5b}$$

$$m : \sum F_x = 0 \tag{5.5c}$$

$$\sum F_y = T - mg = -ma, \tag{5.5d}$$

which we can, with some work, find an expression for the coefficient of kinetic friction as,

$$\mu_k = \frac{mg - (M+m)a}{Mg}. \tag{5.6}$$

Setup I: Static Friction

In this setup we will slowly increase the inclination of our board until we reach the critical point where our block is on the verge of sliding down the board. We will then use the measurements to calculate the coefficient of static friction of the different materials on the block.

Procedure

1. Set the support rod up so that the crossbar is as low as possible.

2. Place one end of the board on the support rod crossbar and the other end on table.

3. Place the block so its largest smooth surface is on the board.

4. Raise the height of the board by 1 cm until the block just starts to slide.

5. Measure and record the height and length as shown in Figure 5.2 into the data table.

6. Repeat steps 1–5 for the small smooth side, and both large and small rough sides of the block.

Analysis

1. Using the lengths found earlier calculate the coefficients of static friction of each side of the block using Equation (5.3).

2. Average the coefficients of the smallest and largest smooth sides together.

3. Average the coefficients of the smallest and largest rough sides together.

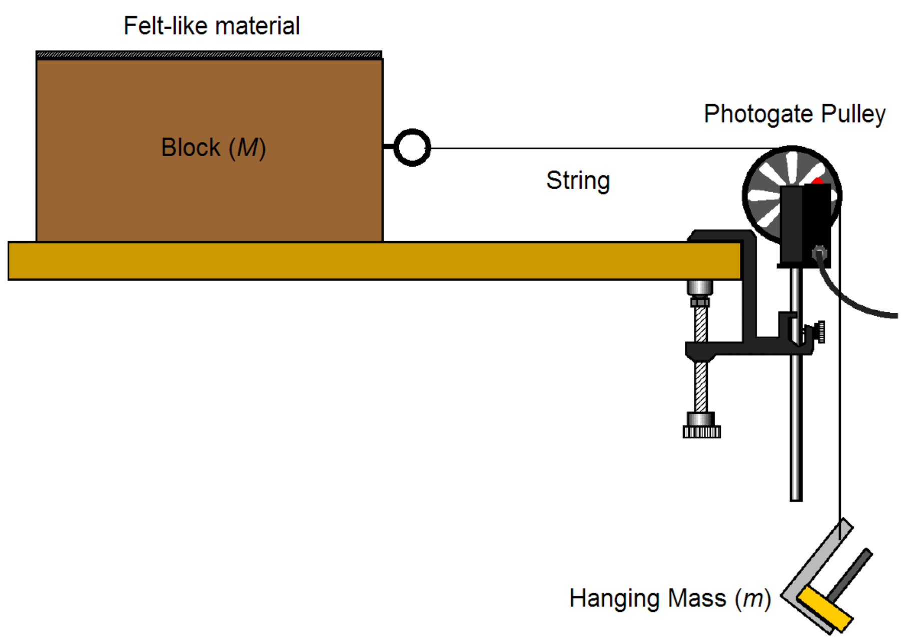
Question 1. Why is the equation for the coefficient of static friction independent of the weight of the block?

Setup II: Kinetic Friction

For this setup, the Photogate Pulley will measure the motion of the block sliding on a horizontal surface. The block is connected by a string to a hanging mass, We will vary the mass, surface area of the block, the type of material between the block and the surface, and the amount of hanging mass to change the circumstances of the block. Capstone will then record and display the data by plotting the speed vs time, the slope of which will be the average acceleration of the block for each trial.

1. Start Capstone.

2. Mount the Photogate Pulley at the edge of the board and table top as shown.



3. Attach the Photogate Pulley to the 850UI.

4. Under "Hardware Setup" connect the "Photogate with Pulley" in the appropriate port. Click "Hardware Setup" to hide it.

5. Double-click "Graph" in the toolbar on the right.

6. For the vertical axis choose "Linear Speed (m/s)."

Procedure

1. Measure and record the mass of the block of wood (M).

2. Use a piece of string that is about 10 cm longer than the distance from the top of the board to the floor. Attach one end to the string to the block. Attach the mass hanger to the other end of the string.

3. Run #1 & #2—Large, Smooth Surface

   (a) Place the block so its largest smooth surface is on the horizontal surface.

   (b) Put enough mass on the the mass hanger so that the block will slide on the surface without needing an initial push. Measure and record the value of the total hanging mass (m).

(c) Pull the block away from the Photogate Pulley until the hanging mass is almost up to the pulley. Hold the block in place. Turn the pulley so the Photogate's beam is not blocked. (The LED on the Photogate will be off when the beam is clear.)

(d) Click on the Record button and release the block. Click on the Stop button to end data collection just before the block hits the pulley. Stop the block so that it does not hit the Photogate Pulley.

(e) Repeat steps 3c–3d.

4. Run #3—Different Mass of Block

    (a) Double the mass of the block by placing a mass approximately equal to that of the block on top of the block.

    (b) Measure and record the total mass of the block system (M).

    (c) Double the hanging mass. Measure and record the total hanging mass (m).

    (d) Repeat steps 3c–3d.

5. Run #4—Different Surface Area

    (a) Remove the extra mass from the block.

    (b) Place the block so that its smallest smooth side is on the horizontal surface. Adjust the height of the Photogate Pulley so that the string remains level with the horizontal surface.

    (c) Use the same hanging mass we used in step 3 (*not* Run #3) so we can compare the runs.

    (d) Repeat steps 3c–3d.

6. Runs #5–#7—Different Hanging Mass

    (a) Return the block so the largest smooth side is on the horizontal surface.

    (b) Put an amount of mass on the hanger that is larger than the amount we used in step 3. Measure and record the total hanging mass (m).

    (c) Repeat steps 3c–3d.

    (d) Repeat the process using two larger hanging masses each larger than the previous. Measure and record the total hanging mass (m) each time.

7. Runs #8 & #9—Different Surface Material

    (a) Place the block so that its largest rough side is on the horizontal surface.

(b) Put enough mass on the mass hanger so that the block will slide on the surface without needing an initial push. Measure and record the value of the total hanging mass (m).

(c) Repeat steps 3c–3d to see how the different material affects the coefficient of kinetic friction.

(d) Place the block so its smallest rough side is on the horizontal surface.

(e) Repeat steps 3c–3d using the same hanging mass as we did for the largest rough side so we can compare the two runs.

Analysis

1. Using the Select Data Run button  display only Run #1.

2. For all data runs we will use a linear fit. The slope of the fit line will be the average acceleration of the block.

3. If necessary, use the Selection Tool  to highlight only the relevant portion of the data.

4. Record the acceleration in the data table.

5. **Print** a copy of the graph for all group members.

6. Repeat steps 1–4 for each of the remaining runs.

7. Calculate μ_k for each run using Equation (5.6).

Question 2. How does the coefficient of kinetic friction vary with the mass of the block? Is this variation significant?

Question 3. How does the coefficient of kinetic friction vary with the area of contact between the block and the horizontal surface? Is this variation significant?

Question 4. How does the coefficient of kinetic friction vary with the type of material between the block and the horizontal surface? Is this variation significant?

Question 5. When we used the different type of material, how does the coefficient of kinetic friction vary with the area of contact between the block and the horizontal surface? Is this variation significant?

Question 6. How does the coefficient of kinetic friction vary as the speed varied due to the different hanging masses? Is this variation significant?

Question 7. When the mass of the block is increased, does the force of kinetic friction increase? Why?

Chapter 5: **Static & Kinetic Friction**

1. **(1)** Title Page
2. **(5)** Purpose
3. **(16)** Theory for both Static and Kinetic Friction—Should include free-body diagrams.
4. **(2)** Data Sheet
5. **(6)** Sample Calculations for Kinetic Friction
6. **(2)** Graph from Kinetic Friction
7. **(14)** Answers to questions.
8. **(4)** Conclusion

Data Sheet

Static Friction

Large Smooth Side: Height=__________m Coefficient μ_s=__________

Length=__________m

Small Smooth Side: Height=__________m Coefficient μ_s=__________

Length=__________m

Large Rough Side: Height=__________m Coefficient μ_s=__________

Length=__________m

Small Rough Side: Height=__________m Coefficient μ_s=__________

Length=__________m

Average Smooth Sides Coefficient μ_s=__________

Average Rough Sides Coefficient μ_s=__________

Kinetic Friction

Mass of block (M):__________kg (Except Run #3)

__________kg (Run #3)

Run #	Hanging Mass m (kg)	Acceleration a (m/s^2)	Coefficient μ_k
1			
2			
3			
4			
5			
6			
7			
8			
9			

Chapter 6

Archimedes' Principle

Goal

Today we will preform experiments to determine the density of several unknown objects and an unknown fluid by using the concepts we learned in Chapter 4 and Archimedes' Principle, which we will cover today.

Equipment

- 850 Universal Interface
- PASCO Capstone Software
- Force Sensor and lab pole for mounting the Force Sensor
- Two spheres: one less dense than water, one more dense than water. One 500 g mass. One irregular object of either aluminium or copper.
- Tall can for submerging objects in water.
- Vernier calipers
- String
- Container of a solution other than water at the lecture desk
- Triple-beam balance

Theory

In order to find the density of our various objects we will use *Archimedes' Principle* which states that when a body is completely or partially immersed in a fluid, the fluid exerts and upward buoyant force on the body equal to the weight of the fluid displaced by the body. First we will need to know the weight of the object.

From Figure 6.1 we can see that the net force on the object m is,

$$\Sigma F = T - mg = 0. \tag{6.1}$$

Thus we can identify the true weight of the object as $T = mg$. If we were to immerse our object in water and examine the forces acting on it now we can see from Figure 6.2 we can write,

$$\Sigma F = T' + B - mg = 0, \tag{6.2}$$

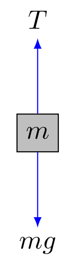


Figure 6.1: Object in Air.

where T' is the new scale reading and B is called a *buoyant force*. By solving Equation (6.2) and substituting in the true weight we have,

$$B = T - T'. \tag{6.3}$$

But we know from Archimedes' Principle that we can write the buoyant force in terms of the weight of the displace fluid,

$$B = m_w g, \tag{6.4}$$

where m_w is the mass of the displaced *water*, not the mass of the object. We can define *density* as the mass per unit volume or,

$$\rho = \frac{m}{V}. \tag{6.5}$$

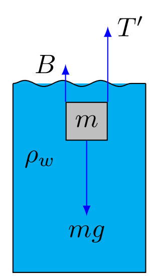


Figure 6.2: Object in Water.

If we use this definition of density we can rewrite Equation (6.4) as,

$$B = \rho_w V g. \tag{6.6}$$

We can use the total volume V of the object in Equation (6.6) because we are assuming that the object is completely submerged. However, given how we will be preforming the experiment, we want all our equations to be in terms of tensions that we can measure with the Force Sensor. To do this we need to look at Equation (6.5) again. If we take Equation (6.5) for the object and solve for V we have,

$$\begin{aligned} V &= \frac{m}{\rho_{obj}} \\ &= \frac{T}{\rho_{obj}\, g}, \end{aligned} \tag{6.7}$$

where in the second line we used the fact that the mass of the object, m, is related to the tension of the string in air, T. Substituting Equation (6.7) into Equation (6.6) and simplifying gives us,

$$B = \frac{\rho_w}{\rho_{obj}} T, \tag{6.8}$$

which when equated to Equation (6.3) and solved for ρ_{obj} gives us our final expression,

$$\rho_{obj} = \frac{T}{T - T'} \rho_w. \tag{6.9}$$

To find the density of the unknown fluid we simply need to take one more step. By using our result from Equation (6.9) and a similar result that derives from Figure 6.3 we should be able to get the following equation for the density of the fluid,

$$\rho_{fl} = \frac{T - T''}{T - T'} \rho_w. \tag{6.10}$$

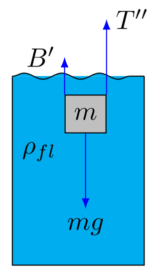


Figure 6.3: Object in Unknown Fluid.

Procedure

1. Arrange the equipment so that the can rests on top of the lift and directly below the Force Sensor. Attach the object to the Force Sensor via a string.
2. Fill the can about $^2/_3$–$^3/_4$ full of water.
3. Measure the diameter of the two spheres with the vernier calipers.
4. Open Capstone. Attach the Force Sensor to the 850UI and setup Capstone to use the Force Sensor in the appropriate port.
5. Calibrate the Force Sensor as we did on page 32, making sure to set the object's weight as a *negative* value.
6. Create a graph in Capstone to display the value of the Force Sensor. Click on the Statistics button  to display the mean of the data.

7. Measure the masses of the two spheres using the scale.
8. Zero the Force Sensor by pressing the tare button.
9. Determine the weight (T) of the the heavier sphere using the Force Sensor. The weight will be the mean value of our force data on the graph.
10. Using the lift, raise the can up towards the Force Sensor until the sphere is completely submerged in the water.
11. Determine the new tension (T') in the string. Click on the Select Data Run button  so that both T and T' values are shown on the graph.
12. Dry the sphere and remove the wet string.
13. Repeat steps 7–12 for the remaining three objects. Print a graph for each object. n.b., to submerge the lighter sphere we need to tie it tightly to the heavier one. Use the two spheres together to measure the forces T and T'.
14. Repeat steps 7–12 for the heavier sphere, but using the unknown fluid at the lecture desk instead of water. *Caution:* The fluid used may be poisonous, such as methyl alcohol or hydrochloric acid. Handle the container with care. Wash and dry the sphere after this step.

Analysis

1. Using the diameter measurements, determine the volume of the two spheres. Remember that the volume of a sphere is given by,

$$V_{sphere} = \frac{4}{3}\pi r^3.$$

2. Determine the densities of the two spheres using the mass and volume.
3. From the tensions measured in procedure steps 9,11, and 13 calculate the densities of the four samples using Equation (6.9). For the lighter sphere we will need to subtract the appropriate corresponding tensions of the heavier sphere from T and T' before calculating the density.
4. Using Equation (6.10) and the measurements from procedure step 14 determine the density of the unknown fluid.
5. Compare the two methods of calculating the density for the two spheres as determined in analysis steps 2 and 3. Using the mass/volume (step 2) result as the standard, give a percent discrepancy for each sphere.
6. Compare the density of the irregular object with either copper (8960kg/m^3) or aluminium (2700kg/m^3) as appropriate for the sample.

7. Compare the density of the unknown fluid with those on the list displayed in the laboratory classroom. Identify the liquid and give a percent discrepancy.

Chapter 6: **Archimedes' Principle (Major Report)**

1. **(5)** Title page (Experiment name, date, author, lab partner names)
2. **(10)** Objective/Goal/Purpose — Should have two parts: What is the overall physics principle or principles we are using and what are we actually determining in this lab.
3. **(25)** Theory — Theory section included both complete sentences and equations. For this report, free body diagrams are required. Name the physics principles we use. Name and define any physical quantities that are used. Derive any equations that we use in our calculations from first principles. List, but do not derive the volume of a sphere formula. Be sure to include any error formulas we use (percent discrepancy).
4. **(10)** Equipment — Make a list of the equipment used and a sketch of the experimental setup. If it is not obvious identify what each part does in the experiment. e.g., what is the paper clip used for?
5. **(10)** Procedure — Give a brief description of each step of the experiment.
6. **(25)** Data & Data Analysis — Present the data in a nicely organized table including units. Make sure the number of significant figures reflects the precision of the measurements. Present the results of the analysis in an organized way, clearly identifying what each number represents. Give a sample calculation for each new quantity that is presented.
7. **(10)** Error Analysis — Discuss at least three sources of experimental error. Human error is *not* an acceptable source. Is the error large or small? Is it systematic or random? Give reasons for each. Also, discuss what could be done to reduce the errors in each case.
8. **(5)** Conclusion — What actual quantities were measured to within what error? (Percent discrepancy.) Were the underlying physical principles verified? (Were the errors small enough?)

Data Sheet

Diameters of spheres:

$d_H =$ ____________ $\pm$ ________ $d_L =$ ____________ $\pm$ ________

Masses of spheres:

$m_H =$ ____________ $m_L =$ ____________

Object	Heavy sphere	Heavy & Light spheres	500 g mass	Irregular object
T (N)				
T' (N)				
T'' (N)		—	—	—

Chapter 7

Energy

Goal

Today we will investigate the Work-Energy Theorem by measuring the work done on an object as well as the change in kinetic energy of the object. We will also investigate the conservation of energy as we turn potential energy into kinetic energy.

Equipment

- 850 Universal Interface
- PASCO Capstone Software
- Force Sensor, Photogate with Pulley, Photogate
- Dynamics track, cart, end stop for track
- Cart-mountable picket fence
- Stand for Force Sensor
- String
- Masses, mass hanger
- Triple-beam Balance
- Phillips screwdriver

Theory

For an object with mass m that experiences a force parallel $F_{||}$ to its displacement $\mathbf{x}$, the work done on the object is,

$$W = F_{||}x = Fx\cos\theta = \mathbf{F}\cdot\mathbf{x}. \tag{7.1}$$

In our experiments, we will assume that F and θ (the angle between the directions of $\mathbf{F}$ and $\mathbf{x}$) are constant; also we will assume that $\mathbf{F}$ and $\mathbf{x}$ will be in the same direction, thus simplifying Equation (7.1) to,

$$W = Fx. \tag{7.2}$$

Using Newton's Second Law and one of the kinematic equations, or a little calculus, we can develop what is called the Work-Energy Theorem,

$$W = \Delta KE = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2. \tag{7.3}$$

This will be left as an exercise in lab.

When studying springs, it is often helpful to use Hooke's Law,

$$\mathbf{F} = -k\mathbf{x}, \tag{7.4}$$

which describes the force on the end of a spring that has been displaced (compressed or stretched) a distance x. (The minus sign implies that the force always points back toward the spring's equilibrium position.) The spring constant k, can be determined experimentally by applying different forces to compress or stretch the spring to different distances. Then if we were to plot the applied force versus the distance, the slope of our graph would be k.

This spring force is a conservative force, meaning that it has a potential energy associated with it. The elastic potential energy of a spring is given by,

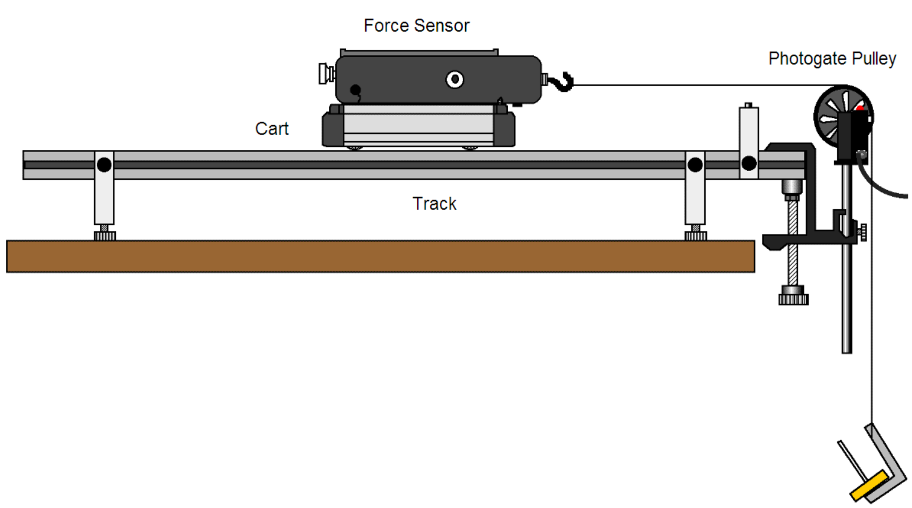
$$PE_{spring} = \frac{1}{2}kx^2. \tag{7.5}$$

If the energy is conserved, i.e., if there are no losses, such as friction, then any decrease in the elastic potential energy in the compressed spring will be completely converted into kinetic energy when the spring pushes against an object of mass m.

Setup I: Work-Energy Theorem

For this activity, the force sensor measures the force applied to the cart by a string that is suspended over a pulley with an object of known mass at the end of the string. The Photogate Pulley measures the motion of a cart as it is pulled. Capstone will display the force applied, and the distance and speed of the object. The program integrates the area under a curve of force versus distance to determine the work done. (Equation (7.2) treats the simple case where F is constant during the whole displacement. Calculus teaches how to extend Equation (7.2) to the case of a varying force by the process called "integration" which is the same as calculating an area.) The program calculates the final kinetic energy. The final kinetic energy is compared to the work done.

1. Start Capstone
2. Connect the Force Sensor and Photogate Pulley to the 850UI.
3. Open the file titled "Energy1" by clicking on . The file should be located in ..\Documents\Phys Lab Files\Phys1154.
4. Measure and record the mass of both the cart and the Force Sensor.
5. Use a #0 Phillips head screwdriver to mount the Force Sensor onto the accessory tray of the cart.
6. Put adjustable feet on both ends of the track. Using the feet, adjust the height to level the track so that cart will not roll one way or another.
7. Put one end stop at the end of the track and place the cart next to the end stop.
8. Mount the Photogate with Pulley on the end of the track so that the top edge of the pulley is approximately the same height as the hook on the Force Sensor that is mounted on top of the cart.



9. In Capstone, click on "Calibration." We will use a two-standard calibration for the Force Sensor.

10. For the first calibration point, press the tare button on the side of the force sensor to zero the sensor. Type in 0 N for the standard value and click "Set Current Value to Standard Value."

11. Use a piece of string that is about 10 cm longer than the distance from the top of the Photogate Pulley to the floor. Connect one end of the string to the Force Sensor's hook and hang the string over the Photogate Pulley.

12. Attach an object of known mass m to the end of the string so that the bottom of the object is just above the floor when the end of the cart is against the end stop.

13. Enter the object's weight in Newtons as the second calibration point as a *positive* value and click "Set Current Value to Standard Value."

14. Press "Next" then "Finish" to complete the calibration. Finally, click "Calibration" to hide the window.

Procedure

1. Pull the cart away from the Pulley so that the object on the end of the string is just below the Pulley. Put enough mass on the hanger so that the cart will move down the track.

2. Turn the pulley so the beam of the Photogate Pulley is not blocked. (The LED on the bottom will turn off when the gate is unblocked.)

3. Click on "Record" to begin data collection.

4. Release the cart.

5. Click on "Stop" to end data collection just before the cart reaches the end stop.

Analysis

1. At the bottom of the Linear Speed/Kinetic Energy Table will be the maximum value of the velocity, record this value.

2. The Kinetic Energy calculation has an arbitrary mass in it. We need to enter *our* mass of the cart and sensor. To do this, click on "Calculator" in the toolbar on the left. The second line should read "mass=0 kg." Enter the correct mass here. Click on "Calculator" again to hide the window. The data will automatically update with the correct mass.

3. At the bottom of the Kinetic Energy column record the maximum value of the Kinetic Energy.

4. On the Force vs. Distance graph, click on the “Display area under active data” button . Record the area as the work done.
5. **Print** the table and graph for each group member.

Question 1. What is the initial kinetic energy of the cart? Why?

Question 2. What is the value for the change in kinetic energy ΔKE?

Question 3. Why is the area under the Force vs. Distance graph interpreted as the work done on the cart by the hanging mass?

Question 4. What is the percent discrepancy between the change of kinetic energy ΔKE and the work done holding ΔKE as the standard?

Question 5. What are the possible reasons for any differences? Discuss any possible sources of error and explain why the sources are large or small.

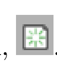
Question 6. In the calibration of the Force Sensor, we chose pulling on the hook to be a positive instead of the usual negative. Why was this a sensible choice for this experiment?

Setup II: Conservation of Energy

For this section we will have a short pre-lab activity before we begin our experiment in earnest. We will use the Force Sensor to measure the force that compresses the plunger spring in the dynamics cart. We will need to measure the amount of distance that the spring compresses and using this information we will plot a graph of force vs distance in Capstone. The slope of the line that our data will make will be the spring force constant, k.

Pre-lab: Determining the Spring Constant.

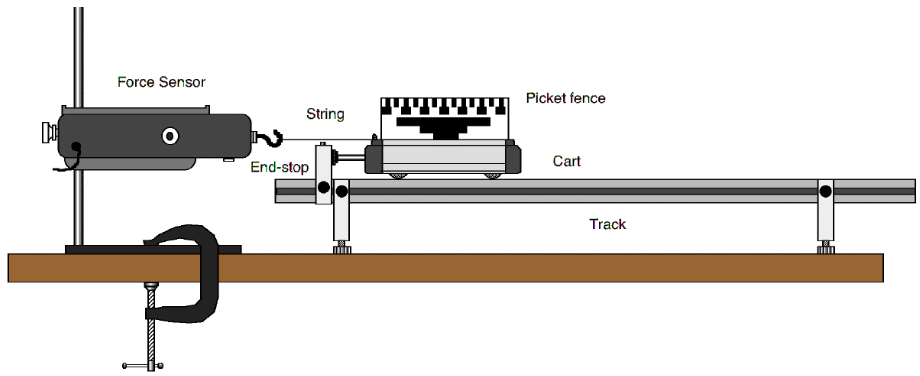
Pre-lab Setup

1. In Capstone delete all data runs from the previous setup.
2. Under “Hardware Setup” remove the Photogate with Pulley by clicking on the icon then pressing “Delete” on the keyboard.
3. Create a new page by clicking the New Page button, . Select the “Table & Graph” template.
4. In the table, for the first column click on “<Select Measurement>” and choose “Force (N).”
5. For the second column, click on “<Select Measurement>” and mouse over “Create New” then choose “User Entered Data.”

6. Name this “Compression” press Tab and choose units of meters by typing “m.” Capstone will automatically understand the units.

7. For the graph create a Force vs. Compression plot by choosing “Force (N)” for the vertical axis and “Compression (m)” for the horizontal axis.

8. Arrange the track and Force Sensor as shown below but do not connect the cart yet.



Pre-Lab Data Collection

1. Put the five-pattern picket fence in the accessory tray of the cart so that the pattern of 1 cm opaque bands is at the top.

2. Measure the mass of the cart plus the picket fence. Record the mass in the data table.

3. Place the cart so the end of its plunger bar is against the end stop. Tie one end of the string to the small hole in the end cap of the cart just above the plunger.

4. Tie the other end of the string to the Force Sensor’s hook.

5. Slide the track itself away from the force sensor so the string is taut and the plunger is against the end stop but the plunger spring is *not* compressed. Make a mark on the edge of the track to indicate the initial position of the end of the cart.

6. In Capstone, next to the Record button click on the button labeled “Continuous Mode” and change it to “Keep Mode.”

7. Click on the Preview button to begin data sampling.

8. Press the tare button on the side of the Force Sensor to zero the sensor.

9. In the table, type in "0" under the Compression column, then click on "Keep" to store the present Force Sensor value.

10. Slide the track away from the Force Sensor so the plunger spring is compressed 5 mm (0.005 m) against the end stop. Use the mark on the track as a reference guide. Make sure the string between the sensor and the cart is kept taut. *Hold* the track in this new position.

11. Enter "0.005" into the table under Compression on the next row and click the Keep button to store the new value.

12. Repeat steps 10–11 moving the track away from the sensor in 5 mm increments until the plunger spring has been compressed 20 mm.

13. Press the Stop button to finish data collection.

Pre-lab Analysis

1. Rescale the data in the graph as necessary.

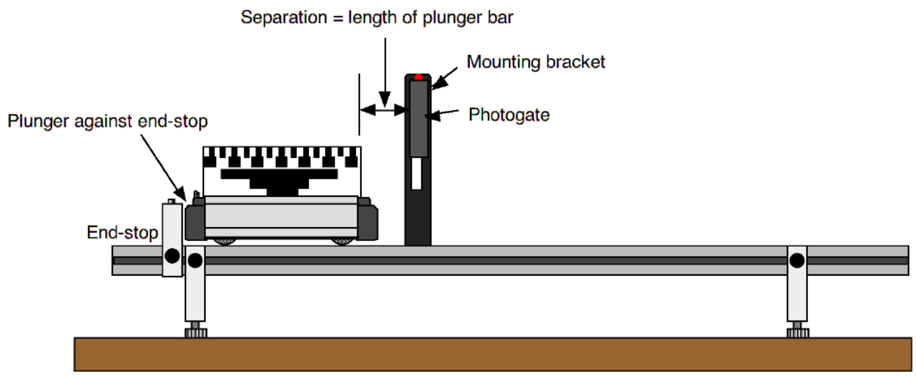
2. Apply a Linear Fit to the data. The slope of this line is the spring constant k that we are looking for. Record this value in the data table.

3. **Print** a copy of the table and graph for each member of the group.

4. Remove the string from the Force Sensor's hook. Disconnect the Force Sensor and wrap the cord. Set the sensor aside.

Now, for the experiment itself we will use the Photogate to measure the motion of the cart just after it is pushed by the plunger spring as it relaxes. Using Capstone we can then calculate the initial velocity and maximum kinetic energy of the cart. Finally, we will be able to compare our measured kinetic energy against the calculated elastic potential energy.

1. Setup the track, cart, and Photogate as shown below. Put the cart on the track so the end of the plunger bar is pressed against the end stop but the spring is not compressed. Position the Photogate so that it is as close as possible to the edge of the picket fence.



2. Adjust the height of the Photogate so the beam will be blocked by the pattern of 1 cm opaque bands at the top edger of the picket fence.

3. Connect the Photogate to the 850UI.

4. Start a new experiment in Capstone . When prompted to save changes, choose "Discard."

5. Under "Hardware Setup" add the Photogate to the appropriate port.

6. Click on "Timer Setup" and choose "Pre-Configured Timer" for the Photogate, Ch 1.

7. Select "Picket Fence," make sure that "Speed" is checked.

8. For the flag spacing make sure to change the value to 0.01 m.

9. Click "Next" then "Finish." Finally, click on "Timer Setup" again to hide the window.

10. We now want to setup a calculated value of kinetic energy based on our velocity measurements of the cart. Click on "Calculator" on the left.

11. In the first line type "Kinetic Energy = 0.5*mass*[Speed (m/s)]ˆ2" and give it units of "J." (Once "[" is typed we can simply select "Speed" and Capstone will link the appropriate data set for the calculation.)

12. The second line should now read "mass =." Type in the mass (in kg) of the cart plus the picket fence into this line and give it units of "kg."

13. To make sure that our values are displayed properly, we will need to increase the display precision. To do this, click on the line containing our kinetic energy equation. Now, click on the properties button . In the window that opens, under "Numerical Format" increase the "Number of Decimal Places" value to 3.

14. Click on "Calculator" again to hide the window.

15. Double-click on "Graph" in the toolbar on the right.

16. Create a Speed vs. Time graph.

17. Now to create a second plot below our Speed vs. Time graph, click on the "Add New Plot Area" button . Select our "Kinetic Energy (J)" calculation for the vertical axis on the bottom graph.

Procedure

1. Measure the length of the plunger relative to the end cap of the dynamics cart. Record the plunger length x in the data table. n.b., When the cart is placed so the end of the plunger bar is pressed against the end stop, we can measure the length of the plunger by measuring the distance from the end stop to the cart.

2. Completely compress the plunger spring on the cart and lock the spring in position by pushing the end of the plunger bar upward slightly so that one of its notches will catch on the metal rail inside the cart.

3. Put the plunger end of the cart against the end stop. The separation between the edge of the picket fence furtherest away from the end stop and the Photogate should be the length of the plunger bar when the spring is not compressed.

4. Click on the Record button to begin data collection.

5. Use the end of a pencil or similar object to tap down on the plunger release button. The cart will be pushed away from the end stop by the plunger spring.

6. Click on the Stop button to end data collection.

7. Repeat steps 2–6 two more times for a total of three runs.

8. Add additional mass to the cart and update the mass in the calculator. Usually the slotted masses can fit under the picket fence block in the accessory tray of the cart. The Photogate height may need to be adjusted so that it is still blocked by the 1 cm opaque bands. Record the mass of the cart, picket fence, and additional masses in the data table.

9. Repeat steps 2–6 three more times for a total of six runs altogether.

Analysis

1. Select Data Run #1 by clicking on the "Select Data Run" button .

2. Click in the Kinetic Energy vs. Time graph.

3. Using the Statistics button , display the Mean and Standard Deviation. Record these values in the data table.

4. **Print** a copy of this graph for each group member.

5. Hide Run # 1 and show Run #2. Record the Mean and Standard Deviation in the data table.

6. Repeat step 5 for each data run.

7. Calculate the average kinetic energy from the results in the data table.

8. Calculate the elastic potential energy using Equation (7.5).

Question 7. What is the percent difference between the average kinetic energy and the elastic potential energy?

Question 8. Which energy was larger in each case, the elastic potential energy or the kinetic energy of the cart?

Question 9. What are possible reasons for the differences, if any?

Question 10. When the mass of the cart was increased, did the kinetic energy change? Should it change? Explain.

Question 11. How constant is the kinetic energy? (How does the standard deviation compare to the mean?)

Chapter 7: **Energy**

1. **(1)** Title Page
2. **(3)** Purpose
3. **(8)** Theory for Setup I (Work-Energy Theorem).
4. **(4)** Procedure for Setup I. Hit the highpoints.
5. **(4)** Data, Tables/Graphs, for both setups..
6. **(22)** Answers to all questions.
7. **(8)** Conclusion for Setup I. What were the goals of the experiment? Were they achieved? To within what error? Were there any major problems/errors? (n.b., human error is not an acceptable response. Be creative.) What could be done differently to improve or expand this experiment?

Data Tables

Setup I

Item	Value
Mass (cart & sensor)	kg
v_f (maximum)	m/s
KE_{max}	J
Work (F vs. x)	J

Setup II

Item	Value
Mass (cart & fence)	kg
Spring Constant k	N/m
Plunger Length x	m
Mass (cart, fence, & extra mass)	kg

Run #	1	2	3	4	5	6	Average
KE (J)							
Std. Dev. (J)							

Elastic Potential Energy $= PE_{elastic} = \frac{1}{2}kx^2 =$ _________ J

Chapter 8

Impulse & Momentum

Goal

Today we will study an elastic collision and measure the change in momentum during the collision as well as the impulse. Then we will investigate Newton's Third Law by looking at the forces acting on two carts as they interact during a collision.

Equipment

- 850 Universal Interface
- PASCO Capstone Software
- Motion Sensor, two Force Sensors
- Dynamics Track, two carts with rubber bumpers
- Lab post for mounting Force Sensor
- Mass hanger, masses
- Triple-beam balance
- Mass bar

Theory

In a similar way that we developed the Work-Energy Theorem last time, we can develop what is called the Impulse-Momentum Theorem. First though, let's define what we mean by momentum. *Momentum* is the product of mass and velocity,

$$\mathbf{p} = m\mathbf{v}. \tag{8.1}$$

Under a collision an object will undergo a change in momentum, which requires a force applied on the object. We can relate an applied force at a specific instant of time to the change of momentum through the following,

$$\mathbf{F}_{net} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d}{dt}(m\mathbf{v}) = \frac{d\mathbf{p}}{dt}, \tag{8.2}$$

which as it turns out is how Newton originally stated his second law of motion in *Principia*. As usual in calculus notation, $d\mathbf{p}$ and dt refer to infinitely small changes of $\mathbf{p}$ and t. Using this we can now define the impulse of the force over a finite interval of time Δt as,

$$\mathbf{J} = \mathbf{F}_{net}\Delta t, \tag{8.3}$$

provided $\mathbf{F}_{net}$ is a constant force. Using what we know from Equation (8.2) we can show that,

$$\mathbf{F}_{net} = \frac{\Delta\mathbf{p}}{\Delta t},$$

which if we substitute into Equation (8.3),

$$\mathbf{J} = \mathbf{F}_{net}\Delta t = \Delta\mathbf{p} = m\mathbf{v}_f - m\mathbf{v}_i, \tag{8.4}$$

a specific case of the Impulse-Momentum Theorem. For a generalized Impulse-Momentum Theorem we need to include cases where the net force is not constant. To do this we simply integrate both sides of Equation (8.2) with respect to time,

$$\int_{t_0}^{t} \mathbf{F}_{net}\, dt = \Delta\mathbf{p}. \tag{8.5}$$

n.b., for those without calculus training, for the purposes of this class to take an integral of a function means to find the area between the curve (in this case $F_{net}(t)$ vs t) and the horizontal axis of our graph.

For the second part of the experiment we will be studying Newton's Third Law. This is simply stated as,

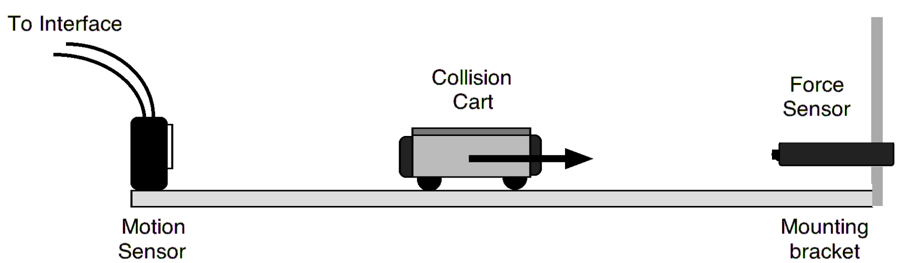
$$\mathbf{F}_{12} = -\mathbf{F}_{21}, \tag{8.6}$$

meaning that the force of object 1 on object 2 is equal in magnitude but opposite in direction to the force of object 2 on object 1.

Setup I: Colliding one cart into a Force Sensor.

For this activity, the motion sensor will measure the motion of a cart before and after it collides with a bumper that is mounted on the front of a force sensor. The force sensor will measure the force during the collision. Capstone calculates the velocity of the cart before and after the collision, and the integral of force as a function of time during the collision.

1. Connect the Motion Sensor and Force Sensor to the 850UI.
2. Start Capstone.
3. Under “Hardware Setup” add the Motion Sensor and Force Sensor to the appropriate ports.
4. In the toolbar at the bottom of the window next to the timer, change the focus to the Force Sensor by selecting it in the drop-down box. Now, increase the sampling rate to 500 Hz.
5. Calibrate the Force Sensor as we did on page 32, making sure to set the object’s weight as a *negative* value.
6. Carefully measure the mass of the cart and record it in the data table.
7. Create a calculated result for the momentum of the cart. To do this, click on the “Calculator” button on the left.
8. In the first line type “Momentum = mass*[Velocity (m/s)]” and give it units of “kg*m/s” (Once “[” is typed we can simply select “Velocity” and Capstone will link the appropriate data set for the calculation.)
9. The second line should now read “mass =.” Type in the mass (in kg) of the cart into this line and give it units of “kg.”
10. Click on “Calculator” again to hide the window.
11. Create a graph of Force vs. Time.
12. Create a new plot area below the Force vs. Time plot by clicking . Choose Momentum as the vertical axis measurement for this plot.
13. Mount the Force Sensor horizontally as shown below. Make sure that the rubber bumper is on the end of the Force Sensor’s hook.



14. Raise the end of the track that is opposite to the end with the Force Sensor about 1.5 cm so the cart will have roughly the same initial velocity each trial at the moment it first contact the force sensor.
15. Place the Motion Sensor at the raised end of the track so that it can measure the motion of the cart.

16. Brace the Force Sensor end of the track so that the track will not move during the collision.

Procedure

1. Press the tare button on the side of the Force Sensor to zero the sensor.
2. Place the cart on the track at least 20 cm from the front of the Motion Sensor.
3. Click on Record to begin data collection and at the same time release the cart so that it rolls down towards the Force Sensor.
4. Click on Stop to end the data collection after the cart has rebounded *once* from the collision with the Force Sensor's bumper. n.b., it is best to allow the cart to begin to roll back up the track before ending data collection.

Analysis

1. Scale the data to fit the graph.
2. On the Force vs Time graph, click on the Selection Tool, . Move and resize the rectangle to only highlight the spike, the region of the graph that corresponds to the collision.
3. Press the scale to fit button, , this will scale the graph to only show the points inside the Selection Tool. Resize the rectangle as needed so that only the spike is included.
4. Click on the "Display area under active data" button, . Record the area in the data table.
5. On the Momentum vs Time graph, click on the Selection Tool,  and move the rectangle to highlight the collision which should occur at the same times as it did in the Force vs Time graph. (It isn't critical to have the rectangle perfect for this graph, so long as the collision is included inside the Selection Tool.
6. Under Statistics , check the"Minimum" and "Maximum" choices and uncheck everything else. Finally click the Σ to display the information.
7. **Print** a copy of the graph for each group member.
8. Using the information from the Momentum vs Time graph, calculate the change in momentum.

9. Calculate the percent discrepancy between the impulse and the change in momentum, holding the impulse as the standard.

Question 1. What is the unit of impulse given in the Force vs Time graph? What is the unit of momentum given in the Momentum vs Time graph? Show that these units are equivalent.

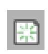
Question 2. Why do the maximum and minimum momenta have opposite signs?

Setup II: Collision of Two Carts.

For this activity, Force sensors are attached to two carts. The carts are placed on a track, and the Force Sensors measure the force of interaction during collisions. Capstone displays Force vs Time for both sensors simultaneously. We will compare the area under the Force vs Time curve for one Force Sensor with the area above the Force vs Time curve for the second Force Sensor.

1. Disconnect the Motion Sensor, wrap the cord and put aside.

2. Connect a second Force Sensor to the 850UI.

3. Click the New Page button  and create a graph.

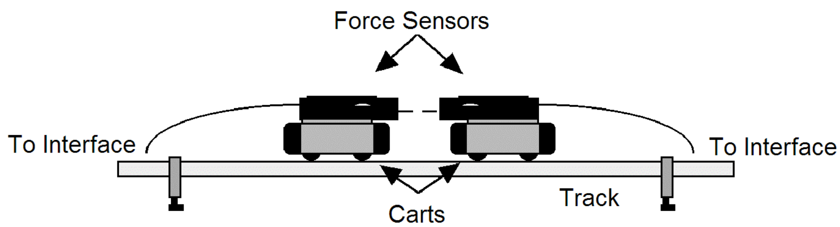
4. We will need to display the data from both Force Sensors simultaneously on this graph. Click on the Add New y-axis to plot button, .

5. Select "Force, Ch A (N)" for one of the axes and "Force, Ch B (N)" for the other.

6. As we have already calibrated the Force Sensor in Channel A, we only need to calibrate the sensor in Channel B. Calibrate this sensor as we did before (see page 32) however this time make sure that the weight of the mass is recorded as a *positive* value.

7. Under "Data Summary" in the toolbar on the left, click on the line labeled "Force, Ch A (N)" and then click on the Properties button . In the "Numerical Format" section, change the Number of Decimal Places to 4. Click "OK" to close the window. Repeat this step for "Force, Ch B (N)."

8. Set up the carts and track as shown below. Make sure that the rubber bumpers are attached to the hooks of the sensors.



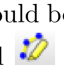
Newton's Third Law: Collisions

Procedure I

1. For this experiment it is important to zero the sensors by pressing the tare button before we begin data collection each time.

2. Move the carts to opposite ends of the track.

3. Click on the Record button. Push the two carts together, allowing them to collide at approximately equal speeds near the center of the track. Click the Stop button to end the data collection.

4. On the graph there should be two narrow spikes representing the collision. Use the Selection Tool  to highlight one of the peaks. As we did in Setup I, use the Scale to Fit  to help trim the selection so that only the peak is selected. Click the 'Display area under active data" button, .

5. Repeat step 4 for the other peak. To switch between data sets, simply click the icon in the legend.

6. If both peaks look reasonably similar and have minimums of 0 N, then proceed to the analysis. If not, for example there are multiple peaks or one sensor has minimum readings that are not zero., then remove the data and repeat steps 1–5.

Analysis

1. Record the areas from both peaks in the data table.

2. **Print** the graph for each group member.

3. Calculate the average impulse readings and enter the result in the data table.

4. Calculate the *relative* discrepancy. This compares how individual readings differ from the average. Since there are only two readings they have the same relative discrepancy. The equation is defined on page 134 but we will repeat it here for convenience.

$$\text{Relative discrepancy} = \frac{|\text{experimental value} - \text{average value}|}{\text{average value}} \times 100\%$$

Procedure II

1. Add a mass bar to the cart connected to the Force Sensor in channel A.
2. Repeat the data collection and analysis from above. Do not print a graph for this run but enter the data and calculations in the data table.

Procedure III

1. Keep the mass bar on the Channel A cart.
2. Repeat the data collection and analysis from above. However, this time leave the heavier cart at rest near the middle of the track and only move the lighter cart. Do not print a graph for the run but enter the data and calculations in the data table.

Procedure IV

1. Keep the mass bar on the Channel A cart.
2. Repeat the data collection and analysis from above. However, this time leave the lighter cart at rest near the middle of the track and only move the heavier cart. Do not print a graph for the run but enter the data and calculations in the data table.

Question 3. Why is the area under the Force vs Time graph the impulse?

Question 4. Which cart experiences more force when both are moving with equal masses?

Question 5. Which cart experiences more force when one cart remains at rest? Do the masses of the carts matter? What are the relative discrepancies?

Chapter 8: **Impulse & Momentum**

1. **(1)** Title Page
2. **(6)** Purpose of both setups.
3. **(12)** Theory — Should cover both setups.
4. **(4)** Data sheets
5. **(3)** Graphs
6. **(8)** Sample calculations for both setups
7. **(10)** Answers to all questions.
8. **(6)** Conclusion of both setups.

Data Tables

Setup I

Item	Value
Mass of cart	kg
Impulse	N s
Momentum before collision	kg m/s
Momentum after collision	kg m/s
Percent Uncertainty	%

Setup II

Run #1: Two equal mass carts.

Impulse of cart #1	N s
Impulse of cart #2	N s
Average Impulse	N s
Relative Discrepancy	%

Run #2: Cart #1 heavier, equal speeds.

Impulse of cart #1	N s
Impulse of cart #2	N s
Average Impulse	N s
Relative Discrepancy	%

Run #3: Cart #1 heavier and at rest.

Impulse of cart #1	N s
Impulse of cart #2	N s
Average Impulse	N s
Relative Discrepancy	%

Run #4: Cart #1 heavier, lighter cart at rest.

Impulse of cart #1	N s
Impulse of cart #2	N s
Average Impulse	N s
Relative Discrepancy	%

Chapter 9

Centripetal Force

Goal

Today we will examine the force that causes an object to follow a circular path. In our first experiment, we will create a centripetal force by whirling around at constant angular velocity an object attached to a spring. We will compare this centripetal force to the force needed to stretch the spring itself. In our second experiment, we will plot the centripetal force on a pendulum bob using a Force Sensor then compare it to the force that we will calculate from the speed of the pendulum as measured by a Photogate.

Equipment

- 850 Universal Interface
- PASCO Capstone Software
- Centripetal Force Apparatus
- Masses and mass hanger
- Meter Stick
- Timer
- Triple-beam Balance
- Force Sensor and Photogate
- Lab post, rod and clamps for Force Sensor
- Pendulum bob and string

Theory

In order to have an object follow a curved path we need to have an acceleration towards the center of the circle. We call this center-seeking acceleration a centripetal acceleration. The magnitude is given by,

$$a_c = \frac{v^2}{r}. \tag{9.1}$$

Let's quickly prove this. We know that an acceleration is defined as a change in velocity,

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i}. \tag{9.2}$$

As the triangle in Figure 9.1a is similar to the triangle in Figure 9.1b the ratios of their sides will be equal,

$$\frac{\Delta v}{v} = \frac{\Delta s}{r}.$$

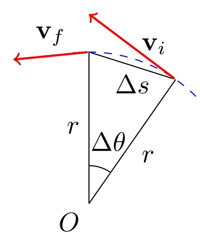
Therefore we can solve for Δv,

$$\Delta v = \frac{\Delta s}{r} v. \tag{9.3}$$

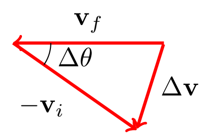
But $\Delta s = v\Delta t$ so if we substitute this and Equation (9.3) into (9.2) we have,

$$a_c = \frac{\Delta v}{\Delta t} = \frac{\Delta s}{\Delta t}\frac{v}{r} = \frac{v^2}{r},$$

which is exactly Equation (9.1).



(a)



(b)

Figure 9.1

Unfortunately, due to how we are preforming the experiment in Setup I, Equation (9.1) isn't useful to us in it's current form. We need to convert our equation into terms that we can directly measure. First let's redefine our linear velocity v in terms of angular velocity ω,

$$v = \omega r. \tag{9.4}$$

Angular velocity is the change of angular position (angle θ) with time. By substituting Equation (9.4) into (9.1) we have,

$$a_c = \omega^2 r. \tag{9.5}$$

The SI units of angular velocity are radians per second which unfortunately is not a unit that we can easily measure by eye so we need one more conversion. We will convert our units of angular velocity from rad/s to revolutions per second as for every one revolution we have displaced 2π radians. Therefore we can write our conversion as,

$$\omega = 2\pi n, \tag{9.6}$$

where n is the angular velocity in rev/s. Finally if we substitute Equation (9.6) into (9.5) we have our final expression,

$$a_c = 4\pi^2 n^2 r. \tag{9.7}$$

Equation (9.2) and Figure 9.1b show that $\mathbf{a}_c$, like $\Delta\mathbf{v}$, points in towards the center of the circle. Since we are interested in the centripetal *force*, we simply need to multiply Equation (9.7) by the mass of our object m,

$$F_c = ma_c = 4\pi^2 n^2 mr. \tag{9.8}$$

In the second setup (See page 84), a pendulum bob swings through a Photogate. We don't need any of the conversions we just developed because our sensors will be able to measure v directly. We do however need to be aware of all the forces at play.

As the pendulum swings, the centripetal force in the string causes the bob to follow a circular path. At the bottom of the swing the net force on the bob is the combination of the tension in the string and the weight of the object. Therefore we can write our net force equation based off Figure 9.2 as,

$$F_c = \Sigma F = T - mg = ma, \tag{9.9}$$

where T is the tension in the string, m the mass of the bob, and F_c the centripetal force given by,

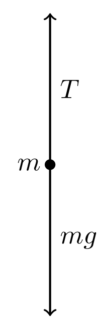


Figure 9.2

$$F_c = \frac{mv^2}{r}. \tag{9.10}$$

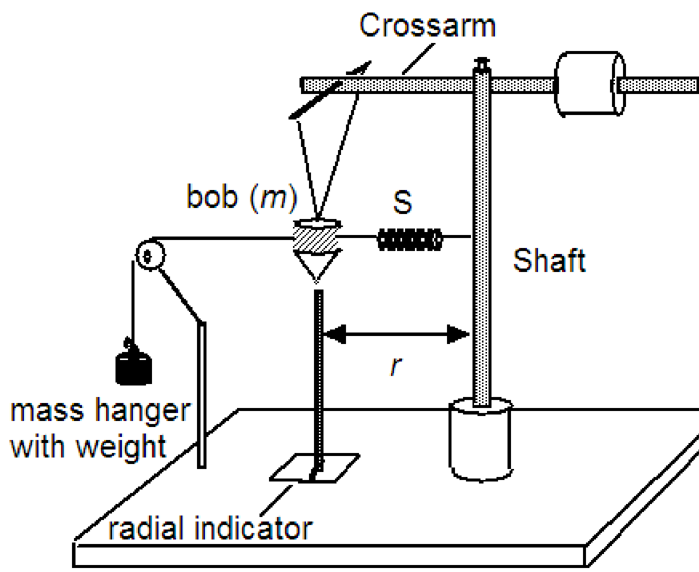
Setup I: Spring applied Centripetal Force

In the experiment apparatus, the centripetal force is applied to the bob (of mass m) by a spring, 'S,' attached between the mass and the shaft. The rotation causes the spring to stretch. The force necessary to stretch the spring the same amount can be measured directly using Hooke's Law, which states that the force needed to stretch (or compress) a spring is proportional to the amount, x, the spring is stretched (or compressed),

$$F_s = -kx,$$

where k is called the spring constant and is a measurement of the stiffness of the spring.

We will be rotating the mass in a circle of radius r and determining its angular velocity by counting the number of revolutions per second. Using Equation (9.8) we will calculate the centripetal force F_c. We will also determine



the force F_s by attaching a hanging mass to the spring that will stretch to the equivalent distance. The forces F_c and F_s should have equal values within measurement uncertainty.

Procedure

1. Remove the bob (the rotating mass) and determine its mass.

2. Adjust the radial indicator's position so that it is closest to the shaft.

3. Adjust the crossarm at the top of the shaft so that the bob—without the spring attached—hangs over the radial indicator. This is extremely important since when we begin rotating, the string must be vertical so that it only supports the weight of the bob. If the string is not vertical during rotation, the tension in the string would add an unknown horizontal component of force on the bob, and would invalidate our calculations.

4. Attach the spring between the bob and the shaft.

5. Attach a mass hanger with a string and hook to the opposite side of the bob.

6. Add enough mass to the hanger to cause the bob to realign with the radial indicator and record this mass, including the mass of the hanger. The weight of this mass is F_s. This should also be the same force that the spring will apply to the bob under rotation.

7. As we are making all of our measurements by hand, we will also need to determine the uncertainty of our readings by hand as well. The total uncertainty will be the inherent uncertainty of the masses themselves plus the far greater uncertainty from the sensitivity of the apparatus to changes of mass on the hanger. To determine this, add mass in very small increments (e.g., 1 g increments) and determine the minimum mass necessary to cause a barely noticeable movement of the bob away from the exact alignment with the radial indicator. This uncertainty will not be the same for different masses so we will need to measure it each time we move the radial indicator.

8. Remove the mass hanger from the string and wrap the string up so that it will be out of the way when we rotate the bob.

9. Rotate the shaft until the bob is again lined up with the radial indicator as it moves around in a circle.

10. Record the time for 20 revolutions over the radial indicator using a timer. Do this three times and calculate the average and standard deviation of the angular velocity n. The formula for standard deviation can be found on page 130.

11. To test the affect of a greater rotating mass, add 100 g to the bob, then repeat steps 9–10.

12. Measure the distance r from the center of the shaft to the center of the radial indicator.

13. Move the radial indicator so that it is midway to its furthest position then repeat steps 3–12.

14. Move the radial indicator to its furthest position from the shaft and repeat steps 3–12.

Analysis

1. Calculate the force F_c using equation (9.8) for all six trials.

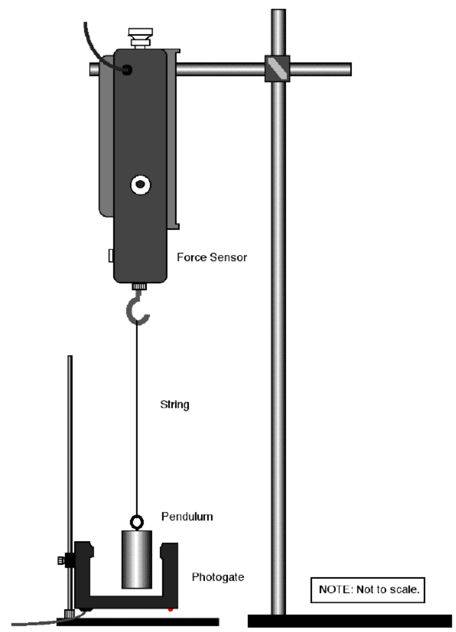
2. Calculate the average of the two forces F_c (for different masses of the bob—see Procedure, step 11) at each radius.

3. Compare the average values of F_c found in the step above to F_s for the corresponding radius by calculating the percent discrepancy using F_s as the standard.

Question 1. When the mass of the bob was increased by 100 g and the radius was held constant did the speed vary appropriately? In what way?

Setup II: Centripetal Force on a Pendulum.

For this setup, the Force Sensor will measure the centripetal force on a pendulum bob as it swings back-and-forth. The Photogate will measure the time that the pendulum bob blocks the Photogate beam. We will need to enter the value for the diameter of the pendulum bob. Capstone will calculate and display the speed of the pendulum bob and the centripetal force on the pendulum. We can then use this data to calculate centripetal force based on the measured speed of the pendulum bob. Finally we can compare the calculated and measured values of the centripetal force.



1. Start Capstone.
2. Attach a Photogate and a Force Sensor to the 850UI and add the sensors to the appropriate ports in Capstone.
3. Calibrate the Force Sensor as we did on page 32.
4. We need to set up the Photogate timer for our experiment. Click on the "Timer Setup" button on the left.
5. We will be using a Pre-Configured Timer. When Capstone asks us what type of timer we would like to create, select "One Photogate (Single Flag)."
6. Check "Speed" and uncheck all other options.
7. Measure the diameter of the pendulum bob and enter it in Capstone as the Pendulum Width.
8. Name the timer as "Tangential Speed," click Save, and then click "Timer Setup" to hide the window.
9. Create a Graph display and add a second plot area to the display 
10. For the upper plot area select "Force (N)" then select "Speed (m/s)" for the lower plot area.

Procedure

1. Measure the mass (m) of the bob and record it in the data table.

2. Use a piece of string that is about one meter long. Tie one end of the string to the Force Sensor hook and the other to the pendulum bob.

3. Position the Photogate so that the bob blocks the beam when at rest. Adjust the length of the string so that the center of mass of the bob is a the same height as the Photogate's beam.

4. Measure the length (r) of the pendulum from the Force Sensor's hook to the bob's center of mass. Record this length in the data table.

5. When all the equipment is set up, check the alignment by pull the pendulum bob straight to the side about 15–20 cm. Carefully release the bob so that it swings through the Photogate as smoothly as possible. Adjust the Photogate position as necessary.

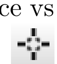
6. Before we begin data collection, press the tare button on the Force Sensor while the pendulum bob is *at rest* and hanging *straight downwards.*

7. Pull the pendulum bob to the side, click on the Record button and release the bob. Record data for at least 20 seconds then click Stop.

8. The graph will display plots of the pendulum's centripetal force and tangential velocity. The force plot should be sinusoidal while the velocity plot should be linear. If this is not the case, delete the data run and try again.

9. **Print** the graph for each group member.

Analysis

1. Click on the Force vs Time graph and rescale to fit the data. Click on the Coordinate Tool  and move it to the first or second peak of the plot. Record the value of the centripetal force as shown by the tool.

2. We want to measure the tangential velocity at the same time at which the force is being measured. Click on the Speed vs Time graph and use the Coordinate Tool to select the data point that occurs at the same time as the data point we have selected in the Force vs Time graph. Record the value of the speed at this data point.

3. Repeat steps 1–2 for four more peaks on the Force vs Time plot.

4. For each tangential speed value calculate the centripetal force using Equation (9.10).

5. Calculate the percent difference between the measured centripetal force and the calculated centripetal force for each point.

Question 2. Why is it important to tare the Force Sensor before we started data collection for this experiment?

Question 3. What are the possible reasons for the differences between the measured and calculated values of centripetal force? Is each possible source of error big or small? Does it have a significant effect on the data?

Chapter 9: **Centripetal Force**

1. **(1)** Title Page
2. **(4)** Purpose
3. **(10)** Theory of both setups
4. **(5)** Procedure of Setup I
5. **(8)** Data Sheet of Setup I
6. **(5)** Data Processing Sheet for Setup I. Show sample calculations.
7. **(1)** Graph from Setup II
8. **(6)** Data Processing Sheet for Setup II. Show sample calculations.
9. **(6)** Answer all questions
10. **(4)** Conclusion

Data Tables

Setup I

Data Sheet

Be sure to include appropriate units.
Mass of bob______________
Position 1: $r_1 =$ ______________
Mass on spring (including hanger) $m_s =$ ______________ $\pm$ ________

Bob alone. Remember $n_i = 20/t_i$.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________

Bob + 100 g.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________
Position 2: $r_2 =$ ______________
Mass on spring (including hanger) $m_s =$ ______________ $\pm$ ________

Bob alone.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________

Bob + 100 g.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________
Position 3: $r_3 =$ ______________
Mass on spring (including hanger) $m_s =$ ______________ $\pm$ ________

Bob alone.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________

Bob + 100 g.

$n_1 =$ ______________, $n_2 =$ ______________, $n_3 =$ ______________

$n_{average} =$ ______________ $\pm$ ________

Data Processing

Radius	$F_s (= m_s g)$
r_1	
r_2	
r_3	

Radius	F_c Bob only
r_1	
r_2	
r_3	

Radius	F_c Bob + 100 g
r_1	
r_2	
r_3	

Radius	Average F_c for both masses
r_1	
r_2	
r_3	

Setup II

mass of pendulum bob $m =$ ______________ kg

length of pendulum $r =$ ______________ m

Peak	F_c Measured (N)	v (m/s)	F_c Calculated (N)	% difference
1				
2				
3				
4				
5				

Chapter 10

Torque

Goal

Today we will investigate the angular counterpart to force, torque. In our experiments we will see how the placement of the applied force can affect the torque on an object and how torque on an object affects the rotational dynamics on that object.

Equipment

- PASCO Capstone Software
- 850 Universal Interface (850UI)
- Pegboard levers
- Masses
- Meter stick
- String
- Support frame
- Torque problem at lecture table
- Rotary Motion Sensor (RMS) and small pulley to be mounted on RMS
- Silver Disk, White Disk
- Triple-beam Balance

Theory

When we apply a force on an elongated object a distance r from some point of rotation we will create a torque on the object. As in Figure 10.1 the point that the body rotates around is called the fulcrum. The force F on the object is applied along a line we call the "Line of action." The distance r is called the lever arm which is always measured *from* the fulcrum *to* the line of action of the force. In general the torque vector of the force with respect to the fulcrum is defined as,

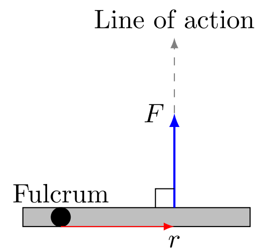


Figure 10.1

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, \tag{10.1}$$

where this is the vector product of the vectors $\mathbf{r}$ and $\mathbf{F}$. One of the properties of vector products is that the resultant must always be *orthogonal* or mutually perpendicular to both parent vectors, so we can handle the direction and magnitude of our torque separately. The magnitude of torque is,

$$\tau = rF \sin\theta, \tag{10.2}$$

where θ is the angle between the vectors $\mathbf{r}$ and $\mathbf{F}$. For most of our experiment $\theta = \pi/2$ so Equation (10.2) simplifies to,

$$\tau = rF. \tag{10.3}$$

To determine the direction of the torque we use what is known as the "right-hand rule." This simple mnemonic is true for all vector products and is setup as follows: The index finger of the right hand is pointed in the direction of the first vector of the product ($\mathbf{r}$), the middle finger is pointed in the direction of the second vector ($\mathbf{F}$), then the thumb will be pointing in the correct direction of the resultant vector ($\boldsymbol{\tau}$).

It is worth to note that for a body that is in translational and rotational equilibrium, meaning that the the sums of the forces and the torques are both zero, the point that we choose for the fulcrum is arbitrary. We can choose any point on the body and the sum of the torques will still be zero. This is extremely advantageous for us because we can then choose a fulcrum that makes our calculations easier.

To study dynamic toque we first need to expand on what we stared in Chapter 9. We had from equation (9.4),

$$v = \omega r. \tag{10.4}$$

We also know that acceleration is the change in velocity, thus we can write our tangential acceleration in terms of angular acceleration α,

$$a_{tan} = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha. \tag{10.5}$$

Now we can return to equation (10.2) and identifying that $F \sin\theta$ picks out the component of the applied force tangential to the angular motion we have,

$$\tau = rF_{tan} = r(ma_{tan}) = rm(r\alpha) = mr^2\alpha. \tag{10.6}$$

Since our body is a rigid object we can consider it as a collection of small pieces. Each piece of mass m_i will rotate around a circle centered on the axis of rotation with the torque on each particle given by equation (10.6). Thus the net torque on the entire body will be the sum of the individual torques,

$$\sum \tau = \left(\sum m_i r_i^2\right) \alpha = I\alpha, \tag{10.7}$$

where we define $I = \sum m_i r_i^2$ to be the moment of inertia of the body and describes how mass is distributed around a rotating body. We will explore the concept of moments of inertia in more detail in Chapter 11.

Setup I: Static Torque

Procedure

1. Support the pegboard lever at the center by hanging it on a string attached to a hook ring. Verify that the board is approximately balanced in horizontal position at that point.

2. Attach a 200 g mass to the third hole from the center using the lower line of holes. Calculate the mass needed at the sixth hole from the center on the opposite side to again balance the board. Hang that mass from the board and test for balance. Remember that there are uncertainties involved and make a list of their causes.

3. Remove the last mass added and calculate what mass must be added at the fourth hole from the center on the side opposite the 200 g mass to re-balance the board. After testing the result, derive a simple formula that works for any situation.

4. Remove all added masses and move the supporting string to the sixth hole from the center on the top row of holes. Add enough mass at the eighth hole on the same side to balance the lever board. Recall that the weight of the board can be approximated to a point at the center, use the formula derived in the previous step to determine the mass of the board.

5. Use the triple-beam balance to measure the mass of the lever board.

6. At the lecture desk is a rod and ball setup that is in rotational equilibrium. Record the scale values (in the proper units) and angles of the supporting strings using the protractor available. Make all length measurements deemed necessary to determine the mass of the bar and the mass of the ball hanging from the bar. Assume the center of the mass of the bar is at the center of the bar.

Analysis

1. Determine the relative discrepancy in the mass of the lever board by comparing the results from steps 4–5.
2. Determine the mass of the bar and the ball hanging from the bar.

Question 1. Explain why door handles are typically placed furthest from the hinges.

Question 2. Work is defined by the product of force and displacement while torque is defined by the product of force and distance. Are work and torque equivalent? Explain why or why not.

Setup II: Dynamic Torque

For this setup, we will use the Rotary Motion Sensor (RMS) to measure the angular acceleration of our system which when combined with other measurements will allow us to determine the torque on the system.

1. Open Capstone.
2. Connect the RMS to the 850UI then setup Capstone to record from the sensor (click on the left-most port that the sensor is plugged into).
3. Create a graph of Angular Acceleration vs Time.
4. Mount the RMS on the vertical rod in the table so that the disks will be on top.
5. Tie the string around the smallest, top step of the clear plastic RMS pulley. Then, feed the string down through the notch into the middle step and wrap the string around the middle step.

Procedure

1. Measure the mass of the silver disk and record it. Measure the diameter of the silver disk three different times from different locations on the disk and record the average.
2. Attach the silver disk to the RMS.
3. Lay the string over the black pulley at the end of the RMS.

4. Attach the 50 g mass hanger (feel free to experiment with extra masses but 50 g should do the job) to the string.

5. Click the Record button to start data collection and release the hanging mass.

6. Stop data collection when the hanging mass is at its lowest point.

7. Use the Selection Tool  to highlight the constant portion of the data corresponding to the string being unwound.

8. Display the mean and standard deviation of the data using the Statistics button . Record this into the data sheet.

9. Reset the RMS but this time wind the string around the lowest, largest step of the clear RMS pulley. The silver disk may need to be removed to access the pulley.

10. Repeat steps 2–8 with the string on the larger step of the RMS pulley.

11. We now want to see how changing the moment of inertia affects the measured torque. To do this we will replace the silver disk for the white disk. Measure the mass of the white disk an record it. Measure the diameter three times from different locations on the disk and record the average.

12. Remove the silver disk, remember to return the screw to the RMS then place the white disk on the RMS.

13. Repeat steps 2–10.

14. Use the Select Data Run button  to display all four runs. **Print** a copy of this graph for each group member.

Analysis

1. Calculate the moments of inertia for both the silver and white disks. The moment of inertia of a solid disk is given by,

$$I_{disk} = \frac{1}{2}mr^2,$$

   where r is the radius of the disk.

2. Calculate the torque for each of the four runs above using equation (10.7).

3. Average the torques for both disks that had the string around the smaller step of the RMS pulley. Repeat this for the torques at the larger step of the RMS pulley.

Question 3. How constant is the torque for runs on the same step of the RMS pulley? What is the relative discrepancy?

Question 4. Say we were to use a RMS pulley with an even larger radius than the large step on our pulley. How should the torque compare to our previous runs?

Chapter 10: **Torque**

1. **(1)** Title Page
2. **(5)** Purpose
3. **(10)** Theory—Should cover both setups
4. **(5)** Procedure for Setup II
5. **(2)** Graph
6. **(4)** Data Sheet
7. **(10)** Data Analysis with sample calculations shown.
8. **(8)** Answers to all questions
9. **(5)** Conclusion

Data Sheets

Static Torque

Mass needed to counter 200 g at third hole:______________

Uncertainties:

Mass needed to counter 200 g at fourth hole:______________

Formula:

Mass needed to counter weight of the board:______________

Mass of board (calculated):______________

Mass of board (measured):______________

Lecture Desk Problem:

Force on string:______________, Angle:______________

Force on string:______________, Angle:______________

Sketch with lengths indicated:

Dynamic Torque

Mass of Silver Disk:______________

Average Diameter of Silver Disk:______________

Mass of White Disk:______________

Average Diameter of White Disk:______________

	Angular Acceleration (rad/s^2)	
Object	Small Step	Large Step
Silver Disk		
White Disk		

Chapter 11

Angular Momentum

Goal

Today we will continue our discussion of angular motion this time looking at the counterpart to momentum, angular momentum. We will also investigate moments of inertia further which we introduced in Chapter 10. In our experiment we will use the conservation of angular momentum to calculate the moment of inertia of several different objects.

Equipment

- PASCO Capstone Software
- 850 Universal Interface (850UI)
- Rotary Motion Sensor
- Silver Disk, White Disk, Rod with masses, Odd-Shaped Object
- Triple-beam Balance

Theory

The last rotational quantity we want to investigate in this class is angular momentum. As can be assumed angular momentum $\mathbf{L}$ is related to linear momentum $\mathbf{p}$ in the same way that torque is related to force,

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}. \tag{11.1}$$

As it was with torque (Equation (10.2)) the magnitude of angular momentum is,

$$L = rp \sin \theta, \tag{11.2}$$

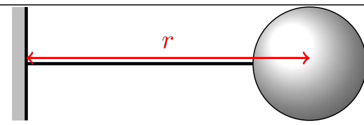
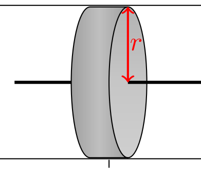
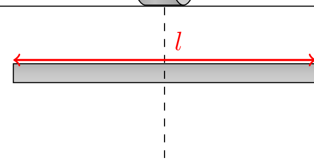
Point Mass (at a distance r from the axis of rotation)		$I = mr^2$
Solid Disk (of radius r rotating about an axis through the center of the disk)		$I = \frac{1}{2}mr^2$
Thin Rod (of length l rotating about an axis perpendicular to the rod and through its center)		$I = \frac{1}{12}ml^2$

Table 11.1

and if $\theta = \pi/2$ then Equation (11.2) simplifies to

$$L = rp = mvr \tag{11.3}$$

We can however write an expression completely in terms of angular quantities which will be more useful to us given our equipment. Like we did in Chapter 10, let us consider our body as a collection of small pieces. Each piece of mass m_i will rotate around a circle centered on the axis of rotation with a velocity v_i. This tangential velocity vector $\mathbf{v}_i$ will always be perpendicular to a position vector $\mathbf{r}_i$ as we saw in Chapter 9. We also know from Chapter 9 that $v_i = r_i\omega$. Therefore we can write Equation (11.3) as,

$$L_i = m_i(r_i\omega)r_i = m_i r_i^2 \omega. \tag{11.4}$$

Now the total angular momentum L of the body will be the sum of all the individual angular momenta L_i of the particles,

$$L = \sum L_i = \left(\sum m_i r_i^2\right)\omega = I\omega, \tag{11.5}$$

where we define $I = \sum m_i r_i^2$ to be the moment of inertia of the body and describes how mass is distributed around a rotating body. Table 11.1 provides the formulas of the various geometries that we will use in our experiment. An important property about moments of inertia is that it is an additive quantity, meaning that if two or more objects are combined into one compound object then then total moment of inertia will be the sum of the individual moments.

$$\sum I = I_1 + I_2 + \cdots$$

The *conservation of angular momentum* states that in the absence of a net external torque, the total angular momentum of the system is constant. In our experiment we will be colliding two objects together. One object will always be the silver disk that is attached to the Rotary Motion Sensor. We will begin each collision by spinning the silver disk . Then we will drop onto it the second object, which will initially have zero angular velocity. All of our collisions will be perfectly inelastic collisions, meaning that the objects will stick together and rotate with the same final angular velocity. Thus,

$$\begin{aligned} L_i &= L_f \\ I_1\omega_i &= (I_1 + I_2)\omega_f. \end{aligned} \tag{11.6}$$

For our experiment we will use the Rotary Motion Sensor (RMS) to measure the angular velocity of our system before and after we drop an object on the rotating silver disk.

1. Open Capstone.
2. Connect the RMS to the 850UI then setup Capstone to record from the sensor (click on the left-most port that the sensor is plugged into).
3. Create a graph of Angular Velocity vs Time.
4. Mount the RMS on the vertical rod in the table so that the disks will be on top.

Procedure

1. Measure the mass of the silver disk and record it. Measure the diameter of the silver disk three different times from different locations on the disk and record the average.
2. Attach the silver disk to the RMS.
3. Measure the mass of the white disk and record it. Measure the diameter three different times from different locations on the disk and record the average.
4. Give the silver disk a spin. Click on the Record button to start data collection. After about 25 data points have been recorded, drop the white disk onto the spinning disk. After the collision click the Stop button to end the data collection. Rescale the data. If the collision on the graph is not sharp enough, then delete the data run and repeat this step.
5. Click on the Coordinates Tool  and move it to the data point immediately before the collision. Record the angular velocity at this point in the data table as the initial angular velocity. Now move the tool to the data point immediately after the collision. Record the angular velocity at this point as the final angular velocity.

6. Repeat steps 3–5 with the silver disk and the rod. For the rod we will need to measure its mass (without the attached masses) and overall length, the mass of each attachable mass removed from the rod and the distance of each attachable mass from the center of rotation. Make sure the masses are attached at the outer set of holes.

7. Repeat steps 3–5 with the silver disk and the odd-shaped object. We only need to measure the mass of this object.

8. Show all data runs with the Select Data Run button . **Print** out a copy of this graph for each group member.

Optional: Repeat steps 3–5 with the silver disk and the rod. This time, move the masses to the inner set of holes on the rod. **Print** a copy of this run for each group member.

Analysis

1. We will use the white disk run to determine an experimental value for the moment of inertia of the silver disk and compare it to a theoretical value.

   (a) Using the appropriate formula from Table 11.1, calculate the theoretical moments of inertia for both disks.

   (b) Using Equation (11.6) with the measured angular velocities and the theoretical moment of inertia for the *white* disk calculate the moment of inertia for the *silver* disk.

   (c) Calculate the percent discrepancy of the experimental value for the moment of inertia of the silver disk with its theoretical value.

2. For the the rod/masses run we will determine an experimental value of the moment of inertia of the rod with attached masses and compare it to a theoretical value.

   (a) To determine the theoretical moment of inertia of the rod with attached masses, we must first calculate the individual moments of inertia for each piece using the appropriate formulas from Table 11.1. (We can treat the attachable masses as point masses.) Calculate the total moment of inertia by summing each individual moment together.

   (b) Using Equation (11.6) with the measured angular velocities and the *experimental* moment of inertia for the silver disk, calculate the moment of inertia for the rod/masses.

   (c) Calculate the percent discrepancy of the experimental moment of inertia of the rod/masses with its theoretical value.

3. For the odd-shaped object run we will determine an experimental value for the moment of inertia for the object. We will then use this value to determine the *radius of gyration.* If we consider revolving off center a point mass of the same mass as our object and with the same moment of inertia then the radius of gyration r_G is given by,

$$I_{odd} = mr_G^2 \tag{11.7}$$

   (a) Using Equation (11.6) with the measured angular velocities and the experimental moment of inertia of the silver disk, calculate the moment of inertia of the odd-shaped object.

   (b) Using Equation (11.7) with the moment of inertia calculated in the previous step, determine the radius of gyration of the odd-shaped object.

Optional: For the optional rod/masses data, repeat analysis step 2.

Question 1. Why is it better to use the experimental value for the moment of inertia of the silver disk rather than the theoretical value? (Hint: What is not being considered in the theoretical formula?)

Question 2. The rotational kinetic energy is given by $KE = \frac{1}{2}I\omega^2$. Is energy conserved in these three runs? Explain your answer.

Question 3 (*Optional*)**.** How is the moment of inertia with the attached masses changed by moving the masses to the inner holes? How should the final angular velocity change by moving the masses inwards?

Chapter 11: **Angular Momentum**

1. **(1)** Title Page
2. **(5)** Purpose
3. **(10)** Theory
4. **(5)** Procedure
5. **(1)** Graph with all three mandatory runs displayed.
6. **(5)** Data Sheet
7. **(14)** Data Analysis with sample calculations shown.
8. **(4)** Answers to the mandatory questions.
9. **(5)** Conclusion

Data Sheet

Mass of Silver Disk:______________

Average Diameter of Silver Disk:______________

Mass of White Disk:______________

Average Diameter of White Disk:______________

Mass of Rod:______________

Length of Rod:______________

Mass of Attachable Mass 1:______________ Mass of Attachable Mass 2:______________

Distance from center of rotation to attachable masses:______________

Mass of Odd-Shaped Object:______________

(Optional) Distance from center of rotation to attachable masses:______________

Object	ω_i (rad/s)	ω_f (rad/s)
White Disk		
Rod with Masses		
Odd-Shaped Object		
Rod with Masses (Optional)		

Chapter 12

Simple Harmonic Motion

Goal

Today we will be investigating what is possibly one of the most utilized models in physics. The harmonic oscillator can be used to describe the vibrations of strings, the back-and-forth of a spring, the swing of a pendulum, the oscillation of atoms in a molecule, and many other periodic motions.

Equipment

- PASCO Capstone Software
- 850 Universal Interface (850UI)
- Force Sensor, Photogate, and Motion Sensor
- Two Springs
- Dynamics Track and Cart
- Lab stand for Force Sensor
- Mass Bar for Cart
- Masses
- Triple-beam balance

Theory

We have used Hooke's Law several times this semester but have never studied the law itself in detail. To remind ourselves, Hooke's Law relates the force of an object attached to a spring with the displacement of the spring from what is called the equilibrium position as,

$$\mathbf{F} = -k\mathbf{x}, \tag{12.1}$$

where k is called the spring-force constant. The negative sign indicates that this is what is called a *restoring* force meaning that the force is always directed towards the *equilibrium position*—the point where the system has minimum potential energy.

As Hooke's Law is a force we can equate Equation (12.1) to Newton's Second Law,

$$ma = -kx. \tag{12.2}$$

Equation (12.2) is called the harmonic equation and in its standard form (and using differentials) is,

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0. \tag{12.3}$$

With suitable choice of the zero of time, the general solution of Equation (12.3) can be written as follows,

$$x = A\cos(\omega t), \tag{12.4}$$

where ωt can be regarded as an angle, and the rate at which it increases is the so-called angular frequency,

$$\omega = \sqrt{\frac{k}{m}}. \tag{12.5}$$

As the angular frequency ω is related to the (ordinary or "cycles-per-second") frequency ν by,

$$\omega = 2\pi\nu,$$

and frequency is related to the period T by,

$$T = \frac{1}{\nu},$$

we can determine the period of oscillation by,

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}. \tag{12.6}$$

For the first setup of our experiment we have two springs connected independently to the cart (borrowing a term from electronics, they are connected "in parallel") the effective spring constant is the sum of the two individual spring constants, thus Equation (12.6) is,

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}. \tag{12.7}$$

Setup I: SHM of a Cart with Two Springs

In this setup, the Force Sensor will measure the force that stretches each spring as it is pulled a short distance away from the Force Sensor. We will measure the amount of distance that the spring stretches and we will use Capstone to graph the force versus the distance. The slope of this line is the spring constant k.

Then we will use the Photogate to measure the motion of the cart that is oscillating back-and-forth as it is pulled by springs attached at each end. Capstone will calculate and display the period of oscillation which we can compare against our theoretical period.

1. Open Capstone.

2. Connect the Force Sensor to the 850UI and setup Capstone to read from it.

3. Choose the "Table & Graph" template.

4. For the first column in the table choose "Force (N)" and for the second column click on "<Select Measurement>" and under "Create New" choose "User-Entered Data." Name this User-Entered Data "Distance" and give it units of "m."

5. Set up the graph so that the vertical axis measures "Force (N)" and the horizontal axis measures "Distance (m)."

6. Put the five-pattern picket fence in the accessory tray of the cart so that the 2.5 cm single opaque band is at the top.

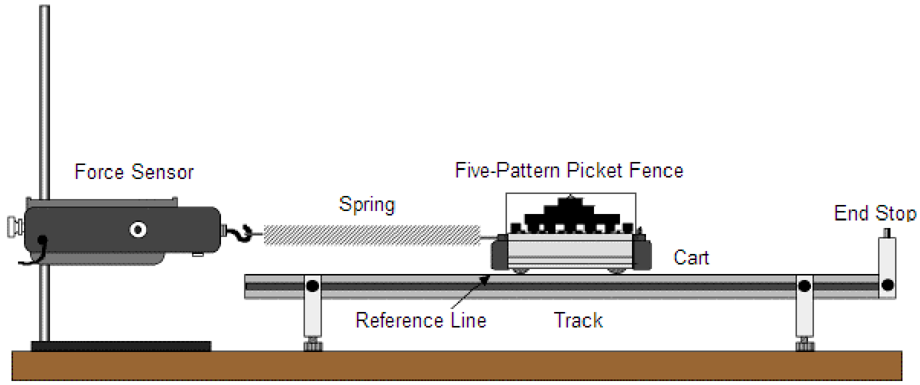
7. Measure the mass of the cart and picket fence. Record this in the data table.

8. Level the track by using the adjustable feet so that the cart will not roll one way or another.

9. Mount the Force Sensor horizontally on a support rod at one end of the track.

10. Attach an end stop to the end of the track that is opposite of the Force Sensor.

11. Connect a spring to the end of the cart facing the Force Sensor using the small holes on the black end caps of the cart. Attach the other end of the spring to the Force Sensor's hook.



12. Position the cart so the spring is in its relaxed length (neither stretched nor compressed). Put a mark on the track to indicate the initial position we will use this as a reference line for our readings.

13. In Capstone, using the button next to the timer, change the recording mode from "Continuous" to "Keep." Click on "Preview" to begin data collection.

14. Press the tare button on the Force Sensor.

15. With the cart in the correct position, enter the initial position of 0 m into the table and click on the "Keep Sample" button to store the reading.

16. Pull the cart 2 cm farther away from the Force Sensor. Hold the cart at this position then enter in the distance of 0.02 m into the table and click on "Keep Sample."

17. Continue to move the cart in 2 cm increments until the near edge of the cart is 24 cm from the reference line.

18. Once at 24 cm, click on the Stop button to end the data collection.

19. Rescale the graph and apply a linear fit. The absolute value of the slope of the line is the spring constant for this spring. Record the spring constant in the data table as k_1.

20. Disconnect the spring and replace it with the second spring. Make sure to keep track of which spring is which so we do not find the spring constant of the same spring twice.

21. Repeat steps 12–19 for the second spring. Record the spring constant of this spring as k_2.

22. **Print** a copy of the graph showing both data sets (use the Select Data Run  button to show both runs).

Procedure

1. Unplug the Force Sensor then connect a Photogate to the 850UI.

2. Open the file titled "Harmonics1" by clicking on . The file should be located in ..\Documents\Phys Lab Files\Phys1154.

3. Connect both springs to the cart and the end stops on the track.

4. When the springs are attached at opposite ends of the track, the cart should be near the middle of the track.

5. Position the Photogate so that it is aligned with the the center of the cart when the cart is at rest near the middle of the track. Adjust the height of the Photogate so the beam is blocked by the 2.5 cm opaque band at the top edge of the picket fence.

6. Pull the cart about 25 cm from its equilibrium position at the middle of the track.

7. Click on the Record button. Data collection should begin once the Photogate is first blocked.

8. Release the cart so that it can move back-and-forth through the Photogate.

9. After the cart completes at least four or five oscillations, click the the Stop button.

10. The table will display the mean and standard deviation. Record these values from the elapsed time column in the data table.

11. Measure the mass of the cart's extra mass bar. Record the mass in the data table.

12. Add the mass bar to the cart and repeat steps 5–11.

Analysis

1. Calculate the theoretical periods (Equation (12.7)) and enter the results in the table provided.

2. Calculate the percent discrepancies using the the theoretical period as the standard for each run.

Setup II: SHM of a Mass on a Spring

In this setup, a Motion Sensor will measure the motion of a mass that is suspended from the end of the spring. Capstone will record the motion and display the position and velocity of the oscillating mass. The period of oscillation is then measured and compared to the theoretical value.

1. From Setup I we have measured the spring constants of both springs. Choose one and copy the spring constant for that spring onto the data sheet for this setup.

2. Start a new activity  to clear all data from the previous setup.

3. Remove all equipment from the previous setup then connect a Motion Sensor and setup Capstone to record from it.

4. Create a graph then add a second plot area . Select the top graph to measure Position and the bottom graph to measure Velocity.

5. Using a support rod and clamp, suspend the spring so that it can move freely up and down.

6. Put a mass hanger on the end of the spring, then add about 200 g to the hanger. Enough so that the spring will oscillate in a smooth motion. Record the total mass attached to the spring on the data sheet.

7. Place the Motion Sensor on the floor directly below the mass hanger.

8. Adjust the height of the spring so that the minimum distance from the mass hanger to the Motion Sensor is greater than 20 cm at the bottom of the mass hanger's movement.

Procedure

1. Pull the mass down about 5–10 cm then release it. Let it oscillate a few times so that any unwanted motion can dampen out.

2. Click on the Record button to begin data collection. Continue recording for about 10 seconds then click on the Stop button to end data collection.

n.b., If the data points do not appear on the plots for position and velocity, click on the rescale button . The position curve should be sinusoidal. If it is not, check the alignment of the Motion Sensor and the bottom of the mass hanger at the end of the spring. You may need to increase the reflecting area of the mass hanger by attaching a circular paper disk (about 6 cm diameter) to the bottom of the mass hanger. Erase the bad data set and repeat the data collection.

Analysis

1. Rescale the data as necessary.

2. Use the Coordinates Tool  to find the average period of oscillation of the masses. Move the tool to the first peak of the Position vs Time graph, record the time value into the data table.

3. Move the Coordinates Tool to each consecutive peak in the plot and record the time value.

4. Find the period of each oscillation by calculating the difference between the times for each consecutive peak. Find the average of the periods. Record the result in the data table.

5. Calculate the theoretical period using Equation (12.6).

6. Calculate the percent discrepancy between the experimental and theoretical results.

Chapter 12: **Simple Harmonic Motion**

1. **(10)** Procedure

2. **(15)** Questions

3. **(8)** Data Sheets

4. **(2)** Graph of Spring Constants

5. **(15)** Data Analysis for both setups with sample calculations shown

Simple Harmonic Motion Worksheet

Name:______________________________

Setup I

Write the procedure for finding the spring constant.

1. Do the theoretical values of the periods agree within one standard deviation of the experimental periods? Within two standard deviations?

2. Does the period change with mass as the theoretical formula indicates? In what way? (Describe the nature of the dependence on mass.)

Setup II

1. How does your calculated value for the period of oscillation compare to the measured value for the period of oscillation? Find the percent discrepancy between your calculated value and measured value.

2. When the position of the mass is farthest from the equilibrium position, what is the velocity of the mass?

3. When the absolute value of the velocity of the mass is greatest, where is the mass relative to the equilibrium position?

Data Tables

Setup I

Item	Value
Mass of Cart	kg
Mass of Mass Bar	kg
Spring Constant k_1	N/m
Spring Constant k_2	N/m
Exp. Period (cart alone)	s
Std. Dev. (cart alone)	s
Exp. Period (cart & mass)	s
Std. Dev. (cart & mass)	s

	Theoretical Period
Cart	s
Cart & Mass Bar	s

Setup II

Spring Constant $k =$ ______________ N/m

Hanging mass = ______________ kg

Peak #	1	2	3	4	5	6	7
Time (s)							
Period (s)	——						

Average Period of Oscillation=______________ s

Calculated Period of Oscillation = ______________ s

Percent Discrepancy = ______________

Chapter 13

Calorimetry

Goal

For our final experiment we will study calorimetry, a topic found in thermal physics which is another branch of physics.

Equipment

- Styrofoam calorimeters
- Thermometer
- Triple-beam balance
- Copper, lead, and aluminium samples
- Water ice

Theory

The amount of energy required to raise the temperature of a sample by one degree Celsius (or equivalently one Kelvin) without causing a change of phase is called the heat capacity of that substance. The heat capacity per unit mass is called the specific heat of the substance. Historically, the amount of heat required to raise the temperature of one gram of water from 14.5°C to 15.5°C was defined to be 1 calorie, which in SI units is equivalent to 4.186 J. The Calorie that is used in food labeling is 1000 calories or 4186 J. In order to measure this quantity we will use the conservation of energy also known as the first law of thermodynamics,

$$\Delta U = W + Q, \tag{13.1}$$

where ΔU is the increase in total internal energy of a closed system, W is the work done on the system and Q is the heat added to the system. When there is no work done on the system, such as the assumption of our experiment then the change in energy is equal to the heat added to the system alone. The heat added to a system of mass m in terms of specific heat c and temperature T is,

$$Q = mc\Delta T. \tag{13.2}$$

As we will be placing hot metal samples in relatively cool water, the heat will transfer from the metal sample into the water since heat always tends to flow from a hotter to a cooler body. We can describe this by the following equation,

$$Q = m_w c_w \Delta T_w + m_m c_m \Delta T_m = 0, \tag{13.3}$$

in which the first term describes the heat added to the water and the second term describes that of the metal sample. Using Equation (13.3) we can then determine the specific heat of our metal sample given that we know all other data.

In the second part of the experiment, we will melt ice in liquid water. If we were to graph the temperature of the ice versus the amount of heat put into the system we would see a graph similar to Figure 13.1. Most notable are the segments of Figure 13.1 at the melting/freezing (0°C) and boiling/condensation (100°C) points as the graph reaches plateaus at both these points. As the temperature doesn't change in these regions, Equation (13.2) becomes nonsensical therefore we need a different expression to describe the heat transfer in the system. We handle this problem by introducing a constant known as the *latent heat* which describes the energy that is being absorbed or released during one of these transitions. The latent heat of fusion, the constant for melting/freezing, denoted by L_f is related to the heat by,

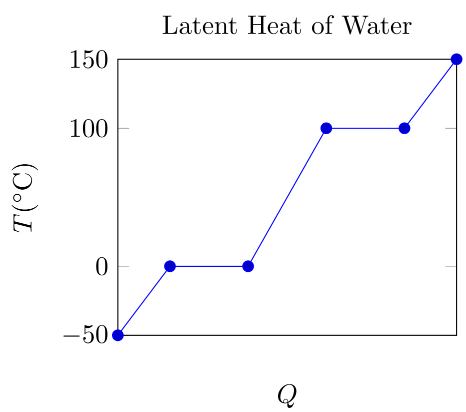


Figure 13.1

$$Q = mL_f \tag{13.4}$$

The ice and water can be regarded as a "closed" system, i.e., they do not exchange any energy with the lab because they are contained within a rigid, well-insulated calorimeter, a vessel for measuring heat flows. Hence there should be zero net heat flow, $Q = 0$, for the water-ice system. The ice (mass m_i) starts out (in a warm lab) on the verge of melting, so we can assume its initial temperature is 0°C. When we put it in the calorimeter with warm water (mass m_w), the ice melts at 0°C($Q_1 = m_i L_f$), then heats up by an additional temperature change ΔT_i($Q_2 = m_i c_w \Delta T_i$). Note that in this second term we use the specific heat of water ($c_w = 4186$ J/kg°C), not that of solid ice ($c_i = 2093$ J/kg°C) because the ice is now in the form of liquid melt water. The mass of this melt water is the same as that of the original ice (m_i), so even though it mixes with the warm water already in the calorimeter, we can identify the total heat it absorbs with help of the mass m_i. Finally, we must include the heat transfer to the warm water as it cools down ($Q_3 = m_w c_w \Delta T_w < 0$). Adding these three heats,

$$Q = m_i L_f + m_i c_w \Delta T_i + m_w c_w \Delta T_w = 0. \tag{13.5}$$

Using Equation (13.5) we can then solve for the latent heat.

Setup I: Specific Heat of a Metal

Procedure

1. Record all masses in kilograms. Determine the mass of the calorimeter, thermometer, and stirring rod.
2. Fill the calorimeter about $^2/_3$ full of water from the tap in the room.
3. Determine the mass of the water by obtaining the mass of the entire system of calorimeter, water, rod, and thermometer. The calorimeter, rod, and thermometer take very little heat so we can consider them to be negligible to the thermodynamic process.
4. At the lecture desk have the instructor place a hot metal sample carefully into the calorimeter and record the initial temperature of the sample.
5. Use the stirring rod to keep the water circulated so the heat will be evenly distributed. Read the temperature at one minute intervals until the system comes to its equilibrium temperature i.e., when the temperature stops changing.
6. Determine the mass of the system with the sample added to find the mass of the sample.
7. Bring the sample back to the lecture desk, empty the calorimeter and get fresh water then repeat steps 1–6 for another sample of a different type.

Setup II: Latent Heat of Fusion

1. Repeat the above procedure except use a small chunk of ice cubes instead of the metal sample. Make sure to dry the ice with a paper towel before placing it into the calorimeter.

Analysis

1. Use Equation (13.3) to calculate the specific heat of the two metal samples. Compare with the standard value of the sample. The standard values are as follows:

$$c_{\text{Pb}} = 128 \text{ J/kg}^\circ\text{C}, \qquad c_{\text{Al}} = 900 \text{ J/kg}^\circ\text{C}, \qquad c_{\text{Cu}} = 387 \text{ J/kg}^\circ\text{C}.$$

2. Use Equation (13.5) to calculate the latent heat of fusion for water. Compare with the standard value of 3.33×10^5 J/kg.

Chapter 13: **Calorimetry**

1. **(50)** Complete the following data worksheet displaying sample calculations for all analysis items.

Calorimetry Data Sheet

Name:______________________

Attach a sheet to this one with all calculations on it (or use the back of this sheet).

Setup I

Metal 1: Copper Lead Aluminium

Mass of cup, thermometer, stir rod, and lid:___________

With water:___________ Mass of water (m_w):__________

Initial temperature of sample ($T_{i,m}$):___________

Mass of calorimeter assembly and sample:___________

Mass of sample (m_m):___________

Calculated specific heat (c_m):___________ J/kg°C

Percent error:___________

Metal 1	
Time (min)	Temperature (°C)
0	
1	
2	
3	
4	
5	
6	
7	

Metal 2: Copper Lead Aluminium

Mass of cup, thermometer, stir rod, and lid:___________

With water:___________ Mass of water (m_w):__________

Initial temperature of sample ($T_{i,m}$):___________

Mass of calorimeter assembly and sample:___________

Mass of sample (m_m):___________

Calculated specific heat (c_m):___________ J/kg°C

Percent error:___________

Metal 2	
Time (min)	Temperature (°C)
0	
1	
2	
3	
4	
5	
6	
7	

Setup II

Mass of cup, thermometer, stir rod, and lid:___________

With water:___________ Mass of water (m_w):__________

Initial temperature of ice: $T_{i,i} = 0°C$

Mass of calorimeter assembly and sample:___________

Mass of ice (m_i):___________

Calculated latent heat (L_f):___________ J/kg

Percent error:___________

Ice Water	
Time (min)	Temperature (°C)
0	
1	
2	
3	
4	
5	
6	
7	

Appendix A

Measurements & Uncertainty

Measurements

The direct result of any experiment in the natural sciences is a measurement or a set of measurements–the "raw data". The measurements are usually in the form of a numerical statement resulting from the "reading" of some sort of measuring device. The most familiar examples at this point are rulers, which measure length, protractors, which measure angles, and clocks, which measure time but there are others with which the student will become acquainted later; voltmeters, ammeters, thermometers, etc. Regardless of what physical quantity is being measured, or what type of device is being used, the accuracy of the reading is ultimately dependent upon the ability of a human being to read the instrument. Hence, the measurement can only be as accurate as the scale of the measuring instrument and the limitations of the experimenter permit. No reading is ever exact.

Because of this, certain conventions have been adopted by scientists and engineers to indicate the accuracy of their measurements. If an engineer measures the length of a room and reports that he finds it to be 12 meters he means that it is larger than 11.5 meters and less than 12.5 meters. The implication is that the measuring device could only be read to the nearest half meter (perhaps a string with knots tied at one meter intervals was used). Under the circumstances, it would not be justifiable to state that the room was found to be 12.00 meters long. Such a statement implies that the room is at least 11.995 meters long and no longer than 12.005 meters, i.e., it implies that the experimenter can read the measuring device to at least 0.005 meters—or to the nearest half centimeter. With a meter stick graduated in centimeters, a reported length of 12.35 meters would be justified.

The two digits in the reading 12 meters and the four digits 12.35 are called *significant* figures because they contain meaningful information. In the later case, the first three digits in the number 12.35 are called *certain* numbers–there is no doubt about their values. The last digit, 5, is uncertain—it is a result of an estimate on the part of the observer who has mentally divided the smallest scale division—one centimeter—into two halves and has judged that the last digit is between 4.5 and 5.5. It would obviously be foolish, with a meter stick graduated in centimeters, to claim a measurement of, say, 9.563728 meters. The numbers beyond 6 are merely guesses—with no real physical significance at all. It is very important that the student develop a "sense of significance" when recording data. When a thermometer can be read only to the nearest half degree, don't record temperatures to the nearest hundredths of a degree. On the other hand, don't underestimate wer ability. If we can read a voltmeter to a tenth of a volt, do so. It is quite possible that if the measurements are recorded only to the nearest volt, we may miss the very effect we are looking for.

Difficulty sometimes arises in trying to determine whether or not a zero should be regarded as a significant figure or not. The reason for this difficulty is that zeros are placeholders. Whenever zeros appear sandwiched between significant numbers, such as 20.003 cm., there is no question; the zeros are significant numbers. However, when zeros appear between a decimal point and the first significant number such as 0.000312, the zeros are not significant. The only significant numbers in this reading are 312. A similar problem arises with trailing zeros. An example of this would be 93,000,000. The zeros are presumably not significant; unless we have other information to the contrary, we can probably assume they only indicate the size of the number.

Now consider a couple of more complicated cases. What are the significant figures in the numbers 0.0003120 and 93,000,000.00? In the first case, there are four significant figures, 3120. The last zero in this case does not serve as a decimal place indicator. In fact it has no purpose in being there at all unless it indicates the accuracy of the number. Similarly, the two zeros behind the decimal in 93,000,000.00 are not necessary to the position the decimal, they can only be there to indicate the accuracy of the number. In this case there are ten significant figures! The zeros in front of the decimal are now sandwiched between significant numbers and are therefore significant numbers. This is an important distinction; the number 93,000,000 merely indicates that the reading lies between 92,500,000 and 93,500,000—a possible uncertainty of 1,000,000. But the number 93,000,000.00 means that the reading lies between 92,999,999.995 and 93,000,000.005, a fantastic difference in accuracy!

In most cases there is not really much difficulty in using the proper number of significant figures to express a measurement. But it is surprising how often significant figures are neglected in expressing a final result obtained from calculations involving measurements. For example, if one measures the length of a room as 10.3**5** meters and the width as 5.2**1** meters and then wished to find the area, a direct calculation gives:

$$10.35 \text{ m} \times 5.21 \text{ m} = 53.9235 \text{ m}^2$$

But how many of these 6 digits are significant? The results lies between these two values:

$$10.345 \text{ m} \times 5.205 \text{ m} = 53.85 \text{ m}^2, \text{and}$$
$$10.355 \text{ m} \times 5.215 \text{ m} = 54.00 \text{ m}^2.$$

Hence we can only say that the area of the room is 54. m^2—a value with 2 significant figures. To quickly estimate the accuracy of a result, we generally follow this crude rule-of-thumb: In multiplication or division with several numbers, the result has as many significant figures as the least accurate of the numbers involved. We can see that in this case, where we multiplied a 4 s.f. and a 3 s.f. number together—"s.f." stands for the number of significant digits—our rule-of-thumb predicts 3 significant figures for the product. This is incorrect—the product, 54. m^2, has only 2. But in this course, the rule-of-thumb will be good enough for our purposes and we should get into the habit of using it for all our calculated results.

There are two ways to record the number of s.f. in a number:

1. We can underline of the smallest significant digit, e.g., $12.3\underline{5}$, or when typing, we can bold the digit, e.g., $9\mathbf{3}, 000, 000$; or

2. Use scientific notation:

$$1.235 \times 10^1, \qquad 9.3 \times 10^6.$$

   This shows at once that these numbers have 4 s.f. and 2 s.f., respectively.

In addition and subtraction one should only retain numbers out to the first column containing an uncertainty. In dropping the first number beyond an uncertainty, proper procedures for rounding numbers should be observed. If the first number to be dropped is greater than 5, the last number retained should be increased by one. If the first number to be dropped is less than 5, no change is made in the numbers to be retained. If the first number to be dropped is exactly 5, increase the preceding digit by one if it's odd, but leave it unchanged if it is even. For example, 43.945 would round to 43.94 while 26.835 would round to 26.84.

Uncertainty and Evaluation of Data

Experiments are rarely designed and performed just for something to do or for the benefit of the experimenter alone. The experiment and its result are usually of interest to others, even if only because it has been assigned as a problem. The measurements and final results must therefore be reported, usually in written form, so they may be useful to others. In scientific and engineering reports this involves a detailed statement of how the data was obtained and what calculations, if any, were made. For numerical results it is also extremely important that the final result be accompanied by an estimate of the uncertainty of the result.

Uncertainties, or errors in measurement, can be grouped roughly into two categories: *systematic* errors and *random* errors. Systematic errors are those which arise mainly from deficiencies in the measuring device and they almost always tend to make *all* of the readings too high or all of the readings too low. For example, consider two experimentalists, Alice and Bob who have decided to measure the rate at which a given mass of iron cools. (This involves keeping the room temperature fairly constant, guarding against air currents, etc., but we will assume that all of this has been properly taken care of). The measurements consist of obtaining temperature readings of the iron at regularly spaced intervals of time, say one minute apart. Experimenter Bob has a well-oiled, properly calibrated stop watch, but suppose that Alice has inadvertently left her highly accurate timer in a jar of molasses overnight, so that it still runs but at a slower rate that normal. The outcome of the experiment is obvious. Even though Alice's timer may be read to four significant figures and Bob's may only give two, the reporting measurements of the two experiments will be vastly different. Alice will maintain that the object cools much more rapidly than Bob claims, because Alice's "minutes" are all too long, she has a systematic error in his timing device. Systematic errors are also common in electrical meters that have not been properly "zeroed" or whose moving parts are not properly lubricated. Happily, systematic errors can usually be eliminated by making certain that the measuring device is being used and cared for as it should be and by "calibrating" it—making sure, for example, that it reads zero when it is supposed to or that its readings agree with those of some "standard" devices.

The second class of errors—random errors—arise from less definable sources, but can generally be attributed to the physical limitations of the observer and measuring device referred to in the previous section. The fact is, if one measures the length of a block of wood using different meter sticks (all with the same smallest scale division), one will not always obtain the same reading. Or if the masses of several ten-gram masses are measured on a balance, it will be found that the masses do not come out to be "exactly" ten grams in every case, nor will all of the measured values of mass be the same. In case of the block of wood, the experimenter has a dilemma. When he has finished taking, say ten readings, his data sheet may look like this:

Length of block (cm)
5.32
5.30
5.31
5.33
5.32
5.31
5.32
5.33
5.33
5.32

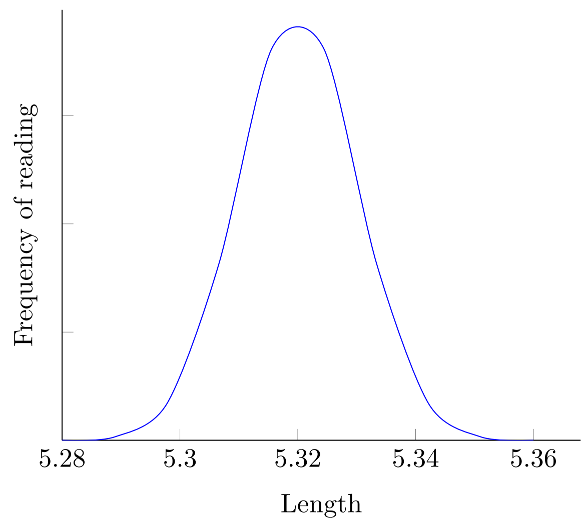


Figure A.1

Which, if any, of these readings are to be trusted and what is the significance of the differences between the various readings? If we assume that the errors involved in the measurements are entirely random in nature, the implication is that there will be as many readings that are too high as there are readings that are too low. Furthermore, (and even more important) the probability of making a measurement which is too large by a given amount (say .01 cm) is *equal to the probability* of getting a reading which is too small by the same amount (.01 cm). If one could plot the probability of getting a certain reading (on the y-axis), the curve should be that shown below. It is a completely symmetric, bell-shaped curve which is known as the Gaussian or "normal" distribution. Note that in making this graph the probability of obtaining a certain reading has been equated to the frequency with which the reading turns up in the data, which is exactly what the term "probability" implies in this case—given ten or a hundred or a thousand readings, the value that should turn up most frequently is the most probable or "best" value.

It turns out, for the Gaussian distribution, that the most probable or "best" value—the value at which the curve peaks—is just the familiar arithmetic average. Mathematically, the average value, $\bar{x}$, of a set of n numbers is written,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

This just means we add all of the different numbers to get $x_1 + x_2 + x_3 + \cdots + x_n$ then divide by n. If we add all of the readings of the length of a block of wood given previously and divide by the number of readings, n, which is 10 in this case, we get

$$L = 5.319 \text{ cm}$$

as the "best" or most probable value. This is something we might have surmised by examining the original set of readings. This is the value we would be justified in quoting as the result of our length measurements. But what is the significance of the measurements that do *not* agree with the best average? Are these simply wrong results, to be discarded? Not at all. Remember, these readings were the consequence of real physical limitations of the observer and the measuring device and they should serve to tell us something about the precision of the result we have just found. To do this, we will return once more to the Gaussian distribution. We will assume the Gaussian distribution to be valid regardless of what sort of measurement is being made or what measuring device is being used. The width of the curve and the height of the central peak will depend on the number of readings taken and on the scale of the instrument being read. It is not difficult to see that if all of 100 readings for example result in the very same identical value, the Gaussian plot becomes a "spike", a curve with essentially zero width and there can be no uncertainty associated with such a set of readings. If some, but not many, of the 100 readings are not identical with the rest, the curve begins to have some finite width and the peak "shorter" since some of the readings formerly contributing to the "spike" have now been shifted to the right or left in the widening process. The area under the curve remains the same as it was. If we now imagine that for the same 100 readings, more and more of them turn out to be different from the "most frequent" one, it appears that the Gaussian curve will spread out more and more (in a symmetric way, so that the conditions of randomness are fulfilled) and the height of the peak decreases. For such a curve, there must be a high degree of uncertainty. Hence, *the wider the curve of the Gaussian distribution, the higher the uncertainty of the best value.*

Putting this statement in terms of a numerical uncertainty is something of an arbitrary process. The most common measure of uncertainty is the standard deviation, σ (sigma), given by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

where $\bar{x} - x_i$ are the deviations of the individual readings and n is the total number of readings. The reason for the definition and common usage of the standard deviation is connected with the mathematical equation for the Gaussian curve itself. Regardless of the width of the Gaussian plot, 68% of the area under the curve lies between the values of $\bar{x} + \sigma$ and $\bar{x} - \sigma$ on the graph and 95% lies between $\bar{x} + 2\sigma$ and $\bar{x} - 2\sigma$. Once the standard deviation has been determined for a given "system" (using a measuring device with a given scale) then any subsequent reading made using a device of similar accuracy will have a 68% probability of being within $\pm\sigma$ of the "best" value. The deviation and σ for the sample of 10 readings listed earlier are given below.

x_i	Deviations, $x_i - \bar{x}$	$(x_i - \bar{x})^2$
5.32	0.001	1×10^{-6}
5.30	−0.019	4×10^{-4}
5.31	−0.009	8.1×10^{-5}
5.33	0.011	1.21×10^{-4}
5.32	0.001	1×10^{-6}
5.31	−0.009	8.1×10^{-4}
5.32	0.001	1×10^{-6}
5.33	0.011	1.21×10^{-4}
5.33	0.011	1.21×10^{-4}
5.32	0.001	1×10^{-6}

$$\sum_{i=1}^{10}(x_i - 5.319)^2 = 8.9 \times 10^{-4}$$

Now,

$$\bar{x} = 5.319 \qquad \sigma = \sqrt{\frac{8.9 \times 10^{-4}}{10-1}} = 0.009944 \approx 0.01$$

A random length measurement will, as we have said, fall within $\pm\sigma$ of $\bar{x}$ about 68% of the time. Symbolically we could represent this typical range of individual length measurements by writing $\bar{x} \pm \sigma$. In this case, rounded to its level of significance, that equals 5.32 ± 0.01 m. This tells us that individual length measurements usually range from 5.31 m to 5.33 m.

But notice that σ does *not* give a correct indication of how fuzzy $\bar{x}$ is. It overestimates the uncertainty by a sizable factor. This is because $\bar{x}$ is not an individual value. It is an average over 10 values, each of which deviates randomly to one side or another of the "ideal length measurement, L." L is what we would get if we made an infinite number of measurements and averaged them.

Let us imagine that we take 10 measurements and average them to get our first estimate of L_o—call it $\bar{x}_1$. Next, we do another 10 measurements and get $\bar{x}_2$. We repeat this procedure over and over, generating a huge number of average length values, $\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots$. If we plot these we get a peaked curve very similar to the Gaussian of Figure A.1. These cluster around L with a standard deviation called s_E, called the standard error of the mean. If our sample consists of n measurements, then it turns out that,

$$s_E = \frac{\sigma}{\sqrt{n}}.$$

Notice that s_E is always smaller than σ, and the bigger our sample size, n, the smaller s_E gets. This reflects the tendency of fluctuations to cancel our when we average a bunch of measurements.

For our example, the uncertainty of the average length is,

$$s_E = \Delta\bar{x} = \frac{0.009944 \text{ m}}{\sqrt{10}} = 0.00314 \text{ m}.$$

Since our original average value $\bar{x}_1 = 5.319$ m likely falls about 68% of the time within one s_E of the ideal value, our best guess for L can be expressed as a range,

$$\begin{aligned} L &= \bar{x}_1 \pm \Delta\bar{x} = \bar{x}_1 \pm s_E \\ &= 5.319 \pm 0.003 \text{ m}. \end{aligned}$$

In general, the standard error is given by,

$$s_E = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n(n-1)}}.$$

Note that the standard error, s_E, has been rounded to *one significant figure.* Remember that the standard error controls how many figures are kept in the mean value so the mean value should not be rounded before the standard error is calculated. The laboratory student is expected to be able to calculate the mean and standard error as done in the above example even though the standard error function is built into the calculator being used. Incidentally, some calculators give two standard deviations, σ_n and σ_{n-1}. The latter is to be used in this course—the former is used in social sciences where n is often very large. The student is still expected to round the numbers for the final answer correctly. *Remember, the standard error retains one significant figure and the location of this figure determines the way the mean is rounded.* It is quite possible for the standard error to indicate that the mean has more digits than any single measurement, as our calculation illustrates.

Often the standard error is referred to as the numerical uncertainty of the reading and called ΔL (in this case, since we are talking about a length measurement, so that the measurement is $L \pm \Delta L$, where L is the "best value" and ΔL the standard error.) A relative uncertainty can also be obtained from the following formula

$$\text{Relative uncertainty } = \frac{\Delta L}{L} \times 100\%$$

In the case of the wooden block,

$$\text{Relative uncertainty } = \frac{0.003}{5.319} \times 100\% = 0.06\%,$$

which is a fairly accurate reading. If we had tried to measure the block with a meter stick whose smallest scale division was centimeters, we would likely have obtained a best value of 5.3 cm and estimate $\sigma = 0.1$ cm and $\Delta L = .1 \text{ cm}/\sqrt{n}$. The relative uncertainty in this case would have been $2\%/\sqrt{n}$. Hence scale division plays a crucial role in determining the accuracy of the final result of a set of measurements. Incidentally, relative uncertainty may be expressed as a decimal.

There will be instances in which time or the nature of the experiment will not permit repeated measurements of a given quantity. In such a case the scale division itself represents the best estimate of the uncertainty of a single reading. For example, if the smallest scale division on a meter stick is 1 mm then clearly our readings will be uncertain by about 0.5 mm (= 0.05 cm), and we are measuring something that we judge to be 3.25 cm long, the best estimate of percentage uncertainty would be (0.05 cm/3.25 cm)100% = 1.5%. However, if we needed to measure an object 7 mm long with such a meter stick, and only one reading can be taken then we could incur a percentage error of (0.5 mm/7 mm)100% = 7%—a considerable larger error. It would be better, in such a case, to use a special instrument such as micrometer calipers, which can normally be read to 0.01 mm with an inherent uncertainty of 0.005 mm. As part of the "sense of significance" we mentioned earlier, the student should get in the habit of constantly thinking of the "size" of the measurement being made relative to the smallest scale division of the measuring device. This often allows the experimenter to recognize the "weak points" in an experiment.

The errors we have been discussing are truly errors in the sense that they involve definite limitations involved in the measuring process. Often, at the end of an experiment, it is possible to compare the result of the experiment to some standard value which can be found in a handbook or calculated from a theoretical equation (for example, the acceleration due to gravity or the elastic properties of copper rod, etc.) It is all too common to regard any differences between such a "standard value" and the value found in the laboratory as an "error". Strictly speaking, this is not so. The values for any physical constant in a handbook are usually averages compiled from several experimental sources and obtained under various conditions of temperature, atmospheric pressure, etc. The difference between an experimental value and a standard value should really be regarded as a discrepancy rather than as an error. (In scientific terminology, a discrepancy denotes a difference between a pair of results without implying which value is right or wrong.) The comparison of an experimental and a standard value is usually done by means of a percentage discrepancy defined as follows,

$$\text{Percent discrepancy } = \frac{|\text{experimental value} - \text{standard value}|}{\text{standard value}} \times 100\%.$$

In many cases there will be no "standard value" with which to compare with experimental results. However, in some experiments we will be asked to find the same physical quantity in two or three different ways. (For example, one can find the initial velocity of a projectile by finding how far it travels or by finding how much energy it has as it leaves the ground). In such cases, it is more likely than not that the results obtained using different methods will not agree with each other. Obviously one result will turn out to be more reliable than another and the student should always thoroughly analyze the various experiments involved to determine why one might be more accurate than another. This, most often, will involve non-mathematical discussion. In addition, the student should calculate the relative discrepancy between the various values obtained. The formula for relative discrepancy is the same as that for percent discrepancy except that in place of the "standard value" one uses the *average* of the various experimental results.

$$\text{Relative discrepancy} \ = \frac{|\text{experimental value} - \text{average value}|}{\text{average value}} \times 100\%$$

The phrase "percent error" is sometimes used for percent discrepancy and "percent difference" for relative discrepancy.

The final topic for this section involves the propagation of errors; how one handles uncertainties when a calculation is to be made with experimentally determined quantities. As a simple example, suppose we have determined the masses of two lumps of clay and found them to be,

$$m_1 = 56.2 \pm 0.1 \text{ g}$$

and

$$m_2 = 39.9 \pm 0.3 \text{ g}$$

respectively, for mass "one" and mass "two" and we now wish to calculate the mass we would obtain by rolling the two lumps together. The "best value" of the separate lumps,

$$m_T = m_1 + m_2 = 95.5 \text{ g}.$$

But what about uncertainty? If we consider the meaning of the numerical uncertainty for m1, it means that there is a reasonably good chance that mass m_1 is larger than 56.1 g and smaller than 56.3 g. Likewise, m_2 must be between 39.0 g and 39.6 g. The sum of these two masses could be as low as $56.1{+}39.0 = 95.1$ g or as large as $56.3 + 39.6 = 95.9$ g, i.e. $m_T = 95.5 \pm 0.4$ g. Since $0.4 = 0.1 + 0.3$ the uncertainties in this case are found to add. This is the result we would have found if we had simply assumed,

$$m_T \pm \Delta m_T = m_1 \pm \Delta m_1 + m_2 \pm \Delta m_2,$$

and since,

$$m_T = m_1 + m_2,$$

then

$$\Delta m_T = \Delta m_1 + \Delta m_2.$$

However, if we had wanted to find the difference between m_1 and m_2 we would again find that the uncertainties add! The best value of the difference is 16.9 g but the lowest value we could get would be $56.1 - 39.6 = 16.5$ g and the largest value would be $56.3 - 39.0 = 17.3$ g. Hence the final value is $m_d = 16.9 + 0.4$ g.

Next, consider the case in which we have found a length and width of a rectangle experimentally and we want to use these values to calculate an area. Assume that

$$L \pm \Delta L = 6.32 \pm 0.06 \text{ cm} \quad \text{and} \quad w \pm \Delta w = 3.05 \pm 0.05 \text{ cm}.$$

The "best value" of the area is,

$$A = Lw = 6.32 \times 3.05 = 19.276 \text{ cm}^2 \approx 19.3 \text{ cm}^2.$$

It is logical to assume that,

$$\begin{aligned} A \pm \Delta A &= (L \pm \Delta L)(w \pm \Delta w) \\ &= Lw \pm w\Delta L \pm L\Delta w \pm \Delta L \Delta w. \end{aligned}$$

Assuming $\Delta L \Delta w$ is very small (it is 0.003 in this example) so we can neglect it and since $A = Lw$ it must be true that,

$$\Delta A = w\Delta L + L\Delta w.$$

Suppose we divide both sides of this equation by A,

$$\begin{aligned} \frac{\Delta A}{A} &= \frac{w\Delta L}{A} + \frac{L\Delta w}{A} \\ &= \frac{w\Delta L}{Lw} + \frac{L\Delta w}{Lw} \\ &= \frac{\Delta L}{L} + \frac{\Delta w}{w}. \end{aligned}$$

Thus the relative uncertainty of the product of two numbers is simply the sum of the relative uncertainties of the individual terms in the product. To find ΔA as a number the relative uncertainty $(\Delta A/A)$ is multiplied by A. Using the above values for the length and width we have,

$$\frac{\Delta L}{L} + \frac{\Delta w}{w} = \frac{0.06}{6.32} + \frac{0.05}{3.05} = 0.009 + 0.02 = 0.03$$

Thus, $\Delta A/A = 0.03$ so,

$$\Delta A = 0.03 \times 19.3 = 0.6.$$

We can write our formula then as,

$$\Delta A = \left(\frac{\Delta L}{L} + \frac{\Delta w}{w}\right) A.$$

Note that for the case $A = x/y$, the relative uncertainty must again be added even though it would appear that they should subtract, for much the same reasons that we found when we subtracted two numbers. If in doubt always revert to the original equation and substitute numerical values to find the lowest and highest possible values of the quantity we are calculating. This procedure is always correct.

For those of us who know calculus, the equation above may be obtained in terms of the differentials dA, dl and dw using the standard procedure for differentiating the product of two numbers. If $A = wL$ then,

$$dA = wdL + Ldw$$

and

$$\frac{dA}{A} = \frac{dL}{L} + \frac{dw}{w}.$$

In fact the laws of differentiation may usually be used in determining the propagation of errors in calculations involving experimental values.

As another example, what error is incurred by taking the square root of an experimental number? Let,

$$R = \sqrt{X} = X^{1/2},$$

then,

$$dR = \frac{1}{2}X^{-1/2}dX,$$

therefore,

$$\frac{dR}{R} = \frac{1}{2}\frac{dX}{X}.$$

So the relative uncertainty in R is *half* of the relative uncertainty in X.

Those of we who do not have an understanding of calculus will still be expected to make use of the formulas given in this section for calculating propagated errors. Some additional formulas are given below. They are easily verified using the rules of calculus.

1. If $Z = X^2$, $\frac{\Delta Z}{Z} = 2\frac{\Delta X}{X}$.
2. If $Z = X^n$, $\frac{\Delta Z}{Z} = n\frac{\Delta X}{X}$.
3. If $Z = X^n Y^p$, $\frac{\Delta Z}{Z} = n\frac{\Delta X}{X} + p\frac{\Delta Y}{Y}$.
4. If $Z = \log X$, $\Delta Z = \frac{1}{X}\Delta X$.
5. If $Z = \sin X$, $\Delta Z = \cos X \Delta X$.

Appendix B

Report Guidelines

Any new instructions given by the instructor will, of course, supersede those given here.

It is important to note that the report is to be an individual project even though the data was taken as a group. This does not mean that other members of the group cannot be contacted and asked questions but the report should be in the words of its author. If texts or other written material is used, any material from such sources utilized in the report should be cited in the report by either using footnotes or a bibliography. Data sheets must accompany the report so that the raw data is represented unaltered. Photocopied work will be considered as representing academic dishonesty as will any work that appears to be too close to that of another person.

The report should be clear, concise, and complete (self-explanatory). It should not contain references to this manual. When deriving equations, explain what is being done in both words and mathematical notation. For examples of this, look at the theory sections in this manual. Remember when doing calculations that a numerical answer without units is the same as having no answer at all. If an answer is truly unitless such as a ratio of the same quantity make sure to indicate that there are no units. If there are a large number of similar calculations, show one sample then place the results in a table. In addition to the general guidelines for a minor report, a major report should be more detailed in all sections. A report should include the following sections:

1. Objective:
   State what the experiment is attempting to do. Is the object to measure a given quantity, to verify a given number such as g $(= 9.8\text{m/s}^2)$, to verify a concept, or to determine the relationship between several variables? It might be a combination of these.

2. Theory:
   All formulas used in the experiment are to be derived or given as definitions. For example, in the centripetal force experiment the formula $a = v^2/r$ would be derived as would the other equations used. However a

formula such as $\rho = m/V$ is a defining equation for the density and should be given as such.

In deriving equations, descriptive wording should be used to tell what is being done. For example, do not say "$v = 2\pi r/t$," instead say "The speed is given by the distance around a circular path divided by the time it takes to complete the circle. Instead of dividing by the time for one revolution, we can multiply $2\pi r$ by n, the number of revolutions per unit time. Then $v = 2\pi nr$." Also discuss the concepts fully, explain what the terms mean. e.g., explain what centripetal force, torque, buoyant force, etc. mean.

3. Method and equipment (*Major Report*):
State the general method to be used to meet the objective stated above. An example might be: "Numerous measurements of the length and width of a metal plate will be used to study the relation between the magnitude of both quantities, the magnitude of the uncertainties, and the relative uncertainties. Measurements of the same rod with three different instruments having inherently different precision will be used to determine the relation between the precision of the measurement and the relative uncertainty."

   This section also includes a list of equipment used to preform the experiment.

4. Procedure:
Briefly outline everything that was done to preform the experiment. This should be in the words of the author only.

5. Data Processing:
A table of all raw measurements should be shown here. Show how all calculations were done in this section. Include any graphs asked for as well as any interpretations requested. A sample of each different calculation including both numbers and units must be included. Generally, only one sample of each type of calculation is needed. If there is any difference, other than numerical values, then a new sample should be shown.

6. Error Analysis (*Major Report*): Discuss the reason for any large relative uncertainties. If the results of two or more calculations are to be compared and the results do not overlap, determine either the relative discrepancy or the percent discrepancy as explained in the Uncertainty section on page 127. Also discuss the reasons for discrepancies, especially any large discrepancies.

7. Questions and Problems:
Any questions or problems stated in the instructions of the experiment should be answered.

8. Conclusions:
What relationships between quantities did the experiment reveal? What

physics principles or concepts were verified? Any suggestions for improving the experiment as a learning experience would be appreciated and should be given here.