

Selective Exposure and Electoral Competition

Avidit Acharya*, Peter Buisseret†, Adam Meirowitz‡, Floyd Zhang§

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Abstract

We theoretically study how selective exposure to information by voters impacts electoral competition between two policy motivated candidates. Each candidate has stochastic valence that is realized after the candidates choose platforms. In our model of selective exposure voters receive information about the candidates' valences that is slanted to reflect their ideological preferences. Existing work predicts that selective exposure intensifies platform polarization relative to settings where voters obtain information from a neutral source. We instead show that it can reduce platform polarization.

Keywords: electoral competition, media consumption, polarization

Supplementary material for this article is available in the online appendix.

*Department of Political Science, Stanford University, Stanford CA 94305, *Email:* avidit.acharya@gmail.com.

†Department of Government, Harvard University, Cambridge MA 02138, *Email:* pbuisseret@fas.harvard.edu.

‡Department of Political Science, Yale University, New Haven CT 06520, *Email:* adam.meirowitz@yale.edu.

§Graduate School of Business, Stanford University, Stanford CA 94305, *Email:* floydjz@stanford.edu.

1 Introduction

In many democracies voters acquire information from sources that reflect their ideological predispositions. Empirical work establishes this pattern of selective exposure in the contexts of newspapers (Gentzkow and Shapiro, 2010), scientific publications (Zhang, 2023), TV and radio networks (Iyengar and Hahn, 2009, Broockman and Kalla, 2023, Martin and Yurukoglu, 2017), social media (González-Bailón et al., 2023, Nyhan et al., 2023), and face-to-face interactions (Gentzkow and Shapiro, 2011, Mutz and Martin, 2001). There are also theoretical reasons to expect voters to receive information disproportionately from sources that conform to their prior beliefs. A Bayesian agent assesses the uncertain quality of a source by comparing the signal to her prior (Acemoglu, Chernozhukov and Yildiz, 2016, Gentzkow and Shapiro, 2006). Non-Bayesian theories also posit that human cognition exhibits confirmation bias and motivated reasoning (Greenwald and Ronis, 1978, Leeper and Slothuus, 2014, Thaler, 2023).

How does this selective exposure shape electoral competition? Does selective exposure necessarily increase platform polarization relative to an environment in which all voters are equally informed? To address this question, we pursue a novel extension of the Calvert-Wittman model of electoral competition between policy-motivated candidates (Wittman, 1983, Calvert, 1985). Two candidates—a “liberal” and a “conservative”—announce platforms. A valence shock capturing events like partisan swings, scandals and how the campaigns unfold is then realized for each candidate. Voters learn about valence and cast ballots. The winning candidate implements her announced platform.

We assume liberal (resp. conservative) voters only learn the liberal (resp. conservative) candidate’s valence when it is better than expected, and only learn the conservative (resp. liberal) candidate’s valence when it worse than expected. A natural interpretation is that voters receive news from different and biased media sources. Bernhardt, Krasa and Polborn (2008) show how profit-maximizing media suppress stories seen as unfavor-

able by their audiences, generating this pattern of information segmentation by voter ideology. Broockman and Kalla (2022) call this “partisan coverage filtering”.¹ Our main result shows that selective information exposure may *reduce* platform polarization. It highlights that segmentation of voters’ information according to ideology need not lead to more polarized elections.

To see why, recognize that candidates face a trade-off when selecting platforms: moderation increases the probability of winning while extremism increases the value of winning. How do voters’ preferences over candidates respond to platforms? In a ‘no learning’ benchmark where voters do not observe valences, voting is driven entirely by platforms. This yields full median convergence. In a ‘full learning’ benchmark where voters learn valences, the candidates instead anticipate the possibility of large ex-post differences in relative valence that make voters’ preferences less responsive to platforms. This softens platform competition and induces divergent platforms.

Selective exposure is an intermediate case. On average, selective exposure reduces voters’ perceived differences in candidate quality. Selective exposure about valence makes voters’ choices more sensitive to differences in the candidates’ policies than in the full learning benchmark, but not as much as in the no learning benchmark. Candidates respond by competing more aggressively on policy. They converge more than in the full-information setting but not fully as in the no learning setting. This finding is robust to several natural extensions—including a setting in which selective exposure to information is uncorrelated with voters’ preferences.

In related work, Glaeser, Ponzetto and Shapiro (2005) and Virág (2008) assume that liberal voters are better-informed about the liberal candidate and conservatives are better-informed about the conservative candidate, and show that platforms are more polarized under selective information exposure. Chan and Suen (2008) assume the media

¹In addition to demand-driven accounts, media outlets may bias their reporting due to reputation-building considerations (Pant and Trombetta, 2019).

provides information about a policy-relevant state. Unlike these papers, voters in our model know the candidates' policies but are uncertain about their valence.² In Wittman (2007), the possibility of securing endorsements from interest groups leads candidates to diverge from the median voter's preferred policy

In concurrent work, Hu, Li and Segal (2019) examine platform polarization under common or personalized information with rationally inattentive voters and show that personalized media can increase polarization. Their purely office-seeking candidates trade off support from moderate versus extreme voters that respond differently to information. Our policy-motivated candidates trade off their preference for extreme policy with moderation's benefit to their winning odds.

2 Model

Two candidates, L and R , choose platforms x_L and x_R prior to an election. The policy space is $X = [-1, 1]$. A unit mass of citizens hold ideal points that are uniformly distributed on X . If candidate $j \in \{L, R\}$ implements platform x_j , voter i with ideal point y_i 's payoff is

$$U^j(x_j, v_j, y_i) = v_j - |x_j - y_i|,$$

where v_j is candidate j 's valence. Valences v_L and v_R are independently and uniformly distributed on $[0, 1]$. Candidates care exclusively about policy. Like voters, they have linear policy losses; L 's ideal policy is -1 and R 's ideal policy is 1 .

²Polborn and Yi (2006) and Bernhardt, Krasa and Polborn (2008) also assume voters are uncertain about valence, but these papers presume politicians' policy platforms are fixed. Duggan and Martinelli (2011) study how a single (possibly biased) media outlet optimally filters information to voters about a challenger's fixed but unknown platform. Oliveros and Várdy (2015) show how the possibility of abstention encourages even moderately polarized voters to consume centrist outlets.

We analyze three models. In each the interaction proceeds as follows. First, the candidates simultaneously announce platforms x_L and x_R . Second, valences are realized, and revealed according to a process described below. Third, each voter votes for one of the candidates. Finally, the majority winner implements her policy announcement. Ties are broken by a fair coin toss. We study Nash equilibria with sincere voting.

Our two benchmarks are the case of full learning, in which voters learn the realization of each candidate's valence before voting, and the case of no learning in which voters obtain no information about valence. The following result (proven in both Wittman and Calvert) characterizes the equilibrium to these models.

Proposition 1. *These benchmark models have a unique equilibrium:*

(i) *In the model with no learning $x_L^* = x_R^* = 0$.*

(ii) *In the model with full learning $x_L^* = -\frac{1}{4}$ and $x_R^* = \frac{1}{4}$.*

3 Selective Exposure

We now compare these benchmarks to the setting where a voter's information about the valences is mediated by her ideology. A liberal voter with ideal policy $y_i \leq 0$ learns L 's valence if and only if it is better-than-expected (i.e., if and only if $v_L \geq \mathbb{E}[v_L]$), and learns R 's valence if and only if it is worse-than-expected (i.e., if and only if $v_R < \mathbb{E}[v_R]$). Analogously, a conservative voter learns R 's valence if and only if it is better-than-expected and learns L 's valence if and only if it is worse-than-expected.

We assume that voters are fully rational and understand this information environment. Our main result characterizes the unique symmetric equilibrium.

Theorem 1. *With selective exposure, there exists a unique symmetric pure strategy Nash equilibrium, in which $x_L^* = -\frac{1}{6}$ and $x_R^* = \frac{1}{6}$. Policies therefore diverge more than in the no-learning benchmark, but less than in the setting with full learning.*

In the appendix we relax the assumption that voters fully account for the informational environment, instead allowing them to have misspecified beliefs. Here, we focus on developing the intuition by way of a local analysis that leads to necessary conditions for platform locations. The conditions are uniquely satisfied by the locations $-\frac{1}{6}$ and $\frac{1}{6}$.³

A voter with ideal policy y_i prefers R if and only if $|x_L - y_i| - |x_R - y_i| + \mathbb{E}[v_R|I_i] - \mathbb{E}[v_L|I_i] \geq 0$, where I_i is i 's information set. To derive a symmetric profile of platforms $(-x, x)$, we conjecture $x_L = -x$ and $x_R = x + \Delta$, for $\Delta \geq 0$. So, Δ captures a possible movement towards its base for R and away from the symmetric profile. We require a value of x for which no such deviation is desirable.

Figure 1 plots L 's valence on the horizontal axis, and R 's valence on the vertical axis. We begin with the bottom left-hand corner, which corresponds to both candidates having worse-than-expected valence, $v_L, v_R < 1/2$. Conservatives learn L 's valence but not R 's, and liberals learn R 's valence but not L 's. Liberals therefore believe that the L 's valence is $\mathbb{E}[v_L|v_L < 1/2] = 1/4$; conservatives similarly believe R 's valence is $1/4$.

If $v_R \leq \frac{1}{4} + \Delta$ then R 's valence is bad enough that *all* liberals vote for L . The election yields a tie if $v_L \leq \frac{1}{4} - \Delta$ so that L 's valence is sufficiently bad that *all* conservatives vote for R . Otherwise, L wins. So, with probability $(\frac{1}{4} + \Delta)(\frac{1}{4} - \Delta)$ the candidates tie.

If $v_R > \frac{1}{4} + \Delta$, then some but not all liberals vote for R .⁴ In particular, we can identify an indifferent liberal 'swing' voter with ideal policy $y_L = \frac{1}{2}(\Delta - v_R + \frac{1}{4})$. Liberals with ideal policies to the left of y_L vote for L and liberals with ideal policies to the right of y_L vote for R . Similarly, conservatives with ideal points to the right of $\frac{1}{2}(\Delta + v_L - \frac{1}{4})$ vote

³The proof presented in the Appendix considers both small and large deviations-which matter here.

⁴That not all liberals vote for R in this case is an equilibrium property: it is true only under the conjecture x is not too close to zero. Theorem 1's proof verifies that this conjecture is satisfied in the unique symmetric equilibrium.

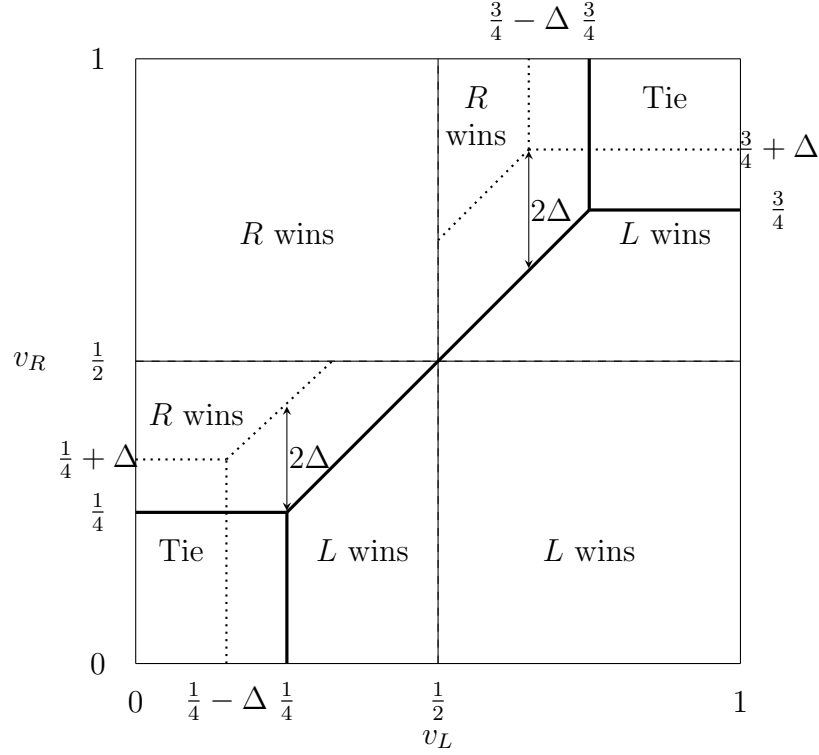


Figure 1: Election outcomes for valence pairs $(v_L, v_R) \in [0, 1]^2$, when $x_L = -x$ and $x_R = x + \Delta$; the thick lines represent $\Delta = 0$, and the dashed lines represent $\Delta > 0$.

for R , and the rest vote for L . Since ideal policies are uniformly distributed on $[-1, 1]$, R wins so long as $v_L < v_R - 2\Delta$.

Putting all of this together, the probability that both candidates have worse-than-expected valence *and* that R wins is

$$\phi(\Delta) \equiv \left(\frac{1}{4} - \Delta\right) \left(\frac{1}{4} + \Delta\right) \frac{1}{2} + \int_{\frac{1}{4} + \Delta}^{\frac{1}{2}} \int_0^{\tilde{v}_R - 2\Delta} d\tilde{v}_L d\tilde{v}_R = \frac{1}{8} - \frac{3\Delta}{4} + \Delta^2. \quad (1)$$

The top right quadrant in Figure 1 corresponds to the setting where both candidates' valences are instead *above* their expectations, and it is symmetric to the bottom left quadrant, so R 's probability of winning is again given by (1). The top left-hand quadrant corresponds to $v_R > \frac{1}{2} > v_L$, in which case for sufficiently small $\Delta > 0$ R *always* wins. Notice that, in this context, segmentation *weakens* the responsiveness of R 's victory

prospects to small platform changes. The bottom right-hand panel is the opposite case in which $v_R < \frac{1}{2} < v_L$, so that small enough $\Delta > 0$ ensures L wins.

We conclude that Right’s probability of winning from $x + \Delta$ for $\Delta > 0$ small is $\pi^{SE}(x, \Delta) \equiv 2\phi(\Delta) + \frac{1}{4}$. Right therefore maximizes

$$\Pi_R^{SE}(\Delta, x) = \pi^{SE}(x, \Delta)(x + \Delta - 1) + (1 - \pi^{SE}(x, \Delta))(-x - 1), \quad (2)$$

yielding that $\partial \Pi_R^{SE}(0, x)/\partial \Delta = 0$ if and only if $x = 1/6$.

4 Discussion

In the Appendix, we pursue three extensions. In the first voters fail to account for the fact that information is selectively censored. Polarization is still less than in the full-learning benchmark but platform divergence increases smoothly with the degree of imperfect information processing. This is because voters that fail to account for censoring over-rely on “no news” signals about valence and this softens policy competition. The second extension extends our results to a setting where voters base a turnout decision on the intensity of their preference for the preferred candidate. Here we find that increasing the cost of voting increases the sensitivity of the electoral outcome to policy and thereby decreases equilibrium divergence.⁵ The final extension shows that our qualitative results extend to a setting in which voters’ information is imperfect but uncorrelated with their ideology.

By re-casting questions about how information sources impact voter behavior in the canonical Calvert-Wittman framework where mass and elite behavior is simultaneously determined, our analysis complements important but narrower empirical studies of se-

⁵In the no-learning benchmark adding costly voting results in platform divergence; the most moderate voters abstain, thereby decreasing the electoral cost to some platform divergence.

lective exposure’s effects on voter preferences and behavior (recently, Broockman and Kalla, 2022). We hope it will spur future investigations into its effects on both voters’ and politicians’ behavior.

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Biographical Statements

Avidit Acharya is a Professor of Political Science at Stanford University, Stanford, CA 94305.

Peter Buisseret is an Assistant Professor of Government at Harvard University, Cambridge, MA 02138.

Adam Meirowitz is a Professor of Political Science at Yale University, New Haven, CT 06520.

Floyd Zhang is a PhD student in the Graduate School of Business, Stanford University, Stanford, CA 94305.

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A Proofs of Propositions

Proof of Proposition 1. We begin with full learning. Given $x_L \leq 0 \leq x_R$, let $\pi_R^{FL}(x_L, x_R)$ denote R 's probability of winning the election. R 's expected utility is then

$$\begin{aligned}\Pi_R^{FL}(x_L, x_R) &= \pi_R^{FL}(x_L, x_R)(x_R - 1) + (1 - \pi_R^{FL}(x_L, x_R))(x_L - 1) \\ &= x_L - 1 + \pi_R^{FL}(x_L, x_R)(x_R - x_L).\end{aligned}\tag{3}$$

Similarly, L 's expected utility is

$$\begin{aligned}\Pi_L^{FL}(x_L, x_R) &= (1 - \pi_R^{FL}(x_L, x_R))(-1 - x_L) + \pi_R^{FL}(x_L, x_R)(-1 - x_R) \\ &= -1 - x_L + \pi_R^{FL}(x_L, x_R)(x_L - x_R).\end{aligned}\tag{4}$$

We next derive $\pi_R^{FL}(x_L, x_R)$. Recognize that R wins the election if and only if she is preferred by a majority of voters—in particular, the median voter with ideal policy zero. This voter prefers R over L if and only if $v_R - x_R \geq v_L + x_L$, i.e., if and only if $v_R - v_L \geq x_R + x_L$. So, $\pi_R^{FL}(x_L, x_R)$ is the probability that $v_R - v_L \geq x_R + x_L$.

To obtain the probability of this event recall that each of v_L and v_R are independently and uniformly realized from $[-1, 1]$. Their difference $Z = v_R - v_L$ therefore has a triangular distribution with support $[-1, 1]$ and mode 0. Its cumulative distribution function is:

$$F_Z(z) = \begin{cases} 0 & \text{if } z < -1 \\ z + \frac{1}{2} + \frac{z^2}{2} & \text{if } -1 \leq z < 0 \\ z + \frac{1}{2} - \frac{z^2}{2} & \text{if } 0 \leq z \leq 1 \\ 1 & \text{if } z > 1. \end{cases}$$

So, $x_L \leq 0 \leq x_R$ implies

$$\begin{aligned}\pi_R^{FL}(x_L, x_R) &= \Pr(v_R - v_L \geq x_R + x_L) = 1 - F(x_R + x_L) \\ &= \begin{cases} \frac{1}{2}(x_R + x_L - 1)^2 & \text{if } x_R + x_L \geq 0 \\ \frac{1}{2} - x_R - x_L - \frac{1}{2}(x_L + x_R)^2 & \text{if } x_R + x_L < 0. \end{cases}\end{aligned}\quad (5)$$

A pure strategy \hat{x}_R is a best response only if it solves the first-order condition associated with (3):

$$\frac{\partial \pi_R^{FL}(x_L, \hat{x}_R)}{\partial x_R}(\hat{x}_R - x_L) + \pi_R^{FL}(x_L, \hat{x}_R) = 0. \quad (6)$$

Similarly, a pure strategy \hat{x}_L is a best response only if it solves the first-order conditions associated with (4):

$$-1 + \frac{\partial \pi_R^{FL}(\hat{x}_L, x_R)}{\partial x_L}(\hat{x}_L - x_R) + \pi_R^{FL}(\hat{x}_L, x_R) = 0. \quad (7)$$

Using the winning probability derived in expression (5), direct computation verifies that a unique pair $x_L = -\frac{1}{4}$ and $x_R = -\frac{1}{4}$ satisfying these first-order necessary conditions. For existence, it is easy to verify that second-order conditions for candidate R hold for all $x_R \geq 0$ when $x_L = -\frac{1}{4}$, and similarly second-order conditions for candidate L hold for all $x_L \leq 0$ when $x_R = \frac{1}{4}$. Thus, the pair $(x_L, x_R) = (-\frac{1}{4}, \frac{1}{4})$ is the unique equilibrium.

The no learning result follows because if voters obtain no information, candidate R wins if and only if $|x_L - y_i| - |x_R - y_i| + \mathbb{E}[v_R] - \mathbb{E}[v_L] \geq 0$. So, R wins the election whenever $|x_R| < |x_L|$. Because valences are never learned, the candidates compete exclusively on platforms, and the median voter theorem applies. \square

Proof of Theorem 1. We prove the Theorem in a more general framework that parameterizes voter bias in Bayesian inference. Specifically, we assume that for $\beta \leq 1$,

a liberal voter’s belief about R (or a conservative voter’s belief about L) when she does not directly learn that candidate’s valence is

$$\beta \frac{3}{4} + (1 - \beta) \frac{1}{2}$$

The interpretation is that when a voter receives no news, her posterior is a convex combination of (1) the “correct” Bayesian posterior, with weight β , and (2) a “naive” posterior—the prior belief—that fails to account for how the outlet censors information about valence.

Similarly, a conservative voter’s belief about R (or a liberal voter’s belief about L) when she does not directly learn that candidate’s valence is

$$\beta \frac{1}{4} + (1 - \beta) \frac{1}{2}.$$

Recognize that our benchmark model corresponds to the special case of $\beta = 1$. All other aspects of the model are unchanged.

Theorem 1*. *There is a threshold $\bar{\beta} \approx 0.456612$ such that:*

1. *For all $\beta > \bar{\beta}$, a unique symmetric pure strategy Nash equilibrium exists, in which $x_L = -\frac{1}{2(2+\beta)}$ and $x_R = \frac{1}{2(2+\beta)}$.*
2. *For all $\beta < \bar{\beta}$, no symmetric pure strategy Nash equilibrium exists.*

We emphasize that the case of $\beta = 1$ corresponds to Theorem 1 in the main text.

Proof of Theorem 1*, part 1. For convenience, we work with a re-parameterized posterior expectation given no news, namely $b(\beta) = \frac{\beta}{4}$. The posterior expectation is then either $\frac{1}{2} + b$ or $\frac{1}{2} - b$.

Given the media strategies and consumption choices, we can partition realizations of the candidates’ valences v_L and v_R into four qualitatively distinct patterns of inference

for citizens on the left (“liberals”) and right halves (“conservatives”) of the policy space. In particular, each of the following four events occurs with probability $\frac{1}{4}$.

- *Event A*: $\max\{v_L, v_R\} < \frac{1}{2}$ so citizens with $y_i > 0$ evaluate the candidates at expected valences of $(v_L, \frac{1}{2} - b)$ and citizens with $y_i < 0$ evaluate the candidates at expected valences of $(\frac{1}{2} - b, v_R)$.
- *Event B*: $\min\{v_L, v_R\} > \frac{1}{2}$ so citizens with $y_i > 0$ evaluate the candidates at expected valences of $(\frac{1}{2} + b, v_R)$ and citizens with $y_i < 0$ evaluate the candidates at expected valences of $(v_L, \frac{1}{2} + b)$.
- *Event C*: $v_L < \frac{1}{2} < v_R$ so citizens with $y_i > 0$ evaluate the candidates at expected valences of (v_L, v_R) and citizens with $y_i < 0$ evaluate the candidates at expected valences of $(\frac{1}{2} - b, \frac{1}{2} + b)$.
- *Event D*: $v_R < \frac{1}{2} < v_L$ so citizens with $y_i > 0$ evaluate the candidates at expected valences of $(\frac{1}{2} + b, \frac{1}{2} - b)$ and citizens with $y_i < 0$ evaluate the candidates at expected valences of (v_L, v_R) .

We refer to voters with $y_i \leq 0$ as *liberal* voters and voters with $y_i \geq 0$ as *conservative* voters.

Suppose that the candidates locate at platforms $(-x, x)$. We conjecture and prove existence of a symmetric pure strategy equilibrium in which $x = \frac{1}{4(1+2b)}$. We then show (1) this is the unique pure strategy symmetric equilibrium whenever it exists, and (2) whenever it does not exist there are no symmetric pure strategy equilibria.

Fixing $x_L = -x$, we derive necessary conditions on x for candidate R to not have a profitable deviation to $x + \Delta$ for $\Delta \neq 0$. Because the expected utility to R captures different effects to moderating or becoming more extreme, this step requires considering separately the cases of deviations with $\Delta > 0$ and $\Delta < 0$. It also requires distinct consideration of ‘small’ and ‘large’ $|\Delta|$.

Right-side deviations: Consider the case of a deviation to $x + \Delta$ with $\Delta \geq 0$. The payoffs to R for the three possible outcomes (R wins, L wins, and the vote is tied) are:

$$V_R(R \text{ wins}) = x + \Delta - 1, \quad V_R(L \text{ wins}) = -x - 1, \quad V_R(\text{tie}) = -1 + \frac{\Delta}{2}.$$

Now consider the four events that partition realizations of v .

Event A: In this event, $v_L, v_R < 1/2$. Under the conjecture $x = \frac{1}{4(1+2b)}$, for any $\Delta \geq 0$ voters with ideal policies greater than or equal to $x + \Delta$ strictly prefer R for all $v_L < 1/2$. Similarly, voters with ideal policies less than or equal to $-x$ strictly prefer L for all $v_R < 1/2$.

For any $\Delta \in (0, \frac{1}{2} - b)$, conservatives with ideal $y = 0$ vote L if $v_L - x > \frac{1}{2} - b - (x + \Delta)$ which is equivalent to $v_L > \frac{1}{2} - b - \Delta$ and so the probability that conservatives with ideal $y = 0$ vote L in *Event A* is $2(b + \Delta)$. Similarly, liberals with ideal $y = 0$ vote R if $v_R > \frac{1}{2} - b + \Delta$, and the probability of this event is $2(b - \Delta)$. Observe that v_R and v_L are independent conditional on *Event A*. Define *Sub-Event A₀* as the sub-event in which both liberal and conservative voters with ideal point $y = 0$ vote for R and L , respectively—which happens with probability $4(b^2 - \Delta^2)$ conditional on *Event A*—then R wins if $v_R - v_L > 2\Delta$; otherwise L wins. Thus, conditional on *Sub-Event A₀* the probability that R (resp. L) wins is $\frac{b-\Delta}{2(b+\Delta)}$ (resp. $\frac{b+3\Delta}{2(b+\Delta)}$).

For $\Delta \in [b, \frac{1}{2} - b]$ R cannot win; L wins as long as a conservative voter with $y = 0$ votes L , which happens with probability $2(b + \Delta)$. Finally, if $\Delta > \frac{1}{2} - b$ then L wins. Combining these cases yields

$$\text{prob}(R \text{ wins} | A) = \begin{cases} 2(b - \Delta)(1 - 2(b + \Delta)) + 2(b - \Delta)^2 & \Delta \leq b \\ 0 & b < \Delta, \end{cases}$$

$$\text{prob}(L \text{ wins}|A) = \begin{cases} (1 - 2(b - \Delta))2(b + \Delta) + 2(b - \Delta)(b + 3\Delta) & \Delta \leq b \\ 2(b + \Delta) & b < \Delta \leq \frac{1}{2} - b \\ 1 & \frac{1}{2} - b < \Delta. \end{cases}$$

and the probability of a tie is $1 - \text{prob}(L \text{ wins}|A) - \text{prob}(R \text{ wins}|A)$.

Event B: As we noted in previous *Event A*, $x > \frac{b}{2}$ implies that all voters with ideal policies greater than or equal to $x + \Delta$ prefer R , while all voters with ideal policies less than or equal to $x - \Delta$ prefer L . Conservative voters with ideal $y = 0$ vote for L if $\frac{1}{2} + b - x > v_R - x - \Delta$. This occurs if $v_R < \frac{1}{2} + b + \Delta$ and so the probability that R voters with $y = 0$ vote for L in *Event B* is $2(b + \Delta)$. Liberal voters with $y = 0$ vote for R if $v_L - x < \frac{1}{2} + b - x - \Delta$. This occurs if $v_L < \frac{1}{2} + b - \Delta$ and so the probability that liberals with $y = 0$ vote for R is $2(b - \Delta)$. Also define *Sub-Event B*₀ analogous to *Sub-Event A*₀. We see that the probabilities of the three payoff relevant events conditional on *Event B* are identical to the corresponding probabilities conditional on *Event A*.

Event C: For sufficiently small $\Delta > 0$ (namely $b > \Delta > 0$), R always wins. But with larger deviations, the outcome depends on the realization of v_L and v_R . Recall that conservative voters condition on the realizations of v_L, v_R and liberal voters condition on neither, treating R 's valence advantage as $2b$. If $b < \Delta < 2b$, a measure of $b - \frac{1}{2}\Delta$ (resp. $\min\{0, \frac{1}{2}(\Delta - (v_R - v_L))\}$) of liberals (resp. conservatives) vote for the opposite party, and R 's winning probability is $8(\Delta - b)^2$. If $\Delta > 2b$, no liberal votes for R , so the election will be tied if no conservative votes for L either (which happens if $v^R - v^L > \Delta$). Otherwise, L wins. Thus we obtain:

$$\text{prob}(R \text{ wins}|C) = \begin{cases} 1 & \Delta \leq b \\ 1 - 8(\Delta - b)^2 & b < \Delta \leq 2b \\ 0 & 2b < \Delta, \end{cases}$$

$$\text{prob}(L \text{ wins}|C) = \begin{cases} 0 & \Delta \leq b \\ 8(\Delta - b)^2 & b < \Delta \leq 2b \\ 2\Delta^2 & 2b < \Delta \leq \frac{1}{2} \\ 1 - 2(1 - \Delta)^2 & \frac{1}{2} < \Delta. \end{cases}$$

Event D: As long as $\Delta > 0$, L voters with $y = 0$ vote for L and because $b > 0$ R voters with $y = 0$ vote for L . Thus the outcome in *Event D* is a certain L win.

Now we can combine the above observations to obtain R 's expected utility from $\Delta \geq 0$, which we denote $\mathbb{E}V_R^+$. We normalize the expected utility by multiplying it by 2 in all cases below.

If $\Delta < b$:

$$\begin{aligned} \mathbb{E}V_R^+ \propto & \underbrace{[2(b - \Delta)(1 - 2(b + \Delta)) + 2(b - \Delta)^2 + \frac{1}{2}]}_{\text{probability that R wins}}(x + \Delta) + \\ & \underbrace{[2(b + \Delta)(1 - 2(b - \Delta)) + 2(b - \Delta)(b + 3\Delta) + \frac{1}{2}]}_{\text{probability that L wins}}(-x) + \\ & \underbrace{[(1 - 2(b - \Delta))(1 - 2(b + \Delta))]}_{\text{probability of a tie}}(\frac{\Delta}{2}). \end{aligned} \quad (8)$$

If $b < \Delta \leq \min\{2b, \frac{1}{2} - b\}$:

$$\begin{aligned} \mathbb{E}V_R^+ \propto & \frac{1}{2}[1 - 8(\Delta - b)^2](x + \Delta) + [2(b + \Delta) + \frac{1}{2}8(\Delta - b)^2 + \frac{1}{2}](-x) + \\ & [1 - 2(b + \Delta)](\frac{\Delta}{2}). \end{aligned} \quad (9)$$

Assuming $b \leq \frac{1}{6}$, if $2b \leq \Delta < \frac{1}{2} - b$:

$$\mathbb{E}V_R^+ \propto [2(b + \Delta) + \frac{1}{2}2\Delta^2 + \frac{1}{2}](-x) + [(1 - 2(b + \Delta)) + \frac{1}{2}(1 - 2\Delta^2)](\frac{\Delta}{2}). \quad (10)$$

Assuming $b > \frac{1}{6}$, if $\frac{1}{2} - b \leq \Delta < 2b$:

$$\mathbb{E}V_R^+ \propto \frac{1}{2}[1 - 8(\Delta - b)^2](x + \Delta) + [1 + \frac{1}{2}8(\Delta - b)^2 + \frac{1}{2}](-x). \quad (11)$$

If $\max\{2b, \frac{1}{2} - b\} < \Delta \leq \frac{1}{2}$:

$$\mathbb{E}V_R^+ \propto [1 + \frac{1}{2}(2\Delta^2) + \frac{1}{2}](-x) + [\frac{1}{2}(1 - 2\Delta^2)](\frac{\Delta}{2}). \quad (12)$$

Finally, if $\frac{1}{2} < \Delta$:

$$\mathbb{E}V_R^+ \propto [1 + \frac{1}{2}(1 - 2(1 - \Delta)^2) + \frac{1}{2}](-x) + [\frac{1}{2}(2(1 - \Delta)^2)](\frac{\Delta}{2}). \quad (13)$$

Left-side deviations: Consider now the case of a deviation to $x + \Delta$ with $\Delta \leq 0$. Recall our conjecture $x = \frac{1}{4(1+2b)}$ and note that the parties' win probabilities for *Events* A and B are the same as in the case of $\Delta \geq 0$ as long as $\Delta > -\frac{b}{3}$. Under the conjecture, if $-b \leq \Delta \leq -\frac{b}{3}$, R can lose all voters to the right of 0, which reduces R 's winning probability under *Sub-Event* A_0 (B_0). For $i \in \{L, R\}$, define $M(\Delta, b) = \text{prob}(v^L > (\frac{1}{2} - b) + 2x + \Delta > v^R | A_0, v^L < v^R - 2\Delta) = \text{prob}(v^R < (\frac{1}{2} + b) - 2x - \Delta < v^L | B_0, v^R > v^L + 2\Delta)$; $N(\Delta, b) = \text{prob}(\min\{v^R, v^L\} > (\frac{1}{2} - b) + 2x + \Delta | A_0, v^L < v^R - 2\Delta) = \text{prob}(\max\{v^R, v^L\} < (\frac{1}{2} + b) - 2x - \Delta | B_0, v^R > v^L + 2\Delta)$. Thus for either *Event* $\mathcal{E} = A, B$,

$$\text{prob}(R \text{ wins} | \mathcal{E}) = \begin{cases} 2(b - \Delta)(1 - 2(b + \Delta)) + 2(b - \Delta)^2 & -\frac{b}{3} \leq \Delta \\ 2(b - \Delta)(1 - 2(b + \Delta)) \\ \quad + 4(b^2 - \Delta^2)(1 - M(\Delta, b) - N(\Delta, b)) & -b \leq \Delta < -\frac{b}{3} \\ 2(b - \Delta) & b - \frac{1}{2} \leq \Delta < -b \\ 1 & \Delta < b - \frac{1}{2}, \end{cases}$$

$$\text{prob}(L \text{ wins}|\mathcal{E}) = \begin{cases} (1 - 2(b - \Delta))2(b + \Delta) + 2(b - \Delta)(b + 3\Delta) & -\frac{b}{3} \leq \Delta \\ (1 - 2(b - \Delta))2(b + \Delta)M(\Delta, b) & -b \leq \Delta < -\frac{b}{3} \\ 0 & \Delta < -b. \end{cases}$$

Event C: As long as $\Delta < 0$, conservatives vote for R and because $b > 0$, liberals vote for R ; thus R wins.

Event D: For $\Delta \geq -b$, L wins for sure. When $\Delta < -b$, R wins if the valence difference is small. The winning probabilities are given by:

$$\text{prob}(R \text{ wins}|A) = \begin{cases} 0 & -b \leq \Delta \\ 8(\Delta + b)^2 & -b - \frac{1}{4} < \Delta \leq -b \\ 1 - 2(1 - 2(\Delta + b))^2 & -b - \frac{1}{2} \leq \Delta < -b - \frac{1}{4} \end{cases}$$

$$\text{prob}(L \text{ wins}|A) = \begin{cases} 1 & -b \leq \Delta \\ 1 - 8(\Delta + b)^2 & -b - \frac{1}{4} < \Delta \leq -b \\ 2(1 - 2(\Delta + b))^2 & -b - \frac{1}{2} \leq \Delta < -b - \frac{1}{4} \end{cases}$$

$$\text{prob}(R \text{ wins}|D) = 0, \quad \text{prob}(L \text{ wins}|D) = 1 - 2\Delta^2, \quad \text{prob}(\text{tie}|D) = 2\Delta^2$$

First Order Conditions: Differentiating the right side of Equation (8) with respect to Δ yields the following first-order necessary condition for an optimal deviation:

$$1 + 12\Delta^2 - (4 + 8b)x - (4 + 8b - 16x)\Delta = 0 \tag{14}$$

which holds at $\Delta = 0$ if and only if $x = \frac{1}{4(1+2b)}$. This conjectured symmetric Nash equilibrium $(-\frac{1}{4(1+2b)}, \frac{1}{4(1+2b)})$ yields 0 expected utility for both parties by symmetry. Recall that utility was normalized above.

For future reference, we define

$$\Delta' = \min \left\{ \frac{1}{12}(-1 + 8b - \frac{2 - \sqrt{15 + 36b - 12b^2 - 48b^3 + 64b^4}}{1 + 2b}), 2b, \frac{1}{2} - b \right\}, \quad (15)$$

and recognize that when $x = \frac{1}{4(1+2b)}$ as required by local necessary conditions (namely, that (14) holds at $\Delta = 0$), evaluating the right of (9) at Δ' reveals that the expression is strictly positive if and only if $b < \bar{b} \approx 0.114153$ or equivalently $\beta < \bar{\beta} = 4\bar{b} \approx 0.456612$.

Existence. Without loss and invoking symmetry, we focus on possible deviations by R . Fixing $x = \frac{1}{4(1+2b)}$ we argue that R 's expected utility, as expressed in the proof of Part 1, is weakly lower for any $\Delta \neq 0$. Recall that expected utility is 0 at the conjectured symmetric equilibrium, where $\Delta = 0$. We must consider separately deviations to the right ($\Delta > 0$) and deviations to the left ($\Delta < 0$).

Right-side deviations. Suppose that $\Delta < b$, and observe that (8) is cubic in Δ and $\Delta = 0$ is a local maximum. To make sure $\Delta = 0$ is also the global maximum over $[0, b]$, it suffices to compare $\Delta = 0$ with $\Delta = b$. Plugging in $x = \frac{1}{4(1+2b)}$, we see that the expression on the right of (8) equals 0 at $\Delta = 0$ and is negative at $\Delta = b$ for $b > 0$.

Now suppose that $\Delta < \min\{2b, \frac{1}{2} - b\}$, and notice that the expression on the right of (9) is cubic in Δ and goes to $-\infty$ as $\Delta \rightarrow \infty$. Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$1 - 16b^3 - 12\Delta - 24\Delta^2 + 4b^2(-3 + 16\Delta) + b(10 + 24\Delta - 48\Delta^2) = 0$$

The greater root $\frac{1}{12}(-1 + 8b - \frac{2 - \sqrt{15 + 36b - 12b^2 - 48b^3 + 64b^4}}{1 + 2b})$ is the local maximum. Since the local minimum $\frac{1}{12}(-1 + 8b - \frac{2 + \sqrt{15 + 36b - 12b^2 - 48b^3 + 64b^4}}{1 + 2b})$ is always less than $2b$, it suffices to check that R 's expected payoff is weakly negative at Δ' if $b > \bar{b}$ where Δ' is defined in (15) and \bar{b} is defined in the text immediately afterwards. Simple numerical calculations verify that this is the case.

Now suppose that $b \leq \frac{1}{6}$ and $2b \leq \Delta < \frac{1}{2} - b$, and note that the expression in (10) is cubic in Δ and goes to $-\infty$ as $\Delta \rightarrow \infty$. Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$1 - 8b^2 - 10\Delta - 6\Delta^2 + 2b(1 - 8\Delta - 6\Delta^2) = 0.$$

The local maximum (minimum) is the greater (smaller) root and always (never) in $[2b, \frac{1}{2} - b]$ if $b \geq \bar{b}$. It thus suffices to check that at the local maximum R 's expected utility is weakly negative. This is indeed the case.

Now suppose that $b > \frac{1}{6}$ and $\frac{1}{2} - b \leq \Delta < 2b$. Expression (11) is cubic in Δ and goes to $-\infty$ as $\Delta \rightarrow \infty$. Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$2 - 16b^3 - 8\Delta - 24\Delta^2 + 8b^2(-1 + 8\Delta) + 2b(5 + 16\Delta - 24\Delta^2) = 0$$

The local minimum is the smaller root and always less than $\frac{1}{2} - b$ for any $b \in [\bar{b}, \frac{1}{4}]$. It thus suffices to show the local maximum value is no greater than 0 for any $b \in [\bar{b}, \frac{1}{4}]$. Simple calculations show it is weakly negative.

Now suppose $\max\{2b, \frac{1}{2} - b\} < \Delta \leq \frac{1}{2}$. Expression (12) is cubic in Δ , and goes to $-\infty$ as $\Delta \rightarrow \infty$. Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$\frac{1}{2} - \frac{\Delta}{2 + 4b} - 3\Delta^2 = 0.$$

Since the interval $[\max\{2b, \frac{1}{2} - b\}, \frac{1}{2}]$ is always strictly between the two roots of the first-order condition for any $b \in [\bar{b}, \frac{1}{4}]$, one only needs to consider R 's expected utility at the outer limit $\frac{1}{2}$. Simple numerical calculations shows it is negative for any $b \in [\bar{b}, \frac{1}{4}]$.

Finally, suppose $\Delta > \frac{1}{2}$. Expression (13) is cubic in Δ and goes to ∞ as $\Delta \rightarrow \infty$. Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$\frac{1}{2}(-3 + 3\Delta)\Delta + b(1 - 4\Delta + 3\Delta^2) = 0.$$

The local maximum is always less than $\frac{1}{2}$, and the local minimum exceeds the policy space. It thus suffices to check the limit of (13) as $\Delta \rightarrow \frac{1}{2}^+$. Simple numerical calculations show it is negative for any $b \in [\bar{b}, \frac{1}{4}]$.

Left-side deviations: First we argue that $\Delta < 0$ cannot be a profitable deviation. When $x = \frac{1}{4(1+2b)}$ as conjectured, equation (8) is locally maximized at $\Delta = 0$ and monotonically increasing in Δ for $\Delta < 0$. Inspecting the wining probabilities for $\Delta < 0$ in the proof of Part 1 shows that for any $-b \leq \Delta < 0$, R 's wining (losing) probability is weakly lower (higher) than the corresponding probability incorporated in (8) given the same Δ value. This means such deviations would not be profitable for R .

Since parties cannot move across 0, we can disregard the cases where $\Delta < b - \frac{1}{2}$ or $\Delta < -b - \frac{1}{4}$ as $x = \frac{1}{4(1+2b)} \leq \frac{1}{4}$. So the only case left to consider is $-b - \frac{1}{4} \leq \Delta < -b$. In this case,

$$\begin{aligned} \mathbb{E}V_R^- \propto & (2(b - \Delta) + \frac{1}{2}8(\Delta + b)^2)(x + \Delta) + \frac{1}{2}(1 - 8(\Delta + b)^2)(-x) \\ & + (1 - 2(b - \Delta))\frac{\Delta}{2} \end{aligned} \quad (16)$$

This expression is cubic in Δ and goes to ∞ in Δ . Its first-order condition, after plugging in $x = \frac{1}{4(1+2b)}$, is:

$$1 + 16b^3 + 4\Delta + 24\Delta^2 + 4b^2(3 + 16\Delta) + 2b(7 + 12\Delta + 24\Delta^2) = 0.$$

The left side has no real root for $b \in [\bar{b}, \frac{1}{4}]$. This implies (16) monotonically increases in Δ for $b \in [\bar{b}, \frac{1}{4}]$, and it suffices to check its limit as $\Delta \rightarrow -b - \frac{1}{4}^-$, which is negative.

We conclude that if $b > \bar{b}$, $x_R = \frac{1}{4(1+2b)}$ and $x_L = -x$ is a symmetric pure strategy Nash equilibrium.

Uniqueness: We proved that $(-\frac{1}{4(1+2b)}, \frac{1}{4(1+2b)})$ is a symmetric pure strategy Nash equilibrium if $b > \bar{b}$. We establish that there is no other symmetric pure strategy equilibrium $(-x, x)$ for *any* b by considering two possibilities: $x \geq \frac{b}{2}$, or $x < \frac{b}{2}$. In both cases, we rule out another equilibrium by restricting attention to local (i.e., small) rightward deviations by candidate R , i.e., from $x \geq 0$ to $x + \Delta$ for $\Delta \geq 0$.

Suppose, first, there is another symmetric equilibrium satisfying $x \geq \frac{b}{2}$. Under that conjecture, R 's expected payoff from $0 \leq \Delta < b$ is again given by Equation (8). It must therefore satisfy the corresponding first-order necessary condition at $\Delta = 0$, which we already verified has a unique solution $x = \frac{1}{4(1+2b)}$. We conclude that there is no other symmetric equilibrium satisfying $x \geq \frac{b}{2}$.

Suppose, second, there is another symmetric equilibrium satisfying $x < \frac{b}{2}$. Under this conjecture, R 's probability of winning in the *Events A* and *B* is modified from our earlier analysis. The reason is that for small enough $\Delta > 0$ we can have $\frac{1}{2} > v_L > 2x + \Delta + \frac{1}{2} - b$, whereby *all* conservative voters prefer L . Similarly, for small enough $\Delta > 0$ we can have $\frac{1}{2} > v_R > 2x + \Delta + \frac{1}{2} - b$, where *all* liberal voters prefer R . Straightforward computation yields the probability that $v_L < \frac{1}{2}$ and $v_R < \frac{1}{2}$ and R wins is:

$$\frac{1}{8} - \Delta^2 + \Delta \left(b - 4x - \frac{1}{2} \right). \quad (17)$$

The probability of $v_L, v_R > \frac{1}{2}$ and R wins is the same as (17). Finally, it is easy to verify that—just as in our analysis under the conjecture $x > \frac{b}{2}$ — R always wins whenever $v_R > \frac{1}{2} > v_L$ whenever $\Delta > 0$ sufficiently small, and similarly R always loses whenever

$v_R < \frac{1}{2} < v_L$. We conclude that for $\Delta > 0$ sufficiently small (in particular small enough that $x + \Delta < \frac{b}{2}$), R 's expected utility is

$$\left[2 \left(\frac{1}{8} - \Delta^2 + \Delta \left(b - 4x - \frac{1}{2} \right) \right) + \frac{1}{4} \right] (2x + \Delta).$$

Differentiating with respect to Δ yields a first-order necessary condition for an optimal deviation; evaluating that condition at $\Delta = 0$ yields the following condition on x :

$$x = \frac{1}{16} \left(\sqrt{4b^2 - 4b + 9} + 2b - 1 \right).$$

For all $b \leq .25$, this violates the conjecture that $x < \frac{b}{2}$. We conclude that there is no symmetric pure strategy equilibrium $(-x, x)$ satisfying $x < \frac{b}{2}$.

Proof of Theorem 1*, part 2. The previous part showed that $(-x, x)$ is a pure strategy equilibrium only if $x = \frac{1}{4(1+2b)}$ as required by local necessary conditions (namely, that (14) holds at $\Delta = 0$). Evaluating the right of (9) at Δ' reveals that the expression is strictly positive whenever $b < \bar{b}$, yielding a profitable rightward deviation by R to Δ' .

□

B Extensions

B.1 Abstention

Our benchmark model presumes that all voters cast ballots. Are our findings robust to settings in which some voters abstain? To examine this question, we modify our benchmark model by presuming that a voter with ideal policy $y_i \in [-1, 1]$ casts her ballot for party $j \in \{L, R\}$ if and only if

$$U^j(x_j, v_j, y_i) - U^{-j}(x_{-j}, v_{-j}, y_i) \geq \kappa,$$

where $\kappa \in [0, .25]$ can be interpreted as a cost (or opportunity cost) of voting. This voting heuristic is commonly referred to as *abstention from indifference* (Guttman, Hilger and Shachmurove, 1994) whereby a voter turns out only if the intensity of her preference for her preferred candidate is large enough. All other aspects of our benchmark model are preserved.

Full Learning Benchmark. We start with a setting in which voters learn the realization of each candidate's valence before casting their ballots.

Proposition 2. *With full learning, $x_L^* = -\frac{1}{4}$ and $x_R^* = \frac{1}{4}$ is a Nash equilibrium.*

Proof. A voter with ideal policy y_i votes for party R if and only if: $|x_L - y_i| - |x_R - y_i| + v_R - v_L \geq \kappa$. She votes for party L , instead, if and only if: $|x_L - y_i| - |x_R - y_i| + v_R - v_L \leq -\kappa$. Let

$$y_R^* \equiv \begin{cases} \frac{v_L - v_R + x_L + x_R + \kappa}{2} & \text{if } v_R - v_L \in [\kappa - (x_R - x_L), \kappa + (x_R - x_L)] \\ +1 & \text{if } v_R - v_L < \kappa - (x_R - x_L) \\ -1 & \text{if } v_R - v_L > \kappa + (x_R - x_L) \end{cases}$$

denote the unique ideal point of the voter that is indifferent between voting for R and abstaining. Here, recognize that when $v_R - v_L < \kappa - (x_R - x_L)$ *no* voter with ideal policy $y_i \in [-1, 1]$ wants to vote for R and so we define the indifferent voter type to be $y_i = 1$. Recognize further that when $v_R - v_L > \kappa + (x_R - x_L)$, *every* voter wants to vote for R and so we define the indifferent type to be $y_i = -1$.

Similarly, let

$$y_L^* \equiv \begin{cases} \frac{v_L - v_R + x_L + x_R - \kappa}{2} & \text{if } v_R - v_L \in [x_L - x_R - \kappa, x_R - x_L - \kappa] \\ -1 & \text{if } v_R - v_L > x_R - x_L - \kappa \\ +1 & \text{if } v_R - v_L < x_L - x_R - \kappa. \end{cases}$$

Conjecture a strategy profile satisfying $x_R - x_L > \kappa$. Then, R wins the election if $v_R - v_L > x_R - x_L - \kappa$ and L wins the election if $v_R - v_L < \kappa - (x_R - x_L)$. If, instead, $\kappa - (x_R - x_L) \leq v_R - v_L \leq x_R - x_L - \kappa$, we have $x_L \leq y_L^* \leq y_R^* \leq x_R$, and R wins if $y_R^* + y_L^* < 0$, which is equivalent to $v_R - v_L > x_R + x_L$. Since the candidates' valences are uniformly distributed, R 's probability of winning is $\pi^{FL}(x, \Delta) \equiv \frac{1}{2}(1 - x_L - x_R)^2$. R therefore maximizes $\Pi_R^{FL}(x_L, x_R) \equiv \pi^{FL}(x_L, x_R)(x_R - x_L)$ while L maximizes $\Pi_L^{FL}(x_L, x_R) \equiv (1 - \pi^{FL}(x_L, x_R))(x_R - x_L)$. This yields a pair $x_L = -\frac{1}{4}$ and $x_R = \frac{1}{4}$. Notice $x_R - x_L = 1/2$, and so our assumption that $\kappa < \frac{1}{4}$ implies that the conjecture $x_R - x_L > \kappa$ is satisfied. Given the symmetric pair, a sufficient condition for no profitable non-local deviation is that party R does not prefer \tilde{x}_R such that $\tilde{x}_R - (-1/4) < \kappa$. This deviation requires $\tilde{x} < 0$; from R 's perspective, *any* lottery over platforms $x_L = -\frac{1}{4}$ and $\tilde{x}_R < 0$ is worse than the lottery over platforms induced by $x_R = \frac{1}{4}$ and $x_L = -\frac{1}{4}$. ■

No Learning Benchmark. Now voters obtain no information about valence. A voter with ideal policy $y \in (x_L, x_R)$ that is indifferent between voting R and abstaining has ideal policy $y_R^* = \frac{x_L + x_R + \kappa}{2}$ and a voter $y \in (x_L, x_R)$ that is indifferent between L and abstaining has ideal policy $y_L^* = \frac{x_L + x_R - \kappa}{2}$.

Proposition 3. *With no learning, the unique equilibrium has $x_L^* = -\frac{\kappa}{2}$ and $x_R^* = \frac{\kappa}{2}$.*

Proof. First, we claim that there is no Nash equilibrium in which $|x_R - x_L| < \kappa$. To see why, recognize that whenever the platforms satisfy this condition, no voter turns out and the parties tie. Party R could therefore re-position to a new x'_R satisfying $x'_R > x_R$, $|x'_R - x_L| < \kappa$, and benefit from a strictly better policy lottery.

We, thus, conclude that in any Nash equilibrium, $|x_R - x_L| \geq \kappa$. If $x_R - x_L > \kappa$, every voter casts her ballot: R could deviate to $x'_R \in (x_R - \epsilon, x_R)$ for $\epsilon < x_R - x_L - \kappa$, thereby winning with probability one and securing a strictly greater policy payoff. We conclude that in an equilibrium $x_R - x_L = \kappa$. This implies that in any Nash equilibrium, every voter with ideal policy $y_i \geq x_R$ casts her ballot for R while every voter with ideal policy $y_i \leq x_L$ casts her ballot for L . And voters between the platforms abstain. Thus R wins with positive probability if and only if $|x_R| \leq |x_L|$, and L wins with positive probability if and only if $|x_L| \leq |x_R|$. If either inequality is strict, then the losing candidate has a profitable deviation. We must therefore have $|x_R| = |x_L|$, and thus $x_L < 0$ and $x_R = -x_L$. Each party therefore wins with probability one half. Combining this condition with $x_R = -x_L$ yields the necessary condition for an equilibrium that $x_R = \frac{\kappa}{2}$ and $x_L = -\frac{\kappa}{2}$. Notice that at this strategy profile every voter with type $y_i \in (x_L, x_R)$ abstains. If party R were to re-position to $x'_R > x_R$, she loses the election with probability one and suffers a strictly worse policy payoff. If party R were to instead re-position to $x'_R < x_R$, she either ties with party L or loses the election with probability one: regardless, her expected policy payoff is strictly worse as a consequence of the deviation.

We conclude that the pair $x_L = -\frac{\kappa}{2}$ and $x_R = \frac{\kappa}{2}$ is the unique Nash equilibrium. \blacksquare

Selective Exposure. Recognize that the paper's benchmark is a special case in which $\kappa = 0$. We verify the following result:

Proposition 4. *With selective exposure, there exists $\kappa^* > 0$ such that for all $\kappa \in [0, \kappa^*)$, a symmetric pure strategy Nash equilibrium exists in which $x_L^* = -\frac{1}{6+8\kappa}$ and $x_R^* = \frac{1}{6+8\kappa}$.*

Proof. Recognize that voter y_i prefers to vote for R rather than abstain if and only if $|x_L - y_i| - |x_R - y_i| + \mathbb{E}[v_R|I_i] - \mathbb{E}[v_L|I_i] \geq \kappa$, where I_i denotes i 's information set. Call this indifferent conservative voter $y_R^+ \geq 0$ and the corresponding liberal voter $y_R^- \leq 0$. Similarly, a voter y_i prefers to vote for L rather than abstain if and only if $|x_L - y_i| - |x_R - y_i| + \mathbb{E}[v_R|I_i] - \mathbb{E}[v_L|I_i] \leq -\kappa$. Call this indifferent conservative voter $y_L^+ \geq 0$ and the corresponding liberal voter $y_L^- \leq 0$.

Conjecture that $x_L = -x$ and $x_R = x + \Delta$, for $\Delta \geq 0$. Our analysis replicates the benchmark, but we account for the possibility of up to four interior indifferent voter ideal points.

Case 1: $v_L, v_R < 1/2$. Suppose both candidates have worse-than-expected valence, $v_L, v_R < 1/2$. This implies that conservatives learn L 's valence, but do not learn R 's valence, while liberals learn R 's valence, but do not learn L 's valence. Liberals therefore believe that the L 's valence is $\mathbb{E}[v_L|v_L < 1/2] = 1/4$, while conservatives believe R 's valence is $1/4$. Then:

$$y_R^+ = \begin{cases} \frac{\Delta + v_L - \frac{1}{4} + \kappa}{2} & \text{if } \frac{1}{4} - \Delta - \kappa \leq v_L \leq 2x + \Delta + \frac{1}{4} - \kappa \\ 1 & \text{if } 2x + \Delta + \frac{1}{4} - \kappa < v_L \\ 0 & \text{if } v_L < \frac{1}{4} - \Delta - \kappa. \end{cases}$$

Here, we define $y_R^+ = 1$ whenever no conservative voter prefers to vote for R , while $y_R^+ = 0$ when all conservative voters prefer to vote for R . Proceeding similarly:

$$y_L^+ = \begin{cases} \frac{\Delta + v_L - \frac{1}{4} - \kappa}{2} & \text{if } \frac{1}{4} - \Delta + \kappa \leq v_L \leq 2x + \Delta + \frac{1}{4} + \kappa \\ 1 & \text{if } 2x + \Delta + \frac{1}{4} + \kappa < v_L \\ 0 & \text{if } 0 \leq v_L < \frac{1}{4} - \Delta + \kappa, \end{cases}$$

$$y_R^- = \begin{cases} \frac{\Delta - v_R + \frac{1}{4} + \kappa}{2} & \text{if } \Delta + \frac{1}{4} + \kappa \leq v_R \leq 2x + \Delta + \frac{1}{4} + \kappa \\ -1 & \text{if } v_R > 2x + \Delta + \frac{1}{4} + \kappa \\ 0 & \text{if } 0 \leq v_R < \Delta + \frac{1}{4} + \kappa, \end{cases}$$

and

$$y_L^- = \begin{cases} \frac{\Delta - v_R + \frac{1}{4} - \kappa}{2} & \text{if } \Delta + \frac{1}{4} - \kappa \leq v_R \leq 2x + \Delta + \frac{1}{4} - \kappa \\ -1 & \text{if } v_R > 2x + \Delta + \frac{1}{4} - \kappa \\ 0 & \text{if } 0 \leq v_R < \Delta + \frac{1}{4} - \kappa. \end{cases}$$

Conjecture that $x > \frac{1+4\kappa}{8}$. This conjecture implies that all cut-off types are interior to their respective sub-intervals whenever $v_R < \frac{1}{2}$ and $v_L < \frac{1}{2}$.

Notice that if $y_R^+ = 0$ and $y_L^- = 0$, the parties tie. This occurs when $v_L \leq \frac{1}{4} - \Delta - \kappa$ and $v_R \leq \Delta + \frac{1}{4} - \kappa$. If $y_R^+ = 0$ and $y_L^- < 0$, R wins with probability one. This occurs when $v_L \leq \frac{1}{4} - \Delta - \kappa$ and $v_R > \Delta + \frac{1}{4} - \kappa$. If $y_R^+ > 0$ and $y_L^- = 0$, L wins with probability one. This occurs when $v_R \leq \Delta + \frac{1}{4} - \kappa$ and $v_L > \frac{1}{4} - \Delta - \kappa$.

Putting these first cases together, whenever $v_L \leq \frac{1}{4} - \Delta - \kappa$, the probability that $v_R < \frac{1}{2}$ and R wins the election is:

$$\frac{1}{2} \Pr[v_R < \Delta + 1/4 - \kappa] + \Pr[\Delta + 1/4 - \kappa < v_R < 1/2] = \frac{1}{8}(3 - 4\Delta + 4\kappa).$$

The remaining cases correspond to $y_R^+ > 0$ and $y_L^- < 0$. There are four subcases.

First, if $v_L \in [1/4 - \Delta - \kappa, 1/4 - \Delta + \kappa]$ and $v_R \in [1/4 + \Delta - \kappa, 1/4 + \Delta + \kappa]$, then $y_R^+ \in (0, x + \Delta]$, $y_L^+ = 0$, $y_R^- = 0$ and $y_L^- \in [-x, 0)$. R wins the election if and only if $1 - y_R^+ \geq y_L^- - (-1)$, which is equivalent to $v_R - v_L \geq 2\Delta$.

Second, if $v_L \in [1/4 - \Delta - \kappa, 1/4 - \Delta + \kappa]$ and $v_R \geq 1/4 + \Delta + \kappa$, then R wins if and only if $1 - y_R^+ + 0 - y_R^- > y_L^- - (-1)$, which is always true.

Third, if $v_L > 1/4 - \Delta + \kappa$ and $v_R \in [1/4 + \Delta - \kappa, 1/4 + \Delta + \kappa]$, then R wins if and only if $1 - y_R^+ > y_L^+ - 0 + (y_L^- - (-1))$, which is equivalent to $v_R > \frac{-1+8v_L+12\Delta-8\kappa}{2}$, which is always false.

Fourth, if $v_L > 1/4 - \Delta + \kappa$ and $v_R > 1/4 + \Delta + \kappa$, R wins if and only if $1 - y_R^+ + 0 - y_R^- > y_L^+ - 0 + (y_L^- - (-1))$, which is equivalent to $v_R - v_L > 2\Delta$.

Putting all of this together, the probability of the joint event that $v_R < 1/2$ and $v_L < 1/2$ and R wins the election is

$$\int_0^{\frac{1}{4}-\Delta-\kappa} \left[\frac{1}{8}(3 - 4\Delta + 4\kappa) \right] d\tilde{v}_L + \int_{\frac{1}{4}-\Delta-\kappa}^{\frac{1}{2}-2\Delta} \left[\int_{\tilde{v}_L+2\Delta}^{\frac{1}{2}} d\tilde{v}_R \right] d\tilde{v}_L = \frac{1}{8} + \Delta^2 - \frac{\Delta}{4}(3 + 4\kappa).$$

Case 2: $v_L, v_R > 1/2$. Suppose, next, that both candidates have better-than-expected valence, $v_L, v_R > 1/2$. This implies that conservatives learn R 's valence, but do not learn L 's valence, while liberals learn L 's valence, but do not learn R 's valence. Liberals therefore believe that the R 's valence is $3/4$, while conservatives believe L 's valence is

$\frac{3}{4}$. As in the previous step, we define the indifferent cut-off voter types:

$$y_R^+ = \begin{cases} 1 & \text{if } v_R \leq \frac{3}{4} - 2x - \delta + \kappa \\ \frac{\Delta + \frac{3}{4} - v_R + \kappa}{2} & \text{if } \frac{3}{4} + \delta + \kappa \geq v_R \geq \frac{3}{4} - 2x - \delta + \kappa \\ 0 & \text{if } v_R \geq \frac{3}{4} + \delta + \kappa, \end{cases}$$

$$y_L^+ = \begin{cases} 1 & \text{if } v_R \leq \frac{3}{4} - 2x - \Delta - \kappa \\ \frac{\Delta + \frac{3}{4} - v_R - \kappa}{2} & \text{if } \frac{3}{4} + \Delta - \kappa \geq v_R \geq \frac{3}{4} - 2x - \Delta - \kappa \\ 0 & \text{if } v_R \geq \frac{3}{4} + \Delta - \kappa, \end{cases}$$

$$y_R^- = \begin{cases} -1 & \text{if } v_L \leq \frac{3}{4} - 2x - \Delta - \kappa \\ \frac{\Delta + v_L - \frac{3}{4} + \kappa}{2} & \text{if } \frac{3}{4} - \Delta - \kappa - 2x \leq v_L \leq \frac{3}{4} - \Delta - \kappa \\ 0 & \text{if } \frac{3}{4} - \Delta - \kappa < v_L, \end{cases}$$

and

$$y_L^- = \begin{cases} -1 & \text{if } v_L \leq \frac{3}{4} - 2x - \Delta + \kappa \\ \frac{\Delta + v_L - \frac{3}{4} - \kappa}{2} & \text{if } \frac{3}{4} - \Delta + \kappa - 2x \leq v_L \leq \frac{3}{4} - \Delta + \kappa \\ 0 & \text{if } \frac{3}{4} - \Delta + \kappa < v_L. \end{cases}$$

Under the supposition that $x > \frac{1}{8} - \frac{\kappa}{2}$, we have $\frac{3}{4} - \Delta - 2x - \kappa < \frac{1}{2}$. Thus, the analysis of the case $v_L, v_R < 1/2$ is the same as $v_L, v_R > 1/2$ and R 's probability of winning is the same.

Case 3: $v_R > \frac{1}{2} > v_L$ or $v_L > \frac{1}{2} > v_R$. These cases are symmetric, so we focus on the first. As before, we obtain the indifferent cut-off types:

$$y_R^+ = \begin{cases} 1 & \text{if } v_R \leq v_L - 2x - \Delta + \kappa \\ \frac{\Delta + v_L - v_R + \kappa}{2} & \text{if } v_L + \Delta + \kappa \geq v_R \geq v_L - 2x - \Delta + \kappa \\ 0 & \text{if } v_R \geq \frac{3}{4} + \Delta + \kappa, \end{cases}$$

$$y_L^+ = \begin{cases} 1 & \text{if } v_R \leq v_L - 2x - \Delta - \kappa \\ \frac{\Delta + v_L - v_R - \kappa}{2} & \text{if } v_L + \Delta - \kappa \geq v_R \geq v_L - 2x - \Delta - \kappa \\ 0 & \text{if } v_R \geq v_L + \Delta - \kappa, \end{cases}$$

$$y_R^- = \begin{cases} -1 & \text{if } \frac{1}{4} \leq \frac{3}{4} - 2x - \Delta - \kappa \\ \frac{\Delta + \frac{1}{4} - \frac{3}{4} + \kappa}{2} & \text{if } \frac{3}{4} - \Delta - \kappa - 2x \leq \frac{1}{4} \leq \frac{3}{4} - \Delta - \kappa \\ 0 & \text{if } \frac{3}{4} - \Delta - \kappa < \frac{1}{4}, \end{cases}$$

and

$$y_L^- = \begin{cases} -1 & \text{if } \frac{1}{4} \leq \frac{3}{4} - 2x - \Delta + \kappa \\ \frac{\Delta + \frac{1}{4} - \frac{3}{4} - \kappa}{2} & \text{if } \frac{3}{4} - \Delta + \kappa - 2x \leq \frac{1}{4} \leq \frac{3}{4} - \Delta + \kappa \\ 0 & \text{if } \frac{3}{4} - \Delta + \kappa < \frac{1}{4}. \end{cases}$$

Recognize that R wins the election unless $y_R^+ = 0$ and $y_L^- = 0$, in which case the parties tie. But $y_L^- = 0$ occurs only if $\frac{3}{4} - \Delta + \kappa < \frac{1}{4}$, which fails Δ small enough.

Putting all of this together, we have

$$\pi^{SE}(x, \Delta) \equiv 2 \left(\frac{1}{8} + \Delta^2 - \frac{\Delta}{4}(3 + 4\kappa) \right) + \frac{1}{4},$$

and R maximizes $\pi^{SE}(x, \Delta)(2x + \Delta)$, yielding

$$x = \frac{1}{6 + 8\kappa}.$$

We recall our conjecture that $x > \frac{1+4\kappa}{8}$. This is satisfied by the interior solution if $\kappa < \frac{\sqrt{5}-2}{4}$. Furthermore, we showed that in our benchmark setting with $\kappa = 0$ any large deviation (where “large” is defined in the proof of Theorem 1) makes R strictly worse off. We conclude that the local conditions (i.e., first-order conditions) are sufficient for party R for $\kappa > 0$ small enough. ■

B.2 Unbiased Imperfect Learning

Our benchmark model focuses on biased learning: voters only learn their ex-ante preferred candidate’s valence when it is above its expected value, and learn the other candidate’s valence only when it is below the expected value. In this extension we maintain incomplete information but untether voters’ information from their ideological leanings.

Specifically, we modify our benchmark mode as follows. Each voter learns both valences with probability $\mu \in (0, 1)$, and learns neither valence with complementary probability $1 - \mu$. We say a voter is ‘informed’ if she learns both valences; otherwise she is ‘uninformed’. All other assumptions are unchanged from our benchmark model.

Notice that the setting $\mu = 1$ corresponds to our ‘full learning’ benchmark, while $\mu = 0$ corresponds to ‘no learning’. To facilitate direct comparison with Theorem 1—our main result—recall that in our selective exposure model every voter learns at least one candidate’s valence with probability one half in that model. So, we can use $\mu = \frac{1}{2}$ in the current extension as a point of comparison.

Proposition 5. *In the unbiased learning extension, the unique symmetric equilibrium is as follows:*

- (i) *With full learning, i.e., $\mu = 1$, we have $x_L^* = -\frac{1}{4}$ and $x_R^* = \frac{1}{4}$.*
- (ii) *With no learning, i.e., $\mu = 0$, we have $x_L^* = x_R^* = 0$.*
- (iii) *With intermediate learning, i.e., $\mu = 1/2$, we have $x_L^* = -\frac{1}{8}$ and $x_R^* = \frac{1}{8}$.*

We therefore obtain the same insights from our main presentation: a reduction of information about valence intensifies platform competition and therefore platform convergence. In the proposition's proof we show more generally that a pure strategy symmetric equilibrium—when it exists—takes the form $x_L^* = -\frac{\mu}{4}$ and $x_R^* = \frac{\mu}{4}$.

Proof of Proposition 5. Suppose the platforms are $x_R \geq 0 \geq x_L$. An uninformed voter i with ideal policy y_i (weakly) prefers R over L if and only if $y_i \geq \frac{x_R + x_L}{2}$. Thus, R 's support from the uninformed voters is

$$(1 - \mu) \frac{1 - \frac{x_L + x_R}{2}}{2}.$$

Consider, instead, the informed voters. An informed voter i with ideal policy y_i (weakly) prefers R over L if and only if $-|y_i - x_R| + v_R \geq -|y_i - x_L| + v_L$. This yields three cases.

1. if $v_L < v_R + x_L - x_R$ all informed voters prefer R over L .
2. if $v_L \in [v_R + x_L - x_R, v_R + x_R - x_L]$ all informed voters with ideal policies $y_i \geq \frac{x_L + x_R}{2} - \frac{v_R - v_L}{2}$ prefer R , and the rest prefer L .
3. if $v_L > v_R + x_R - x_L$ all informed voters prefer L over R .

If $v_L < v_R + x_L - x_R$ then R wins all the informed voters, and is strictly preferred by a majority of *all* voters if and only if

$$(1 - \mu) \frac{1 - \frac{x_L + x_R}{2}}{2} + \mu - \frac{1}{2} > 0 \iff \mu - (1 - \mu) \frac{x_L + x_R}{2} > 0. \quad (18)$$

Similarly, if $v_L > v_R + x_R - x_L$ then L wins all the informed voters, and is strictly preferred by a majority of *all* voters if and only if

$$(1 - \mu) \frac{1 - \frac{x_L + x_R}{2}}{2} + 0 - \frac{1}{2} < 0 \iff \mu + (1 - \mu) \frac{x_L + x_R}{2} > 0. \quad (19)$$

Finally, if $v_L \in [v_R + x_L - x_R, v_R + x_R - x_L]$ the informed voters split between the candidates and R wins a majority of *all* voters if and only if

$$(1 - \mu) \frac{1 - \frac{x_L + x_R}{2}}{2} + \mu \frac{1 - \frac{x_L + x_R - (v_R - v_L)}{2}}{2} - \frac{1}{2} \geq 0 \iff v_L \leq v_R - \frac{x_L + x_R}{\mu}. \quad (20)$$

Conjecture a symmetric equilibrium, i.e., in which $x_L = -x$ and $x_R = x \geq 0$. At this strategy profile the expected policy outcome is zero and both (18) and (19) are satisfied for any $\mu > 0$. We analyze deviations by candidate R to $\tilde{x} \in [0, 1]$, recognizing that a deviation outside this interval is trivially unprofitable. Note that L 's conditions are symmetric. We separately analyze “small” and “large” deviations.

Small deviations. We define a small deviation as a location \tilde{x} for which conditions (18) and (19) continue to hold when $x_L = -x$ and $x_R = \tilde{x} \in [0, 1]$. For any small deviation candidate R wins a majority if and only if (20) holds, which is equivalent to $v_L \leq v_R - \frac{\tilde{x} - x}{\mu}$.

We start with a small rightward deviation to $\tilde{x} > x$. If $0 \geq 1 - \frac{\tilde{x} - x}{\mu}$, i.e., if $1 \leq \frac{\tilde{x} - x}{\mu}$, then R loses for any realized valence and the deviation is unprofitable. So, we restrict attention to small rightward deviations $\tilde{x} \in (x, \mu + x)$. Then, R 's probability of winning

is

$$\pi_R(\tilde{x}, -x) = \int_{(\tilde{x}-x)/\mu}^1 \left(v_R - \frac{(\tilde{x}-x)}{\mu} \right) dv_R$$

R 's payoff is therefore

$$\begin{aligned} \pi_R(\tilde{x}, -x)(\tilde{x}_R - 1) + [1 - \pi_R(\tilde{x}, -x)](-x - 1) \\ = -1 - x + \pi_R(\tilde{x}, -x)(\tilde{x} + x), \end{aligned}$$

which is strictly concave in $\tilde{x} \in (x, \mu + x)$ with unique maximum $\tilde{x} = \frac{\mu-x}{3}$.

Consider, instead, a small leftward deviation to $\tilde{x} \in (0, x)$. Then, R 's probability of winning is

$$\pi_R(\tilde{x}, -x) = \int_0^{1+(\tilde{x}-x)/\mu} \left(v_R - \frac{(\tilde{x}-x)}{\mu} \right) dv_R + \int_{1+(\tilde{x}-x)/\mu}^1 dv_R.$$

With this probability of winning, we verify that R 's payoff is strictly concave in $\tilde{x} \in [0, x)$ and direct calculation yields the unique solution to R 's FOC:

$$\tilde{x} = \frac{1}{3} \left(\sqrt{7\mu^2 - 4\mu x + 4x^2} - 2u + x \right).$$

Recognize that the FOCs corresponding to the small left and right deviations coincide if and only if $x = \frac{\mu}{4}$. We've shown that this is a necessary condition for a symmetric equilibrium, and is also preferred by R to any small deviation when $x_L = -\frac{\mu}{4}$.

Large deviations. We define a large deviation as a location \tilde{x} for which either (18) or (19) fails. We start with a large deviation to the left, i.e., $\tilde{x} < x$. For any such deviation (18) still holds so we are considering cases in which (19) fails, i.e., $\tilde{x} \leq x - \frac{2\mu}{1-\mu}$. We already recovered the necessary condition for a symmetric equilibrium $x = \frac{\mu}{4}$, so that $\tilde{x} < x - \frac{2\mu}{1-\mu}$ is equivalent to $\tilde{x} < \frac{\mu}{4} - \frac{2\mu}{1-\mu}$. But this implies $\tilde{x} < 0$ for any $\mu \in (0, 1)$, so that the expected policy outcome is to the left of zero, and the deviation is unprofitable.

Now consider a large deviation to the right, i.e., $\tilde{x} > x$. For any such deviation, (19) still holds, so it must be that (18) fails. Using the necessary local condition that $x = \frac{\mu}{4}$, failure of (18) is equivalent to $\tilde{x} \geq x + \frac{2\mu}{1-\mu}$. If that inequality is strict, i.e., if $\tilde{x} > x + \frac{2\mu}{1-\mu}$, then a strict majority of voters support L for any realized valence. This implies that R loses the election with probability one so the deviation is unprofitable.

Suppose, finally, R deviates to $\tilde{x} = x + \frac{2\mu}{1-\mu} = \frac{\mu}{4} + \frac{2\mu}{1-\mu}$. At the profile $x_L = -\frac{\mu}{4}$ and $x_R = \tilde{x}$, R loses the election whenever she fails to win the support of all informed votes, i.e., whenever $v_L > v_R - \tilde{x} - x = v_R - \frac{\mu}{4} - \frac{2\mu}{1-\mu} - \frac{\mu}{4} = v_R - \frac{\mu(5-\mu)}{2(1-\mu)}$. Letting $g(\mu) \equiv \frac{\mu(5-\mu)}{2(1-\mu)}$, observe that $v_L \geq v_R - g(\mu)$ for any $(v_L, v_R) \in [0, 1]^2$ if $g(\mu) \geq 1$, i.e., if $\mu \geq \frac{1}{2}(7 - \sqrt{41})$. In that case, R loses with probability one and the deviation is unprofitable. If $g(\mu) < 1$, however, R 's probability of winning at platform $\tilde{x} = \frac{\mu}{4} + \frac{2\mu}{1-\mu}$ is

$$\pi_R \left(\frac{\mu}{4} + \frac{2\mu}{1-\mu}, -\frac{\mu}{4} \right) = \frac{1}{2} \int_{g(\mu)}^1 (v_R - g(\mu)) dv_R.$$

To understand why, recognize that when $v_L \leq v_R - g(\mu)$, (18) holds with equality. Candidate R wins the support of every informed voter and a sufficient share of the uniformed that the candidates tie; R wins with probability one half.

The deviation is profitable for R if and only if the expected policy outcome that the deviation induces lies strictly to the right of zero. This holds if and only if

$$\frac{1}{2} \int_{g(\mu)}^1 (v_R - g(\mu)) dv_R \left(\frac{\mu}{4} + \frac{2\mu}{1-\mu} \right) + \left(1 - \frac{1}{2} \int_{g(\mu)}^1 (v_R - g(\mu)) dv_R \right) \left(-\frac{\mu}{4} \right) > 0. \quad (21)$$

Numerical analysis yields a unique threshold $\mu^* \approx .151$ such that (21) holds if and only if $\mu \in (0, \mu^*)$. Putting all of this together, we conclude that there exists $\mu^* \approx .151$ such that if $\mu \geq \mu^*$ then the unique symmetric equilibrium is $x_L^* = -\frac{\mu}{4}$ and $x_R^* = \frac{\mu}{4}$, and if $0 < \mu < \mu^*$, no symmetric equilibrium exists.

Additional References

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