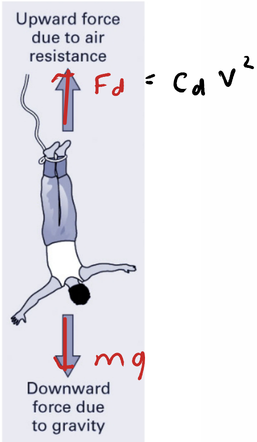
Model Function Example

Assuming a bungee jumper is in mid-flight, from Newton's second law that the change in velocity is determined by the gravitational forces acting on the jumper versus the drag force.

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2 \quad ①$$



To determine the velocity of the jumper over time, we need to solve the differential equation above.

- Analytical (Exact, closed form) Solution
- Numerical (Estimated) Solution

Euler's method

$$\frac{dy}{dt} = F(y, t)$$

$$m \frac{dv}{dt} = mg - c_d v^2$$

$$\frac{dv}{dt} = g - \frac{C_d v^2}{m} \quad ① \text{ O.E} \quad \rightarrow \frac{dv}{dt} = F(v)$$

Analytical soln:

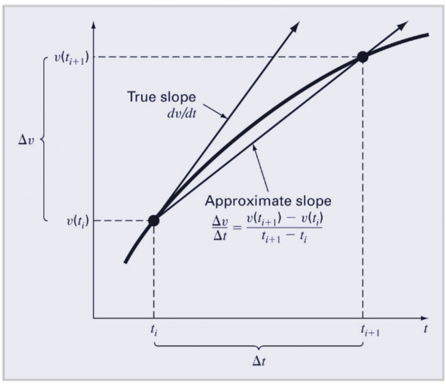
$$v(t) = \sqrt{\frac{gm}{C_d}} \tanh\left(\sqrt{\frac{g C_d}{m}} t\right)$$

Numerical Methods

Numerical methods are those in which the mathematical problem is reformulated so it can be solved by arithmetic operations

To solve the problem using a numerical method, note that the time rate of change of velocity can be **approximated** as:

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$



$$g - \frac{C_d}{m} v(t)^2 = \frac{v(t + \Delta t) - v(t)}{\Delta t} = \frac{dv}{dt}$$

Euler's Method

Substituting the finite difference into the differential equation gives

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

$$\boxed{\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c_d}{m} v^2}$$

Solve for

$$\boxed{\begin{gathered} v(t_{i+1}) = v(t_i) + \left(g - \frac{c_d}{m} v(t_i)^2\right)(t_{i+1} - t_i) \\ \text{new} = \text{old} + \text{slope} \times \text{step} \end{gathered}}$$

Classroom Example

Solve for

$$v(t_{i+1}) = v(t_i) + \left(g - \frac{c_d}{m} v(t_i)^2\right)(t_{i+1} - t_i)$$

$$\text{new} = \text{old} + \text{slope} \times \text{step}$$

Simulation Settings	
m [kg]	80
g [m/sec^2]	9.81
DELTA_t [sec]	0.5
initial velocity [m/sec]	0

time [sec]	velocity [m/sec]	drag coefficient [kg/m]	c_d/m*v^2 [m/sec^2]	dv/dt [m/sec^2]	
0	0	0.25	0	9.81	
0.5					
1					
1.5					
2					

$$v(\quad) = 0 + \left[9.81 - \frac{c_d}{80}(0^2)\right](0.5 - 0)$$

$$v(0.5) = 0 + \left[9.81 - \frac{c_d}{80}\right($$

Classroom Example

Solve for

$$v(t_{i+1}) = v(t_i) + \left(g - \frac{c_d}{m} v(t_i)^2\right)(t_{i+1} - t_i)$$

$$\text{new} = \text{old} + \text{slope} \times \text{step}$$

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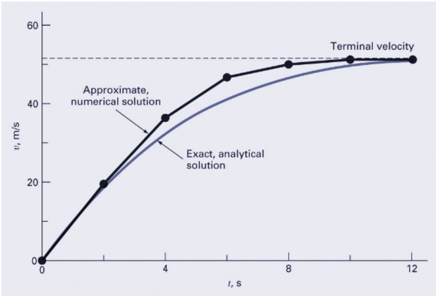
time [sec]	velocity [m/sec]	drag coefficient [kg/m]	c_d/m*v^2 [m/sec^2]	g [m/sec^2]	
0	0.000	0.25	0.000	9.8100	4.905
0.5	4.905	0.25	0.075	9.8100	9.772
1	9.772	0.25	0.298	9.8100	14.52
1.5	14.528	0.25	0.660	9.8100	19.103
2	19.103	0.25	1.140	9.8100	23.438

Numerical Results

As shown in later chapters, the efficiency and accuracy of numerical methods will depend upon how the method is applied.

Applying the previous method in 2 s intervals yields:



Example

(problem 1.26)

An RLC circuit consists of three elements: a resistor (R), an inductor (L), and a capacitor (C). The flow of current across each element induces a voltage drop. Kirchhoff's second voltage law states that the algebraic sum of these voltage drops around a closed circuit is zero,

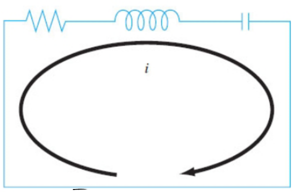
$$iR + L\frac{di}{dt} + \frac{q}{c} = 0$$

where i = current, R = resistance, L = inductance, t = time, q = charge, and C = capacitance. In addition, the current is related to charge as in

$$\frac{dq}{dt} = i \qquad q = it$$

(a) If the initial values are i(0) = 0 and q(0) = 1 C, use Euler's method to solve this pair of differential equations from t = 0 to 0.1 s using a step size of Δt = 0.01 s. Employ the following parameters for your calculation: R = 200 Ω, L = 5 H, and C = 10^{-4} F.

(b) Develop a plot of i and q versus t.



$$\frac{dq}{dt} = i \quad ①$$

$$iR + L\frac{di}{dt} + \frac{q}{c} = 0$$

$$\frac{di}{dt} = -\frac{q}{LC} - \frac{iR}{L} \quad ②$$

$$q = it$$

$$\frac{\Delta i}{\Delta t} = \frac{-q}{LC} - \frac{iR}{L} \qquad \frac{\Delta q}{\Delta t} = i$$

$$\frac{\Delta i}{\Delta t} = \frac{i(t+\Delta t) - i(t)}{\Delta t} \qquad \frac{\Delta q}{\Delta t} = \frac{q(t+\Delta t) - q(t)}{\Delta t}$$

$$-\frac{q}{LC} - \frac{iR}{L} = \frac{i(t+\Delta t) - i(t)}{\Delta t} \qquad i = \frac{q(t+\Delta t) - q(t)}{\Delta t}$$

$$q(t+\Delta t) = i\Delta t + q(t)$$

$$i(t+\Delta t) = \left(\frac{-q(t)}{CL} - \frac{i(t)R}{L}\right)\Delta t + i(t)$$