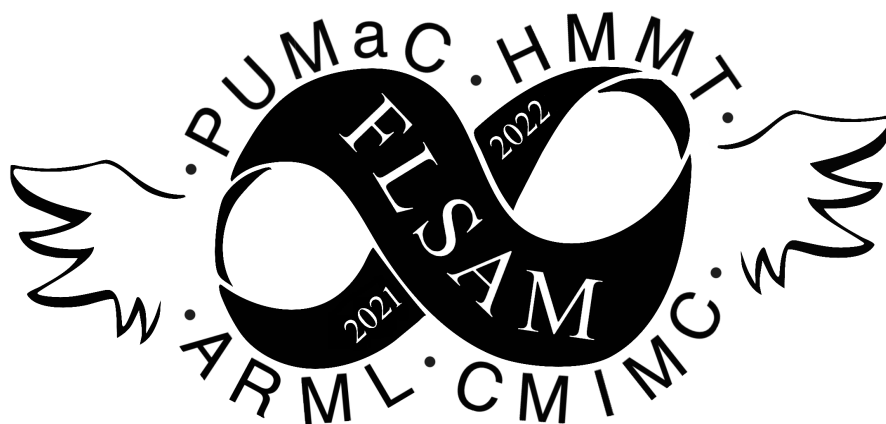


# Florida Student Association of Mathematics



## 2022 ARML Tryout

Set 5 of 7

Welcome to the **2022 FLSAM ARML Tryout!** The tryout will consist of **7 sets** of **2 problems each**. You will have **10 minutes** to work on each set. Write your name and answers directly on each problem set. Scoring is based on the number of correct answers; there is no penalty for wrong answers. Good luck!

**Round 5**

Name: \_\_\_\_\_

9. \_\_\_\_\_

10. \_\_\_\_\_

9. Consider the set  $S = \{1, 2, 3, \dots, 20\}$ . Adam selects 3 distinct elements  $a, b, c$  from  $S$ . Let  $d = \gcd(a, b, c)$ . If the probability that  $d$  is prime is  $\frac{m}{n}$  for relatively prime positive integers  $m$  and  $n$ , find  $100m + n$ .

10. Three points  $A, B, C$  are marked on a sphere  $\Psi$  with diameter at least  $\pi$  is such that the shortest distance between  $A, B$  traveling on the surface of  $\Psi$  is 2, the shortest distance between  $B, C$  is 3, and the distance between  $C, A$  is 4. In space  $\angle ABC = 90^\circ$ ; that is, the angle between line segments  $\overline{AB}, \overline{BC}$  is  $90^\circ$ . What is the ratio of the surface area to the radius of  $\Psi$ ?