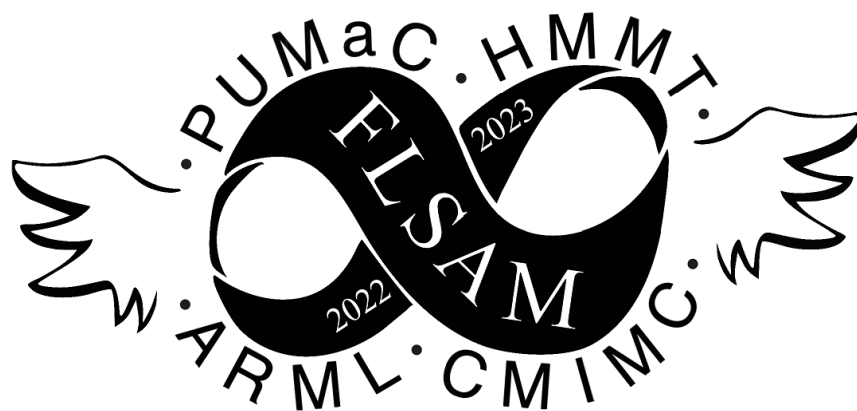# Florida Student Association of Mathematics

2022-2023 Everything Tryout

January 21-22, 2023

Round 1: Algebra and Number Theory

Welcome to the **2022-2023 FLSAM HMMT-PUMaC-CMIMC Tryout!** This is round one of three, each of which will consist of six problems to be completed in 40 minutes. Scoring is simply based on correct answers; there is no penalty for wrong answers. Good luck!

1. The number 3375343 is the product of two primes. Compute the smaller one.

2. Let $a > 1$ be a real number such that $r = a^3 + \frac{1}{a^3}$ and $s = a^2 + \frac{1}{a^2}$ are both integers. Compute the minimum value of rs.

3. Let 2^N be the largest power of 2 which divides $1! \cdot 2! \cdot 3! \cdots 63!$. Compute N.

4. Let F_k denote the kth Fibonacci number defined by $F_1 = F_2 = 1$ and $F_k = F_{k-1} + F_{k-2}$ for $k > 2$, and let P be the unique polynomial with degree 99 satisfying $P(k) = F_k$ for $k = 1, 2, \ldots, 100$. Given that $P(101)$ can be expressed as $F_m + n$ for nonnegative integers m, n with $n < F_{m-1}$, compute $m^2 + n$.

5. Consider function f satisfying $f(0) = 0$ and $f(n) = f(n - p) + 1$ for all integers $n > 1$, where p is the smallest prime divisor of n. Find the sum of all k such that the number of composite solutions to $f(n) = k, 1 < n \le 500$ is maximized.

6. Let $P(x) = 2(x - 1)^2$. Given that there are n values of $x \ge \frac{1}{2}$ satisfying

$$\underbrace{P(P(\ldots P(x) \ldots))}_{2023 \ P's} = x,$$

find the sum of the digits in the base-2 representation of n.