Chapter 4 - Solutions

Arithmetic Sum:

$$\textit{first element}: \sum_{i=1}^{1} i = \frac{(1)(2)}{2} = 1 \quad , \quad \textit{assume}: \sum_{i=1}^{n-1} i = \frac{(n-1)(n)}{2}$$

$$\Rightarrow \sum_{i=1}^{n} i = n + \sum_{i=1}^{n-1} i = n + \frac{(n-1)(n)}{2} = \frac{n^2+n}{2} = \frac{(n)(n+1)}{2}$$

Geometric Sum:`

$$S_n = \sum_{i=0}^{n} r^i = 1 + r + ... + r^n$$

$$rS_n = r + r^2 + ... + r^{n+1} \quad \Rightarrow \quad S_n - rS_n = 1 - r^{n+1}$$

$$S_n = \frac{1-r^{n+1}}{1-r}$$

Reinforcement

4.3. $2n^3 \geq 40n^2 \Rightarrow n \geq 20$

4.8. $2^{10}, 2^{\log n}, 3n + 100\log n, 4n, n\log n, 4n\log n + 2n, n^2 + 10n, n^3, 2^n$

4.9. $O(n)$ 4.10. $O(n)$ 4.11. $O(n^2)$ 4.12. $O(n)$ 4.13. $O(n^3)$

4.16.

$$d(n) \in O(f(n)) \quad \Rightarrow \quad d(n) \leqslant c_1 f(n) \quad , \quad \forall n \geq n_1$$

$$e(n) \in O(g(n)) \quad \Rightarrow \quad e(n) \leqslant c_2 g(n) \quad , \quad \forall n \geq n_2$$

$$c_0 = \max(c_1, c_2) \quad , \quad n_0 = \max(n_1, n_2)$$

$$d(n) + e(n) \leq c_1 f(n) + c_2 g(n) \leq c_0 f(n) + c_0 g(n) = c_0 (f(n) + g(n)) \ , \ \forall n \geq n_0$$

$$\Rightarrow \quad d(n) + e(n) \in O(f(n) + g(n))$$

Creativity

4.45. Sum for all elements in the range $[0, n-1] \Rightarrow sum = \sum_{i=0}^{n-1} i = \frac{(n-1)n}{2}$

```
public static int find_missing_number (int[ ] arr)
{
        int arr_total = 0, n=arr.length;
        int sum = n*(n-1)/2;
        for ( int i = 0; i < n; i++)
                arr_total += arr[i];
        return (sum - arr_total);
}
```