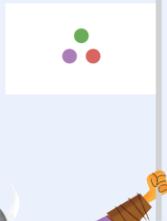


# An introduction to the **julia** language for researchers

Universidad de Sevilla Oct 2023

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# Introduction

Course repository in Github:

[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia)



# Objectives of this workshop

- Get a broad overview of what is Julia and what it is good for
- *Don't get lost in the details*, use “GPT” tools generously (not only for Julia)
- Experiment with notebooks and think of educational and personal uses
- Learn some basic concepts about parallelism and High Performance Computing
- Have fun and enjoy!

# What is Julia?



[What makes Julia so awesome?](#)

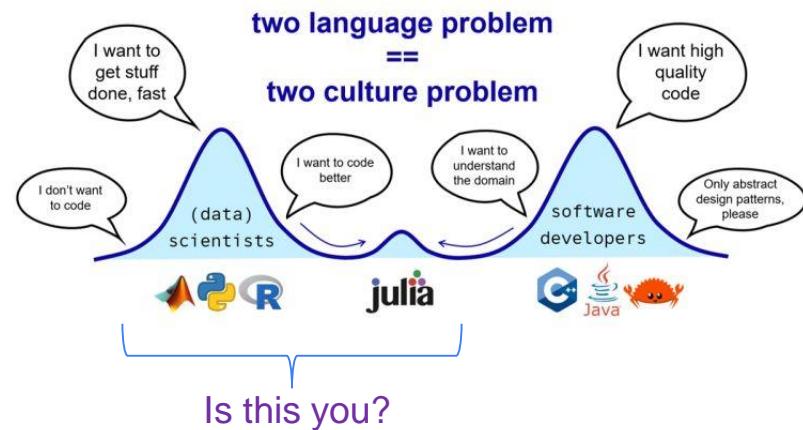
Julia is a language for **Scientific Computing**

**Scientific Computing** is the number crunching required by scientists (and Engineers) to either generate or process large amounts of numerical (or other) data.

Julia intends to solve the “two language problem”, whereby “domain experts” write rough prototypes in an “easy” language like Matlab or Python and then a team of professional programmers has to convert the code to C or equivalent for performance.

With Julia, a scientist can write high performance software from day 1.

The “**Julia**” name makes internet search a nightmare, instead, use “**Julialang**” in Google etc...



Why is it called Julia?

In an interview with [InfoWorld](#) in April 2012, Karpinski said of the name "Julia": "*There's no good reason, really. It just seemed like a pretty name.*"

*Yes, sure...*



# Where does it come from? How much does it cost?

Julia started as a project at MIT to develop a “high level language” (i.e. easy) but with high performance in numerical computations. It is heavily influenced by Lisp, Python, Matlab and others



Here be dragons

Dall-e

2009

2012

2015

2017

2018

2023

Work on Julia was started in 2009, by [Jeff Bezanson](#), [Stefan Karpinski](#), [Viral B. Shah](#), and [Alan Edelman](#)

[Blogpost explaining the mission of Julia](#)

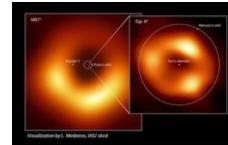


[YouTube](#) Alan Edelman and his dog, an important contributor to Julia, it seems

Julia is completely free and open source

Julia enters the “petaflops” club

[State of Julia](#)



High resolution black hole visualization (coming soon, these are current resolutions)

[JuliaHub](#) created, the company which develops and maintains Julia and provides (paid) HPC services

Julia 1.0 released

Julia gets 20<sup>th</sup> position in the Tiobe index

		Aug 2023	Aug 2022	Change	Programming Language	Ratings	Change
1	1	Python	Python	-2.30%		13.23%	-2.30%
2	2	C	C	-3.35%		11.41%	-3.35%
3	4	C++	C++	+0.48%		10.63%	+0.48%
4	3	Java	Java	-2.34%		10.33%	-2.34%
5	5	C#	C#	+1.64%		7.04%	+1.64%
6	8	JavaScript	JavaScript	+0.85%		3.29%	+0.85%
7	6	Visual Basic	Visual Basic	-2.28%		2.63%	-2.28%
8	9	SQL	SQL	-0.61%		1.52%	-0.61%
9	7	Assembly language	Assembly language	-1.41%		1.34%	-1.41%
10	10	PHP	PHP	-0.09%		1.27%	-0.09%
11	21	Scratch	Scratch	+0.82%		1.22%	+0.82%
12	15	Go	Go	+0.20%		1.16%	+0.20%
13	17	MATLAB	MATLAB	+0.27%		1.09%	+0.27%
14	18	Fortran	Fortran	+0.24%		1.02%	+0.24%
15	31	COBOL	COBOL	+0.93%		0.96%	+0.93%
16	16	QBASIC	QBASIC	+0.01%		0.92%	+0.01%
17	19	Ruby	Ruby	+0.08%		0.91%	+0.08%
18	11	Swift	Swift	-0.35%		0.90%	-0.35%
19	22	Rust	Rust	+0.32%		0.89%	+0.32%
20	26	Julia	Julia	+0.41%		0.85%	+0.41%

# Why it may be interesting for you? (or not)



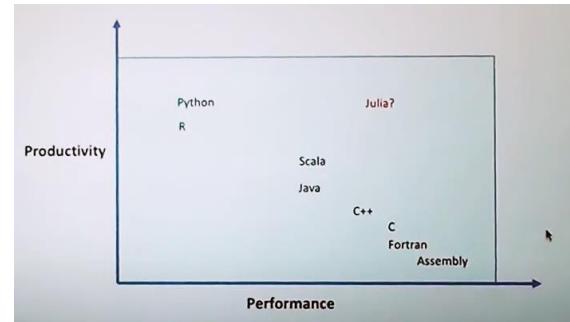
[Julia in 100 seconds](#)

## THE GOOD

Speed and productivity. Hundreds of times faster than Python and as fast as C

Easy to learn and use. Plenty of resources and a good ecosystem.

Extremely elegant language, feels like mathematics and it's excellent to express mathematical concepts.



From the Celeste project

## THE BAD

No generic way to compile executable files ( [although it can be done in some cases](#))

Not very easy support, or ecosystem, for 3D graphics, particularly for real time

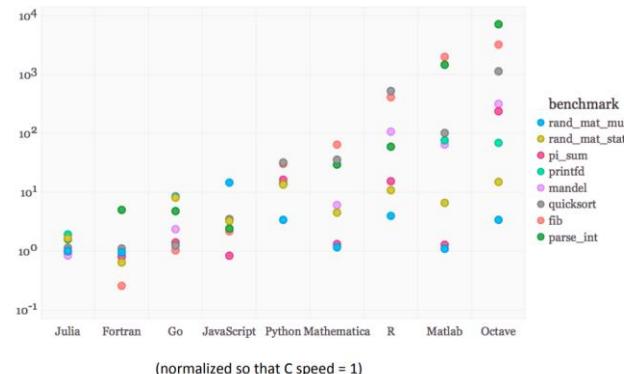
“Time to first plot”. Due to the compilation process Julia may seem quite slow the first time the code is executed (although its getting better with each version).

## THE UGLY

Its name (*for a computer language*)

## Performance on synthetic benchmarks

[ loops, recursion, etc., implemented in most straightforward style ]



(normalized so that C speed = 1)

# Some specific characteristics of Julia (1/2)

**Julia is Not Object Oriented (OOP)**, by a good design choice. Julia is mainly oriented to Functional Programming (FP), although it is not a “pure functional” language. Side effects are allowed (although discouraged)

The encapsulation of OOP can be reproduced using Modules and polymorphism is in fact a weak form of multiple dispatch



[Multiple Dispatch](#) A function in Julia can have many “methods” which are triggered depending on the type of the inputs. Julia will look at the types of the inputs and will “dispatch” the appropriate method. [Multiple dispatch is extremely effective.](#)

Let’s define:

```
function add(x::Int, y::Int) = x + y  (a method which adds integer numbers in the usual way)
```

and then (in the same session);

```
function add(x::Fruit, y::Fruit) = "juice"  (a method “specialized” on data of type “Fruit”, returns the word “juice”)
```

The function `add` has now two methods (two specializations), but it’s still a unique function in Julia

# Some specific characteristics of Julia (2/2)

**Composability** is a consequence of functional programming and allows to safely “compose”, or plug, functions with functions. This facilitates scalability and code abstractions. Watch the great  [Alan Edelman](#) explain composability.

`(sin ∘ cos)(x)` (*composed function of cos with sin applied to x, which is similar, but different from*)

`sin(cos(x))` (*just watch the video...*)

## Just-in-time (JIT) compilation with LLVM.

When executed for the first time, the Julia code is parsed and “lowered” to increasingly optimized intermediate representations (IR) by the LLVM compiler.

At the end, highly optimized machine code for the current hardware is produced and run. Subsequent executions run on already compiled code, so the first execution is slower



[LLVM in 100 seconds](#)



LLVM is also the compiler of C, C++, Lisp, Fortran, Kotlin, Ruby, Swift, Lua, Rust and many others, so it's state of the art nowadays

# Language comparison:

For a “cheatsheet” comparing basic syntax between Matlab, Python and Julia use:

<https://cheatsheets.quantecon.org/>

Julia is most similar to Matlab, code conversion is almost trivial in simple cases

However, note that Matlab is much slower than Julia in most cases

Matlab is significantly slower than Julia on simple evaluation -  
MATLAB Answers – MATLAB Central (mathworks.com)

Julia is typically more than 100 times faster than Python (in fair comparisons – generally much more)

Julialang: Comparing the Speed of Julia, C, and Python  
(copyprogramming.com)

Check this official Julia documentation page for more information if you are transitioning from another language

Noteworthy Differences from other Languages · The Julia Language

## Matlab

Reshape (to 5 rows, 2 columns)

```
A = reshape(1:10, 5, 2)
```

```
A = A.reshape(5, 2)
```

```
A = reshape(1:10, 5, 2)
```

Convert matrix to vector

```
A(:)
```

```
A = A.flatten()
```

```
A[:, :]
```

Flip left/right

```
fliplr(A)
```

```
np.fliplr(A)
```

```
reverse(A, dims = 2)
```

Flip up/down

```
flipud(A)
```

```
np.flipud(A)
```

```
reverse(A, dims = 1)
```

Repeat matrix (3 times in the row dimension, 4 times in the column dimension)

```
repmat(A, 3, 4)
```

```
np.tile(A, (4, 3))
```

```
repeat(A, 3, 4)
```

Preallocating/Similar

```
x = rand(10)  
y = zeros(size(x, 1), size(x, 2))
```

```
x = np.random.rand(3, 3)  
y = np.empty_like(x)  
  
# new dims  
y = np.empty((2, 3))
```

```
x = rand(3, 3)  
y = similar(x)  
# new dims  
y = similar(x, 2, 2)
```

N/A similar type

Broadcast a function over a collection/matrix/vector

```
f = @(x) x.^2  
g = @(x, y) x + 2 + y.^2  
x = 1:10  
y = 2:11  
f(x)  
g(x, y)
```

Functions broadcast directly

```
def f(x):  
    return x**2  
def g(x, y):  
    return x + 2 + y**2  
x = np.arange(1, 10, 1)  
y = np.arange(2, 11, 1)  
f(x)  
g(x, y)
```

Functions broadcast directly

```
f(x) = x^2  
g(x, y) = x + 2 + y^2  
x = 1:10  
y = 2:11  
f.(x)  
g.(x, y)
```

Functions broadcast directly

# The future of Julia

Before you invest your career on a new computer language you may want to know if you are making the best choice.

Today the “safe choice” is **Python** but **Julia** was slated to be the “**Python killer**” due to its performance (about 100 times faster than **Python**) and its elegance.

And then, in May 2023, the creator of LLVM, the **Julia** compiler (and also of Apple’s Swift language, Chris Lattner), made public the “**Mojo**” project (talk about bad name choices...)

**In theory**, **Mojo** will be as fast as **Julia** but 100% compatible with **Python** syntax and will be oriented to machine learning (which is just like saying to numerical analysis).

*If **Mojo** succeeds many of the programming techniques covered in this course will still be of use.*



[Mojo in 100 seconds](#)



[Chris Lattner on Mojo and Julia](#)



# Mandelbrot in Julia vs Mojo

This code comparison shows that even a rather verbose **Julia** code is significantly more concise than **Mojo**, for a comparable performance

*Your call...*

using Plots

```
const xn = 960
const yn = 960
const xmin = -2.0
const xmax = 0.6
const ymin = -1.5
const ymax = 1.5
const MAX_ITERS = 200

function mandelbrot_kernel(c)
    z = c
    for i = 1:MAX_ITERS
        z = z * z + c
        if abs2(z) > 4
            return i
        end
    end
    return MAX_ITERS
end
```

Julia

```
function compute_mandelbrot()
    result = zeros(yn, xn)
```

```
    x_range = range(xmin, xmax, xn)
    y_range = range(ymin, ymax, yn)
```

```
    Threads.@threads for j = 1:yn
        for i = 1:xn
            x = x_range[i]
            y = y_range[j]
            result[i, j] = mandelbrot_kernel(complex(x, y))
        end
    end
    return result
end
```

```
result = compute_mandelbrot()
```

```
x_range = range(xmin, xmax, xn)
y_range = range(ymin, ymax, yn)
heatmap(x_range, y_range, result)
```

```
from benchmark import Benchmark
from complex import ComplexSIMD, ComplexFloat64
from math import pi
from python import Python
from runtime.Icl import num_cores, Runtime
from algorithm import parallelize, vectorize
from tensor import Tensor
from util.index import Index
```

```
alias width = 960
alias height = 960
alias MAX_ITERS = 200

alias min_x = -2.0
alias max_x = 0.6
alias min_y = -1.5
alias max_y = 1.5

alias float_type = DType.float64
alias SIMD_width = SIMDwidthof[float_type]()

def show_plot(tensor: Tensor[float_type]):
    alias scale = 10
    alias dpi = 64

    np = Python.import_module("numpy")
    plt = Python.import_module("matplotlib.pyplot")
    colors = Python.import_module("matplotlib.colors")

    numpy_array = np.zeros((height, width), np.float64)

    for row in range(height):
        for col in range(width):
            numpy_array.itemset((col, row), tensor[col, row])

    fig = plt.figure(1, (scale * height / width), dpi)
    ax = fig.add_axes([0.0, 0.0, 1.0, 1.0], False, 1)
    light = colors.LightSource(315, 10, 0, 1, 0)

    image = light.shade(
        numpy_array, plt.cm.hot, colors.PowerNorm(0.3), "hsv", 0, 0, 1.5
    )
    plt.imshow(image)
    plt.axis("off")
    plt.show()
```

Mojo

```
fn mandelbrot_kernel SIMD[
    SIMD_width: Int,
](c: ComplexSIMD[float_type, SIMD_width]) -> SIMD[float_type, SIMD_width]:
    """A vectorized implementation of the inner mandelbrot computation."""
    var z = ComplexSIMD[float_type, SIMD_width][0, 0]
    var iters = SIMD[float_type, SIMD_width][0]

    for i in range(MAX_ITERS):
        if not in_set_mask.reduce_or():
            break
        in_set_mask = z.squared_norm() <= 4
        iters = in_set_mask.select(i + 1, iters)
        z = z.squared_add(c)

    return iters
```

```
fn parallelized():
    let t = Tensor[float_type](height, width)
```

```
@parameter
fn worker(row: Int):
    let scale_x = (max_x - min_x) / width
    let scale_y = (max_y - min_y) / height
```

```
@parameter
fn compute_vector SIMD_width: Int | col: Int:
    """Each time we operate on a 'SIMD_width' vector of pixels."""
    let cx = min_x + (col + iota[float_type, SIMD_width]()) * scale_x
    let cy = min_y + row * scale_y
    let c = ComplexSIMD[float_type, SIMD_width](cx, cy)
    t.data().SIMD_set(c, SIMD_width)(
        row * width + col, mandelbrot_kernel SIMD[ SIMD_width ](c)
    )
```

```
# Vectorize the call to compute_vector where call gets a chunk of pixels.
vectorize[ SIMD_width, compute_vector ](width)
```

```
with Runtime() as rt:
```

```
@parameter
fn bench_parallel SIMD_width: Int |():
    parallelize[worker](rt, height, 5 * num_cores())
```

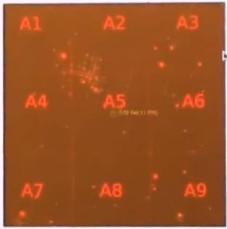
```
alias SIMD_width = SIMDwidthof[DType.float64]()
let parallelized = Benchmark().run(bench_parallel[ SIMD_width ]()) / 1e6
print("Parallelized:", parallelized, "ms")
```

```
try:
    _ = show_plot(t)
except e:
    print("Failed to show plot:", e.value)
```

```
def main():
    parallelized()
```

# Notable examples of Julia in industry and academia

## Celeste project



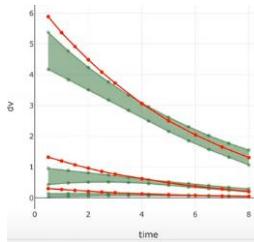
Catalogue of 180 million stars and galaxies in the visible universe (created in 15 minutes with 9000 nodes, 100\_000 cores)

Brought Julia to the “PetaFlop” club (after Fortran and C), first high level language to join the club

Key project for the maturity of Julia in High Performance Computing

[JuliaCon 2017 video](#)

## Pharma

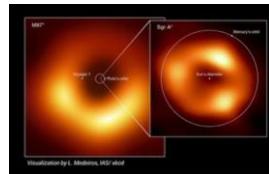


The pharmaceutical industry is a strong user of Julia mainly as a consequence of the proprietary libraries and companies built on the “DifferentialEquations” package.

In pharma, stiff differential equations representing chemical reactions are the norm and Julia excels at this. At least two covid vaccines were developed with the help Julia

[Pumas video](#)

## Black Holes



In 2017 the first synthetic image of the black hole at the center of the Messier 87 galaxy was released. This was generated using Python and C code and took about a week.

The lead researcher has now switched to Julia and can do the same job on a laptop in 15 minutes...

[Imaging Black Holes with the Event Horizon Telescope 2023](#)

[High performance Black Hole Imaging 2022](#)

## Boeing

A news article from Forbes magazine. The main headline is "Deepak Vinchhi, on plans at JuliaHub after recent funding from a VC firm backed by Boeing". The article discusses the strategic investment made by Boeing in JuliaHub. The sidebar includes links to other news stories and a navigation menu for the website.

There's something going on between Boeing and Julia...

A venture capital company owned by Boeing has invested 13 million dollars in JuliaHub, the company which maintains and develops Julia.

Boeing was sponsor of the [JuliaCon 2023](#)

Many other companies are using Julia at different levels, we'll see how this evolves in the future

# Installation



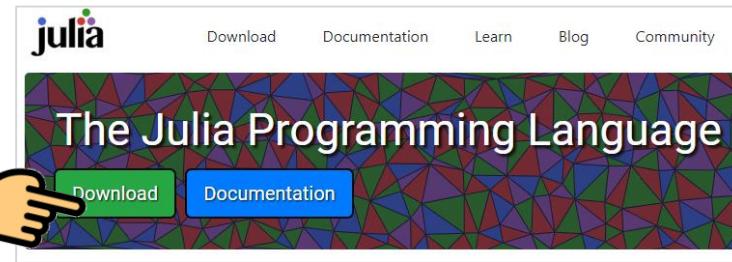
# Installing Julia



[How to install Julia and Visual Studio Code](#)

1 Go to [The Julia Programming Language web page \(julialang.org\)](https://julialang.org)

2 Click on the “Download” button and then on your preferred version. If you are in windows on a decent computer (64bits processor) select the version highlighted below



Download Julia

Star 43,340

Please star us on [GitHub](#). If you use Julia in your research, please [cite us](#). If possible, do consider [sponsoring us](#).

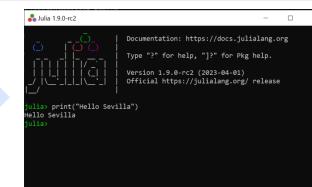
Current stable release: v1.9.3 (August 24, 2023)

Checksums for this release are available in both [MD5](#) and [SHA256](#) formats.

Windows [help]	64-bit (installer), 32-bit (portable)	32-bit (installer), 32-bit (portable)
macOS x86 (Intel or Rosetta) [help]	64-bit (.dmg), 64-bit (.tar.gz)	
macOS (Apple Silicon) [help]	64-bit (.dmg), 64-bit (.tar.gz)	
Generic Linux on x86 [help]	64-bit (glibc) (GPG), 64-bit (musl) <sup>[1]</sup> (GPG)	32-bit (GPG)
Generic Linux on ARM [help]	64-bit (AArch64) (GPG)	
Generic Linux on PowerPC [help]	64-bit (little endian) (GPG)	
Generic FreeBSD on x86 [help]	64-bit (GPG)	
Source	Tarball (GPG)	Tarball with dependencies (GPG)
		Github

3 Accept all the proposed settings during the installation. Click on “Add to path”

4 After the installation find the Julia icon in your desktop and double-click on it, it will open the Julia REPL



“The REPL”, an instance of Julia running

# REPL and package manager



The Julia REPL

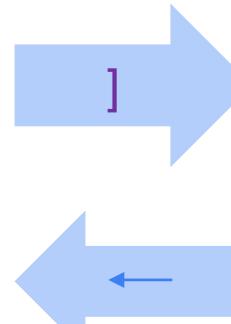
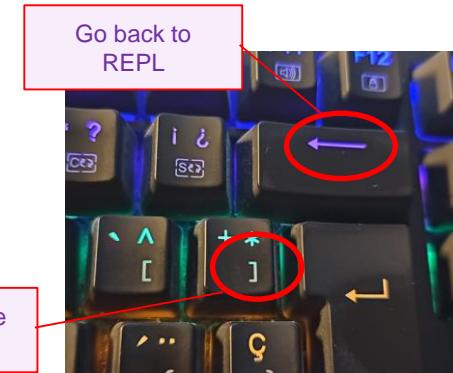
“REPL” stands for Read Evaluate Print Loop and it’s where you are just after starting Julia.

It is the most basic way in which Julia can be written using the “console”

The package manager is a “mode” of the REPL in which you can add “libraries” (packages)

It is accessed from the REPL by typing: **]**

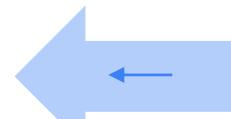
To exit the package manager and go back to the REPL type: **←**



```
Julia 1.9.0-rc2
Documentation: https://docs.julialang.org
Type "?" for help, "]?" for Pkg help.
Version 1.9.0-rc2 (2023-04-01)
Official https://julialang.org/ release

julia> print("Hello Sevilla")
Hello Sevilla
julia>
```

REPL



```
Julia 1.9.0-rc2
Documentation: https://docs.julialang.org
Type "?" for help, "]?" for Pkg help.
Version 1.9.0-rc2 (2023-04-01)
Official https://julialang.org/ release

julia> print("Hello Sevilla")
Hello Sevilla
(@v1.9) pkg>
```

Name of the current environment (where the packages will be added). In this case, the global environment

Package manager

# Basic usage of the package manager



[Managing Julia packages](#)

The Julia package manager enables to install “packages” (libraries of code), update them, delete them and various other actions.

**It is easily the best package manager of any language today.**

To add a package, inside the package manager (after typing `]` in the REPL) type:

`add name_of_the_package`

```
(@v1.9) pkg> add pluto
  Updating registry at `C:\Users\USUARIO\.julia\registries\General.toml`
  Downloading [=====] 91.9 %
```

*In this case we are adding the “Pluto” package, which we will use for this course*

To check the status (versions) of the packages type: `st`

To update all packages to the latest version type: `update`

# >>> Add required packages now <<<

Packages (libraries) are downloaded from an internet repository (very often Github) when “added” in the package manager, so you need an internet connection.

After downloading, the packages are pre-compiled for your specific hardware by Julia. This takes a while, so this is a good time to add various packages that we will use in this course.

- 1- Copy the following block of text from the box below (select with the mouse and right-click “copy” or type “Ctrl-c”)

```
add Pluto PlutoUI Plots
```

- 2- Enter the package manager (type `]`) and paste the text. If “Ctrl+v” does not work, just do a mouse right-click after `pkg>`
- 3- Press `enter` (if the installation does not start on its own) and let Julia download and precompile the packages.

The box below has packages that take longer to install, open a new Julia REPL (click on the desktop icon), enter the package manager and copy the text to add them.

```
add Flux BenchmarkTools Images JuMP Cbc Statistics DataFrames Sound
```

- 4- While the installation of the second group of packages goes on, go back to the first Julia session and exit the package manager (`←`)

# Pluto notebooks for productive Julia development



Pluto is a Julia “program” which creates an automatically opens a locally hosted **webpage**, connected with Julia, where you can write and execute Julia code and add other non-code information in a very pleasant way.

Formally, Pluto is a notebook, like **Jupyter** (which means **Julia**, **Python** and **R**). You can also use Jupyter to write Julia notebooks (and many people do) but the main advantage of Pluto is that it is **reactive**.

A **reactive notebook** makes sure that the state of the code you see is consistent, i.e., when you change a value, all the code which needs to change in response will be re-evaluated. A good example is an Excel sheet where the values of some cells depend on the values of others. When you change some of those values, the rest of the cells are automatically updated.

**Sharing fully interactive notebooks**

Goal: Send someone a link to play with your notebook on the web.

Requirements: no web architecture/container skills & free (as in beer).

Possible solution

Pluto back-end is lightweight

Runs on free 512MB heroku server ([demo](#))

make it one click



[Watch the great Fons introducing Pluto to the world](#)

[Watch more Fons, another peculiar character in the Julia world](#)

**Using JavaScript inside Pluto**

You have already seen that Pluto is designed to be interactive. You can make fantastic exploratory documents using just the basic inputs provided by Pluto, together with the wide range of visualization libraries that Julia offers.

However, if you want to take your exploratory document one step further, then Pluto offers a great framework for combining Julia with HTML, CSS and JavaScript.

using Pluto; #, HyperTextLiteral,

```
function() {
    alert("Hello!");
    // Note: for some reason, alert() does not work in the browser
    // console.log("Hello!");
}
//>>> run()
Hello!
```

wow = [1, 2, 3]
wow \* [1, 2]

Prerequisites

**Pluto is also open source and free:**  
Github homepage <https://github.com/fonsp/Pluto.jl>



# Installing and starting Pluto

Pluto homepage <https://plutojl.org/>

If you have already added the Pluto package (explained in the previous pages), you have access to Pluto. To start it type in the REPL:

```
import Pluto; Pluto.run()      (adding a package is like buying a book and “importing” a package is like opening the book)
```

`Pluto.run()` calls the run function of the Pluto package and will open an empty notebook as a tab in your internet browser (it will even start the browser if required).

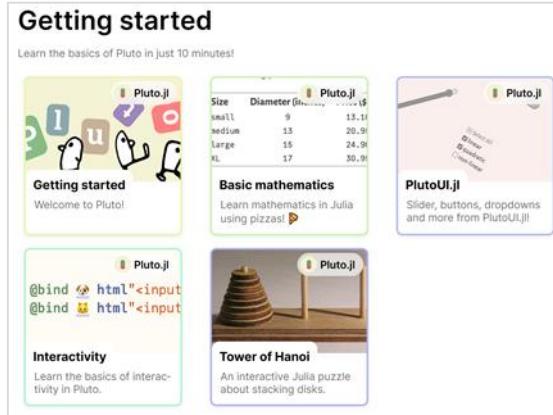
If you put the full path name or the url of an existing notebook in the brackets it will open it.

All the instructions are well explained in the Pluto homepage <https://plutojl.org/>

Go to

<https://featured.plutojl.org/>

for a good collection of Pluto examples running online



If you want to edit these examples click on the button at the top right;

Edit or run this notebook

and follow the instructions



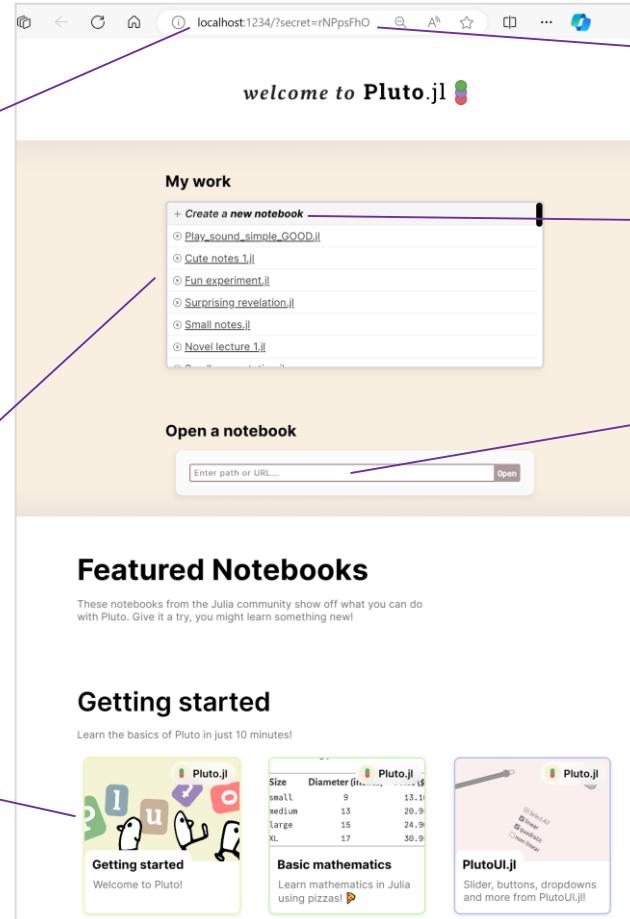
# Exploring Pluto

Pluto is a web page “served” by Julia, in this case in port 1234

These are previous notebooks on which you have worked, just click to open. The funny names (e.g. “cute notes”) are given automatically by Pluto to notebooks from which you exit without giving a name

A good way to learn Pluto is to open the example notebooks and play with it

## Pluto start page



The “secret” is a way to differentiate different Pluto sessions in different tabs, don’t worry about it

To **create** a notebook just click here

To **open** a Pluto notebook paste the full path to the file (or navigate by typing the path in the field) or paste a url of a file hosted in a cloud service

Both:

C:\Users\USUARIO\Lorenz\_Euler.jl

and

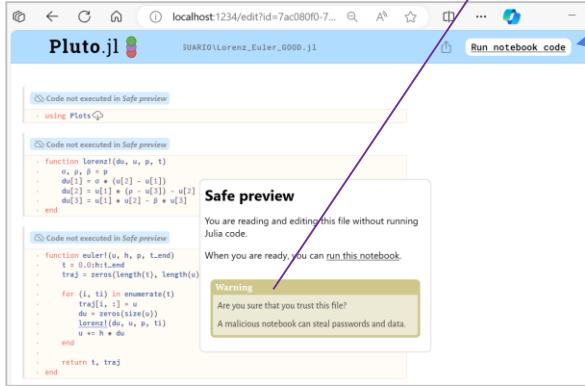
[https://github.com/MyRepo/MyProject/blob/main/Lorenz\\_Euler.jl](https://github.com/MyRepo/MyProject/blob/main/Lorenz_Euler.jl)

are well formatted names of possible Pluto (Julia) files



# Exploring Pluto

If you get this message when trying to open a file, and you trust the source, just click “Run notebook code” (top right)



Pluto.jl

```
localhost:1234/edit?id=7ac080f0-7...
```

Run notebook code

Code not executed in Safe preview.

```
using Plots
```

Code not executed in Safe preview.

```
function lorenz!(du, u, p, t)
    α, β = p
    du[1] = u[2] * (u[3] - u[1])
    du[2] = u[1] * (p - u[3]) - u[2]
    du[3] = u[1] * u[2] + β * u[3]
end
```

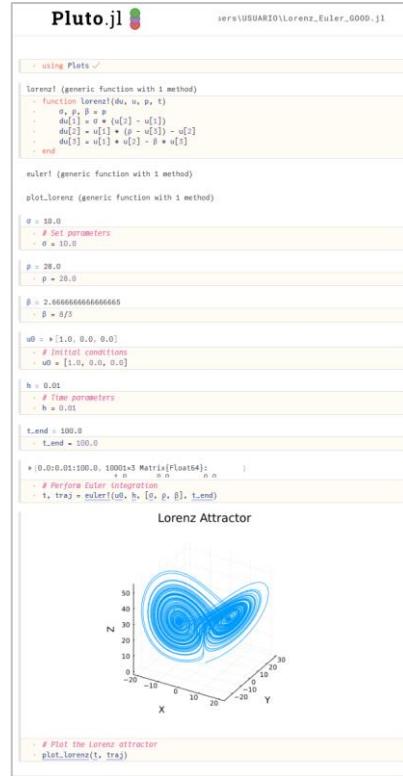
Code not executed in Safe preview.

```
function euler!(u, h, p, t_end)
    t = 0.0
    u0 = zeros(3)
    traj = zeros(length(t), length(u))
    for (i, t_i) in enumerate(t)
        traj[i, :] = u
        du = zeros(size(u))
        lorenz!(du, u, p, t_i)
        u += h * du
    end
    return t, traj
end
```

**Safe preview**  
You are reading and editing this file without running Julia code.  
When you are ready, you can run this notebook.

**Warning**  
Are you sure that you trust this file?  
A malicious notebook can steal passwords and data.

Use “Control+mouse\_wheel” or “Control+” or “Control-” to zoom in and out in Pluto (like in any other web page)



Pluto.jl

```
svers\USUARIO\Lorenz_Euler_0000.jl
```

using Plots

```
lorenz! (generic function with 1 method)
```

```
function lorenz!(du, u, p, t)
    α, β = p
    du[1] = u[2] * (u[3] - u[1])
    du[2] = u[1] * (p - u[3]) - u[2]
    du[3] = u[1] * u[2] + β * u[3]
end
```

```
euler! (generic function with 1 method)
```

```
plot.lorenz (generic function with 1 method)
```

```
u = [10.0
      # Set parameters
      0.01
      28.0
      0.001]
```

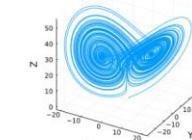
```
p = 10.0
      # Time parameters
      0.01
      100.0
      0.001
```

```
u0 = [1.0, 0.0, 0.0]
      # Initial conditions
      u0 = [1.0, 0.0, 0.0]
```

```
h = 0.01
      # Time parameters
      h = 0.01
      t_end = 100.0
      t_end = 100.0
```

```
# Perform Euler integration
t, traj = euler!(u0, h, [α, β], t_end)
```

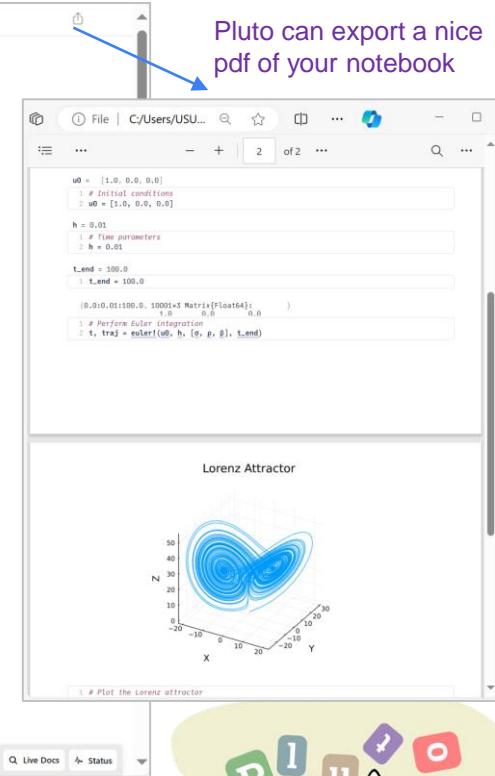
Lorenz Attractor



```
# Plot the Lorenz attractor
plot.lorenz(t, traj)
```

Pluto notebooks are pure Julia files but with some additional information (including the names and versions of the packages specified by the `using` keyword). A Pluto file can be run in Julia directly (unlike in the case of Jupyter notebooks)

Pluto is **REACTIVE**: the order of the cells in Pluto is not important, Pluto will track the dependencies of the variables and functions and execute everything that depends on anything which changes (e.g. a variable value changes or a cell is recompiled with “Shift+Enter”)



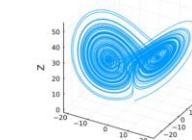
Pluto can export a nice pdf of your notebook

File | C:/Users/USU...

... 2 of 2 ...

Initial conditions: u0 = [1.0, 0.0, 0.0]; Time parameters: h = 0.01; t\_end = 100.0; t\_end = 100.0; Plot the Lorenz attractor

Lorenz Attractor



# Exploring Pluto

Exit the notebook (it will be saved with a funny name if you don't save it) and go back to the start page

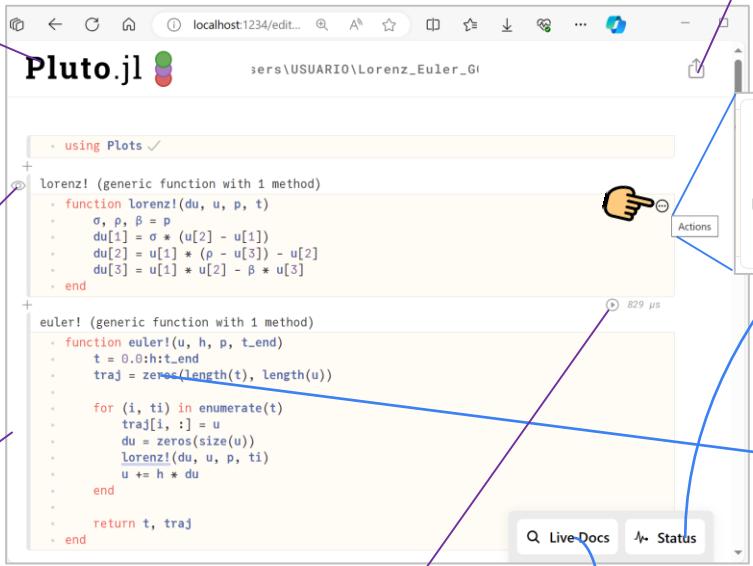
Hide/show the code in the cell. The results of the cell, which could be none, will always be shown above the cell

Click on the margin of the cell to move it up or down in the notebook.  
**Pluto does not care about the order of the cells**

An important peculiarity of Pluto is that a cell can only hold one "block of code" (excluding comments).

This is either a single variable assignment, a function definition or a "begin...end" block

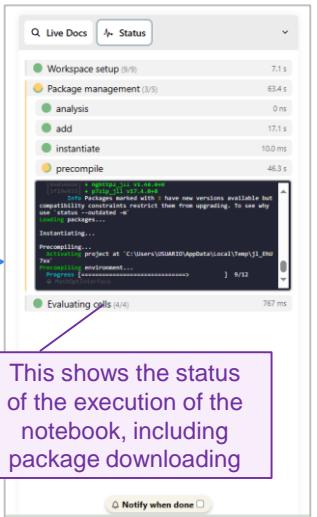
If you type several lines, Pluto will propose to either automatically split them or automatically wrap them inside a `begin...end` block



```
using Plots ✓
+
lorenz! (generic function with 1 method)
+   function lorenz!(du, u, p, t)
+     σ, ρ, β = p
+     du[1] = σ * (u[2] - u[1])
+     du[2] = u[1] * (ρ - u[3]) - u[2]
+     du[3] = u[1] * u[2] - β * u[3]
+   end
+
euler! (generic function with 1 method)
+   function euler!(u, h, p, t_end)
+     t = 0.0:t_end
+     traj = zeros(length(t), length(u))
+
+     for (i, ti) in enumerate(t)
+       traj[i, :] = u
+       du = zeros(size(u))
+       lorenz!(du, u, p, ti)
+       u += h * du
+     end
+
+     return t, traj
+   end
```

Export the notebook, including as a pdf file

Disable cell prevents the execution of the code, use to debug



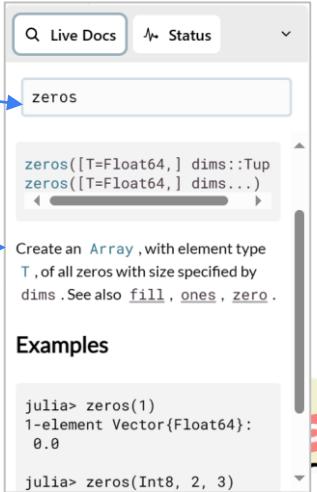
Live Docs ▾ Status

- Workspace setup (9/9) 7.1s
- Package management (3/3) 65.4s
- analysis 0ms
- add 17.1s
- instantiate 10.0 ms
- precompile 46.3s

Evaluating cells (4/4) 767 ms

Notify when done

This shows the status of the execution of the notebook, including package downloading



Live Docs ▾ Status

zeros

zeros([T=Float64,] dims::Tuple{Int64,...})

zeros([T=Float64,] dims...) Create an `Array`, with element type `T`, of all zeros with size specified by `dims`. See also `fill`, `ones`, `zero`.

Examples

```
julia> zeros(1)
1-element Vector{Float64}:
 0.0

julia> zeros(Int8, 2, 3)
```

To execute a cell, click here or, better, type "Shift+Enter"

In theory you can also stop the execution with the stop button, but don't count on it.

You will get the cell execution time at the end

Click on a word in the code to get help, if available

# Error messages in Julia

A large part of the time spent in developing software is used in fixing errors (“debugging”)

Error messages are always frustrating and not always very clear but [Julia tries to be as helpful as possible](#).

You can always copy the error message type and search in google, most likely you are not the first to encounter it...

“**no method matching**” is very typical, it happens when a function is called with input of a type for which there is no method (so check what you are sending to the function)

Julia REPL

```
ERROR: LoadError: MethodError: no method matching +(::Int64, ::String)
Closest candidates are:
  +(::Any, ::Any, ::Any, ::Any...) at operators.jl:591
  +(::T, ::T) where T<:Union{Int128, Int16, Int32, Int64, Int8, UInt128, UInt16,
  +(::Union{Int16, Int32, Int64, Int8}, ::BigInt) at gmp.jl:537
  ...
Stacktrace:
 [1] top-level scope
   @ ~/Learning/jul/first.jl:41
in expression starting at /Users/demo/Learning/jul/first.jl:41
```

Julia tries to help by proposing types of inputs for which there are methods

Pluto

```
BoundsError: attempt to access 3-element Vector{Float64} at index [31]
1. getindex @ essentials.jl:13 [inlined]
2. lorenz! @ Other: 5 [inlined]
3. euler!(::Vector{Float64}, ::Float64, ::Vector{Float64}, ::Float64) @ Other: 8
4. top-level scope @ Local: 2 [inlined]
  # Perform Euler integration
  t, traj = euler!(u0, h, [σ, ρ, β], t_end)
```

```
UndefVarError: `t` not defined
  # Plot the Lorenz attractor
  plot_lorenz(t, traj)
```

In Pluto a certain cell can give an error which is caused on a different cell. Click on the “Other...” to go to the line potentially causing the error.

All the cells affected by the error will show an error too, in this case “t” is not defined because its creating failed somewhere else, just go to the original error, fix it, and try to execute cell by cell to find when things fail

# Getting help in Julia (and you will need it)



[Getting Help in Julia](#)

Depending on where you are you can get immediate help from Julia in two ways:

In the REPL: Type “?” to enter the help mode and type the name of the function for which you need help

```
julia> # Type ? and then enter the keyword for which you want help
help> zeros
search: zeros count_zeros set_zero_subnormals get_zero_subnormals leading_zeros trailing_zeros zero iszero RoundToZero
zeros([T=Float64,] dims::Tuple)
zeros([T=Float64,] dims...)
Create an Array, with element type T, of all zeros with size specified by dims. See also fill, ones, zero.
```

In Pluto: Click on the button and then click on the keyword you are interested in.

In order for a function in Julia to provide help it has to be written with “Doc strings” like;

Write the “doc strings” just above the function definition with exactly the format shown

```
"""
ms2kt(v)

Convert a speed v from meters per second (m/s) to knots (kt)

# Examples
```julia-repl
julia> ms2kt(1)
1.94384449
```
"""

ms2kt(v) = v*1.94384449
```

If you click on “ms2kt” in the Pluto code or type it in the Live Docs form you will see this ->

The screenshot shows the Pluto interface with the "Live Docs" tab selected. In the search bar, "ms2kt" is typed. Below the search bar, the function signature "ms2kt" is listed. Underneath it, the docstring "Convert a speed v from meters per second (m/s) to knots (kt)" is displayed. A section titled "Examples" shows the command "julia> ms2kt(1)" followed by the output "1.94384449".

The [Julia official documentation page](#) is an excellent source of information and examples. If you are like me you can download a [pdf file](#) with all the language documentation and print all of its 1644 pages to read in the beach.

# First Experiments



# Course notebooks (Rémy Vezy)



[Github repository for this course](#)

A good place to start learning the basic Julia syntax is a course developed by Rémy Vezy, who is a researcher in plant (vegetal...) structure.

He uses Pluto and has a collection of notebooks in Github accompanied by videos in YouTube

The screenshot shows a GitHub repository page for 'VEZY/julia\_course'. The repository has 2 branches and 0 tags. The 'Code' dropdown menu is open, showing options like 'Go to file' and 'Code'. The 'About' section of the README contains the text: 'Julia course: from total beginner to power user!'. Below this, it says: 'This repository is a collection of resources to teach everything about the Julia programming language. These resources are used in the [Julia: from total beginner to power user](#) YouTube playlist.' It also provides 'Direct links to the notebooks' and instructions for using them with Pluto. The repository has 47 commits, with the most recent being 'VEZY Remove cache' by '889f3de' last month. The 'Languages' section shows Julia at 99.7%, CSS at 7.4%, JavaScript at 2.8%, and HTML at 0.1%. A large orange hand icon points to the 'About' section of the README.

The screenshot shows the 'Clone' section of the GitHub repository page. It includes options for 'Local' and 'Codespaces', and a 'Clone' section with 'HTTPS' and 'GitHub CLI' tabs. The URL 'https://github.com/VEZY/julia\_course.git' is listed. Below it are buttons for 'Open with GitHub Desktop' and 'Download ZIP'. A large orange hand icon points to the 'Download ZIP' button. The entire screenshot is timestamped '2 years ago'.

# Opening the course notes in Pluto

In the following slides, an icon like  denotes that the text behind is the url to a Pluto notebook. Copy the text and paste it into the “Open a notebook” field in the Pluto start page.

**Direct links to the notebooks** 

You can use the notebooks either by using the direct links provided here, or by downloading/cloning the repository (see below).

To use the direct link, open julia, then type `using Pluto`, and execute the line of code provided below. If you need to install Pluto first, see below.

1. Variables and basic types in Julia



```
book = "https://raw.githubusercontent.com/VEZY/julia_course/main/content/1-variables_and_basic_types.jl"
```

welcome to Pluto.jl 

**My work**

- + Create a new notebook
- ① 00\_LIN\_FSD\_TOPO\_V0.119.jl
- ② 00\_LIN\_FSD\_TOPO\_V0.136.jl
- ③ Tiny\_theory.jl
- ④ 00\_LIN\_FSD\_TOPO\_V0.135.jl
- ⑤ 00\_LIN\_FSD\_TOPO\_V0.134.jl
- ⑥ Wild\_notes.jl
- ⑦ Important lecture.jl

**Open a notebook**

Enter path or URL...

 [https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/1-variables\\_and\\_basic\\_types.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/1-variables_and_basic_types.jl)

# Basic Julia syntax (1/2) (Rémy Vezy)

These videos and Pluto notebooks below are an excellent starting point to learn basic Julia syntax



1- [Variables and basic types:](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/1-variables\\_and\\_basic\\_types.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/1-variables_and_basic_types.jl)



2- [Arrays](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/2-arrays.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/2-arrays.jl)



3- [Tuples](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/3-tuples.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/3-tuples.jl)



4- [Dictionaries](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/4-dictionnaries.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/4-dictionnaries.jl)



5- [Basic operators](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/5-basic\\_operators.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/5-basic_operators.jl)

# Basic Julia syntax (2/2) (Rémy Vezy)



6- [String operators](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/6-string\\_operators.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/6-string_operators.jl)



7- [Compound expressions \(begin, let...\)](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/7-compound\\_expressions.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/7-compound_expressions.jl)



8- [Conditional statements \(if...else...\)](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/8-conditional\\_statements.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/8-conditional_statements.jl)



9- [For loops \(each index, enumerate...\)](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/9-for\\_loops.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/9-for_loops.jl)



10- [Functions](#)

[https://raw.githubusercontent.com/VEZY/julia\\_course/main/content/10-functions.jl](https://raw.githubusercontent.com/VEZY/julia_course/main/content/10-functions.jl)

# Linear Algebra and array virtuosity

Arrays are an extremely important data structure in Julia and the language provides an efficient syntax to use arrays and linear algebra support without the need to add any package.

It's very important to be "fluent" in the syntax around arrays. Here are some examples of usual code snippets (try yourself to follow these examples and play freely)

`A = rand(3,3); B = rand(3,3)` creates two 3x3 arrays of random Float64 numbers between 0 and 1. ";" here means "continue"

`inv(A)` same as `A^-1` calculates the inverse of A. `transpose(A) == A'` = `true` ways to compute the transpose of A

`b = rand(3)` creates a vector with random numbers. Vectors are **always column vectors** and Julia is "column major" (more later)

`x = A\b` solves the linear system  $Ax = b$

`A+B` adds all the elements of A and B, elementwise

`A^2` computes the operation `A * A`

`A.^2` computes the square of each individual element of the array (i.e.,  $a_{11}^2 \ a_{12}^2 \ a_{13}^2$  etc...)

`A[:,2]` extracts the elements intersection between all the elements in the first dimension and the elements of index 2 in the second dimension (so, in this case it returns the second column). `A[3,:]` returns the third row

`collect(1:5)` creates an array (in this case a column vector) from a range. `collect` is very general to create arrays from iterators

# Special symbols and idiosyncrasy (1/5)

The idiosyncrasy of a computer language can make the code look ugly and not easy to understand. Julia is quite reasonable in this aspect but it still uses some non-intuitive (but generally non arbitrary) notation.

The following is a list, ordered roughly by frequency of occurrence, of some symbols which are used in Julia:

**begin ... end** defines a block of code, necessary in Pluto but normally not very common in Julia.

( ) Function parameters, as in **function add(x,y) = x + y**

[ ] Array indices, as in **A[3,4]** Indices can also be given by a range, **A[1:2,4]** (elements in the first and second rows, column 4). This notation extends to **A[:, 4]**, which represents the elements in all the rows from the fourth column)

; After a statement it means “don’t show the result in REPL or Pluto”, it denotes the separator for keyword arguments in functions and the separator for rows in an array definition **A = [ 1 2 ; 3 4 ]**

(x, ) definition of tuple of a single element. Tuples are very important and, for example, the inputs to a function are a tuple.

# Special symbols and idiosyncrasy (2/5)

Julia is “1” indexed, i.e., the first element of an iterator V (e.g. a vector) is `V[1]` (unlike in C and other languages)

- Broadcasting operator (“dot operator”). It means that the operator which it precedes shall be applied elementwise to the arrays, as in `B .= A .* C`, where A, B and C are arrays of the same dimension and the result is an array of the products of the corresponding elements of B and C ( $a_{11} \cdot c_{11}$   $a_{12} \cdot c_{12}$  etc...)

`{}` Denotes the type of a variable, as in `Vector{Float64}(undef, 3)`, which defines a vector of 3 dimensions of type `Float64`

`@` used to identify a macro, which is a function which operates on code and returns executable code. Common examples are `@time`, which returns the time taken to run a function and `@.`. Which “vectorizes” all the operations in its line (more later)

`::` Declares the variable type for an input in a function, as in `add(x::Float32, y::Float32)`

— Optional thousand separator, just for clarity, as in `1_000_000` to represent one million. `1000000` also works.

`%` remainder of an integer division, or modulo operator; `8 % 3` returns `2`. Same as `mod(8,3)`

`÷` result of an integer division; `10 ÷ 3` returns `3`

# Special symbols and idiosyncrasy (3/5)

`# This is a comment` everything after `#` is treated as a comment in the code in the same line

`"""`

`I am`

`a comment`

a multiline comment, also used for “doc strings”

`"""`

`#=`

`I am also`

`a comment`

alternative multiline comment

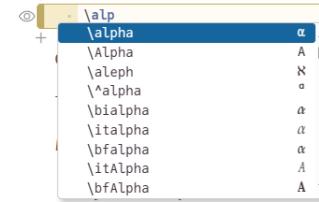
`=#`

! Convention for a mutating function (called “Bang!”). Mutating functions are those which change the value of the input passed to them. This is not “pure functional programming”, as this is a “side effect”, but can save memory allocations and make the code more efficient, for example in the function `mul!(Y, A, B) -> Y` which calculates the matrix-matrix or matrix-vector product \$AB\$ and stores the result in Y, overwriting the existing value of Y.

`\` escape character for \, for example to write file paths as in `c:\file.txt` equivalent to `c:\file.txt` in Windows

# Special symbols and idiosyncrasy (4/5)

Special symbols, Greek letters. Can be written by typing, for example `\alpha + tab` and choosing the option you want



- ☺ Emojis can be selected after typing “Win + .” and can be used as variables and function names

- Composition of functions, written by typing “`\circ + tab`”, as in `(sin ∘ cos)(2)`

`md"># ...` Markdown block, Pluto will print a non-code text above the cell of different size depending on how many “#s” are used, as in `md### Set Operating Point for calculations ↗` “

Set Operating Point for calculations ↗  
- `md### Set Operating Point for calculations ↗` “

`==` Is equal? Note that `=` is used for assignment; `3 == 3` -> true    `a = 3`; `a -> 3`

`!=` not equal

`≈` “Is approximately equal” (`\approx + tab`). Used to compare numbers (particularly floating point) ignoring the machine truncation error

# Special symbols and idiosyncrasy (5/5)

\$ Denotes that the variable value shall be interpolated in a string (and also in the input to some macros like `@btime`)

A screenshot of a terminal window. The script contains:  
```python  
v = 3  
+ v = 3  
  
. print("The value of v is = \$v")  
  
The value of v is = 3  
```  
The last line, "The value of v is = 3", is highlighted in green.

|> “Pipe operator”, very popular in functional programming. `|>(x, f)` or `x |> f` means `f(x)` and it allows composition or concatenation of functions with a simple syntax (like a pipe), as in `f(g(x))` written as `x |> g |> f`

`f(x) = x^2 ; g(x) = x/2`      `; 4 |> f |> g = 8`      `; |>(4,f) = 16`

... “Splat operator” ..., represents a sequence of arguments. ... can be used in function definitions, to indicate that the function accepts an arbitrary number of arguments. ... can also be used to apply a function to a sequence of arguments as in

`add([1, 2, 3]...)`

=> Is used in Dictionaries to assign values to keywords as in `Dict("A"=>1, "B"=>2)`

# Warming up



# Intermediate topics (MIT Computational Thinking)

A fantastic learning resource is the “Computational Thinking” course from MIT

<https://computationalthinking.mit.edu/Fall23/>

The image shows a screenshot of a web browser displaying the MIT Computational Thinking course website. The browser window has a title bar with the URL <https://computationalthinking.mit.edu/Fall23/>. The main content area of the browser shows the course homepage with sections like "Introduction to Computational Thinking", "Julia: A Fresh Approach to Computing", "Choose your track:", "WELCOME", "MODULE 1: IMAGES, TRANSFORMATIONS, ABSTRACTIONS", and "Highlights". A red box highlights the "1.1 Images as Data and Arrays" section under the "MODULE 1" heading. A blue box highlights the "Real-world problems" section, which includes a graph titled "Global CO<sub>2</sub> emissions". A yellow hand icon points to the "Real-world problems" section.

**Section I.1**

**Images as Data and Arrays**

Lecture Video

INIT x MIT INTRO TO COMPUTATIONAL THINKING Spring 2021

Initializing packages

When running this notebook for the first time, this could take up to 15 minutes. Hang in there!

```
begin
    using Colors, ColorVectorSpace, ImageShow, FileIO, ImageIO
    using PlutoUI
    using PlutoTeachingTools
    using HypertextLiteral: @htl, @htlattr
end
```

Images as examples of data all around us

Welcome to the Computational Thinking using Julia for Real-World Problems, at MIT in Spring 2021!

To be able to edit code and run cells, you need to run the notebook yourself. Where would you like to run the notebook?

This notebook takes about 3 minutes to run.

In the cloud (experimental)

binder

Binder is a free, open source service that runs scientific notebooks in the cloud! It will take a while, usually 2-7 minutes to get a session.

On your computer

(Recommended if you want to store your changes.)

1. Copy the notebook URL:

<https://computationalthinking.mit.edu/Fall23/>

A yellow hand icon points to the "Copy the notebook URL" button.

Click on a lecture, edit the notebook, copy the url and paste it into the “Open notebook” cell in Pluto

# MIT Computational Thinking key lectures (intermediate level)

Below is a selection of memorable online lectures from the “Computational Thinking” MIT course, follow the links to the lecture pages (with the videos) and the interactive notebooks



## 1.1- Images as data and Arrays



[https://computationalthinking.mit.edu/Fall23/generated\\_assets/hw1\\_395530a6.jl](https://computationalthinking.mit.edu/Fall23/generated_assets/hw1_395530a6.jl)



## 1.4- Transformations with images



[https://computationalthinking.mit.edu/Fall23/generated\\_assets/transforming\\_images\\_b0baefaa.jl](https://computationalthinking.mit.edu/Fall23/generated_assets/transforming_images_b0baefaa.jl)



## 2.9 Optimization



[https://computationalthinking.mit.edu/Fall23/generated\\_assets/optimization\\_5abd7af7.jl](https://computationalthinking.mit.edu/Fall23/generated_assets/optimization_5abd7af7.jl)



## 3.2 Differential equations



[https://computationalthinking.mit.edu/Fall23/generated\\_assets/odes\\_and\\_parameterized\\_types\\_b2f33ff1.jl](https://computationalthinking.mit.edu/Fall23/generated_assets/odes_and_parameterized_types_b2f33ff1.jl)



## 3.6 Snowball Earth and Hysteresis (excellent video)



[https://computationalthinking.mit.edu/Fall23/generated\\_assets/climate2\\_snowball\\_earth\\_bffbd696.jl](https://computationalthinking.mit.edu/Fall23/generated_assets/climate2_snowball_earth_bffbd696.jl)

# Interactive notebooks for education



[https://github.com/flt-acdesign/Low\\_speed\\_AC\\_performance/blob/main/21\\_04\\_08\\_Low\\_speed\\_perfo\\_v\\_0.0.1.jl](https://github.com/flt-acdesign/Low_speed_AC_performance/blob/main/21_04_08_Low_speed_perfo_v_0.0.1.jl)

This Pluto notebook is a simple example of an interactive resource to teach aircraft performance.

The students are expected to extend the code to plot additional graphics

Below is an example of how to document functions in Julia so that Pluto can provide “Live Docs”. Some of the plots are interesting to explore too.

## Aircraft Aerodynamic functions

```
begin
    # Aircraft aerodynamic coefficients, drag, power required and helper functions

    # NOTE: the text below, with exactly the format used, corresponds to the Julia
    # "docstrings" standard. The documentation needs to be exactly on the line above the
    # function definition. the "live docs" button at the bottom right of Pluto will show
    # the documentation of the function when the cursor is over the function name anywhere
    # in the code

    #
    """
    Vs1gTAS(W, h, CLmax, Sw)

    Calculate stall speed as TAS at 1g from weight (W) in Newtons, altitude (h) in
    meters, aircraft maximum lift coefficient (CL) and wing reference area (Sw) in m^2

    # Examples
    """julia-repl
    julia> Vs1gTAS(80000, 4000, 2.1, 20)
    48.241248922681834
    """
    """
    Vs1gTAS(W, h, CLmax, Sw) = ((W)/(ρ(h)*CLmax*Sw))^0.5

    #
    """
    Vs1gEAS(W, CLmax, Sw)
```

Q Live Docs ⌂ Status

Vs1gTAS

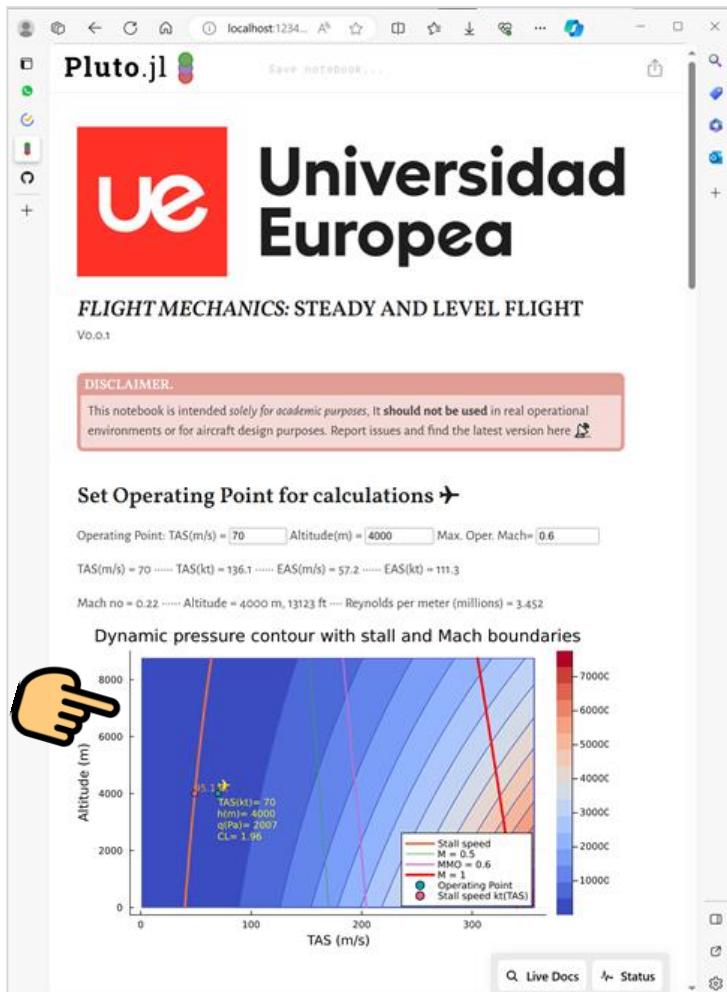
Vs1gTAS(W, h, CLmax, Sw)

Calculate stall speed as TAS at 1g from weight (W) in Newtons, altitude (h) in meters, aircraft maximum lift coefficient (CL) and wing reference area (Sw) in m<sup>2</sup>

Examples

```
julia> Vs1gTAS(80000, 4000, 2.1, 20)
48.241248922681834
```

Check the  
code for  
this plot for  
inspiration



# Enter chatGPT (or Bing chat or...)

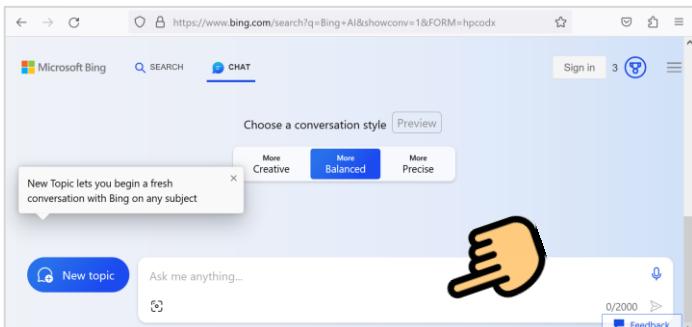
The world will never be the same since the advent of ChatGPT (and many other Large Language Models since).

Although probably the most robust automatic code generation occurs for Python, you can use ChatGPT to generate code in Julia

However, be prepared to edit the code manually or keep asking for refinements (“Regenerate”) until it works. **Note that you may get different answers every time as the system keeps on “learning”!**

The link below opens the Bing chat, which is not too bad for code generation and is completely free

<https://www.bing.com/search?q=Bing+AI&showconv=1&FORM=hpcodx>



This is called the “prompt” and should always start with “please” and be concrete, concise and cover all the points for which you need an answer in a logical way

A screenshot of the Microsoft Bing Chat interface. The top navigation bar shows "Microsoft Bing", "SEARCH", and "CHAT" (which is underlined). On the right, there is a user profile for "RAULCARLO..." with a notification count of "25". A blue button at the bottom right says "Please write a julia code to plot the mandelbrot set". Below this, a message history shows a checkmark indicating "Searching for: julia code mandelbrot set" and another checkmark for "Generating answers for you...". A message from "Julia" provides a sample Julia code for plotting the Mandelbrot set. The code uses the "Plots" package and defines a "mandelbrot" function that iterates up to 100 times to determine the number of iterations required for a point to escape the circle of radius 2. It then generates a heatmap of the resulting data. A note explains that the code uses the "Plots" package and the "mandelbrot" function to calculate iterations and plot the heatmap. A final message says "I hope this helps! Let me know if you have any other questions."

This code does not create the Mandelbrot set, ask ChatGPT to rewrite it using complex numbers...

# Mandelbrot set – Raw GPT3.5 code

The code below has been generated automatically by GPT3.5 (the free version of ChatGPT). Copy it into a cell of a new Pluto notebook and click Shift+ENTER

```
using Plots

function mandelbrot(c, max_iter)
    z = c
    for i in 1:max_iter
        if abs2(z) > 4.0
            return i - 1
        end
        z = z * z + c
    end
    return max_iter
end

function plot_mandelbrot(x_center, y_center, zoom, width, height, max_iter)
    x_min = x_center - 0.5 / zoom
    x_max = x_center + 0.5 / zoom
    y_min = y_center - 0.5 / zoom
    y_max = y_center + 0.5 / zoom

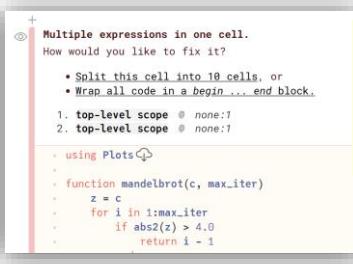
    x = LinRange(x_min, x_max, width)
    y = LinRange(y_min, y_max, height)

    mandelbrot_set = [mandelbrot(complex(r, i), max_iter) for r in x, i in y]

    heatmap(x, y, mandelbrot_set, c=:viridis, xlabel="Re", ylabel="Im", aspect_ratio=:equal)
end

# Set parameters
x_center = -0.5
y_center = 0.0
zoom = 1.0
width = 800
height = 800
max_iter = 100

# Plot Mandelbrot set
plot_mandelbrot(x_center, y_center, zoom, width, height, max_iter)
```



When you get this message in Pluto, click on “Split this cell into...”

Default (GPT-3.5)

UE please write a julia code to plot the mandelbrot set with center in coordinates x,y and a zoom level of z

Certainly! You can use the 'Plots' library in Julia to create a plot of the Mandelbrot set with a specified center and zoom level. First, make sure you have the 'Plots' package installed by running `import Pkg; Pkg.add("Plots")` if you haven't done so already.

Here's a simple Julia code to generate and plot the Mandelbrot set:

```
using Plots

function mandelbrot(c, max_iter)
    z = c
    for i in 1:max_iter
        if abs2(z) > 4.0
            return i - 1
        end
        z = z * z + c
    end
    return max_iter
end

function plot_mandelbrot(x_center, y_center, zoom, width, height, max_iter)
    x_min = x_center - 0.5 / zoom
    x_max = x_center + 0.5 / zoom
    y_min = y_center - 0.5 / zoom
    y_max = y_center + 0.5 / zoom

    x = LinRange(x_min, x_max, width)
    y = LinRange(y_min, y_max, height)

    mandelbrot_set = [mandelbrot(complex(r, i), max_iter) for r in x, i in y]

    heatmap(x, y, mandelbrot_set, c=:viridis, xlabel="Re", ylabel="Im", aspect_ratio=:equal)
end

# Set parameters
x_center = -0.5
y_center = 0.0
zoom = 1.0
width = 800
height = 800
max_iter = 100

# Plot Mandelbrot set
plot_mandelbrot(x_center, y_center, zoom, width, height, max_iter)
```

This code defines a `'mandelbrot'` function to calculate the Mandelbrot set's iteration count for a given complex number '`c`', and a `'plot_mandelbrot'` function to create a plot. 

If you don't like the code, regenerate it 

# A refined version of Mandelbrot starting from GPT code

Open this code in Pluto



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/mandelbrot\\_zoom\\_refined.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/mandelbrot_zoom_refined.jl)

```
using Plots ✓, PlutoUI ✓ # Load required packages (For plotting and interactivity)
1000
begin
    const width = 600 # Number of pixels in width of image
    const aspect_ratio::Float64 = 9/16 # Aspect ratio (defined inversely as normal)
    const height = Int64(round(width * aspect_ratio)) # Image height as integer value
    const max_iter = 1000 # Maximum number of iterations for the Mandelbrot equation
end

mandelbrot (generic function with 1 method)
# Mandelbrot algorithm for a complex point "c" with Re and Im components of type
# BigFloat in order to capture "arbitrarily" small numbers
function mandelbrot(c::Complex{BigFloat}, max_iter::Int)
    z = c # start at the point from the complex plane fed to the function
    for i in 1:max_iter # Loop up to max_iter (it may not get there...)
        if abs2(z) > 4.0 # If the length of the vector exceeds 4
            return i - 1 # exit the function returning the previous iteration number
        end
        #z = z ^2 + c # z * z changed to z^2 for readability
        z = muladd(z, z, c) # Calling BLAS library operators (25% faster than above)
    end
    return max_iter # If the iterative process does not diverge, return max_iter
end
```

**using** makes all the functions of the packages available in this program. Adding a package is like buying the book, using is like reading it

Using **const** before the value of a variable makes it a constant (it cannot be changed during the execution of the code) and, more importantly, it stores the value “closer” to the CPU, which is useful to improve speed of the code if the value is called millions of times (not this case).

The **mandelbrot** function takes a complex number  $x$  and the maximum number of iterations in the algorithm. The complex number has real and imaginary components of type “**BigFloat**”, which in Julia represents a number of arbitrary precision (a large number of bits, user defined). This is required to zoom into extremely small windows, smaller than about  $10^{-16}$

The **muladd** function multiplies  $z$  by itself (in this case) and adds  $c$ , storing the result in  $z$ . This is a “non allocating” function and is faster as it does not require memory access. In this case, given the number of times this operation is performed, it is worth it as it gives a 25% increase in speed.

# A refined version of Mandelbrot

```
generate_mandelbrot (generic function with 1 method)
  • # This function calls the Mandelbrot algorithm for each point in the rectangle of the
    # complex plane defined by its center coordinates and a "delta" ( $\delta$ ) on each side of the
    # center. Center and delta use the BigFloat type for arbitrary precision

  • function generate_mandelbrot(x_center::BigFloat,
    y_center::BigFloat,
    δ::BigFloat,
    width, aspect_ratio::Float64, max_iter)

    # Generate ranges of x and y coordinates
    x = LinRange(x_center - δ, x_center + δ, width)
    y = LinRange(y_center - δ * aspect_ratio, y_center + δ * aspect_ratio,
      Int(round(width * aspect_ratio)))

    """
    Matrix comprehension method
    mandelbrot_set = [mandelbrot(complex(BigFloat(r), BigFloat(i)), max_iter) for r in x,
    i in y]
    """

    # Explicit CPU multithreaded loop
    mandelbrot_set = zeros(length(x), length(y)) #initialize empty array to hold result

    # Loop for all the points in the rectangle x,y.
    # The "@inbounds" macro disables the internal checking of out of bounds in the arrays
    # The "@Threads" macro breaks down the loops into smaller loops sent to each core

    @inbounds Threads.@threads for index_r in 1:length(x)
      @inbounds Threads.@threads for index_i in 1:length(y)
        mandelbrot_set[index_r, index_i] = mandelbrot(
          complex(BigFloat(x[index_r]), BigFloat(y[index_i])), max_iter)
      end
    end

    return mandelbrot_set # This function returns an array with the number of iterations
    # before divergence for each point in the domain of the complex plane under study
  end
```

This function also takes **BigFloat**, it is important to specify the types of the inputs to all functions for the highest performance in Julia. This is called having “type stability”. Otherwise Julia will infer the types from the values, but in some cases it will not be able and will use “Any” which takes a lot of memory and is very slow.

The LinRange function generates a collection of elements equally spaced between the first and second input value, with the third input value stating how many elements are required. Type \delta+tab to get a  $\delta$

A “matrix comprehension” generates the elements of an array by applying a function to an iterator as in `[i^2 for i in 1:4]`, which creates an array of squares of all the integers between 1 and 4. It is not used here because we will generate the array of complex points using multithreading, which can be faster

The **@inbounds** macro tells Julia to not check whether the indices of the arrays are within the array limits. In this case we are sure and we can disable, if we are not sure and there is a problem we can crash the program due to memory overrun

The **Threads.@treads** macro tells Julia to break the loop into as many processes as threads are available in the machine. Simple loops are [“embarrassingly parallelizable”](#) and are a good candidate for multithreading.

# A refined version of Mandelbrot

```
Set zoom 10^zoom =  -0.4
· md"Set zoom 10^zoom = $($@bind zoom Slider(-.6:.01:7 ;default=-.4, show_value=true) ) "
6 = 2.511886431509580130949643717030994594097137451171875
· δ = BigFloat(1/(10^zoom)) # Convert the "zoom" into the width of the interval

xcenter =  -1.75 ycenter =  0.0
·
· md"
· xcenter = $($@bind x_center Slider(-3:.01:1; default = xcvideo , show_value=true) )
· ycenter = $($@bind y_center Slider(-1:.01:1; default = ycvideo , show_value=true) )
· "
108.18144970414201
· begin
·
· #setprecision(BigFloat, 500) # Enable this line to set arbitrary precision on the
BigFloat type. Default value is 128 bits.
·
· # The array to be plotted is the return value of the generate_mandelbrot function,
note that the array needs to be transposed in order to obtain the usual orientation
of the Mandelbrot set
array_to_plot = transpose(generate_mandelbrot(
·
·
·
·
·
BigFloat(x_center),
BigFloat(y_center),
δ,
width, aspect_ratio, max_iter))

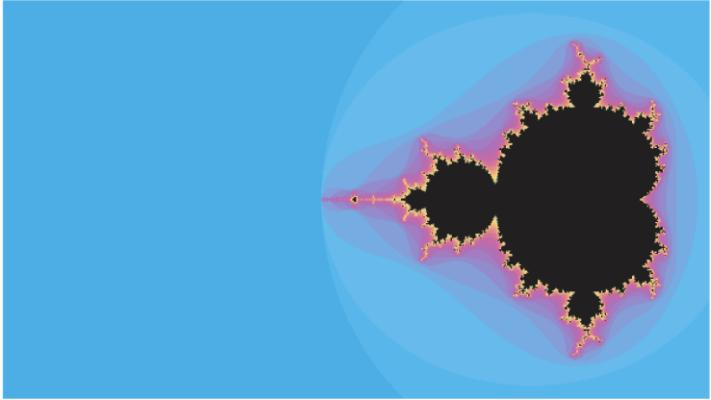
· # Calculate the average value of the array with the number of iterations. This is
used to set ad-hoc a color scale for plotting
· average = (sum(array_to_plot)/length(array_to_plot))
·
· end
```

This code creates a slider which can assign a value to the variable `zoom` in the range defined by -0.6 to 7 in steps of 0.1, and it shows the value next to the slider. You need the [PlutoUI](#) package for this

The `setprecision(BigFloat, 500)` is commented to improve performance for normal levels of zoom. When you zoom a lot you may need higher floating point precision and the previous statement sets the representation of floating point numbers at 500 bits (normally in Julia they have 64, as in [Float64](#)). The default number of bits for `BigFloat` is 128 bits.

Here you can see how flexible the Julia parser is to spaces and newlines. This can help a lot to make the code clearer).

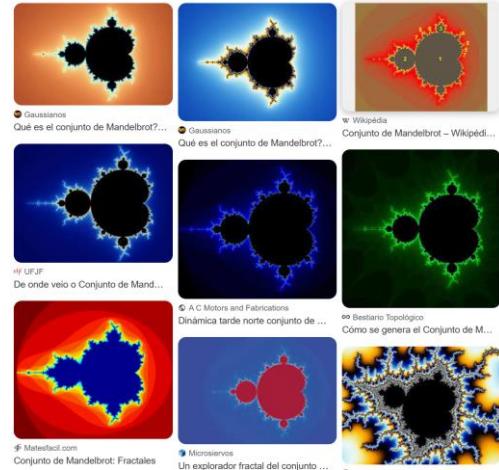
# A refined version of Mandelbrot



```
begin
    image = heatmap( # Assign the result of a "heatmap" plot to "image"
        array_to_plot,
        c=:cmyk, # Color scheme
        clim=(0, average /4), # Min and max values of the color range
        aspect_ratio=1,
        ticks=false,
        showaxis=false,
        legend=:none,
        size =(width, Int64(round(width * aspect_ratio))) # Image size
        #dpi=1200
    )

    savefig("mandelbrot_heatmap.png") # Save the current plot in a png file
    image
end
```

And here it is in a small part of its beauty... compare it with some of the results from other people. The color schemes are totally arbitrary and their selection comes down to aesthetics.



The image variable now holds the complete **heatmap** picture. A heatmap is a color picture, as shown above. It can be used to plot matrices as images. The parameters passed to the heatmap function specify various options, including the color scheme.

Go to the Plots documentation to see other [color schemes](#). Change the scheme **:cmyk** for others, for example **:flag**

This saves the image in .png format

# A refined version of Mandelbrot

```
. #save_animation() # Invoke this function to write a gif file with an animation

save_animation (generic function with 1 method)
  .# Create an animation with various images constructed with the Mandelbrot algorithm
  # for variations of one parameter, in this case, the zoom value
  .
  .function save_animation()
  .
  .# The @animate macro loops with the generation of the images
  anim_evolution = @animate for zoom in -.7:1:30
  .
  .array_to_plot = transpose(generate_mandelbrot( # Calculate the array to plot
  .                                BigFloat(x_center),
  .                                BigFloat(y_center),
  .                                BigFloat(1/(10^zoom)),
  .                                width, aspect_ratio, max_iter))
  .
  .heatmap( # Generate a "heatmap" image of the array to be plotted
  .        array_to_plot,
  .        c=:cmyk,
  .        clim=(0, average/5),
  .        aspect_ratio=1,
  .        ticks=false,
  .        showaxis=false,
  .        legend=:none,
  .        dpi=1200)
  .
  end
  .
  gif(anim_evolution, "mandelbrot_anim.gif", fps = 10) # Save the animation as .gif
  .
end
```

Remove the comment in this line to save the animation of a zoom into the set as a gif file

The animation is produced by the @animate macro which is an implicit loop on the variable zoom, in the range -0.7 to 30 in steps of 1. Note that a zoom of 30 means that you have amplified the image by 30 orders of magnitude. With normal BigFloat numbers (128 bits) you can zoom about 70 orders of magnitude, for higher zooms you need to increase the number of bits.

For more information on the Mandelbrot set watch:

[Numberphile](#)

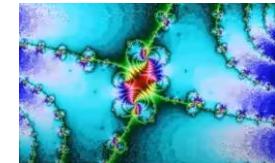
[Three blue one brown \(must watch\)](#)

[Veritasium \(excellent\)](#)

The initial coordinates are specified here and are taken from the video description of this YouTube video:



[Hardest Mandelbrot Zoom in 2017 - 750 000 000 iterations! - YouTube](#)



Coordinates from this video [https://www.youtube.com/watch?v=aSg2Db3jF\\_4](https://www.youtube.com/watch?v=aSg2Db3jF_4)

```
. begin
.   # Default coordinates in the complex plane of the center of the image
.   xcvideo =
.   BigFloat(-1.7499576837060935036022145060706997072711057972625207793024283782028600
.   8082972804887218672784431700831100544507655659531379747541999999999
.
.   yctvideo =
.   BigFloat(0.0000000000000000278793706563379402178294753790944364927085054500163081
.   379043930650189386849765202169477470552201325772332454726999999999
.
.   md" Coordinates from this video https://www.youtube.com/watch?v=aSg2Db3jF\_4"
```

# Lorenz Attractor – Euler method

[Visualizing the Lorenz Attractor in Julia | by Daryl Tng | Medium](#)

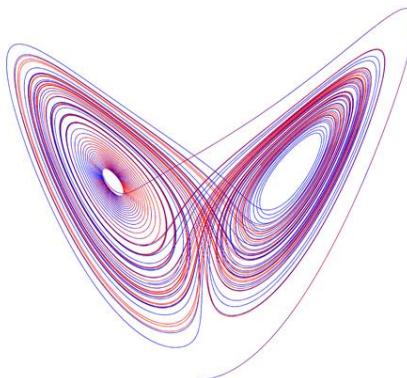
The Lorenz system of equations is very famous in mathematical and popular culture. It opened the field of Chaos which loosely means that infinitesimal variations of the initial conditions will give rise to totally different behaviours of a “chaotic” system (of which there are many, starting by the atmosphere, which is the origin of this system).

The phase plot of the Lorenz system is the “Lorenz attractor” which is created an popular association of chaos to the “[butterfly effect](#)” (“*Does the flap of a butterfly in Brazil set off a tornado in Texas?*”)

The Lorenz system

$$\begin{aligned}\frac{dx}{dt} &= \sigma y - \sigma x, \\ \frac{dy}{dt} &= \rho x - xz - y, \\ \frac{dz}{dt} &= xy - \beta z.\end{aligned}$$

The 3 governing equations of the Lorenz system



[Lorenz system – Wikipedia](#) with a Julia code!

Please write a julia code to plot and rotate the lorenz attractor using the euler method

Certainly! Below is a simple Julia code to plot and rotate the Lorenz attractor using the Euler method. The code uses the “Plots” library for plotting.

```
julia
using Plots

function lorenz!(du, u, p, t)
    σ, ρ, β = p
    du[1] = σ * (u[2] - u[1])
    du[2] = ρ * u[1] * (u[3] - u[2])
    du[3] = u[1] * u[2] - β * u[3]
end

function euler!(u, h, p, t_end)
    t = 0.0:h:t_end
    traj = zeros(length(t), length(u))

    for (i, t_i) in enumerate(t)
        traj[i, :] = u
        du = zeros(size(u))
        lorenz!(du, u, p, t_i)
        u += h * du
    end

    return t, traj
end

function plot_lorenz(t, traj)
    plot3d(
        traj[:, 1], traj[:, 2], traj[:, 3],
        xlabel="X", ylabel="Y", zlabel="Z",
        title="Lorenz Attractor",
        legend=false,
    )
end

# Set parameters
σ = 10.0
ρ = 28.0
β = 8/3

# Initial conditions
u₀ = [1.0, 0.0, 0.0]

# Time parameters
h = 0.01
t_end = 100.0

# Perform Euler integration
t, traj = euler!(u₀, h, [σ, ρ, β], t_end)

# Plot the Lorenz attractor
plot_lorenz(t, traj)
```

Make sure you have the “Plots” library installed by running “`using Pkg; Pkg.add("Plots")`” in the Julia REPL if you haven’t installed it already.

You can adjust the parameters such as “`σ`”, “`ρ`”, “`β`”, “`u₀`”, “`h`”, and “`t_end`” to explore different behaviors of the Lorenz attractor.

# Lorenz Attractor – Euler method

Copy the code into a new Pluto notebook



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Lorenz\\_Euler\\_GOOD.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Lorenz_Euler_GOOD.jl)

```
• using Plots ✓

lorenz! (generic function with 1 method)
• function lorenz!(du, u, p, t)
•     σ, ρ, β = p
•     du[1] = σ * (u[2] - u[1])
•     du[2] = u[1] * (ρ - u[3]) - u[2]
•     du[3] = u[1] * u[2] - β * u[3]
• end
```

```
euler! (generic function with 1 method)
• function euler!(u, h, p, t_end)
•     t = 0.0:h:t_end
•     traj = zeros(length(t), length(u))
•
•     for (i, ti) in enumerate(t)
•         traj[i, :] = u
•         du = zeros(size(u))
•         lorenz!(du, u, p, ti)
•         u += h * du
•     end
•
•     return t, traj
• end
```

```
plot_lorenz (generic function with 1 method)
• function plot_lorenz(t, traj)
•     plot3d(
•         traj[:, 1], traj[:, 2], traj[:, 3],
•         xlabel="X", ylabel="Y", zlabel="Z",
•         title="Lorenz Attractor",
•         legend=false,
•     )
• end
```

We'll solve this problem with pretty basic Julia, we just need Plots for plotting the attractor (Lorenz did not have that luxury in the 60's)

The system of non-linear differential equations is isolated in this function. The state of the system is stored in a vector  $u[3]$ .  $du[3]$  corresponds to the vector  $du/dt$ .

Note how the parameters of the differential equation are inferred from a vector  $p$

The `euler!` function is mutating, this means that it takes the state vector and operates on it "in place" (without memory allocations). This is important for performance but it's not pure functional programming. However there is no merit in pure almost anything if you have good reasons not to be pure...

The returned value is a "tuple", a group of two values: the time array (a collection) and an array with the trajectory in phase space.

This function plots the phase state in 3D by providing a single array for each component of the trajectory.

# Lorenz Attractor – Euler method

```
o = 10.0
· # Set parameters
· o = 10.0
```

```
p = 28.0
· p = 28.0
```

```
β = 2.6666666666666665
· β = 8/3
```

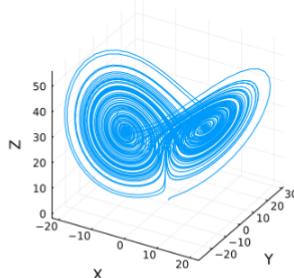
```
u0 = [1.0, 0.0, 0.0]
· # Initial conditions
· u0 = [1.0, 0.0, 0.0]
```

```
h = 0.01
· # Time parameters
· h = 0.01
```

```
t_end = 100.0
· t_end = 100.0
```

```
► (0.0:0.01:100.0, 10001x3 Matrix{Float64};)
· # Perform Euler integration
· t, traj = euler!(u0, h, [o, p, β], t_end)
```

Lorenz Attractor



```
· # Plot the Lorenz attractor
· plot_lorenz(t, traj)
```

This part of the code sets the initial value and parameters of the system and is written in a not very elegant way because Pluto has expanded each line into its own cell.

A better way is to group all these cells into a block of code

This cell assigns to t and traj the result of the euler! Function call with the parameters defined above, containing the phase trajectory.

And this is the plot of the Lorenz attractor.

For more information watch:

[Chaos | Chapter 7 : Strange Attractors - The butterfly effect – YouTube](#)

# Lotka-Volterra differential equations using Euler method

Open this code in Pluto



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Lotka\\_Volterra\\_Euler\\_GOOD](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Lotka_Volterra_Euler_GOOD)

```
using Plots ✓

lotka_volterra_euler (generic function with 1 method)
• function lotka_volterra_euler(α, β, γ, δ, u₀, T, dt)
•     t_values = 0:dt:T
•     u_values = zeros(length(t_values), length(u₀))
•     u_values[1, :] .= u₀
•
•     for i in 2:length(t_values)
•         du = lotka_volterra(u_values[i-1, :], α, β, γ, δ)
•         u_values[i, :] = u_values[i-1, :] + dt * du
•     end
•
•     return t_values, u_values
• end

lotka_volterra (generic function with 1 method)
• function lotka.volterra(u, α, β, γ, δ)
•     du1 = α * u[1] - β * u[1] * u[2]
•     du2 = δ * u[1] * u[2] - γ * u[2]
•
•     return [du1, du2]
• end

▶ (0.1, 0.02, 0.1, 0.01)
• # Parameters
• α, β, γ, δ = 0.1, 0.02, 0.1, 0.01

u₀ = ▶ [100.0, 20.0]
• # Initial conditions
• u₀ = [100.0, 20.0]

T = 1000.0
• # Time span and step size
• T = 1000.0

dt = 0.1
• dt = 0.1

▶ (0.0:0.1:1000.0, 10001×2 Matrix{Float64}):
    100.0      20.0
• # Solve using Euler method
• t_values, u_values = lotka.volterra_euler(α, β, γ, δ, u₀, T, dt)
```

This code has been created automatically by ChatGPT and works right out of the box, probably because the Lotka-Volterra problem is very well covered in examples and the AI had plenty of opportunity to train.

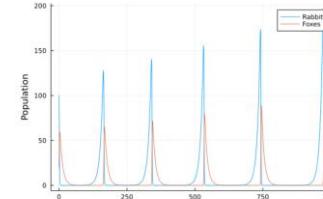
Here it was explicitly asked to use the Euler method (to prevent GPT from proposing packages for differential equations, which we will see later).

The **Lotka–Volterra equations**, also known as the **Lotka–Volterra predator–prey model**, are a pair of first-order **nonlinear differential equations**, frequently used to describe the **dynamics of biological systems** in which two species interact, one as a **predator** and the other as prey. The populations change through time according to the pair of equations:

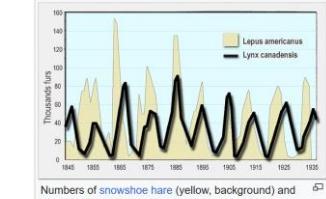
$$\begin{aligned}\frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y,\end{aligned}$$

where

- the variable  $x$  is the **population density** of prey (for example, the number of **rabbits** per square kilometre);
- the variable  $y$  is the population density of some **predator** (for example, the number of **foxes** per square kilometre);
- $\frac{dy}{dt}$  and  $\frac{dx}{dt}$  represent the instantaneous growth rates of the two populations;
- $t$  represents time;
- The prey's **parameters**,  $\alpha$  and  $\beta$ , describe, respectively, the maximum prey **per capita** growth rate, and the effect of the presence of predators on the prey growth rate.
- The predator's parameters,  $\gamma$ ,  $\delta$ , respectively describe the predator's **per capita** death rate, and the effect of the presence of prey on the predator's growth rate.



```
# Plot the results
plot(t_values, u_values, label=["Rabbits", "Foxes"], xlabel="Time",
      ylabel="Population", legend=:topright)
```



Numbers of snowshoe hare (yellow, background) and Canada lynx (black line, foreground) furs sold to the Hudson's Bay Company. Canada lynxes eat snowshoe hares.

# Getting serious with Julia



# Julia Types

## [Types - The Julia Language](#)

The “type” of a variable refers to the information that it can contain and it is required by the computer to store the value in memory.

Different types require different numbers of “bits” of memory and in the case of numbers can represent different ranges of numbers with different level of precision

In Julia it's not obligatory to declare the types of variables and constants (the compiler will detect them), but it is good practice to help Julia store the data in an orderly way and as close to the CPU as possible

```
a = 3; typeof(a)      -> Int64
```

```
b = 3.0 ; typeof(b)  -> Float64
```

This is the default type for numbers when you write a “decimal point” anywhere (0.4, .4, 3., 3.0). It has a “precision” of `eps(Float64)` -> 2.220446049250313e-16

Languages that detect the types are called “dynamically typed” (like Julia, Python, Basic...) and those which need explicit declaration of the types of variables are called “statically typed” (like C, Fortran, Typescript...).

Julia is considered to be “dynamically typed” but it has the performance of a “statically typed” language, specially if you help her...

To declare a variable, function input or function output value of a certain type use `::` as in ;

```
x::Int = 10      ;           function sinc(x)::Float64
```

# The Julia type system

“Any” is the most general and memory-expensive type and is the default value when you declare an empty array like

In Julia only “concrete” types can hold values.

“Abstract” types are “supertypes” in the type system to group related concrete types.

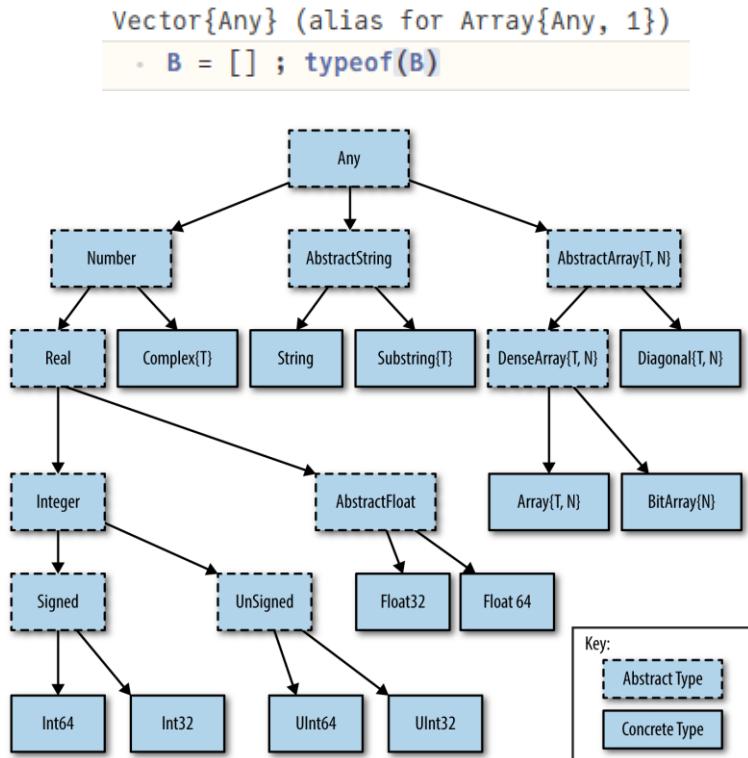
A function defined with inputs of the abstract type “Number” will work with any of its subtypes.

The hierarchy can be explored with `<:` as follows;

```
julia> Integer <: Number
true
```

```
julia> Integer <: AbstractFloat
false
```

If you want to specialize it for a particular concrete type you can do it creating a new method (more later)



# Multiple dispatch: Functions and Methods



[01x07] Julia's Secret Sauce? Methods,  
Multiple Dispatch and More; Tutorial 7/13  
Julia for Beginners - YouTube

A key characteristic of Julia is “multiple dispatch” which means that a function with the same name can have various implementations with different input types. Each implementation is called a “method” of the function.

Julia will “dispatch” or call the corresponding method to the input types and if no method exist will give a “*no method matching*” error.

For example, the Julia function \* can work in very different ways in these two cases (note that  $*(x,y) = x*y$  );

```
julia> *(3,4)      Usual number multiplication operating on integers
12
```

```
julia> *("Hello ", "Sevilla")    String concatenation (The Julia manual explains why "*" instead of the usual "+")
"Hello Sevilla"
```

To know how many methods a function "f" has implemented type `methods(f)`

You can write a new method for the \* function by just stating the types of the inputs

```
function *(x::Fruit, y::Fruit)    where Fruit is a custom type, or a structure, that you have created.
```

# Type safety

Although Julia can infer the types by “their looks” (3 is Int and 3.0 is Float), performance and “code safety” (avoiding errors) is improved if the types are explicitly specified

Making sure that Julia can know the types of data flowing through the code is called “type safety” and is a key performance enhancer as it allows proper and efficient memory allocation.

A classical example where this is not observed is;

```
function myfun(a; tol = 2)
    x = (a > 42 ? a : 42.)  the type of the return variable will change at run time depending on the value of one input
    return x + tol
end
```

The macro `@code_warntype` will analyze the code and identify the types flowing, in this case there is a “Union”, which is inefficient

```
julia> @code_warntype myfun(2)
MethodInstance for myfun(::Int64)
  from myfun(a; tol) @ Main REPL[8]:1
Arguments
  #self#:Core.Const(myfun)
  a::Int64
Body::Union{Float64, Int64}
1 - %1 = Main.:(var"#myfun#3")(2, #self#, a)::Union{Float64, Int64}
└   return %1
```

# LLVM and intermediate representations of code

Julia with the LLVM compiler is extremely efficient because it interprets the code in various phases, optimizing the code representation at each level (this is called [code lowering](#))

The last phase is machine code but the intermediate representations (IR) can be used (by advanced coders) to optimize the code and understand bottlenecks. The IR can be inspected as in the example below:

```
my_fun (generic function with 1 method)
• my_fun(x) = x + x^2
```

Function definition, untyped

```
CodeInfo(
1 - %1 = Main.var"workspace#22".:^
  %2 = Core.apply_type(Base.Val, 2)
  %3 = (%2]()
  %4 = Base.literal_pow(%1, x, %3)
  %5 = x + %4
      return %5
)
```

```
• @code_lowered my_fun(3)
```

First level of code lowering by LLVM. To carry on type :  
`@code_llvm my_fun(3)` and `@code_native my_fun(3)`

```
CodeInfo(
1 - %1 = Base.mul_int(x, x)::Int64
  %2 = Base.add_int(x, %1)::Int64
  return %2
)                                     ⇒ Int64
```

```
• @code_typed my_fun(3)
```

Lowered code showing the type interpretation for the given input values

# Macros and metaprogramming

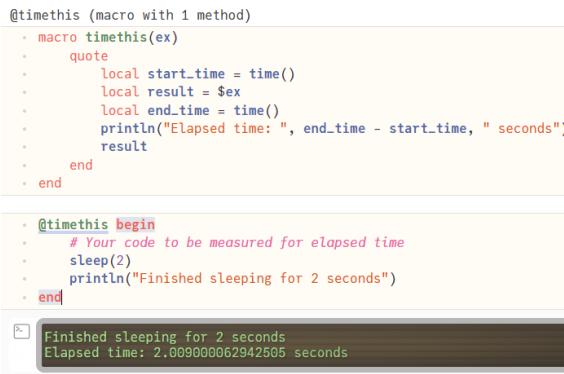
Macros are functions that take code as input, transform it and output new code which is immediately evaluated as part of the program flow.

“Metaprogramming” refers to this technique where a running program processes itself as if it was data.

In Julia, the name of macros is called with a “@” prefix just to differentiate them from normal functions

```
@timethis (macro with 1 method)
- macro timethis(ex)
-   quote
-     local start_time = time()
-     local result = $ex
-     local end_time = time()
-     println("Elapsed time: ", end_time - start_time, " seconds")
-     result
-   end
- end

- @timethis begin
-   # Your code to be measured for elapsed time
-   sleep(2)
-   println("Finished sleeping for 2 seconds")
- end
```



Finished sleeping for 2 seconds  
Elapsed time: 2.009000062942505 seconds

A simple example is a macro which introduces additional code around a function or block of code to measure the time elapsed to evaluate a function.

This is done by the base Julia macro `@time`, but its not the best way to measure performance because it can include compilation time etc..

Metaprogramming is an advanced topic but any user can benefit from already available macros in base Julia and many packages. Very commonly used macros in Julia are `@time`, `@btime` and `@benchmark` (from BenchmarkTools), `@. ,` `@inline` and others

# Inlined functions (the reason why Julia can be faster than C)

```
sq(x) = x^2
```

```
for i in 1:1000_1000
    sq(i)
end
```

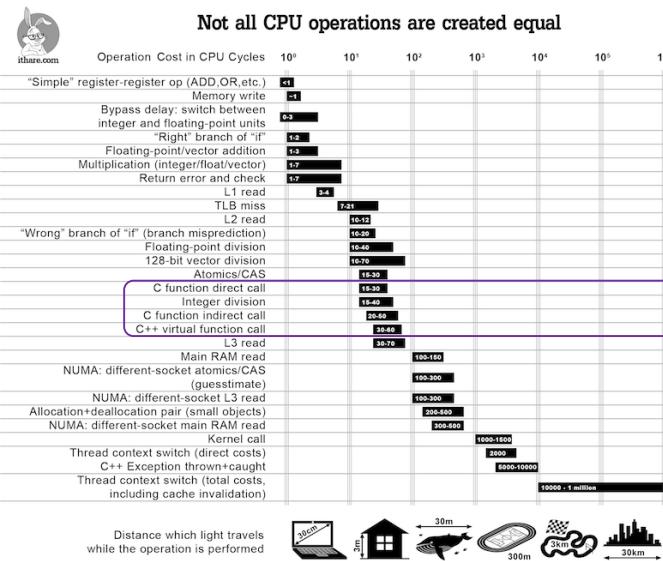
In this trivial (and unrealistic) example, the function `sq` would be called a million times. It is unrealistic because the LLVM compiler is smart enough to embed the code of the

function in the loop when the code is lowered, saving the function call, which is expensive. This is called “[inlining](#)” and for larger functions can be “suggested to the compiler with the macro `@inline` as in this example:

Defining the function as `@inline sq(x) = x^2`

would effectively result in the loop becoming :

```
for i in 1:1000_1000
    x^2      which avoids the function call
end      (although in this simple case the inlining will happen anyway)
```



Calling a function takes about 50 CPU cycles.

Inlining is the reason why Julia can appear to be faster than (old) C in some benchmarks, due to the avoidance of function calls

# The Julia ecosystem



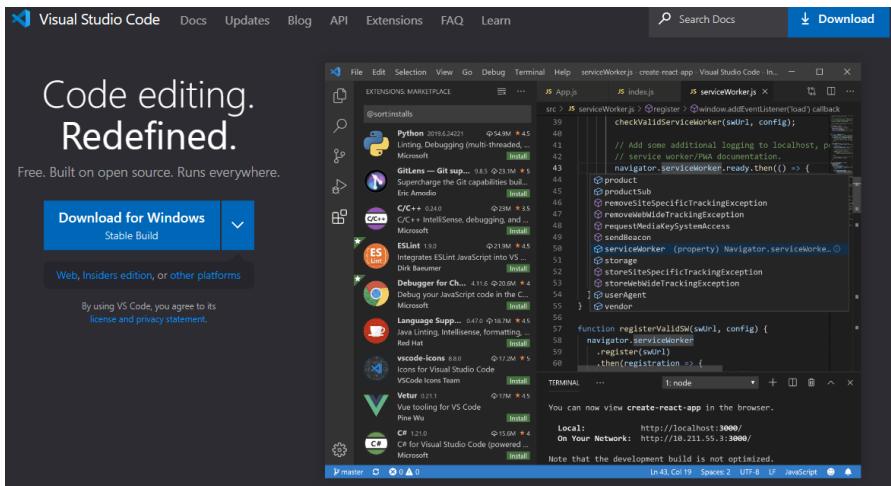
# Visual Studio Code



[10x06] How to use Julia  
in VS Code - YouTube

Although Pluto is even more powerful than it seems, when the code becomes too complex or takes long to execute it shows its limitations

The “serious” way to code in Julia is using [Visual Studio code](#) (or “Vscode”). In fact this is the case now for most open source languages



The Julia extension for Vscode is continuously being developed and offers many advanced features



[What's new in the Julia extension for VS Code | Sebastian Pfitzner | JuliaCon 2023 - YouTube](#)

In this workshop we will stick to Pluto even for the advanced topics

# Julia Packages

There are thousands of Julia packages freely available, you can find a curated list here [Julia Packages](#) (as you can see, in Oct 2023 Pluto is the most popular of all)

Not all the packages are equally well documented or useful. Eventually the eco-system will converge to a reduced number of “good” packages per domain.

The source code of packages and, in most cases, the documentation (if it exists) is located in a Github repository.

The complete Julia language lives in a [Github repository](#) (and can be explored and even improved by you)

You can also create packages for the Julia eco-system, check [5. Creating Packages · Pkg.jl \(julialang.org\)](#)

Trending Packages		⋮
Pluto.jl	★ 4500	
Simple reactive notebooks for Julia		
Plots.jl	★ 1710	
Powerful convenience for Julia visualizations and data analysis		
Flux.jl	★ 4122	
Relax! Flux is the ML library that doesn't make you tensor		
Makie.jl	★ 1978	
Visualizations and plotting in Julia		
IJulia.jl	★ 2629	
Julia kernel for Jupyter		
ModelingToolkit.jl	★ 1212	
An acausal modeling framework for automatically parallelized scientific machine learning (SciML) in Julia. A computer algebra system for integrated symbolics for physics-informed machine learning and automated transformations of differential equations		
PrecompileTools.jl	★ 128	
Reduce time-to-first-execution of Julia code		
VoronoiFVM.jl	★ 134	
Solution of nonlinear multiphysics partial differential equation systems using the Voronoi finite volume method		

# Creating local environments. Project.toml and Manifest.toml

[How to setup Project Environments in Julia | by René | Towards Data Science](#)

So far we have just added packages in the package manager without taking care of where these are installed. This is a “global” installation and makes the packages available to all Julia programs as long as all the paths are set correctly in the installation.

But as packages evolve, their new versions can be incompatible with dependent packages installed by other packages, which can lead to compatibility problems if all the packages “live” in the same “environment”

To create a local environment for each project you start do the following:

- 1- Using a console (terminal) go to the folder where you want to create for your project called **MyProject**
- 2- Start Julia by typing **julia** and enter the package manager (type **]** )
- 3- Type **generate MyProject** This will create a directory called **MyProject** that contains a **Project.toml** file and a folder for the source code. **Project.toml** contains general information about the project creation.
- 4- type activate **MyProject**
- 5- Add the packages you need, for example type **add Pluto Sound** At this point the **Manifest.toml** file will appear, which contains all the information relative to the packages installed in this environment. When you’re done go back to REPL

```
C:\> Users > USUARIO > kk_test_3 > MyProject > Manifest.toml
 1  # This file is machine-generated - editing it directly is not advised
 2
 3  julia_version = "1.9.0-rc2"
 4  manifest_format = "2.0"
 5  project_hash = "f407a7b03eae89275544b134432c5ae3b949a76"
 6
 7  [[deps.AbstractFFTs]]
 8  deps = ["LinearAlgebra"]
 9  git-tree-sha1 = "d92ad398961a3ed262d8bf04a1a2b8340f915fef"
10  uid = "621f4979-c628-5d54-868e-fcf4e3e8185c"
11  version = "1.5.0"
12
```

You don't need to edit these files but they contain all the information to guarantee the consistency of the libraries (packages) used in your project. Even if the package versions keep evolving, you can always use the correct ones to which the **Manifest.toml** file points

[10. Project.toml and Manifest.toml · Pkg.jl \(julialang.org\)](#)

# Image convolution using Images

<https://github.com/JuliaImages/Images.jl>

Open this code in Pluto



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/gaussian\\_convolution\\_GOOD2\\_withlayout.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/gaussian_convolution_GOOD2_withlayout.jl)

Let's use the images.jl package to perform the convolution of an image obtained from a random matrix . In this case, the kernel is a Gaussian 2D function and therefore the resulting image will be “blurred”.

```
• using Images ✓ , PlutoUI ✓  
• using Plots ✓  
  
gaussian_kernel (generic function with 1 method)  
• function gaussian_kernel(o)  
•     k = 2 * ceil(Int, 3.0 * o) + 1  
•     kernel = [exp(-(i^2 + j^2) / (2 * o^2)) for i in -k:k, j in -k:k]  
•     kernel /= sum(kernel)  
•     return kernel  
• end  
  
• heatmap(gaussian_kernel(o))  
  
gaussian_convolution (generic function with 1 method)  
• function gaussian_convolution(image, o)  
•     kernel = gaussian_kernel(o)  
•     return imfilter(image, kernel, "circular")  
• end
```

A heatmap visualization of a 2D Gaussian kernel. The plot shows a central bright yellow/orange spot at approximately (45, 45) and a smooth, symmetric decay towards the edges. The axes range from 0 to 100, with major ticks at 20, 40, 60, and 80. A vertical color bar on the right indicates values from -0.003 to 0.003.

Increasing the width of the Gaussian, determined by sigma, increases the blurring effect, use the slider to play with the code

convolution 3blueone

```
• # Example usage  
• img = rand(100, 100); # Replace this with your image data  
  
• @bind o Slider(1:10)  
  
• smoothed_img = gaussian_convolution(img, o);  
  
Original Image  


A heatmap showing a noisy, high-contrast pattern of red and black pixels, representing a raw image with significant noise.

  
Smoothed Image  


A heatmap showing the same noisy pattern but with much smoother, blurred features, demonstrating the convolution effect. A color bar on the right shows values from 0.475 to 0.520.

  
• # Display the original and smoothed images using Plots  
• plot()  
•     heatmap(img, c=:fire, title="Original Image"),  
•     heatmap(smoothed_img, c=:fire, title="Smoothed Image"),  
•     layout=(1,2),  
•     size=(800,400)  
• )
```

# JuMP - Modeling language for Optimization

Open this code in Pluto



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/JuMP\\_example\\_GOOD](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/JuMP_example_GOOD)

```
• using JuMP ✓  
• using Cbc ✓ # or GLPK if you prefer GLPK  
  
model = A JuMP Model  
    Feasibility problem with:  
    Variables: 0  
    Model mode: AUTOMATIC  
    CachingOptimizer state: EMPTY_OPTIMIZER  
    Solver name: COIN Branch-and-Cut (Cbc)  
• # Create a JuMP model  
• model = Model(Cbc.Optimizer) # or Model(GLPK.Optimizer) if you prefer GLPK  
  
x  
• # Define variables  
• @variable(model, x >= 0)  
  
y  
• @variable(model, y >= 0)  
  
3x + 2y  
• # Define the objective function  
• @objective(model, Max, 3x + 2y)  
  
2x + y ≤ 20  
• # Define constraints  
• @constraint(model, constraint1, 2x + y <= 20)  
  
4x - 5y ≥ -10  
• @constraint(model, constraint2, 4x - 5y >= -10)  
  
• # Solve the optimization problem  
• optimize!(model)
```

The JuMP package is very mature (it has even its own yearly conference).

It is a domain language built on Julia and allows to perform mathematical optimization of problems expressed in an intuitive form

The example code has been generated by ChatGPT3.5

UE please write an example of mathematical optimization using the JuMP package in Julia

AI Sure, I'll provide a simple example of mathematical optimization using the JuMP package in Julia. In this example, we'll solve a linear programming problem:

Maximize  $3x + 2y$   
Subject to  
 $2x + y \leq 20$   
 $4x - 5y \geq -10$   
 $x \geq 0, y \geq 0$

Make sure you have the JuMP and a solver (like Cbc.jl or GLPK.jl) installed. You can install them using the following commands:

Pluto code for the plot


[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Illustrate\\_JuMP\\_solution\\_GOOD](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Illustrate_JuMP_solution_GOOD)

The problem formulated is a linear program, where the objective function is a plane and the constraints are linear inequalities. To visualize the problem and the result use the notebook above.

```

- # Solve the optimization problem
- optimize!(model)

status = OPTIMAL::TerminationStatusCode = 1
- # Check the optimization status
- status = termination_status(model)

- println("Optimization Status: $status")
Optimization Status: OPTIMAL

- # Display the results
- println("Optimal value of x: ", value(x))

Optimal value of x: 6.428571428571429

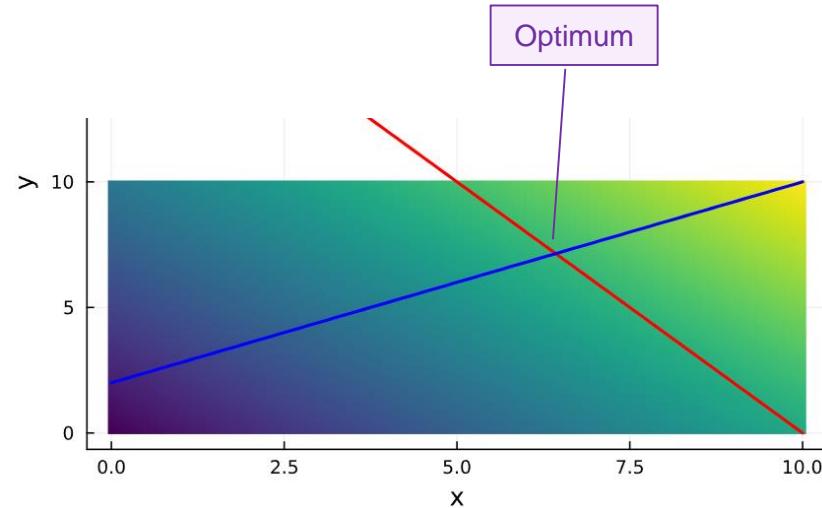
- println("Optimal value of y: ", value(y))

Optimal value of y: 7.142857142857143

- println("Optimal objective value: ", objective_value(model))

Optimal objective value: 33.57142857142857

```



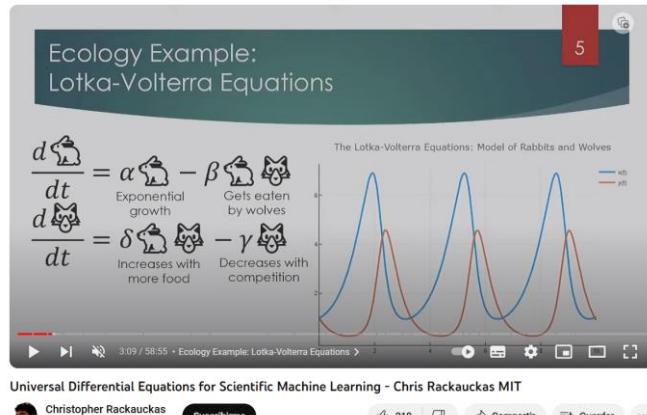


# Differential Equations

This is possibly the most mature and powerful package in the Julia ecosystem

It was started by Dr Chris Rackauckas around 2015, when he was a PhD student in Mathematical Biology

The package is extremely well written and documented and is the base for various other free and proprietary packages like SciML, PUMAS



[Introduction to solving differential equations with Julia \(a Classic\)](#)



[Solving partial differential equations with Julia \(JuliaCon2018\)](#)



[Universal Differential Equations for Machine Learning](#)



[Dr Chris Rackauckas YouTube channel](#)

# Lotka-Volterra DifferentialEquations (all from Bing GPT)

**prompt>** please write a program to solve the lokta volterra equations with differentialequations in julia

```
using DifferentialEquations
```

```
# Define the Lotka-Volterra equations
function lotka_volterra(du, u, p, t)
```

```
    x, y = u
    α, β, δ, γ = p
    du[1] = dx = α*x - β*x*y
    du[2] = dy = -δ*y + γ*x*y
end
```

```
# Set initial conditions and parameters
u₀ = [1.0, 1.0]
p = [1.5, 1.0, 3.0, 1.0]
```

```
# Set up the problem and solve it
tspan = (0.0, 10.0)
prob = ODEProblem(lotka_volterra, u₀, tspan, p)
sol = solve(prob)
```

```
# Print the solution
println(sol)
```

This prints the solution, note that if you want a plot you need to ask for it!

**prompt>** please please explain the previous code line by line

"This line imports the DifferentialEquations package, which provides a suite of tools for solving differential equations in Julia

This function defines the Lotka-Volterra equations. The function takes four arguments: du, u, p, and t. du is the output array that stores the time derivatives of the state variables x and y. u is the input array that stores the current values of x and y. p is an array that stores the parameters of the model. Finally, t is the current time.

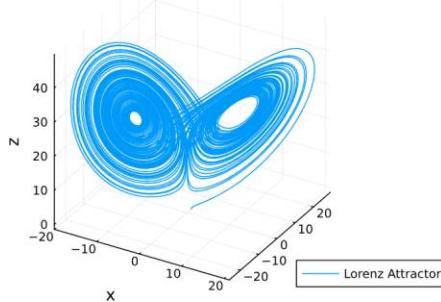
These lines set the initial conditions and parameters for the Lotka-Volterra equations.

These lines set up the problem using ODEProblem and solve it using solve. The solution is stored in sol."

The picture of Dr Rackaukas came also with the Bing response



# Lorenz attractor – DifferentialEquations package



```
begin
    using DifferentialEquations
    using Plots

    function lorenz!(du, u, p, t)
        σ, ρ, β = p
        du[1] = σ * (u[2] - u[1])
        du[2] = u[1] * (ρ - u[3]) - u[2]
        du[3] = u[1] * u[2] - β * u[3]
    end

    # Set parameters
    σ = 10.0
    ρ = 28.0
    β = 8/3

    # Set initial conditions
    u₀ = [1.0, 0.0, 0.0]

    # Set time span
    tspan = (0.0, 100.0)

    # Define the Lorenz system
    lorenz_prob = ODEProblem(lorenz!, u₀, tspan, [σ, ρ, β])

    # Solve the ODE
    sol = solve(lorenz_prob, Tsit5())

    # Plot the Lorenz attractor
    plot(sol, vars=(1, 2, 3), label="Lorenz Attractor", xlabel="x", ylabel="y",
         zlabel="z", legend=:bottomright)
end
```

Any Ordinary Differential Equations (ODE) problem has the same interface in the DifferentialEquations package.

This is the example solving the Lorenz attractor (also from ChatGPT)

Default (GPT-3.5)

UE Please write a julia code to plot the lorenz attractor

Certainly! The Lorenz attractor is a set of chaotic solutions to a system of three ordinary differential equations. Here's a simple Julia code using the `DifferentialEquations` and `Plots` packages to simulate and plot the Lorenz attractor:

First, make sure you have the required packages installed. You can install them by running:

```
julia
using Pkg
Pkg.add("DifferentialEquations")
Pkg.add("Plots")
```

Now, you can use the following code:

```
julia
using DifferentialEquations
using Plots

function lorenz!(du, u, p, t)
    σ, ρ, β = p
    du[1] = σ * (u[2] - u[1])
    du[2] = u[1] * (ρ - u[3]) - u[2]
    du[3] = u[1] * u[2] - β * u[3]
end

# Set parameters
σ = 10.0
ρ = 28.0
β = 8/3

# Set initial conditions
u₀ = [1.0, 0.0, 0.0]

# Set time span
tspan = (0.0, 100.0)

# Define the Lorenz system
lorenz_prob = ODEProblem(lorenz!, u₀, tspan, [σ, ρ, β])

# Solve the ODE
sol = solve(lorenz_prob, Tsit5())

# Plot the Lorenz attractor
plot(sol, vars=(1, 2, 3), label="Lorenz Attractor", xlabel="x", ylabel="y",
     zlabel="z", legend=:bottomright)
```

This code defines the Lorenz system using the `lorenz!` function, sets up the parameters, initial conditions, and time span, and then solves the system using the `solve` function from the `DifferentialEquations` package. Finally, it uses the `Plots` package to visualize the Lorenz attractor in 3D.

Machine Learning is a part of mathematics/computer science which focuses on the development of statistical algorithms which can exploit large amounts, generally high dimensional, data to create predictive models.

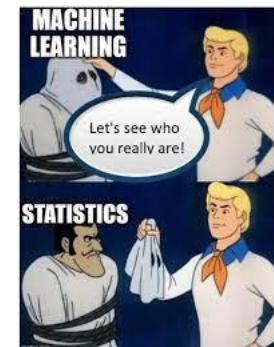
The mathematical models (simple regressions, Neural Networks, decision trees, Bayes, Support Vector Machines...) tend to require an iterative process to adjust the model parameters to the available data. This process is usually called "learning".

Julia has various packages for Machine Learning but the most mature is Flux. It is not as powerful as Tensorflow or Pytorch but the code is completely open and you can do things with Flux which are not possible with these other frameworks (like hacking the algorithms...)

Flux exploits all the power of Julia's composability to use automatic differentiation to train the models.

We shall focus on understanding Deep Neural Networks in Flux.

Useful videos:  [Building Deep Learning models in Flux](#) , [Flux introduction \(Doggo...\)](#) ,



# But what is a Deep Neural Network?



[Three Blue One Brown Neural networks – YouTube](#)

It's nonlinear algebra.

The language of Neural Networks is full of terminology that can obfuscate what things really are.

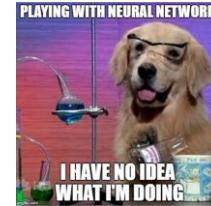
What is a neuron?

The equation of a line,  $y = w \cdot x + b$ , is linear. This is studied in Linear Algebra.  $w$  is the slope and  $b$  is the intercept with the  $y$  axis at  $x = 0$ . In machine learning these are called “weight” and “bias”.

The equation  $y = \text{sigma}(w \cdot x + b)$ , where sigma is a nonlinear function (like a sigmoid), represents a “bent line”. The nonlinear function sigma is called the “activation function”. The previous equation is the equation of a single “neuron”.

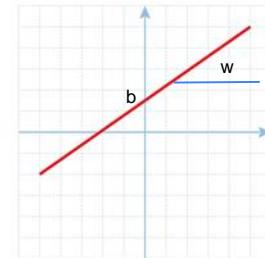
If we compose linear functions with linear functions we get a linear function (principle of superposition, a key tenet of linearity)

If we compose nonlinear functions with nonlinear functions we get a neural network.

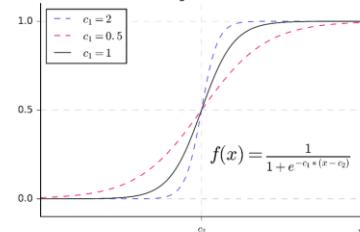


Linear Function

$$y = wx + b$$



Basic Sigmoid Function



# But what is a Deep Neural Network?



[Neural Networks Pt. 3: ReLU In Action!!! - YouTube](#)

## The simplest neural network

A very simple (and useful) activation function is the **ReLU** – Rectified Linear Unit. (another fancy name for something as simple as saying “if it’s negative make it 0”).

## The ReLU Activation Function...



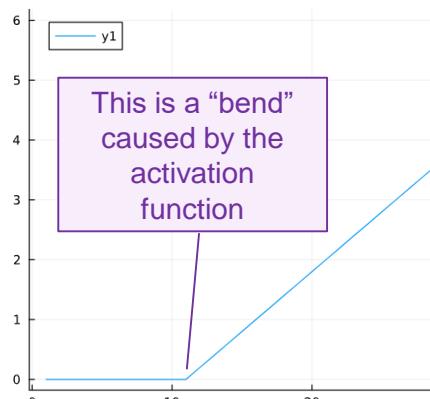
$$f(x) = \max(0, x)$$

...Clearly Explained!!!

```
relu (generic function with 1 method)
  · relu(x) = max(0, x)
```

```
y (generic function with 1 method)
  · y(x, w, b) = w * x + b
```

```
neuron (generic function with 1 method)
  · neuron(x, w, b) = relu(y(x, w, b))
```



```
· plot([neuron(i, 2, 2) for i in -2:1:2])
```

Any nonlinear activation function “bends” the line defined by the equation:

$$y = w \cdot x + b$$

The composition of a neuron function with another neuron function rotates and shifts the curves at each level of nesting (the effect of the “weights” and “biases”) and the activation functions apply successive “bends” to the curves.

If you create a concatenation of neuron compositions you get a “deep” one-dimensional neural network (in this case with 2 “layers”) of equation:

$$\text{“Prediction”} = \text{DNN}(x) = \text{ReLU}(w_2 \cdot \text{ReLU}(w_1 \cdot x + b_1) + b_2) \quad \text{for } x \text{ in test range.}$$

If you can now find the values of the w’s and the b’s that minimize the error in the prediction you have just trained your neural network.

# Linear regression “loss” space

Fitting a Line · Flux (fluxml.ai)

Pluto code



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Linear\\_regression\\_error\\_space\\_GOOD.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Linear_regression_error_space_GOOD.jl)

```
- using Plots ✓

rmse (generic function with 1 method)
- # Function to compute root mean square distance
- function rmse(points, w, b)
-     n = length(points)
-     distances = [abs(y - w * x - b) for (x, y) in points]
-     return sqrt(sum(distances.^2) / n)
- end

generate_points (generic function with 2 methods)
- # Function to generate random 2D points
- function generate_points(n, w_true, b_true, noise=0.5)
-     x_vals = sort(randn(n))
-     y_vals = w_true .* x_vals .+ b_true .+ noise * randn(n)
-     return zip(x_vals, y_vals)
- end

- Enter cell code...

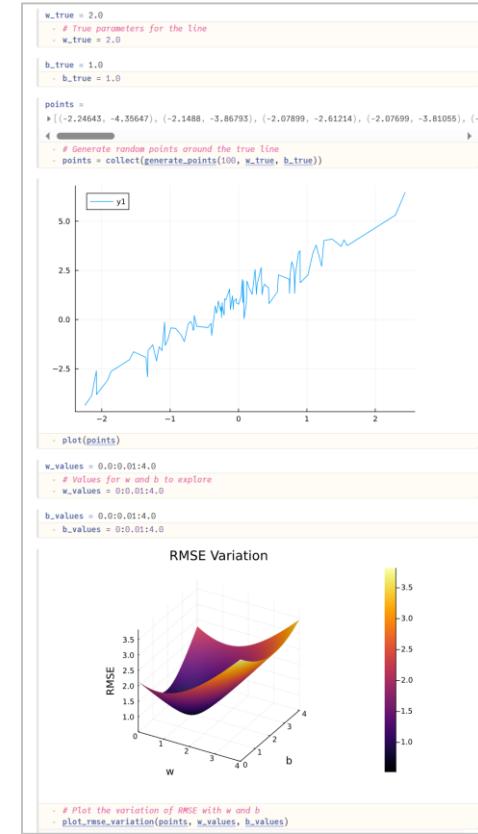
plot_rmse_variation (generic function with 1 method)
- # Function to plot the variation of RMSE with w and b
- function plot_rmse_variation(points, w_vals, b_vals)
-     rmse_matrix = zeros(length(w_vals), length(b_vals))
-
-     for (i, w) in enumerate(w_vals)
-         for (j, b) in enumerate(b_vals)
-             rmse_matrix[i, j] = rmse(points, w, b)
-         end
-     end
-
-     print(rmse_matrix)
-
-     surface(w_vals, b_vals, rmse_matrix, xlabel="w", ylabel="b", zlabel="RMSE",
-             title="RMSE Variation")
-
-     #heatmap(w_vals, b_vals, rmse_matrix, xlabel="w", ylabel="b", zlabel="RMSE",
-             title="RMSE Variation", c = :roma100)
-
-     #rmse_matrix
- end
```

“Training” the Neural Network becomes an optimization problem where we want to minimize some form of aggregate error of the model with respect to the training set of points (we want to find the “best” model parameters, the w’s and the b’s)

Let’s plot the error (“loss”, in AI parlance) function of a purely linear approximation to a cloud of points as a function of the parameters w and b of the line

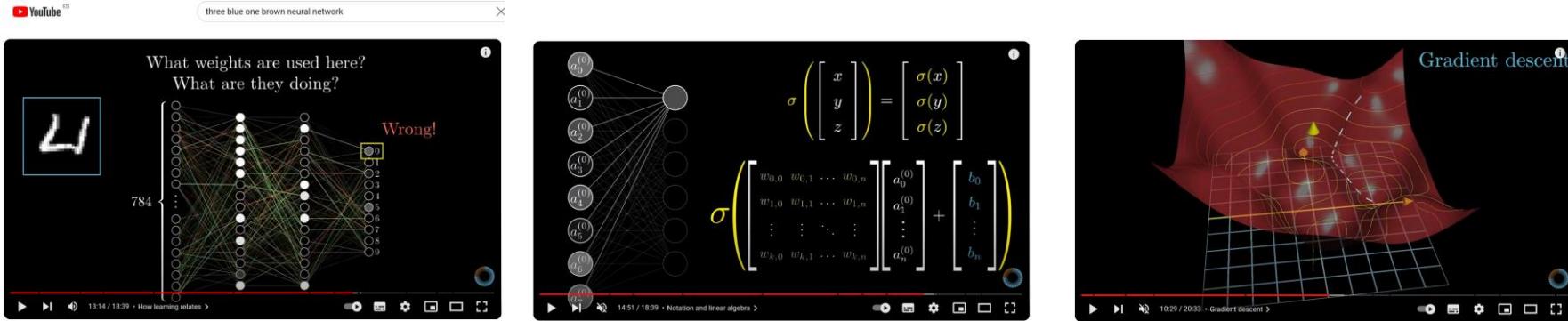
This code has been generated using ChatGPT3.5 with the prompt below (after “regenerating” 3 times) and manually edited.

Prompt: Please write a julia code that computes the root mean square distance between a cloud of 2D points and a line defined as  $y = w \cdot x + b$  and then plot the variation of the root mean square distance as a function of the parameters w and b



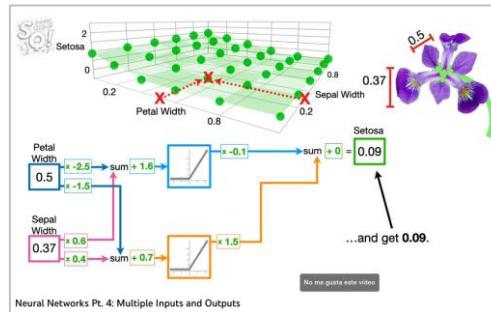
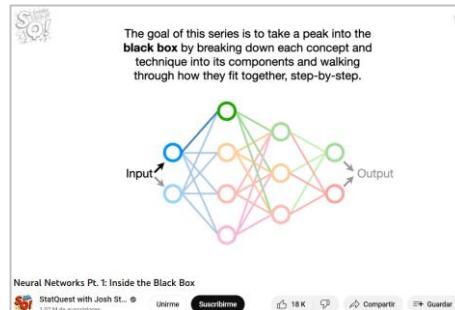
# Introductory resources to learn about Neural Networks

One of the best introductions is that by Grant Sanderson's YouTube channel ([3Blue 1Brown](#)) [here](#), [here](#) and [here](#)



The best introductory YouTube channel for machine learning is Josh Starmer's "[StatQuest](#)"

The [Deep Learning series of videos](#) is excellent (although you need to either get used or ignore the songs)



Don't get the wrong impression from watching a single video, StatQuest explains really advanced topics in a clear way

# Learning a simple function with a simple Neural Network

Pluto code



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Flux\\_quadratic\\_function\\_GOOD\\_MUSIC.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Flux_quadratic_function_GOOD_MUSIC.jl)

The simplest “complex” neural network is the [multilayer perceptron](#).

Mathematically it is just a sequence of applications of a non-linear function (e.g. sigma) to the results of a linear transformation ( $W \cdot x + b$ ), where  $W$  is a matrix of weights and  $b$  is a vector and these are different in each layer

```
• using Flux √ , Plots √ , PlutoUI √ , Sound √ # import libraries for machine
  learning and plotting

1x100 Matrix{Float32}:
-1.77889  1.64984  0.295516 -0.134305  0.115341 ...  0.941935 -0.47048 -0.519015
• begin
•   # Generate training data
•   x_train, y_train = generate_data(100) # obtain the data for training (100 points)
•   # convert the data into a row vector for Flux
•   Y_train = Float32.(reshape(y_train, 1, :)) # Flux prefers Float32 data (for GPU)
•   X_train = Float32.(reshape(x_train, 1, :))
• end

ground_truth (generic function with 1 method)
• # Define the ground truth, the underlying true data
• ground_truth(x) = 3 * x^3

generate_data (generic function with 1 method)
• # Generate the training data using the ground truth function and add noise
• function generate_data(n)
•   x_vals = randn(n) # generate an array of random numbers
•   y_vals = ground_truth.(x_vals) + randn(n) # add some Gaussian noise to the truth
•   return x_vals, y_vals # return the training points as x and y values
• end
```

Generation of the training data and reshaping of the arrays to conform to the format expected by Flux

This function defines the (generally not known) underlying true function of the data

The generate\_data function adds some Gaussian noise to the underlying law of the data

# Learning a simple function with a simple Neural Network

10

```
• @bind n_neurons Slider(1:20, default = 10, show_value = true )
```

86.5 ms

```
model = Chain(  
    Dense(1 => 10, relu), # 20 parameters  
    Dense(10 => 10, relu), # 110 parameters  
    Dense(10 => 1), # 11 parameters  
)
```

# Total: 6 arrays, 141 parameters, 948 bytes.

```
• # Define the neural network as a multilayer perceptron  
• model = Chain( # stack layers of "neurons"  
•     Dense(1, n_neurons, relu), # a layer with 1 input, n_neurons "neurons"  
•     Dense(n_neurons, n_neurons, relu), # a layer with n_neurons inputs and n_neurons "neurons"  
•     Dense(n_neurons, 1) # output layer, taking n_neurons inputs from the previous  
•     layer into a single value  
• )
```

The slider allows to select a different number of “neurons” in the hidden layers. This is really the dimensionality in which the network operates in those layers

The model is built as a sequence of layers, defined as the dimensions of the input and output of the linear transformations and stating the name of the non-linear function (“Activation function”). In this case, we use ReLU

The loss function is “mse” (mean squared error) and is selected from a variety of possible choices offered by Flux

The type of optimizer (in this case a simple gradient descent, normally ADAM is used) and the number of “epochs” (iterations) are defined here

This function returns the absolute error between the ground truth and the model at a given point, used to play a sound...

# Learning a simple function with a simple Neural Network

```
begin
    # Train the neural network using Flux
    for epoch in 1:num_epochs    # iterate the training "num_epochs" times
        Flux.train!(loss,
                    Flux.params(model), # specify the loss function to use
                    [(X_train, Y_train)], # pass the model weights and biases
                    optimizer) # use the optimizer function defined before

        current_loss = Flux.mse(model(X_train), Y_train) # current loss
        training_loss[epoch] = current_loss # store loss per iteration

        # Make a "chord" with frequencies corresponding to loss and local errors at
        # some points
        beep(hcat([current_loss], [err_Ground_vs_Model(p) for p in [-1 -.5 0 .5
        1].*500] )) # [current_loss and local errors]

    end
end
```



Pluto shows a progress bar for the training at each iteration. On a larger model, this can take a while...

The training process is a simple loop over the number of "epochs"

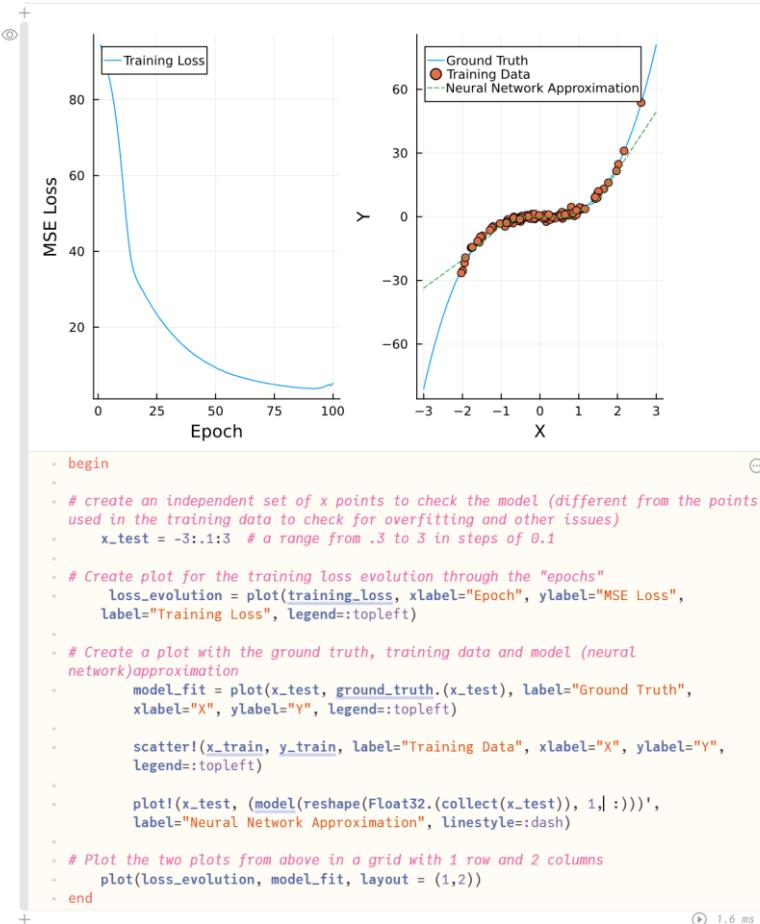
The Flux.train! Function computes internally the derivatives (the gradient) of the loss function with respect to the model parameters and modifies the parameters by a small amount in the opposite direction of the gradient (the gradient tells you, for example, how to move uphill on a Surface)

For each iteration, the loss at that step is recorded in a vector previously initialized. This is used to plot the training history.

The last statement in this block of code computes the absolute error between the underlying function at a collection of points and the model prediction at each step of the training.

A sound, composed of frequencies proportional to the absolute error and the global loss, is played while the model is training. If everything goes well, the sounds should become more grave with each iteration

# Learning a simple function with a simple Neural Network



The curve on the left shows the evolution of the global error (the “loss”) for each iteration of the training process (each “epoch”)

On the right the underlaying curve is plotted with the blue line, the actual points in the training set, which contain noise on top of the underlying function, are shown as circles and the prediction of the model resulting from the training as a dashed green line.

Note that outside from the set of training data the model predict linear behaviour (because we have used a ReLU function)

As the points are generated with a Gaussian random distribution (randn), the model has better accuracy where the points accummulate, near the origin).

A lot of subtleties related to fitting models to real data are covered in this example:

- Noise
- Accummulation of data near regions of interest
- Lack of accuracy of the model outside of the range of the data

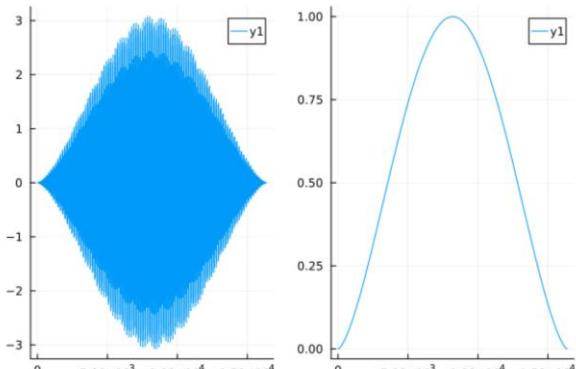
# Playing sounds while learning

Pluto code



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Play\\_sound\\_simple\\_GOOD.jl](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Play_sound_simple_GOOD.jl)

```
beep (generic function with 1 method)
+ function beep(freq_array::Array; t = .02, plot_wave = false)
+ 
+     S = 8192 # sampling rate in Hz
+     scale = 1/t
+ 
+     modu = @. sin( 2π*(1:S*scale) * .5*scale/S )^1.5 # Modulation shape
+ 
+     wave(f) = @. modu * (sin( 2π*(1:S*scale) * f/S ) ) # Define function to create wave
+ 
+     wave_array = reshape([wave(f) for f in freq_array], length(freq_array),1)
+ 
+     wavr = sum(wave_array, dims=1)[1] # right channel
+     wavl = wavr # left channel
+ 
+     soundsc([wavr wavl], S) # scale to unit amplitude
+ 
+     if plot_wave plot(plot(wavr), plot(modu), layout = (1,2)); end
+ 
+ end
```



```
- beep([500 1500 200 150]; t = 2, plot_wave = true) # test
```

As a bonus, this implementation of the code plays a sound as during the training of the network.

The loss and local absolute errors between the underlying function and the model prediction at some points are calculated at each epoch.

The value of the loss is scaled up to match the audible frequency range and waves of frequencies corresponding to these scaled error values are synthetised.

The pure sin waves are modulated with another wave to prevent a “click” at the start and end of the sound.

The superposition of the waves creates a chord which will be evolving towards a grave sound as the training progresses “downhill” in the error functions.

Watch this!



[Watching Neural Networks](#)  
[Learn - YouTube](#)



# DataFrames - library for data processing

Pluto code



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/DataFrames\\_example\\_GOOD](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/DataFrames_example_GOOD)

```
using DataFrames ✓  
using Random ✓, Statistics ✓  
  
TaskLocalRNG()  
# Generate some random data  
Random.seed!(42) # for reproducibility  
  
n = 10  
n = 10  
  
data =  
  
| ID | Name | Age | Score |  
|---|---|---|---|  
| 1 | "Person 1" | 31 | 86 |  
| 2 | "Person 2" | 27 | 37 |  
| 3 | "Person 3" | 27 | 69 |  
| 4 | "Person 4" | 28 | 88 |  
| 5 | "Person 5" | 21 | 7 |  
| 6 | "Person 6" | 32 | 63 |  
| 7 | "Person 7" | 38 | 25 |  
| 8 | "Person 8" | 29 | 72 |  
| 9 | "Person 9" | 23 | 96 |  
| 10 | "Person 10" | 24 | 82 |  
  
data = DataFrame(  
    ID = 1:n,  
    Name = ["Person $i" for i in 1:n],  
    Age = rand(20:40, n),  
    Score = rand(0:100, n)  
)  
  
# Display the DataFrame  
println("Original DataFrame")  
  
Original DataFrame:  
  
| ID | Name | Age | Score |
```

ID	Name	Age	Score
1	Person 1	31	86
2	Person 2	27	37
3	Person 3	27	69
4	Person 4	28	88
5	Person 5	21	7
6	Person 6	32	63
7	Person 7	38	25
8	Person 8	29	72
9	Person 9	23	96
10	Person 10	24	82

```
display(data)  
  
10x4 DataFrame  
Row | ID | Name | Age | Score  
|---|---|---|---|---|  
| 1 | 1 | Person 1 | 31 | 86 |  
| 2 | 2 | Person 2 | 27 | 37 |  
| 3 | 3 | Person 3 | 27 | 69 |  
  
ages = [31, 27, 27, 28, 21, 32, 38, 29, 23, 24]  
# Accessing columns  
ages = data.Age  
  
display(ages)  
  
10-element Vector{Int64}:  
31  
27  
27  
28  
21  
  
["C", "A", "A", "B", "C", "C", "B", "B", "A", "A"]  
# Adding a new column  
data[!, :Grade] = rand(["A", "B", "C"], n)  
  
println("\nDataFrame with a new 'Grade' column:")  
  
DataFrame with a new 'Grade' column:  
  
display(data)  
  
10x5 DataFrame  
Row | ID | Name | Age | Score | Grade  
|---|---|---|---|---|---|  
| 1 | 1 | Person 1 | 31 | 86 | C |  
| 2 | 2 | Person 2 | 27 | 37 | A |  
| 3 | 3 | Person 3 | 27 | 69 | A |
```

This is an AI-generated example of the use of `DataFrames` showing some typical functions

One of the advantages of using Julia instead of Excel is the ability to process and filter enormous quantities of data in a `dataframe` versus a spreadsheet

```
filtered_data =  
  
| ID | Name | Age | Score | Grade | |
|---|---|---|---|---|---|
| 1 | 1 | Person 1 | 31 | 86 | "C" |  
| 2 | 6 | Person 6 | 32 | 63 | "C" |  
| 3 | 7 | Person 7 | 38 | 25 | "B" |  
  
# Filtering data  
filtered_data = filter(row > row.Age > 30, data)  
  
display(filtered_data)  
  
3x5 DataFrame  
Row | ID | Name | Age | Score | Grade  
|---|---|---|---|---|---|  
| 1 | 1 | Person 1 | 31 | 86 | C |  
| 2 | 6 | Person 6 | 32 | 63 | C |  
| 3 | 7 | Person 7 | 38 | 25 | B |
```

# Other notable packages

[Gridap.jl](#) Solving Partial Differential Equations (PDE), including FE, CFD, electromagnetism, etc...

[Ferrite.jl](#) A Finite Elements toolbox

[Waterlily.jl](#) CFD solver using the GPU with kernels

[OrbitalTrajectories.jl](#) A package for orbital mechanics

And many others, just google your field of interest...

The screenshot shows a web browser displaying the Julia Packages website at <https://juliapackages.com/trending?sort=stars>. The page is titled "Trending Packages". On the left, there is a sidebar with navigation links: "All Packages", "Trending" (which is highlighted in blue), and "Developers". Below these are "CATEGORIES" with links to Mathematics, Programming Paradigms, AI, Graphics, File IO, Optimization, Probability & Statistics, and Super Computing. There is also a link to "View more categories". The main content area lists several packages with their names, descriptions, and star counts. The packages listed are:

Package	Description	Stars
Pluto.jl	Simple reactive notebooks for Julia	4500
Flux.jl	Relax! Flux is the ML library that doesn't make you tensor	4122
IJulia.jl	Julia kernel for Jupyter	2629
Makie.jl	Visualizations and plotting in Julia	1978
Plots.jl	Powerful convenience for Julia visualizations and data analysis	1710
ModelingToolkit.jl	An acausal modeling framework for automatically parallelized scientific machine learning (SciML) in Julia. A computer algebra system for integrated symbolics for physics-informed machine learning and automated transformations of differential equations	1212
Revise.jl	Automatically update function definitions in a running Julia session	1053
OhMyREPL.jl	Syntax highlighting and other enhancements for the Julia REPL	676
JET.jl	An experimental code analyzer for Julia. No need for additional type annotations.	571
WaterLily.jl		405

# High performance Julia



# High performance



*"High performance means not wasting a single FloP"*  
*(Celeste Project)*

**It requires:**

Good coding practice

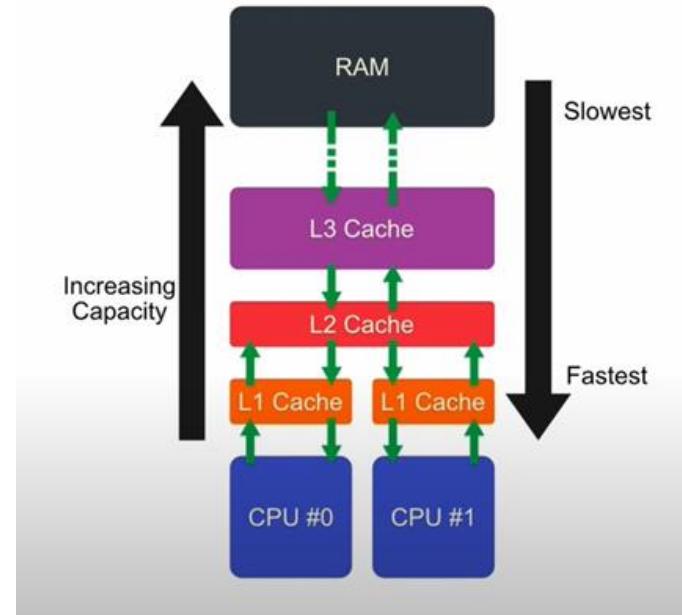
Code optimization (minimize memory allocations and function calls)

Use all of your CPU cores (multithreading)

If you can use all of your GPU cores too (GPU programming)

If it's not enough use all of your neighbours CPU cores (CPU cluster)

And if you're really desperate use all of your neighbours GPU cores  
(GPU cluster, the new standard to come)



# Measuring time and allocations - BenchmarkTools

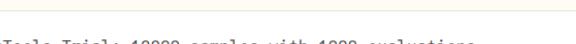
The native @time macro can be misleading as it can measure also the compilation time and does it only once

For proper time and memory allocation measurements, add the `BenchmarkTools` package and use the `@benchmark` macro

```
• using BenchmarkTools ✓

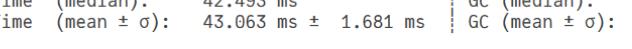
test (generic function with 1 method)
• function test(x::Int)
•     for i in 1:x
•         i^3
•     end
• end
•

BenchmarkTools.Trial: 10000 samples with 1000 evaluations.
Range (min ... max):  2.900 ns ... 15.500 ns | GC (min ... max): 0.00% ... 0.00%
Time (median):        3.100 ns | GC (median):    0.00%
Time (mean ± σ):     3.144 ns ±  0.592 ns | GC (mean ± σ): 0.00% ± 0.00%


Memory estimate: 0 bytes, allocs estimate: 0.
• @benchmark test(100000000000)
```

0 memory used and 0 allocations to loop a trillion times?

The Julia compiler understands that nothing is required and actually doesn't do the loop...

```
• using BenchmarkTools ✓  
• A = rand(1000, 1000) ; b = rand(1000) ;  
  
solve (generic function with 1 method)  
• solve(MyMatrix, MyVector) = MyMatrix \ MyVector  
+  
⑥ BenchmarkTools.Trial: 116 samples with 1 evaluation.  
Range (min ... max): 40.947 ms ... 48.021 ms | GC (min ... max): 0.00% ... 9.79%  
Time (median): 42.493 ms | GC (median): 0.00%  
Time (mean ± σ): 43.063 ms ± 1.681 ms | GC (mean ± σ): 1.51% ± 3.21%  
  
  
40.9 ms Histogram: frequency by time 47.4 ms <  
  
Memory estimate: 7.64 MiB, allocs estimate: 4.  
• @benchmark solve($A, $b)
```

Now we can see the statistical distribution of execution times (due to the CPU doing other things in parallel) and a significant memory allocation

The variable names passed to a function to be benchmarked need to be “interpolated” (just add \$ in front of the variable name)

# Parallelization CPU - Multithreading



[08x03] Intro to Julia Multi-Threading | How to use the Threads.@threads macro - YouTube

To tell Julia how many cores you want to use start Julia with `julia -t=8` (in this case 8 cores)

Normally you want to use all the cores you have, so do `julia --threads=auto`

To know how many cores you have, type (once in a Julia REPL) `Threads.nthreads()`

The easiest and safer way to use all of your cores is when you have loops (or other easily divisible operations)

Example:

```
julia> Threads.@threads for i = 1:10
           a[i] = Threads.threadid()
       end
```

In general, loops tend to be embarrassingly parallelizable so just use the `Threads.@threads` macro in front of the for keyword

To have more control of each thread, use Atomic operations (but this is not so easy and can lead to "[race conditions](#)")

## Race Condition

```
using Base.Threads
function my_sum(numbers::Vector{Int})
    s = 0
    @threads for n in numbers
        s += n
    end
    return s
end
```

## Thread-Safe (with Atomics)

```
function my_sum(numbers::Vector{Int})
    s = Atomic{Int}(0)
    @threads for n in numbers
        atomic_add!(s, n)
    end
    return s[]
end
```

Screenshot from the excellent [MPAGS High Performance Computing in Julia](#) course

# Parallelization GPU - Array (or generic) programming

The CUDA.jl package provides “high level abstractions” (i.e., makes very easy) to program the GPU (but you need a modern Nvidia GPU card). By the way, GPU means Graphics Processing Unit.

There are two levels of abstraction; generic or array programming and kernel programming

Array programming is as simple as declaring an array as of type CUDA, prefixing it as in the example

Two important considerations:

- Moving the arrays from CPU o GPU take time, so move large arrays, do the operation in the GPU and bring it back
- The normal GPU floating point numbers have 32 bits, so there is much lower precision there (at least natively)

```
using CUDA

A = CUDA.fill(1.0f0, 100)
B = CUDA.fill(2.0f0, 100)
C = CUDA.zeros(100)

C .= A .+ B

println(C)
```

These lines declare and initialize the arrays as GPU arrays. You can use rand, zeros....



[CUDA.jl Kernel Programming \(HPC in Julia 10/10\) - YouTube](#)

The element-wise operation generalizes to GPU arrays and this operation, which is embarrassingly parallel, is performed here in the GPU. Just like that.

Excellent resource for this topic

[Lecture 6 \(ethz.ch\)](#)

# Parallelization GPU – Kernel functions



CUDA.jl Kernel Programming (HPC in Julia 10/10) - YouTube

A “kernel” is a small program designed to run in each of the “cores” of a GPU (of which there can be more than 10000 in a 2000€ Nvidia card today)

Kernel programming is hard and inherits a lot of the idiosyncrasies of graphical programming in a GPU (its original purpose).

```
using CUDA

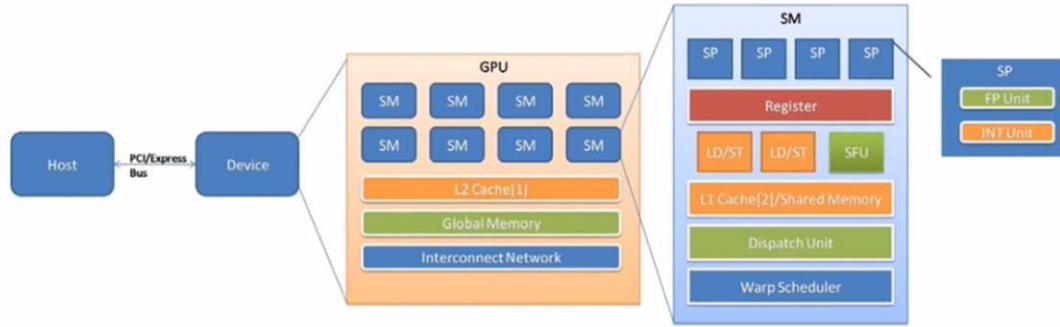
function kernel_example(A::CuArray{Float32}, B::CuArray{Float32}, C::CuArray{Float32})
    i = (blockIdx().x-1) * blockDim().x + threadIdx().x
    C[i] = A[i] + B[i]
    return nothing
end

A = CUDA.fill(1.0f0, 100)
B = CUDA.fill(2.0f0, 100)
C = CUDA.zeros(100)

@cuda threads=100 blocks=1 kernel_example(A, B, C)

println(C)
```

Fig 1: Sivalingam, Karthee “GPU Acceleration of a Theoretical Particle Physics Application”



Probably the hardest part of kernel programming is to understand the indexing required to refer to a single “lane” (core)

Even though Julia is “1 indexed”, in GPU programming is “0 indexed” like C

# Mandelbrot GPU

Pluto code



[https://github.com/flt-acdesign/Mathematical\\_tourism\\_with\\_Julia/blob/main/Mandelbrot\\_GPU\\_CUDA\\_good.ipynb](https://github.com/flt-acdesign/Mathematical_tourism_with_Julia/blob/main/Mandelbrot_GPU_CUDA_good.ipynb)

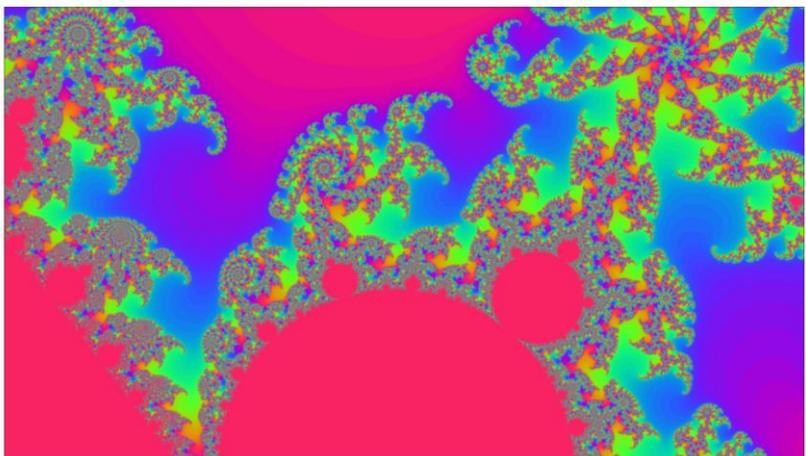
The code in the repository is a Pluto adaptation of a pure Julia code from other repo (details in the code).

It shows two GPU implementations of the Mandelbrot algorithm, one with array processing an another with kernels.

Both run extremely fast (if you have a CUDA.jl-compatible GPU) and the color scheme is quite spectacular.

The original autor himself is not very proud of some parts of the code (particularly he mapping between screen coordinates and complex plane...) but the code shows the use of the Symbolics.jl package too

```
begin
    # Using the CUDA.jl occupancy API, from the example at
    #
    https://github.com/JuliaGPU/CUDA.jl/blob/cb4a6b03ff443a062212c7a8aad260cfbf134410/src/indexing.jl#L37
    Z2 = CuArray{Float64}(undef, height, width);
    # Manual invocation, with fixed set of threads:
    # CUDA.@sync @cuda threads=1024 kernel_mandelbrot!(Z2);
    #
    kernel = @cuda name="mandelbrot" launch=false kernel_mandelbrot!(Z2)
    config = launch_configuration(kernel.fun)
    threads = Base.min(length(Z2), config.threads)
    blocks = cld(length(Z2), threads)
    # The @benchmark macro is optional (again, it causes an error. RCLOct23)
    CUDA.@sync kernel(Z2; threads=threads, blocks=blocks)
end
```



Fijado por doggo dot jl

@viniciusfreitasdealmeida3663 hace 7 meses

Yay, first time getting featured in a video! What an honour :D

I confess I don't remember really well how I implemented the transforms and interpolation expressions, but in retrospective it **does** look a lot like spaghetti code, haha! I'd be happy to go through it and refactor to make it clearer, tough. Also, feel free to give me any feedbacks on how to improve it!

# Explaining code using Large Language Models (e.g. GPT)

LLM frameworks (e.g. ChatGPT) can also be used to explain code.

In the example below, the prompt contains a block of not very simple CUDA.jl code from the previous Mandelbrot example

On the right, ChatGPT3.5 has been able to provide a convincing explanation of the code

Beware! LLMs can be too convincing and have a tendency to “hallucinate”

UE

```
can you explain the following Julia code which uses the CUDA package? Z2 =  
CuArray(Float64)(undef, height, width);  
# Manual invocation, with fixed set of threads:  
# CUDA.@sync @cuda threads=1024 kernel_mandelbrot!(Z2);  
  
kernel = @cuda name="mandelbrot" launch=false kernel_mandelbrot!(Z2)  
config = launch_configuration(kernel.fun)  
threads = Base.min(length(Z2), config.threads)  
blocks = cld(length(Z2), threads)  
# The @benchmark macro is optional (again, it causes an error. RCLOct23)  
CUDA.@sync kernel(Z2; threads=threads, blocks=blocks)
```



Certainly! The provided Julia code involves the use of the CUDA.jl package to perform GPU-accelerated computations for generating the Mandelbrot set. Let's break down the code step by step:

1. `Z2 = CuArray(Float64)(undef, height, width);` : This line creates an uninitialized CUDA array (`CuArray`) of type `Float64` with dimensions `height` by `width`. The Mandelbrot set will be stored in this array.
2. `kernel = @cuda name="mandelbrot" launch=false kernel\_mandelbrot!(Z2)` : This line creates a CUDA kernel object named "mandelbrot" using the `@cuda` macro. The `launch=false` option is used to prevent immediate launching of the kernel.
3. `config = launch\_configuration(kernel.fun)` : This line retrieves the launch configuration for the CUDA kernel. It is used to determine the number of threads and blocks that will be used to execute the kernel.
4. `threads = Base.min(length(Z2), config.threads)` : This line calculates the number of threads to be used. It takes the minimum of the total length of `Z2` (total number of elements in the array) and the number of threads obtained from the launch configuration.
5. `blocks = cld(length(Z2), threads)` : This line calculates the number of blocks to be used. It uses the `cld` function (short for "ceiling division") to ensure that there are enough blocks to cover all elements of the array.
6. `CUDA.@sync kernel(Z2; threads=threads, blocks=blocks)` : This line synchronously launches the CUDA kernel on the GPU with the specified number of threads and blocks. The `@sync` macro ensures that the execution on the GPU is complete before proceeding.

In summary, this code sets up a CUDA array, creates a CUDA kernel object for the Mandelbrot computation, determines the launch configuration, calculates the number of threads and blocks, and then launches the kernel on the GPU using the specified configuration. The result, the Mandelbrot set, is stored in the `Z2` array on the GPU.

# Conclusions

- Julia or Python? -> Do you want to lead or you prefer to follow?
- Julia is an appropriate language for scientists and engineers who require to build high performance algorithms with high productivity
  - (how much time you have left to finish your PhD?)
- Large Language Models represent a step change in the productivity of code development and are a great tool to learn
- Julia + Pluto is a free and powerful educational tool

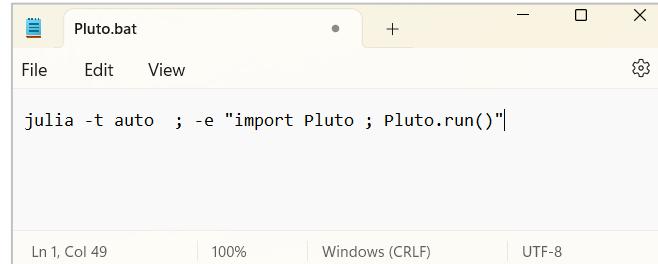
# Backup



# Starting Pluto with one click (in Windows)

Create a file with a text editor and write

```
julia -t auto ; -e "import Pluto ; Pluto.run()"
```



A screenshot of a Windows-style text editor window titled "Pluto.bat". The window has standard minimize, maximize, and close buttons at the top right. Below the title bar is a menu bar with "File", "Edit", and "View". The main area contains the following text:

```
julia -t auto ; -e "import Pluto ; Pluto.run()"
```

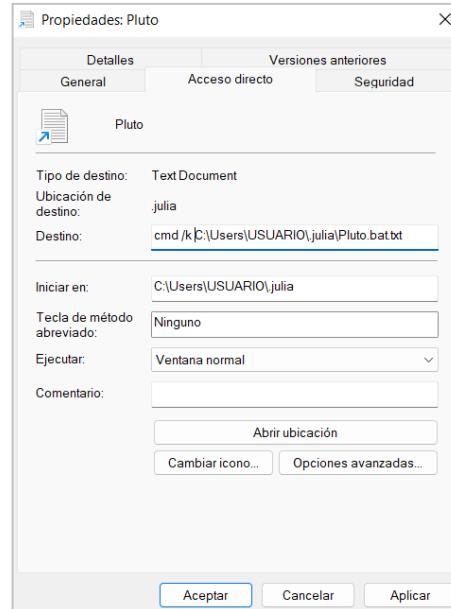
At the bottom of the window, there are status bars for "Ln 1, Col 49", "100%", "Windows (CRLF)", and "UTF-8".

Save it as Pluto.bat

Create a shortcut to this file by right-clicking on the desktop, select the file and edit the target to add in front of the pathname to the Pluto.bat file

```
cmd /k C:\path\to\your\Pluto.bat
```

Save and right-click on the shortcut. A menu will appear with an option “Pin to start”. Now you can launch pluto from the Windows start menu.



# Code Optimization

Type declaration

Columns first

Static arrays

Broadcasting, map, reduce etc... vectorization vs for loops

# Learn Julia



# Basic learning resources

Doggo, MIT Computational Thinking, Rackaukas, Julia conferences

Vectorization

En español

[Fast Track to Julia](#): Syntax summaries

# More resources

Three Blue One Brown

Steve Brunton

Statquest

The coding train

# Profilers