1. 下列各数都是经过四舍五入后得到的近似数，试指出它们是具有几位有效数字的近似值，并确定$x_1^* + x_2^* + x_3^*$和$x_1^* x_2^* x_3^*$的误差限。(4.0 分)

$$x_1^* = 1.1021, x_2^* = 0.031, x_3^* = 385.6$$

答:

x_1^*有 5 位有效数字，x_2^*有 2 位有效数字，x_3^*有 4 位有效数字

$$\eta(x_1^*) = 0.5 * 10^{-4}, \eta(x_2^*) = 0.5 * 10^{-5}, \eta(x_3^*) = 0.5 * 10^{-1}$$
$$\eta(x_1^* + x_2^* + x_3^*) = \eta(x_1^*) + \eta(x_2^*) + \eta(x_3^*) = 0.5055 * 10^{-1}$$
$$\eta(x_1^* x_2^* x_3^*) = |x_1^* x_2^* x_3^*| \left(\frac{\eta(x_1^*)}{|x_1^*|} + \frac{\eta(x_2^*)}{|x_2^*|} + \frac{\eta(x_3^*)}{|x_3^*|} \right) \approx 0.2148$$

2.已知$a^* = 1.1062, b^* = 0.947$都是经过四舍五入后得到的近似值，问$a^* + b^*$和$a^* b^*$各有几位有效数字。(4.0 分)

答:

$$\eta(a^*) = 0.5 * 10^{-4}, \eta(b^*) = 0.5 * 10^{-3}$$
$$\eta(a^* + b^*) = \eta(a^*) + \eta(b^*) = 0.55 * 10^{-3} \leq 0.5 * 10^{-2}$$
$$\eta(a^* b^*) = |b^*|\eta(a^*) + |a^*|\eta(b^*) \approx 0.6 * 10^{-3} \leq 0.5 * 10^{-2}$$
$$a^* + b^* = 2.0532 \approx 2.05, a^* b^* = 1.0476 \approx 1.05$$

故两者均为三位有效数字。

3.求 x，使 3.141 和 3.142 作为 x 的近似值都具有 4 位有效数字。(4.0 分)

答:

$$|x - 3.141| \leq 0.5 * 10^{-3}$$
$$|x - 3.142| \leq 0.5 * 10^{-3}$$
$$\therefore 3.1405 \leq x \leq 3.1415$$
$$3.1415 \leq x \leq 3.1425$$
$$\therefore x = 3.1415$$

4.计算球体体积，要求相对误差限为 1%，问度量半径 R 时的相对误差限为多少？

（4.0 分）

答:

$$|\varepsilon_r(V^*)| = \left|\frac{\varepsilon(V^*)}{V^*}\right| = \left|\frac{\varepsilon\left(\frac{4}{3}\pi(R^*)^3\right)}{\frac{4}{3}\pi(R^*)^3}\right| = \left|\frac{|\frac{4}{3}\pi R^3 - \frac{4}{3}\pi(R^*)^3|}{\frac{4}{3}\pi(R^*)^3}\right|$$

$$= \left|\frac{\frac{4}{3}\pi * |R^3 - (R^*)^3|}{\frac{4}{3}\pi(R^*)^3}\right| = \left|\frac{\varepsilon((R^*)^3)}{(R^*)^3}\right|$$

$$\because \varepsilon(x^*y^*) \le |x^*|\eta(y^*) + |y^*|\eta(x^*)$$

$$\therefore \varepsilon((x^*)^2) \le 2|x^*|\eta(x^*)$$

$$\therefore \varepsilon((x^*)^3) = \varepsilon((x^*)^2 * x^*) \le |x^*|^2\eta(x^*) + |x^*|\eta((x^*)^2)$$

$$\le |x^*|^2\eta(x^*) + |x^*| * 2|x^*|\eta(x^*) = 3|x^*|^2\eta(x^*)$$

$$\therefore |\varepsilon_r(V^*)| = \left|\frac{\varepsilon((R^*)^3)}{(R^*)^3}\right| \le \left|\frac{3|R^*|^2 * \eta(R^*)}{(R^*)^3}\right| = 3\left|\frac{\eta(R^*)}{R^*}\right| = 3\eta_r(R^*)$$

$$\text{又} \because \eta_r(V^*) = 1\%$$

$$\therefore 3\eta_r(R^*) = 1\%$$

$$\therefore \eta_r(R^*) = \frac{1}{3} * 1\% \le 0.334\%$$