BUAD621: Representing Uncertainty

Intro to RV's and Probability Calibration

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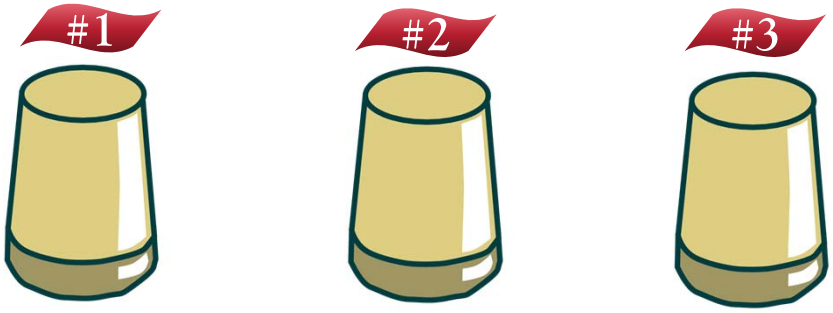
Complete Uncertainty (i.e. ignorance) is Unsettling

Would you pay $2 for the right to choose a cup and receive its contents?

If so, which cup will you choose?



CERTAINTY IS EASIER

Would you pay $2 for the right to choose a cup and receive its contents?

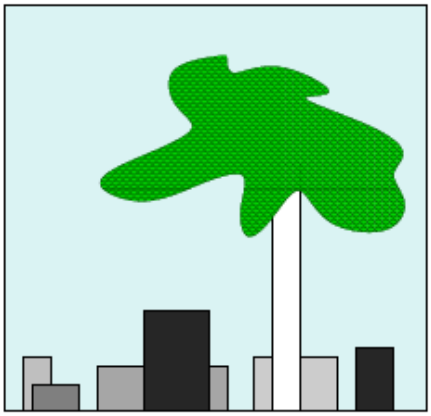
If so, which cup will you choose?

Real-Life is Between Certainty And Ignorance

Would you pay $2 for the right to choose a cup and receive its contents?

If so, which cup will you choose?

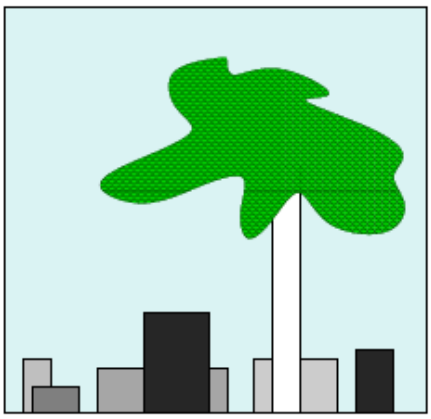
In Real-Life, We Combine Expectations And Data

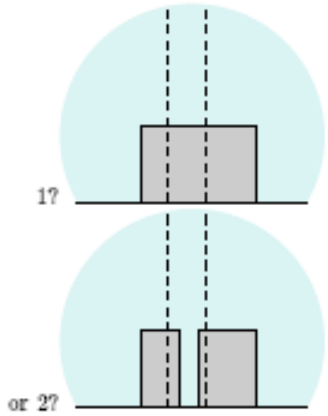


How many boxes are shown in this picture?

In Real-Life, We Combine Expectations And Data

Which is behind the tree?





Data Helps Us To Better Calibrate Our Uncertainty

Beliefs About The World + **Data** = **Updated Beliefs**

1. Beliefs that are consistent with the data become more plausible
2. Beliefs that are inconsistent with the data become less plausible

An Example

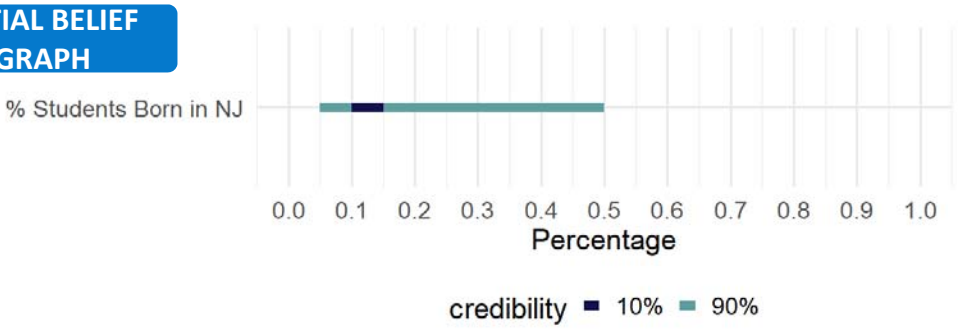
Beliefs About The World + **Data** = **Updated Beliefs**

CURRENT BELIEFS:
What is your belief about the percentage of students in this class born in New Jersey?

An Example

Beliefs About The World + **Data** = **Updated Beliefs**

INITIAL BELIEF GRAPH



An Example

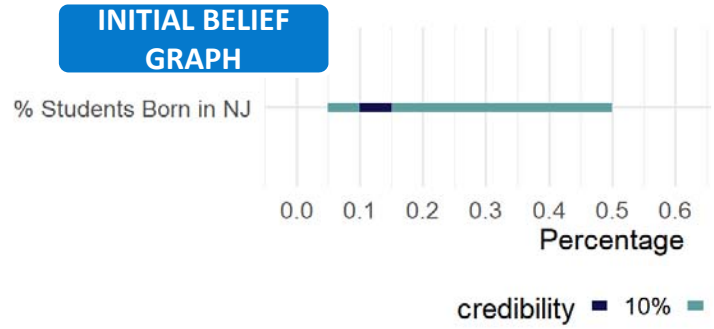
Beliefs About The World + **Data** = **Updated Beliefs**

DATA:
IF YOU ARE SITTING IN THE FIRST ROW OF THE CLASSROOM,
raise your hand and keep it raised if you were born in New Jersey?

An Example

Beliefs About The World + Data = Updated Beliefs

INITIAL BELIEF GRAPH



UPDATED BELIEFS: Interval to the left should get smaller than shown and reflect what you learned from the data.

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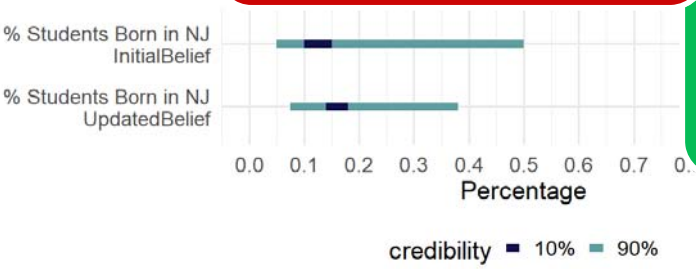
An Example

Beliefs About The World + Data = Updated Beliefs

DATA:
Assume 1 out of 6 polled students were born in NJ



UPDATED BELIEFS: What is your new belief about the percentage of students in this class born in New Jersey?

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Data Helps Us To Better Calibrate Our Uncertainty

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Analytics is Simply Using Data to Inform What "Guesses" Seem to Be Better Than Others

All Data (i.e. measurements) Are Random Variables

Drives A Honda
X

$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

All Data (i.e. measurements) Are Random Variables

What values of X are possible?

Drives A Honda
X

$X \equiv Randomly\ Chosen\ Person\ In\ This\ Room\ Drives\ a\ Honda$

Random Variables Map Outcomes to Real Numbers

$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

What values of X are possible?

$$X \equiv \begin{cases} 0, & if\ person\ does\ not\ drive\ a\ Honda \\ 1, & if\ person\ does\ drive\ a\ Honda \end{cases}$$

Probability Describes Uncertainty in Random Variables

$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

$$X \equiv \begin{cases} 0, & if\ person\ does\ not\ drive\ a\ Honda \\ 1, & if\ person\ does\ drive\ a\ Honda \end{cases}$$

Outcome	Realization (x)	$P(X = x)$
No Honda	0	80%
Honda	1	20%

A Probability Distribution Tells Us Everything

Outcome	Realization (x)	$P(X = x)$
No Honda	0	80%
Honda	1	20%

What values of X are possible?

What is the probability someone drives a Honda?

What is the probability $X = 0$?

What is the probability $X = 2$?

What is the probability $X \leq 2$?

Some Probability Distributions Get Special Names

$$X \sim Bernoulli(\theta)$$

$$X \equiv \begin{cases} 0, & failure \\ 1, & success \end{cases}$$

Outcome	Realization (x)	$P(X = x)$
Failure	0	$1 - \theta$
Success	1	θ

A Probability Distribution Tells Us Everything (revisited)

$$X \sim Bernoulli(\theta = 60\%)$$

What values of X are possible?	What is the probability $X = 0$?	What is the probability $X = 0.6$?	What is the probability $X \leq 1$?

Same questions, just answer without knowing θ

$$X \sim Bernoulli(\theta)$$

What values of X are possible?	What is the probability $X = 0$?	What is the probability $X = 0.6$?	What is the probability $X \leq 1$?

Two Ways of Mathematically Specifying a Bernoulli Random Variable

$$X \sim Bernoulli(\theta)$$ Math Notation

Outcome	Realization (x)	$P(X = x)$
Failure	0	$1 - \theta$
Success	1	θ

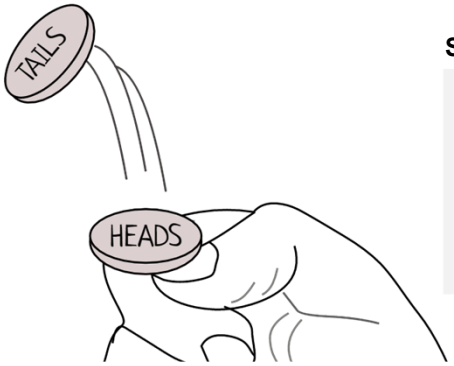
A Lookup Table

Examples of Bernoulli Random Variables

- Will the user click my ad?
- Will the drug lower a patient's cholesterol?
- Will the new store layout increase sales?
- Will the well yield oil?
- Will the customer pay back their loan?
- Will the passenger show up for their flight?
- Is this credit card transaction fraudulent?

The Real World Lacks Mathematical Precision

$$X \equiv Outcome\ of\ Coin\ Flip\ is\ Heads$$



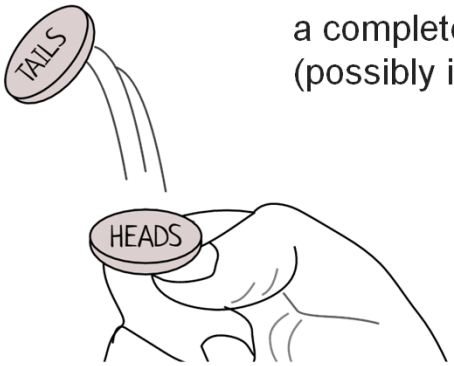
Simulating seven flips of a mathematically perfect coin

```
library(causact)
set.seed(123)
rbern(n=7,prob=0.5)
```

Based only these seven flips, what might you estimate for $P(X = 1)$?

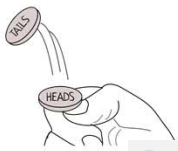
Representative Samples

A **representative sample** is an incomplete collection or subset of data that exhibits similarity to a complete collection of data from an entire (possibly infinite) population.



Do you think seven coin flips:
0 1 0 1 1 0 1
gives a representative sample?

How many flips do you think you need to get a representative sample of coin flips?



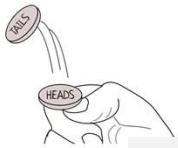
$$X \equiv Outcome\ of\ Coin\ Flip\ is\ Heads$$

```
library(tidyverse)
set.seed(123)

numFlips = 7
df = data.frame(flipNum = 1:numFlips,
                coinFlip = rbern(n=numFlips,prob=0.5)) %>%
  mutate(headsProportion = cummean(coinFlip))

ggplot(df, aes(x = flipNum, y = headsProportion)) +
  geom_point() +
  geom_line() +
  geom_hline(yintercept = 0.5, color = "red")
```


How many flips do you [illegible] representative sample of coin flips?



$$X \equiv Outcome\ of$$

1) Open `numFlips.R`

2) Change the value of:

`numflips`

and see how many flips you need to get a representative sample?

```
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```
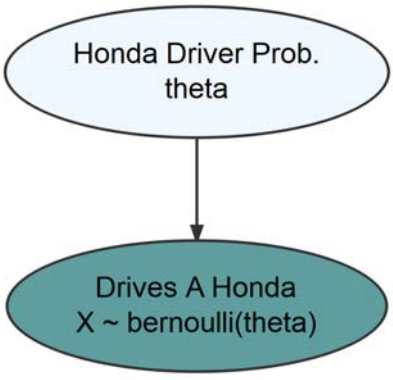

All Data (i.e. measurements) Are Random Variables

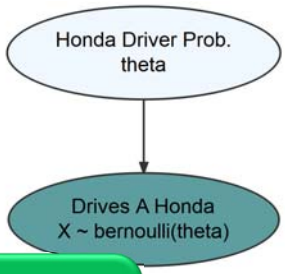
$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

Often Interested in Parameters of Probability Dist



$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

Do we know anything about θ?



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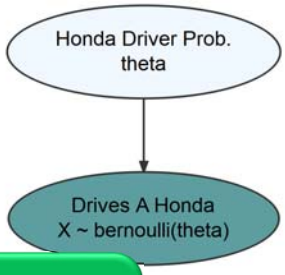
$X \equiv Randomly\ Chosen\ Person\ In\ This\ Class\ Drives\ a\ Honda$

Do we know anything about θ?

What is your 90% confidence interval for θ? Give me a lower bound that has a 5% chance that the true proportion is less than it and an upper bound with a 5% chance that the true proportion is greater than it?



Beliefs About The World + **Data** = **Updated Beliefs**

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Calibrating Your Beliefs

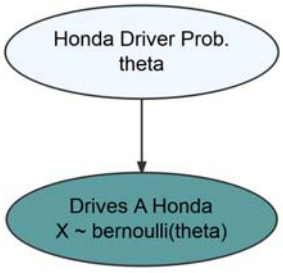
- Create a 90% confidence interval
  - Provide both an upper bound and a lower bound that you believe there is a 90% chance that the answer will be between your bound

1. You will win $1,000 if the true answer is between the numbers you gave for the upper and lower bound

2. You win $1,000 if you draw a red marble from a sack of marbles with 9 red marbles and 1 white marble. (i.e. there is a 90% chance you win $1,000).

Let's Get Calibrated

Poll for true value of θ



See CANVAS for calibration activity