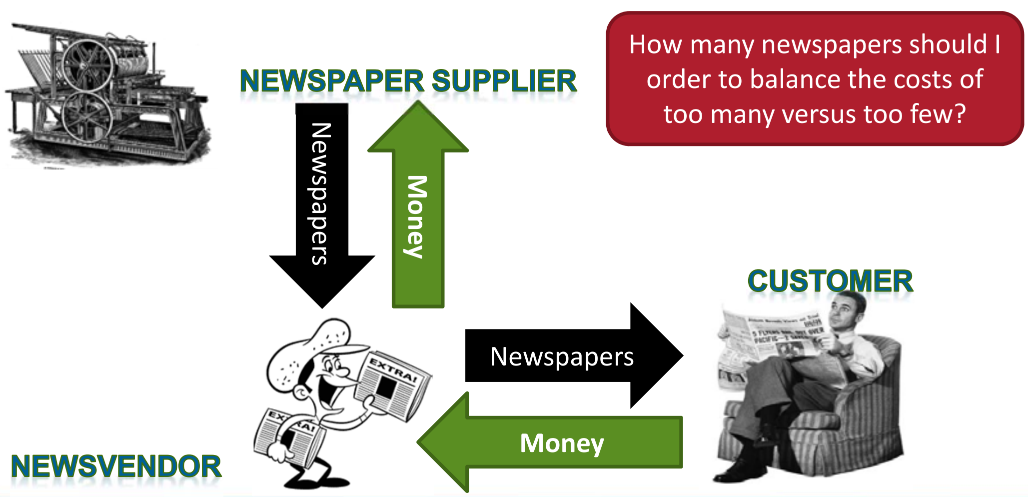
Decision Making Under Uncertainty

A Motivating Example:
Inventory Management for a Short Lifecycle Product – The Newsvendor Model



Inventory Management for a Short Lifecycle Product – The Newsvendor Model

Assumption #1: Inflexible Supply
- ❑ Make order before demand occurs
- ❑ One opportunity to order or procure

Assumption #2: Stochastic Demand
- ❑ Demand is unpredictable

Assumption #3: Supply/Demand Mismatches Are Costly
- ❑ Leftover Inventory: Demand < Supply
- ❑ Lost Sales: Demand > Supply

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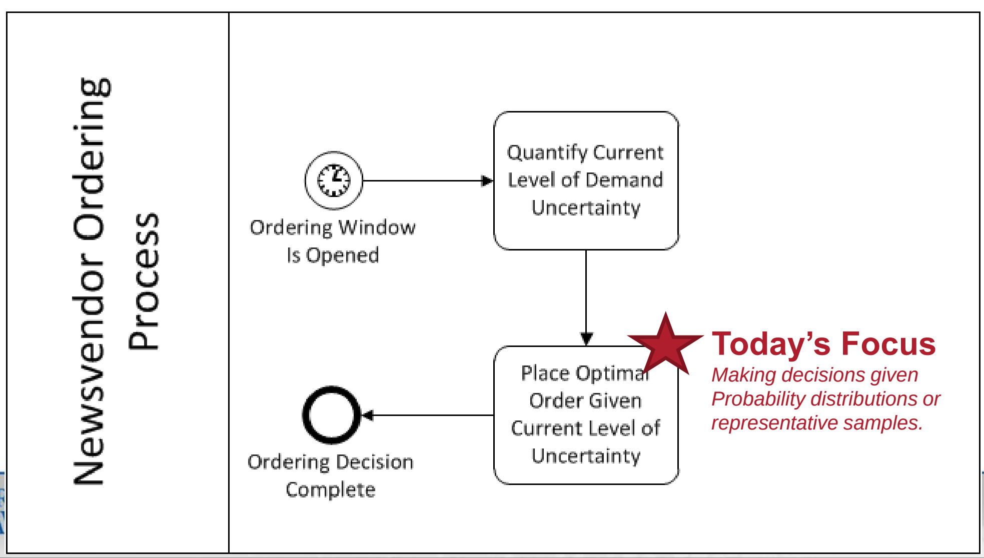
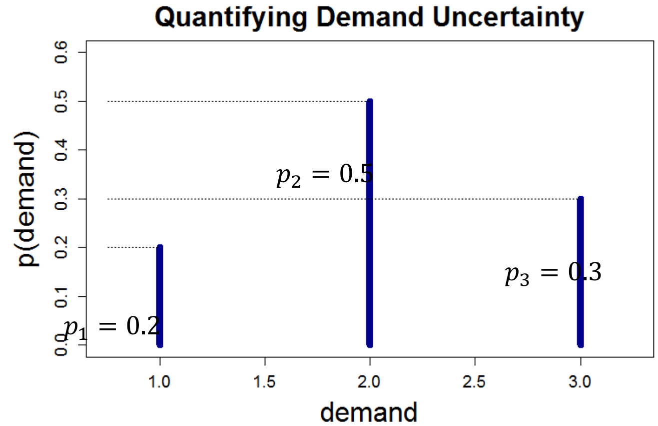
The Typical Newsvendor Ordering Process



Choosing an Order Quantity – The Typical Newsvendor Approach

STEP 1: Quantify Current Level of Demand Uncertainty



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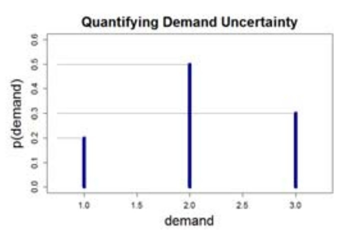

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x	$f(x)$
1	0.2
2	0.5
3	0.3

What is $F(2)$?

Choosing an Order Quantity – The Typical Newsvendor Approach

STEP 2: Place Optimal Order Given Current Level of Uncertainty

If my sales price is \$100 ($p = \$100$) and my cost is \$50 ($c = \$50$), which order quantity ($q \in \{1,2,3\}$) should I choose?

Loss Matrix	Demand #	1 ($p_1 = 0.2$)	2 ($p_2 = 0.5$)	3 ($p_3 = 0.3$)	Exp. Loss
Order Quantity	1	\$0	\$50	\$100	\$55
	2	\$50	\$0	\$50	\$25
	3	\$100	\$50	\$0	\$45

Optimal Order Quantity

Choosing an Order Quantity – The Typical Newsvendor Approach

STEP 2: Place Optimal Order Given Current Level of Uncertainty

If my sales price is \$100 ($p = \$100$) and my cost is **\$1** ($c = \$1$), which order quantity ($q \in \{1,2,3\}$) should I choose?

Loss Matrix	Demand #	1 ($p_1 = 0.2$)	2 ($p_2 = 0.5$)	3 ($p_3 = 0.3$)
Order Quantity	1	?	?	?
	2	?	?	?
	3	?	?	?

Choosing an Order Quantity – The Typical Newsvendor Approach

STEP 2: Place Optimal Order Given Current Level of Uncertainty

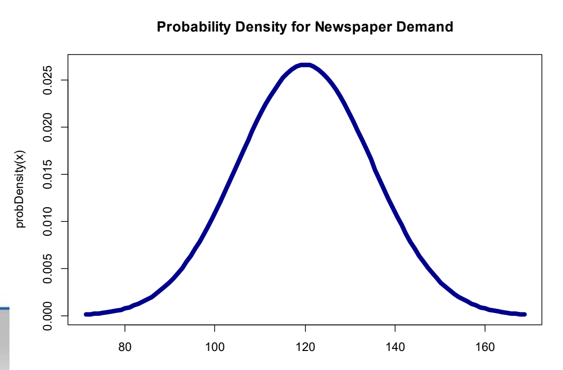
If my sales price is \$100 ($p = \$100$) and my cost is **\$1** ($c = \$1$), which order quantity ($q \in \{1,2,3\}$) should I choose?

Loss Matrix	Demand #	1 ($p_1 = 0.2$)	2 ($p_2 = 0.5$)	3 ($p_3 = 0.3$)	Exp. Loss
Order Quantity	1	\$0	\$99	\$198	\$109
	2	\$1	\$0	\$99	\$30
	3	2	\$1	\$0	\$1

Optimal Order Quantity

Using the normal distribution to choose inventory levels

- Let X be a random variable representing demand for newspapers
- Assume $X \sim N(120,15)$
- How many newspapers should be bought?



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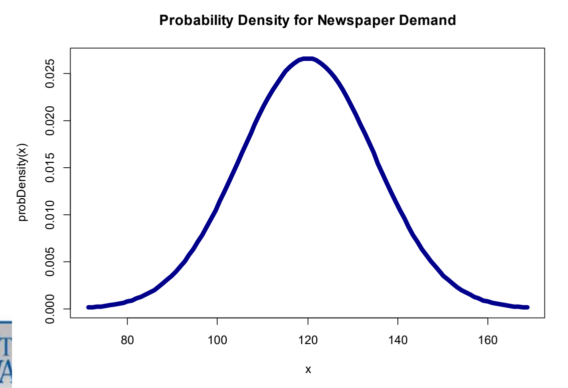
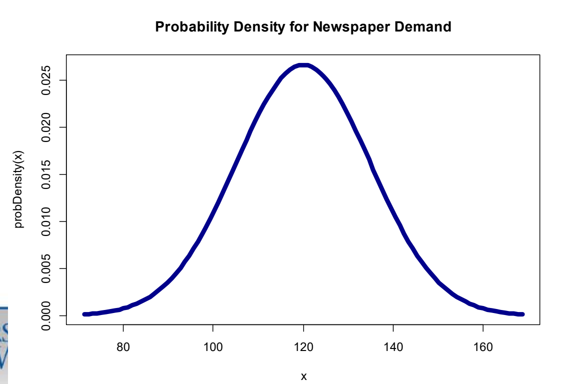


Two Methods:

1. **Rule of Thumb Appraoch:** Pick a **service level** (e.g. 90%) where service level is the probability of not stocking out (i.e. demand ≤ supply)
2. **Decision Theoretic Approach:** Find the **optimal service level** by balancing costs of too few newspapers versus cost of too many.

Method 1: Choose Service Level

- Let X be a random variable representing demand for newspapers
- Assume $X \sim N(120,15)$
- How many newspapers should be bought **if the newsvendor wants to maintain a 75% service level** (i.e. 25% chance of stockout)?



Solution Method:

1. Find quantity, Q, such that $F(Q) = 75\%$.

QUESTION: Is Q larger than 120?

Method 1: Choose Service Level

- Let X be a random variable representing demand for newspapers
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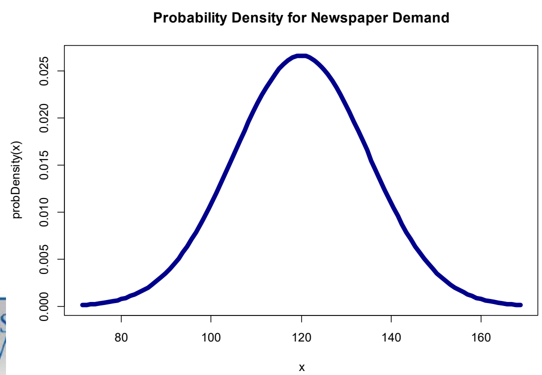

Solution Method:

1. Find quantity, Q, such that $F(Q) = 75\%$.
2. Use inverse CDF function $F^{-1}(Q)$ to find the value of Q such that $P(X \leq Q) = 75\%$.

$$P(X \leq Q) = 75\%$$
$$\rightarrow F(Q) = 75\%$$
$$\rightarrow Q = F^{-1}(75\%)$$

Method 1: Choose Service Level

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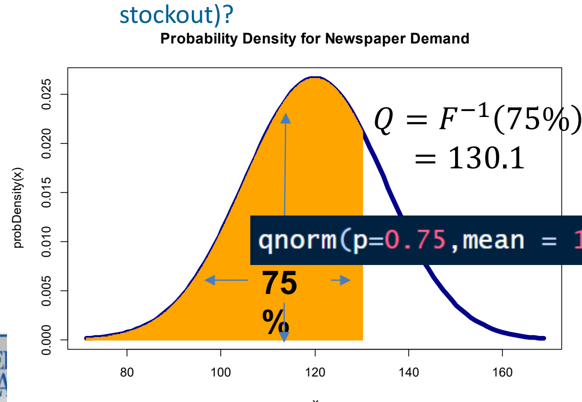


```
qnorm(p=0.75,mean = 120,sd = 15)
```

Find answer using R qfoo function

Solution Method:

1. Find quantity, Q, such that $F(Q) = 75\%$.
2. Use inverse CDF function $F^{-1}(Q)$ to find the value of Q such that $P(X \leq Q) =$

$$P(X \leq Q) =$$
$$\rightarrow F(Q) = 75$$
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CLASS EXERCISE:

- Let X be a random variable representing demand for the official Firefly Music Festival hat.
- Assume $X \sim Binomial(n = 10000$, p = 0.05)
- How many hats should be bought **if the hatvendor wants to maintain a 40% service level** (i.e. 60% chance of stockout)?

Method 2: Find Optimal Service Level

- Let X be a random variable representing demand for newspapers
- Assume $X \sim N(120{,}20)$
- How many newspapers should be bought **if the newsvendor must buy newspapers for $1.00 and must sell them for the listed price of $4.00**? Unsold newspapers are worthless.

Switch to `newsvendor.R`

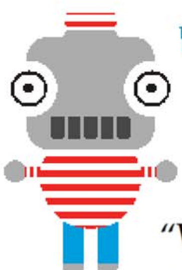
Formal Decision Problem Components

1. **Outcomes –** potential future scenarios

2. **Decision** or **action** - our method of shaping the future

3. **Probabilistic Outcome Model** – a probability distribution of outcomes (distribution may change as a result of actions)

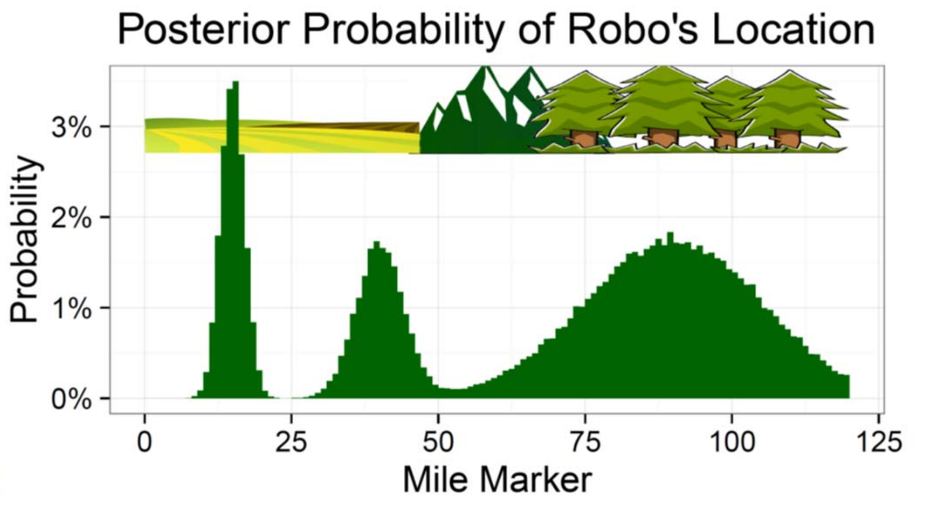
4. **Utility Function:** method to value any combination of outcome and action



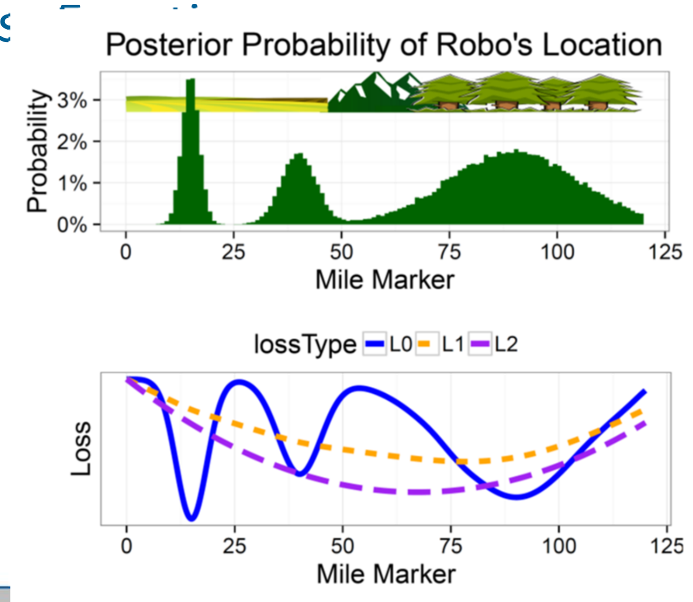
Where's Robo?

"We have lost our robot. He was sent out on a scouting mission and we are not sure where he is. I am sure his battery is dead by now, so our hope of him returning at this point is zero. I do like that little robot, but he can potentially be anywhere on this 120 mile narrow strip of land. As much as I like him, I am not sure he is worth the effort to find. Based on what I know about the robot's behavior, there are some locations that are more plausible to find him than others. In fact, he is most likely to be in the forest, somewhere between mile marker 75 and 120. He might also be hiding in the plains, either around the 15th or the 40th mile. It's not that likely that he's in the mountains, but we can't dismiss it altogether."

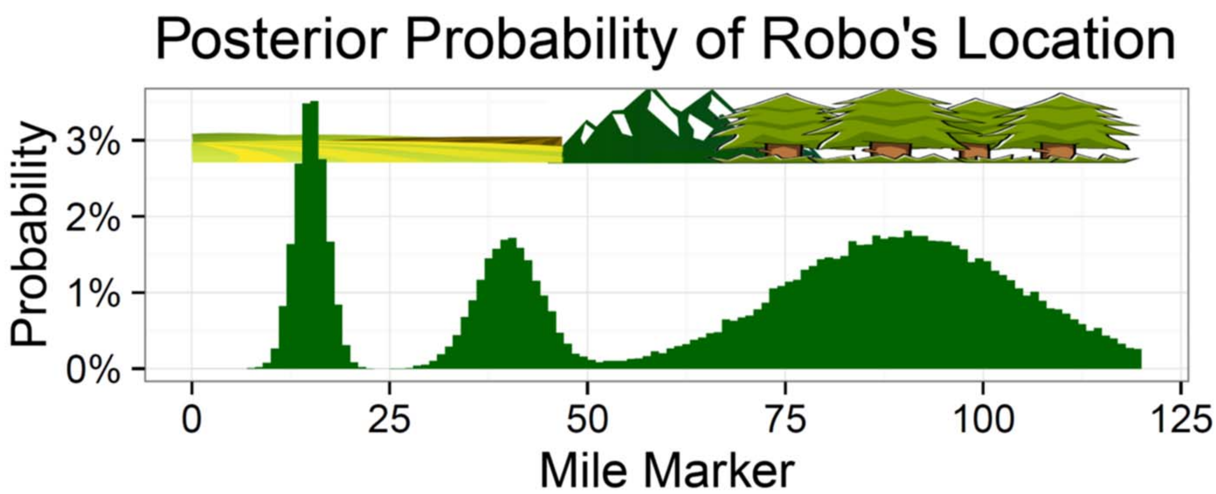
Probability Distributions



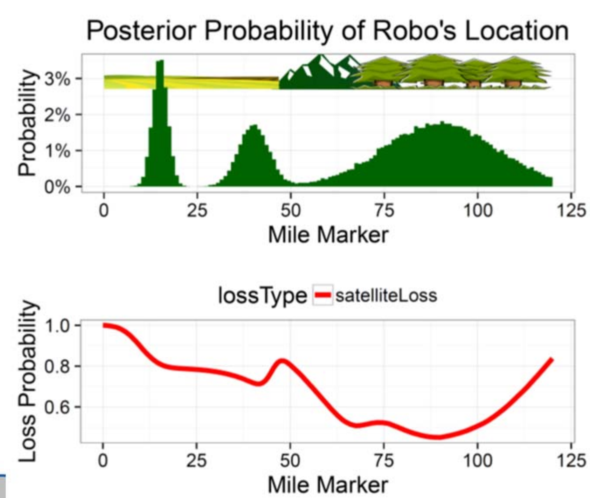
Los



The 30 Mile Satellite Scan

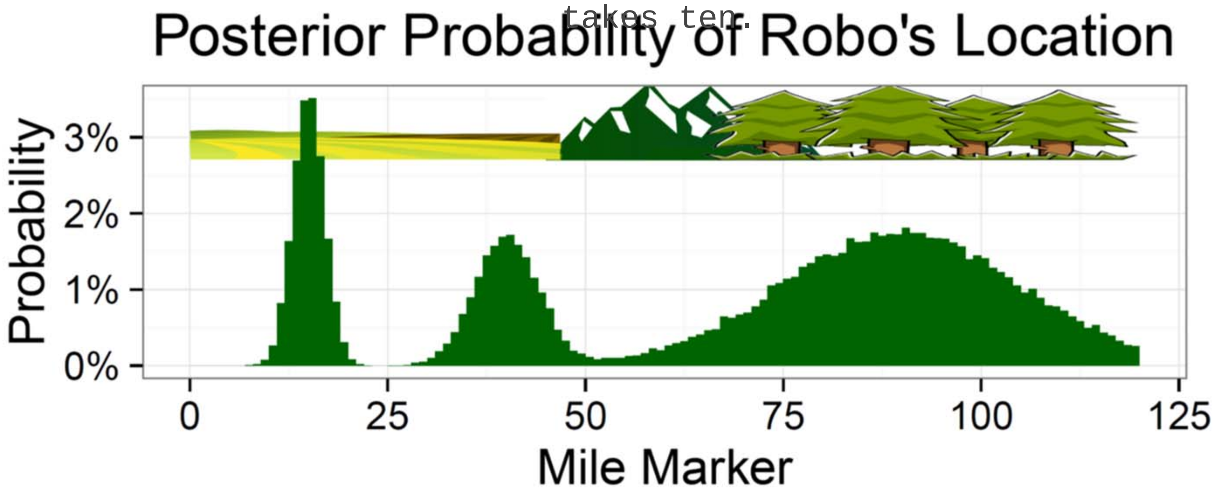


30 Mile Satellite Scan Loss Function



The 24 Hour Walk

a mile of plains takes one hour, a mile of forest takes five hours and a mile of mountains takes ten.



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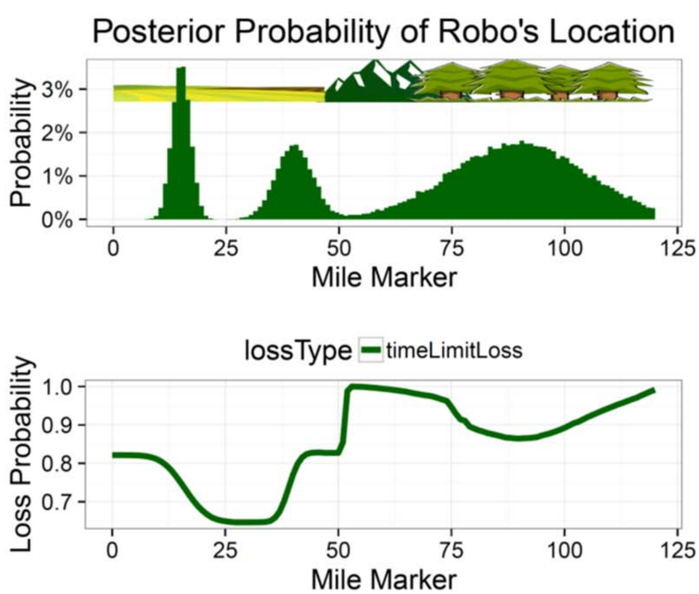

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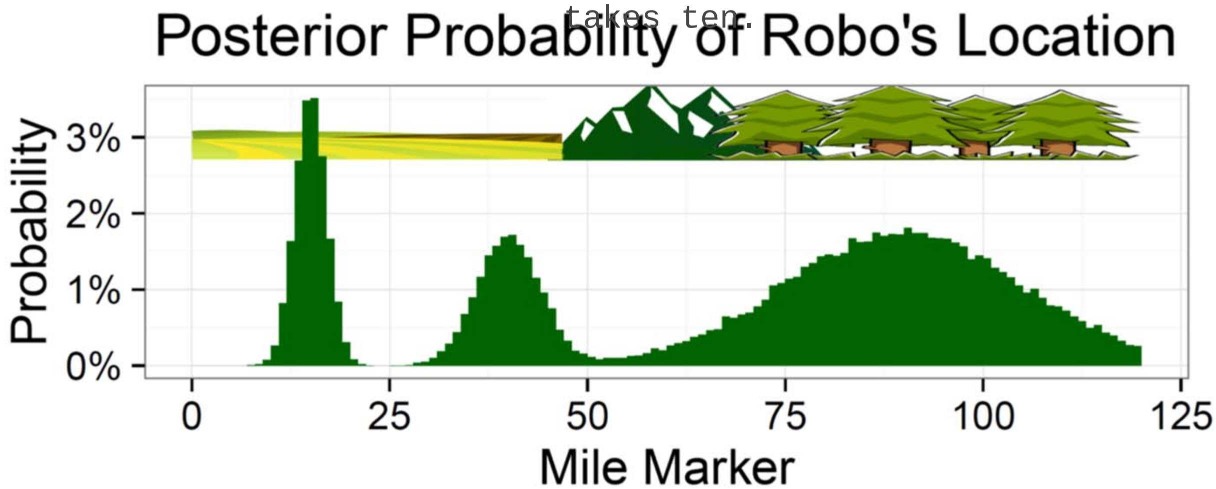
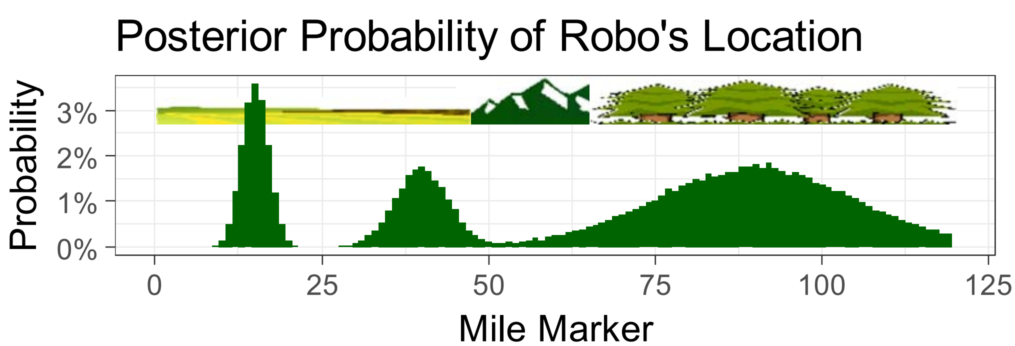
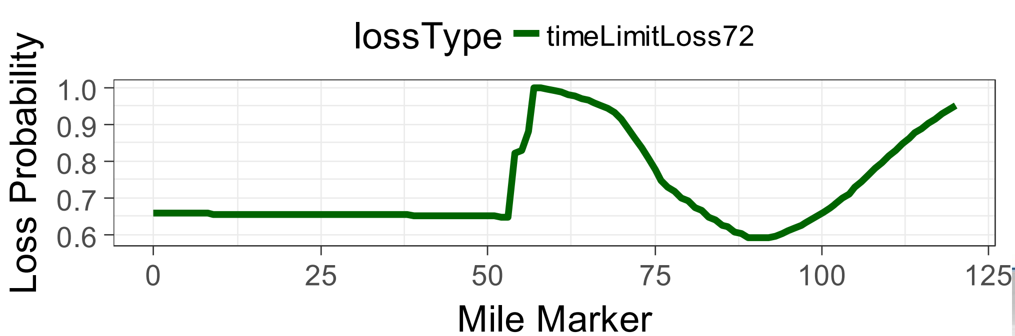
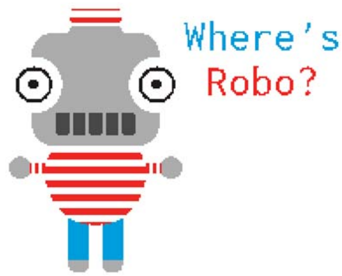
The 72 Hour Walk

a mile of plains takes one hour, a mile of forest takes five hours and a mile of mountains takes ten.









Switch to `statDecTheory.Rmd`