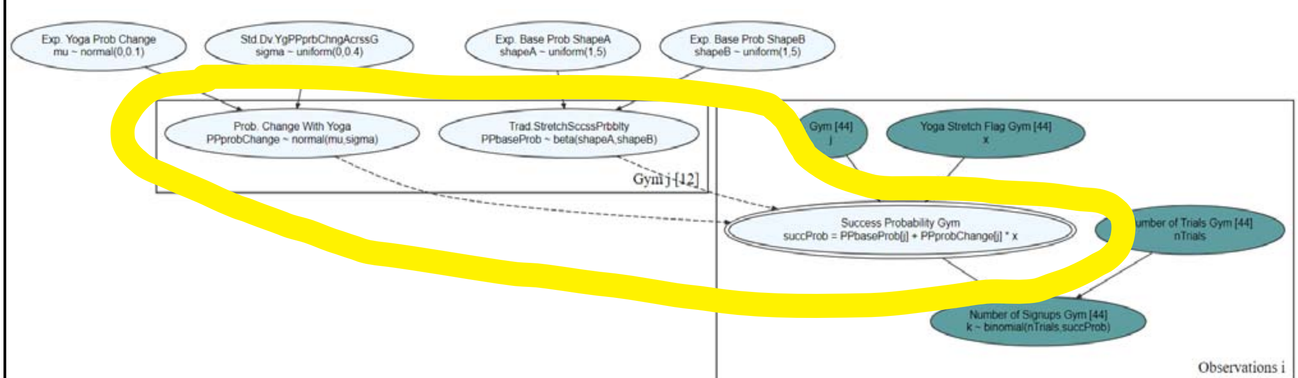
Link Functions

Getting the support of a linear predictor to match the support of a child node

Previous Partial Pooling Model



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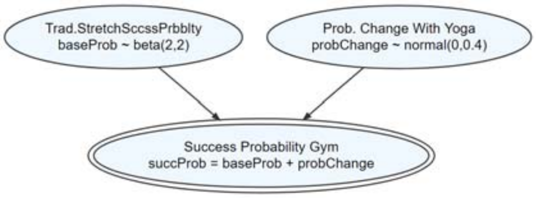
Zoom In on Success Probability Calculation

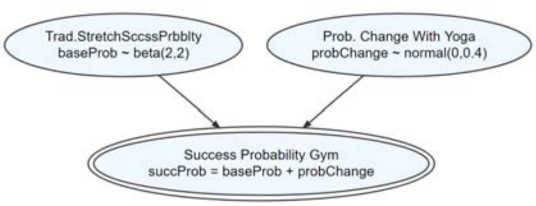
Prior shown as a representative sample

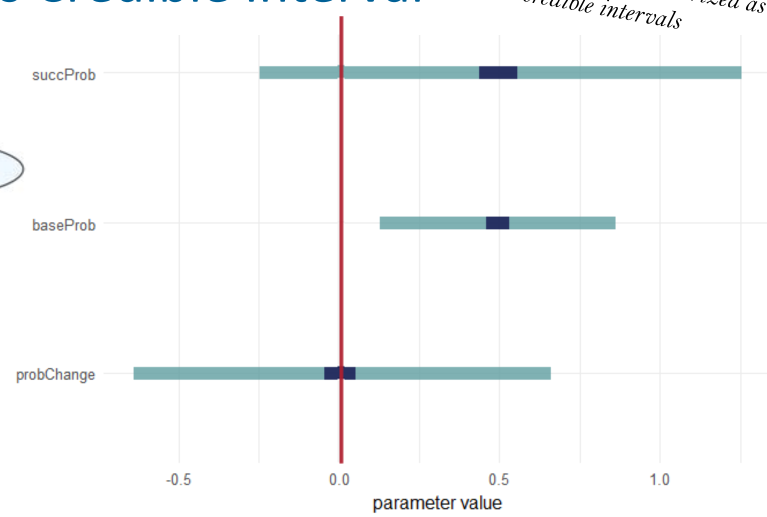



```
# A tibble: 16,000 x 3
   baseProb probChange succProb
      <dbl>      <dbl>    <dbl>
 1    0.708     0.0249    0.733
 2    0.897     0.0261    0.924
 3    0.882     0.269     1.15
 4    0.525    -1.00     -0.476
 5    0.861     0.0421    0.903
 6    0.825    -0.0835    0.742
 7    0.307    -0.544    -0.237
 8    0.904    -0.0183    0.885
 9    0.149    -0.0107    0.139
10    0.149    -0.0107    0.139
# ... with 15,990 more rows
```


Prior as Credible Interval

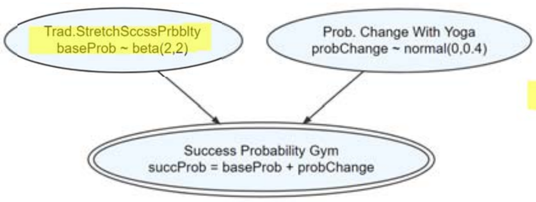
Prior distribution summarized as credible intervals

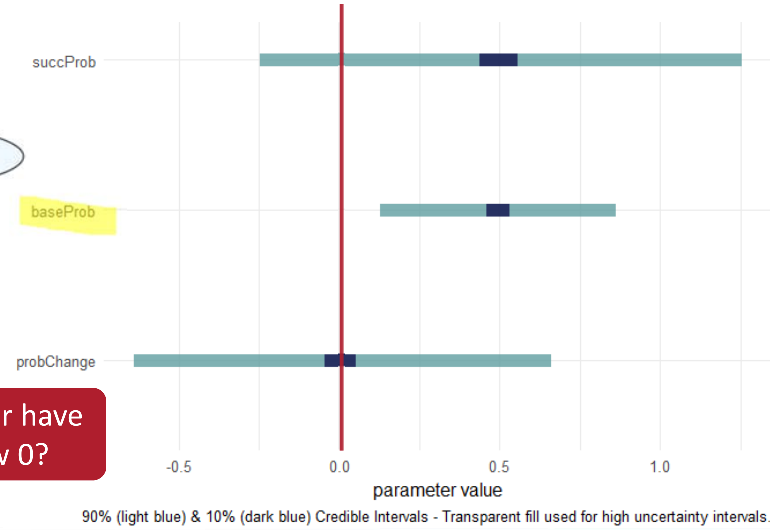




90% (light blue) & 10% (dark blue) Credible Intervals - Transparent fill used for high uncertainty intervals.

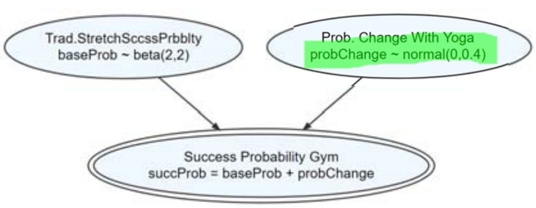
Prior for baseProb

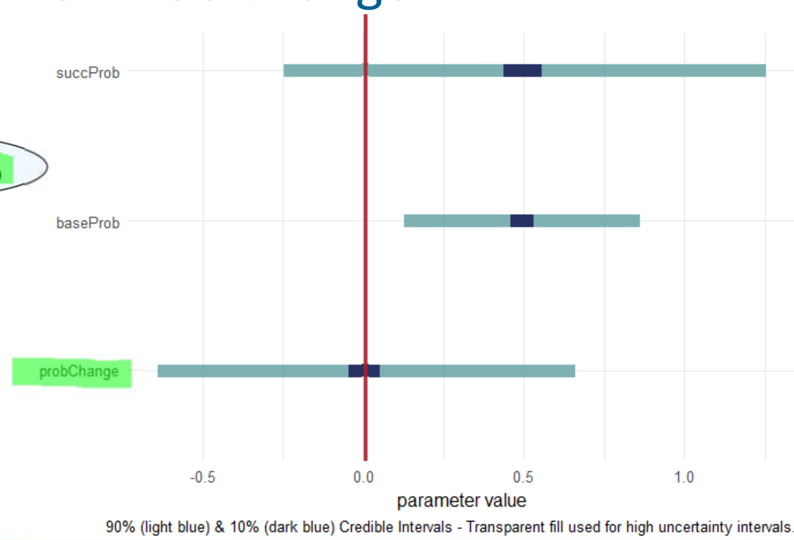




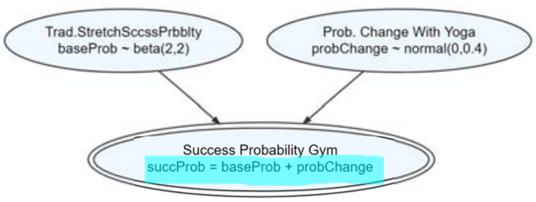
QUESTION: Why will baseProb never have a credible interval that goes below 0?

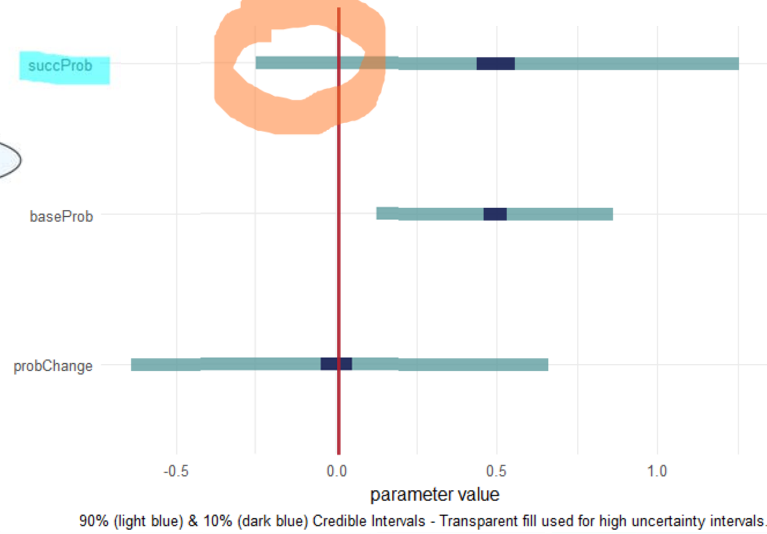
Prior for Prob Change



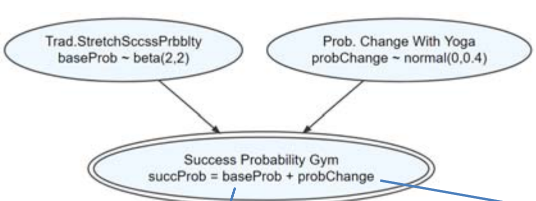


Calculated Prior for succProb





succProb is a Simple Linear Predictor



$$\alpha + \beta_1 \times x_1$$

Assume $x_1 = 1$ *for illustration purposes.*

$$\alpha + \beta_1$$

Linear Predictor Terminology

$$\alpha + \beta_1 \times x_1 + \beta_2 \times x_2 + \cdots + \beta_n * x_n$$

this is called the linear predictor

QUESTION: What term is often used in regression to describe α?

independent variables (e.g. yoga stretch)

unknown parameters

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Linear Predictor Support

$$\alpha + \beta_1 \times x_1 + \beta_2 \times x_2 + \cdots + \beta_n * x_n$$

typical support of prior on linear predictor distribution $(-\infty, \infty)$

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Mismatch between Probability & Linear Predictor Supports

$$\overset{support}{[0,1]} \qquad \overset{support}{(-\infty,\infty)}$$

$$probability \overset{?}{=} \alpha + \beta_1 \times x_1$$

Inverse Link Functions

$$\overset{support}{[0,1]} \qquad \overset{support}{(-\infty,\infty)}$$

$$probability \overset{?}{=} \alpha + \beta_1 \times x_1$$

$$\overset{support}{[0,1]} \qquad \overset{support}{[0,1]}$$

$$probability = f(\alpha + \beta_1 \times x_1)$$

where f is a function that maps values on the $(-\infty,\infty)$ scale to a value between $(0,1)$
For historical reasons, this function, f, is called an inverse-link function.

Inverse Link Functions

$$\overset{\text{support}}{[0,1]} \qquad \overset{\text{support}}{[0,1]}$$

$$probability = f(\alpha + \beta_1 \times x_1)$$

$$probability = \frac{1}{\exp(-(\alpha + \beta_1 \times x_1))}$$

inverse logit function

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What Happens When $\alpha + \beta_1 \times x_1 = 0$

$$probability = \frac{1}{1 + \exp(-(\alpha + \beta_1 \times x_1))}$$

$$= \frac{1}{1 + \exp(0)} = \frac{1}{1 + e^0} = \frac{1}{1 + 1}$$

$$= \frac{1}{2} = 50\%$$

QUESTION: What happens when $\alpha + \beta_1 \times x_1$ approaches infinity?

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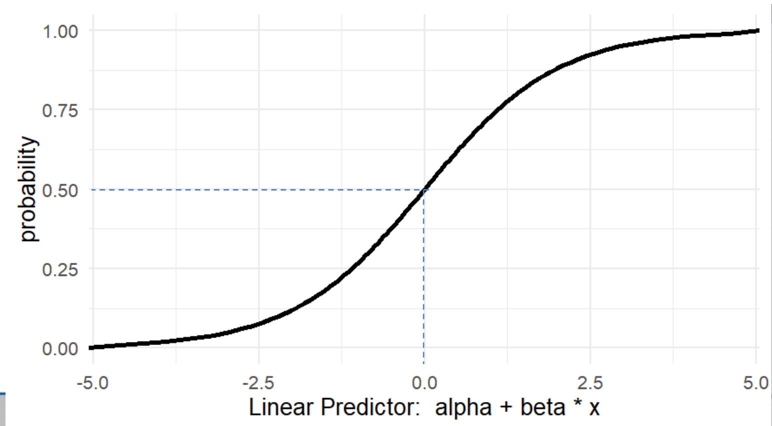
What Happens When

$\alpha + \beta_1 \times x_1 = 0 \quad \rightarrow \quad prob = 50\%$



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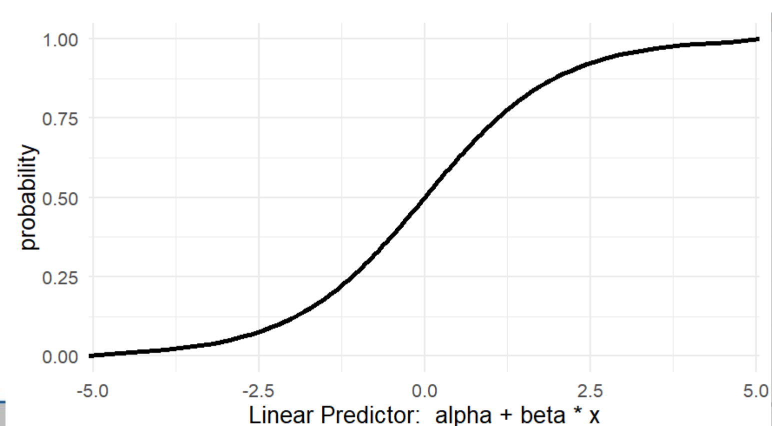
(approximately) What Linear Predictor Yields

$prob = 25\% \quad \rightarrow \quad \alpha + \beta_1 \times x_1 =???$



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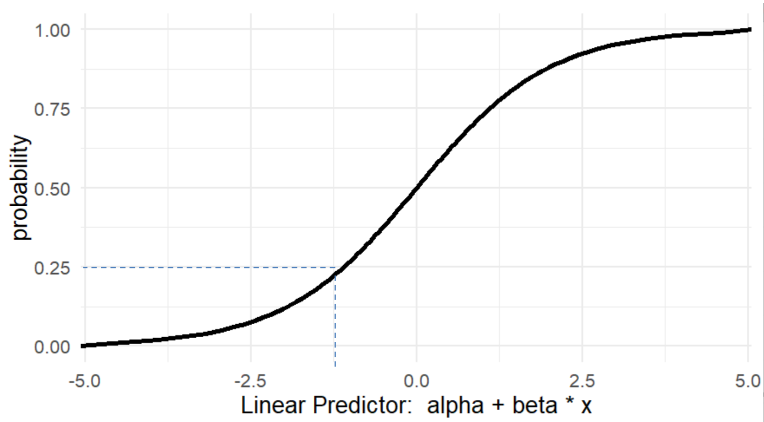
(approximately) What Linear Predictor Yields

$$prob = 25\% \quad \rightarrow \quad \alpha + \beta_1 \times x_1 = \eta$$ *solve for eta (i.e. η)*



$$0.25 = \frac{1}{1 + \exp(-\eta)}$$

$$0.25 + 0.25\exp(-\eta) = 1$$

$$0.25\exp(-\eta) = 0.75$$

$$\exp(-\eta) = 3$$

$$\ln(\exp(-\eta)) = \ln(3)$$

$$\eta = -\ln(3) = -\log(3) \approx -1.09$$

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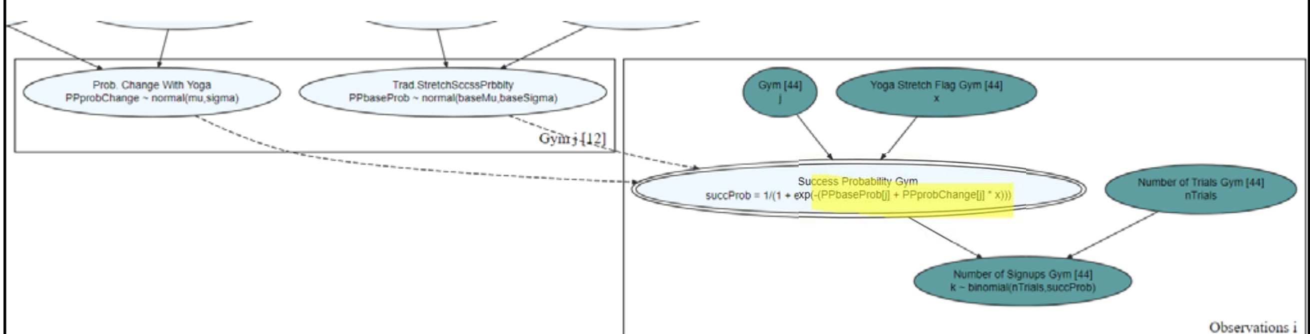
Predicting a Probability With Linear Predictor



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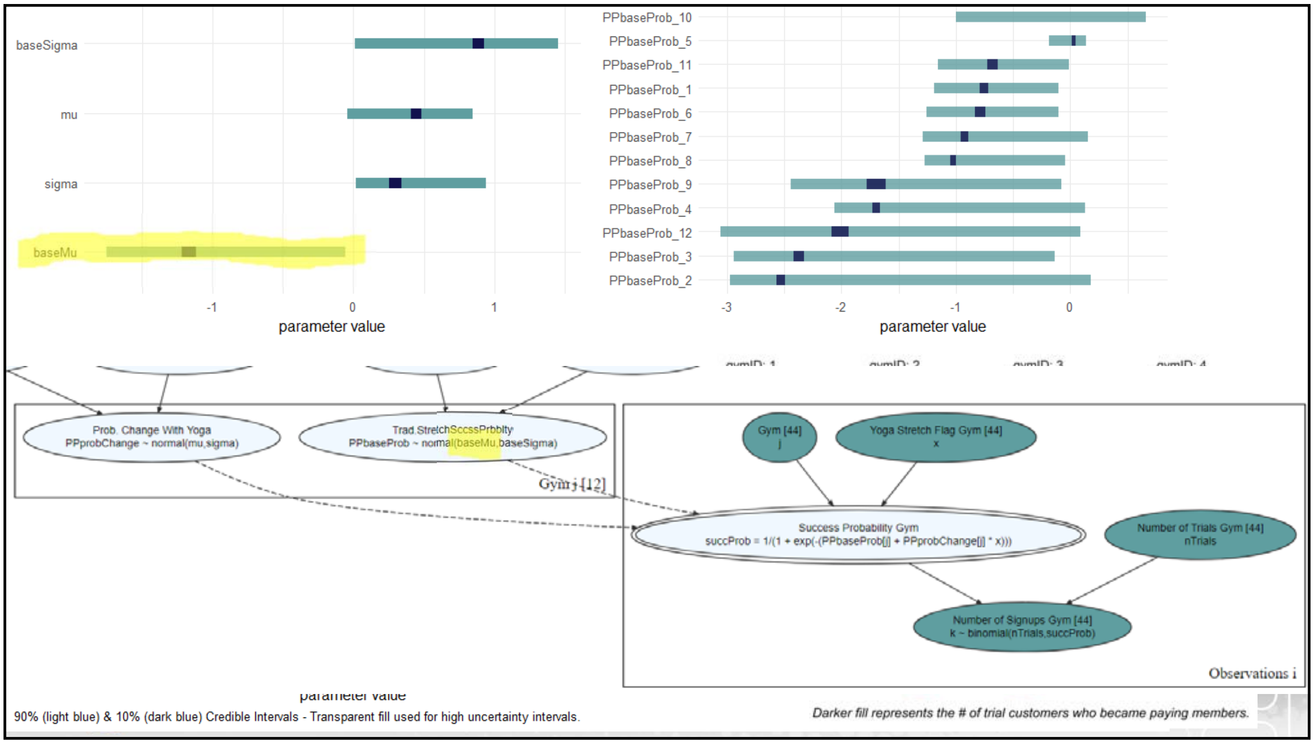
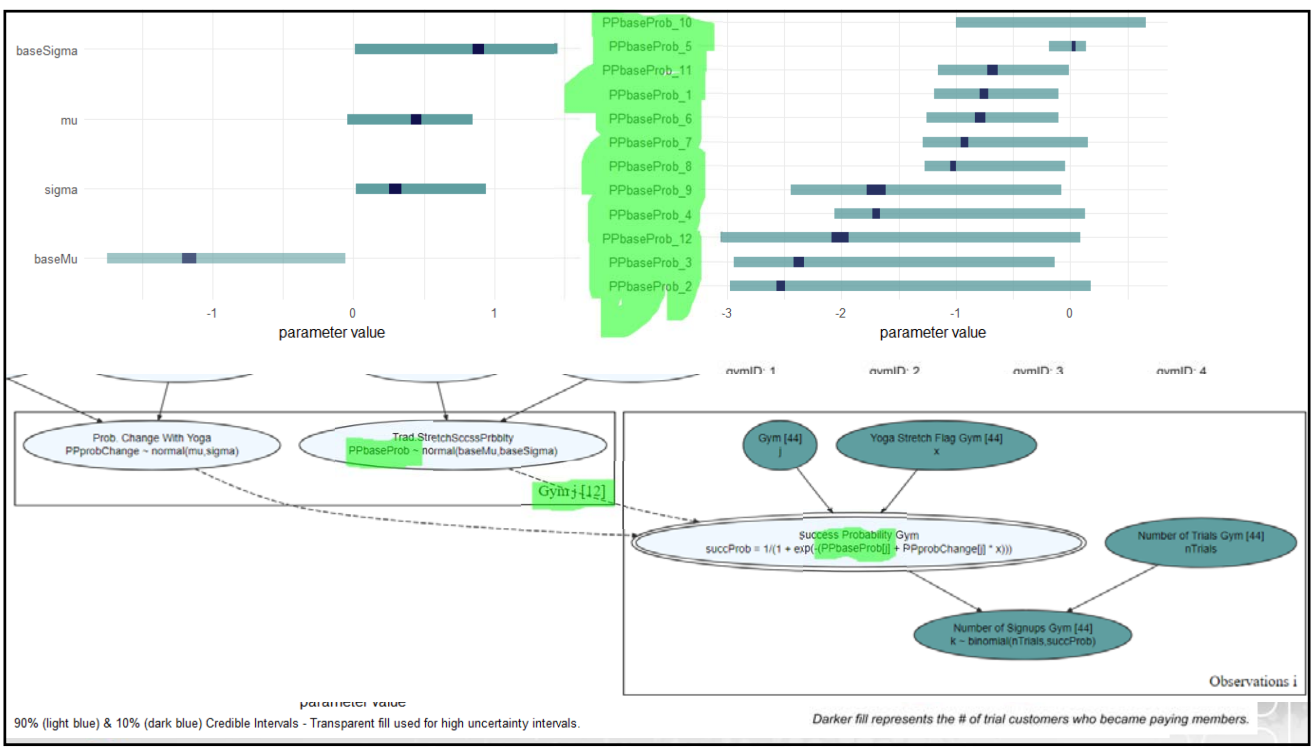

baseSigma
mu
sigma
baseMu
-1
0
1
parameter value
PPbaseProb_10
PPbaseProb_5
PPbaseProb_11
PPbaseProb_1
PPbaseProb_6
PPbaseProb_7
PPbaseProb_8
PPbaseProb_9
PPbaseProb_4
PPbaseProb_12
PPbaseProb_3
PPbaseProb_2
-3
-2
-1
0
parameter value
Prob. Change With Yoga
PPprobChange ~ normal(mu,sigma)
Trad.StretchSccssPrbblty
PPbaseProb ~ normal(baseMu,baseSigma)
Gym j [12]
Gym [44]
j
Yoga Stretch Flag Gym [44]
x
Success Probability Gym
succProb = 1/(1 + exp(-(PPbaseProb[j] + PPprobChange[j] * x)))
Number of Trials Gym [44]
nTrials
Number of Signups Gym [44]
k ~ binomial(nTrials,succProb)
Observations i
90% (light blue) & 10% (dark blue) Credible Intervals - Transparent fill used for high uncertainty intervals.
Darker fill represents the # of trial customers who became paying members.


baseSigma
mu
sigma
baseMu
-1
0
1
parameter value
PPbaseProb_10
PPbaseProb_5
PPbaseProb_11
PPbaseProb_1
PPbaseProb_6
PPbaseProb_7
PPbaseProb_8
PPbaseProb_9
PPbaseProb_4
PPbaseProb_12
PPbaseProb_3
PPbaseProb_2
-3
-2
-1
0
parameter value
Prob. Change With Yoga
PPprobChange ~ normal(mu,sigma)
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PPbaseProb ~ normal(baseMu,baseSigma)
Gym j [12]
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Yoga Stretch Flag Gym [44]
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succProb = 1/(1 + exp(-(PPbaseProb[j] + PPprobChange[j] * x)))
Number of Trials Gym [44]
nTrials
Number of Signups Gym [44]
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Observations i
90% (light blue) & 10% (dark blue) Credible Intervals - Transparent fill used for high uncertainty intervals.
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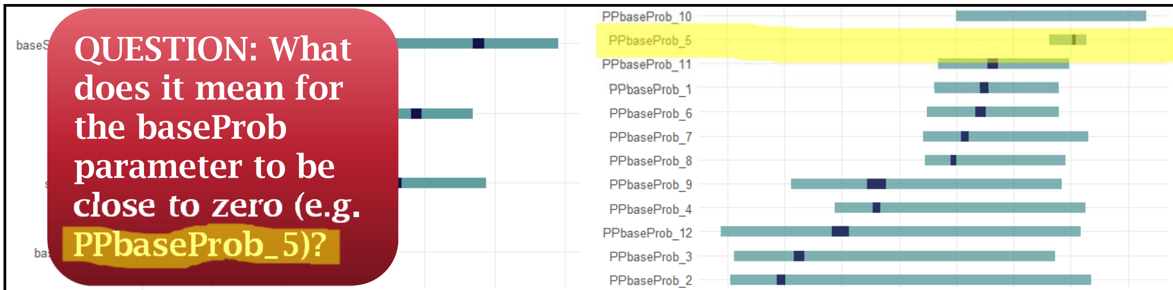
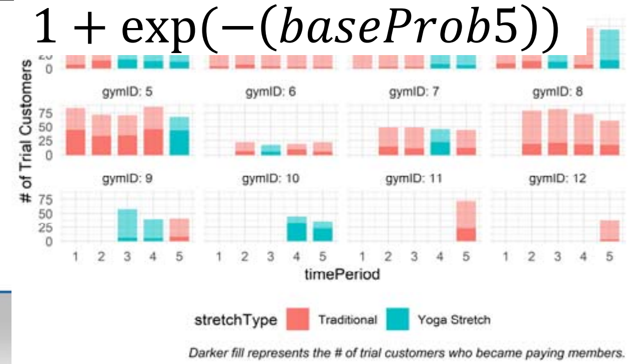


QUESTION: What does it mean for the baseProb parameter to be close to zero (e.g. PPbaseProb_5)?

$$succProb5NoYoga = \frac{1}{1 + \exp(-(baseProb5))}$$



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Link Function Walkthrough

Open `linkFunction.R`

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