Jupyter Notebooks

Madhavan Mukund

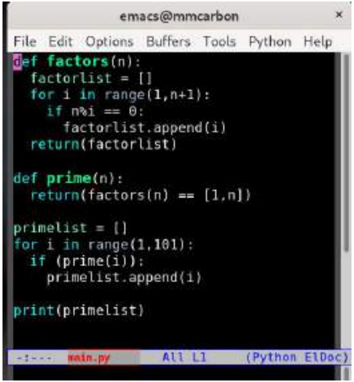
https://www.cmi.ac.in/~madhavan

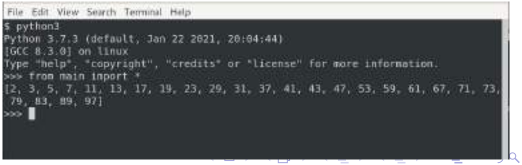
Programming, Data Structures and Algorithms using Python

Week 1

Writing and running code

- Manual
  - Text editor to write code
  - Run at the command line





Writing and running code

- Manual
  - Text editor to write code
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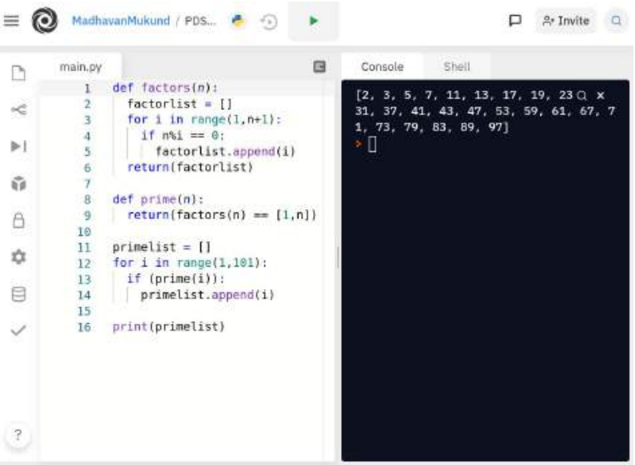
- Integrated Development Environment (IDE)
  - Single application to write and run code
  - On desktop or online, `replit`
  - Quick update-run cycle
  - Debugging, testing, ...



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  - Single application to write and run code
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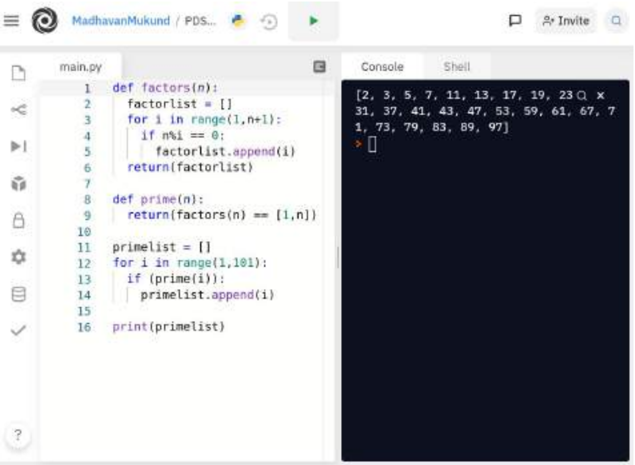
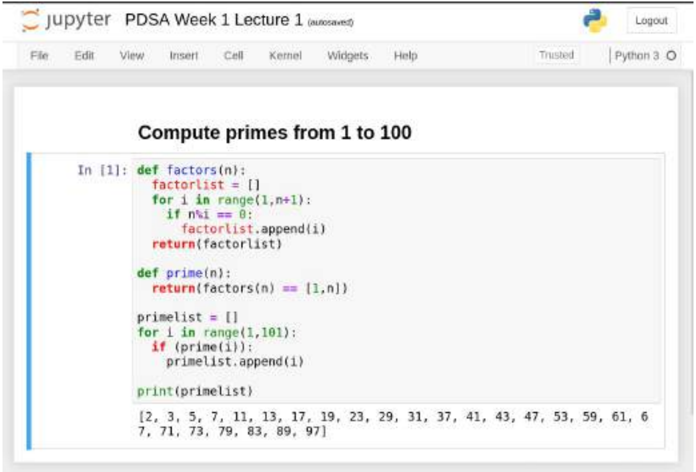
  - Quick update-run cycle
  - Debugging, testing, ...
- What more could one want?



Collaboration

- Share your code
  - Collaborative development
  - Report your results



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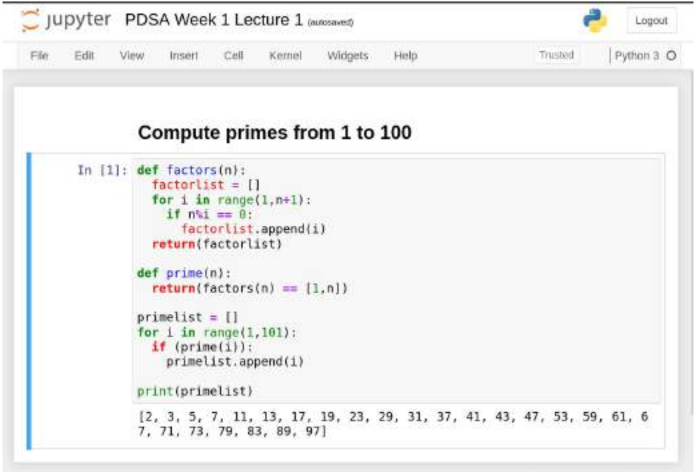

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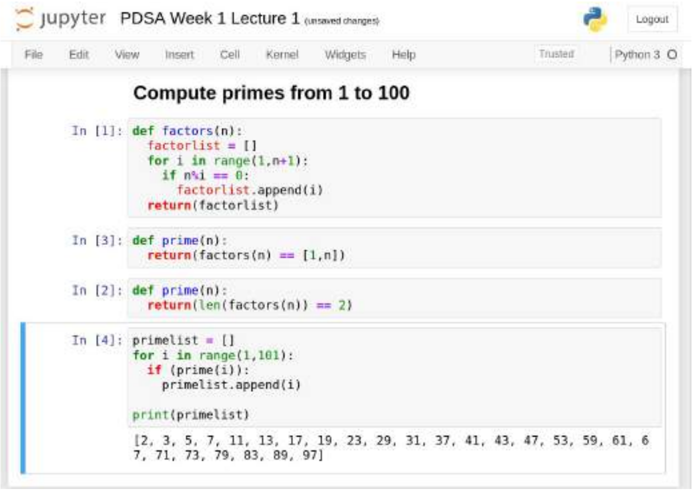

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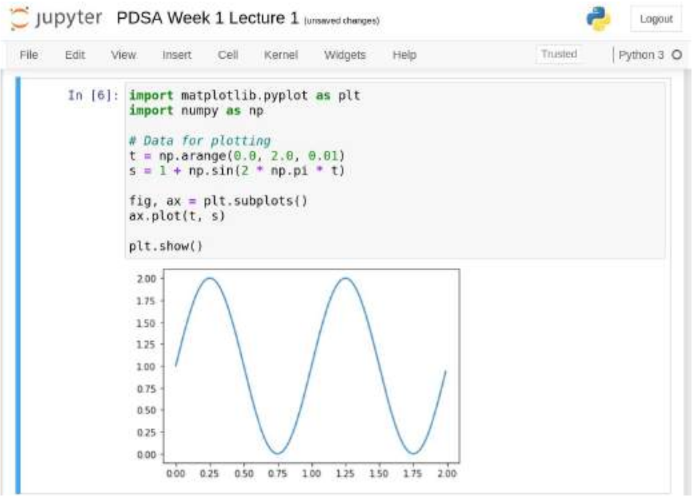
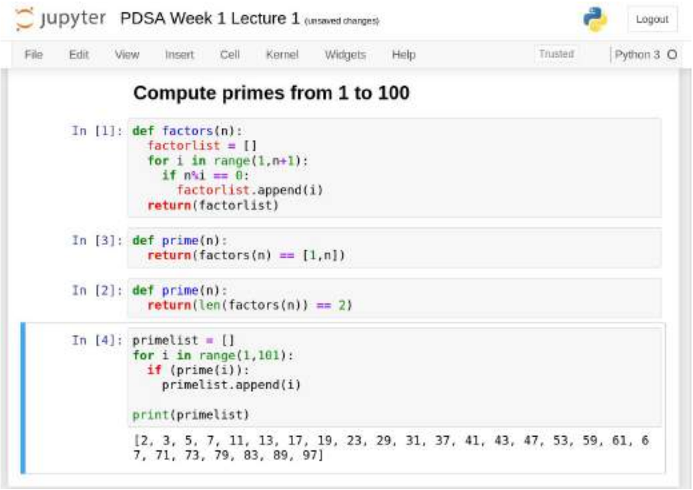
- Documentation
  - Interleave with the code
- Switch between different versions of code
- Export and import your project
- Preserve your output



Jupyter notebook

- A sequence of cells
  - Like a one dimensional spreadsheet



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- A sequence of cells
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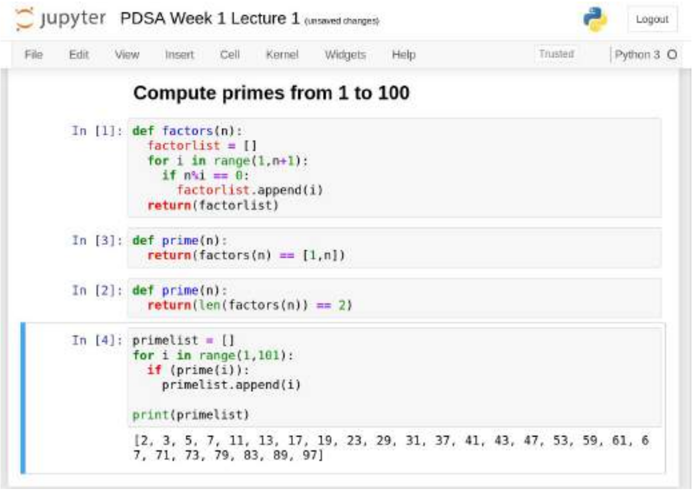
  - Markdown notation for formatting
  - `https://www.markdownguide.org/`



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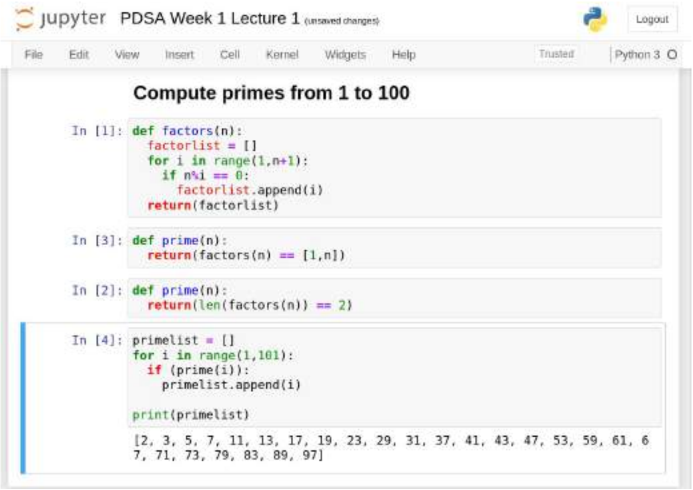

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  - Julia, Python, R
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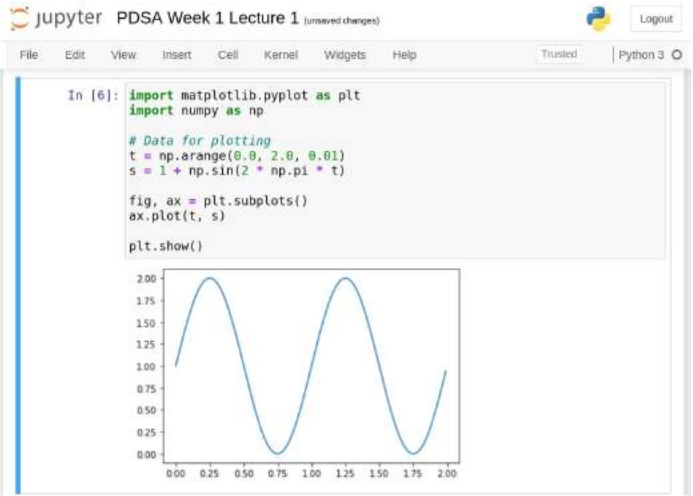

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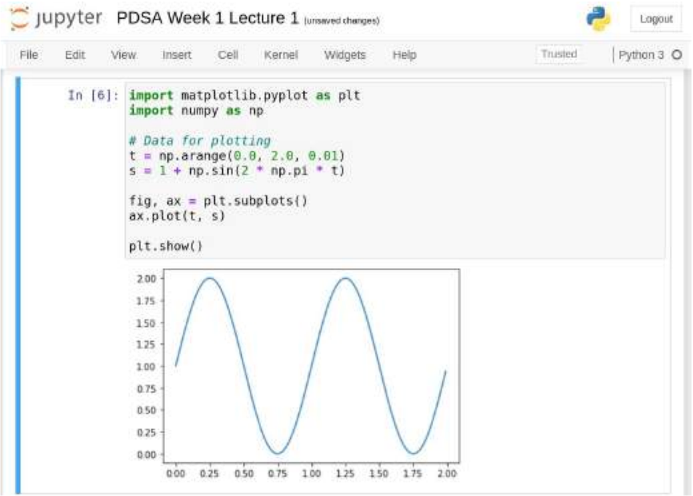
  - Solutions to problems posed on platforms like Kaggle `https://www.kaggle.org`



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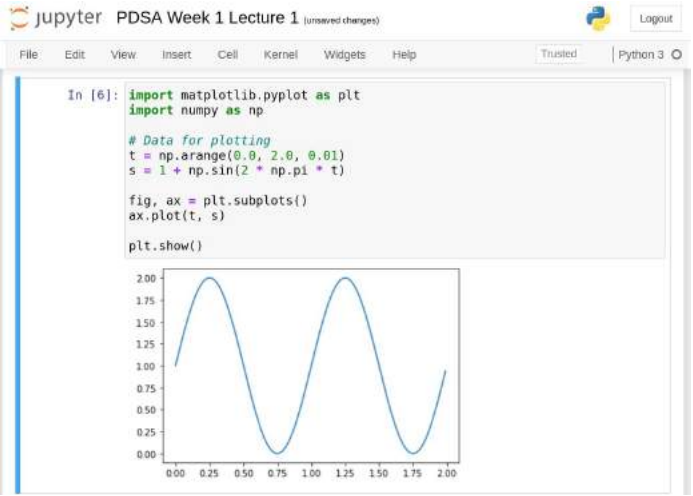
- ACM Software Systems Award 2017



Google Colab

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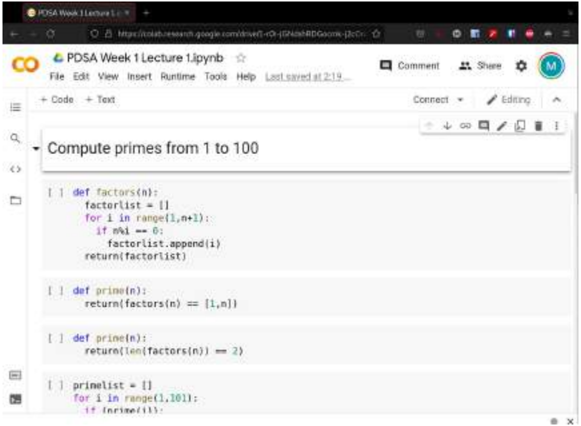

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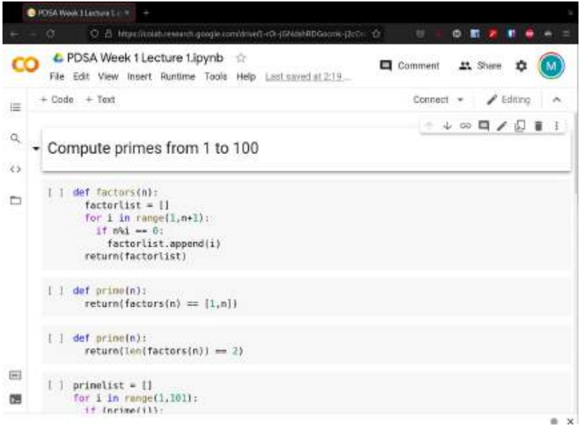

Google Colab

- Google Colaboratory (Colab)
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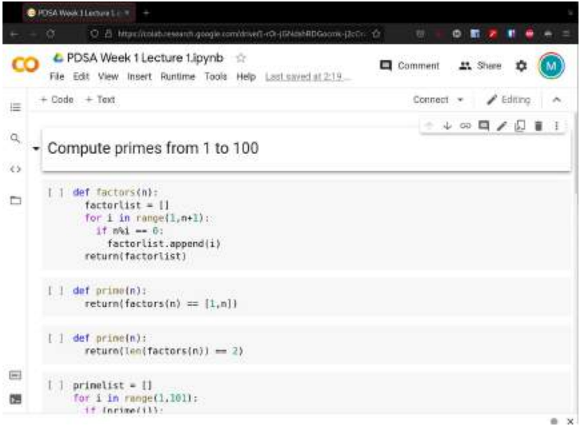
- Customized Jupyter notebook
- All standard packages required for ML are preloaded
  - `scikit-learn`, `tensorflow`
  - Access to GPU hardware



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- Incrementally update and run
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- Useful for collaboration, sharing
- Google Colab — free to use version configured for ML

Python Recap – I

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Programming, Data Structures and Algorithms using Python

Week 1

Computing gcd

- $\gcd(m, n)$ — greatest common divisor
  - Largest k that divides both m and n
  - $\gcd(8, 12) = 4$
  - $\gcd(18, 25) = 1$
  - Also hcf — highest common factor

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  - 1 divides both m and n
- Computing $\text{gcd}(m, n)$
  - $\text{gcd}(m, n) \leq \text{min}(m, n)$
  - Compute list of common factors from 1 to $\text{min}(m, n)$
  - Return the last such common factor

```python
def gcd(m,n):
  cf = []   # List of common factors
  for i in range(1,min(m,n)+1):
    if (m%i) == 0 and (n%i) == 0:
      cf.append(i)
  return(cf[-1])
```

Computing gcd

Points to note

- Need to initialize `cf` for `cf.append()` to work
  - Variables (names) derive their type from the value they hold

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- `range(i,j)` runs from `i` to `j-1`
- List indices run from `0` to `len(l) - 1` and backwards from `-1` to `-len(l)`

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Eliminate the list

- Only the last value of `cf` is important

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Python Recap – II

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Week 1

Checking primality

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- Compute the list of factors of `n`

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  for i in range(1,n+1):
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```

Checking primality

- A prime number n has exactly two factors, 1 and n
  - Note that 1 is not a prime
- Compute the list of factors of `n`
- `n` is a prime if the list of factors is precisely `[1,n]`

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def factors(n):
  fl = []   # factor list
  for i in range(1,n+1):
    if (n%i) == 0:
      fl.append(i)
  return(fl)

def prime(n):
  return(factors(n) == [1,n])
```

Counting primes

- List all primes upto m

```
def primesupto(m):
  pl = []   # prime list
  for i in range(1,m+1):
    if prime(i):
      pl.append(i)
  return(pl)
```

Counting primes

- List all primes upto *m*
- List the first *m* primes
  - Multiple simultaneous assignment

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def primesupto(m):
  pl = []   # prime list
  for i in range(1,m+1):
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  return(pl)

def firstprimes(m):
  (count,i,pl) = (0,1,[])
  while (count < m):
    if prime(i):
      (count,pl) = (count+1,pl+[i])
    i = i+1
  return(pl)
```

Counting primes

- List all primes upto m
- List the first m primes
  - Multiple simultaneous assignment
- `for` vs `while`
  - Is the number of iterations known in advance?
  - Ensure progress to guarantee termination of `while`

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```

Computing primes

- Directly check if n has a factor between 2 and $n-1$

```
def prime(n):
  result = True
  for i in range(2,n):
    if (n%i) == 0:
      result = False
  return(result)
```

Computing primes

- Directly check if n has a factor between 2 and $n-1$
- Terminate check after we find first factor
  - Breaking out of a loop

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```

```
def prime(n):
  (result,i) = (True,2)
  while (result and (i < n)):
    if (n%i) == 0:
      result = False
    i = i+1
  return(result)
```

Computing primes

- Directly check if n has a factor between 2 and $n-1$
- Terminate check after we find first factor
  - Breaking out of a loop
- Alternatively, use `while`
- Speeding things up slightly
  - Factors occur in pairs
  - Sufficient to check factors upto $\sqrt{n}$
  - If n is prime, scan $2, \ldots, \sqrt{n}$ instead of $2, \ldots, n-1$

```
import math
def prime(n):
  (result,i) = (True,2)
  while (result and (i < math.sqrt(n))
    if (n%i) == 0:
      result = False
    i = i+1
  return(result)
```

Properties of primes

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```
def primediffs(n):
  lastprime = 2
  pd = {}  # Dictionary for
           # prime diferences
  for i in range(3,n+1):
    if prime(i):
      d = i - lastprime
      lastprime = i
      if d in pd.keys():
        pd[d] = pd[d] + 1
      else:
        pd[d] = 1
  return(pd)
```

Python Recap – III

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- Can we do better?

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- Suppose d divides m and n
  - $m = ad$, $n = bd$
  - $m - n = (a - b)d$
  - d also divides $m - n$
- Recursively defined function
  - Base case: n divides m, answer is n
  - Otherwise, reduce $gcd(m, n)$ to $gcd(n, m - n)$

```
def gcd(m,n):
  (a,b) = (max(m,n), min(m,n))
  if a%b == 0:
    return(b)
  else
    return(gcd(b,a-b))
```

Computing gcd

- Unfortunately, this takes time proportional to $\max(m, n)$

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  - $\rightarrow$ `gcd(2,9995)`
  - ...
  - $\rightarrow$ `gcd(2,3)`
  - $\rightarrow$ `gcd(2,1)`
  - $\rightarrow$ 1

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- Approximately 5000 steps

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  (a,b) = (max(m,n), min(m,n))
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    return(b)
  else
    return(gcd(b,a-b))
```

Euclid's algorithm

- Suppose n does not divide m

Euclid's algorithm

- Suppose n does not divide m
- Then $m = qn + r$

Euclid's algorithm

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  - If n divides m, $\text{gcd}(m, n) = n$
  - Otherwise, compute $\text{gcd}(n, m \bmod n)$

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def gcd(m,n):
  (a,b) = (max(m,n), min(m,n))
  if a%b == 0:
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Euclid's algorithm

- Suppose n does not divide m
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- Suppose d divides both m and n
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- Can show that this takes time proportional to number of digits in $\max(m, n)$
- One of the first non-trivial algorithms

Exception handling

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Programming, Data Structures and Algorithms using Python

Week 1

When things go wrong

- Our code could generate many types of errors
  - `y = x/z`, but `z` has value `0`
  - `y = int(s)`, but string `s` does not represent a valid integer
  - `y = 5*x`, but `x` does not have a value
  - `y = l[i]`, but `i` is not a valid index for list `l`
  - Try to read from a file, but the file does not exist
  - Try to write to a file, but the disk is full

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  - `y = l[i]`, but `i` is not a valid index for list `l`
  - Try to read from a file, but the file does not exist
  - Try to write to a file, but the disk is full
- Recovering gracefully
  - Try to anticipate errors
  - Provide a contingency plan
  - Exception handling

Types of errors

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- Python flags the type of each error
- Most common error is a syntax error
  - `SyntaxError: invalid syntax`
  - Not much you can do!
- We are interested in errors when the code is running
  - Name used before value is defined
    `NameError: name 'x' is not defined`
  - Division by zero in arithmetic expression
    `ZeroDivisionError: division by zero`
  - Invalid list index
    `IndexError: list assignment index out of range`

Terminology

- Raise an exception
  - Run time error $\rightarrow$ signal error type, with diagnostic information
    `NameError: name 'x' is not defined`
- Handle an exception
  - Anticipate and take corrective action based on error type
- Unhandled exception aborts execution

Terminology

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Handling exceptions

```
try:
   ... ← Code where error may occur
   ...

except IndexError:
   ... ← Handle IndexError

except (NameError,KeyError):
   ... ← Handle multiple exception types

except:
   ... ← Handle all other exceptions

else:
   ... ← Execute if try runs without errors
```

Using exceptions "positively"

- Collect scores in dictionary

```
scores = {"Shefali":[3,22],
          "Harmanpreet":[200,3]}
```

- Update the dictionary
- Batter `b` already exists, append to list

```
scores[b].append(s)
```

- New batter, create a fresh entry

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scores[b] = [s]
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Traditional approach

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if b in scores.keys():
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if b in scores.keys():
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else:
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```

Using exceptions

```
try:
  scores[b].append(s)
except KeyError:
  scores[b] = [s]
```

Flow of control

```
...
x = f(y,z)
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```
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`IndexError` not handled in `h()`

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  h(m)
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`IndexError` inherited from `h()`

```
def h(s):
  ...
  h(s)
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Flow of control

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def f(a,b):
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Flow of control

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Not handled?

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  h(s)
```

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Flow of control

```
...
x = f(y,z)
```

`IndexError` inherited from `f()`

Not handled?

Abort!

```
def f(a,b):
  ...
  g(a)
```

`IndexError` inherited from `g()`

Not handled?

```
def g(m):
  ...
  h(m)
```

`IndexError` inherited from `h()`

Not handled?

```
def h(s):
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  h(s)
```

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Classes and objects

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Programming, Data Structures and Algorithms using Python

Week 1

Classes and objects

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  - Stores some information
  - Designated functions to manipulate the information
  - For instance, stack: last-in, first-out, `push()`, `pop()`

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- Class
  - Template for a data type
  - How data is stored
  - How public functions manipulate data

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- Class
  - Template for a data type
  - How data is stored
  - How public functions manipulate data
- Object
  - Concrete instance of template

Example: 2D points

- A point has coordinates (x, y)
  - `__init__()` initializes internal values `x`, `y`
  - First parameter is always `self`
  - Here, by default a point is at $(0, 0)$

```
class Point:
  def __init__(self,a=0,b=0):
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  - $(x, y) \mapsto (x + \Delta x, y + \Delta y)$

```
class Point:
  def __init__(self,a=0,b=0):
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  def translate(self,deltax,deltay):
    self.x += deltax
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```

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class Point:
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  def odistance(self):
    import math
    d = math.sqrt(self.x*self.x +
                  self.y*self.y)
    return(d)
```

Polar coordinates

- (r, θ) instead of (x, y)
  - $r = \sqrt{x^2 + y^2}$
  - $\theta = \tan^{-1}(y/x)$

```
import math
class Point:
  def __init__(self,a=0,b=0):
    self.r = math.sqrt(a*a + b*b)
    if a == 0:
      self.theta = math.pi/2
    else:
      self.theta = math.atan(b/a)
```

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  def odistance(self):
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```

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- Distance from origin is just r
- Translation
  - Convert (r, θ) to (x, y)
  - $x = r\cos\theta$, $y = r\sin\theta$
  - Recompute r, θ from $(x + \Delta x, y + \Delta y)$

```
def translate(self,deltax,deltay):
  x = self.r*math.cos(self.theta)
  y = self.r*math.sin(self.theta)
  x += deltax
  y += deltay
  self.r = math.sqrt(x*x + y*y)
  if x == 0:
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  - Convert (r, θ) to (x, y)
  - $x = r \cos \theta$, $y = r \sin \theta$
  - Recompute r, θ from $(x + \Delta x, y + \Delta y)$
- Interface has not changed
  - User need not be aware whether representation is (x, y) or (r, θ)

```
def translate(self,deltax,deltay):
  x = self.r*math.cos(self.theta)
  y = self.r*math.sin(self.theta)
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Special functions

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- `__str__()` — convert object to string
  - `str(o) == o.__str()__`
  - Implicitly invoked by `print()`

```
class Point:

  ...

  def __str__(self):
    return(
      '('+str(self.x)+','
         +str(self.y)+')'
    )
```

Special functions

- `__init__()` — constructor
- `__str__()` — convert object to string
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- `__add__()`
  - Implicitly invoked by `+`

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class Point:

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    return(Point(self.x + p.x,
                 self.y + p.y))
```

Special functions

- `__init__()` — constructor
- `__str__()` — convert object to string
  - `str(o) == o.__str()__`
  - Implicitly invoked by `print()`
- `__add__()`
  - Implicitly invoked by `+`
- Similarly
  - `__mult__()` invoked by `*`
  - `__lt__()` invoked by `<`
  - `__ge__()` invoked by `>=`
  - ...

```
class Point:

  ...

  def __str__(self):
    return(
      '('+str(self.x)+','
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Timing our code

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Programming, Data Structures and Algorithms using Python

Week 1

Timing our code

- How long does our code take to execute?
    - Depends from one language to another

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- `perf_time()` is a performance counter
  - Absolute value of `perf_time()` is not meaningful
  - Compare two consecutive readings to get an interval
  - Default unit is seconds

Timing our code

- How long does our code take to execute?
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- Python has a library `time` with various useful functions
- `perf_time()` is a performance counter
  - Absolute value of `perf_time()` is not meaningful
  - Compare two consecutive readings to get an interval
  - Default unit is seconds

```
import time

start = time.perf_counter()
...
# Execute some code
...
end = time.perf_counter()

elapsed = end - start
```

A timer object

- Create a timer class

```
import time
class Timer:
```

A timer object

- Create a timer class
- Two internal values
  - _start_time
  - _elapsed_time

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class Timer:

  def __init__(self):
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```

A timer object

- Create a timer class
- Two internal values
  - `_start_time`
  - `_elapsed_time`
- `start` starts the timer

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import time
class Timer:

  def __init__(self):
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  def start(self):
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A timer object

- Create a timer class
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- `stop` records the elapsed time

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class Timer:

  def __init__(self):
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  def start(self):
    self._start_time = time.perf_counter()

  def stop(self):
    self._elapsed_time =
       time.perf_counter() - self._start_time

  def elapsed(self):
    return(self._elapsed_time)
```

A timer object

- Create a timer class
- Two internal values
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- `start` starts the timer
- `stop` records the elapsed time
- More sophisticated version in the actual code

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- Create a timer class
- Two internal values
  - `_start_time`
  - `_elapsed_time`
- `start` starts the timer
- `stop` records the elapsed time
- More sophisticated version in the actual code
- Python executes 10^7 operations per second

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class Timer:

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Why Efficiency Matters

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Programming, Data Structures and Algorithms using Python

Week 1

A real world problem

- Every SIM card needs to be linked to an Aadhaar card

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```
for each SIM card S:
  for each Aadhaar number A:
     check if Aadhaar details of S
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- How long will this take?
  - M SIM cards, N Aadhaar cards
  - Nested loops iterate $M \cdot N$ times

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- Validate the Aadhaar details for each SIM card
- Simple nested loop
- How long will this take?
  - M SIM cards, N Aadhaar cards
  - Nested loops iterate $M \cdot N$ times
- What are M and N
  - Almost everyone in India has an Aadhaar card: $N > 10^9$
  - Number of SIM cards registered is similar: $M > 10^9$

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  - $(1.17 \times 10^6)/365 \approx 3200$ years!

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  - $(1.17 \times 10^6)/365 \approx 3200$ years!
- How can we fix this?

```
for each SIM card S:
  for each Aadhaar number A:
     check if Aadhaar details of S
     match A
```

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- You propose a date

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- Both algorithms and data structures matter