Machine Vision

Wu Wei

Images Enhancement

01 Gray Level Transformations

02 Histogram Processing

03 Arithmetic/Logical Operation

04 Spatial Filtering

05 Frequency Filtering

PART 5

Frequency Filtering

Frequency Filtering

- Fourier Transform and the Frequency Domain
- Smoothing Frequency Domain Filters
- Sharpening Frequency Domain Filters
- Band Pass Frequency Domain Filters
- Homomorphic Filtering

Frequency Filtering

Fourier was born in Auxerre, France in 1768.

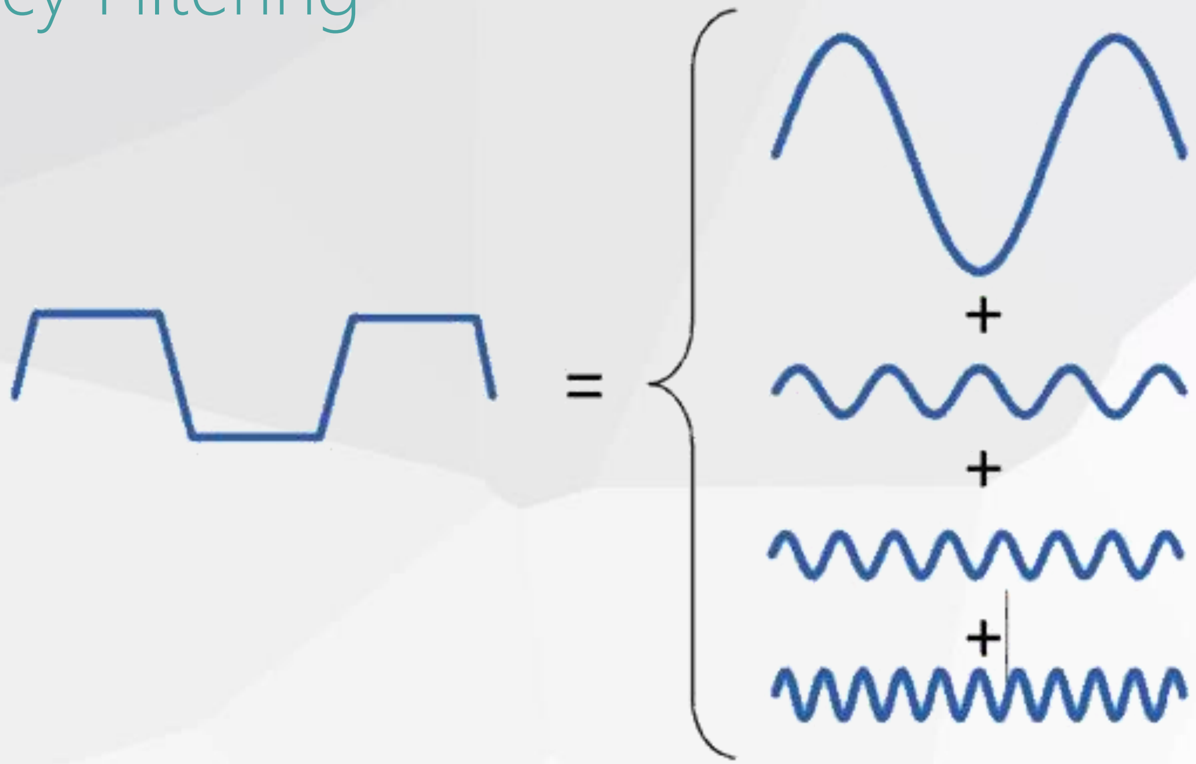
– Most famous for his work “*La Théorie Analitique de la Chaleur*” published in 1822.

– Translated into English in 1878: “*The Analytic Theory of Heat*”.

Nobody paid much attention when the work was first published.

One of the most important mathematical theories in modern engineering.

Frequency Filtering



Any function that periodically repeats itself can be expressed as a sum of sines and cosines of different frequencies each multiplied by a different coefficient – a *Fourier series*

Frequency Filtering

Fourier Transform

The Fourier transform, $F(\omega)$, of a single variable, continuous function, $f(x)$, is defined by the equation

$$F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi\omega x} dx$$

where $j = \sqrt{-1}$, and the *inverse* Fourier transform is

$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{j2\pi\omega x} d\omega$$

Frequency Filtering

Fourier Transform

Extending to two variables, *u* and *v*:

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

and, similarly for the inverse transform,

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv$$

Frequency Filtering

Fourier Transform

The Fourier transform of a discrete function of one variable, $f(x)$, $x = 0, 1, 2, \ldots, M - 1$, is given by the equation

$$F(\omega) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)e^{-j2\pi\omega x/M} \quad \text{for } \omega = 0, 1, 2, \ldots, M-1.$$

Similarly, given $F(\omega)$, we can obtain the original function back using the inverse DFT:

$$f(x) = \sum_{\omega=0}^{M-1} F(\omega)e^{j2\pi\omega x/M} \quad \text{for } x = 0, 1, 2, \ldots, M-1.$$

Frequency Filtering

Fourier Transform

The concept of the frequency domain follows directly from Euler’s formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Substituting this expression into the DFT:

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\left[\cos\frac{2\pi ux}{M} - j\sin\frac{2\pi ux}{M}\right]$$

for $\omega = 0, 1, 2, \ldots, M-1.$

Frequency Filtering

Fourier Transform

The components of DFT are complex quantities. In polar coordinates:

$$F(u) = |F(u)| e^{-j\psi(u)}$$

where

$$|F(u)| = \sqrt{R^2(u) + I^2(u)}$$

is called the *magnitude* or *spectrum* of the FT, and

$$\psi(u) = \tan^{-1}\left[\frac{I(u)}{R(u)}\right]$$

is called the *phase angle* or *phase spectrum* of the FT. $R(u)$ and $I(u)$ are the real and imaginary parts of $F(u)$ respectively.

Another quantity frequently used is the *power spectrum* or *spectral density*:

$$P(u) = |F(u)|^2 = R^2(u) + I^2(u)$$

Frequency Filtering

- Fourier Transform

The DFT of a function (image) $f(x, y)$ of size M x N is given by the equation

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$u = 0, 1, 2, \ldots, M-1,\ v = 0, 1, 2, \ldots, N-1.$

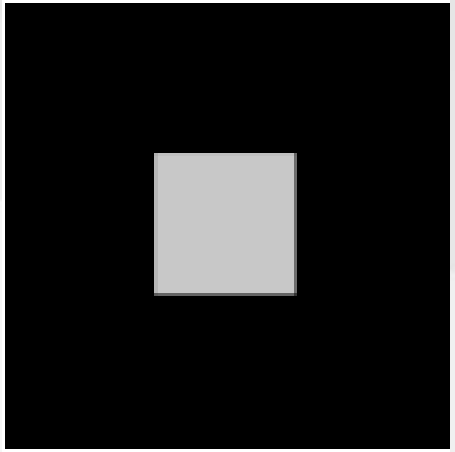
Similarly, the IDFT is

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

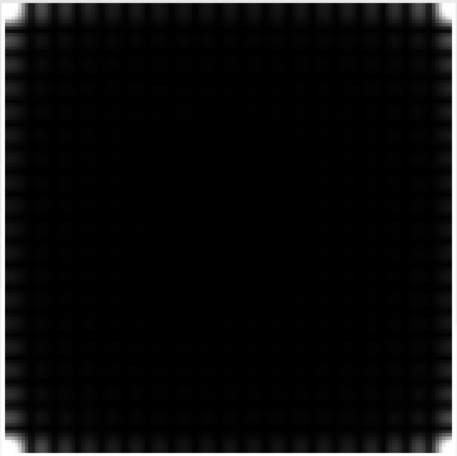
$x = 0, 1, 2, \ldots, M-1,\ y = 0, 1, 2, \ldots, N-1.$

Frequency Filtering

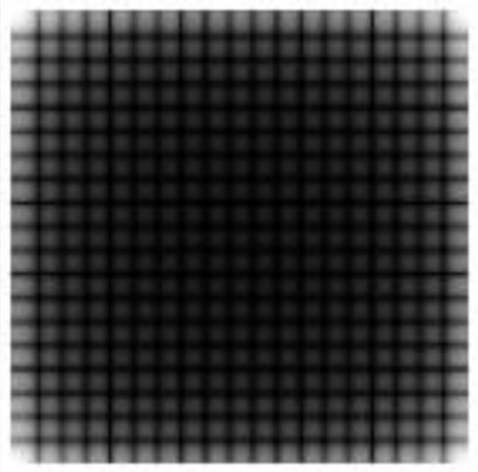
Fourier Transform



Original



DFT spectrum |F(u,v)|



log(1 + |F(u,v)|)

It is common practice to multiply the input image by $(-1)^{x+y}$ prior to computing the FT. It is not difficult to show that

$$\Im\left[f(x,y)(-1)^{x+y}\right] = F(u-M/2, v-N/2)$$

Where $\Im[\cdot]$ denotes the Fourier transform of the argument.
This operation shifts the origin of $F(u, v)$ to frequency coordinates $(M/2, N/2)$, which is the center of the M x N area occupied by the 2-D DFT, called frequency rectangle.

Frequency Filtering

- Fourier Transform

Example:

|F(u)| = 100 4 2 1 0 0 1 2 4

Display in Range([0..100]):

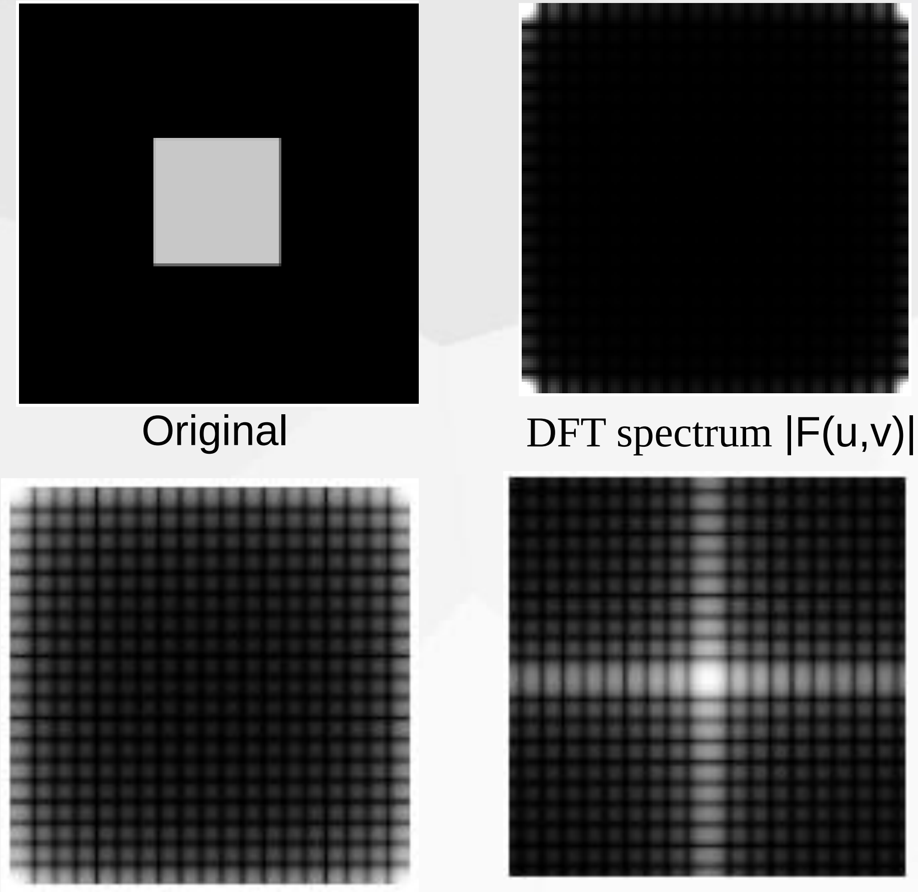
log(1+|F(u)|) = 4.62 1.61 1.01 0.69 0 0 0.69 1.01 1.61

log(1+|F(u)|)/0.0462 =100 40 20 10 0 0 10 20 40

fftshift(log(1+|F(u)|) = 0 10 20 40 100 40 20 10 0

Frequency Filtering

Fourier Transform



Original

DFT spectrum |F(u,v)|

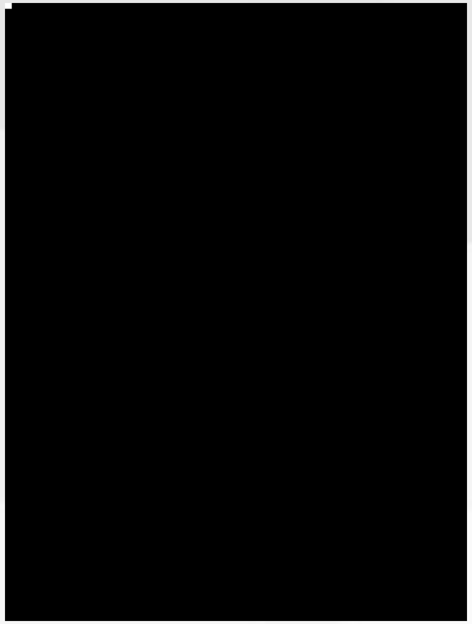
log(1 + |F(u,v)|)

fftshift(log(1 + |F(u,v)|))

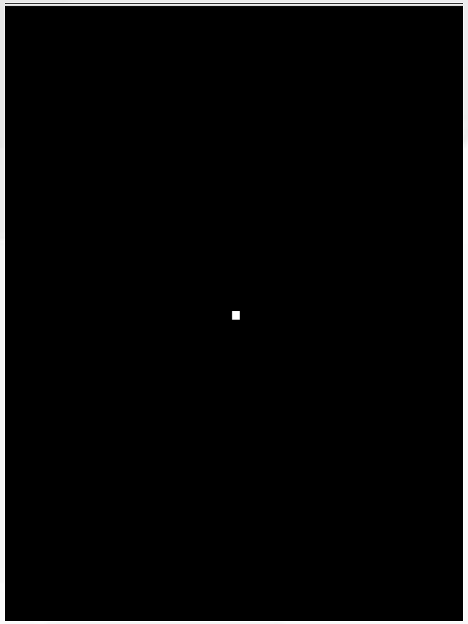
Frequency Filtering

Fourier Transform

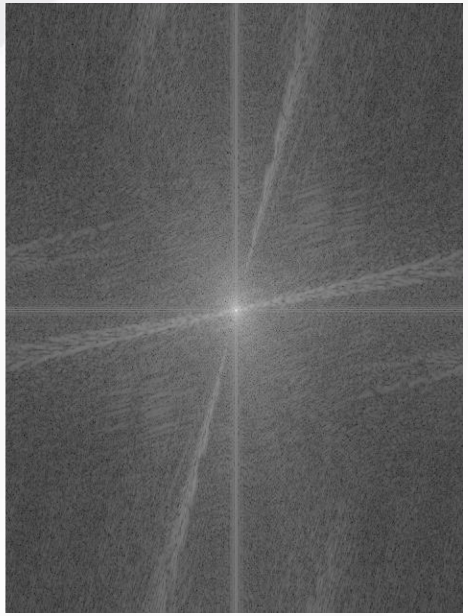
Original



DFT spectrum |F(u,v)|



fftshift(1 + |F(u,v)|)



fftshift(log(1 + |F(u,v)|))

Frequency Filtering

Fourier Transform

magnitude or spectrum of the FT

$$F(u,v) = \sqrt{R^2(u,v) + I^2(u,v)}$$

phase angle or phase spectrum

$$\psi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

power spectrum or spectral density

$$P(u,v) = \left|F(u,v)\right|^2 = R^2(u,v) + I^2(u,v)$$

Frequency Filtering

Fourier Transform

Curious fact

– all natural images have about the same magnitude transform

– hence, phase seems to matter, but magnitude largely doesn’t demonstration

– Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

Frequency Filtering

Fourier Transform

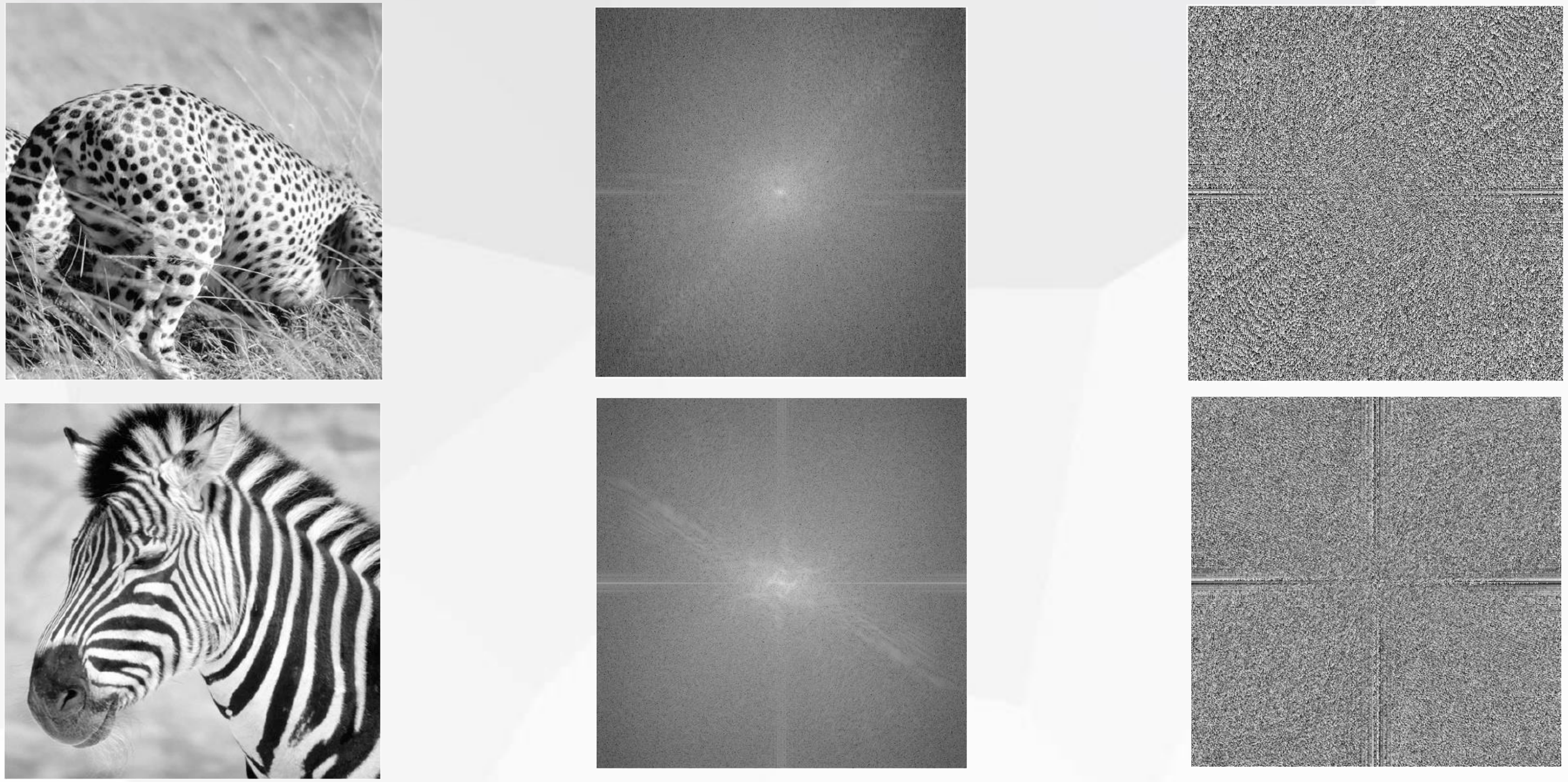
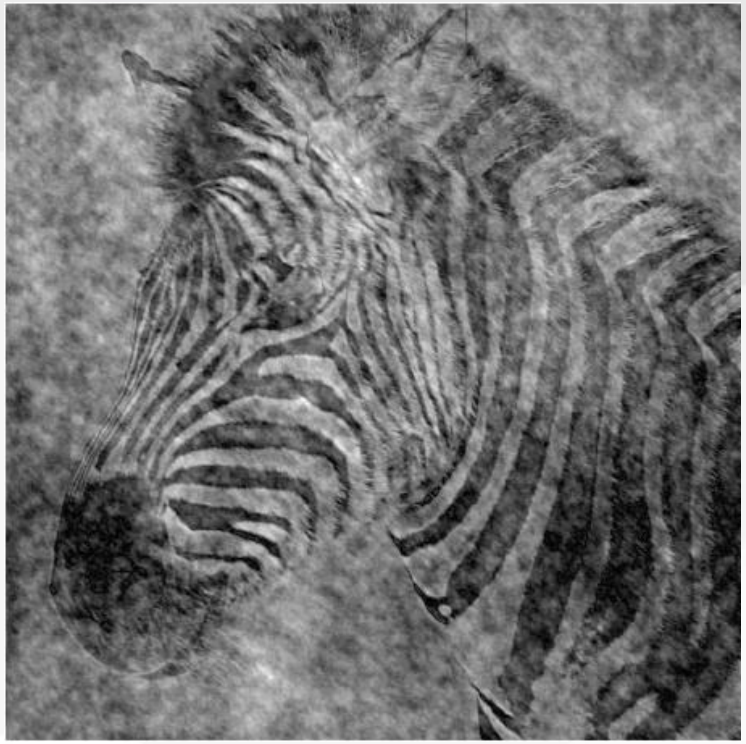


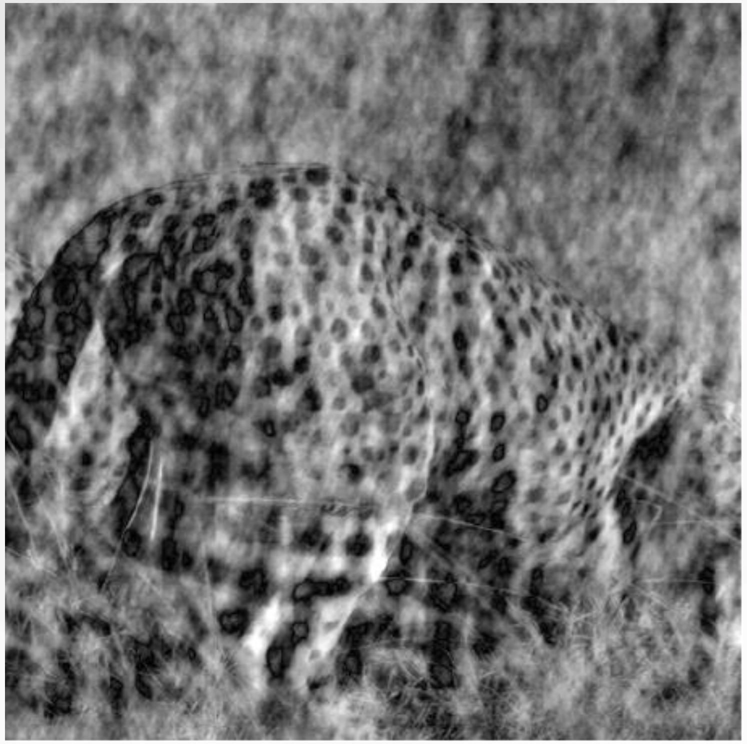
Image | magnitude | phase spectrum

Frequency Filtering

- Fourier Transform



Reconstruction with zebra phase, cheetah magnitude



Reconstruction with cheetah phase, zebra magnitude

Frequency Filtering

Fourier Transform

The value of the transform at (u, v) = (0, 0) is

$$F(0,0)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)$$

which is the average of $f(x, y)$. Sometimes $F(0, 0)$ is called the ***dc*** component of the spectrum.

If $f(x, y)$ is real (an image is naturally is), its FT is conjugate symmetric:

$$F(u,v)=F^{*}(-u,-v)$$

and its spectrum is symmetric:

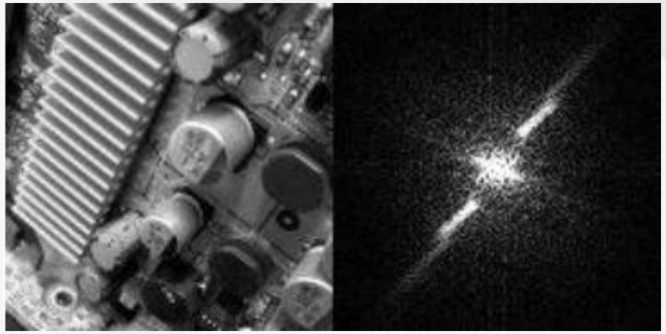
$$|F(u,v)|=|F(-u,-v)|$$

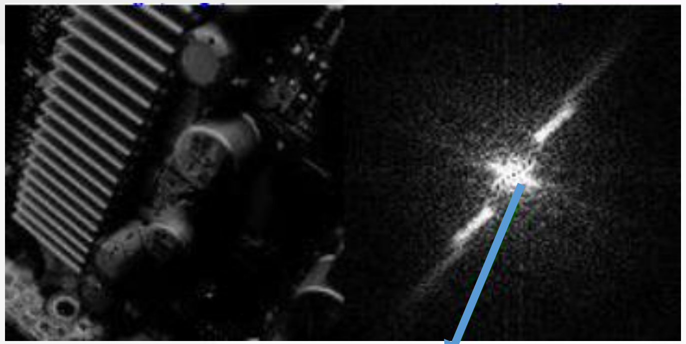
Frequency Filtering

Fourier Transform

The center value (at the origin) of the Frequency Spectrum corresponds to the ZERO frequency component which also referred to as the DC component in an image: Substituting 0,0 to the origin, the Fourier transform function yields to the average/DC component value as follows:

$$F(0,0) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)$$





The center/zero frequency component (DC component) is removed

$f(x,y)$
(Average image)

$F(u,v)$
(DC Component)

Image Enhancement

Frequency Filtering

Fourier Transform

Notice that the Fourier Spectrum for each image at the right contains just a single component, represented by 2 bright spots symmetrically placed about the center.

The dot at the center that represents the (0,0) frequency term or average value of the image. Images usually have a large average value/DC component. Due to low frequencies Fourier Spectrum images usually have a bright blob of components near the center.

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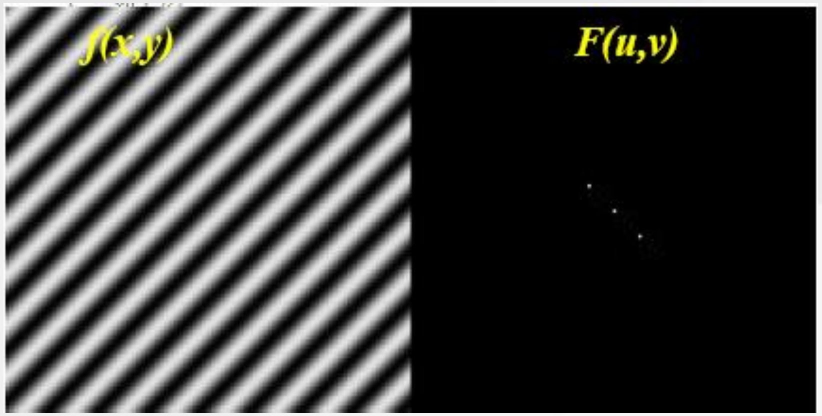
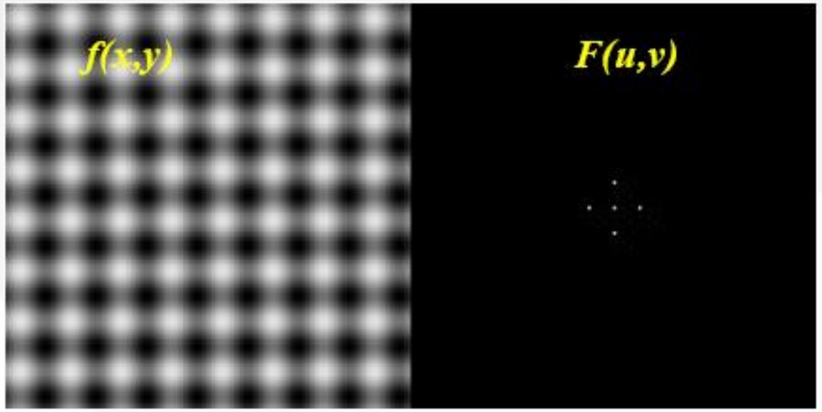
Frequency Filtering

Fourier Transform

Consider the following 2 left images with pure cosines in pure forward diagonal and mixed vertical+horizontal.



One of the properties of the 2D FT is that if you rotate the image, the spectrum will rotate in the same direction



The sum of 2 sine functions, each in opposite (vertical and horizontal) direction

Frequency Filtering

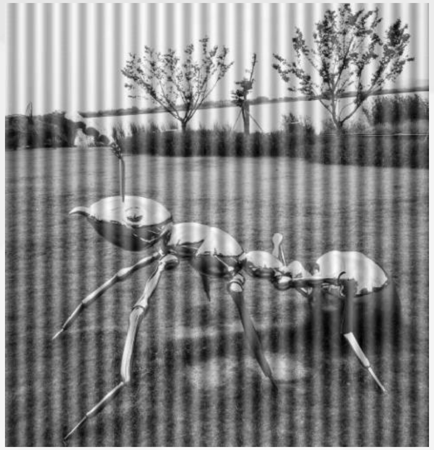
Why do we need representation in the frequency domain?

Original Problem → (Difficult solution) → Solution of Original Problem

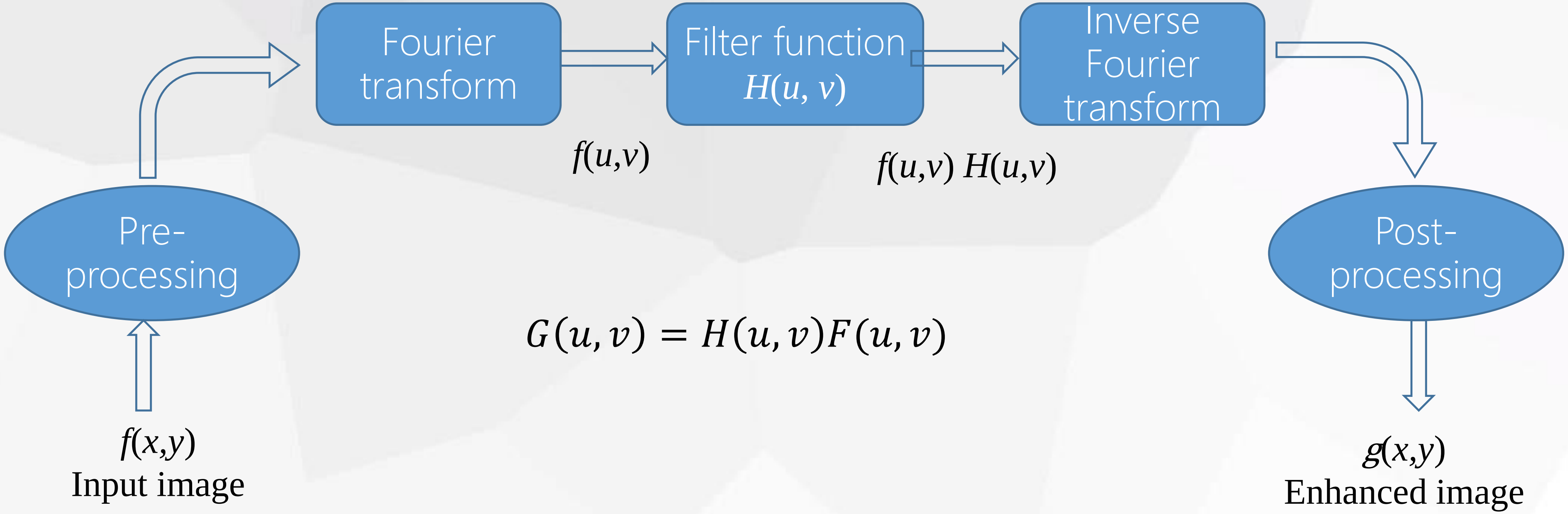
Original Problem → (Fourier Transform) → Problem in Frequency Space

Problem in Frequency Space → (Relatively easy solution) → Solution in Frequency Space

Solution in Frequency Space → (Inverse Fourier Transform) → Solution of Original Problem



Frequency Filtering



$$G(u,v) = H(u,v)F(u,v)$$

Frequency domain filtering operation

Frequency Filtering

Filtering steps:

- $f(x,y)(-1)^{x+y} = f_{new}(x,y)$
- $F(u - {}^{M}/_{2}, v - {}^{N}/_{2}) = \mathscr{F}(f_{new}(x,y))$
- $G(u,v) = H(u,v)F(u - {}^{M}/_{2}, v - {}^{N}/_{2})$
- $G(x,y) = \mathscr{F}^{-1}(G(u,v))$
- $G_{new} = G(x,y)(-1)^{x+y}$

Before filtering, it's necessary to shift the origin of the transfer function to the center of the frequency rectangle occupied by $F(u, v)$.

Frequency Filtering

- Fourier Transform and the Frequency Domain
- Smoothing Frequency Domain Filters
- Sharpening Frequency Domain Filters
- Band Pass Frequency Domain Filters
- Homomorphic Filtering

Frequency Filtering

Smoothing Filter

Frequency Filtering

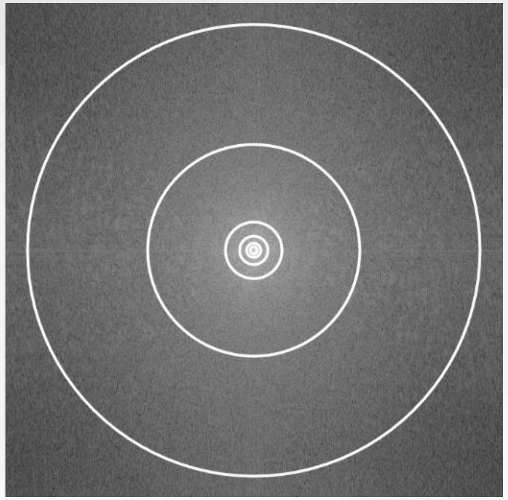
Smoothing Filter

Total image power P_T

$$P_T = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} P(u,v)$$

Percentage of image power

$$\alpha = 100\left[\sum_u \sum_v P(u,v)/P_T\right]$$



Percentage of image power (α) enclosed in circles (large to small) :

99.97% 99.68% 98.23% 96.69% 94.93% 92.71%

Frequency Filtering

Smoothing Filter

99.92 %

99.57%

98.58 %

Blurring - Ideal Low pass Filter

97.04 %

94.82 %

90.26 %

Frequency Filtering

Smoothing Filter

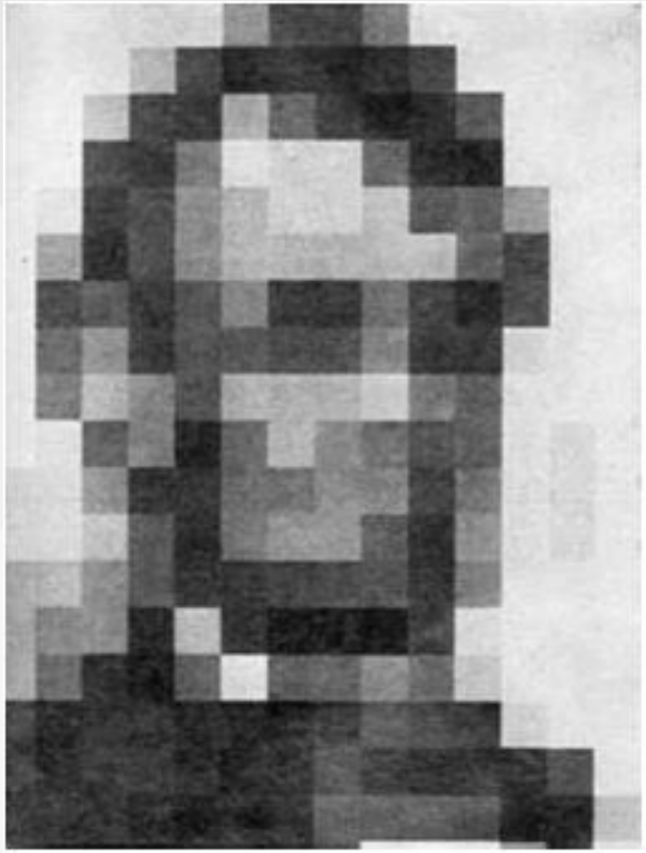
Frequency Filtering

Smoothing Filter

Image Enhancement

Frequency Filtering

Smoothing Filter



Frequency Filtering

Smoothing Filter

Image Enhancement

- Frequency Filtering
  - Smoothing Filter

Frequency Filtering

Smoothing Filter

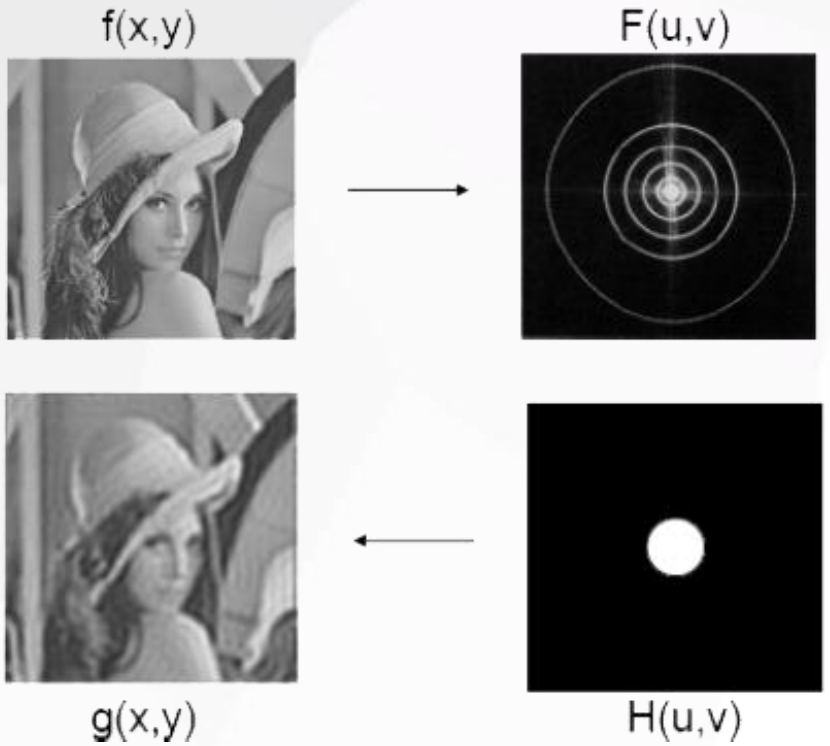
spatial domain   frequency domain

$$f(x,y) \longrightarrow F(u,v)$$

filter

$$G(u,v) = F(u,v) \cdot H(u,v)$$

$$g(x,y) \longleftarrow G(u,v)$$



Frequency Filtering

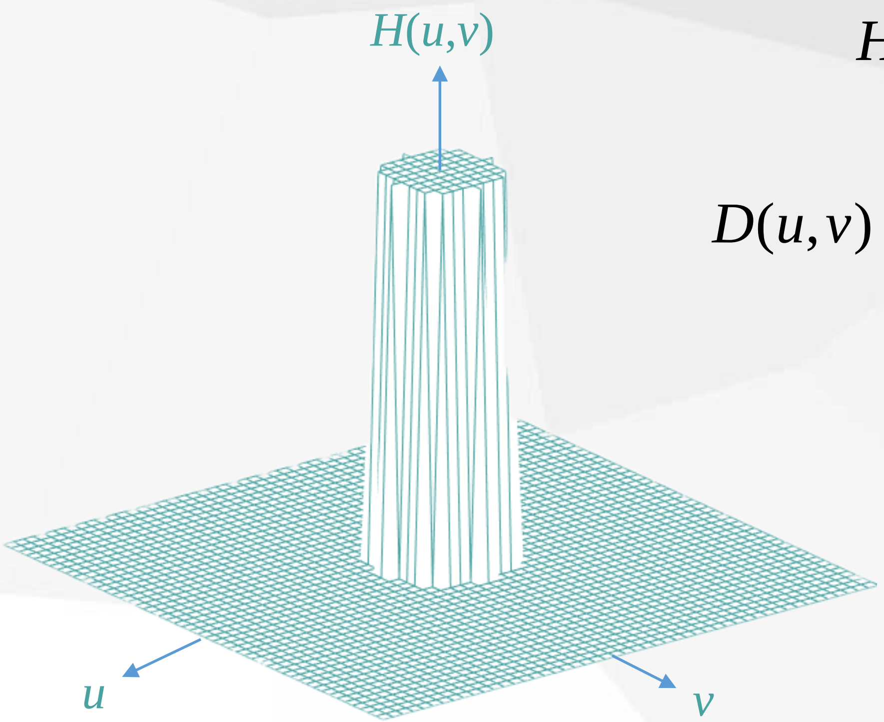
Smoothing Filter

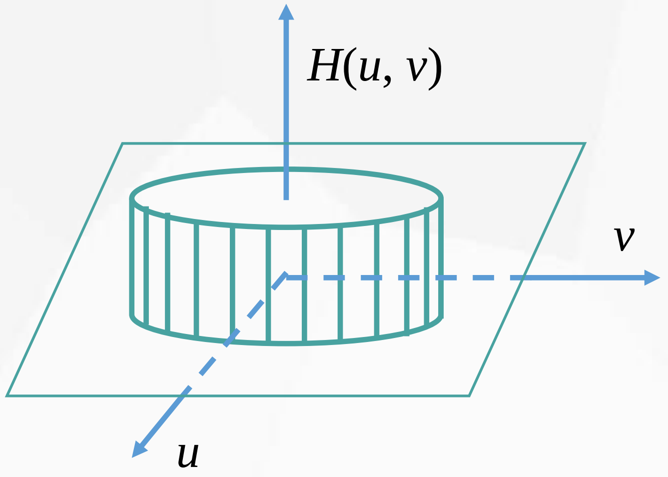
Ideal Low Pass Filter

$$H(u,v)=\begin{cases}1 & D(u,v)\le D_0\\ 0 & D(u,v)>D_0\end{cases}$$

D_0: 截止频率

$$D(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$$





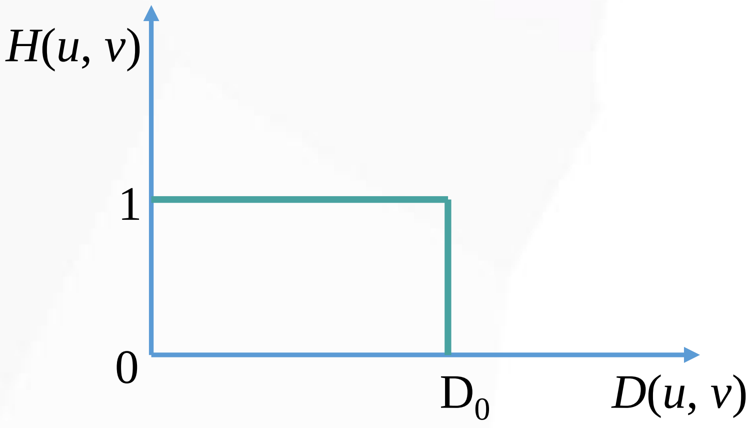
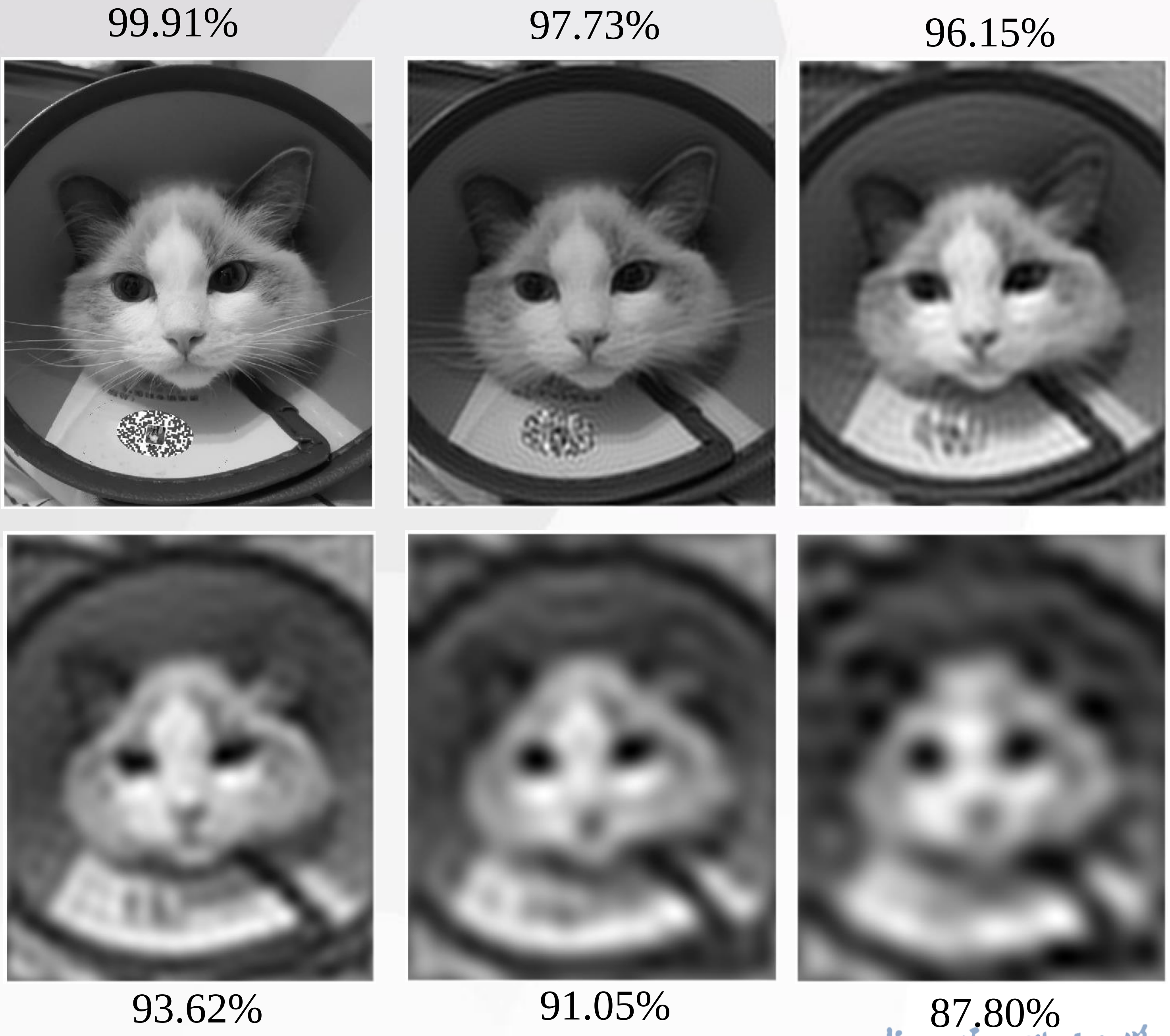


Image Enhancement

Frequency Filtering

Smoothing Filter

- Ideal Low Pass Filter



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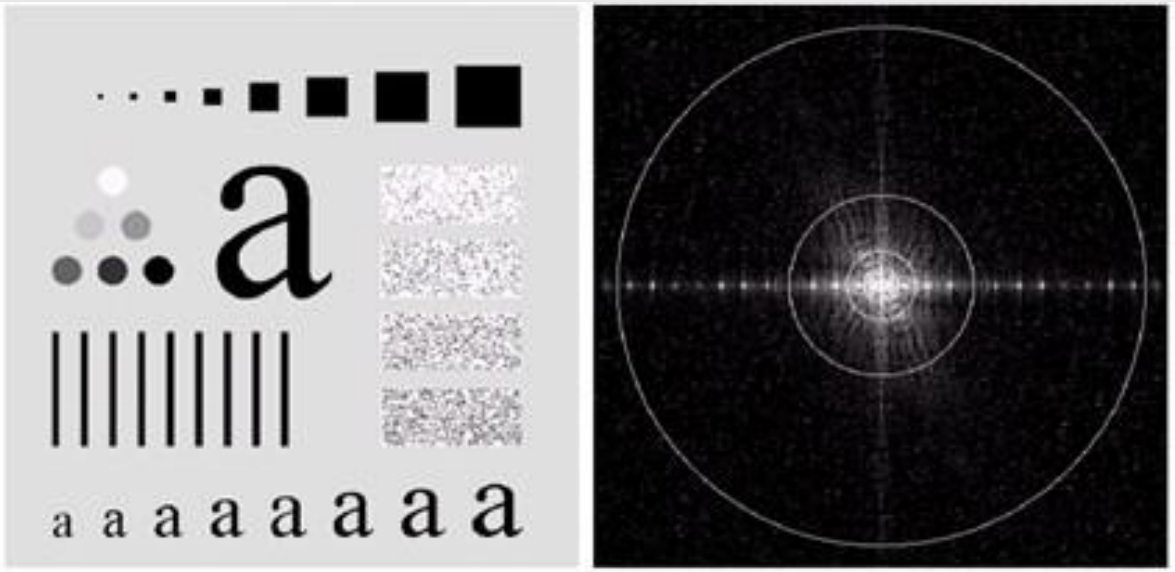
Frequency Filtering

Smoothing Filter

Ideal Low Pass Filter



The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0%, 94.6%, 96.4%, 98.0% and 99.5% of the image power, respectively.

The spectrum falls off rapidly, with 92% of the total power being enclosed by relatively small circle of radius 5.

Frequency Filtering

Smoothing Filter

- Ideal Low Pass Filter

$g = f * h$	$g = f \cdot h$
implies	implies
$G = F \cdot H$	$G = F * H$

Frequency Filtering

Smoothing Filter

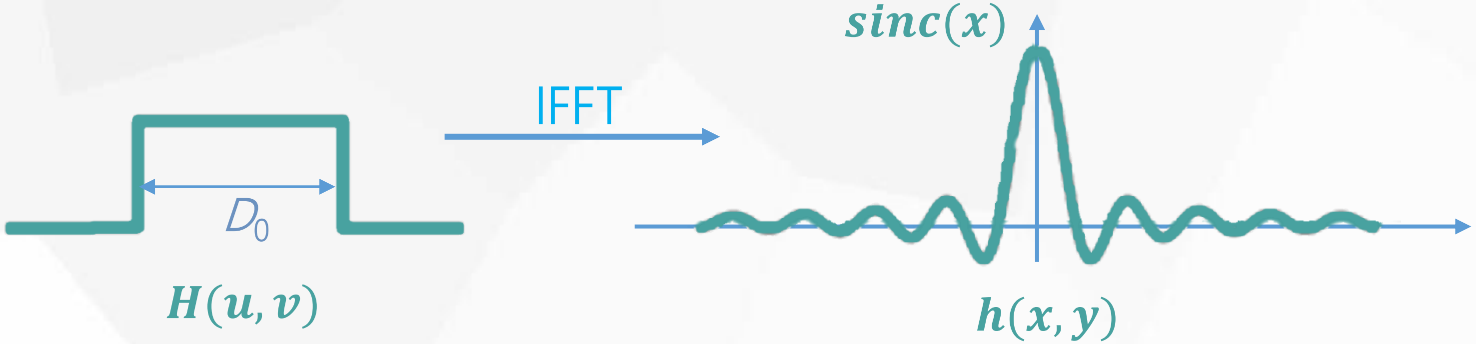
- Ideal Low Pass Filter

The Ringing Problem

$$\boldsymbol{G(u,v) = F(u,v) \cdot H(u,v)}$$

$$\Big\downarrow \text{Convolution Theorem}$$

$$\boldsymbol{g(x,y) = f(x,y) * h(x,y)}$$



$\boxed{\uparrow D_0 \rightarrow \downarrow \text{Ringing radius} + \downarrow \text{blur}}$

Frequency Filtering

Smoothing Filter

- Ideal Low Pass Filter

Most of the sharp detail information in the picture is contained in the 8% power removed by the filter.

Filtered images are characterized by “ringing,” which becomes finer in texture as the amount of high frequency content removed decreases.

The edge information is little contained in the upper 0.5% of the spectrum power in this particular case.

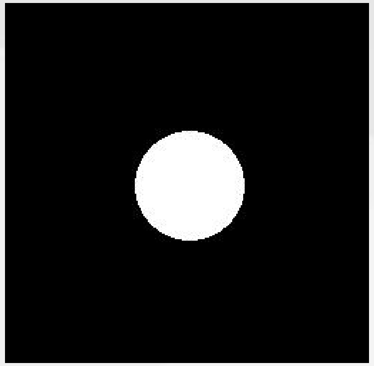
The superimposed circles have radii values of 5, 15, 30, 80, and 230.
The power removed by these filters was 8 %, 5.4 %, 3.6 %, 2 %, and 0.5% of the total, respectively.

Frequency Filtering

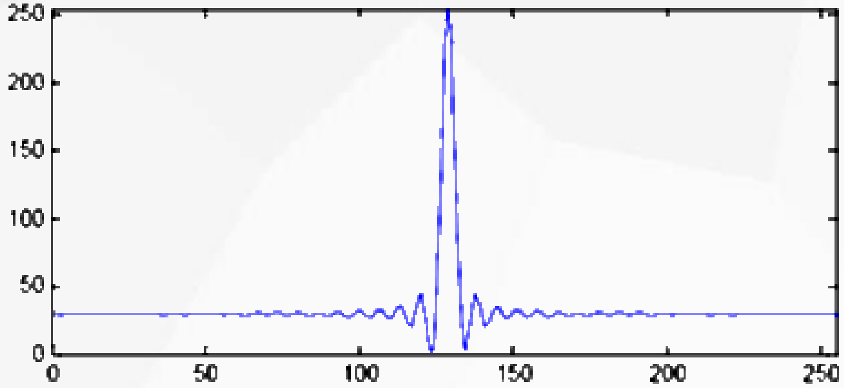
Smoothing Filter

- Ideal Low Pass Filter

Spatial domain



Frequency domain

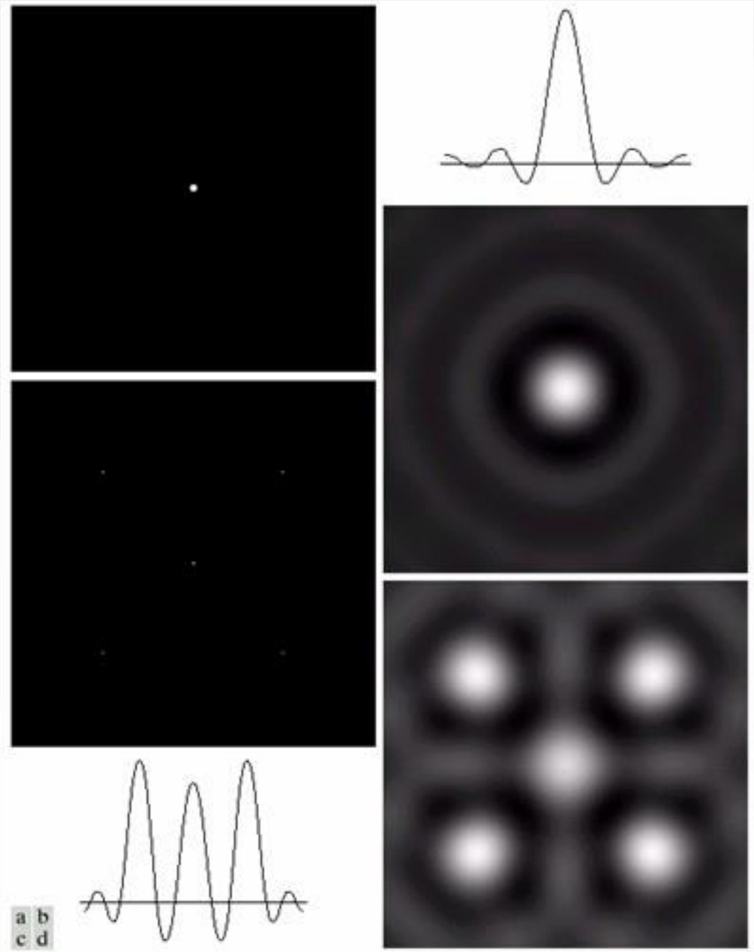


Frequency Filtering

Smoothing Filter

- Ideal Low Pass Filter

The Ringing Problem

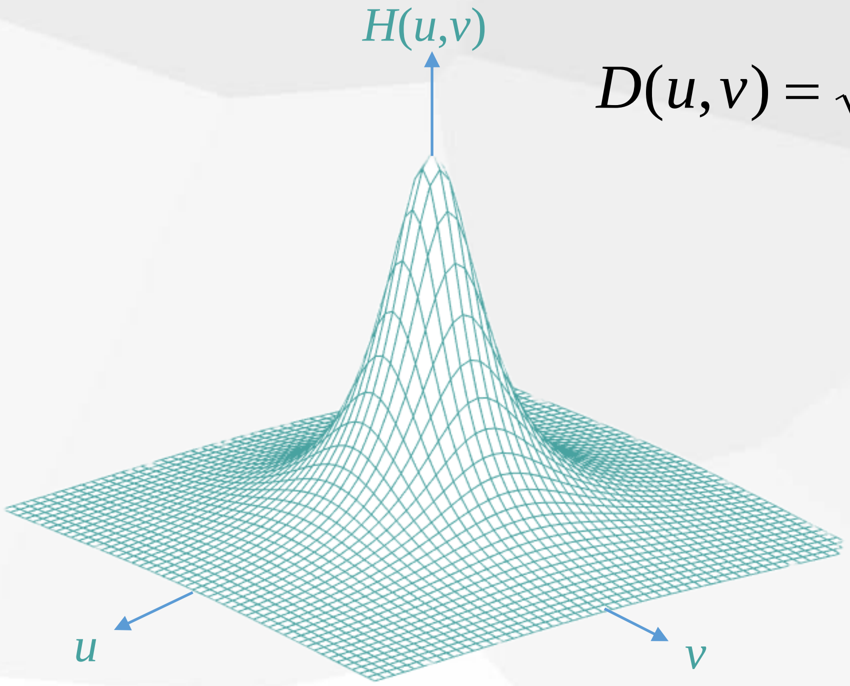


(a) A frequency-domain ILPF of radius 5.
(b) Corresponding spatial filter (note the ringing).
(c) Five impulses in the spatial domain. Simulating the values of five pixels.
(d) Convolution of (b) and (c) in the spatial domain

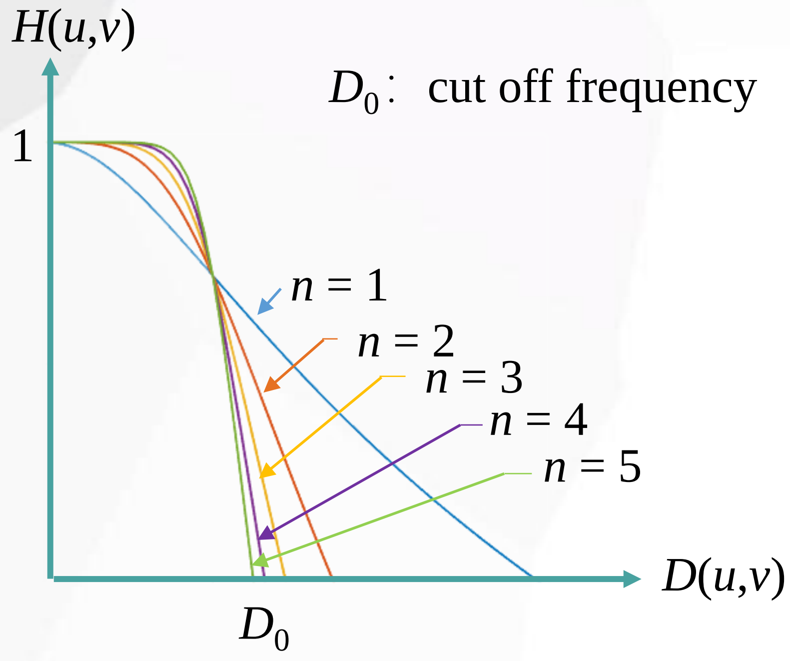
Frequency Filtering

Smoothing Filter

- Butterworth Low Pass Filter



$$D(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$$



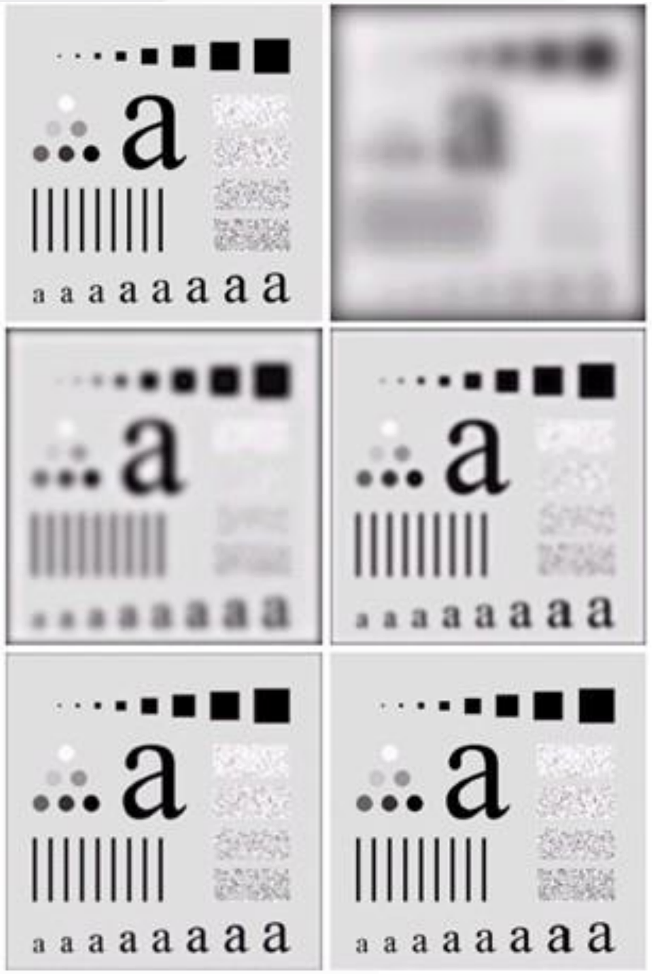
Softer Blurring + no Ringing

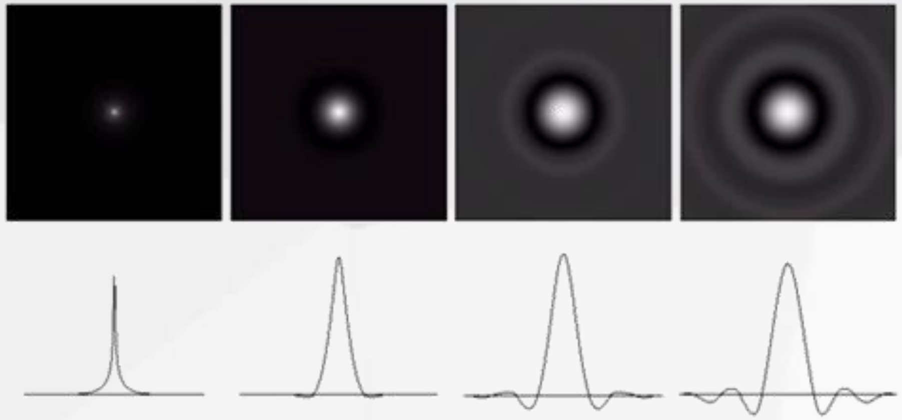
Frequency Filtering

Smoothing Filter

- Butterworth Low Pass Filter

(a) Original image. (b)-(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230 →





(a)-(d) Spatial representation of BLPFs of order 1,2,5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order

a b
c d
e f

Image Enhancement

Frequency Filtering

Smoothing Filter

- Butterworth Low Pass Filter

99.91% 97.73% 96.15%

93.62% 91.05% 87.80%

Frequency Filtering

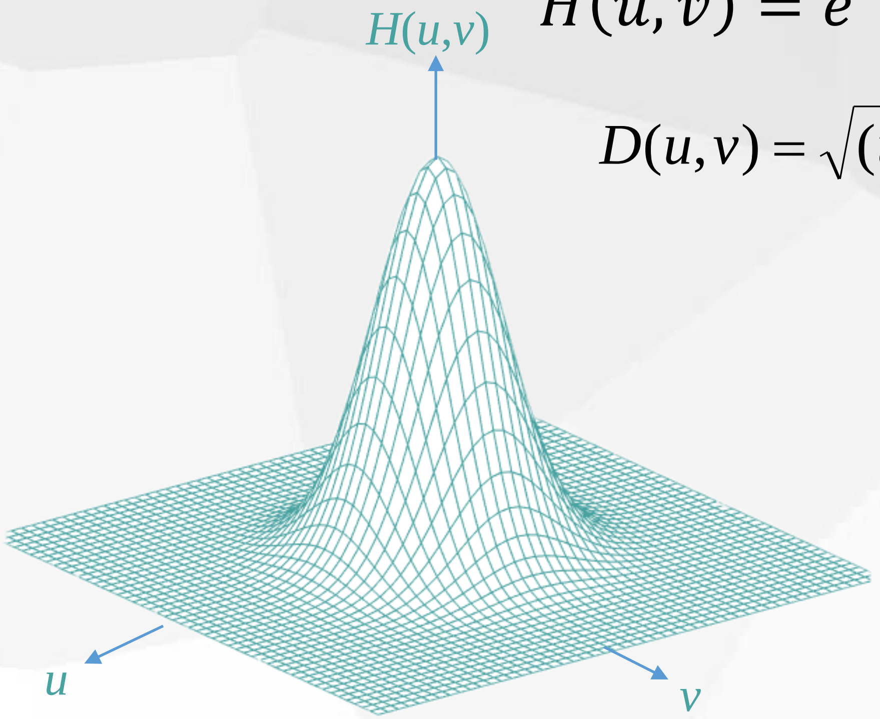
Smoothing Filter

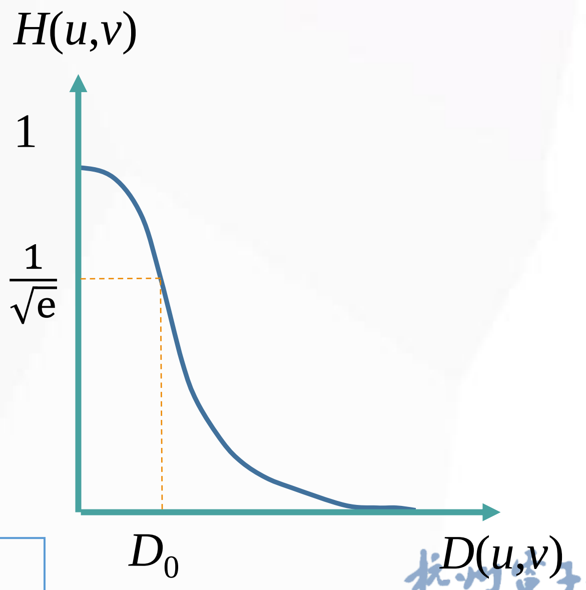
- Gaussian Low Pass Filter

$$H(u,v) = e^{-D^2(u,v)/(2D_0)}$$

$D(u,v) = D_0$, GLPF: 0.607*max

$$D(u,v) = \sqrt{(u - M/2)^2 + (v - N/2)^2}$$





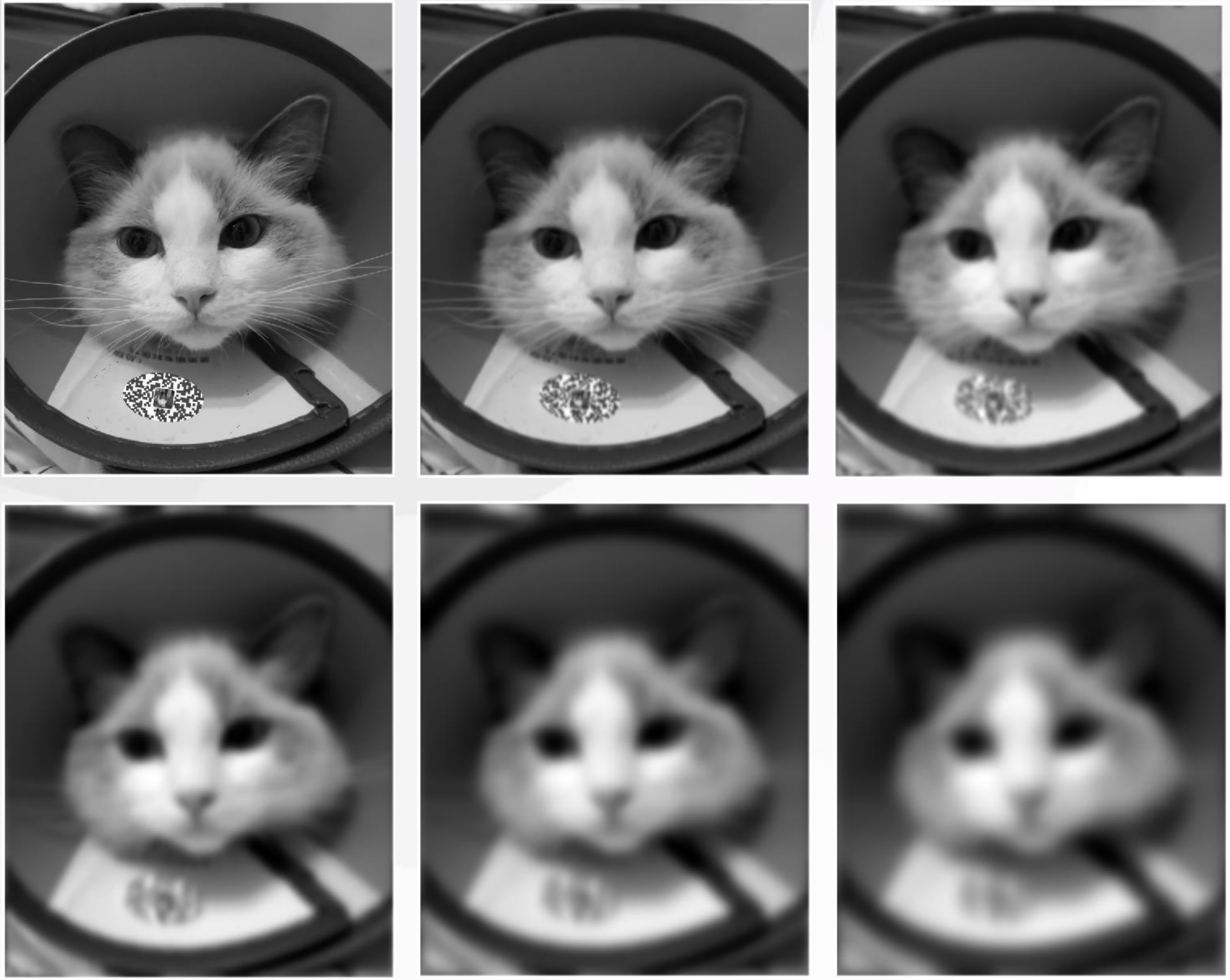
Softer Blurring + no Ringing

Image Enhancement

Frequency Filtering

Smoothing Filter

- Gaussian Low Pass Filter



99.91% 97.73% 96.15%

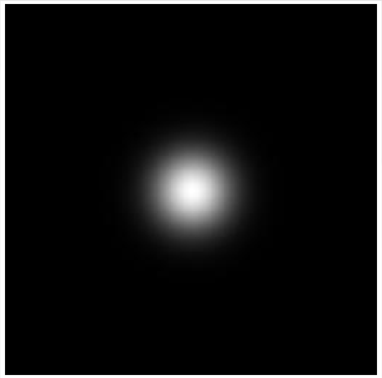
93.62% 91.05% 87.80%

Frequency Filtering

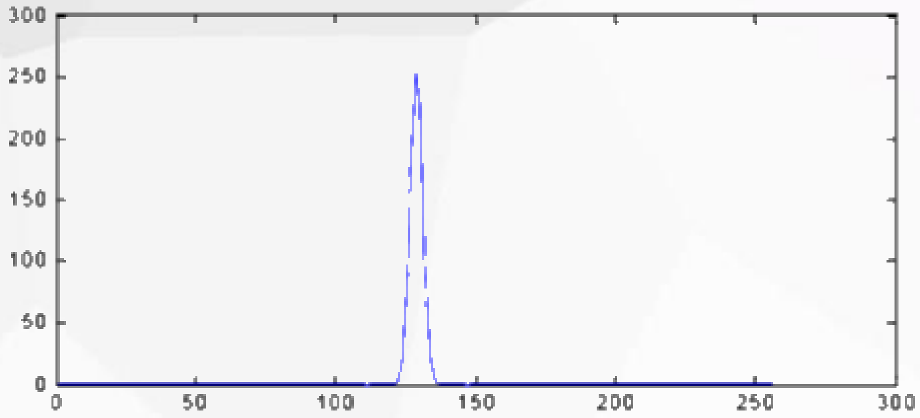
Smoothing Filter

- Gaussian Low Pass Filter

Freq. domain



Spatial domain

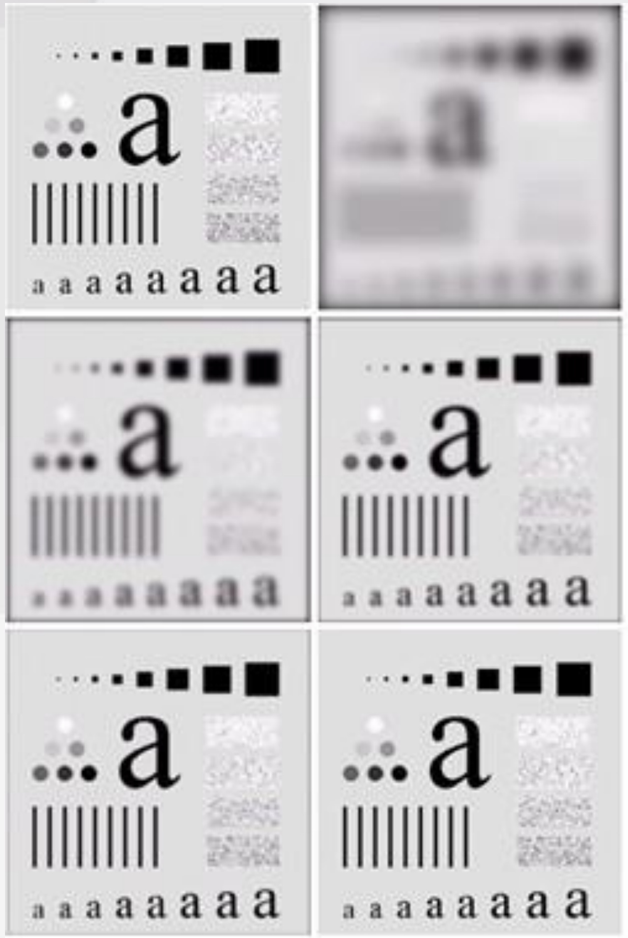


Frequency Filtering

Smoothing Filter

- Gaussian Low Pass Filter

a b
c d
e f



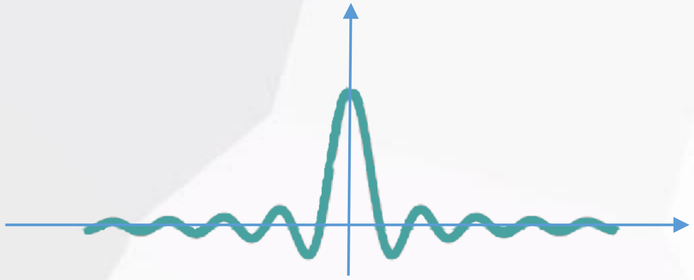
(a) Original image. (b)-(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230.

Frequency Filtering

Smoothing Filter

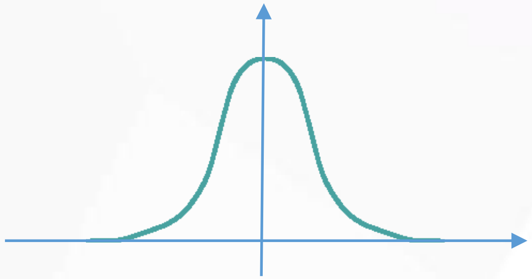
- Gaussian Low Pass Filter

Averaging = convolution with $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$



= point multiplication of the transform with **sinc**:

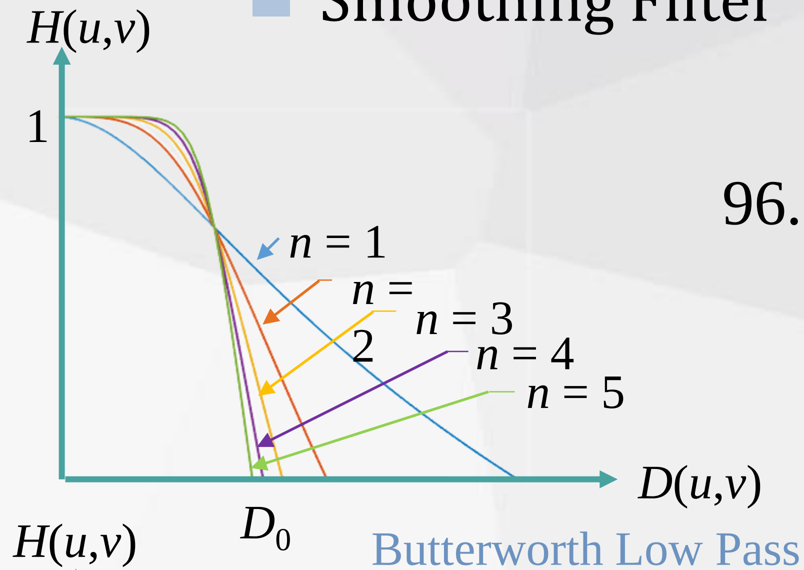
Gaussian Averaging = convolution with $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$



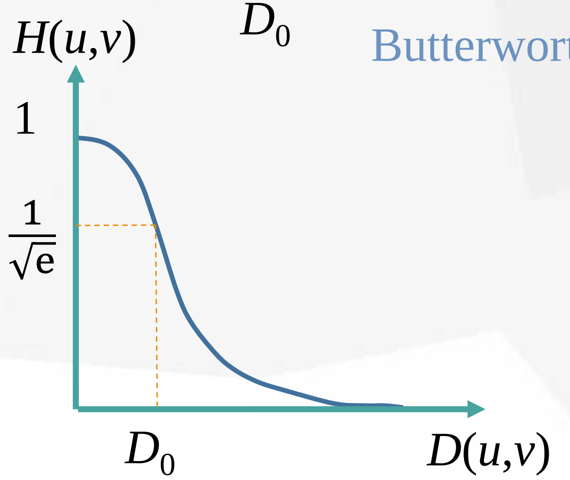
= point multiplication of the transform with a **gaussian**.

Frequency Filtering

Smoothing Filter



Butterworth Low Pass Filter



Gaussian Low Pass Filter

96.15%

93.62%

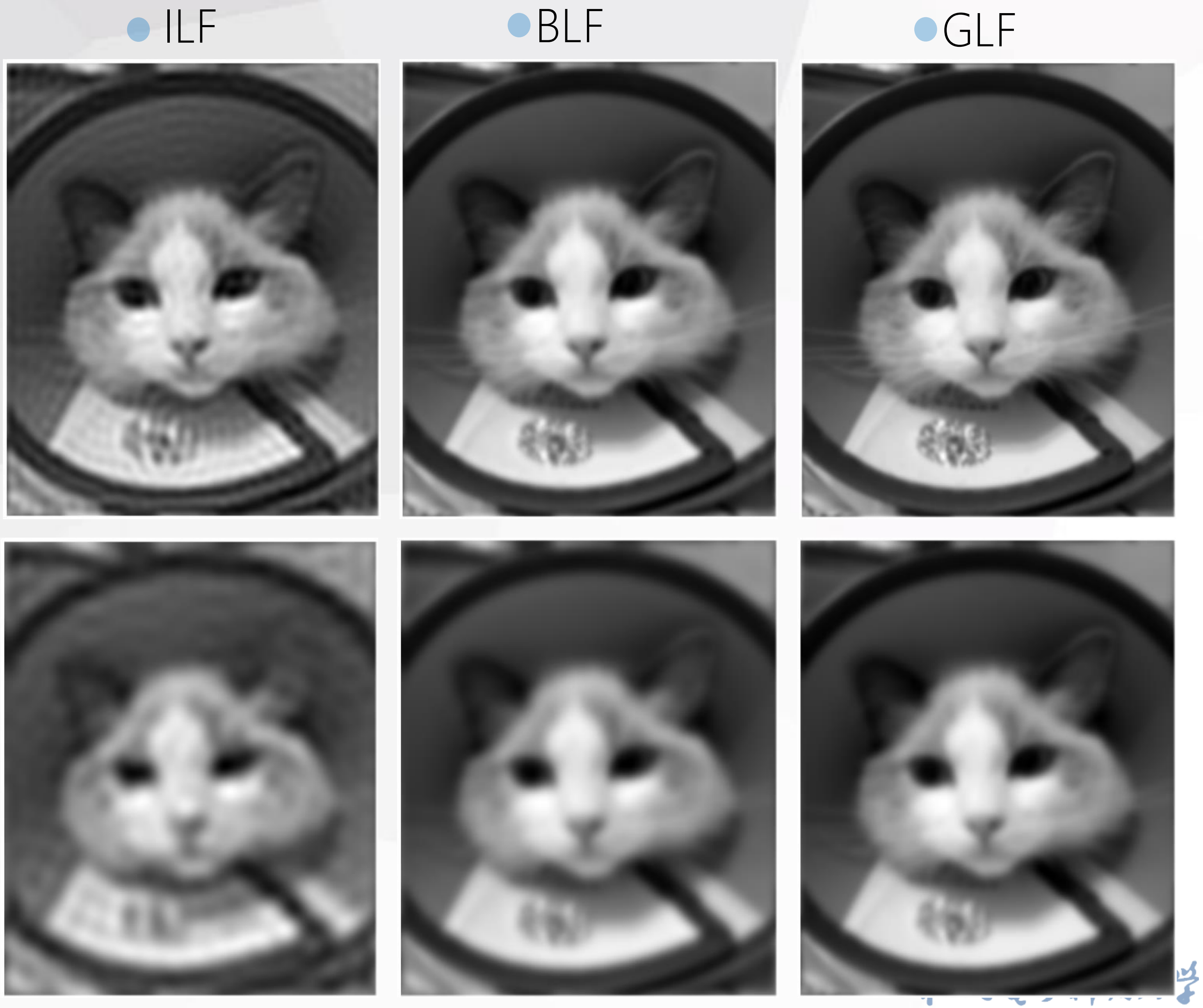
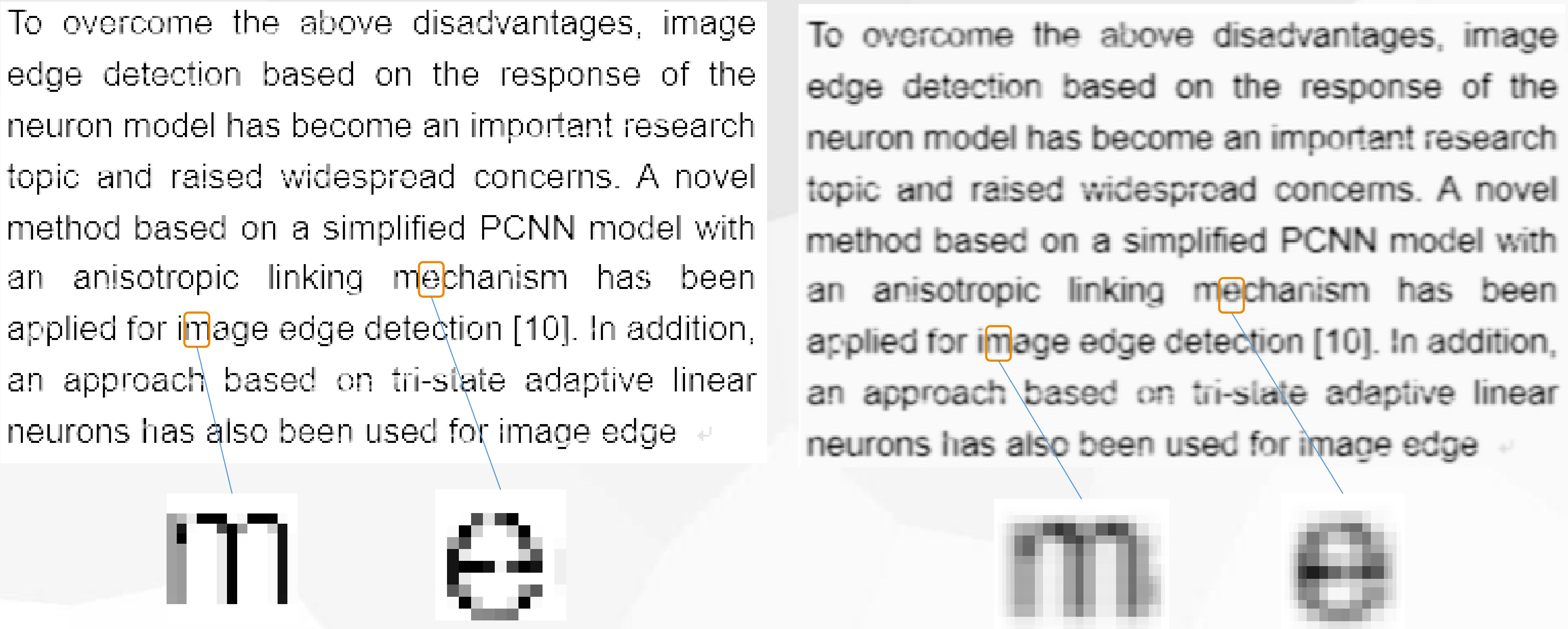


Image Enhancement

Frequency Filtering

Smoothing Filter

An example for character recognition from the field of machine perception



The approach used most often to handle this problem is to bridge small gaps in the input image by blurring it.

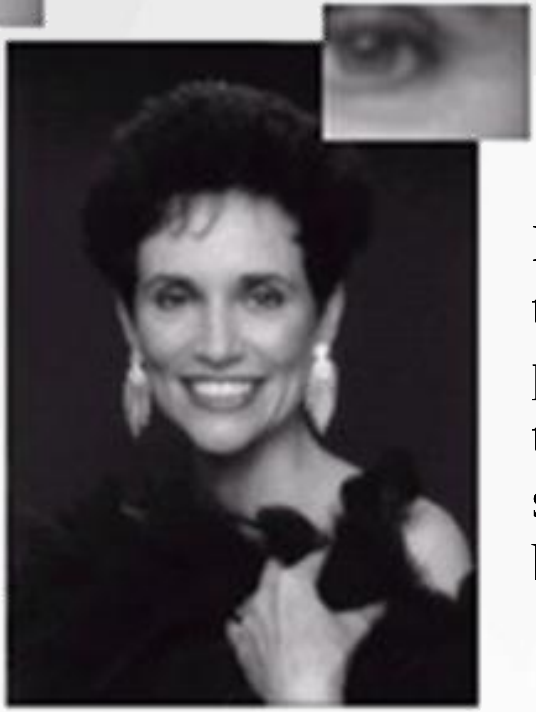
Frequency Filtering

Smoothing Filter

An example of pre-processing for printing and publishing. The typical objective is to produce a smoother, soft-looking result from a sharp original.

Original

GLPF D_0 = 100



GLPF D_0 = 80

For human faces, this “cosmetic” processing reduces the sharpness of fine skin lines and small blemishes.

Frequency Filtering

- Fourier Transform and the Frequency Domain
- Smoothing Frequency Domain Filters
- Sharpening Frequency Domain Filters
- Band Pass Frequency Domain Filters
- Homomorphic Filtering

Image Enhancement

Frequency Filtering

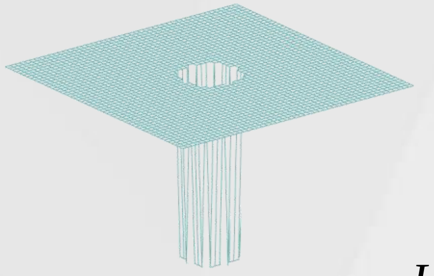
Sharpening Filter

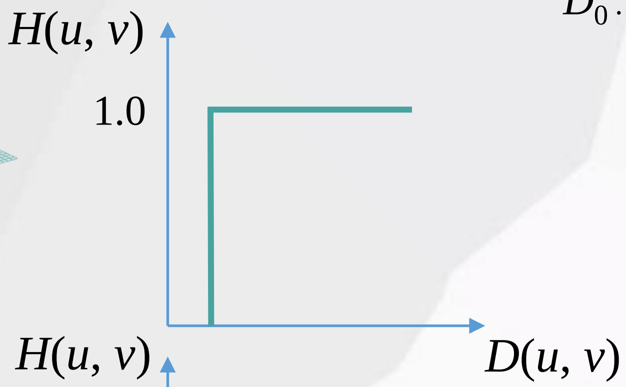
$$D(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$$

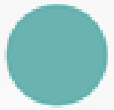
D_0: cut off frequency

- Ideal High Pass Filter

$$H(u,v)=\begin{cases}0 & D(u,v)\le D_0\\ 1 & D(u,v)\ge D_0\end{cases}$$

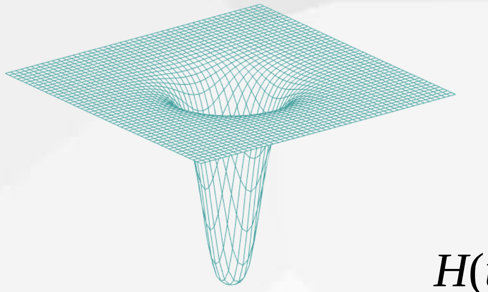


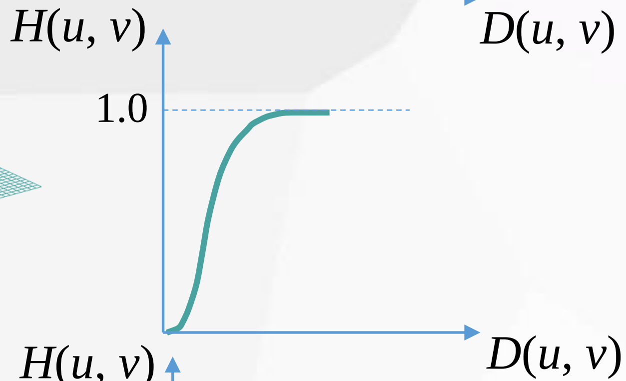




- Butterworth High Pass Filter

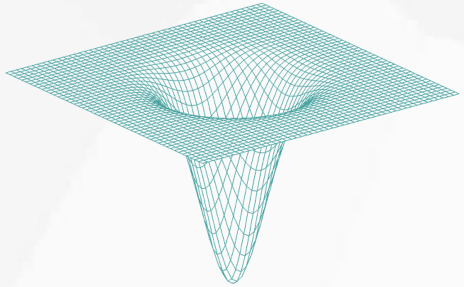
$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^2}$$

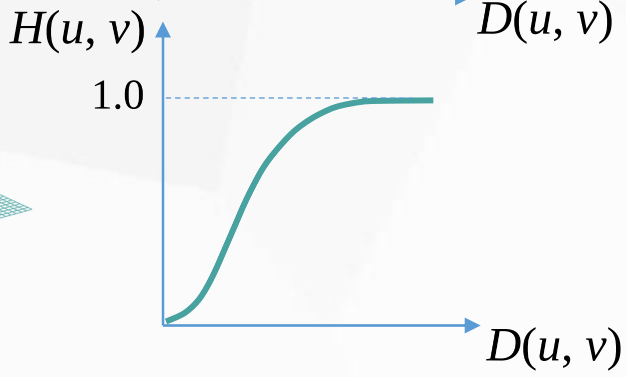


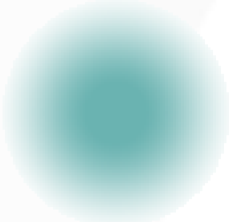


- Gaussian High Pass Filter

$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$



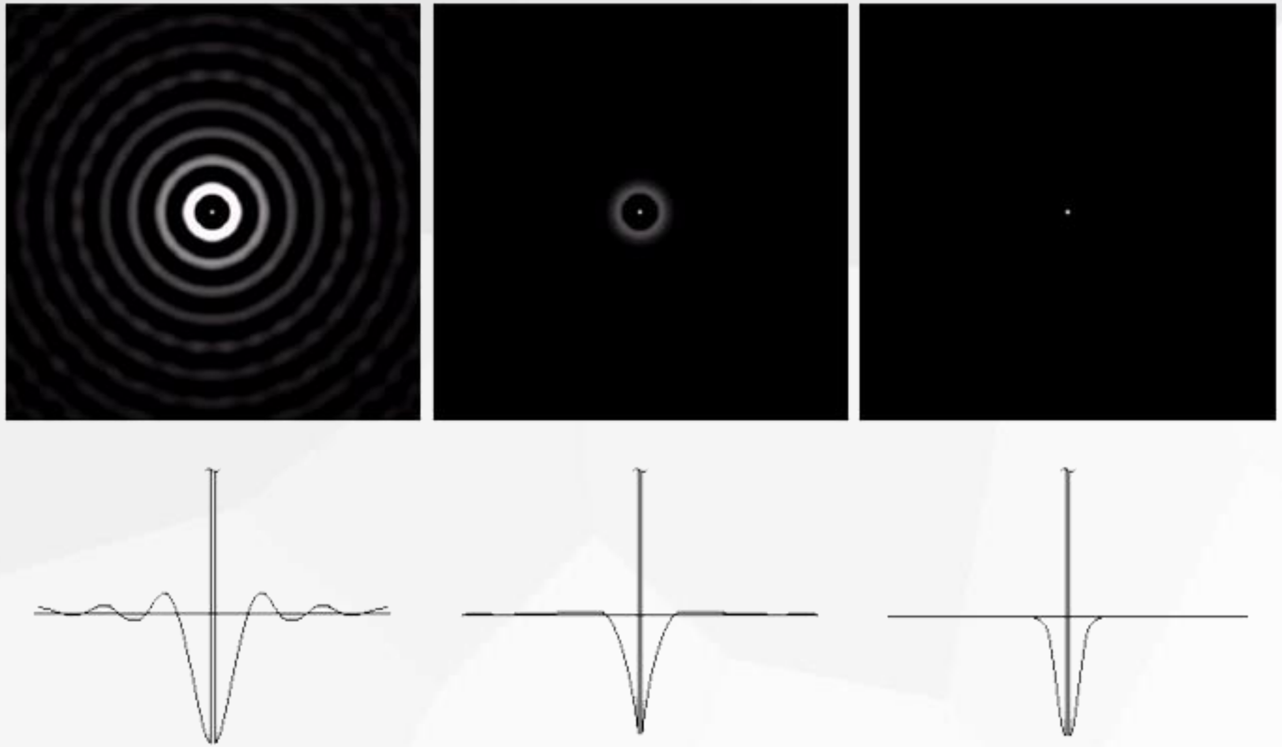




Frequency Filtering

Sharpening Filter

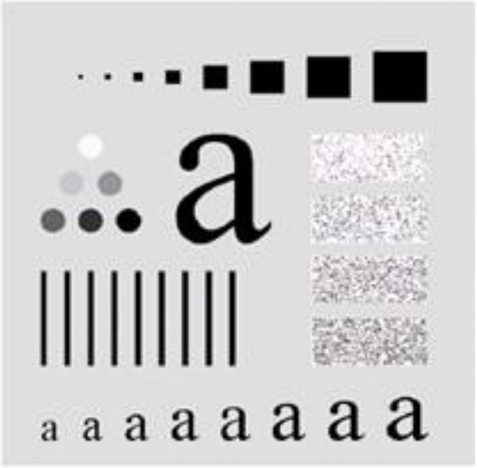
- Ideal
- Butterworh
- Gaussian



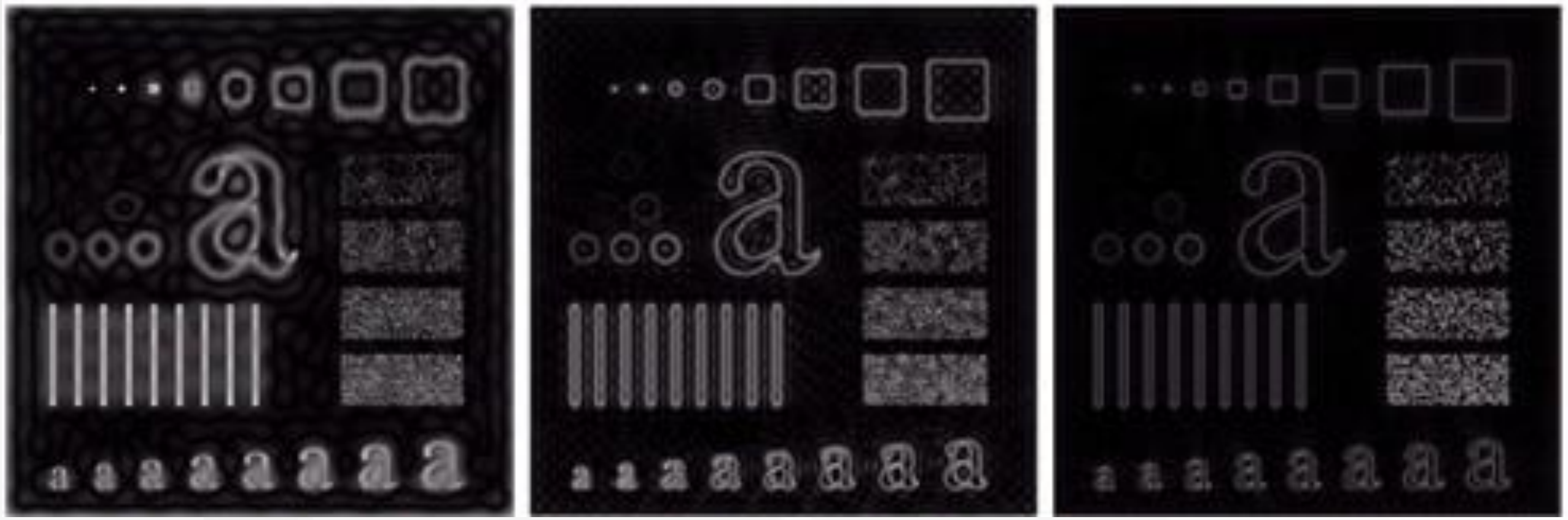
Spatial representations of typical frequency domain highpass filters, and corresponding gray-level profiles

Frequency Filtering

Sharpening Filter



Results of Ideal highpass filter

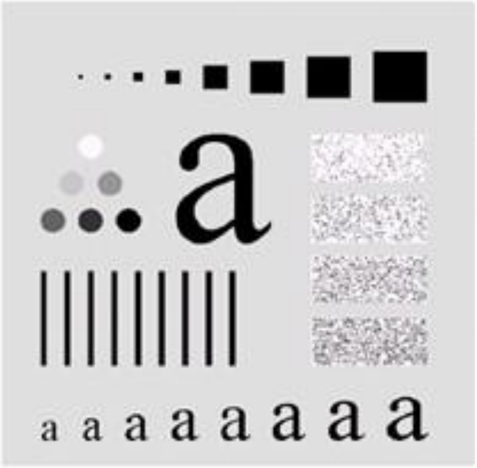


$D_0 = 15$ $D_0 = 30$ $D_0 = 80$

The same ringing properties as ILPFs can be expected.
As D_0 becomes large enough, the result of what a highpass-filtered image should look like can be obtained.

Frequency Filtering

Sharpening Filter



Results of BHPFs

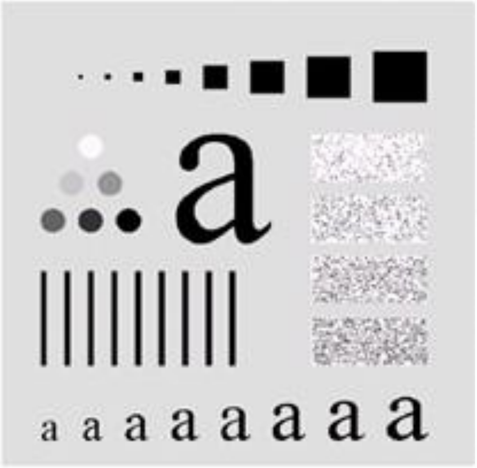
D_0 = 15 D_0 = 30 D_0 =80

These results are much smoother than those obtained with an ILPF.

Image Enhancement

Frequency Filtering

Sharpening Filter



Results of GHPFs

$D_0 = 15$ $D_0 = 30$ $D_0 = 80$

As expected, the results obtained are smoother than the other two.
It is possible to construct GHPFs as the difference of Gaussian lowpass filters which allow more control over the filter shape.

Image Enhancement

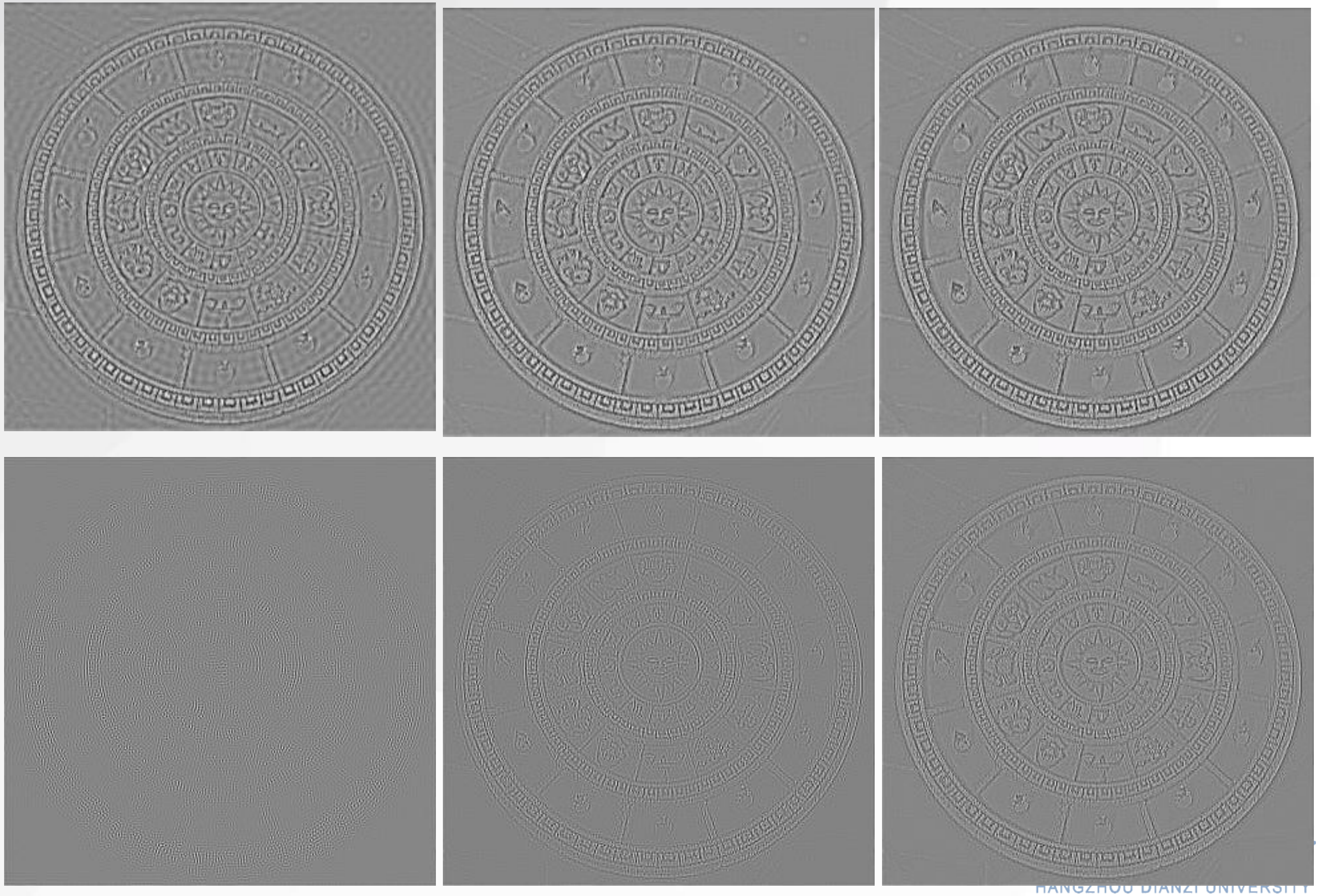
Frequency Filtering

Sharpening Filter

IHPF BHPF GHPF

D_0=40

D_0=120



Frequency Filtering

Sharpening Filter

- How to enhance images by using lowpass filter
- How to enhance images by using highpass filter

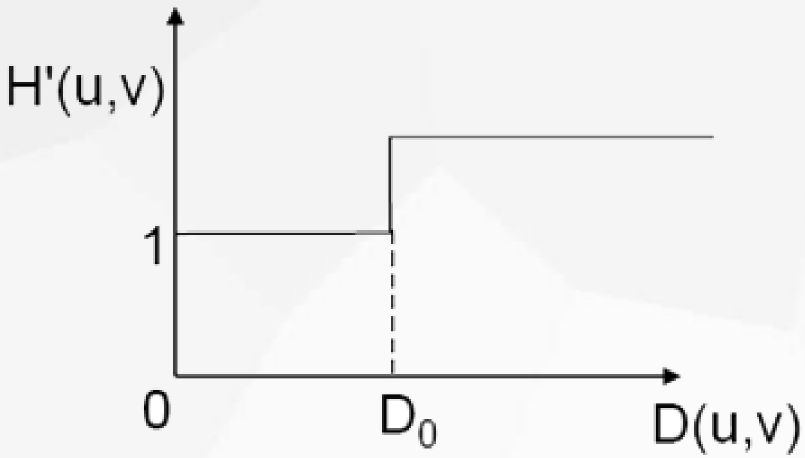
Frequency Filtering

Sharpening Filter

High Frequency Emphasis

Emphasize High Frequency.
Maintain Low frequencies and Mean.

$$H'(u,v) = K_0 + H(u,v)$$ (Typically K_0 =1)



Frequency Filtering

Sharpening Filter

High Frequency Emphasis

Using frequency domain terminology, the ***unsharp masking*** is a highpass filtered image

$$f_{\mathrm{hp}}(x,y) = f(x,y) - f_{\mathrm{lp}}(x,y)$$

and the *high-boost filtering* is

$$\begin{aligned} f_{\mathrm{hb}}(x,y) &= Af(x,y) - f_{\mathrm{lp}}(x,y) \\ &= (A-1)f(x,y) + f_{\mathrm{hp}}(x,y) \end{aligned}$$

In the frequency domain, these become

$$H_{\mathrm{hp}}(u,v) = 1 - H_{\mathrm{lp}}(u,v)$$

and

$$H_{\mathrm{hb}}(u,v) = (A-1) + H_{\mathrm{hp}}(u,v)$$

Sometimes it becomes

$$H_{\mathrm{hfe}}(u,v) = a + bH_{\mathrm{hp}}(u,v)$$

where $a \geqq 0$ (0.25 ~ 0.5) and $b > a$ (1.5 ~2.0).

Frequency Filtering

Sharpening Filter

High Frequency Emphasis

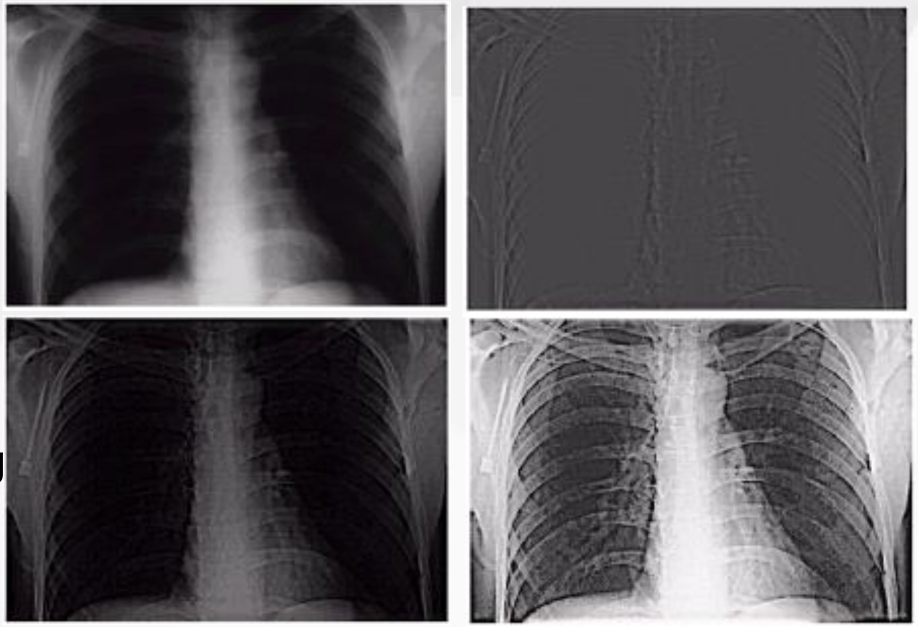
X-ray images generally tend to be slightly blurred, and the gray levels are biased toward the dark end of the gray scale.

(a) A chest X-ray image.

(410*596)

(b) BHFP. n = 2

D_0 = 5%



(c) high-frequency emphasis filtering

a = 0.5

b =0.75

(d) Histogram equalization of (c)

The final enhanced image is a little noisy, this is typical of X-ray images when their gray scale is expanded. This is an example of how spatial domain processing can be complement by using frequency domain filtering.

Frequency Filtering

- Fourier Transform and the Frequency Domain
- Smoothing Frequency Domain Filters
- Sharpening Frequency Domain Filters
- Band Pass Frequency Domain Filters
- Homomorphic Filtering

Frequency Filtering

Sharpening Filter

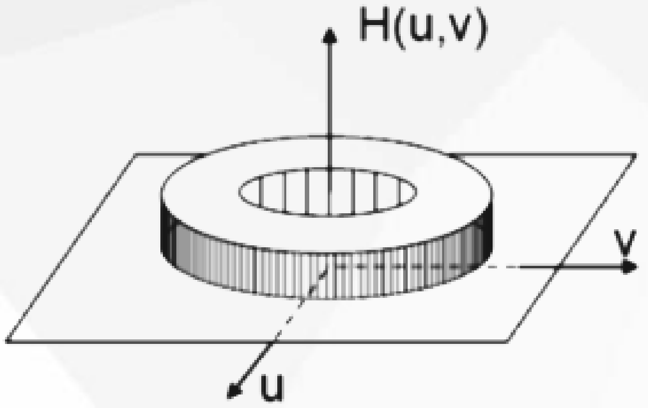
- Band Pass Filtering

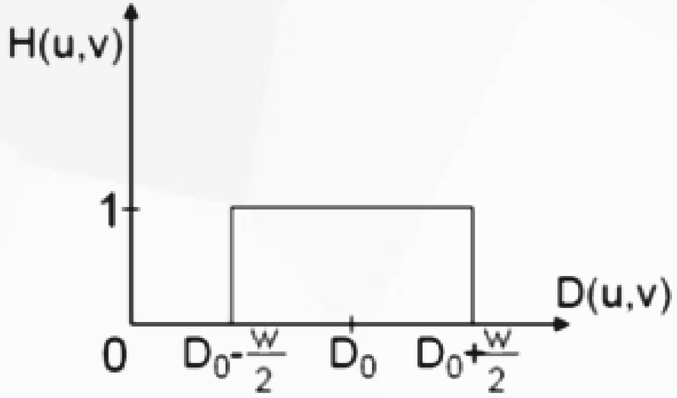
$$H(u,v)=\begin{cases}0 & D(u,v)\le D_0-\frac{W}{2}\\ 1 & D_0-\frac{W}{2}\le D(u,v)\le D_0+\frac{W}{2}\\ 0 & D(u,v)\ge D_0+\frac{W}{2}\end{cases}$$

$D(u,v)=\sqrt{u^2+v^2}$

$D_0=$ cut off frequency

$w=$ band width





Frequency Filtering

Sharpening Filter

- Local Frequency Filtering

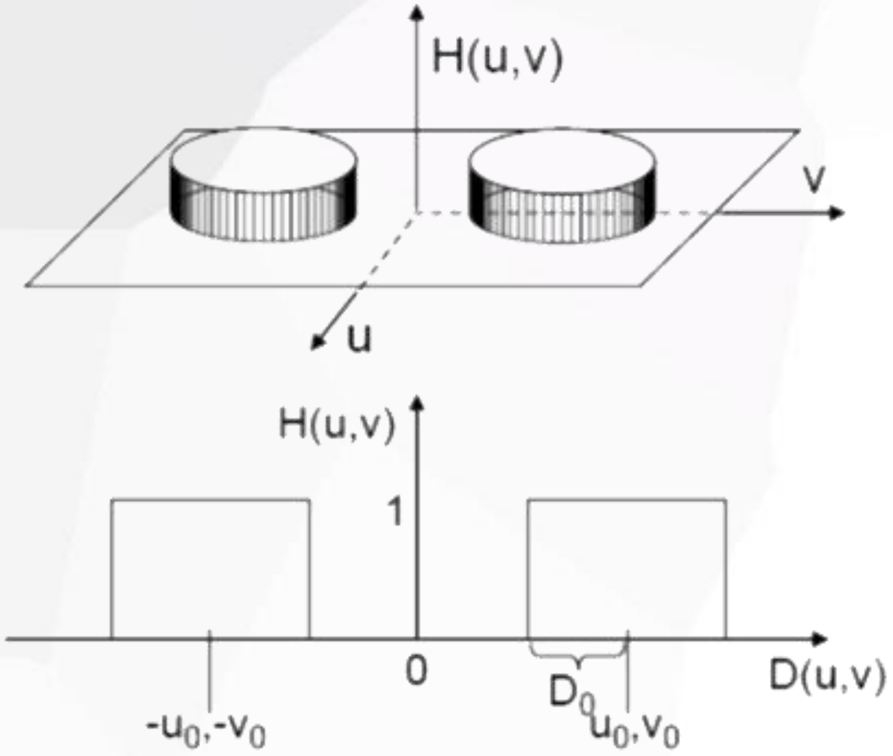
$$H(u,v) = \begin{cases} 1 & D_1(u,v) \le D_0 \;\; or \;\; D_2(u,v) \le D_0 \\ 0 & otherwise \end{cases}$$

$$D_1(u,v) = \sqrt{(u-u_0)^2 + (v-v_0)^2}$$

$$D_2(u,v) = \sqrt{(u+u_0)^2 + (v+v_0)^2}$$

D_0 = local frequency radius

u_0, v_0 = local frequency coordinates

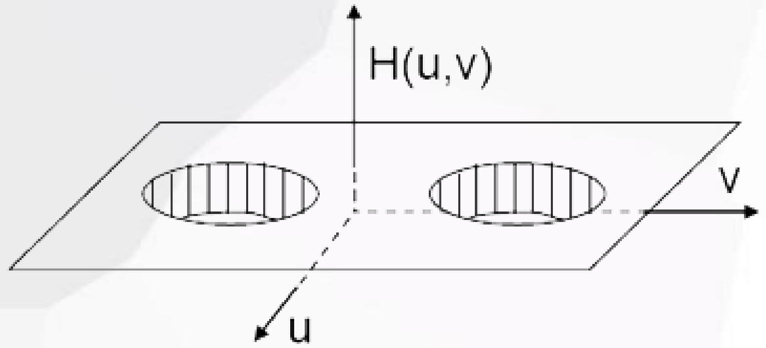


Frequency Filtering

Sharpening Filter

Band Rejection Filtering

$$H(u,v) = \begin{cases} 0 & D_1(u,v) \leq D_0 \quad or \quad D_2(u,v) \leq D_0 \\ 1 & otherwise \end{cases}$$

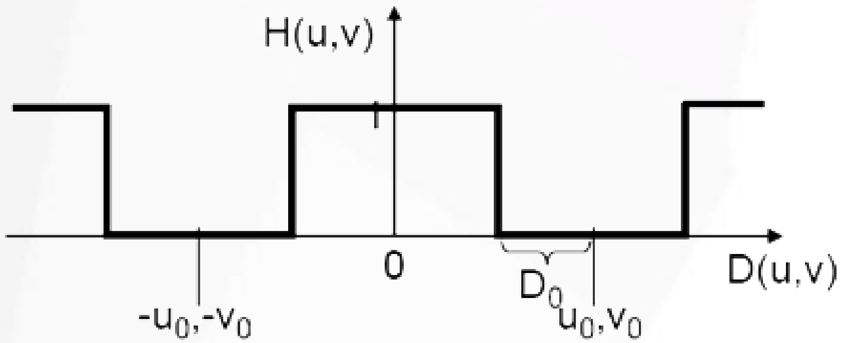


$$D_1(u,v) = \sqrt{(u - u_0)^2 + (v - v_0)^2}$$

$$D_2(u,v) = \sqrt{(u + u_0)^2 + (v + v_0)^2}$$

D_0 = local frequency radius

u_0, v_0 = local frequency coordinates



Frequency Filtering

Sharpening Filter

Band Rejection Filtering

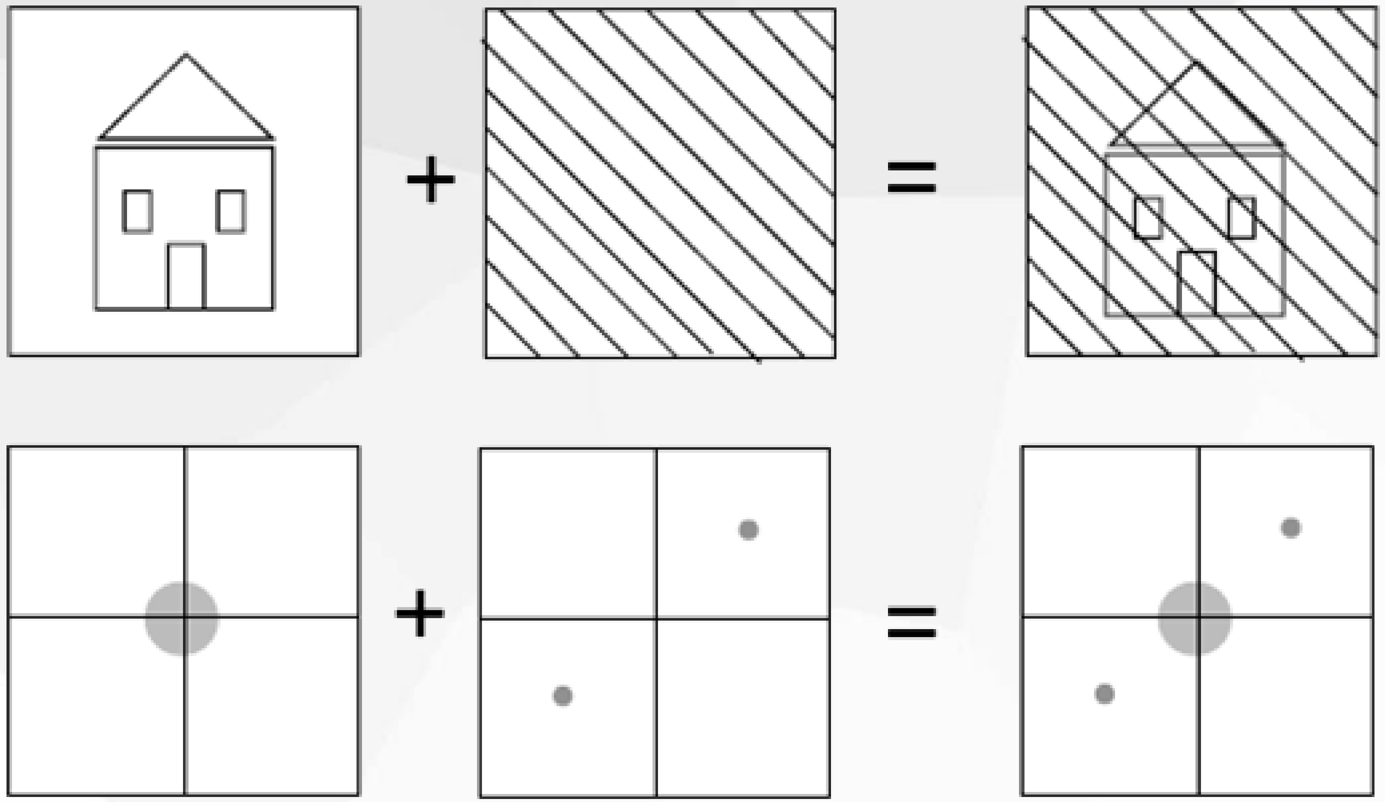


Image Enhancement

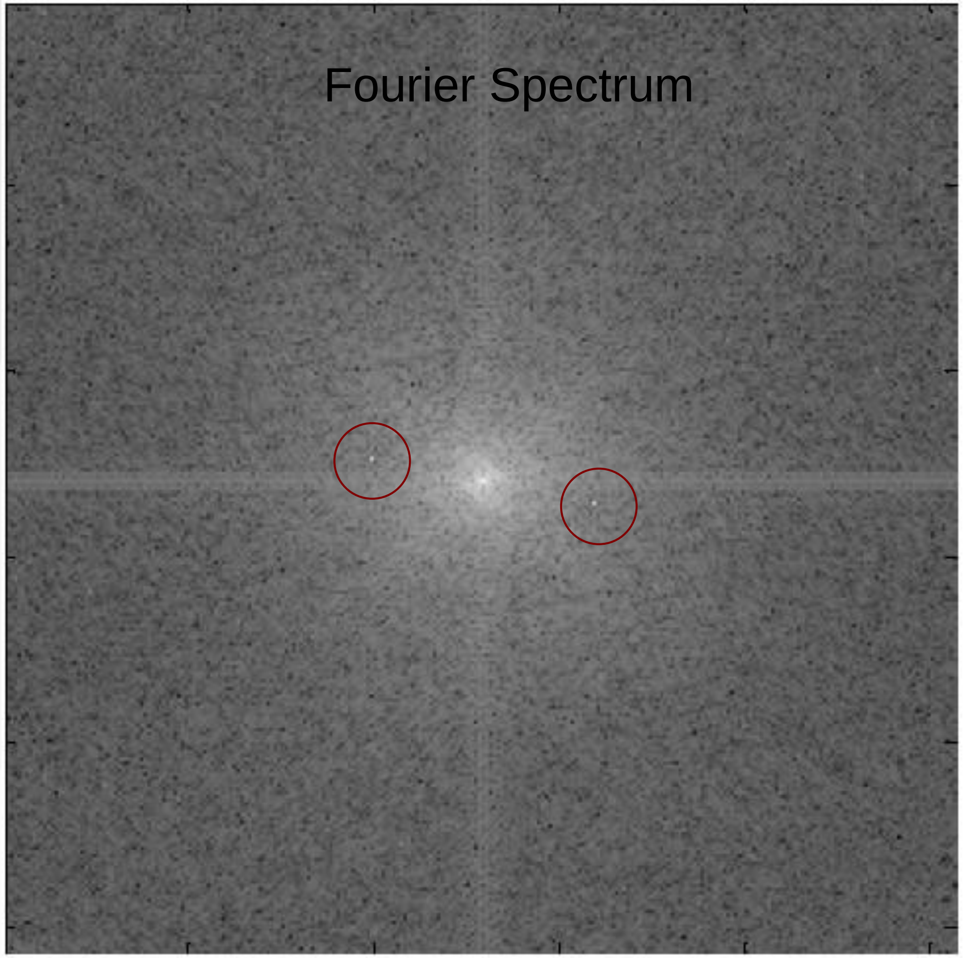
Frequency Filtering

Sharpening Filter

- Band Rejection Filtering

Original Noisy image

Band Reject Filter



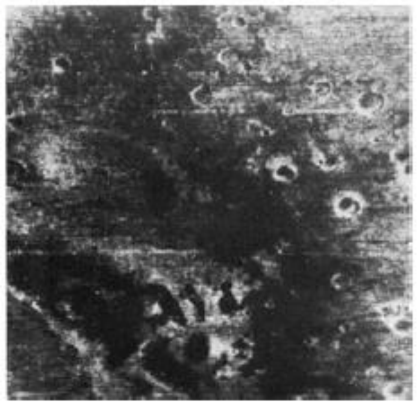
Frequency Filtering

Sharpening Filter

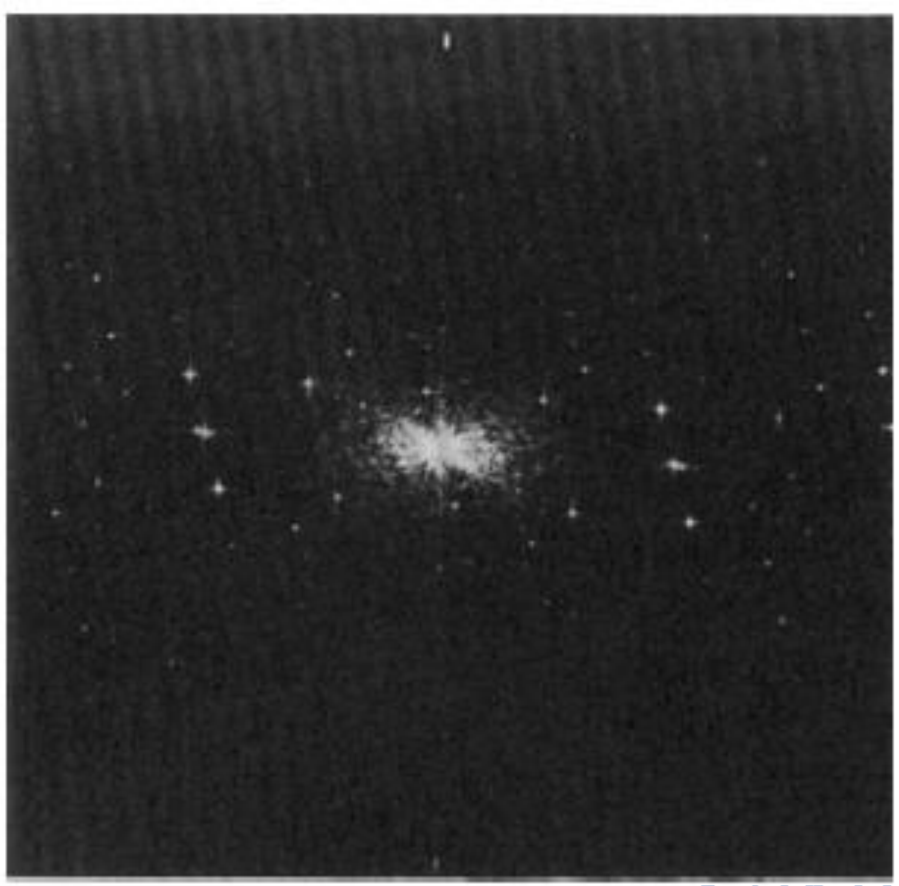
- Band Rejection Filtering

Original Noisy image

Band Reject Filter



Fourier Spectrum



Frequency Filtering

- Fourier Transform and the Frequency Domain
- Smoothing Frequency Domain Filters
- Sharpening Frequency Domain Filters
- Band Pass Frequency Domain Filters
- Homomorphic Filtering

Frequency Filtering

Homomorphic Filtering

(Multiplicative Noise Filtering)

Noise Model:

Image	$i(x, y)$
Noise	$n(x ,y)$
Brightness	$f(x, y) = i(x, y)\cdot n(x, y)$

Assumption: noise ≈ low frequencies

Goal: Clean multiplicative noise

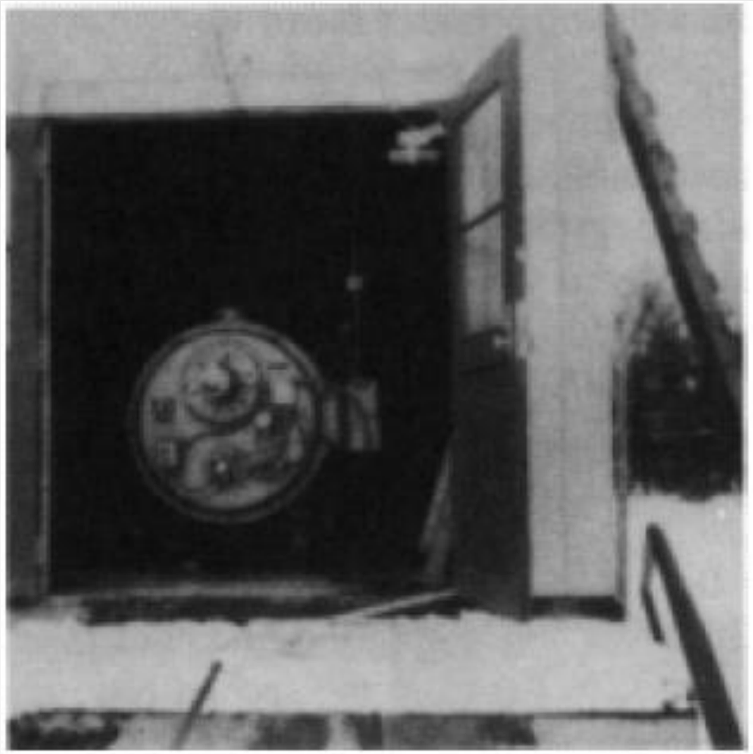
(suppress low frequencies associated with n(x,y))

However:

$$\tilde{F}(i(x,y)\cdot n(x,y)) \neq \tilde{F}(i(x,y))\cdot \tilde{F}(n(x,y))$$

Frequency Filtering

Homomorphic Filtering



Original

Frequency Filtering

Homomorphic Filtering

Reflectance Model:

Surface Reflectance	$i(x, y)$
Illumination	$n(x ,y)$
Brightness	$f(x, y) = i(x, y) \cdot n(x, y)$

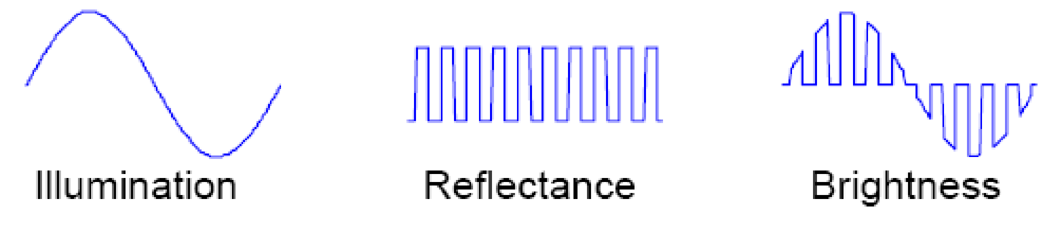
Assumptions:

Illumination changes "slowly" across scene

$\Rightarrow$ Illumination ≈ low frequencies.

Surface reflections change "sharply" across scene

$\Rightarrow$ reflectance ≈ high frequencies.



Goal: Determine i(x,y)

Frequency Filtering

Homomorphic Filtering

Perform: $z(x,y) = \log\big(f(x,y)\big) = \log\big(i(x,y)\cdot n(x,y)\big)$

$$= \log\big(i(x,y)\big) + \log\big(n(x,y)\big)$$

$$\Big\downarrow \mathscr{F}$$

$$Z(u,v) = I(u,v) + N(u,v))$$

Apply low attenuating filter $H(u, v)$

$$S(u,v) = H(u,v)\cdot Z(u,v) = H(u,v)\cdot I(u,v) + H(u,v)\cdot N(u,v)$$

$$\Big\downarrow \mathscr{F}^{-1}$$

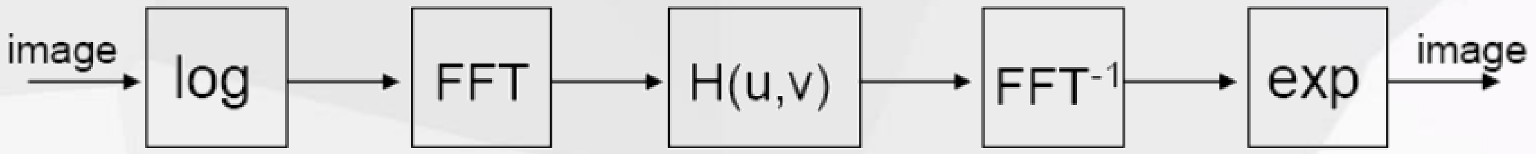
$$s(x,y) = i'(x,y) + n'(x,y)$$

$$\Big\downarrow$$

$$g(x,y) = \exp\big(s(x,y)\big) = \exp(i'(x,y))\cdot\exp(n'(x,y))$$

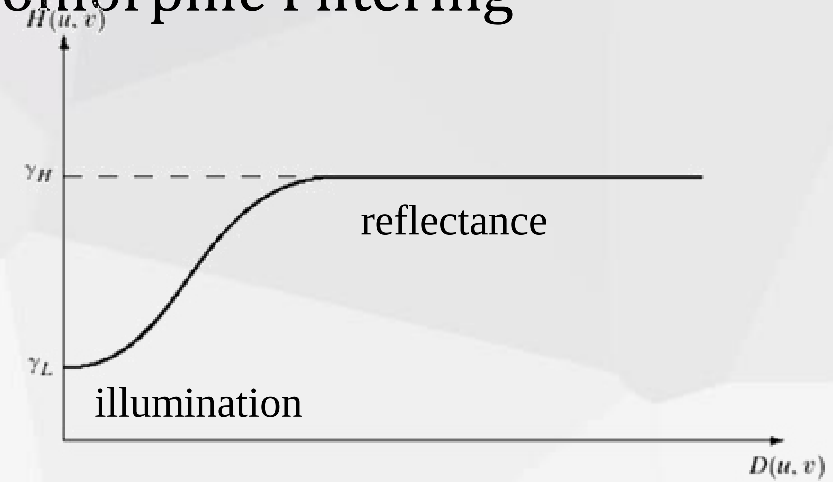
Frequency Filtering

Homomorphic Filtering



Frequency Filtering

Homomorphic Filtering



If the parameters γ_L and γ_H are chosen as γ_L <1 and γ_H >1, the filter function tends to decrease the contribution made by the low frequencies and amplify the contribution made by the high frequencies.

This curve shape can be approximated using the basic form of any highpass filters. Here we choose a slightly modified GHPF:

$$H(u,v) = (\gamma_H - \gamma_L)\left[1 - e^{-c(D^2(u,v)/D_0^2)}\right] + \gamma_L$$

The constant *c* is used to control the sharpness of the slope of the filter function as it transitions between γ_L and γ_H

This type of filter is similar to the *high-frequency emphasis* filter.

Frequency Filtering

Homomorphic Filtering

Original

Filtered

$\gamma_L = 0.5, \gamma_H = 2.0$

A reduction of dynamic range in the brightness, together with an in-crease in contrast, brought out the details of objects inside the shelter and balanced the gray levels of the outside wall. (The enhanced image also is sharper.)

Frequency Filtering

Homomorphic Filtering

Original

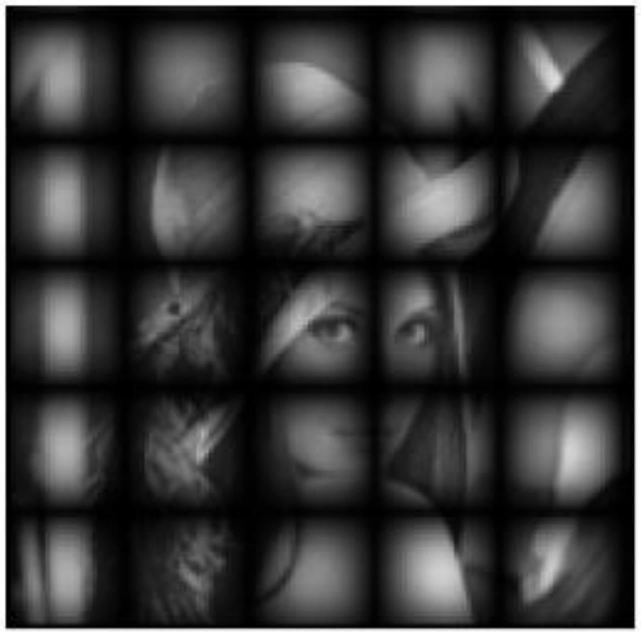
Histogram Equalized

Filtered

Image Enhancement

Frequency Filtering

Homomorphic Filtering



Original

Filtered

Thanks