Machine Vision:
Morphological Image Processing

Wu Wei

ww@hdu.edu.cn

Preview

History

Mathematical Morphology

- ---- was born in 1964 from the collaborative work of Georges Matheron and Jean Serra, at the Ecole des Mines de Paris, France.
- ---- is the analysis of signals in terms of shape. This simply means that morphology works by changing the shape of objects contained within the signal.

Preview

Morphology ’s advantages:

- Preserves edge information
- Works by using shape-based processing
- Computationally efficient
- …

Morphology ’s applications:

- Image enhancement
- Image restoration
- Edge detection
- Texture analysis
- Noise reduction

Outline

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

Preliminaries

Basic concept

- Let A be a set in Z^2. If $a = (a_1, a_2)$ is an element of A, then we write

$$a \in A$$

- if a is not an element of A, we write

$$a \notin A$$

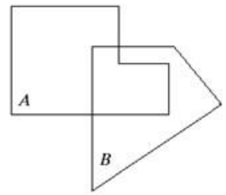
- An empty or *null* set is denoted by the symbol Ø.

A set is specified by the contents of two braces $\{\cdot\}$. The elements of the sets with which we are concerned in this lecture are the *coordinates* of pixels representing the ROI in an image.

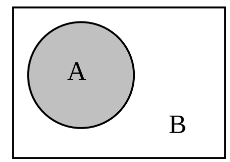
$$C = \{w \mid w = -d, d \in D\}$$

Preliminaries

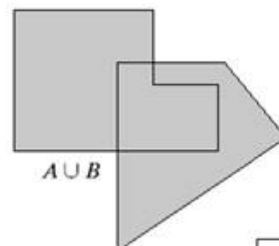
Basic concept



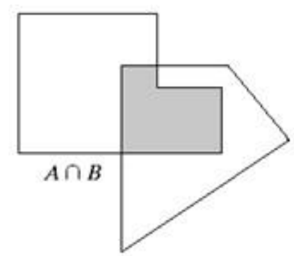
$A \subseteq B$, A is a subset of B



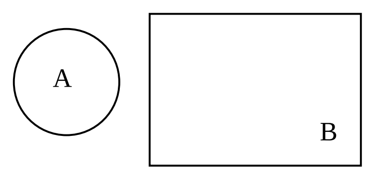
$C = A \cup B$, C is the *union* of A and B



$D = A \cap B$, D is the *intersection* of A and B



$A \cap B = \varnothing$, A and B are *disjoint*

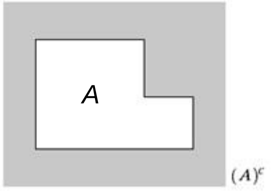


Preliminaries

Basic concept

The *complement* of a set A is the set of elements not contained in A:

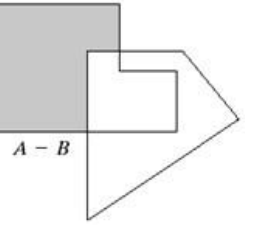
$$A^c = \{w \mid w \notin A\}$$



So if $A \cap B = \varnothing$, then $A \subseteq B^c$

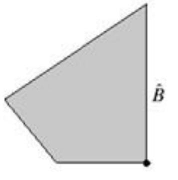
The difference of two sets is defined as

$$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$



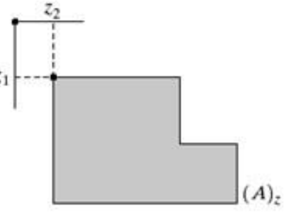
The *reflection* of a set B is defined as

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$



The *translation* of a set A by a point z is defined as

$$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$$

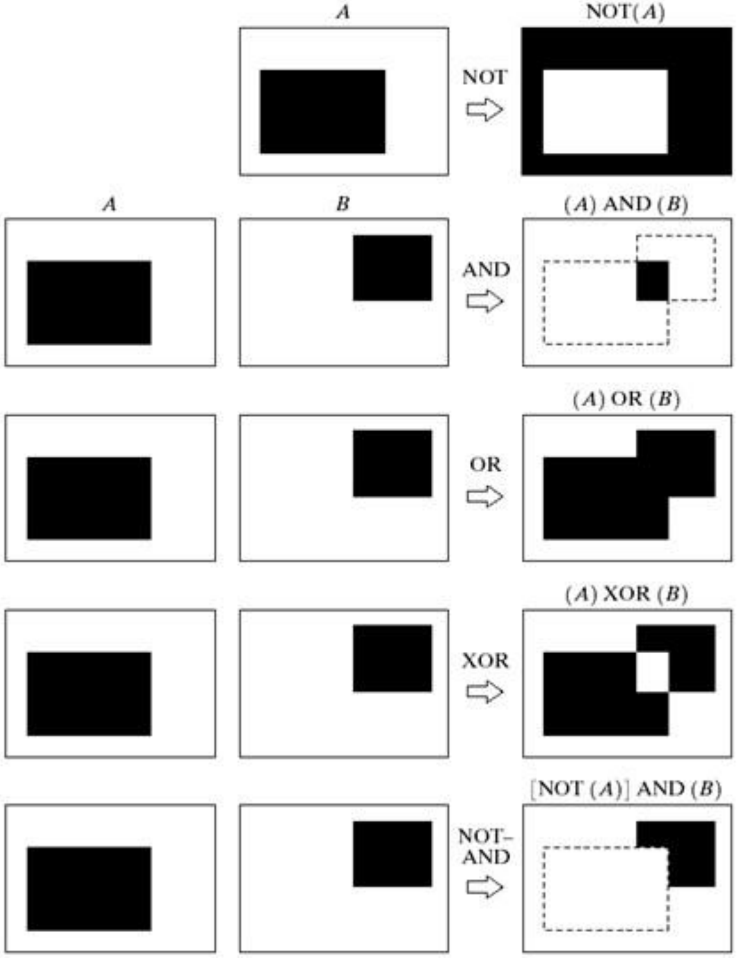


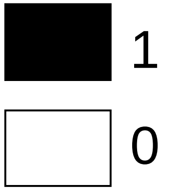
Logic Operations Involving Binary Images

The three basic logical operations

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Logic Operations Involving Binary Images





Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example

Outline

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

Dilation and Erosion

Dilation and *Erosion* are two primitive operations.
They are fundamental to morphological processing.

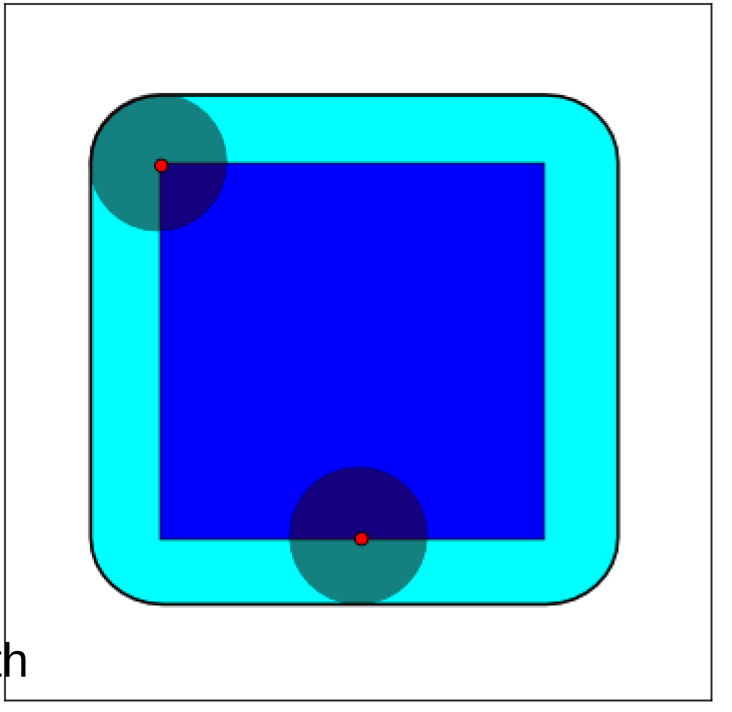
Dilation

The *dilation* of A by B, denoted $A \oplus B$, is defined as

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$$

This equation may be rewritten as

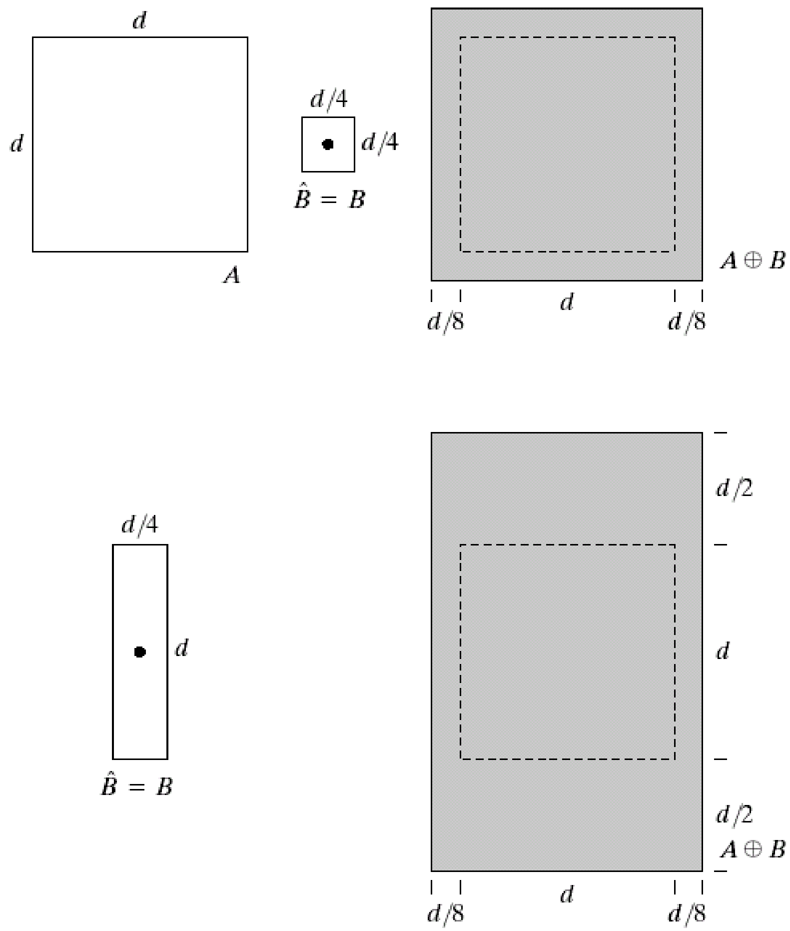
$$A \oplus B = \{z \mid [(\hat{B})_z \cap A] \subseteq A\}$$



Dilation is swelling or enlarging operation. The enlarging is achieved by adding “layers” on an object.

The structuring element is applied to the image by translating it across the image with its center on every point of the image.

Dilation



(a) Set *A*.

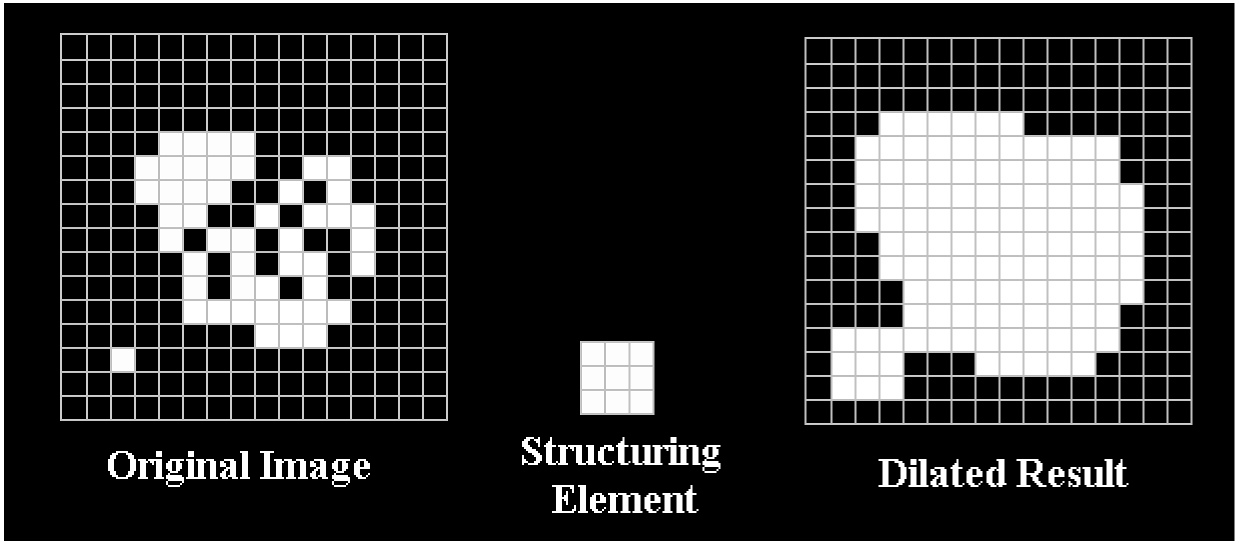
(b) Square structuring element *B* (dot is the center).

(c) Dilation of *A* by *B*, shown shaded.

(d) Elongated structuring element.

(e) Dilation of *A* using this element

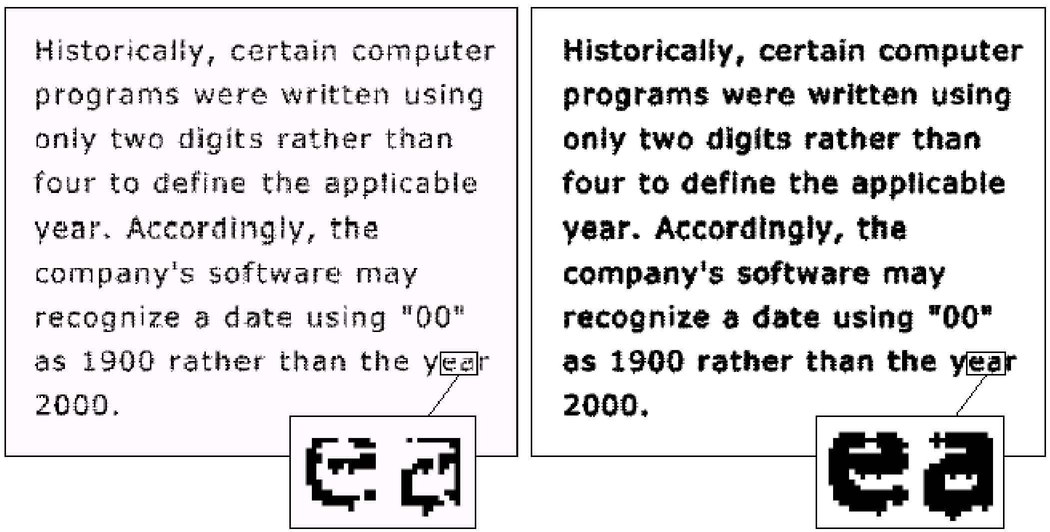
Dilation



Holes and cracks in an object are filled in, and layers are added to objects.

One of the simplest applications of dilation is for bridging gaps.

Dilation



a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

The maximum length of the breaks is known to be two pixels

0	1	0
1	1	1
0	1	0

Dilation

- Example

Original (178x178) | dilation with 3x3 structuring element | dilation with 7x7 structuring element

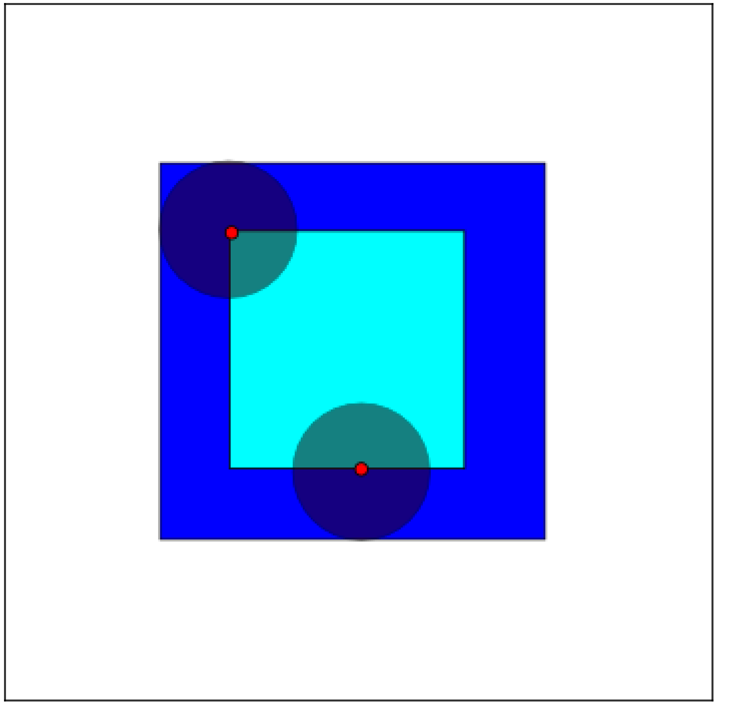
Effects

- Expands the size of 1-valued objects
- Smoothes object boundaries
- Closes holes and gaps

Erosion

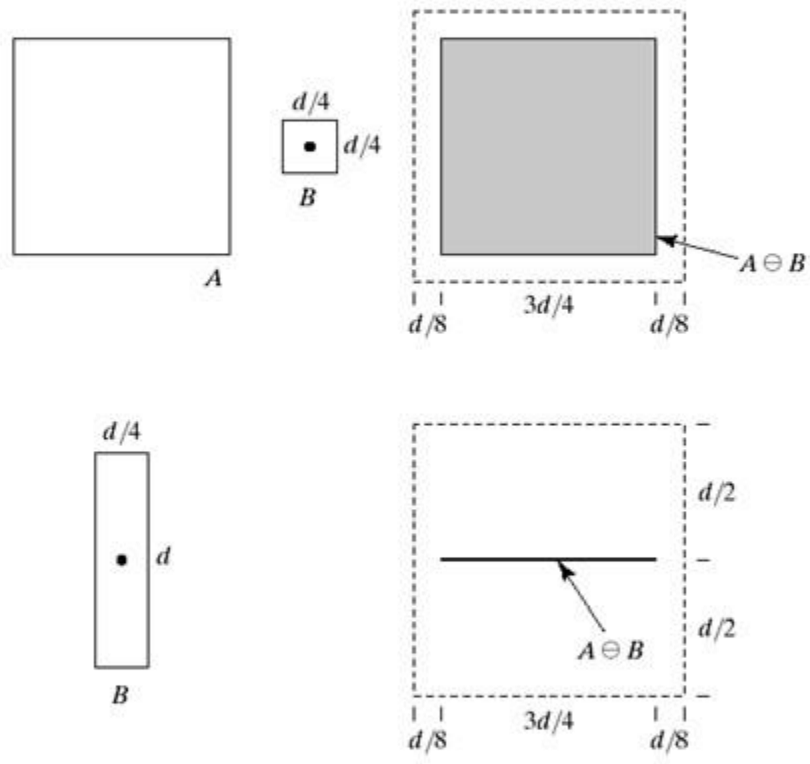
The *erosion* of A by B, denoted $A \ominus B$, is defined as

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$



Erosion is often thought of as shrinking or reducing operation. The reducing is achieved by peeling “layers” off an object.

Erosion



(a) Set A.

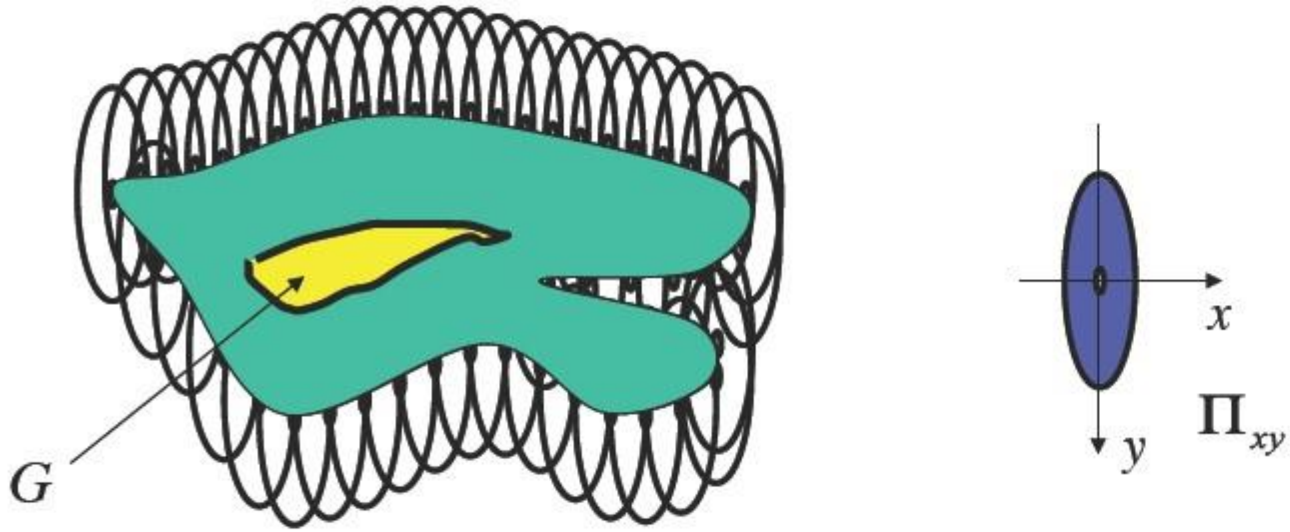
(b) Square structuring element B (dot is the center).

(c) Erosion of A by B, shown shaded.

(d) Elongated structuring element.

(e) Erosion of A using this element

Erosion



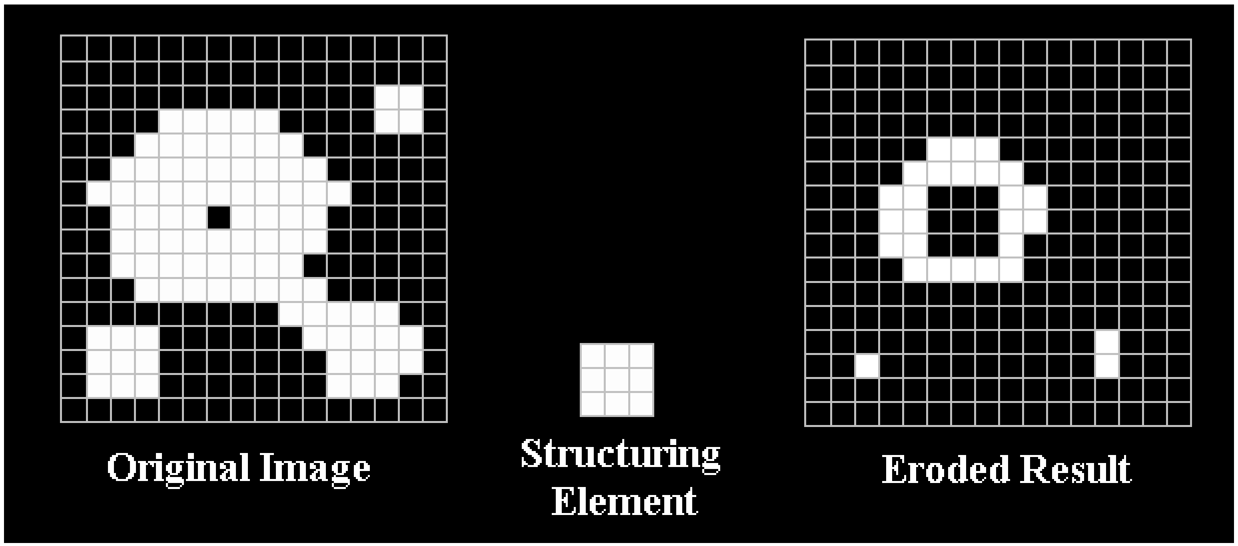
Erosion

- Example

$$\underset{7\times 7}{\begin{matrix}1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\end{matrix}} \ominus \underset{3\times 3}{\begin{matrix}1&1&1\\1&1&1\\1&1&1\end{matrix}} = \underset{5\times 5}{\begin{matrix}1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\end{matrix}}$$

?

Erosion



Small objects are removed, layers are peeled off of lager objects, and connections between thinly connect objects are broken. Holes in an object are enlarged.

One of the simplest applications of erosion is for eliminating irrelevant detail (in terms of size) from a binary image.

Erosion

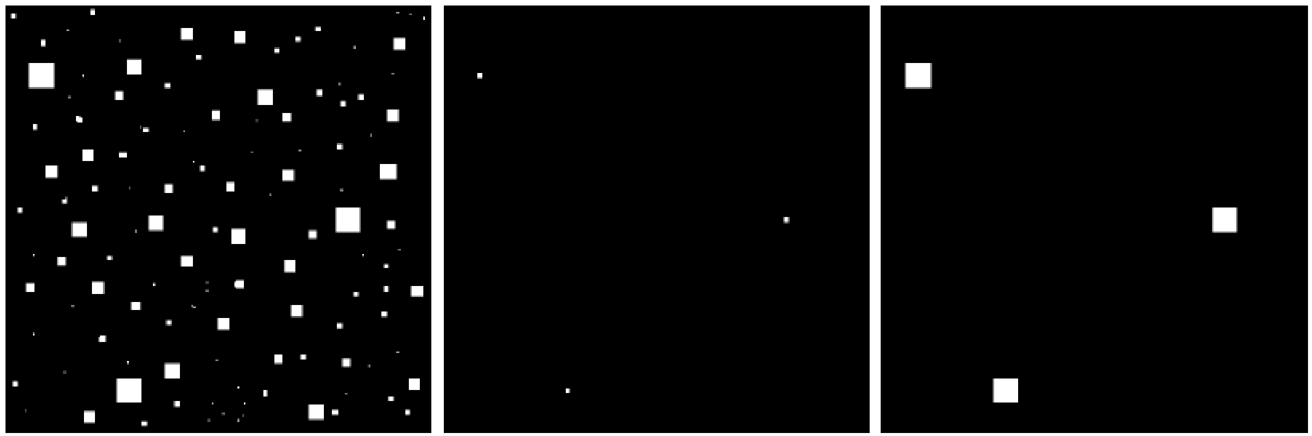
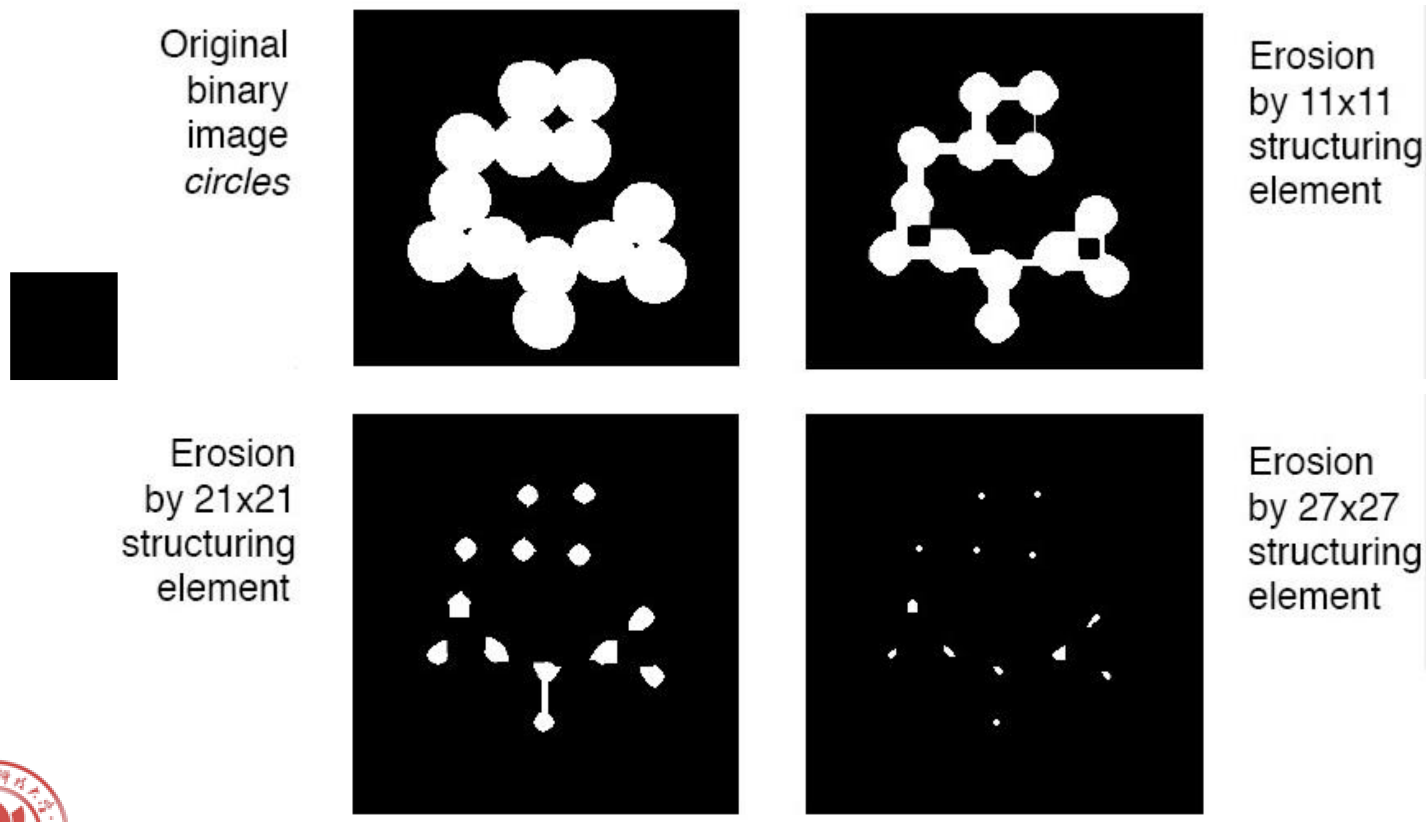


FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

In this example, objects of interest are represented by white pixels, rather than by black pixels as in the previous example. Both repre-sentations are used in practice.

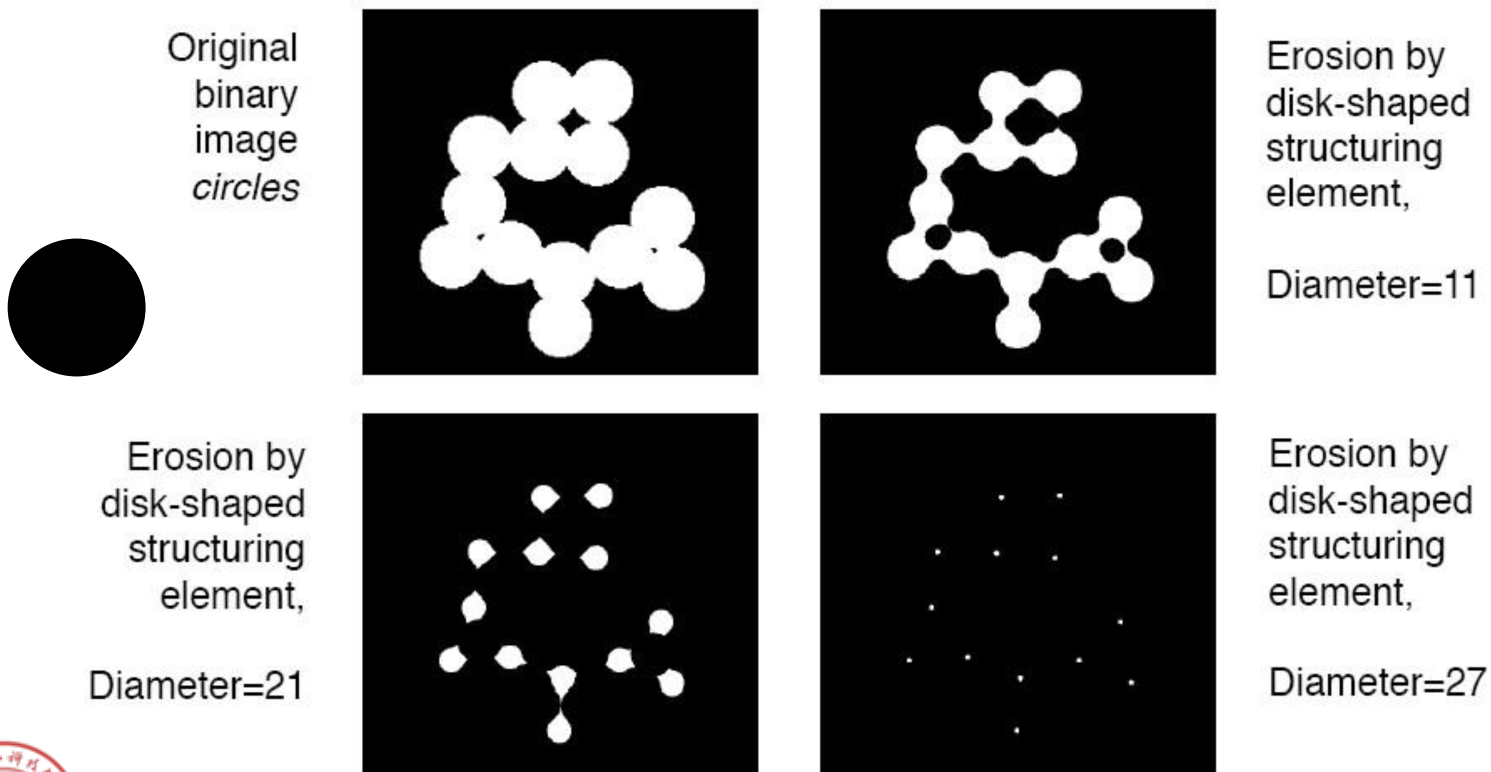
Erosion

Example: blob separation/detection by erosion using square structuring element



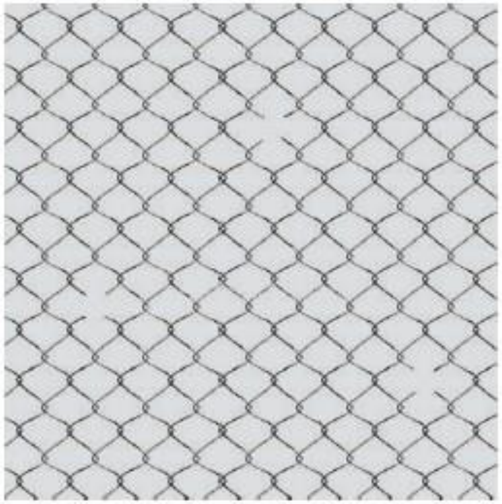
Erosion

Example: blob separation/detection by erosion using disk-shaped structuring element

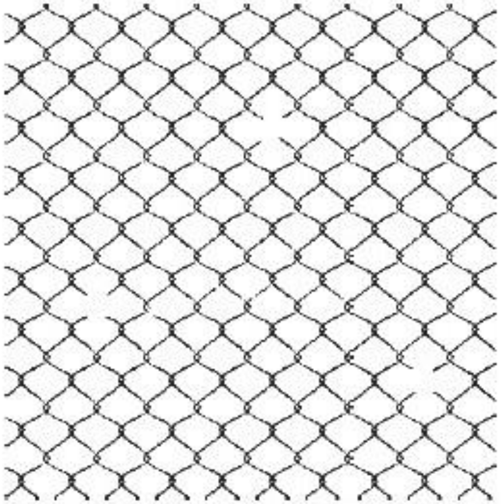


Erosion

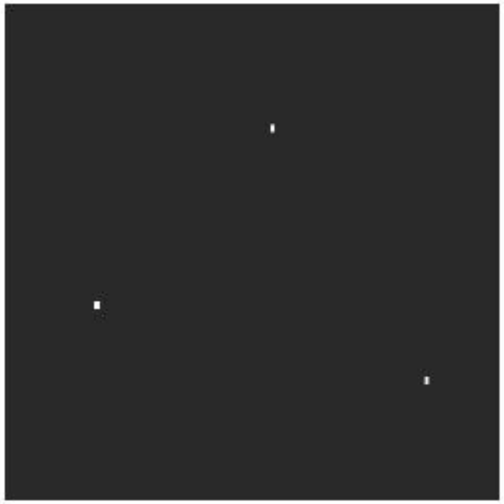
- Example: chain link fence hole detection



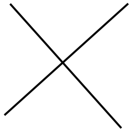
Original grayscale image *Fence*



Fence thresholded using Otsu's method



Erosion with 31x31 "cross" structuring element



Some Properties of Dilation and Erosion

Translation invariant:

$$(A \oplus B)_x = A_x \oplus B$$

Order invariant:

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

Increasing operator:

$$A \subseteq B \Rightarrow A \oplus C \subseteq B \oplus C$$

Scale invariant:

$$rA \oplus rB = r(A \oplus B), \quad r \text{ is a scale factor}$$

Same with erosion.

Some Properties of Dilation and Erosion

Dilation and erosion are duals of each other with respect to set complementation and reflection. That is,

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$ definition of erosion

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c = \varnothing\}^c$$ definition of set

$$(A \ominus B)^c = \{z \mid (B)_z \cap A^c \neq \varnothing\}$$ definition of set complement

$$= A^c \oplus \hat{B}$$ definition of dilation

Outlines

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

Extensions to Gray-Scale Images

Extending to gray-scale images the basic operations of dilation, erosion, opening, and closing.

Developing several basic gray-scale morphological algorithms:

- Boundary extraction via a morphological gradient operation
- Region partitioning based on texture content
- Algorithms for smoothing and sharpening

$f(x, y)$ is the input image, $b(x, y)$ is a structuring element, itself a subimage.

f and b are functions, rather than sets as in the case in binary morphology.

Dilation

Gray-scale dilation of *f* by *b* is defined as

$$(f \oplus b)(s,t) =$$
$$\max\{f(s-x,t-y)+b(x,y) \mid (s-x),(t-y)\in D_f;(x,y)\in D_b\}$$

where D_f and D_b are the domains of *f* and *b*. *f* and *b* overlap.

1	2	2	3	0	3	6	5	2	2	1	0

Input signal (*A*).

1	3	2

Structuring element (*B*).

Although easily to visualize, unlike the binary case, *f*, rather than the structuring element *b*, is shifted. Conceptually there is no difference.

The 2-D Dilation

9	6	3	5	6
6	6	4	0	0
8	6	8	2	1
6	6	3	6	6
6	7	6	7	6

Image (or 2D signal)

Flat Structuring Element (FSE)

0	1	0
1	1	1
0	1	0

N_4 structuring element

1	1	1
1	1	1
1	1	1

N_8 structuring element

shaded indicating the origin

$$(f \oplus b)(s,t) =$$
$$\max\{ f(s-x, t-y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b \}$$

The 2-D Dilation

9	6	3	5	6
6	6	4	0	0
8	6	8	2	1
6	6	3	6	6
6	7	6	7	6

9	6	3	5	6
6	6	4	0	0
8	6	8	2	1
6	6	3	6	6
6	7	6	7	6

9	6	3	5	6
6	6	4	0	0
8	6	8	2	1
6	6	3	6	6
6	7	6	7	6

10				

10	10			

10	10	7	7	7
10	7	7	6	7
9	9	9	9	7
9	8	9	8	7
8	8	8	8	8

dilation result

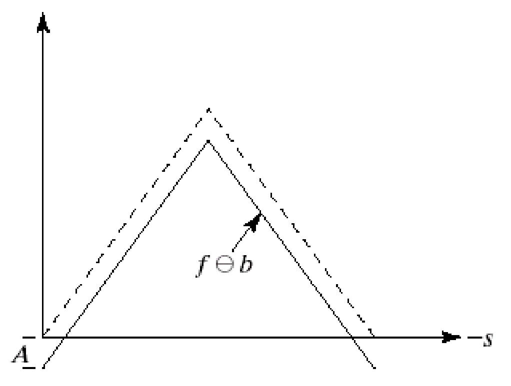
Erosion

Gray-scale erosion of *f* by *b* is defined as

$$(f \ominus b)(s,t) =$$
$$\min\{f(s+x,t+y)-b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

where D_f and D_b are the domains of *f* and *b*. *b* included *in f* .

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



Gray-Scale Dilation and Erosion

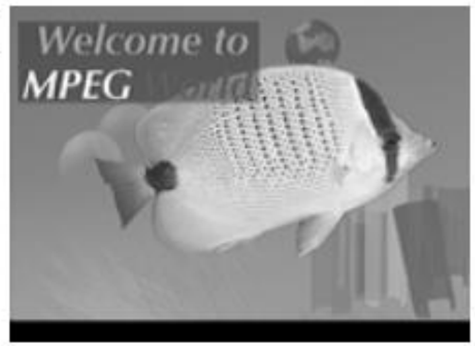
Gray-scale dilation and erosion are duals with respect to function complementation and reflection:

$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

where $f^c = -f(x,y)$ and $\hat{b} = b(-x,-y)$.

Examples of Gray-Scale Dilation and Erosion

Original



Dilation

Erosion

Gray-Scale Dilation and Erosion

The general effect of performing dilation on a gray-scale image is twofold:

- If all values of the structuring element are positive, the output image tends to be brighter than the input.
- Dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element.

The general effect of performing erosion on a gray-scale image is twofold:

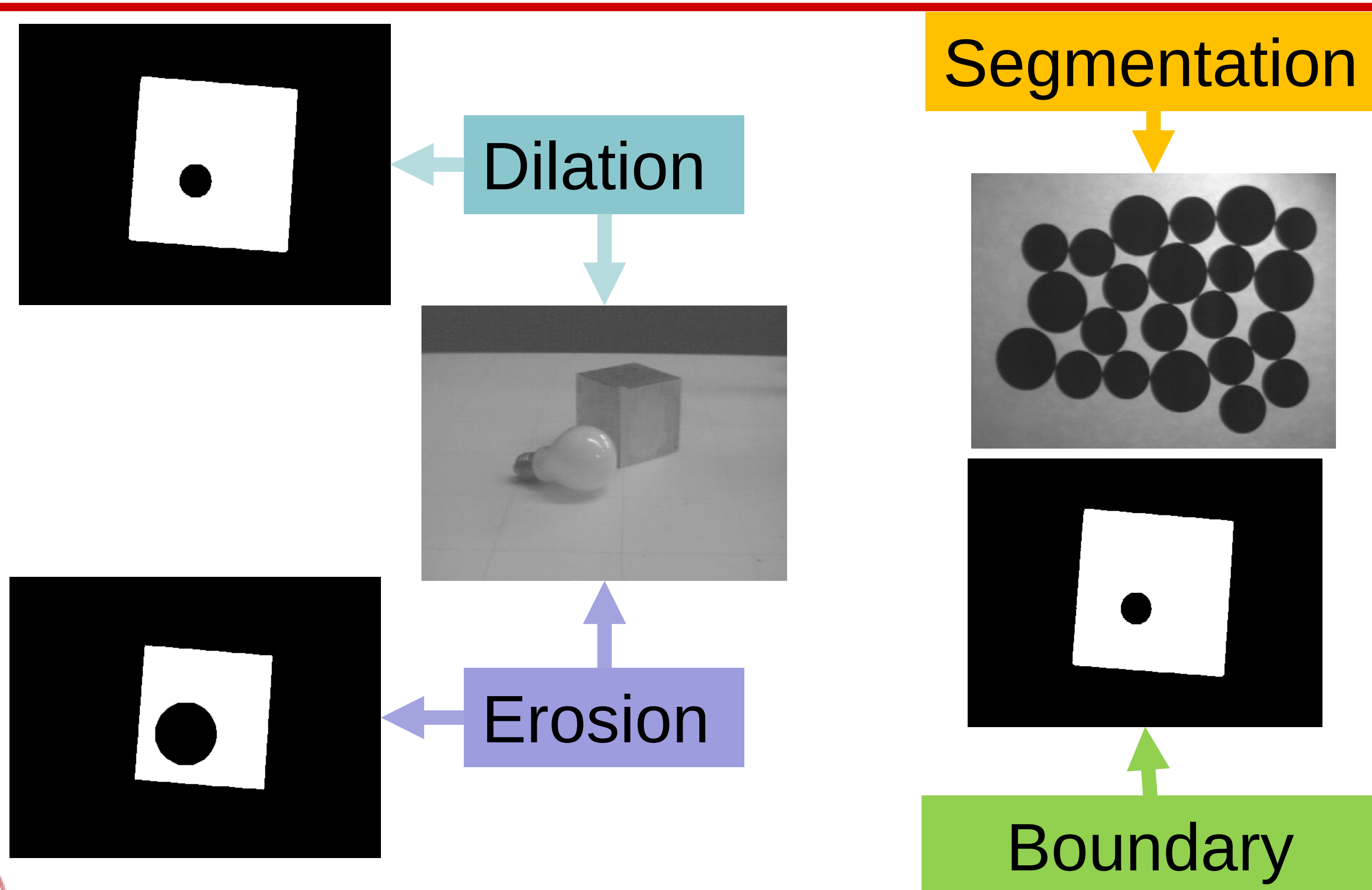
- If all values of the structuring element are positive, the output image tends to be darker than the input image.
- The effect of bright details that are smaller than the structuring element is reduced, depending on the gray-level values surrounding the bright details and the shape of the structuring element.

Examples of Gray-Scale Dilation and Erosion

dilation

erosion

Excises



Outlines

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

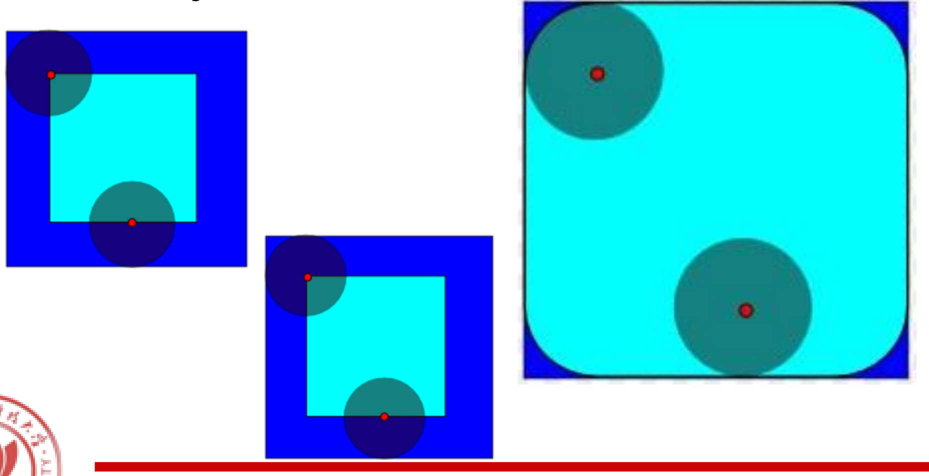
Opening

Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

The opening of set *A* by structuring element *B* is defined as

$$A \circ B = (A \ominus B) \oplus B,$$

Thus, the opening of *A* by *B* is the erosion of *A* by *B*, followed by a dilation of the result by *B*.



The opening of the dark-blue square by a disk, resulting in the light-blue square with round corners.

Opening

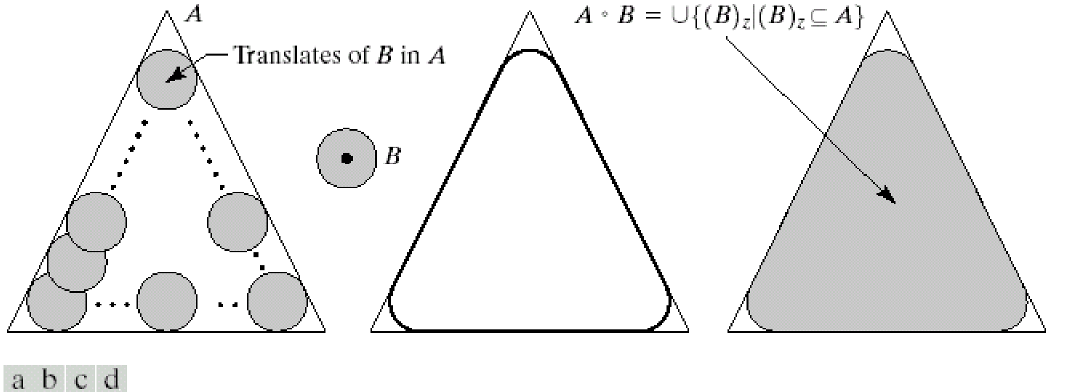


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

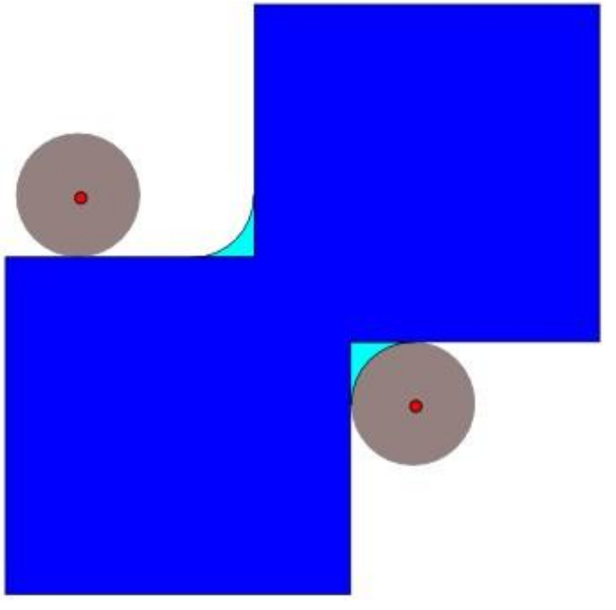
Closing

Closing also tends to smooth sections of contours, fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps.

The closing of set *A* by structuring element *B* is defined as

$$A \bullet B = (A \oplus B) \ominus B,$$

The closing of *A* by *B* is the dilation of *A* by *B*, followed by a erosion of the result by *B*.



The closing of the dark-blue shape (union of two squares) by a disk, resulting in the union of the dark-blue shape and the light-blue areas.

Closing

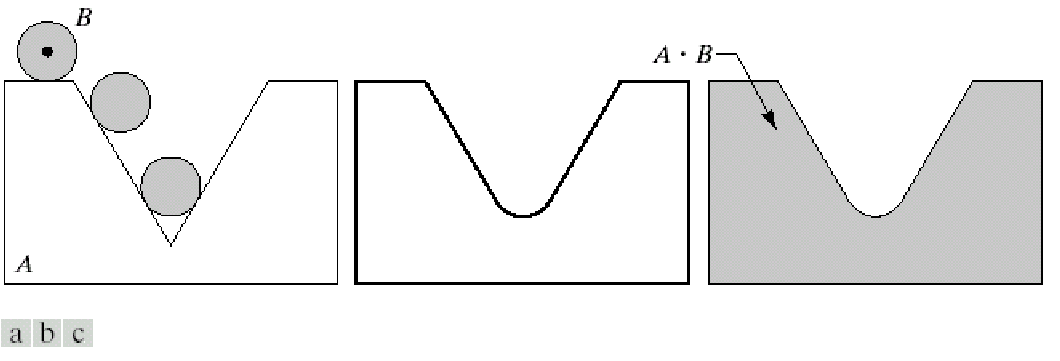
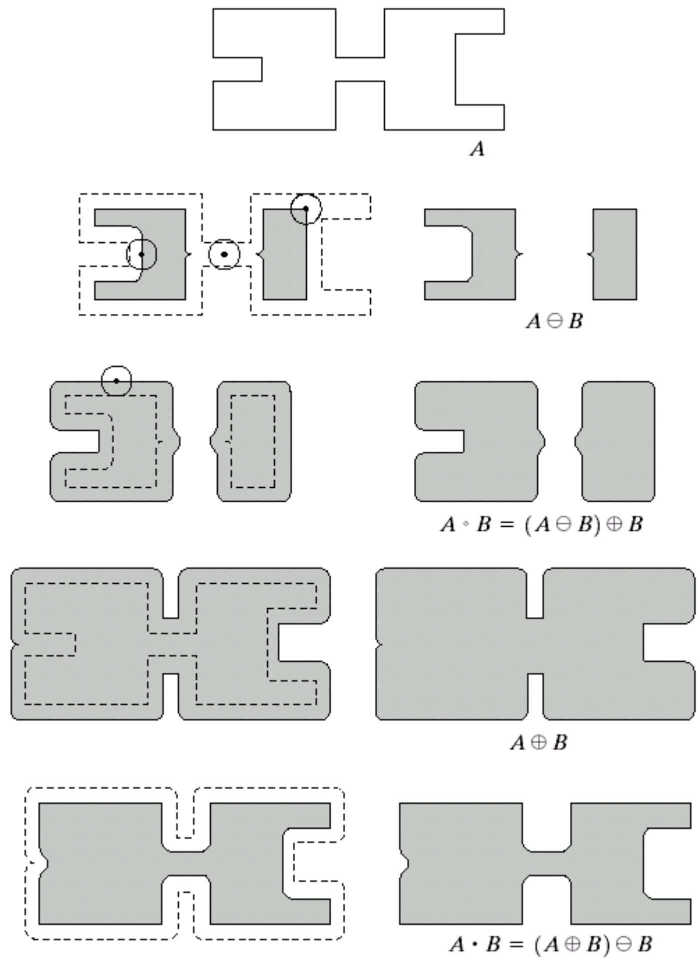


FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Opening and Closing Operation

a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



The bridge and protruding elements were eliminated. The outward pointing corners were rounded, whereas inward pointing corners were not affected.

The inward pointing corners were rounded, whereas outward pointing corners were not affected. The leftmost intrusion on the boundary of *A* was reduced in size significantly.

Properties of Opening and Closing

$$(A \bullet B)^c = A^c \circ \hat{B}$$

The opening operation satisfies the following properties:

(i) $A \circ B$ is a subset (subimage) of A.

(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.

(iii) $(A \circ B) \circ B = A \circ B$ (idempotent)

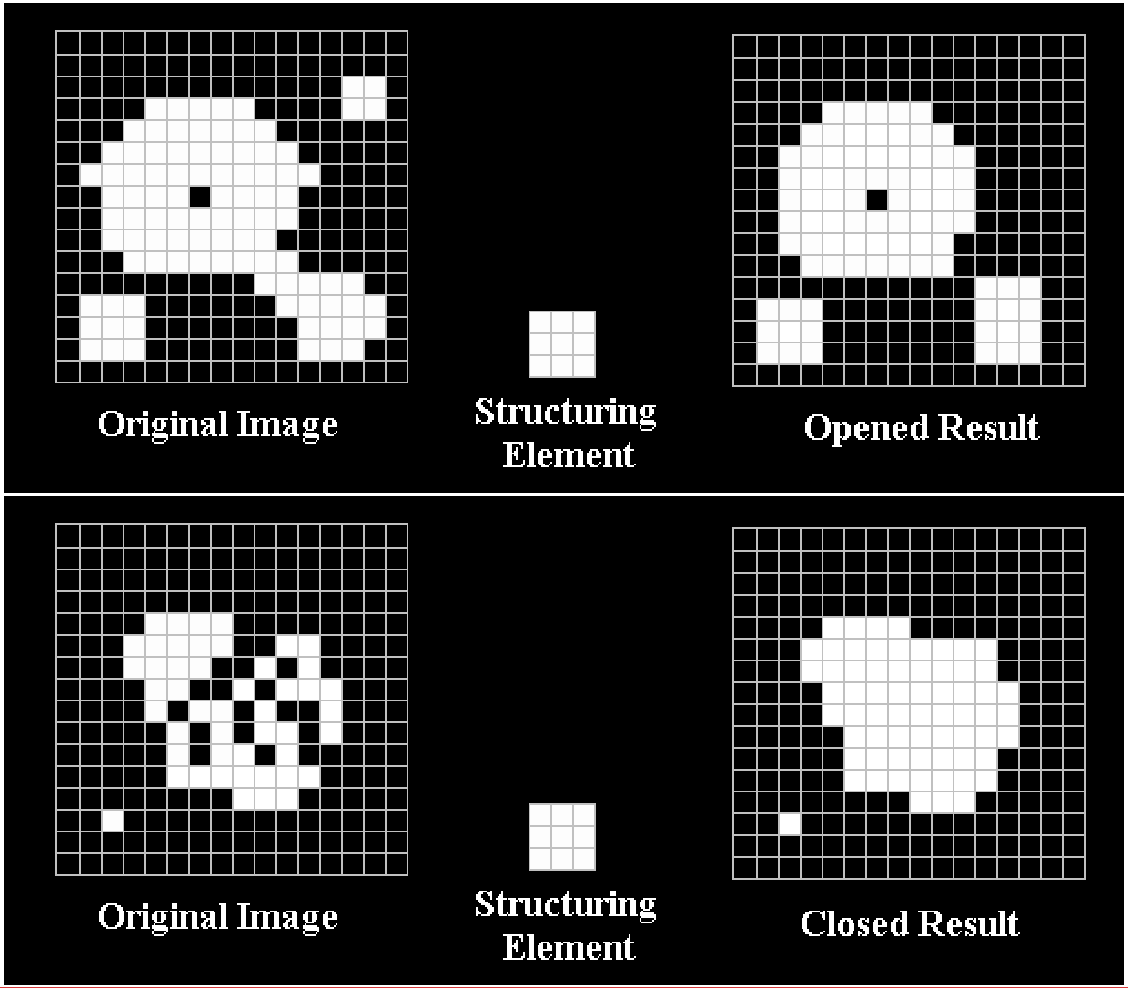
Similarly, The closing operation satisfies the following properties:

(i) A is a subset (subimage) of $A \bullet B$.

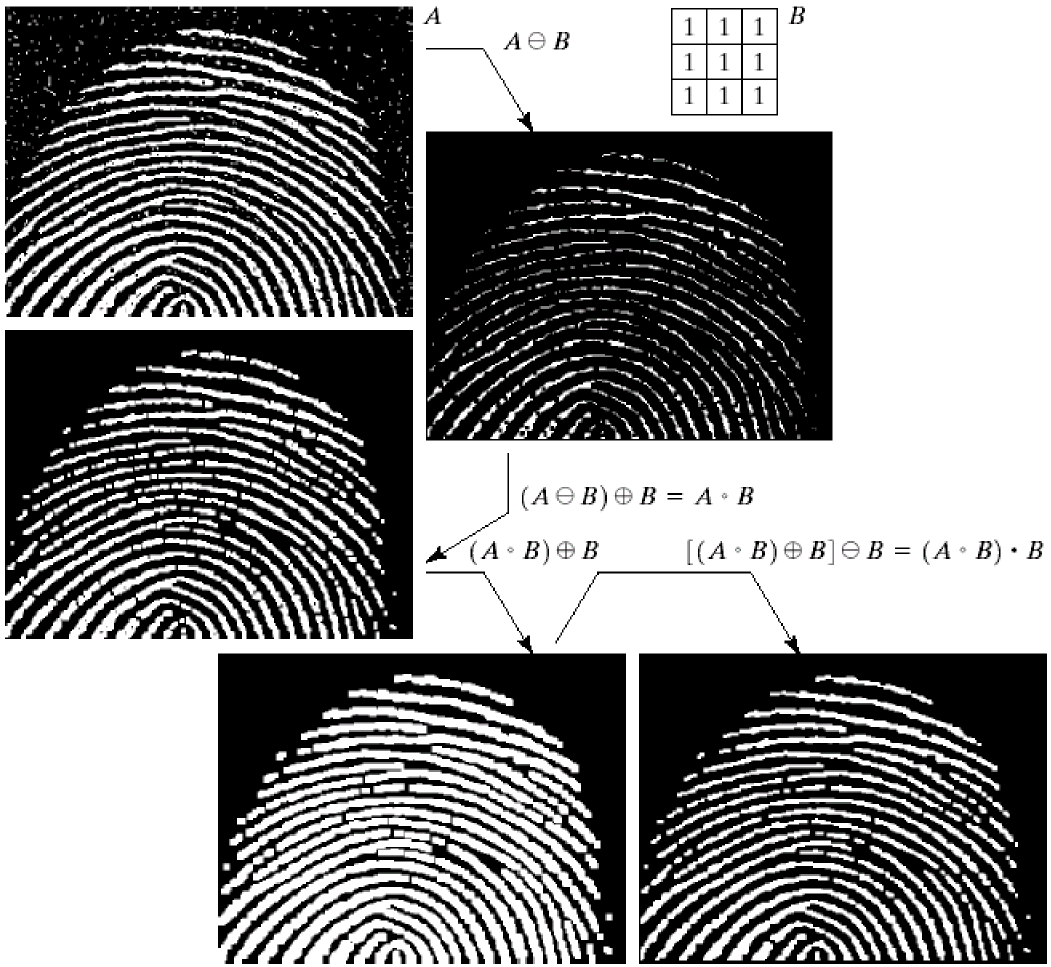
(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.

(iii) $(A \bullet B) \bullet B = A \bullet B$ (idempotent)

Opening and Closing



An Example of Morphological Filter



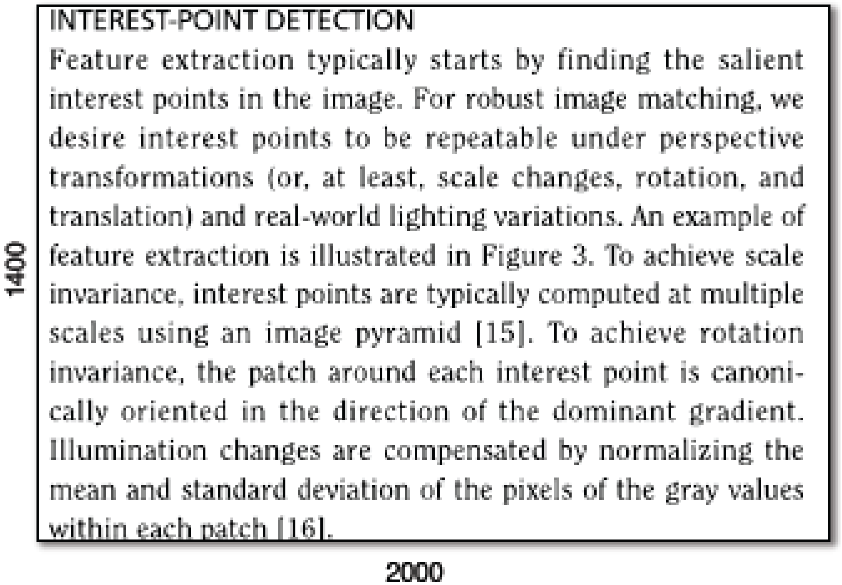
1	1	1
1	1	1
1	1	1

a b c d e f

FIGURE 9.11 (a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Recognition by Erosion

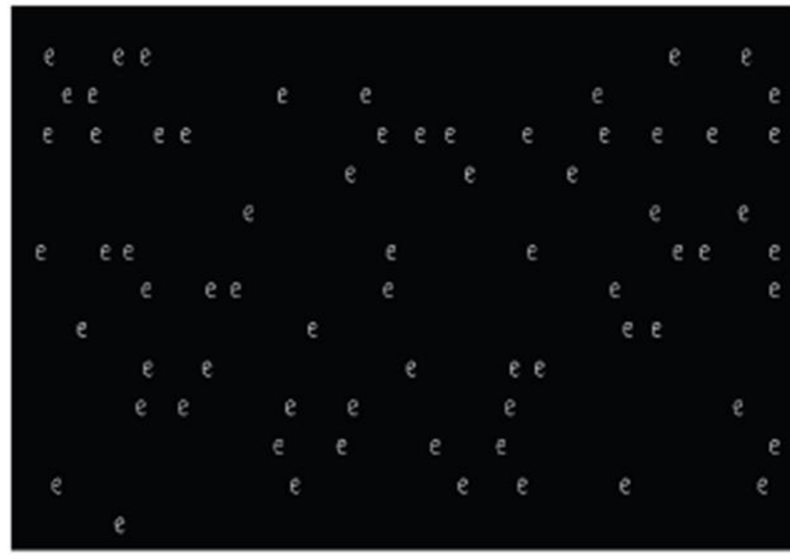
Binary image f



INTEREST-POINT DETECTION

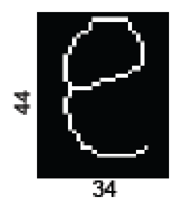
Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

$$open\left(NOT[f],W\right) = dilate\left(erode\left(NOT[f],W\right),W\right)$$



Find **e** in the image

Structuring element W



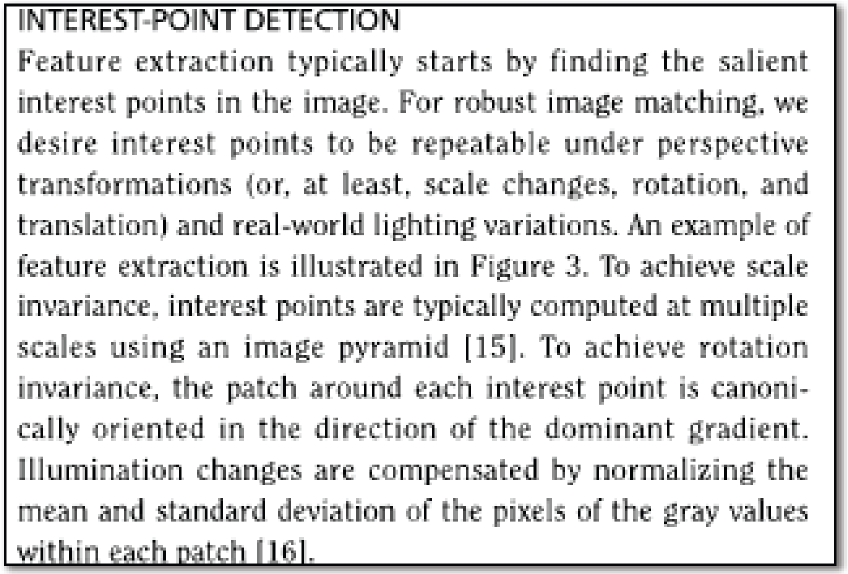
Recognition by Erosion

INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

Recognition by Erosion

Binary image f


INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

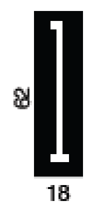
1400

2000

$$open\left(NOT\left[f\right],W\right) = dilate\left(erode\left(NOT\left[f\right],W\right),W\right)$$

Find I in the image

Structuring element W



Recognition by Erosion

INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

Outlines

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

The Hit-or-Miss Transformation

0	0	0	0	0	0	0
0	0	0	1	1	1	0
0	1	1	1	0	0	0
0	0	0	1	1	1	0
0	0	0	1	0	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0

A

	0	
1	1	0
	0	

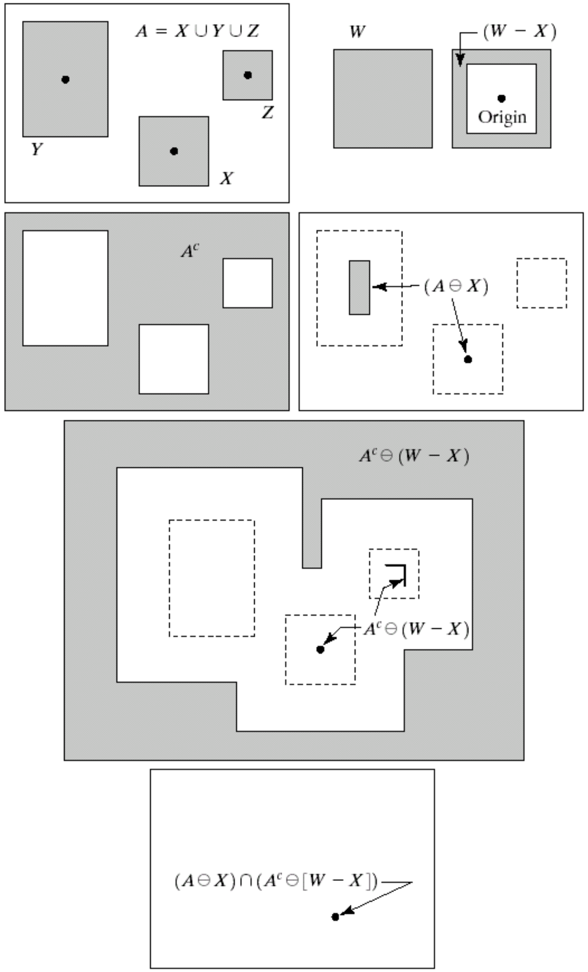
B^1

0	0	0	0	0	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	0
0	0	0	0	0	1	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$A \circledast B^1$

The object B^1 is found in the image A

The Hit-or-Miss Transformation



The Hit-or-Miss Transformation

The morphological hit-or-miss transform is a basic tool for shape detection.

If B denotes the set composed of X and its back-ground, the match (or set of matches) of B in A, denoted as $A \odot B$, is

$$A \odot B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

Letting $B = (B_1, B_2)$, where $B_1 = X$, $B_2 = (W - X)$,

$$A \odot B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

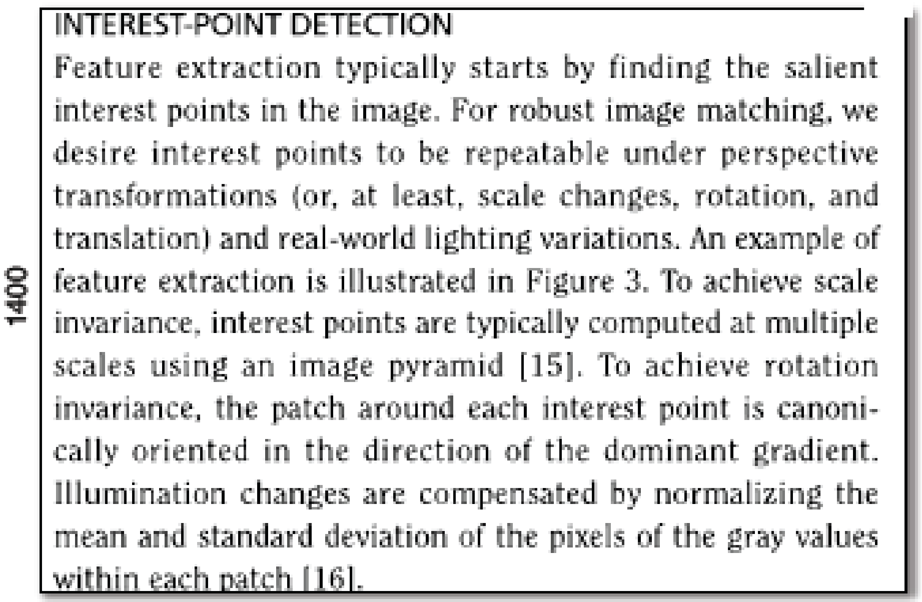
Thus, set $A \odot B$ contains all the (origin) points at which, simultaneously, B_1 founds a match ("hit") in A and B_2 found a match in A^c.

This is called the *morphological hit-or-miss transform.*

Recognition by Hit-or-Miss

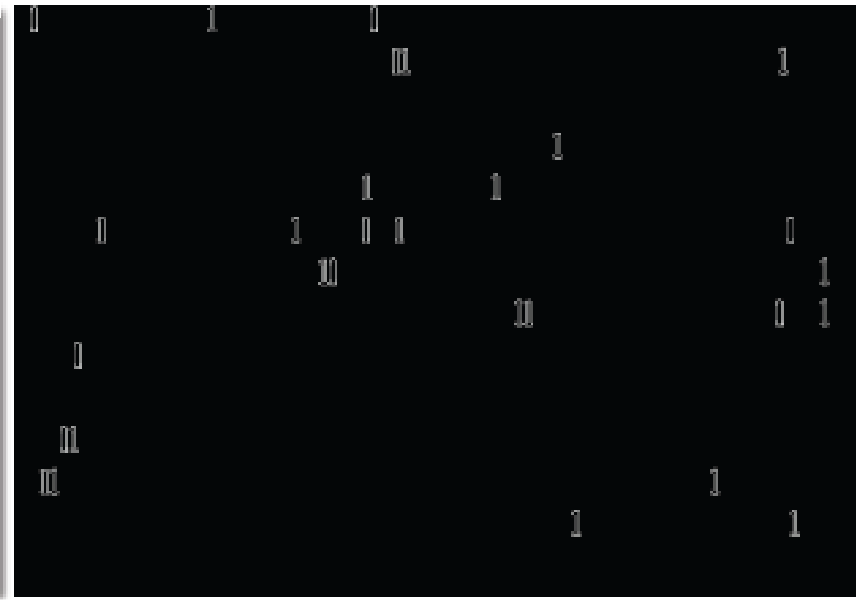
Binary image f

$$dilate\left(erode\left(NOT\left[f\right],V\right)\&\,erode\left(f,W\right),W\right)$$

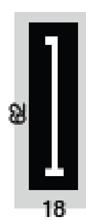


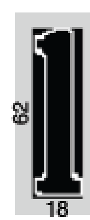
INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].



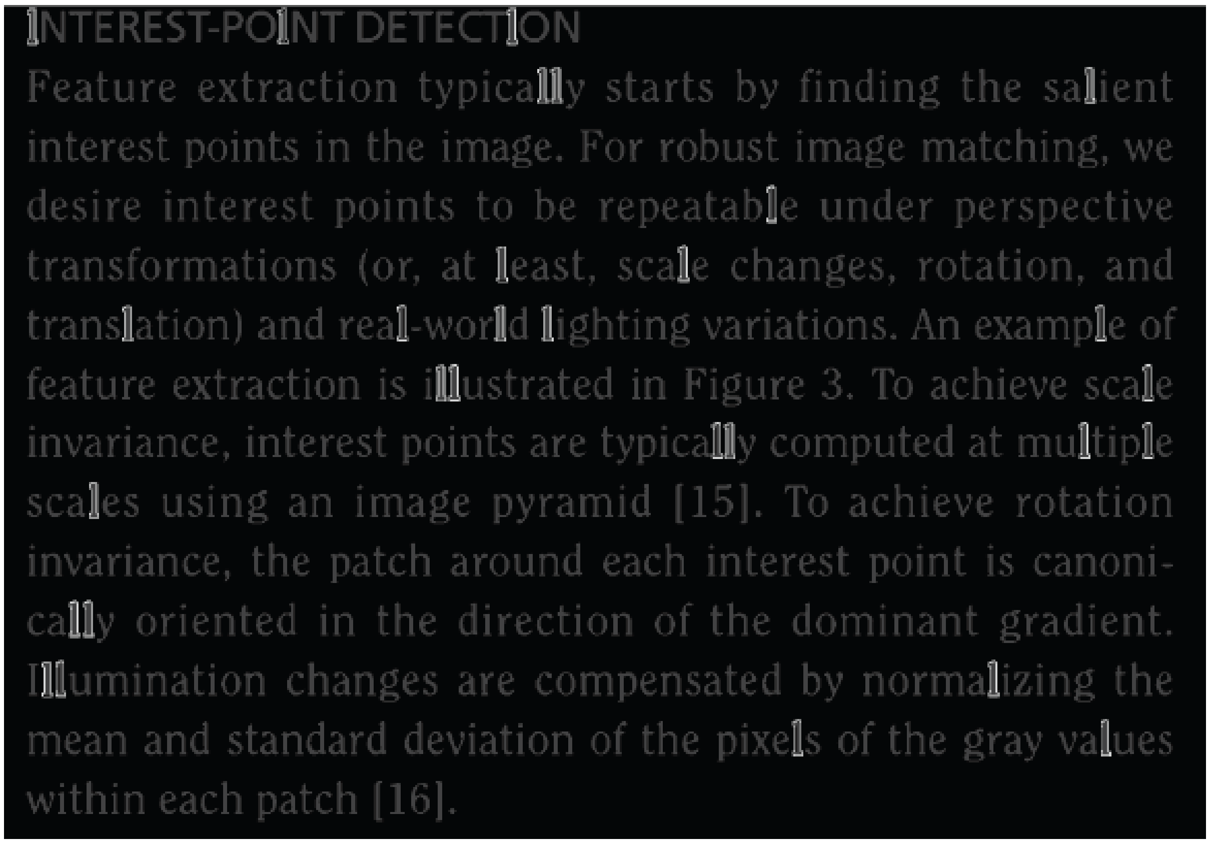
Structuring element V





Structuring element W

Recognition by Hit-or-Miss



INTEREST-POINT DETECTION

Feature extraction typically starts by finding the salient interest points in the image. For robust image matching, we desire interest points to be repeatable under perspective transformations (or, at least, scale changes, rotation, and translation) and real-world lighting variations. An example of feature extraction is illustrated in Figure 3. To achieve scale invariance, interest points are typically computed at multiple scales using an image pyramid [15]. To achieve rotation invariance, the patch around each interest point is canonically oriented in the direction of the dominant gradient. Illumination changes are compensated by normalizing the mean and standard deviation of the pixels of the gray values within each patch [16].

Outlines

- Preliminaries
- Dilation and Erosion
- Opening and Closing
- The Hit-or-Miss Transformation
- Some Basic Morphological Algorithms
- Extensions to Gray-Scale Images

Some Basic Morphological Algorithms

When dealing with binary images, the principal application of morphology is extracting image components that are useful in the representation and description of shape.

- boundaries
- connected components
- convex hull
- skeleton

We also develop several methods that are used frequently in conjunction with above algorithms as pre- or postprocessing steps.

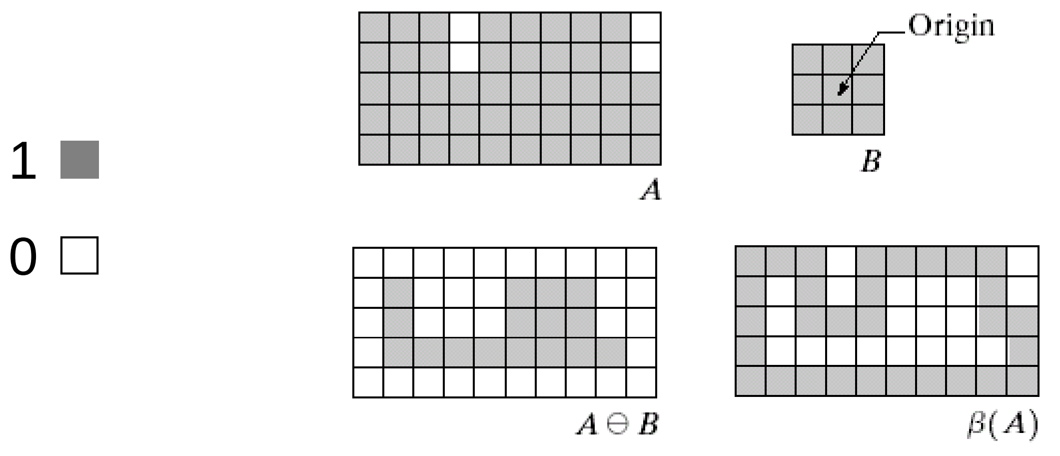
- region filling
- thinning
- thickening
- pruning

Boundary Extraction

The boundary of a set A denoted by β (A), can be obtained by:

$$\beta(A) = A - (A \ominus B)$$

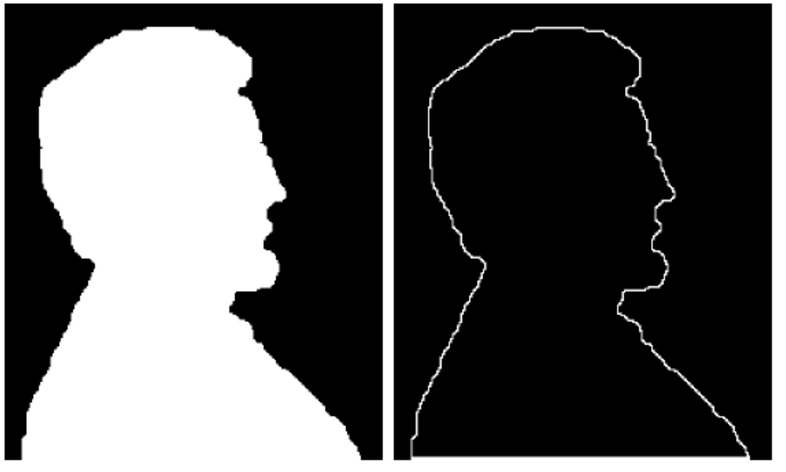
where B is a suitable structuring element.



Boundary Extraction

1 □

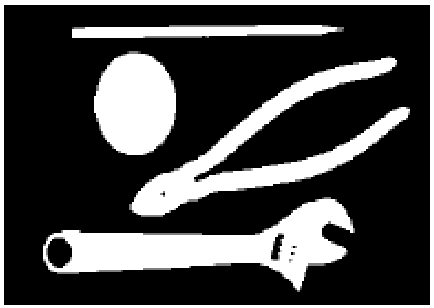
0 ■



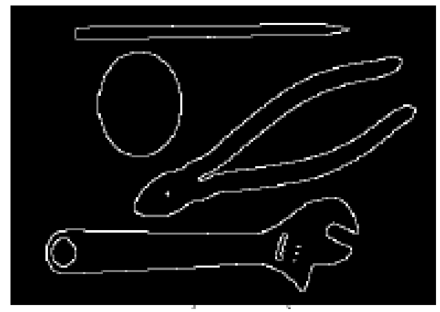
Boundary Extraction

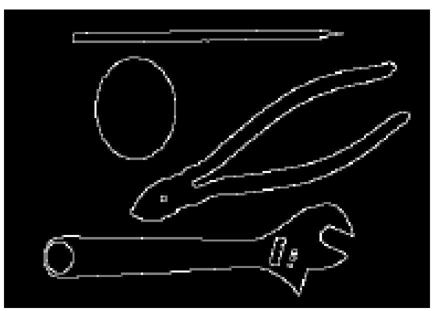
1 □

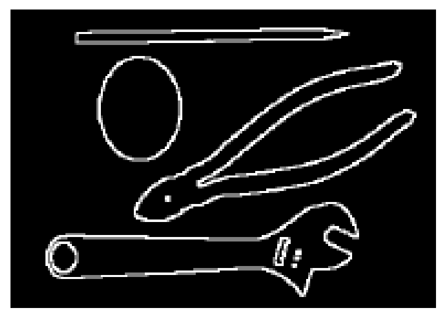
0 ■



$f[x,y]$







Boundary Extraction

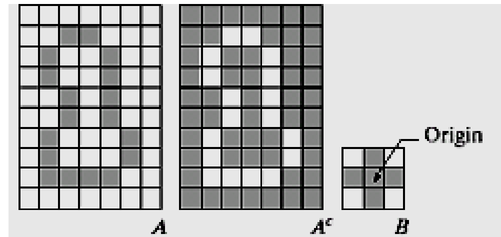
original f

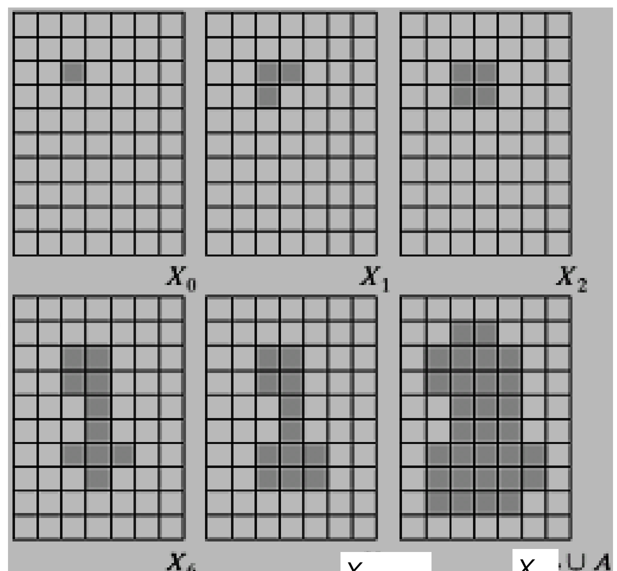
dilation g

g-f

$(g$-$f)$ thresholded

Region Filling





$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3, \ldots$$

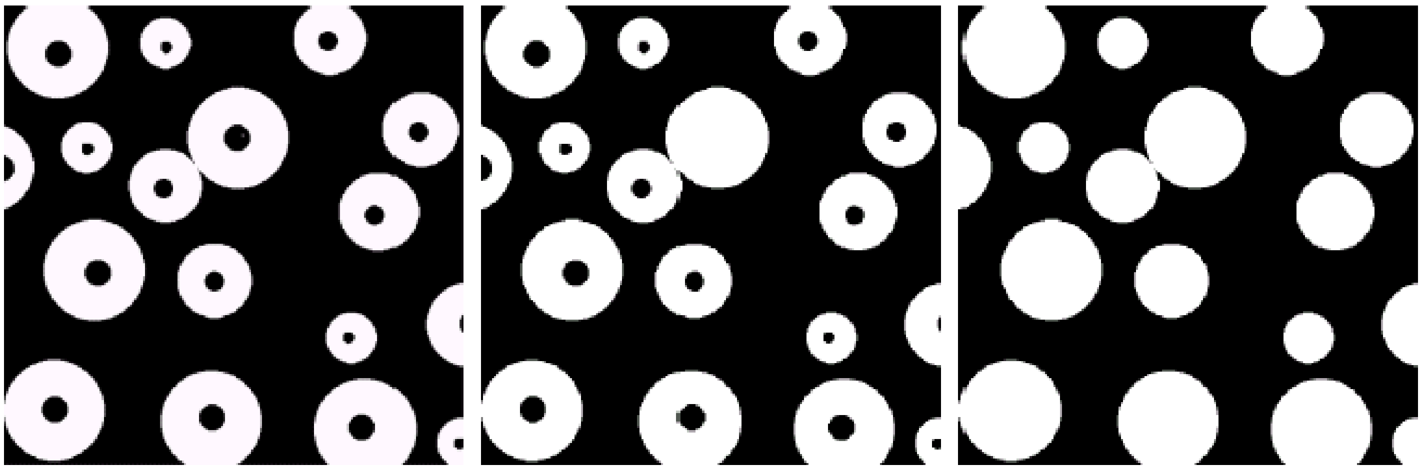
Region Filling

Assuming that a set contains a subset whose elements are 8-connected boundary points of a region, all nonboundary (background) points are labled 0, and we know a start point p inside the boundary, the following procedure will fill the region with 1's:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad k = 1, 2, 3, \ldots$$

where we assign a value of 1 to p to begin, and $X_0 = p$. the algorithm terminates at iteration step k if $X_k = X_{k-1}$.

Region Filling



a b c

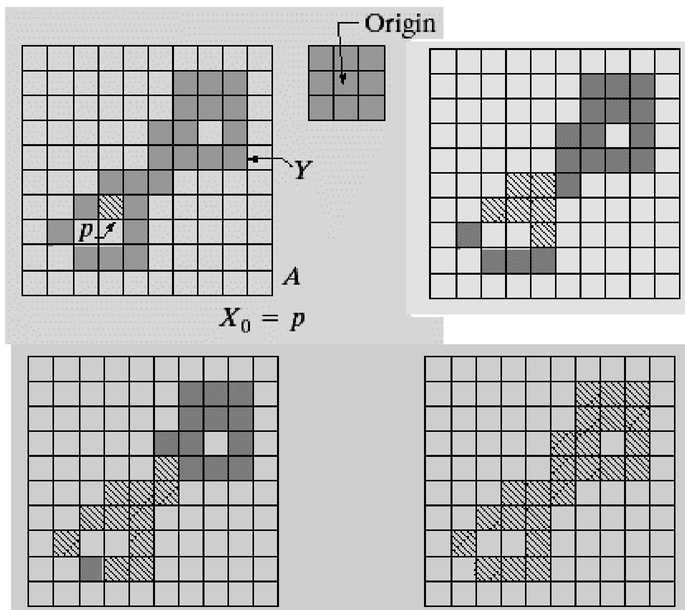
FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Extracting of Connected Components

Let Y represents a connected components contained in a set A, and assume that a point p of Y is known, the following iterative procedure yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k = 1, 2, 3, \ldots$$

where $X_0 = p$. The algorithm converges when $X_k = X_{k-1}$ and we let $Y = X_k$



Extracting of Connected Components

a
b
c d

FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Convex Hull

A set A is said to be *convex* (凸起的) if the straight line segment joining any two points in A lies entirely within A.

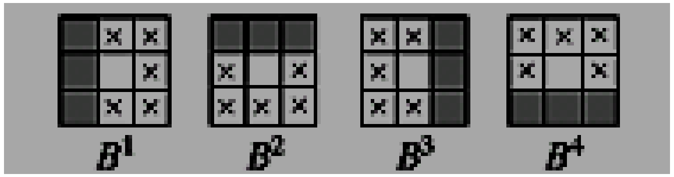
The *convex hull* (凸壳) H of an arbitrary set S is the smallest convex set containing S. $H - S$ is called the *convex deficiency* (凸缺) of S.

The convex hull, $C(A)$, of a set A and its convex deficiency are useful for object description.

$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots$$

with $X_0^i = A$. Now let $D^i = X_{\text{conv}}^i$ when $X_k^i = X_{k-1}^i$, then

$$C(A) = \bigcup_{i=1}^{4} D^i$$

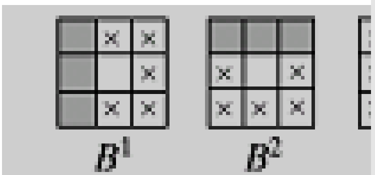


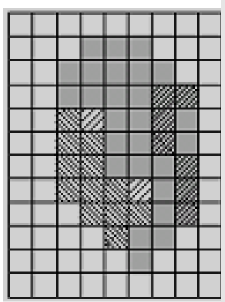
The x indicates "don't care".

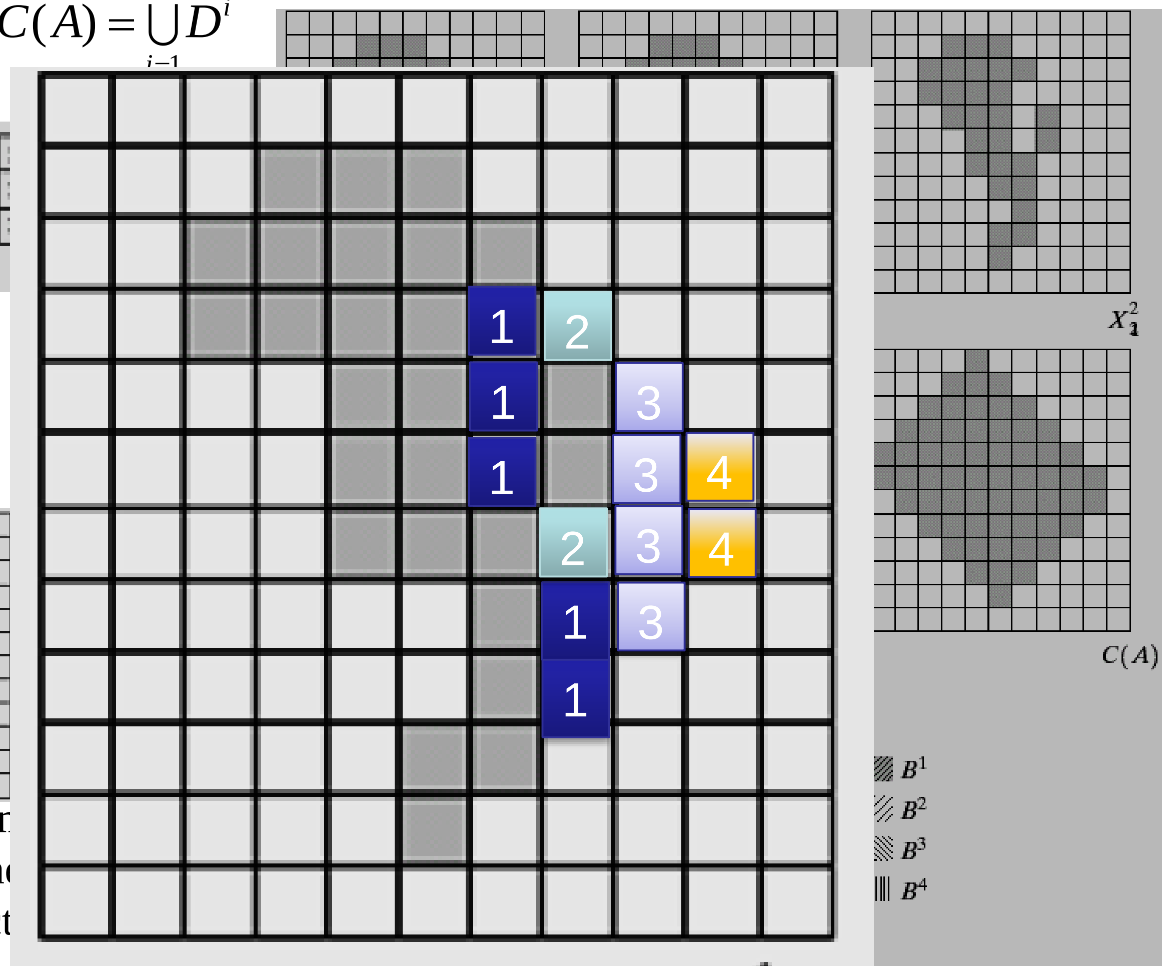
$$X_k^i = (X_{k-1} \circledast B^i) \cup A \quad i = 1, 2, 3, 4 \quad \text{and} \quad k = 1, 2, 3, \ldots$$

with $X_0^i = A$. Now let $D^i = X_{\text{conv}}^i$ when $X_k^i = X_{k-1}^i$, then

$$C(A) = \bigcup_{i=1}^{4} D^i$$







Limiting growth to r
dimensions along th
and horizontal direct

Thinning

The thinning of a set *A* by a structuring element *B* can be defined by:

$$\begin{aligned} A \otimes B &= A - (A \odot B) \\ &= A \cap (A \odot B)^c \end{aligned}$$

A more useful expression for thinning A symmetrically is based on a sequence of structuring elements:

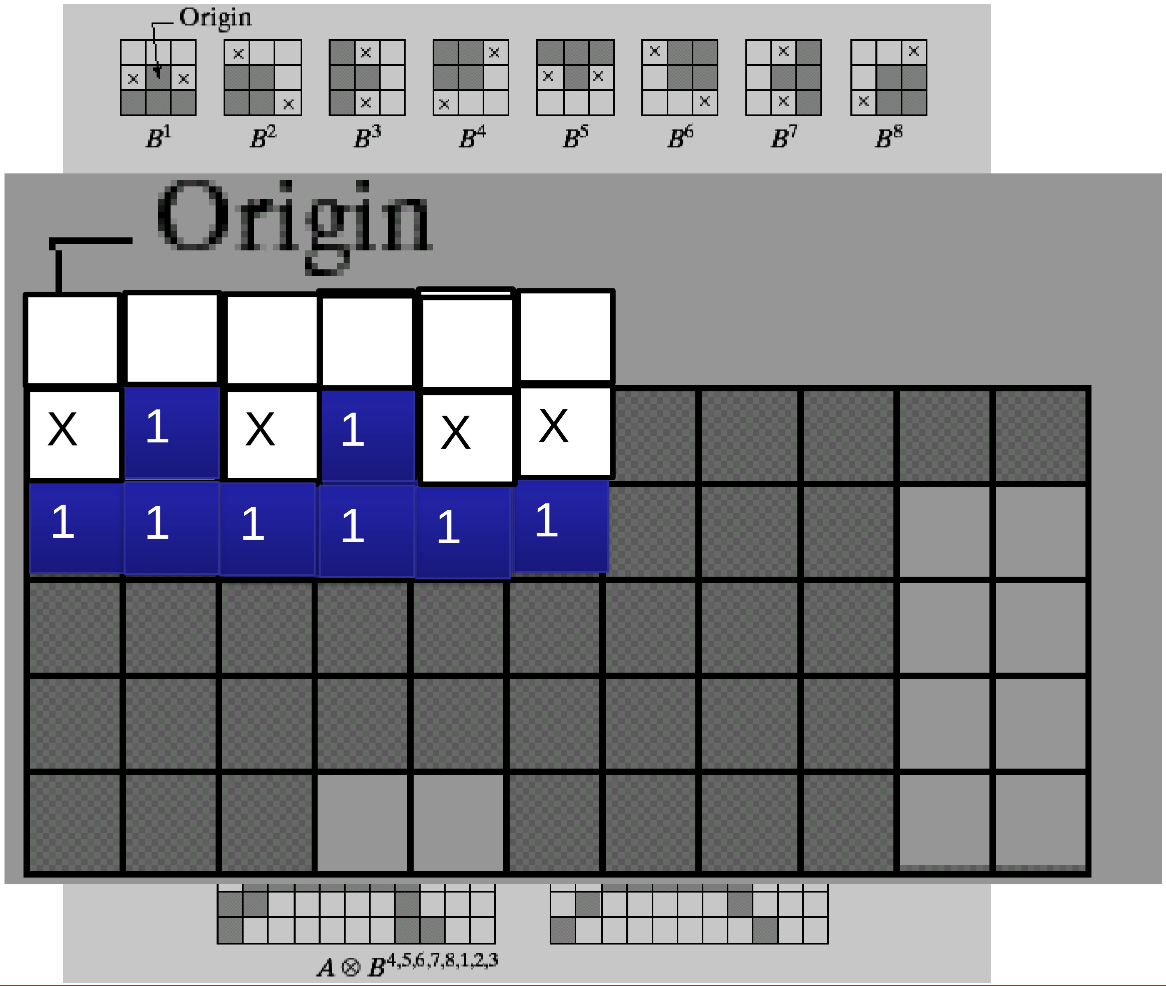
$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

where B^i is a rotated version of $B^{i\text{-}1}$. So,

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

The entire process is repeated until no further changes occur.

Thinning Process



Thickening

Thickening is the morphological dual of thinning and is defined by:

$$A \odot B = A \cup (A \circledast B) \qquad (9.5\text{-}9)$$

where B is a structuring element suitable for thickening. As in thinning, thickening can be defined as a sequential operation:

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n) \qquad (9.5\text{-}10)$$

The structuring elements used for thickening have the same form as those shown in the thinning example, with all 1's and 0's interchanged. But in practice, we often use the following procedure:

$C = A^c$,

thin C,

form C^c as the result

Thickening Process

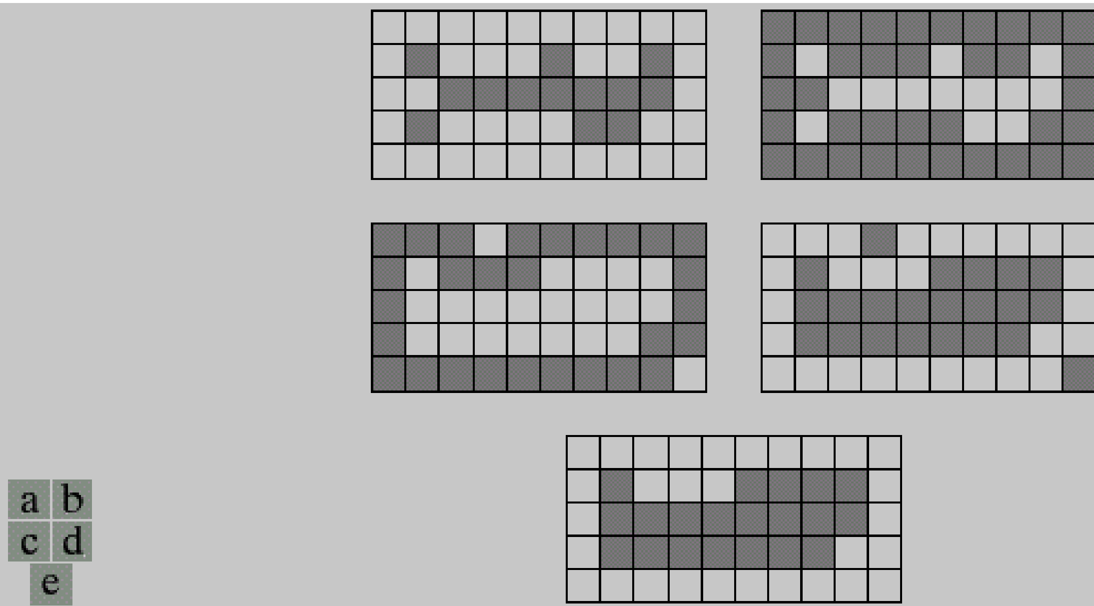


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

Skeletons

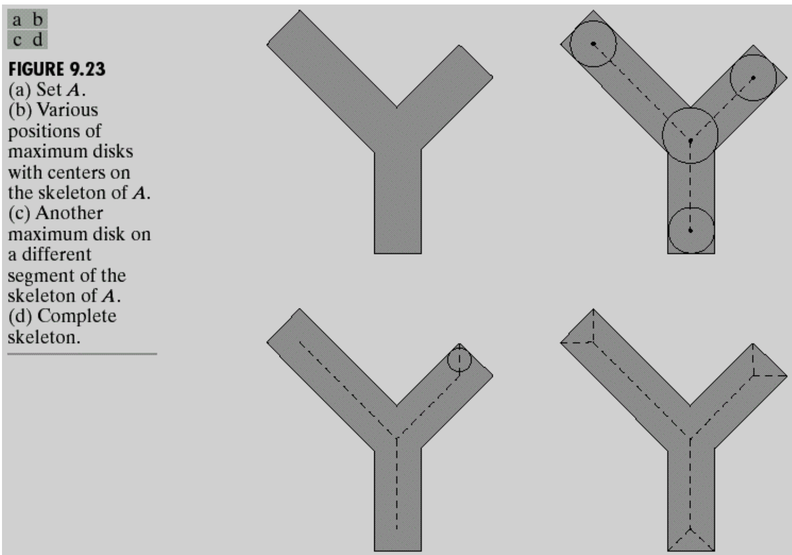


FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

1. If z is a point of a skeleton $S(A)$ of set A, and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk (not necessarily centered at z) containing $(D)_z$ and included in A. the disk $(D)_z$ is called a maximum disk.

2. The disk $(D)_z$ touches the boundary of A at two or more different places.

It can be shown that

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

where

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

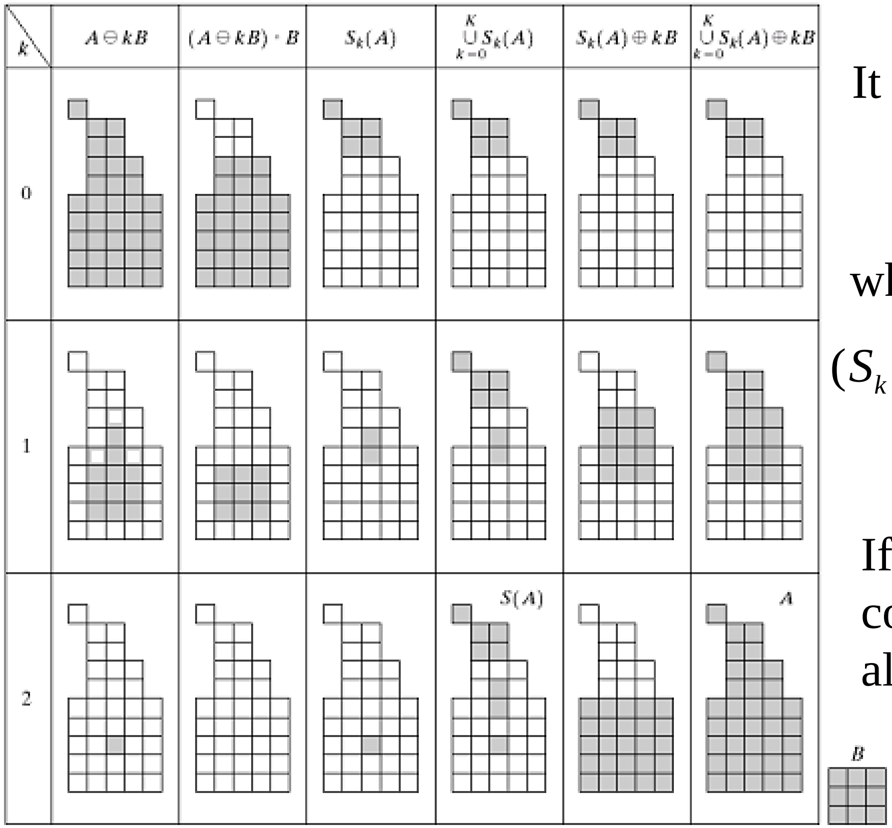
$$(A \ominus kB) = (\ldots((A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

k times erosion

$$K = \max\{k \mid (A \ominus kB) \neq \varnothing\}$$

Skeleton and Reconstruction Procedure

subset of skeleton



$K = 2$

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

It can be shown that:

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where

$$(S_k(A) \oplus kB) = \underbrace{((\ldots(S_k(A) \oplus B) \oplus B) \oplus \ldots) \oplus B}_{k \text{ times dilation}}$$

If the skeleton must be maximally thin, connected, and minimally eroded, other algorithms are needed.

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$(A \ominus kB) = (\ldots(A \ominus B) \ominus B) \ominus \ldots) \ominus B$$

Pruning

Pruning(剪裁) methods are an essential complement to thinning and skeletonizing algorithms because these procedures tend to leave parasitic components that need to be "cleaned up" by post-processing.

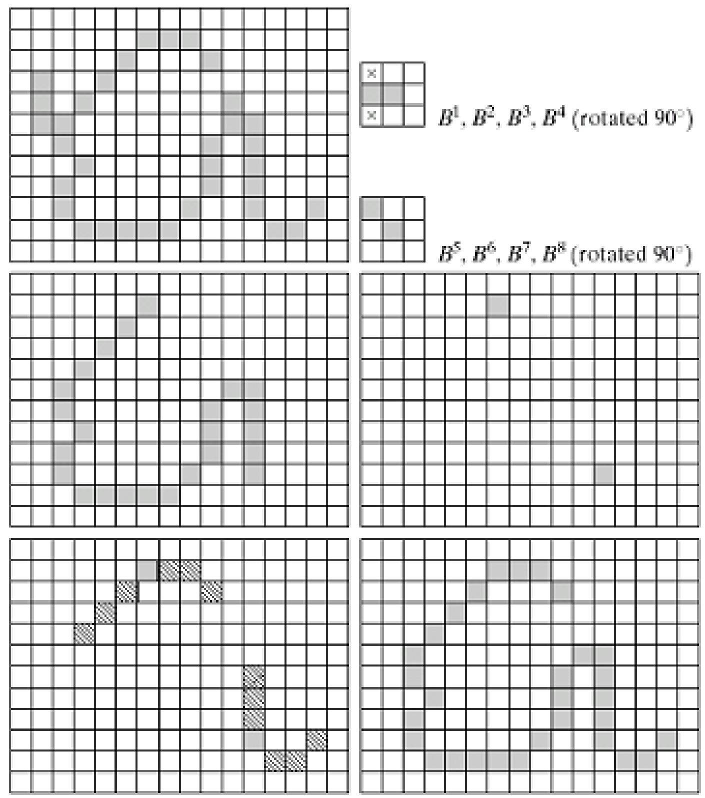
The skeleton of a hand-printed "a".

Thinning three times with structuring elements designed to detect only end points:

$$X_1 = A \otimes \{B\}$$

Conditional dilation of the end points three times with a 3 x 3 structuring element of 1's, H:

$$X_3 = (X_2 \oplus H) \cap A$$



Forming a set containing all end points in X_1 by the same end-point detectors B(hit):

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

Pruned image:

$$X_4 = X_1 \cup X_3$$

Opening and Closing

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

$$(f \bullet b)^c = f^c \circ \hat{b}$$

$$\text{or}: \qquad -(f \bullet b) = -f \circ \hat{b}$$

Let $e \dashv r$ indicates that the domain of e is a subset of r, and also $e(x, y) \leq r(x, y)$ for any (x, y) in the domain of e. Then for the dilation:

(i) $f \circ b \dashv f$.

(ii) If $f_1 \dashv f_2$, then $f_1 \circ b \dashv f_2 \circ b$.

(iii) $(f \circ b) \circ b = f \circ b$

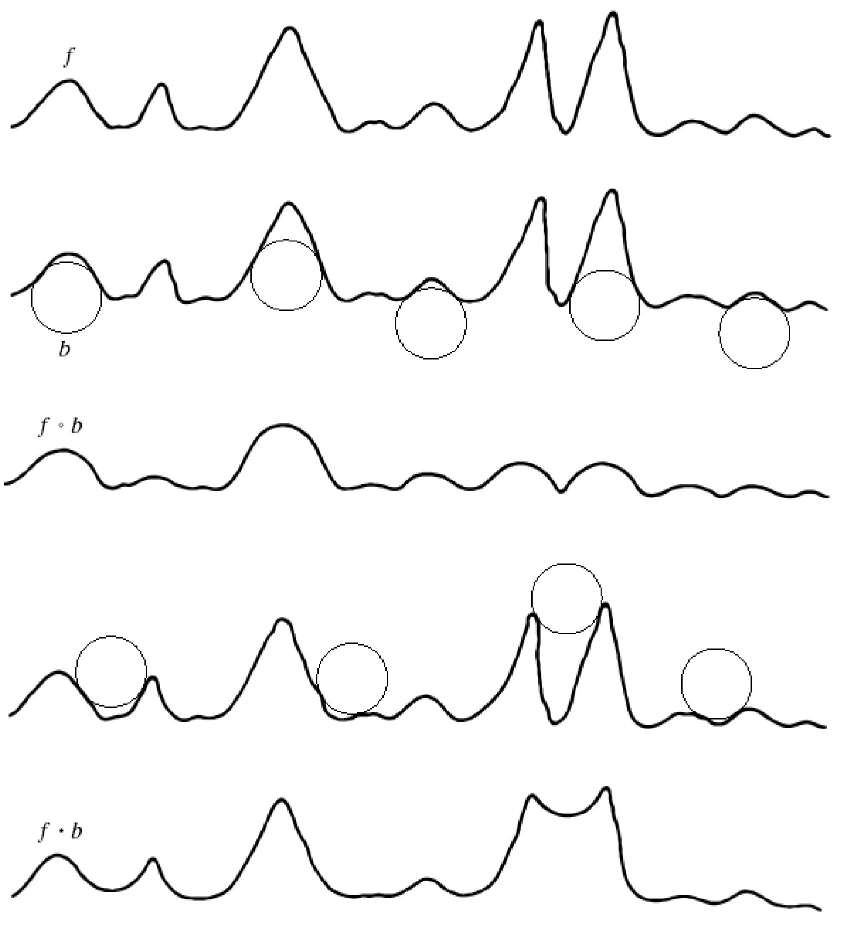
Similarly, for the erosion:

(i) $f \dashv f \bullet b$.

(ii) If $f_1 \dashv f_2$, then $f_1 \bullet b \dashv f_2 \bullet b$.

(iii) $(f \bullet b) \bullet b = f \bullet b$

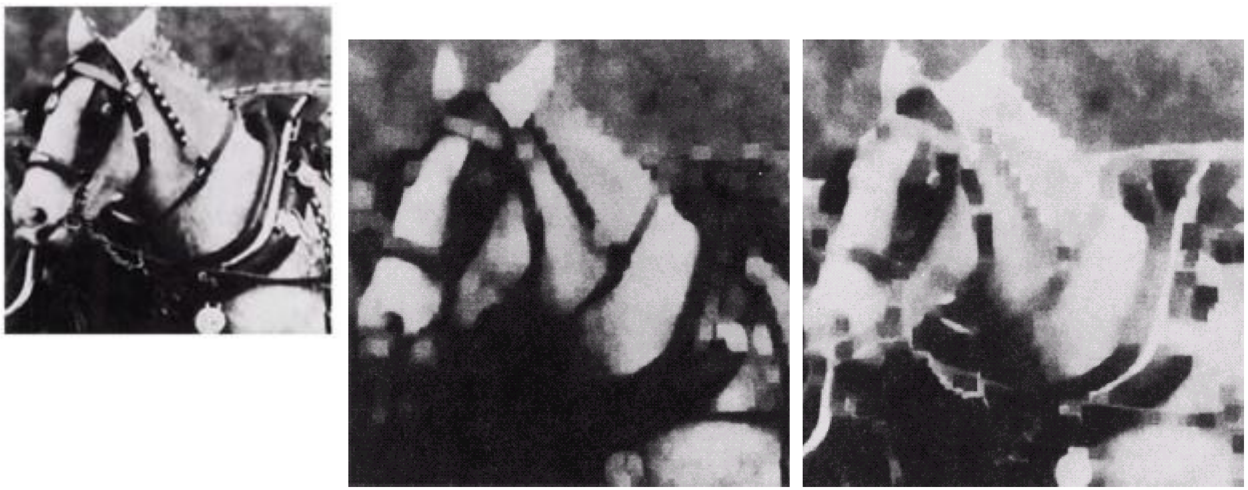
Geometric Interpretation of Opening and Closing



a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

Examples of Opening and Closing



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening decreases the sizes of the small, bright details, with no appreciable effect on the darker gray levels.

Closing decreases the sizes of the small, dark details, with relatively little effect on the bright features.

Examples of Opening and Closing

opening

closing

Morphological Smoothing: Opening-Closing

Both Open-Close and Close-Open remove both high and low valued points (artifacts or noise) while keeping the rest of the image intact, thus smoothing the image. These two operations do not give the same result due to the order of the erosions and dilations.

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

open then close

Example of Open-Close Smoothing

Open close

Close open

Morphological Gradient

The morphological gradient of an image is:

$$g = (f \oplus b) - (f \ominus b)$$

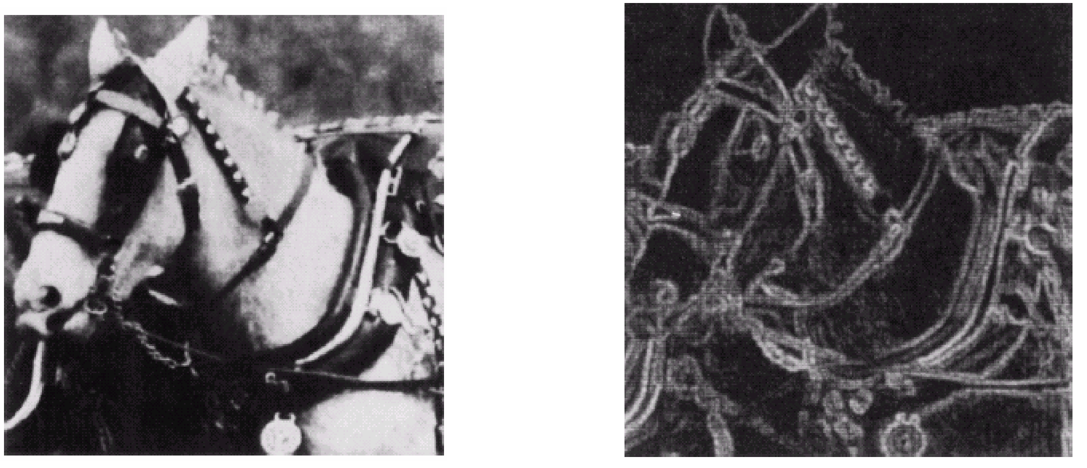


FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

morphological gradient obtained using symmetrical structuring elements tend to depend *less* on edge directionality:

Top-Hat Transformation

The morphological top-hat transformation of an image is defined as:

$$h = f - (f \circ b)$$

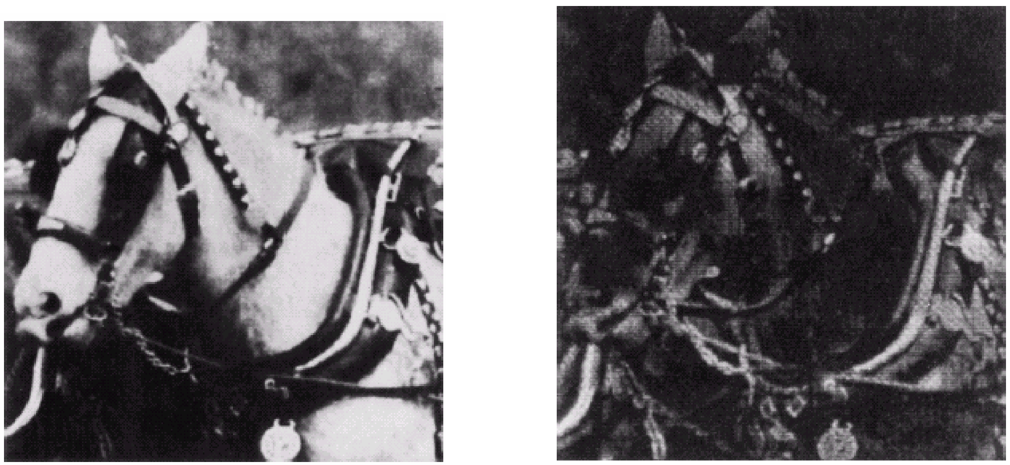
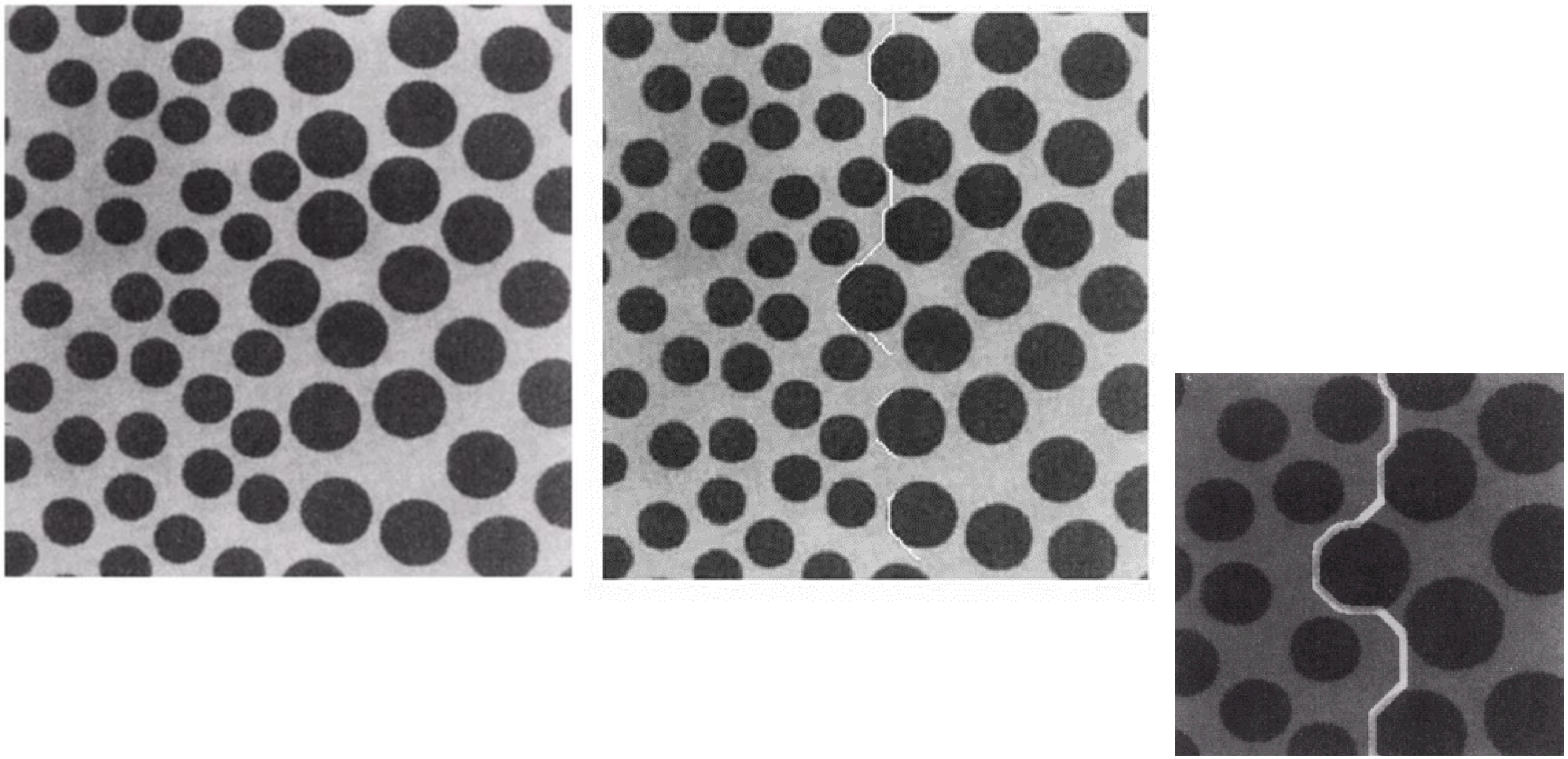


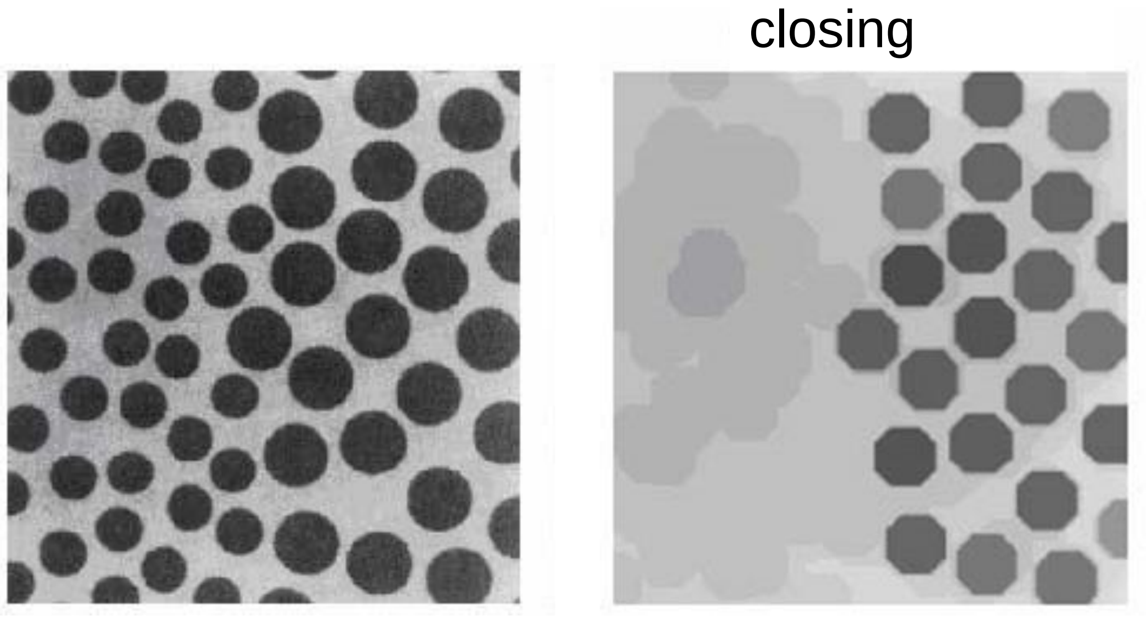
FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

useful for enhancing detail in the presence of shading.

Textural Segmentation

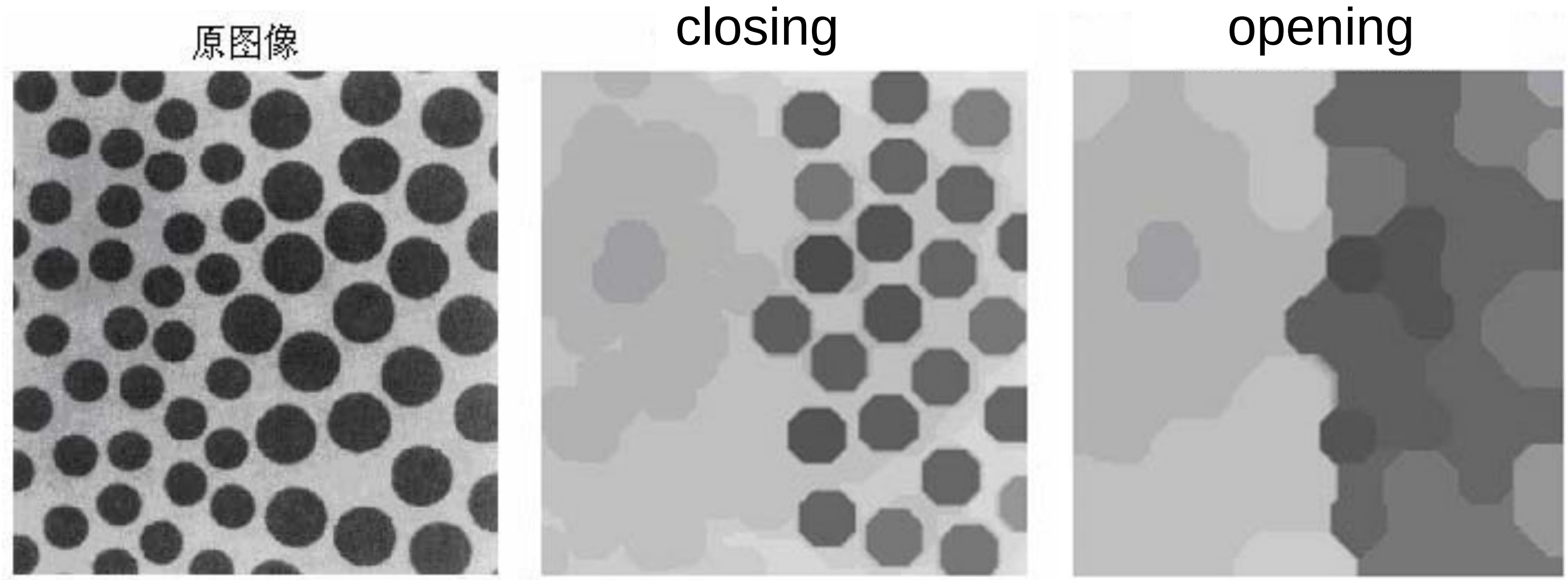


Textural Segmentation



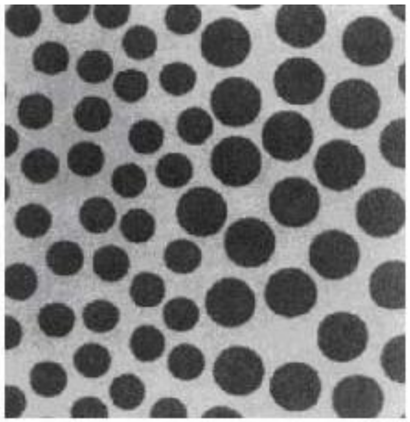
1.Closing the input image by using successively lager structuring element so that the small blobs on the left area are removed from the image, leaving only a light background in the area previously occupied by them.

Textural Segmentation

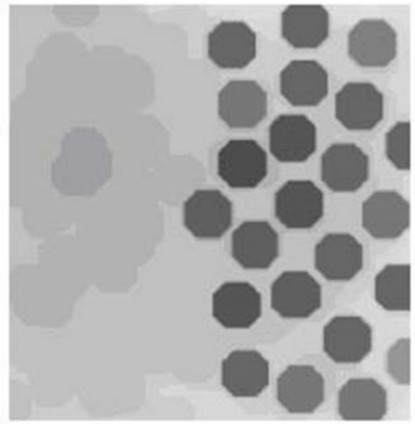


2. A single opening is used to remove the light patches between the large blobs, leaving a dark region on the right area.

Textural Segmentation



closing

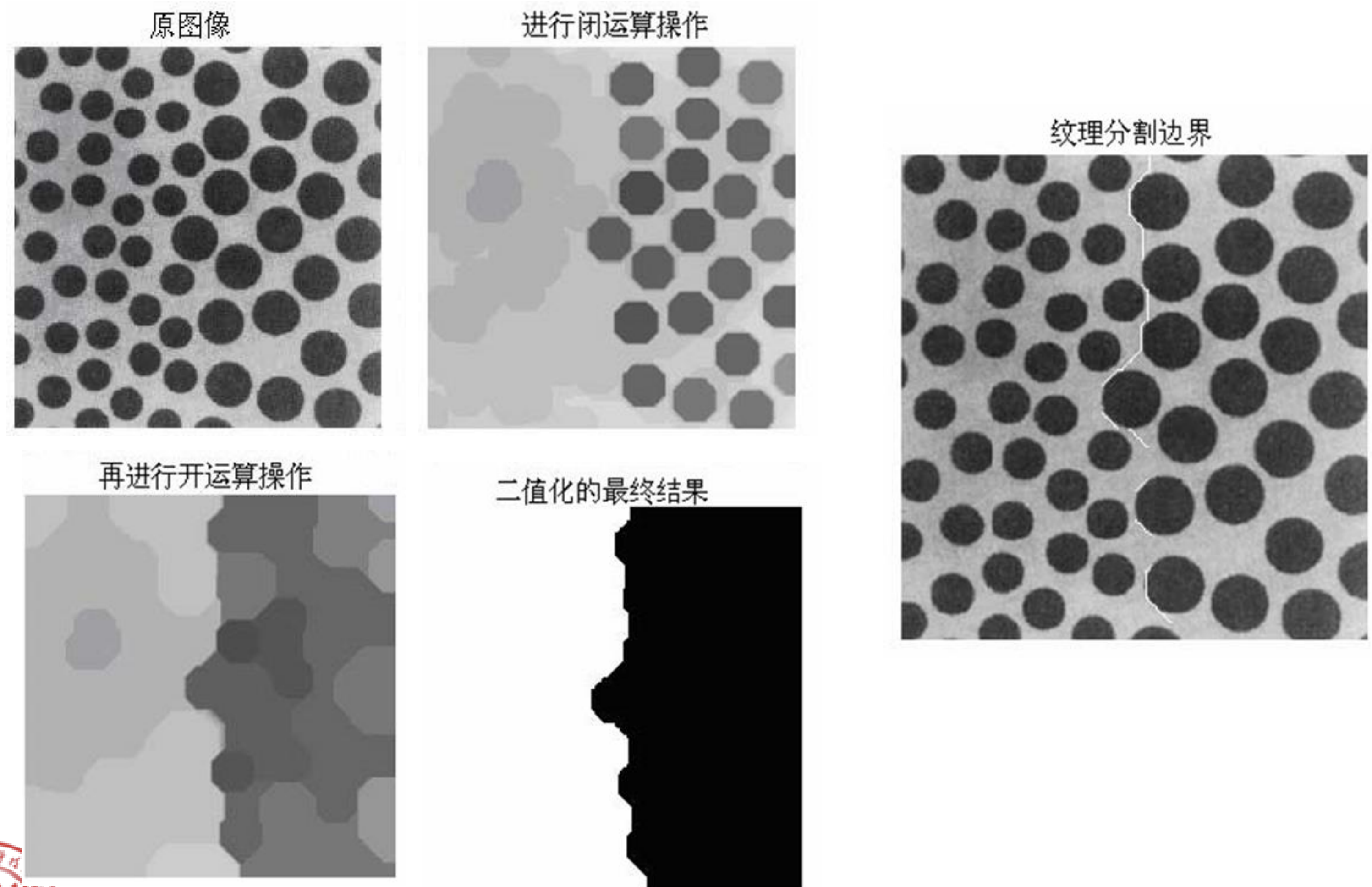


opening

thresholding

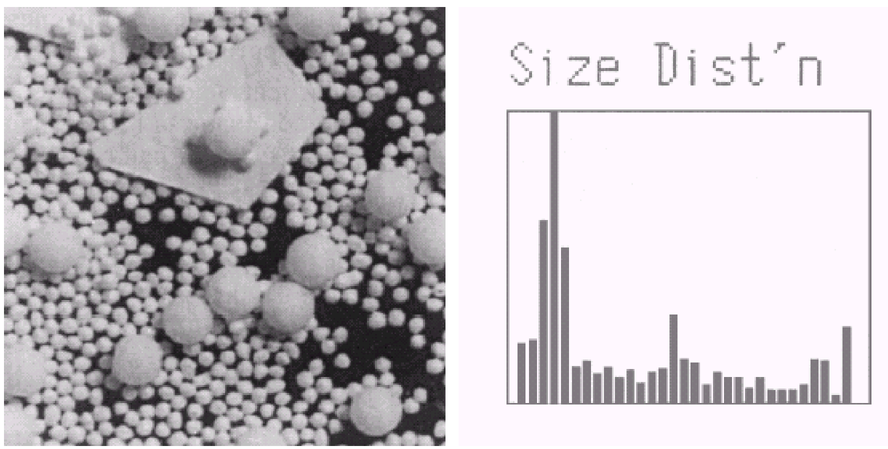
3. A simple threshold than yields the boundary between the two textural regions.

Textural Segmentation



Granulometry

A granule is an element that falls through a '**sieve**' in the same way that small stones fall through a sieve.



a b

FIGURE 9.36
(a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Summary

- Dilation and erosion (**Minkowski Addition** and **Subtraction**) are primitive operations that are the basis for a broad class of morphological algorithms.
- Morphology can be used as the basis for image/signal processing and for developing image segmentation procedures with a wide range of applications, and also play a major role in procedures for image description.

Thanks

The 1-D Dilation

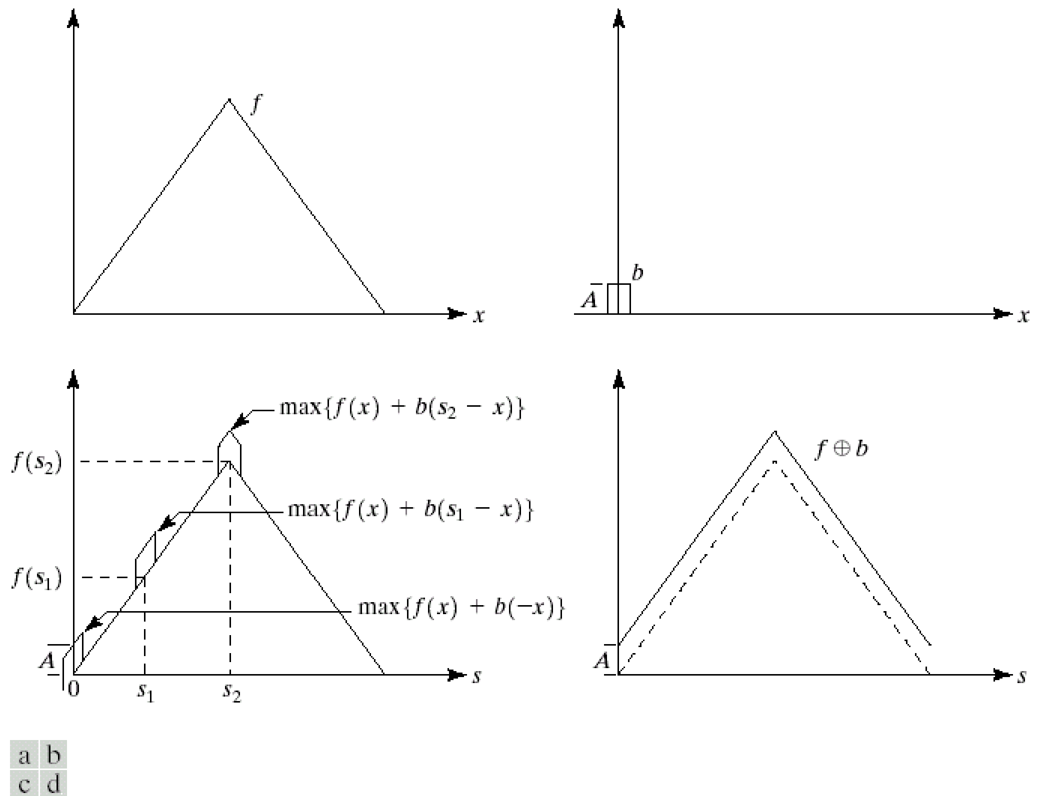


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).