EECS 844 – Fall 2016

Exam 1 Cover page*

Each student is expected to complete the exam individually using only course notes, the book, and technical literature, and without aid from other outside sources.

Aside from the most general conversation of the exam material, I assert that I have neither provided help nor accepted help from another student in completing this exam. As such, the work herein is mine and mine alone.

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Signature Date

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Name (printed) Student ID #

*** Attach as cover page to completed exam.**

EECS 844 Exam 1 (Due September 30)

Data sets can be found at http://www.ittc.ku.edu/~sdblunt/844/EECS844_Exam1

Provide: a) Complete and concise answers to all questions
b) Matlab code with solutions as appropriate
c) All solution material (including discussion, figures, and code) for each problem together (*i.e.* don't put all the plots or code at the end)
d) Email final Matlab code to me in a zip file (all together in 1 email)

** All data time sequences are column vectors with **increasing** time index as one traverses down the vector. (you will need to properly orient the data into "snapshots")

1. For the cost functions below, determine the derivative with respect to $\mathbf{w}^*$. (*Note: the chain rule still holds*)

$$\text{a)}\quad J(\mathbf{w}) = \frac{1}{a^{\left(0.5\mathbf{w}^H\mathbf{R}\mathbf{w}\right)}}$$

$$\text{b)}\quad J(\mathbf{w}) = \|\mathbf{w}\|^3$$

$$\text{c)}\quad J(\mathbf{w}) = \mathrm{Re}\left\{\mathbf{w}^H\mathbf{b}\right\}$$

$$\text{d)}\quad J(\mathbf{w}) = \frac{\left|\mathbf{w}^H\mathbf{R}^{-1}\mathbf{a}\right|^2}{\left(\mathbf{w}^H\mathbf{R}\mathbf{w}\right)\left(\mathbf{a}^H\mathbf{a}\right)} \quad \text{for } \mathbf{R} = \mathbf{R}^H$$

2. For the time-series data in P2.mat, estimate the "temporal" correlation matrix $\mathbf{R}$ by forming a matrix $\mathbf{X}$ of delay-shifted snapshots (each of length M) so that $\mathbf{R} = (1/N)\,\mathbf{X}\mathbf{X}^H$, where N is the number of columns in $\mathbf{X}$ (see Appendix A). Using the $\mathbf{R}$ estimate for $M = 8$,

a) plot the eigenvalues of $\mathbf{R}$ (in dB)
b) determine the condition number
c) compute $\mathbf{R}^{-1}$ and plot its eigenvalues (in dB)
d) discuss how are the two sets of eigenvalues related.

3. Repeat problem 2 using the "diagonally loaded" correlation matrix estimate defined as $\mathbf{R} = (1/N)\, \mathbf{X}\mathbf{X}^H + \sigma^2\mathbf{I}$, where $\sigma^2\mathbf{I}$ is the correlation matrix of white noise. Here set $\sigma^2 = 1$. What do you observe in comparison to the Problem 2 results?

4. For the correlation matrix estimate **R** from Problem 2 with associated eigenvector matrix **V**, transform the data matrix as $\mathbf{X}_{\text{new}} = \mathbf{V}^H\mathbf{X}$. What do you observe about the correlation matrix of this new transformed data? (show plots as appropriate)

5. The dataset P5.mat contains time samples collected from an $M = 12$ element antenna array. Using this data, estimate the "spatial" correlation matrix **R** and subsequently

a) plot the eigenvalues (in dB)
b) determine the condition number
c) compute $\mathbf{R}^{-1}$ and plot its eigenvalues (in dB)
d) discuss how are the two sets of eigenvalues related.

6. The dataset P6.mat contains two length $M = 20$ antenna array filters (i.e. beamformers) for a uniform linear array whose elements are separated by a half-wavelength. The filter 'w_non_adap' is a non-adaptive filter while the filter 'w_adap' is an adaptive filter. Using Appendices B and C, plot the beampatterns of these two filters in terms of electrical angle and spatial angle (plot in dB). Discuss what you observe.

7. For each of the cost functions in Problem 1, apply the linear constraint $\mathbf{w}^H\mathbf{s} = 1$ and <u>solve for the complex Lagrange multiplier</u> (you do <u>not</u> need to solve for the filter). Assume all matrices are PDH. (*In each case, pre-multiply by* $\mathbf{s}^H$ *when solving*)

Appendix A: Generating a matrix of time-series "snapshot" vectors

Given a vector of time samples defined as $\mathbf{x} = [x(1)\ x(2)\ x(3)\ \ldots\ x(K)]^T$, construct the matrix as

$$\begin{aligned}\mathbf{X} &= \left[\mathbf{x}(M)\ \mathbf{x}(M+1) \cdots \mathbf{x}(K-1)\ \mathbf{x}(K)\right] \\ &= \begin{bmatrix} x(M) & x(M+1) & \cdots & x(K-1) & x(K) \\ x(M-1) & x(M) & \cdots & x(K-2) & x(K-1) \\ \vdots & \vdots & & \vdots & \vdots \\ x(2) & x(3) & \cdots & x(K-M+1) & x(K-M+2) \\ x(1) & x(2) & \cdots & x(K-M) & x(K-M+1) \end{bmatrix}\end{aligned}$$

so that the n^{th} column represents the length M vector $\mathbf{x}(n)$.

Appendix B: Generating a steering vector matrix for a uniform linear array (ULA)

Generate a matrix $\mathbf{S}_\theta$ where each column $\mathbf{s}(\theta) = [1\ e^{-j\theta}\ \ e^{-j2\theta}\ \cdots\ e^{-j(M-1)\theta}]^T$ is a spatial steering vector in terms of the electrical angle $-\pi \le \theta \le \pi$ where the number of samples in angle is much greater than M (*e.g.* $10M$). Use the matlab command 'linspace' to get equally-spaced sample values over $-\pi \le \theta \le \pi$. This matrix may alternatively be parameterized in terms of spatial angle by first defining the equal-spaced sampling over $-\pi/2 \le \phi \le \pi/2$ and then converting via $\theta = \pi \sin(\phi)$ for half-wavelength spacing.

Appendix C: Computing the beampattern (or frequency response) of the filter **w**

Using the steering vector matrix from Appendix A, the resulting vector ($\mathbf{S}_\theta^H\,\mathbf{w}$) is the beampattern (or frequency response) for a fixed filter **w** (we generally plot the magnitude of this response in dB). Note that this result is over-sampled (relative to the DFT) to provide better visibility. The steering vector matrix $\mathbf{S}_\theta$ is also used for other signal processing algorithms such as MVDR, MUSIC, or model-based approaches.