**EECS 844 – Fall 2016**
Exam 2 Cover page*

Each student is expected to complete the exam individually using only course notes, the book, and technical literature, and without aid from other outside sources.

I assert that I have neither provided help nor accepted help from another student in completing this exam. As such, the work herein is mine and mine alone.

________________________________________ _________

Signature Date

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Name (printed) Student ID #

*** Attach as cover page to completed exam.**

## EECS 844  Exam 2 (Due October 21)

Data sets can be found at http://www.ittc.ku.edu/~sdblunt/844/EECS844_Exam2

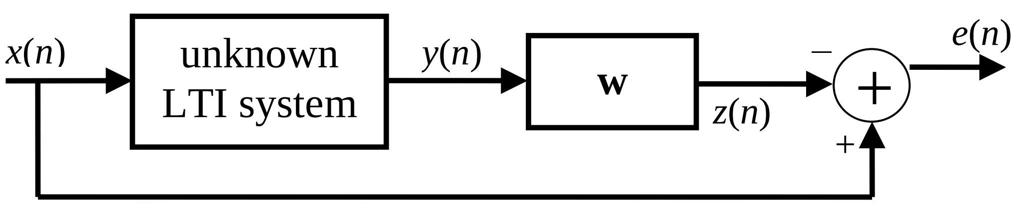
Provide: Complete and concise answers to all questions
Matlab code with solutions as appropriate
All solution material (including discussion and figures) for a given problem together (*i.e.* don't put all the plots or code at the end)
Email final Matlab code to me in a .zip file (all together in 1 email)

** All data time sequences are column vectors with **increasing** time index as one traverses down the vector. (you will need to properly orient the data into "snapshots")

**1**. In the dataset P1.mat are two signals, the input $x(n)$ and desired response $d(n)$ from an unknown system we wish to identify. Construct the Wiener filter and determine the resulting MSE for filter lengths of $M = 0, 1, 2, \ldots, 50$. ($M = 0$ means no filter)

a) Plot the MSE (in dB) as a function of length $M$, using both direct calculation (via average of $|e(n)|^2$) and the analytical MSE. Discuss what you observe.
b) Plot the magnitude and phase of the filter coefficients for the $M = 50$ case.
c) Use the 'freqz' command to plot the frequency response of the $M = 50$ filter.
d) How many filter coefficients are needed to estimate the unknown system?

**2**. For the filter arrangement below, derive the Wiener filter (show math) that minimizes the MSE. What relationship does the Wiener filter **w** have with the unknown system?




**3.** Dataset P3.mat contains 40 time snapshots (in the columns) from an $M = 10$ element uniform linear array (ULA) with half-wavelength element spacing.

*a)* Plot the non-adaptive (spatial) spectrum estimate defined in Appendix A. How many signals do there appear to be?
*b)* Plot the MVDR power (spatial) spectrum estimate in dB in terms of electrical angle $\theta$. Describe how this result relates to the non-adaptive power spectrum (how many signals do there appear to be for this case?).
*c)* Determine the electrical angles of the peaks observed in part *b*).

*d)* For each peak electrical angle from part *c*) plot an MVDR filter beampattern using that electrical angle as the unity gain constraint direction. Also plot the MVDR filter beampattern when the unity gain constraint is in the boresight direction.

**4**. Using dataset P3.mat:

*a)* Plot the magnitude of the raw (unfiltered) time-domain signal corresponding to the first antenna element.

*b)* Plot the magnitude of the non-adaptive filtered time-domain response for electrical angle 0°, where this filter is simply the normalized steering vector applied as $y(n) = (1/M)\, \mathbf{s}^H(\theta_o)\, \mathbf{x}(n)$.

*c)* Plot the magnitude of the filtered time-domain response for the boresight MVDR filter computed for prob. 3d).

*d)* Describe what you observe about the raw and filtered responses.

**5**. Using dataset P3.mat, implement the GSC filter (see Appendix B) that realizes a unity gain constraint in the boresight direction and places a single null. Experiment with different null locations (including using the peak locations from Prob. 3, and others) and compare the resulting beampatterns with the 0° MVDR beampattern from Prob. 3*d*. Explain what you observe.

**6**. For dataset P3.mat we wish to define a broad null over $\theta = -50°$ to $\theta = -15°$ while maintaining a unity gain constraint in the boresight direction. Implement the GSC (see Appendix B) using eigenvector null constraints. Try different numbers of eigenvector constraints and compare the beampatterns of both $\mathbf{w}$ and $\mathbf{w}_q$ to the beampattern of 0° MVDR from Prob. 3*d*. Discuss what you observe.

**7**. Dataset P7.mat contains time-domain samples of a stationary signal captured by an ADC. For a filter length of $M = 10$, form the set of snapshots (should be 291 total).

*a)* Plot the MVDR (frequency) power spectrum when using only the first 10 or 20 snapshots to form the correlation matrix and compare these two cases to when all the available snapshots are used (so three different responses).

*b)* Repeat part *a)* using diagonal loading with unity noise power.

**Appendix A**: Determining a non-adaptive power spectrum

Define the matrix $\mathbf{S}$ comprised of length $M$ steering vectors "over-sampled" in angle as described in Exam 1. Given the set of snapshots $\mathbf{X} = [\mathbf{x}_1 \; \mathbf{x}_2 \cdots \mathbf{x}_L]$, for each electrical angle $\theta$ compute the non-adaptive power spectrum as

$$p(\theta) = \frac{1}{ML} \sum_{\ell=1}^{L} \left| \mathbf{s}^H(\theta) \, \mathbf{x}_\ell \right|^2 .$$

**Appendix B**: Two Matlab techniques to generate an orthogonal complement matrix

For the constraint matrix $\mathbf{C}$ of size $M \times L$:

1) Generate a $M \times (M - L)$ matrix $\mathbf{Z}$ of random complex values.

2) Determine an orthonormal basis for the range of $\mathbf{C}$ as $\mathbf{Q} = orth(\mathbf{C})$ such that $\mathbf{Q}$ spans the same subspace as $\mathbf{C}$.

3) Generate the matrix $\mathbf{P}$ that projects onto the null space of $\mathbf{C}$ as $\mathbf{P} = \mathbf{I} - \mathbf{Q}\mathbf{Q}^H$ (can be done because the columns of $\mathbf{Q}$ are orthonormal).

4) Apply the null projection to the matrix of random values to form an $M \times (M - L)$ orthogonal complement matrix as $\mathbf{C}_a = \mathbf{P}\mathbf{Z}$. Normalize each column of $\mathbf{C}_a$ to have unity gain to avoid scale variations.

An alternative approach is to use the last $(M - L)$ left-singular vectors in $\mathbf{U}$ obtained after applying the singular value decomposition (SVD) as $svd(\mathbf{C}) = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$. Of the two approaches, this one is clearly much simpler (<u>and recommended</u>).