Problem 1

Observation of the Wiener weight solution and the forward linear predictor solution in terms of frequency response shows almost identical responses. The forward LP does exhibit some “lumpiness” in the magnitude response, presumably due to the fact that the LP is using previous samples to form the equivalent cross-correlation matrix (r), as opposed to the Wiener cross-correlation filter (P) which is formed with the desired signal x. The “quality” of r is dependent on the whiteness of the excitation signal x that feeds into the unknown system (which, by inspection of the autocorrelation of x, was shown to be white during the course of solving this problem).

Since x is a white stochastic zero-mean process, the Wiener solution and the forward LP solution both represent the whitening filter that “cancels out” the unknown system’s time correlation of the data (or alternatively, flattens the frequency response of the unknown system).

Problem 4

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| Mismatch filter | Mismatch loss (dB) |
| LS Mismatch Filter | 5.81 |
| LS Mismatch Filter, modified A | 5.37 |
| LS Mismatch Filter, diagonally loaded | 0.14 |
| LS Mismatch Filter, modified and diagonally loaded | 2.19 |

The normalized matched filter is the most basic calculation for deriving a matched filter for x, but as shown in the convolution plot of the NMF with x, it has a peak only about 15-20dB high, and is slightly wider than an impulse.

The LS Mismatch Filter has the sharpest peak out of all the different versions of the mismatched filter, but also the highest mismatch loss (and thus a lower SNR). The peak is about 40dB high, much higher and sharper than the NMF.

By modifying the A matrix to lose the surrounding rows with zeros, the mismatch loss is slightly smaller, but the peak is slightly wider, meaning that the mismatch filter performance is worse for deconvolution. Losing the rows around the impulse in the elementary vector has the effect of widening the deconvolved signal peak slightly.

Diagonal loading of AHA using 2% of the largest eigenvalue of AHA transforms AHA into I (identity matrix), meaning the LS mismatched filter approximates the normalized matched filter. This is supported by the fact that the mismatch loss is almost 0dB. While this maximizes the SNR, it is not ideal for deconvolution.

The combination of diagonal loading and modification of AHA through row removal shows a mismatch loss better than the LS mismatched filter but slightly worse than the diagonally-loaded mismatched filter. This combination seems to have the widest peak of all the mismatched filter (worst deconvolution performance) but preserves the most SNR.

Problem 6

From the non-adaptive power spectrum estimate, there appears to be 3 distinct signals of interest impinging on the array at {-61.5, 43.8, and 53.5} degrees. It can be expected to have the MVDR power spectrum estimate be more precise than the non-adaptive estimate; however, the MVDR power spectrum estimate shows 6 signals at {-61.5, -16.5, 40.5, 45.4, 95.3, and 112.7} degrees, but at very low power levels (almost 290dB below the non-adaptive estimate). This huge discrepancy is due to the autocorrelation matrix R being very ill-conditioned (condition number of 1.34e18).

By using forward/backward averaging, the condition number of the R matrix (denoted Rfb) is decreased by orders of magnitude, to 9823. The corresponding MVDR (denoted by the green Rfb line in the plot) has well defined peaks at {-57.4, -42.4, -17.7, 18.3, 43.0, and 58.0} degrees.

Diagonal loading of the original R matrix (to make the matrix more well-conditioned, in particular condition number = 1312) shows a smoother response (shown in purple) than the unloaded (original R) matrix, but the effect of the dominant diagonal is to smooth out the response; thus, only 4 peaks are seen at {-59.8, -18.3, 43.0, and 49.6} degrees.

Diagonal loading of the Rfb (forward-backward averaged) autocorrelation matrix again “smoothed out” the spectral response of the original Rfb MVDR; the peaks were higher (denoted by the yellow line) and had identical angular locations, but almost hid the unique peaks at -57/-42 degrees and 43/58 degrees. A lower diagonal loading value would likely prevent the blending of the peaks more.

Overall, the signals at -60, -18, 43 and 50 degrees (corresponding to the associated eigenvectors of the 4 dominant eigenvalues) showed up in all the power spectrums, with the exception of the non-adaptive spectrum, which didn’t have enough resolution to identify the two peaks at 43 and 50 degrees.