Problem 1

Observation of the Wiener weight solution and the forward linear predictor solution in terms of frequency response shows almost identical responses. The forward LP does exhibit some “lumpiness” in the magnitude response, presumably due to the fact that the LP is using previous samples to form the equivalent cross-correlation matrix (r), as opposed to the Wiener cross-correlation filter (P) which is formed with the desired signal x. The “quality” of r is dependent on the whiteness of the excitation signal x that feeds into the unknown system (which, by inspection of the autocorrelation of x, was shown to be white during the course of solving this problem).

Since x is a white stochastic zero-mean process, the Wiener solution and the forward LP solution both represent the whitening filter that “cancels out” the unknown system’s time correlation of the data (or alternatively, flattens the frequency response of the unknown system).

Problem 2

As expected, the convergence time for smaller values of mu was much faster than the convergence time of higher values of mu. All three values eventually converge very close to the same minimum cost value of 0.76.

Direct observation of the steering vectors s1 and s2 show that the steering angles are at 39.6 degrees and -21.6 degrees. Examination of the adaptive filter responses show that the filter does in fact converge to have peaks at 38.5 degrees and -20 degrees. However, the larger mu values develop large sidelobes outside the constraints; in fact, at mu=1/10N, the sidelobes are higher than the response of the steering angle responses. At mu=1/300N, the sidelobes are much better behaved. At about 8dB below the constrained steering angles.

Problem 4

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| --- | --- |
| Mismatch filter | Mismatch loss (dB) |
| LS Mismatch Filter | 5.8 |
| LS Mismatch Filter, modified A | 5.2 |
| LS Mismatch Filter, diagonally loaded | 0.14 |
| LS Mismatch Filter, modified and diagonally loaded | 2.2 |

The normalized matched filter is the most basic calculation for deriving a matched filter for x, but as shown in the convolution plot of the NMF with x, it has a peak only about 15-20dB high, and is slightly wider than an impulse.

The LS Mismatch Filter has the sharpest peak out of all the different versions of the mismatched filter, but also the highest mismatch loss (and thus a lower SNR). The peak is about 40dB high, much higher and sharper than the NMF.

By modifying the A matrix to lose the surrounding rows with zeros, the mismatch loss is slightly smaller, but the peak is slightly wider, meaning that the mismatch filter performance is worse for detailed deconvolution. Losing the rows around the impulse in the elementary vector has the effect of widening the deconvolved signal peak slightly.

Diagonal loading of AHA using 2% of the largest eigenvalue of AHA transforms AHA very close into the identity matrix, meaning the LS mismatched filter approximates the normalized matched filter. This is supported by the fact that the mismatch loss is almost 0dB. While this maximizes the SNR, it is not ideal for deconvolution. However, the noise levels (levels outside of the peak) are comparable to that of the modified AHA matrix filter.

The combination of diagonal loading and modification of AHA through row removal shows a mismatch loss better than the LS mismatched filter but slightly worse than the diagonally-loaded mismatched filter. This combination seems to have the widest peak of all the mismatched filter (worst deconvolution performance) but preserves the most SNR.Problem 5

Deconvolution of the received signal y(n) with the known signal x(n) using the previously-generated mismatch filters shows the effects of estimating the unknown system using filters with different resolutions (via peak impulse width) vs. different SNRs (via peak impulse height).

All plots are zoomed in near detected correlation peaks to show details.

The convolution of the normalized matched filter with y(n) shows a single correlated peak at 200 samples, and the peak is unique enough that it is 10dB above the surrounding samples (noise floor), indicating good SNR performance.

In contrast, the remaining four mismatches filters all show four unique peaks at n = {288, 293, 300, and 315}, indicating that the normalized matched filter’s SNR was not good enough to discern the other peaks that the mismatched filters caught. While they all showed more peaks, the peaks were lower relative to the noise floor (not as high as the normalized matched filter’s peak).

* The best performing (in terms of peak “uniqueness” relative to the noise floor) is the LS mismatched filter, as the lowest peak was 5dB above the noise floor.
* The LS mismatched filter with modified A performed similarly, albeit with slightly “fatter” correlation peaks; the noise floor was still below 10dB.
* When the filter matrix was diagonally-loaded, the noise floor increased slightly such that the highest peak was not as high as the previous two (the peaks were slightly fatter), but the lowest peak had the highest noise margin at 5.5dB (see plot). This was expected, since diagonal loading was shown to have the best SNR but worst decorrelator performance (as measured by the peak widths).
* Finally, the diagonally loaded modified A decorrelator had the lowest peak height for the tallest peak, but maintained a reasonable noise margin of 4.4dB for the lowest peak relative to the noise.

Problem 6

From the non-adaptive power spectrum estimate, there appears to be 3 distinct signals of interest impinging on the array at {-61.5, -17.7, and 44.8} degrees. It can be expected to have the MVDR power spectrum estimate be more precise than the non-adaptive estimate; however, the MVDR power spectrum estimate shows 6 signals at {-61.5, -16.5, 40.5, 45.4, 95.3, and 112.7} degrees, but at very low power levels (almost 290dB below the non-adaptive estimate). This huge discrepancy is due to the autocorrelation matrix R being very ill-conditioned (condition number of 1.34e18). It was so low that it is shown on a separate plot.

By using forward/backward averaging, the condition number of the R matrix (denoted Rfb) is decreased by orders of magnitude, to 9823. The corresponding MVDR (denoted by the green Rfb line in the plot) has well defined peaks at {-57.4, -42.4, -17.7, 18.3, 43.0, and 58.0} degrees.

Diagonal loading of the original R matrix (to make the matrix more well-conditioned, in particular condition number = 1312) shows a smoother response (shown in purple) than the unloaded (original R) matrix, but the effect of the dominant diagonal is to smooth out the response; thus, only 4 peaks are seen at {-59.8, -18.3, 43.0, and 49.6} degrees.

Diagonal loading of the Rfb (forward-backward averaged) autocorrelation matrix again “smoothed out” the spectral response of the original Rfb MVDR; the peaks were higher (denoted by the yellow line) and had identical angular locations, but almost hid the unique peaks at -57/-42 degrees and 43/58 degrees. A lower diagonal loading value would likely prevent the blending of the peaks more.

Overall, the signals at -60, -18, 43 and 50 degrees (corresponding to the associated eigenvectors of the 4 dominant eigenvalues) showed up in all the power spectrums, with the exception of the non-adaptive spectrum, which didn’t have enough resolution to identify the two peaks at 43 and 50 degrees.