Beacon Placement for Range-Based Indoor Localization

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***Abstract*—In this paper, we address the problem of range-based beacon placement given a floor plan to support indoor localization systems. Existing approaches for trilateration require three or more beacons to determine a unique position solution. We show that with prior knowledge of the map and a model of beacon coverage, it is possible to uniquely localize with only two beacons. This not only reduces installation cost by requiring fewer nodes, but can also improve robustness. One of the main challenges with respect to beacon placement algorithms is defining a metric for estimating performance. We propose augmenting the commonly used Geometric Dilution of Precision (GDOP) metric to account for indoor spaces. We then use this enhanced GDOP metric as part of a toolchain to compare various beacon placement algorithms in terms of coverage and expected accuracy. When applied to a set of real floor plans, our approach is able to reduce the number of beacons between** 22% **and** 60% (33% **on an average) as compared to standard trilateration.**

I. Introduction

Beacon technologies for indoor localization are rapidly decreasing in price while simultaneously improving in terms of ranging accuracy and energy requirements. Commercial solutions range in cost and performance from Bluetooth Low-Energy (BLE) modules that use Received Signal Strength (RSS) to estimate distance to more precise Time of Flight (TOF) ranging chipsets like [1], Ultra-Wide Band (UWB) [2] and WiFi [3] radios. Other distance sensing technologies like laser ranging and ultrasonic TOF systems are both decreasing in cost and becoming more common in consumer products like digital range finders and consumer robotics products. In order to localize solely based on range to beacons, a system will typically perform trilateration between the receiver/mobile device/tag and three or more fixed beacons that are at known positions. The beacon deployment itself is performed manually where experts working with the system determine the placement of beacons with empirical guidelines for spacing and coverage.

When deploying beacons, an installer faces the conflicting objectives of reducing the number of beacons to be placed and increasing system coverage, accuracy and resilience. The problem of placement indoors differs from Global Positioning System (GPS) in two ways. First, GPS satellites cover enormous areas, whereas indoor systems require comparatively dense placement of beacons, making it challenging to reduce the number of beacons. Second, the satellites are positioned symmetrically around the earth and a subset of them are assumed to be within Line-of-Sight (LOS) at all locations. This simplifies estimating coverage and computing metrics like Geometric Dilution of Precision (GDOP). Indoor systems often have irregular propagation models and barriers that complicate deployment, making it challenging to define a metric to easily compare beacon configurations.

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There are also several practical issues that need to be considered when deploying nodes indoors. First, the coverage of a beacon depends on the ranging-technologies which vary in terms of the maximum range and signal permeability through walls. For instance, acoustic signals are confined to walls, while RF signals exhibit high penetration. Second, indoor spaces have rich semantics that lead to different localization accuracy requirement across different areas. For instance, room-level accuracy might be sufficient in certain areas, while sub-meter accuracy within a large room might be required for audio guides in museums. Third, physical factors often constrain the deployment. For instance, it may be preferable to place beacons with convenient access to power outlets or where they do not disrupt the aesthetics of the space.

In this paper, we address the question - *For a range-based localization system, given the floor plan, where should the beacons be placed?* In order to answer this question, we first discuss how we can compare two beacon *configurations*. A configuration refers to a unique placement of beacons for a given floor plan.

One of the key insights in our work is that beacon coverage information can be used to prune incorrect location solutions in under-defined beacon configurations. When range measurements are received from two beacons, it normally results in two possible location solutions. It is possible to resolve which is the more likely solution if each location is covered by a different set of beacons. For example, if one of the two location solutions does not receive a signal that was likely to be received from a nearby beacon, it indicates that either the beacon was blocked or the alternate solution is more likely. For most of our results, we assume an ideal ray-tracing coverage model since our primary target system is based on acoustic beacons. However, this concept is also applicable to other coverage types given a suitable propagation model. We refer to regions that can be localized in this manner without any ambiguity as *Uniquely Localizable* and introduce a function *UL* that indicates if a location is unambiguously localized by the beacons in coverage. We use the percentage of area uniquely localizable in a floor plan to quantify the quality of a beacon configuration, purely based on coverage.

For any realistic system with noise in the range measurements, in addition to beacon coverage, the beacon geometry also affects the location accuracy. One of our main contributions is that we adapt the concept of GDOP to quantify the location accuracy provided by the beacons. GDOP is a unitless quantity that is a function of the geometry between the beacons and the target, often used to estimate the expected accuracy of GPS due to the location of satellites [4]. When used as a metric for evaluating indoor accuracy, GDOP has two main drawbacks. First, it cannot accurately capture cases where there are multiple ambiguous solutions. In these cases, a localization solver could be completely incorrect and yet an evaluation function would still indicate a confident reading based on geometry. Second, there are circumstances that cause the standard GDOP metric to grow towards infinity. This makes it difficult to normalize over multiple competing configurations. To overcome the first problem, we define a function $GDOP^{UL}$ that is a modified GDOP that also incorporates the Unique Localizability information. For the second problem, we use a function of the Cumulative Distribution Function (CDF) of the $GDOP^{UL}$ curve to quantify the quality of a beacon configuration across the floor plan.

We integrate our evaluation metric into a software toolchain that is able to evaluate a variety of beacon placement algorithms given a floor plan. The system first pre-process the floor plan and segments it into regions based on the coverage of candidate beacon locations. It then uses a greedy approach to place beacons in the largest regions first. We then apply algorithms for placing beacons to optimize for (1) Unique Localizability UL and (2) both GDOP and Unique Localizability $GDOP^{UL}$. The objective in both cases is to place the minimal number of beacons. In the case of $GDOP^{UL}$, the algorithm also accounts for predicted accuracy. We then evaluate this across ten floor plans and the number of beacons placed by our scheme is 22% to 60% (33% on an average) lesser than when coverage by three or more beacons is required. In practice, this results in a large saving in infrastructure and installation cost when provisioning large indoor spaces with range-based beacons.

In summary, this paper makes four main contributions:

1) We introduce and define the concept of *Unique Localizability* and show how we can localize with two beacons instead of three by exploiting the beacon coverage information. We introduce $GDOP^{UL}$, a modified GDOP metric that incorporates the UL information.
2) We show how we can quantify the *quality of a beacon configuration* based on these metrics, where the quality is a function of the localization accuracy across the floor plan.
3) We present an algorithm that provides beacon locations and compare the number of beacons against minimal placement.
4) We design a toolchain that implements the algorithm where the user can provide or draw a floor plan, and specify the design space, such as the accuracy requirement, number of beacons available and beacon coverage information.

II. Related Work

In order to find beacon configurations that minimize the localization error at a given target location, several authors have studied optimal beacon geometries that minimize the Cramér-Rao bound (CRB) [5] or the GDOP [6], [7]. For $N \geq 3$ beacons, optimal placement can be obtained when the adjacent beacons subtend an angle of $\frac{2\pi}{N}$ or $\frac{\pi}{N}$ about the target, and for $N = 2$ beacons, optimal placement is when the two beacons subtend an angle of $\frac{\pi}{2}$ about the target [8], [9].

Though the optimal placement for a single target is well understood, the optimal placement for multiple target locations, a target trace and a target area for range-only systems are still open problems. [10] shows that the generalized sensor placement problem is at least as hard as the k-center problem, which is NP-complete. They present a solution based on integer linear programming for triangulation-based system, but the complexity for trilateration is similar. Due to the complexity of the problem, most proposed solutions involve designing heuristics and utilizing optimization techniques. [11] explores computational-geometry based heuristics for determining location of beacons given a predefined trace of a robot. They define a utility metric that is a function of the number of beacons in range and the convex hull of beacons. They evaluate the quality of a configuration using the integral of the utility function. Our work is similar in the respect that we also account for the coverage of the beacons but differs since the metric they propose is purely based on coverage and not accuracy. [12] proposes a heuristic for *Quality of Trilateration* that is based on the probability that a location estimate is within a given radius of the true location. However, this approach is for a network of nodes where a node localizes itself with respect to three other nodes given a prior distribution on the expected inter-node distances. [13] and [14] propose their own metrics to quantify the quality of a beacon placement, which is a function of the GDOP over the desired localization area and ratio of area which cannot be localized as well as propose a placement algorithms to optimize the metric. [13] classifies areas that are localizable based on whether the GDOP is above or below a threshold and use a genetic algorithm approach for placement. [14] considers the average GDOP over the areas that are localizable and implements a meta-heuristic optimization strategy. These approaches are similar to our work, but do not consider the minimal beacon count placement for areas and require three or more beacons to provide coverage.

[15] looks at the problem of beacon placement while localizing with range-based beacons with limited field of view. They propose a heuristic based on the GDOP being below a certain threshold and propose a placement algorithm for the same, but they consider only a single room case and the beacons are placed on the periphery of the room, avoiding any ambiguity in solution.

The Art Gallery Problem from computational geometry is closely related to our beacon placement problem. Its goal is to find the minimal set of guards such that every region in a floor plan is covered by at least one guard. The range-based beacon placement problem discussed in this paper differs since we require coverage by multiple beacons to localize the target, and the localization accuracy is a function of the beacon locations, even when sufficient coverage is achieved.

III. Quantifying the Quality of a Beacon Configuration

In this section, we discuss our approach for evaluating beacon placement in order to compare candidate placements in terms of expected localization accuracy. We formulate the problem with the following inputs:

- $X = \{X_i | i \in [1, N_x]\}$: Set of points in the discretized

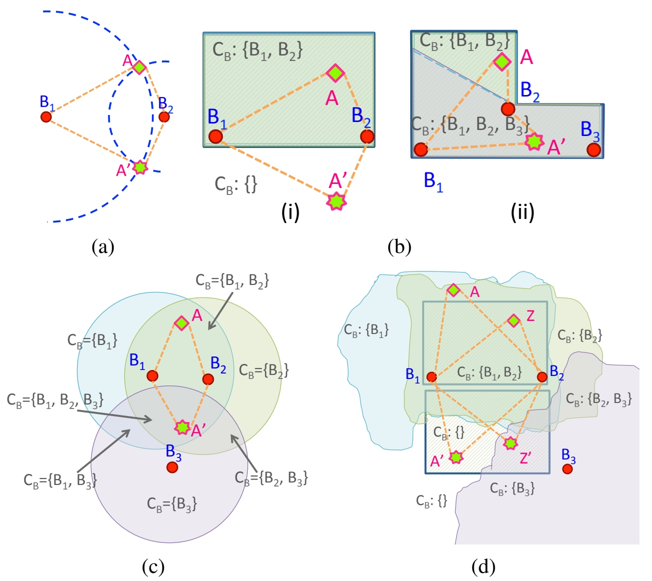


Figure 1. Localizing with two beacons based on beacon coverage set. (a) Localization ambiguity with two beacons (b) Ideal ray-tracing: Infinite range, permeability =0 (c) Unit disk coverage: Fixed finite range, permeability=1 (d) Arbitrary coverage: 0<Permeability<1

floor plan, where X_i denotes the (x, y) coordinates of the i^{th} point, and N_x is the total number of points. In our implementation, we discretize the real-world floor plans using a $20cm \times 20cm$ grid.

- $P = \{P_j | j \in [1, N_p]\}$: Set of candidate beacon locations, where N_p is the total number of locations considered as beacon candidates. In our implementation, we consider all corners or vertices in the floor plan as candidate beacon locations.
- $C_P(X_i)\forall i$: Set of candidate beacon locations that provide coverage to each location X_i, $C_P(X_i) \subseteq P$.

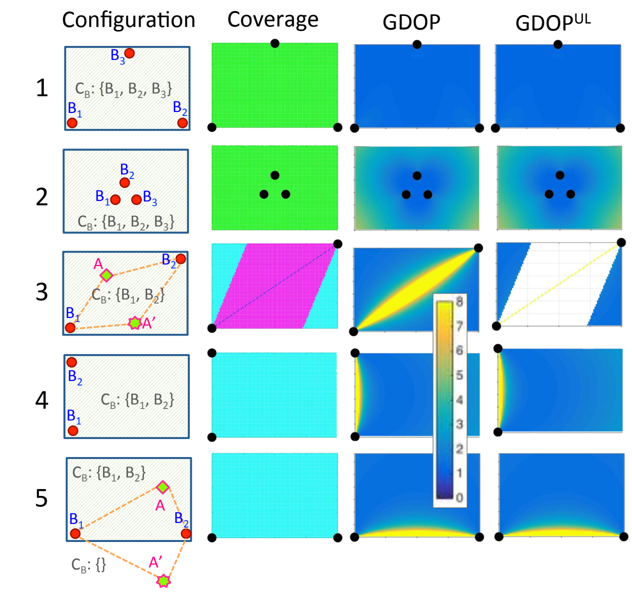
Our objective is to find a beacon configuration, or a subset of the candidate locations $B = \{B_j | j \in [1, N_b], B \subseteq P, N_b \leq N_p\}$, such that the localization accuracy at $X_i, \forall i$ satisfies certain criteria based on how we define the *quality of a beacon configuration*. The set of beacons that provide coverage to X_i, is denoted as $C_B(X_i)$, $C_B(X_i) \subseteq B$. In Section III-A, we show how we localize with two beacons by using the beacon coverage information and introduce the concept of Unique Localization. In Section III-B, we provide the background and definition of GDOP and describe its significance in quantifying the location accuracy. In Section III-C, we combine the concepts of Unique Localizability and GDOP and propose a new function $GDOP^{UL}$. The UL, GDOP and $GDOP^{UL}$ metrics are defined for a single location (X_i). In Section III-D, we propose heuristic metrics for quantifying the quality of a beacon configuration which is a function of the overall localization accuracy provided by the beacons across the entire floor plan $(X_i, \forall i)$. In Section IV, we propose a beacon placement algorithm built on these concepts.

A. Unique Localizability

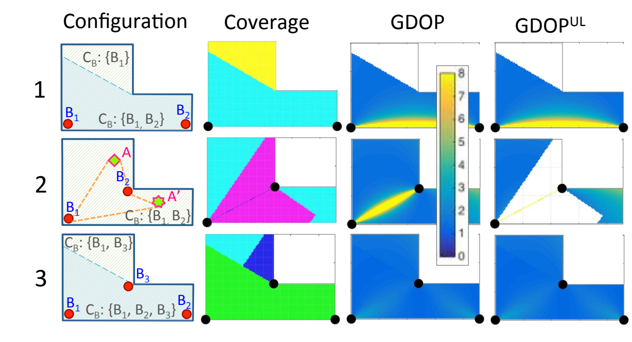
In this section, we formalize our two beacon localization using the concept of *Unique Localizability*.

Coverage models and localizing with two beacons: Consider the two-beacon scenario in Figure 1(a), with beacons B_1 and B_2, with the receiver's true location as A. The location A' also receives the same range measurements and thus we cannot disambiguate between A and A'. However, by making use of beacon coverage information, it is possible to disambiguate between the two locations under the condition that the set of beacons providing coverage is different for A and A'. We explain this with three types of coverage models (1) ray-tracing (2) unit disk and (3) arbitrary coverage, as seen in Figure 1(b)-1(d). The coverage of a beacon is technology and signal-dependent and can vary in terms of range and the penetration/permeability through walls. Figure 1(b) shows ideal ray-tracing coverage, where the beacon has no permeability through walls and the range can be considered to be infinite, or larger than the maximum distance between the beacon and the walls. This type of coverage is common in acoustic or ultrasonic ranging systems. In Figure 1(b)(i), we see that the regions inside the floor plan have $C_B = \{B_1, B_2\}$ and the regions outside the floor plan have $C_B = \{\}$. If the true location is A and the ranges from B_1 and B_2 are received, we can disambiguate the location A from A' since A' cannot receive measurements from B_1 and B_2. In Figure 1(b)(ii), all regions are covered by B_1 and B_2 and in addition, some regions are covered by B_3. Here, though both A and A' receive the same ranges from B_1 and B_2, we can disambiguate them since A' would receive a range measurement from B_3 as well. Figure 1(c) shows a unit disk coverage model where all points within a finite distance of the beacon are within its coverage. This is common for proximity-based technologies such as BLE, or a system that provides a range measurement when the SNR is above a threshold and the SNR is a function of distance from the beacon. Figure 1(d) shows an arbitrary coverage model where the range reduces when the signal penetrates through walls, as is common for several RF-based technologies. In both Figure 1(c) and Figure 1(d), we can resolve the location A from A', and Z from Z' since the two ambiguous locations have a different beacon coverage. *In this manner, if the localization solver incorporates the coverage information of the beacons, we can localize using two beacons, instead of three.* Though a simple concept, this is the key insight that results in around 33% reduction in the number of beacons required across a building floor plan by using our approach for beacon placement. Note that we currently only consider the coverage to be deterministic and binary (in coverage, out of coverage) and not statistical.

Experimental validation of ray-tracing: We adopt a 2D ray-tracing coverage model that most closely matches an acoustic ranging system. Figure 3 shows experimentally obtained results using ultrasonic beacons [16] ranging to a phone, in two different real-world deployments in buildings on our university campus. Figure 3(a) and Figure 3(b) show the coverage of a single ultrasonic beacon with measurements taken uniformly across the entire floor plan. The beacon under test is shown as a blue square. Locations that received a measurement from the beacon are shown with a circle, with radius proportional to the received range measurement. The locations in LOS and NLOS are shown in green and red respectively. In practice, the ultrasonic signals reflect off walls resulting in NLOS measurements, but the ideal ray tracing model assumes no NLOS measurements are received. Although the ray tracing assumption is not perfect, there are techniques to distinguish LOS from NLOS measurements, which we can apply before solving for the final location [16], [17].



(a) Rectangle floor plan



(b) L-shaped floor plan

Figure 2. Comparison of multiple beacon configurations. Legend for *Coverage* column is in Table I

Unique Localizability: We define a location to be *Uniquely Localizable* if in the absence of noise, it can be localized without any ambiguity, when range measurements are received from the beacons that provide coverage to the location. We define the function $UL(X_i, C_B(X_i)) \in \{0,1\}$ which has a binary output, as:

$$UL(X_i, C_B(X_i)) = \begin{cases} 1, \#C_B(X_i) \geq 3 \\ 1, \#C_B(X_i) = 2, C_B(X_i) \neq C_B(X_i') \\ 0, \#C_B(X_i) = 2, C_B(X_i) = C_B(X_i') \\ 0, \#C_B(X_i) \leq 1 \end{cases}$$

where $\#C_B$ denotes the cardinality of the set C_B and X_i' is the reflection of X_i about the line joining the two beacons in the set $C_B(X_i)$. Note that X_i' is defined only when $\#C_B(X_i) = 2$. We subsequently, use the notation $UL(X_i)$ instead of $UL(X_i, C_B(X_i))$.

Table I shows the color coding for the beacon coverage that we have used in Figure 2 and Figure 5. The fourth column shows the value of the binary UL function. Note that the algorithm does not distinguish between Class 2 and 3 points, but they are shown here for visual purposes. Class 2 points have the ambiguous location outside of the floor plan and Class 3 points have the ambiguous location inside but covered by different beacons. We see from the second column of Figure 2(a) that *configurations 1, 2, 4* and *5* have all locations to be Uniquely Localizable either due to coverage by three beacons or in case of two beacons, the ambiguous location is outside the floor plan. In ideal scenarios, any of these configurations will provide exact location estimates and the configurations with fewer beacons then would naturally be desirable. However, in realistic scenarios, these configurations would not all provide the same location accuracy across the floor plan. When range measurements are noisy, the location accuracy is dependent on the error in ranging as well as geometry between beacons and the receiver, as elaborated in the next section.

Class	Color code	Description	UL
1		$\#C_B(X_i) \geq 3$	1
2		$\#C_B(X_i) = 2$, $C_B(X'_i) = \{\}$	1
3		$\#C_B(X_i) = 2$, $C_B(X'_i) \neq C_B(X_i)$, $C_B(X'_i) \neq \{\}$	1
4		$\#C_B(X_i) = 2$, $C_B(X'_i) = C_B(X_i)$	0
5		$\#C_B(X_i) = 1$	0
6		$\#C_B(X_i) = 0$	0

Table I. COVERAGE CLASS LEGEND

B. Geometric Dilution of Precision

A useful guideline to quantify the location accuracy is the Cramér-Rao bound (CRB), the lower bound on the variance in the location that can be achieved by an unbiased location estimator [5]. The results presented in this section are derived from [6], [7], [18]. For 2D trilateration systems, it has been shown [6] that the variance of the positional error $\sigma^2(X_i)$ at location X_i, as defined by $\sigma^2(X_i) = \sigma_x^2(X_i) + \sigma_y^2(X_i)$, corresponding to the CRLB when an unbiased estimator is used is given by:

$$\sigma^2(X_i, C_B(X_i)) = \frac{\sum_{k=1}^{N_b} \sigma_{r,i}^{-2}}{\sum_{k=1}^{N_b-1} \sum_{j=k+1}^{Nb} \sigma_{r,k}^{-2} \sigma_{r,j}^{-2} A_{kj}^2}$$
$$A_{kj} = |\sin(\theta_k - \theta_j)|$$

where $\sigma_{r,k}^2$ is the variance in range measurement of beacon k, N_b is the number of beacons in $C_B(X_i)$, θ_k is the angle between B_k and the X_i.

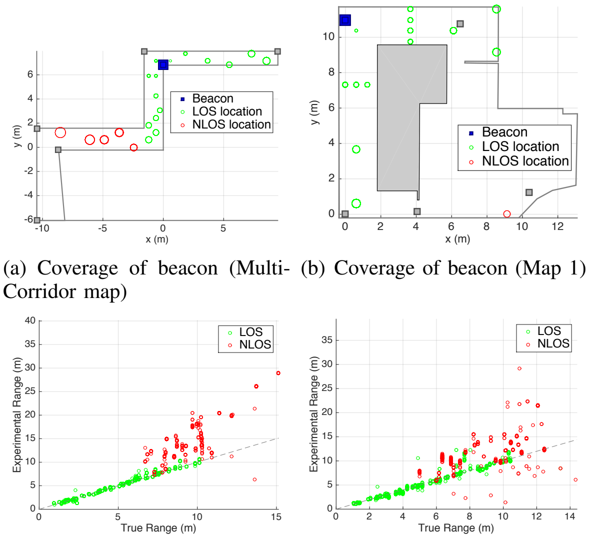
Under the assumption that the range measurements are independent and have zero-mean additive Gaussian noise with constant variance σ_r^2, this reduces to:

$$\sigma(X_i, C_B(X_i)) = \sigma_r \times \sqrt{\frac{N_b}{\sum_{k=1}^{N_b-1} \sum_{j=k+1}^{Nb} A_{kj}}}$$
$$\sigma(X_i, C_B(X_i)) = \sigma_r \times GDOP(X_i, C_B(X_i))$$

The GDOP [4] is a function of the angles between the target X_i and beacons $C_B(X_i)$, and is given by :

$$GDOP(X_i, C_B(X_i)) = \sqrt{\frac{N_b}{\sum_{k=1}^{N_b-1} \sum_{j=k+1}^{Nb} A_{kj}}}$$

The CRB is directly proportional to the GDOP and as seen in Section II, several authors have used GDOP to quantify the location accuracy. We subsequently use the notation $GDOP(X_i)$ instead of $GDOP(X_i, C_B(X_i))$. The third column *GDOP* in Figure 2 shows the GDOP map of the six floor plans. The GDOP is the worst (highest) along the line joining two beacons, and is in general better (lower) when



(a) Coverage of beacon (Multi-Corridor map) (b) Coverage of beacon (Map 1)

(c) Range measurement error (Multi-Corridor map) (d) Range measurement error (Map 1)

Figure 3. Experimental characterization of coverage and range error for acoustic beacons in real-world deployments

the regions are covered by more beacons. As a numeric example, for ranging system with standard deviation of the range measurements of $10cm$, if two beacons subtend an angle of $90°$ at a target location, the resulting GDOP is 1.414 and the 2-D location estimate would have a standard deviation of $14.14cm$.

Experimental validation of zero mean constant variance ranging noise assumption: Figure 3(c) and Figure 3(d) show the distribution of the LOS and NLOS ranges of an acoustic ranging system [16] on two floor plans, for 5-6 beacons with at least 500 range measurements taken uniformly across all regions in the floor plan. We observe that when the beacons are in LOS, the range measurements have nearly zero bias and almost constant variance. There is no appreciable change in variance of the range measurements with distance up to $10m$. This is likely due to the the SNR being sufficiently high that the distance from the beacon does not affect the ranging accuracy.

C. A modified GDOP metric for indoors

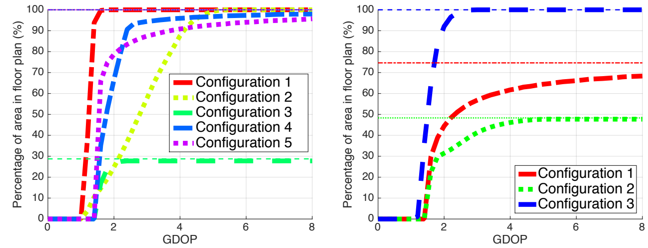
We now formalize our modified GDOP metric to use Unique Localizability, which is given by:

$$GDOP^{UL}(X_i) = \begin{cases} GDOP(X_i), & UL(X_i) = 1 \\ NaN \quad\quad\quad , & UL(X_i) = 0 \end{cases}$$

The fourth column of Figure 2 shows the $GDOP^{UL}$ metric. For most configurations, it is the same as the GDOP metric. But for *configuration 3* of Figure 2(a) and *configuration 2* of Figure 2(b), where there exist *Class 4* locations with the two-beacon ambiguity problem, the $GDOP^{UL}$ is not defined where Unique Localization cannot be achieved. These cases are now numerically handled to avoid providing a confidence on the location estimate when ambiguity exists in the solution.

D. Quality of a beacon configuration (Q)

With our modified GDOP metric, we can now compare beacon configurations like those found in Figure 2(a) and Figure 2(b). We utilize this in our toolchain which has two modes of operation where it can either optimize for Unique Localizability or optimize for both Unique Localizability and GDOP.



(a) Rectangle floor plan (b) L-shaped floor plan

Figure 4. Comparing configurations using *quality of beacon configuration* $Q_{GDOP^{UL}}$

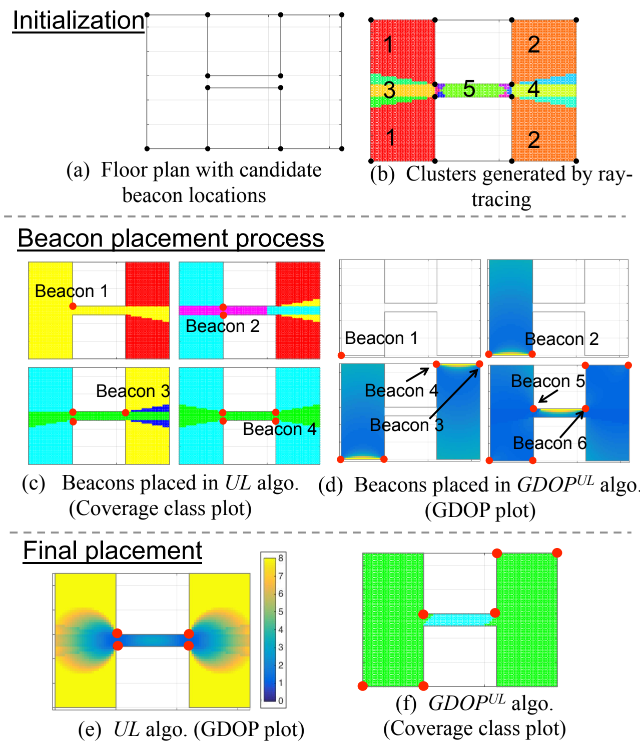
Case 1: UL-based metric Q_{UL}: In this case, we attempt to place beacons such that all regions in the floor plan are Uniquely Localizable without considering the localization accuracy. This could be required in an ideal scenario with no ranging noise where the geometry of the beacons does not affect the localization accuracy, or when the location estimate is averaged over a large number of measurements, resulting in low variance. We define the quality of the beacon configuration B across the floor plan X, as the percentage of area that is Uniquely Localized.

$$Q_{UL}(X, B) = \frac{\sum_{i=1}^{N_x} UL(X_i, C_B(X_i))}{\#X} \times 100$$

where $UL(X_i, C_B(X_i))$ is the binary function that indicates if a location X_i is Uniquely Localizable when covered by the beacons $C_B(X_i)$, as defined in Section III-A and $\#X$ is the cardinality or number of points in the floor plan.

In Figure 2(a), Q_{UL} is 100% for *configurations 1, 2, 4* and *5* and 28.7% for *configuration 3*. In Figure 2(b), the three configurations have Q_{UL} as 74.6%, 48.3% and 100% respectively.

Case 2: UL and GDOP-based metric $Q_{GDOP^{UL}}$: The $GDOP^{UL}$ metric in Section III-C is defined for a single location X_i. In order to quantify the quality of the beacon configuration across all locations in the floor plan, we use a heuristic based on the Cumulative Distribution Function (CDF) of the $GDOP^{UL}$ curve across all locations, as shown in Figure 4. For instance, to compare *configuration 1* and *configuration 2* of the rectangular room, where both configurations have three beacons, we see from Figure 4(a) that 60% of the floor plan has a GDOP less than 1.7 under *configuration 1*, and GDOP less than 3.0 under *configuration 2*. Alternately, 100% of the floor plan has a GDOP under 3.0 in *configuration 1* but only 60% of the floor plan has a GDOP less than 3.0 under *configuration 2*. Hence *configuration 1* is better since it has low overall GDOP than *configuration 2*. For *configuration 3*, where a large part of the floor plan is not Uniquely Localizable, we can see from the CDF plot that only around 27% of the floor plan is localized. We can see from these curves that *configuration 1* is better than *configuration 2* which is better than *configuration 3*. However, it is not obvious how *configuration 4* and *configuration 5* compare since the CDF curves intersect. If the goal is to have 60% of the floor plan with lower GDOP, *configuration 5* is better, but if the goal is to have 90% of the floor plan with lower GDOP, then



(a) Floor plan with candidate beacon locations
(b) Clusters generated by ray-tracing
(c) Beacons placed in UL algo. (Coverage class plot)
(d) Beacons placed in $GDOP^{UL}$ algo. (GDOP plot)
(e) UL algo. (GDOP plot)
(f) $GDOP^{UL}$ algo. (Coverage class plot)

Figure 5. Beacon Placement Process (Legend for coverage in (c),(f) is in Table I)

configuration 4 is better. In our toolchain, the designer can specify the requirement, but by default we use the area under the $GDOP_{UL}$ CDF curve. To compute the area, we need to provide an upper limit on the GDOP. For the plots shown, the upper limit is conservatively chosen to be 8.0, which corresponds to an angle of $1.8°$ between two beacons and a target. This is equivalent to considering the regions with GDOP worse than 8.0 to not be localizable. For the L-shaped room in Figure 2(b), we see from Figure 4(b) that among the configurations with 2 beacons, *configuration 1* is better than *configuration 2* even though *configuration 2* has better coverage, and *configuration 3* with 3 beacons outperforms both these configurations. The same metric can be used even if the target area is a predefined path or a finite set of locations across the floor plan. In these cases, the CDF would be computed only over the desired target locations.

IV. Beacon placement algorithm and toolchain

In this section, we present two beacon placement algorithms based on the concepts described in Section III. The implementation is illustrated by Figure 5 for a floor plan that represents two rooms connected by a corridor. The inputs provided are the floor plan, coverage model of the beacons and the set of candidate beacon locations. The floor plan and obstacles are represented as multiple polygons. We also built a MATLAB-based tool where the user can draw floor plans to aid in prototyping. Figure 5(a) shows the floor plan with candidate beacon locations shown by black circles.

The algorithm has two modes of operation based on whether we want to optimize based on Q_{UL} or $Q_{GDOP^{UL}}$. Our tools provide several design options to the user such as placing beacons until a finite number of beacons are placed or placing beacons until some stop criteria is satisfied. This stop criteria could be in the form of accuracy requirements across the floor plan, for instance $GDOP \leq 4.0$ for 90% of the regions. The stop criteria we have used for both modes of the algorithm is achieving Unique Localization across the entire floor plan.

Step 1: Initialization

- Discretize the floor plan to generate X.
- Apply the beacon coverage model for each P_j to generate $C_P(X_i)$, the set of candidate beacon locations providing coverage of each point on the floor plan.
- Perform clustering on X such that all X_i that have the same beacon coverage, $C_P(X_i)$ belong to one cluster.
- For each cluster, assign:
  - *Localization Status* $= 0$
  - *Size* = number of points in cluster
- Initialize *Selected Beacons* = $\{\}$

Figure 5(b) shows the clusters generated by ideal ray-tracing coverage with five of the largest clusters labeled.
Repeat Steps 2-4 until *Localization Status* of all clusters=1. In every iteration, one beacon is placed.

Step 2: Select cluster
Among the clusters with *Localization Status* = 0, select the cluster with largest *Size*

Step 3: Select subset of candidate locations
Among all candidate beacon locations, select the subset $C_P(X_i)$, where X_i is any point in the cluster. Note that all points in the cluster are covered by the same candidates.

Step 4: Among the subset of candidates, select the candidate that maximizes criteria

- The selection criteria depends on whether we are optimizing for UL or $GDOP^{UL}$. Further, since these metrics are not defined for single-beacon case, we have a different criteria when N_b, the number of beacons already covering the cluster is zero.

(1) UL *and Nb=0*: Select candidate with maximum coverage.
(2) $GDOP^{UL}$ *and Nb=0*: Select candidate with maximum average distance from other candidate locations for this cluster, to provide good geometry.
(3) UL *and* $Nb \neq 0$: Select candidate that maximizes the *quality of beacon configuration* Q_{UL} or the percentage of area Uniquely Localized.
(4) $GDOP^{UL}$ *and* $Nb \neq 0$: Select candidate that maximizes the *quality of beacon configuration* $Q_{GDOP^{UL}}$ or the area under the CDF of the $GDOP^{UL}$ curve.

- Add selected candidate beacon location to the set *Selected Beacons*.

Figure 5(c) shows the placement of beacons in the UL mode with the coverage classes shown and Figure 5(d) shows the placement of beacons in the $GDOP^{UL}$ mode with the $GDOP^{UL}$ values shown. As can be seen, the selection of the first beacon (to localize the cluster labeled 1 in Figure 5(b)) and subsequent beacons is different for both the algorithms.
Step 5: Re-evaluate clusters

- It is possible that a cluster is partially localized by the ambiguity being resolved when the new beacon is placed. In that case, split it into two clusters before the next step. Assign *Size* and *Localization Status* for the new clusters.

- For all clusters: If all the points in the cluster satisfy the stop criteria (achieving Unique Localizability), assign *Localization Status* of cluster=1.

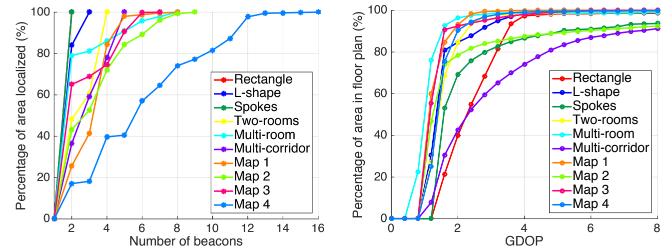
As we can see from Figure 5(c), the entire floor plan is localized with only 4 beacons while optimizing for Unique Localization. However, the beacons are clustered close together and GDOP of the final beacon placement, shown in Figure 5(e) is poor. On the other hand, Figure 5(d) shows the placement while optimizing for GDOP as well, and places two additional beacons.The final configuration has a good GDOP and coverage across the floor plan, as seen in Figure 5(d) and Figure 5(f) respectively. If the design requirement was to only place 4 beacons, the first 4 beacons would have been placed, with the corridor area not localizable but good GDOP in the two rooms.

V. Evaluation

We evaluated our beacon placement algorithm in simulations on 10 floor plans, which are listed in Table II. The first five are smaller floor plans constructed to represent different geometries that could occur within larger floor plans. The next four are real-world floor plans in buildings on our university campus, and the last is a floor plan drawn using our toolchain. The *Rectangle* and *L-shape* floor plans are shown in Figure 2, the *Two-rooms* floor plan is shown in Figure 5 and the remaining seven floor plans are shown in Figure 7.

The first column of Table II, *3-beacon (Minimal)* denotes the minimal number of beacons if all regions are to be covered by at least 3 beacons as required by trilateration. The second column *UL (Minimal)* denotes the minimal number of beacons required under the proposed scheme where we localize with 2 beacons. The results for both of these columns are obtained by brute-force by iterating through every possible beacon configuration. Note that this is not practical for larger floor plans. We see that the proposed scheme results in reducing the number of beacons by 22-60% (33% on an average) *for these floor plans*. The most significant improvement is seen for the *Spokes* floor plan, shown in Figure 7. This is not typical of indoor spaces but we selected this to show when the proposed scheme is most effective. The floor plan is generated such that all points are in LOS of the left bottom and right bottom corners. We can extend this floor plan to generate infinite spokes, and the 3-beacon scheme would require infinite beacons, whereas the proposed UL scheme would require only 2 beacons.

The third column shows the number of beacons placed while optimizing for UL and the fourth column shows the number of beacons placed while optimizing for the $GDOP^{UL}$. We see that the UL algorithm places the same number of beacons as the minimal in most cases, with 1 or 2 additional beacons in some cases (on average 5% more beacons than minimal). The $GDOP^{UL}$ algorithm usually places the same number of beacons as UL, in worst-case it places 2 more beacons. In Figure 7, we show the resulting GDOP map of the final beacon placement with the algorithm in the $GDOP^{UL}$ mode. Due to lack of space, we have not shown the minimal beacon placement and result of UL algorithm for these seven floor plans but the number of beacons for both are shown in Table II.



(a) Area localized with addition of every beacon (UL algo.)

(b) GDOP CDF curve for final placement ($GDOP^{UL}$ algo)

Figure 6. Validation of algorithm's greedy approach and stop criteria

Figure 6 shows the effect of the design choices we have used in the algorithm. Figure 6(a) validates the greedy approach we have adopted, where we iteratively select beacons to localize the largest cluster. For each of the floor plans, we can see the $\%$ of area Uniquely Localized as an additional beacon is added. For instance in Map 4, 90% of the area is localized with 11 beacons and an additional 5 beacons are required for the remaining 10% of the area. For practical purposes, it may be sufficient to have coverage in 90% of the region, since often tracking or filtering would be used in the location solver. Figure 6(b) shows the final CDF of the $GDOP^{UL}$ when the algorithm is in the $GDOP^{UL}$ mode. The final quality of beacon configuration varies across floor plans since the stop criteria is all regions being Uniquely Localizable. We could also have specified a different criteria based on GDOP, such as 90% of the floor plan having a GDOP less than 4.0. We see that across the floor plans, the *Multi-room* floor plan has the overall highest quality of beacon configuration but our algorithm placed 2 beacons more than the minimal number required. The worst quality of beacon configuration is for multi-corridor floor plan and we can see from Figure 7 that the two end corridors have a high GDOP.

In summary, the proposed scheme based on Unique Localizability places between 22% to 60% (33% on average) fewer beacons than a typical trilateration-based scheme. The proposed algorithm in UL mode usually achieves the minimal placement and on an average places 5% more beacons than minimal, but 29% fewer beacons than typical. The proposed algorithm in $GDOP^{UL}$ mode places on an average 14% more beacons than minimal, 23% fewer beacons than typical, and provides a much better quality (GDOP) of beacon placement. The final quality of beacon placement and the number of beacons required varies with the floor plan geometry. The quantitative results apply even for larger floor plans at building-scale since they can be represented as a union of smaller floor plans.

VI. Conclusions

In conclusion, this paper proposes a beacon placement technique and a new metric to compare different indoor placements. Our approach requires a map of the interior space, along with a propagation model for beacons that provide range data that can be used to estimate coverage regions of a beacon location. We show how it is possible to localize with two beacons instead of three beacons and introduce the concept of *Unique Localizability*. We introduce a new formulation of GDOP that incorporates the Unique Localizability information and use these metrics to compare and optimize beacon placements in indoor environments. We integrated our proposed

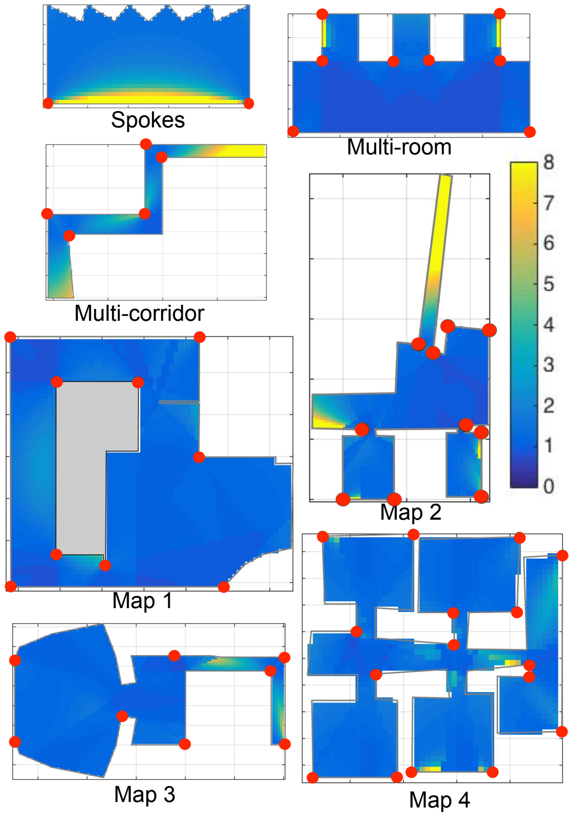


Figure 7. Beacon Placement result and corresponding GDOP for subset of the floor plans with algorithm optimizing for $GDOP^{UL}$

Floor Plan	Beacon Placement Method			
	3-beacon Minimal	**UL Minimal**	**UL Algorithm**	$GDOP^{UL}$ **Algorithm**
Rectangle	3	2	2	2
L-shape	4	3	3	3
Spokes	5	2	2	2
Two-rooms	6	4	4	6
Multi-room	9	6	8	8
Multi-corridor	7	5	5	5
Map 1	11	8	8	9
Map 2	12	8	9	10
Map 3	9	7	7	8
Map 4	21	15	16	16

Table II. NUMBER OF BEACONS PLACED

Quality of Beacon Configuration metrics into a toolchain that can automatically place or refine beacon locations in order to improve accuracy, coverage and/or beacon number. The tool exports a GDOP and coverage map for the final beacon configuration that can be used to visualize expected performance across a floor plan. We believe that these types of automated provisioning tools are going to become critical if indoor localization services become pervasive. In the future, we intend to extend our system to different beacon propagation models like UWB transmissions and validate at scale in real deployments.

REFERENCES

[1] Nanotron: http://www.nantron.com/ (viewed 10/12/2015).

[2] Decawave: http://www.decawave.com/ (viewed 10/12/2015).

[3] *802.11v-2011 - IEEE Standard for Information technology– Local and metropolitan area networks– Specific requirements– Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) specifications Amendment 8: IEEE 802.11 Wireless Network Management*. IEEE Computer Society, 2011.

[4] Pratap Misra and Per Enge. *Global Positioning System: Signals, Measurements and Performance Second Edition*. Lincoln, MA: Ganga-Jamuna Press, 2006.

[5] Steven M Kay. Fundamentals of statistical signal processing, volume i: estimation theory. 1993.

[6] Maurizio Spirito et al. On the accuracy of cellular mobile station location estimation. *Vehicular Technology, IEEE Transactions on*, 50(3):674–685, 2001.

[7] Neal Patwari, Joshua N Ash, Spyros Kyperountas, Alfred O Hero III, Randolph L Moses, and Neiyer S Correal. Locating the nodes: cooperative localization in wireless sensor networks. *Signal Processing Magazine, IEEE*, 22(4):54–69, 2005.

[8] Adrian N Bishop, Barış Fidan, Brian DO Anderson, Kutluyıl Doğançay, and Pubudu N Pathirana. Optimality analysis of sensor-target localization geometries. *Automatica*, 46(3):479–492, 2010.

[9] Naveed Salman, Hemat K Maheshwari, Andrew H Kemp, and Mounir Ghogho. Effects of anchor placement on mean-crb for localization. In *Ad Hoc Networking Workshop (Med-Hoc-Net), 2011 The 10th IFIP Annual Mediterranean*, pages 115–118. IEEE, 2011.

[10] Onur Tekdas and Volkan Isler. Sensor placement for triangulation-based localization. *Automation Science and Engineering, IEEE Transactions on*, 7(3):681–685, 2010.

[11] Ross Allen, Neil MacMillan, Dimitri Marinakis, Rahnuma Islam Nishat, Raziur Rahman, and Sue Whitesides. The range beacon placement problem for robot navigation. In *Computer and Robot Vision (CRV), 2014 Canadian Conference on*, pages 151–158. IEEE, 2014.

[12] Zheng Yang and Yunhao Liu. Quality of trilateration: Confidence-based iterative localization. *Parallel and Distributed Systems, IEEE Transactions on*, 21(5):631–640, 2010.

[13] Tilman Leune, Thorsten Wehs, Marijn Janssen, Christian Koch, and Gerd von Colln. Optimization of wireless locating in complex environments by placement of anchor nodes with evolutionary algorithms. In *Emerging Technologies & Factory Automation (ETFA), 2013 IEEE 18th Conference on*, pages 1–6. IEEE, 2013.

[14] Javier O Roa, Antonio Ramón Jiménez, F Seco, José Carlos Prieto, and J Ealo. Optimal placement of sensors for trilateration: regular lattices vs meta-heuristic solutions. In *Computer Aided Systems Theory–EUROCAST 2007*, pages 780–787. Springer, 2007.

[15] Nicolaj Kirchhof. Optimal placement of multiple sensors for localization applications. In *Indoor Positioning and Indoor Navigation (IPIN), 2013 International Conference on*, pages 1–10. IEEE, 2013.

[16] Patrick Lazik, Niranjini Rajagopal, Bruno Sinopoli, and Anthony Rowe. Alps: A bluetooth and ultrasound platform for mapping and localization. In *The 13th ACM Conference on Embedded Networked Sensor Systems (SenSys 2015)*, SenSys '15, 2015.

[17] Marilynn P Wylie and Jack Holtzman. The non-line of sight problem in mobile location estimation. In *Universal Personal Communications, 1996. Record., 1996 5th IEEE International Conference on*, volume 2, pages 827–831. IEEE, 1996.

[18] Neal Patwari, Alfred O Hero III, Matt Perkins, Neiyer S Correal, and Robert J O'dea. Relative location estimation in wireless sensor networks. *Signal Processing, IEEE Transactions on*, 51(8):2137–2148, 2003.