PS 10 FILIPPOS LYMPEROPOULOS

1. Given that $\dot{y}+y=x$, we perform the laplasse transform on and get

$$Y(s)\cdot s + Y(s) = X((s))$$

$\Rightarrow$ Hence, given that $\dot{y}+y=x \overset{\mathcal{L}}{\Rightarrow} Y(s)\cdot s+Y(s)=X(s)$, we can get

$$H(s)=\frac{Y(s)}{X(s)}=\frac{1}{1+s}$$

$\Rightarrow$ To get the step responce in the s-domain we perform again the transform and get:

$$\int_0^{\infty}\frac{1}{s+1}e^{-st}dt=\frac{1}{s(s+1)}.$$

Using partials fractions we can proceed as follows:

$$\frac{1}{s(s+1)}=\frac{A}{s}+\frac{B}{s+1} \Rightarrow \boxed{1=A(s+1)+B(s)}$$

For $\boxed{s=-1 \Rightarrow B=-1}$ and for $\boxed{s=0 \Rightarrow A=1}$

Therefore, we get $\frac{1}{s(s+1)}=\frac{1}{s}-\frac{1}{s+1}$. If we perform the inverse Laplasse on that we get that $\Rightarrow \frac{1}{s}\overset{\mathcal{L}^{-1}}{\Rightarrow}u(t)$, $\frac{1}{s+1}\overset{\mathcal{L}^{-1}}{\Rightarrow}u(t)e^{-t}$

$$\Rightarrow \boxed{y(t)=u(t)-u(t)e^{-t} \Rightarrow y(t)=u(t)(1-e^{-t})}$$

2. (A) $\dfrac{Y(s)}{Y_{sp}(s)}$, if $k(s) = \dfrac{k_I}{s}$, for any $H(s)$. We proceed as,

$$\lim_{s \to 0} \frac{Y(s)}{Y_{sp}(s)} = \frac{kH(s)}{1+kH(s)} = \frac{\frac{k_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)}{1+\frac{k_I}{s}\left(\frac{1/\tau}{s+1/\tau}\right)} = \frac{k_I\left(\frac{1/\tau}{s+1/\tau}\right)}{s+k_I\left(\frac{1/\tau}{s+1/\tau}\right)} =$$

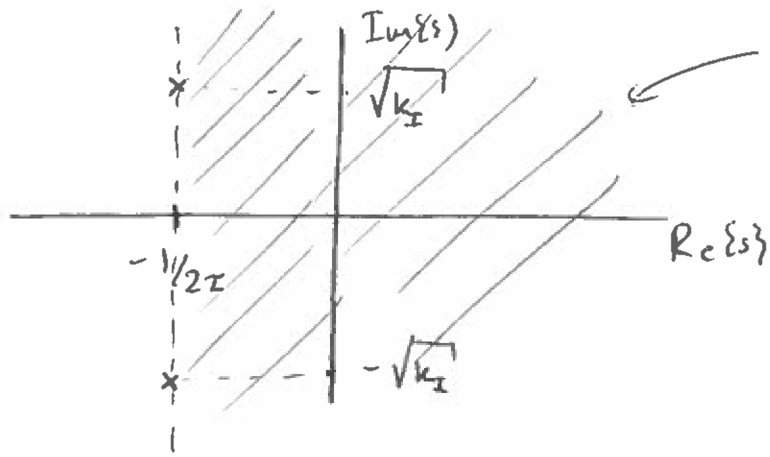
$$= \frac{k_I/\tau}{s(s+1/\tau)+k_I/\tau} = \boxed{\frac{k_I/\tau}{s^2+\frac{s}{\tau}+\frac{k_I}{\tau}}}$$, with no coefficient in the high order term. (s^2)

The result for $\lim\limits_{s \to 0} \dfrac{k_I/\tau}{s^2+\frac{s}{\tau}+\frac{k_I}{\tau}} = 1$, so it doesn't depend on k_I.

(B) Remembering the wonderful equation $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$, we proceed as follows by solving the quadratic

$$s_1, s_2 = \frac{-1/\tau \pm \sqrt{\frac{1}{\tau^2}-\frac{4k_I}{\tau}}}{2} = -\frac{1}{2\tau} \pm \sqrt{\frac{1}{\tau^2}-\frac{k_I}{\tau}} \Rightarrow$$ given that $k >> 1/\tau$

$$\Rightarrow \boxed{s_1, s_2 = -\frac{1}{2\tau} \pm j\sqrt{k_I}}$$, for $\tau > 0$



ROC, for $[-\frac{1}{2\tau}, +\infty)$ defining the system as stable.

3) A) The bode plots generated prove that the system is a high pass. The step responce converges, as the step responce decays. Hence, for $\frac{1}{1+1/s}$, we get the images as given at the attached images.

B) Looking at $\frac{s}{s^2+100s+1} = \frac{1}{s+100+\frac{1}{s}}$, we observe that the system is a bandpass filter. The related images indicated the results. Once again, our step responce decays over time, proving a covergent ROC.

C) $\frac{1}{s+1+\frac{1}{s}}$, resembles a high-pass filter, however, it reminds of a sharp band-pass fiter, with a short bandwidth. Looking at the step responce, the integral once again converges.

D) $\frac{1}{s+0.01+\frac{1}{s}}$ is a very interesting case. The step responce has a perfectly oscillatory behavior leading to zero value. The poles are at a very close position close at zero, and the ROC involves both $Re\{s\}<0$ and $Re\{s\}>0$, yielding that and it's stable.

E) In the case of $\frac{s^2-0.01s+1}{s^2+0.01s+1}$, we observe that the system is pretty unstable. The poles are at zero and the bode plot is a straight line.

F) In that case, looking at the bode plot and observing a drop at a specific value of ω at 1. It is really important to note that the step responce osculates at a very small amplitude variation that sets it to zero

4. (A) $H(s) = \dfrac{1}{s^2 - 0.01s + 1}$

$$\Rightarrow s_1, s_2 = \frac{0.01 \pm \sqrt{0.01^2 - 4}}{2} \Rightarrow s_1, s_2 = \frac{0.01}{2} \pm j\,\frac{\sqrt{0.01^2 - 4}}{2}$$

Using the combine plot method we get a pretty interesting result. The step responce is oscillating increasingly as time goes by. The poles are exactly at 0 ($Re\{s\}=0$), leading to a peak of the bodeplot at 0 Hz.

(B) The system still oscillates and gets bigger. The effect of using proportional control is that the amplitude of this oscillation gets smaller and hence closer to the smaller version of the system. Still the poles are ~~at zero and~~ on the imaginary axis they are at $\pm j$.

(C) Using integral control we observe that the bode plot resembles that of a low pass filter with a peak at higher frequency. The step responce is static, while the pole-zero map had poles at $(0,0)$ and $(0,j)$.

(D) Finally, analyzing the effect of a differential control, we observe that the oscillation decreases exponentially, while looking at the zero-pole map, we observe that the poles are at $-j$ and $+j$ in the y-axis ($Im\{s\}$), and shifted at the left of the y-axis, in the negative values of $\{s\}$, yielding a converging ROC.

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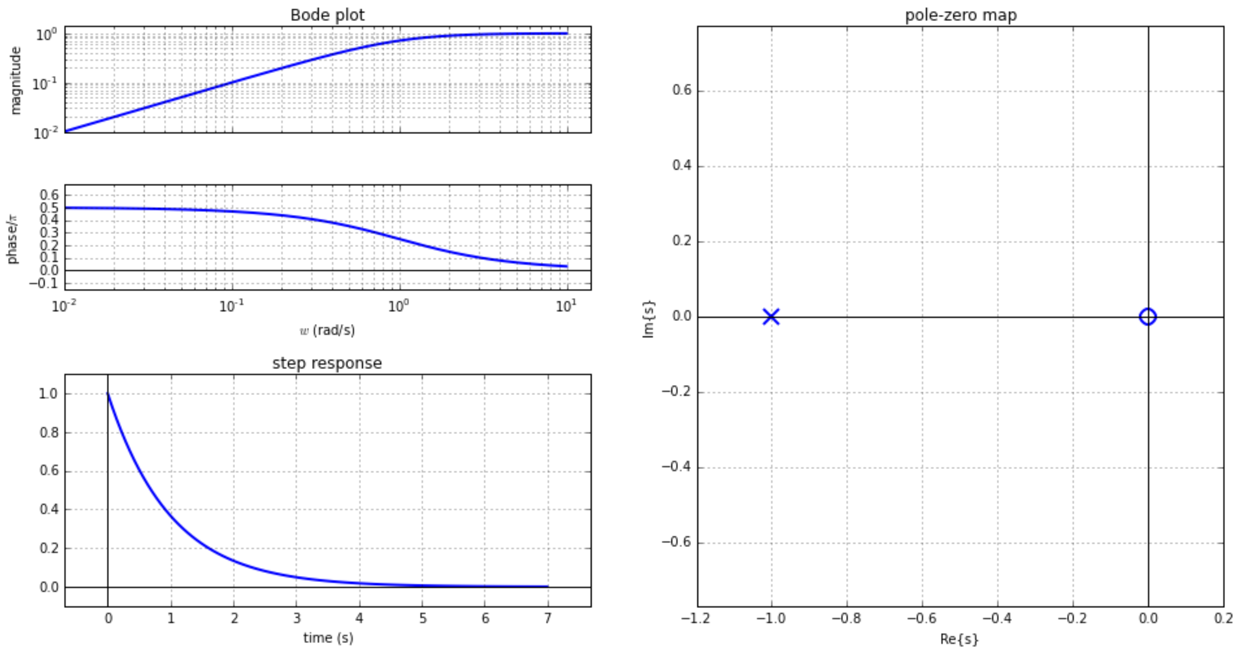
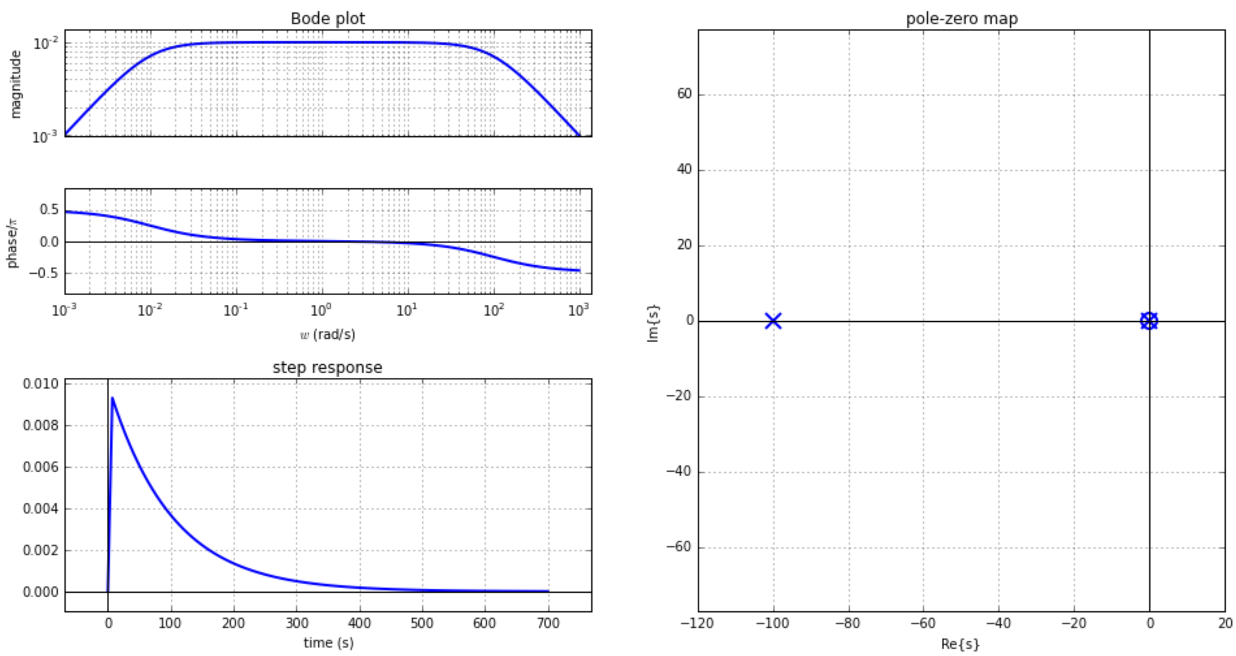
3.A.



3.B.



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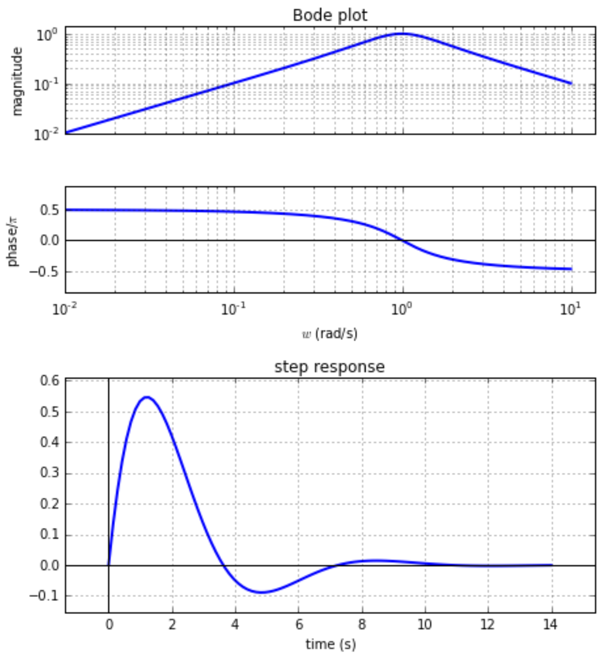
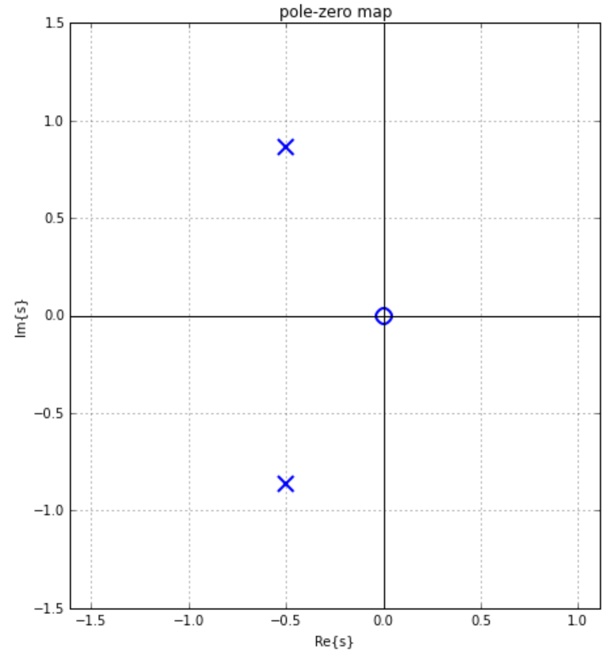
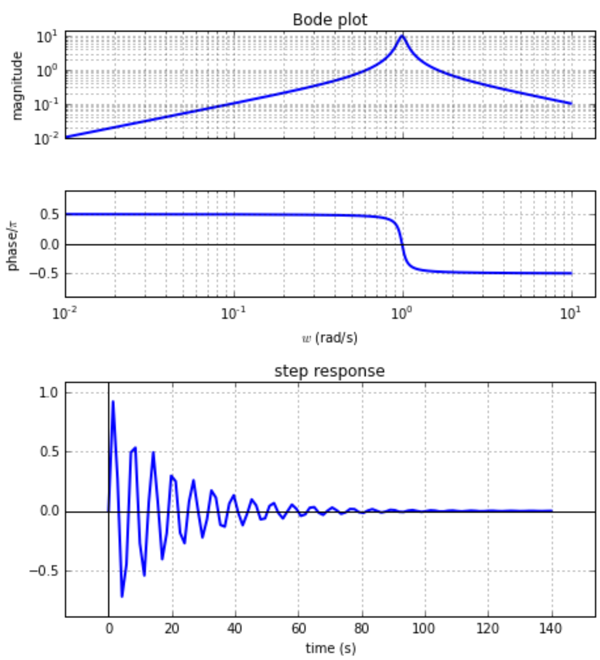
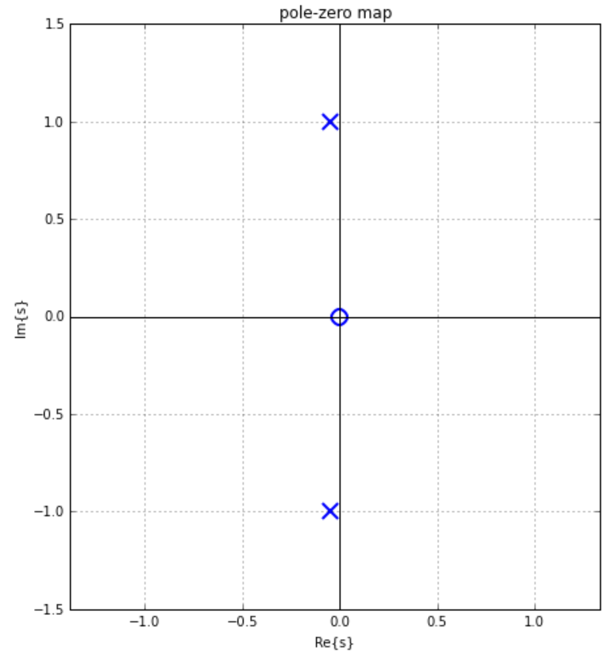
3.C.





3.D.





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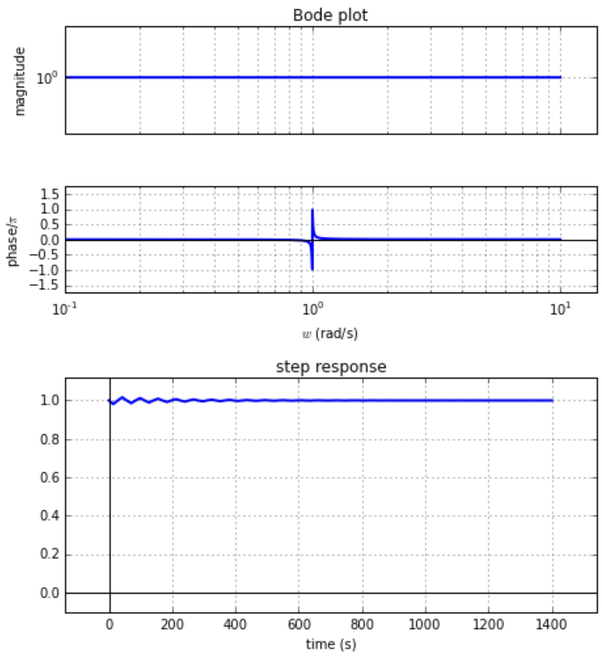
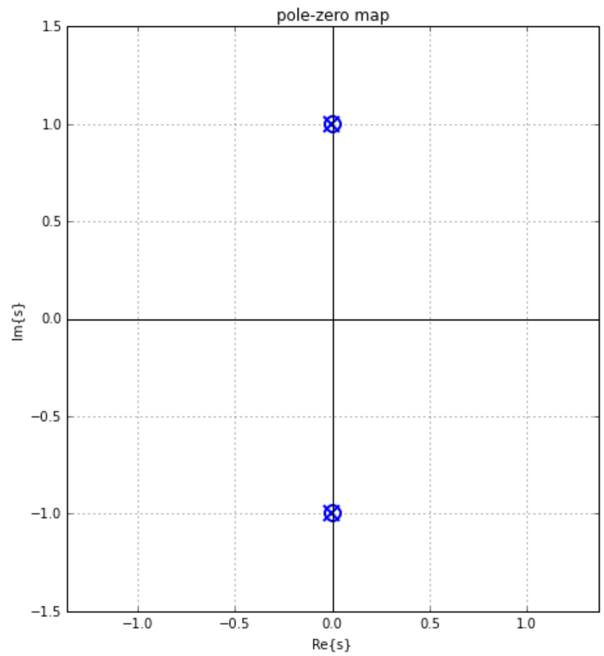
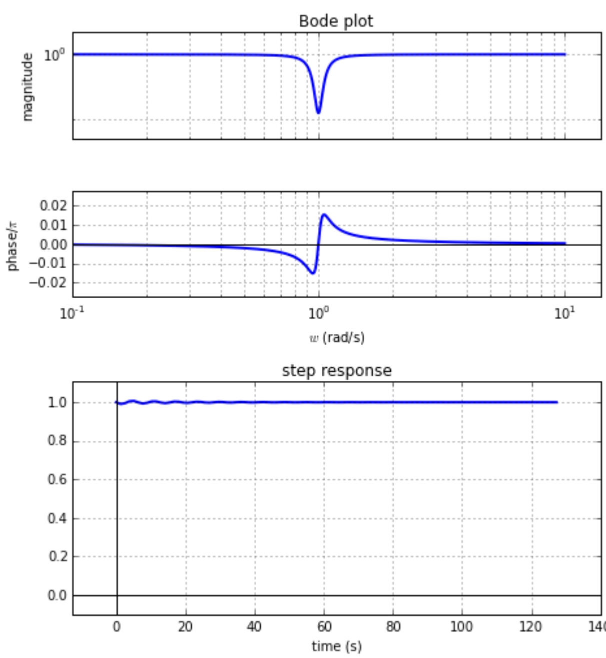
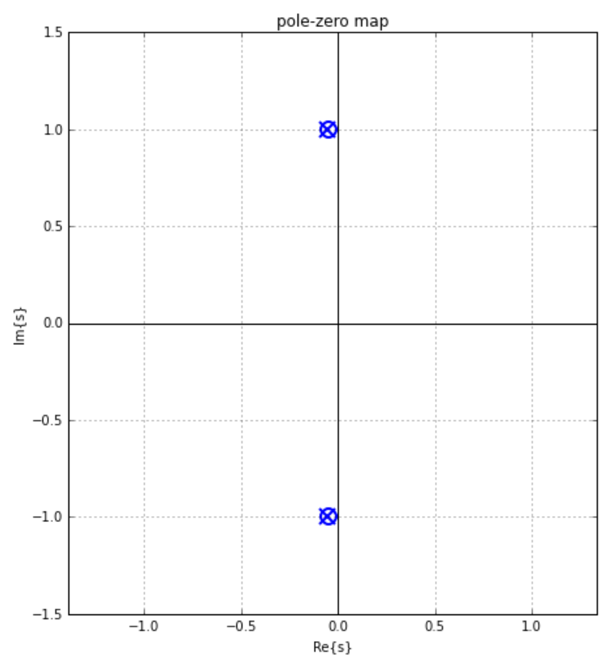
3.E.





3.F.





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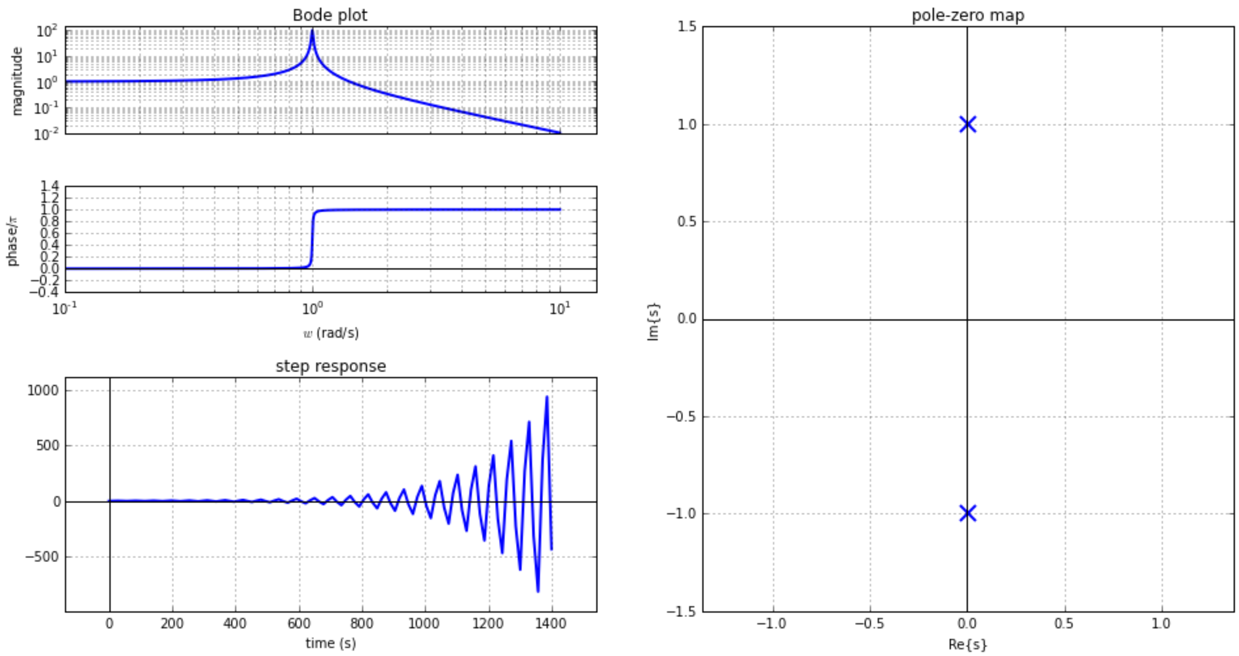
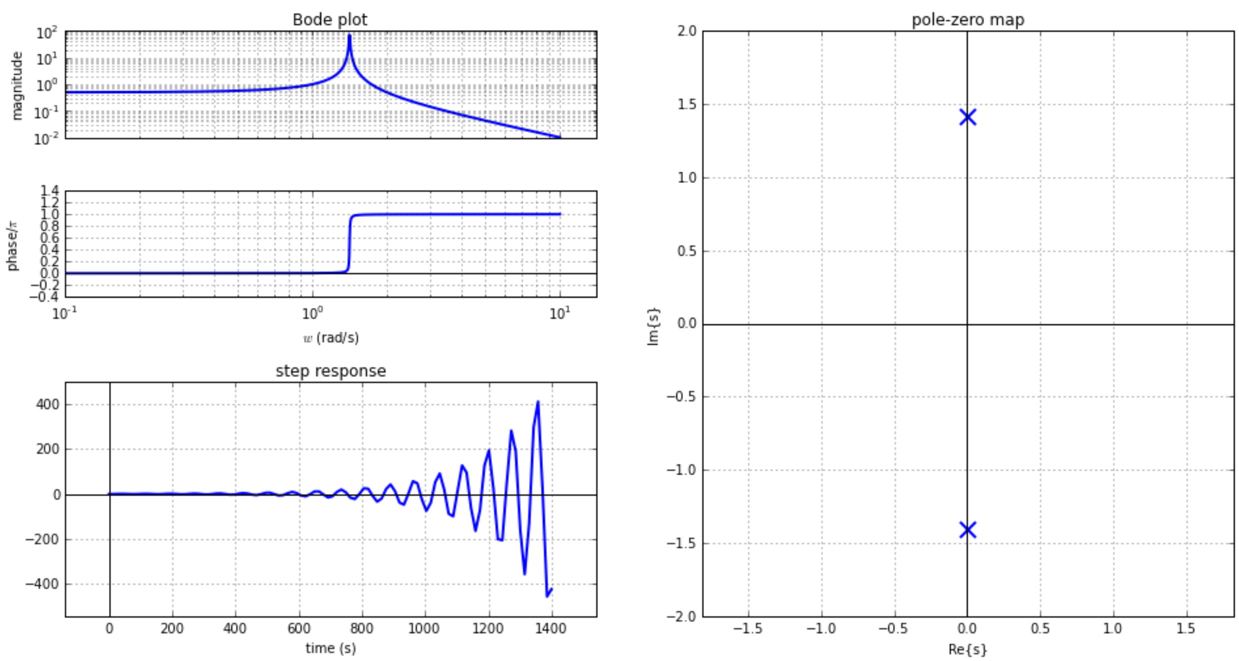
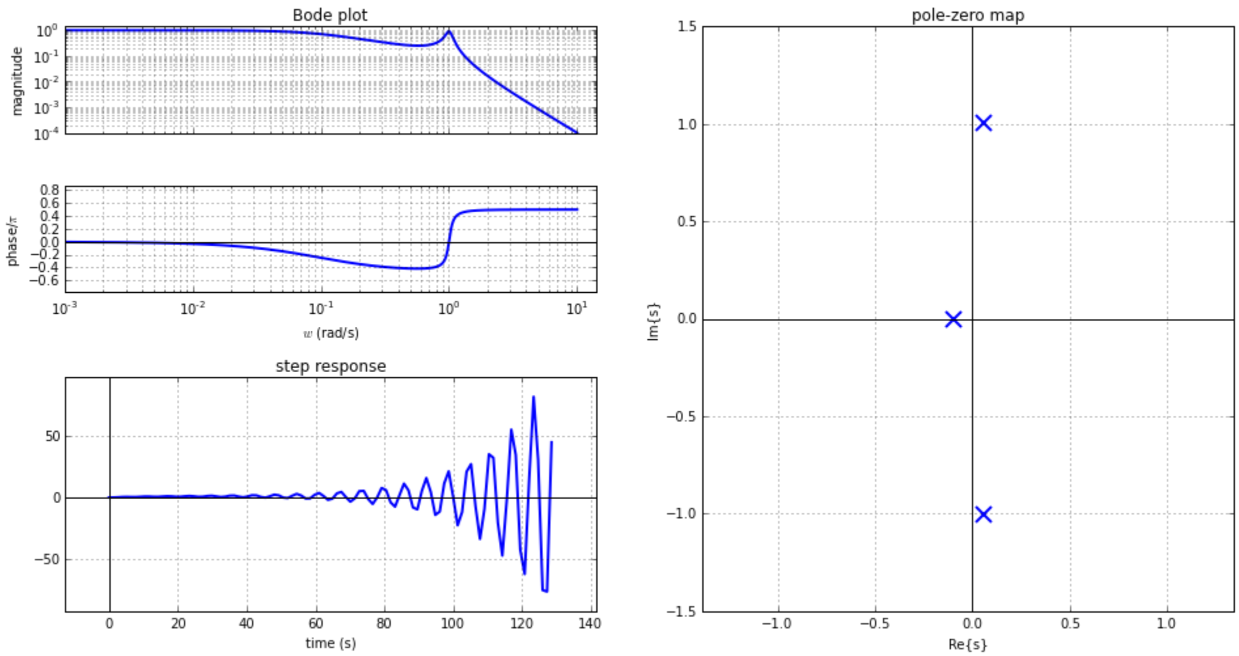
4.A.



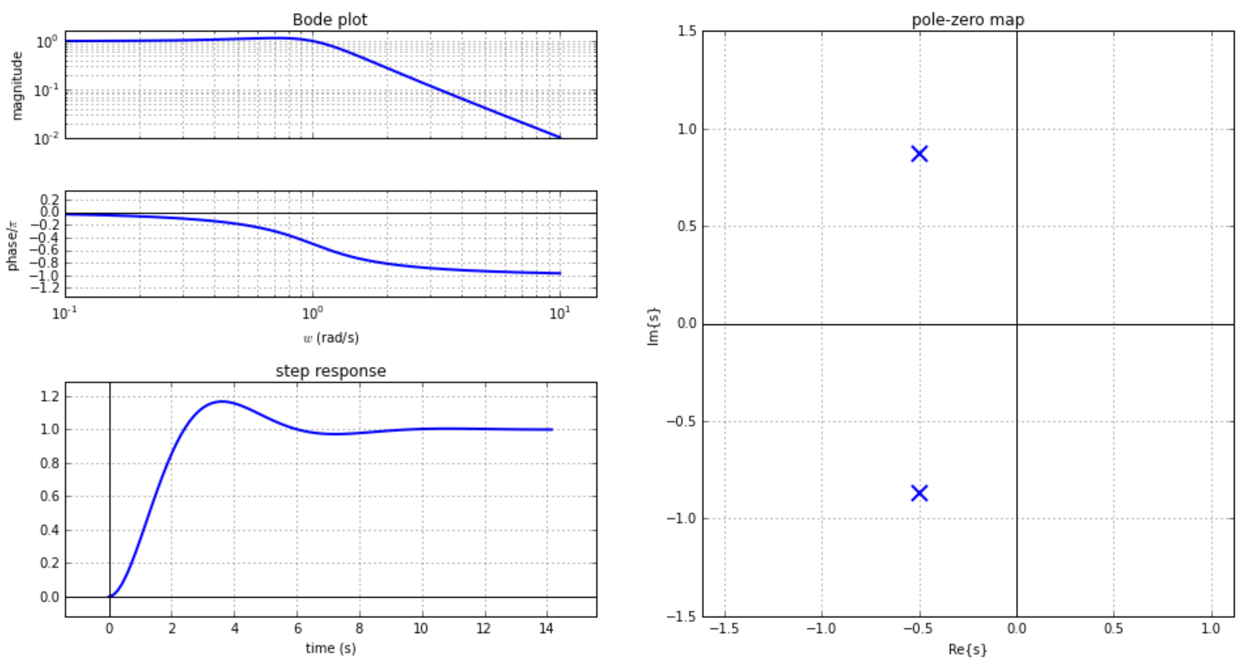
4.B.



4.C.


Bode plot
magnitude
phase/π
w (rad/s)
step response
time (s)
pole-zero map
Im{s}
Re{s}

4.D.


Bode plot
magnitude
phase/π
w (rad/s)
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time (s)
pole-zero map
Im{s}
Re{s}