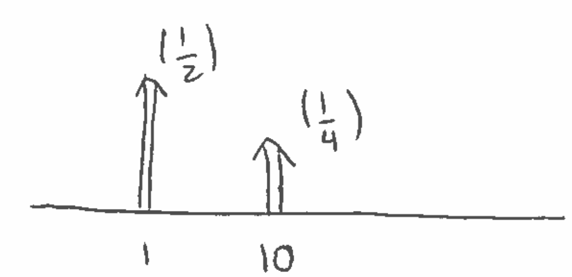
PS06 - Filippos Lymperopoulos

① One can define the concept of impulse responce as the reaction of a system - commonly reffered to as dynamic responce - to an external change (impulse). Zooming into the case of the audio signal of the gunshot (impulse), and the subsequent recording of the the gunshot/violin in the common range (impulse responce), we can point out that several interesting assumptions have been made. The gunshot is an impulse. Since the sound responce of a firing range is an LTI system, the effect this system has on the gunshot and the violins respective recording, is the same. As a resut, convolving a signal with an impulse response will modify the initial signal by that responce, in our case the gun range, so the sound sound of the violin sounds as if it is in the shooting range

② $y(t)=\frac{1}{2}(x(t-1))+\frac{1}{4}x(t-10) \Rightarrow h(t)=\frac{1}{2}\delta(t-1)+\frac{1}{4}\delta(t-10)$

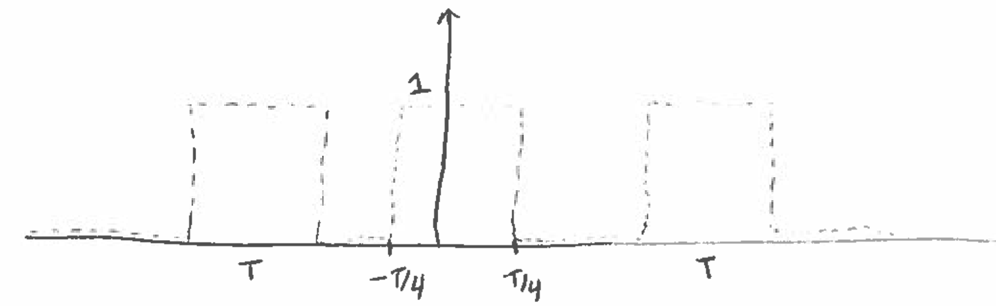


This channel being studied has two impulse responces.

It can be characterized as an echo channel, because if we convolve $x(t)$ with it we will get a result that has an output, where there is a time delay of 1 second at which we can hear half the amplitude of the signal and there is another echo $1/4^{th}$ as strong from the original at $t=10$.

③ Fourier series representation for the square wave depicted below:

(a)



Looking at the square wave above, with period T, we can examine the domain $T/4$ to $5T/4$.

$$C_k = \frac{1}{T}\int_{T/4}^{5T/4} f(t)\cdot e^{-j\omega k t}\,dt, \quad \text{where} \quad \omega = \frac{2\pi}{T}$$

We can compute, using the online software tool "Wolframalpha" that

$$C_k = \begin{cases} \frac{1}{j\pi k}, & \text{when } k = 2n+1,\ n\in\mathbb{Z} \\ 0, & \text{when } k = 2n,\ n\in\mathbb{Z} \\ \frac{1}{2}, & \text{when } k = 0 \end{cases}$$

However, we can look it in terms of sinc function calculation:

Taking the integral from $[-T/2, T/2]$, we cover one period. However, we can limit that from $[-T/4, T/4]$, as the remaining evaluates to zero. Hence,

$$C_k = \frac{1}{T}\int_{-T/4}^{T/4} e^{-j\frac{2\pi}{T}kt}\,dt \Rightarrow C_k = \frac{1}{T}\cdot\frac{1}{-j\frac{2\pi k}{T}}\, e^{-j\frac{2\pi}{T}kt}\Big|_{-T/4}^{T/4} \Rightarrow C_k = -\frac{1}{2\pi j k}\left(e^{-\frac{\pi k j}{2}} - e^{\frac{\pi k j}{2}}\right)$$

$$\Rightarrow C_k = \frac{+1}{2\pi j k}\sin\left(-\frac{k\pi}{2}\right)\cdot 2j \Rightarrow C_k = \frac{+\sin\left(-\frac{k\pi}{2}\right)}{\pi k} \Rightarrow C_k = \frac{\sin\left(-\frac{k\pi}{2}\right)}{\pi k} \Rightarrow$$

$$\Rightarrow \boxed{C_k = \frac{\mathrm{sinc}\left(\frac{k}{2}\right)}{2}}$$

③ (b) Graphs depicted below

(c) Points of discontinuity are great. However, at each point there exist high frequency sine waves with amplitude that decrease significantly. As one adds more and more high frequency components (sine waves), the error decreases approaching zero, however, a discontinuity will always be present. A wave with infinite frequency could approach that discontinuity but it is not prevalent, not possible this can happen

④ (a) Given that $y(t) = x(t-T_1)$, where $|T_1| < T$, knowing that

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot e^{-j\omega k t}\,dt \quad , \text{ where } \omega = \frac{2\pi}{T}$$

$$= \frac{1}{T}\int_{-T/2-W}^{T/2-W} x(t)\, e^{-j\omega k t}\,dt$$

We can look at the case where $W = T_1 \Rightarrow$ we can look that

$$C_k' = \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} x(t-T_1)\, e^{-jk\frac{2\pi}{T}t}\,dt \Rightarrow \text{setting } u = t-T_1 \Rightarrow \frac{du}{dt} = 1 \Rightarrow \boxed{du = dt}$$

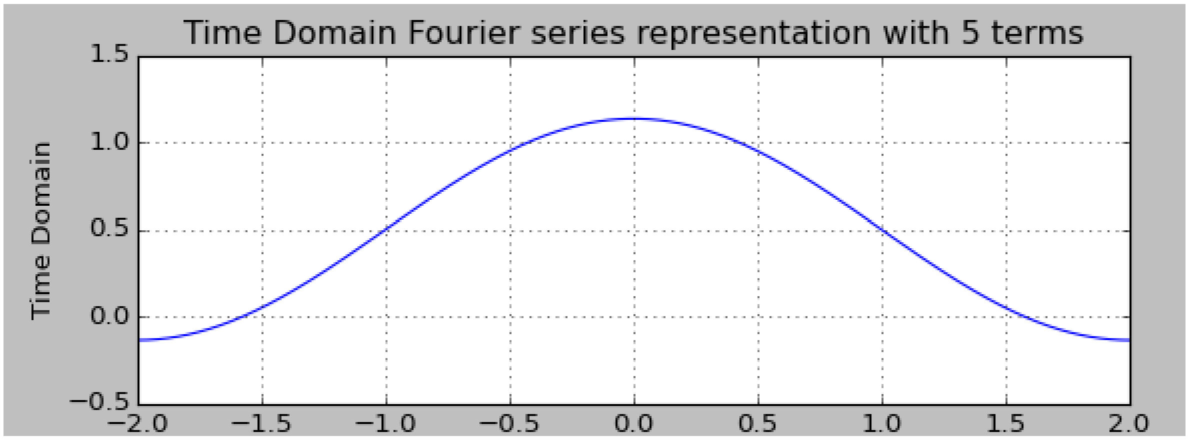
$$\Rightarrow C_k' = \frac{1}{T}\int_{-T/2-T_1}^{T/2-T_1} y(u)\, e^{-jk\frac{2\pi}{T}ku} \cdot e^{-j\frac{2\pi}{T}kT_1}\,du \Rightarrow$$

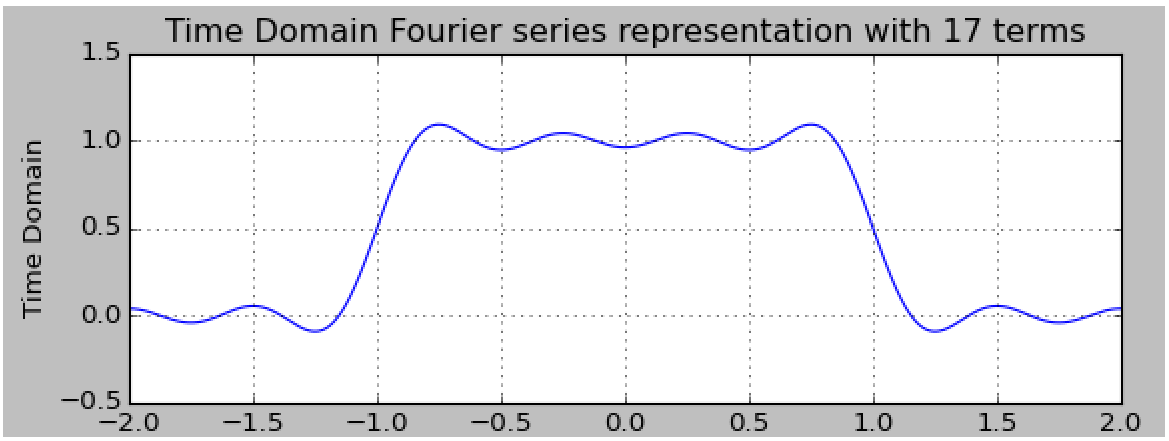
$$\Rightarrow \boxed{C_k' = e^{-j\frac{2\pi}{T}kT_1}\cdot C_k}$$

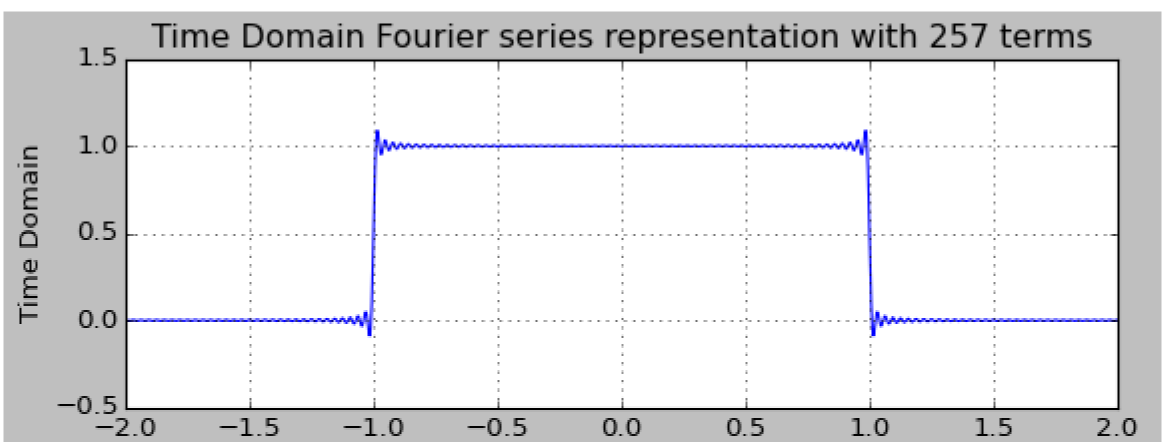
, this basically indicates that the new shifted domain has coefficients C_k that are all multiplied with $e^{-j\frac{2\pi}{T}kT_1}$ where T_1 is any constant for the shift'

(b) Implementing what we determined before we add to our code a factor $\boxed{e^{-\frac{2\pi}{T}jkT_1}}$ to multiply our coefficient C_k and obtain a shifted triangle wave by $T_1=3$, since $T_1<T$, given that $T=4$. The figure of the shifted triangle can be found in the pages that follow.

3 (b)


Time Domain Fourier series representation with 5 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


Time Domain Fourier series representation with 17 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0


Time Domain Fourier series representation with 257 terms
Time Domain
1.5
1.0
0.5
0.0
−0.5
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

4 (b)

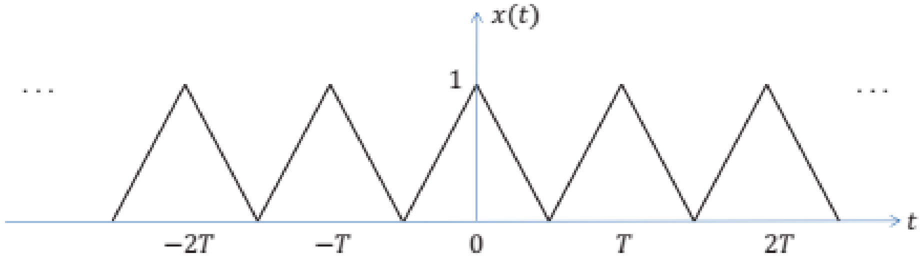
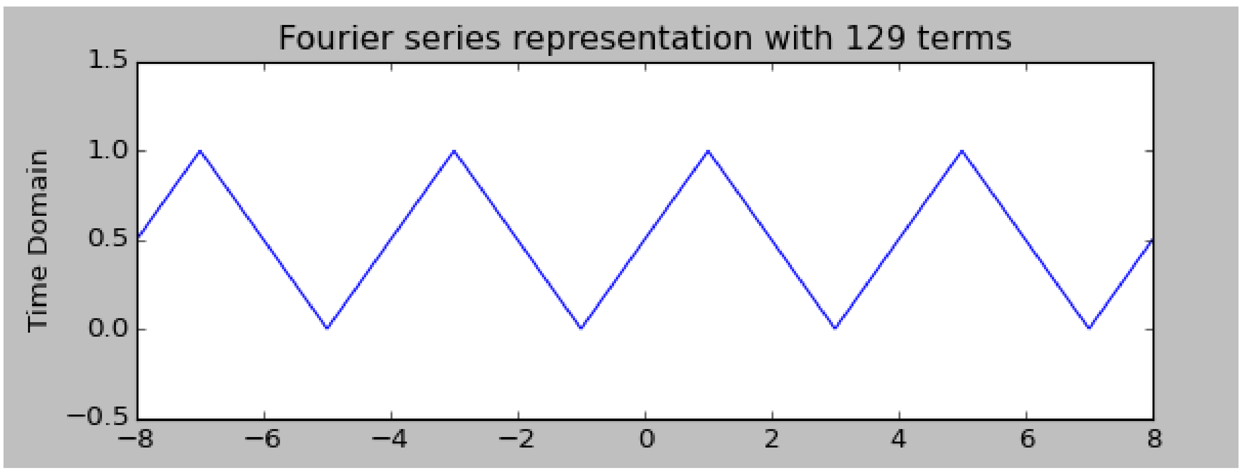


Figure 2: Triangle wave with period T.




```python
def fs_triangle(ts, M=3, T=4):
    # computes a fourier series representation of a triangle wave
    # with M terms in the Fourier series approximation
    # if M is odd, terms -(M-1)/2 -> (M-1)/2 are used
    # if M is even terms -M/2 -> M/2-1 are used

    # create an array to store the signal
    x = np.zeros(len(ts))

    # if M is even
    if np.mod(M,2) ==0:
        for k in range(-int(M/2), int(M/2)):
            # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2))
            if np.mod(k,2)==0:
                Coeff = 0
            if n == 0:
                Coeff = 0.5
            x = x + np.exp(-1j*2*np.pi*k*3/T)*Coeff*np.exp(1j*2*np.pi/T*k*ts)

    # if M is odd
    if np.mod(M,2) == 1:
        for k in range(-int((M-1)/2), int((M-1)/2)+1):
           # if n is odd compute the coefficients
            if np.mod(k, 2)==1:
                Coeff = -2/((np.pi)**2*(k**2))
            if np.mod(k,2)==0:
                Coeff = 0
            if k == 0:
                Coeff = 0.5
            x = x + np.exp(-1j*2*np.pi*k*3/T)*Coeff*np.exp(1j*2*np.pi/T*k*ts)

    return x
```