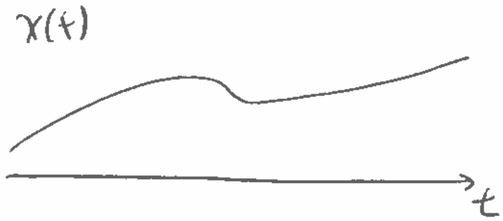
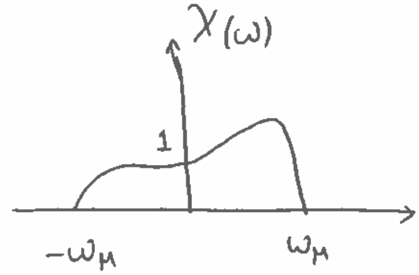
PS08 / FILIPPOS LYMPEROPOULOS

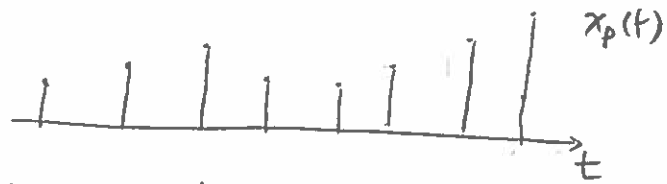
1.



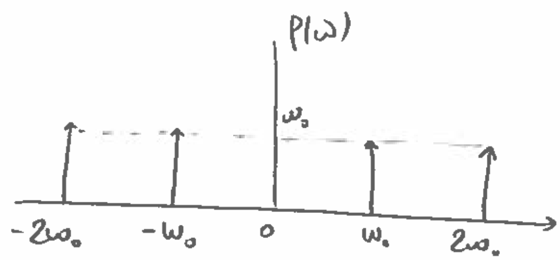


Moreover, we are given that $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$ and $x_p(t) = x(t)\,p(t)$.

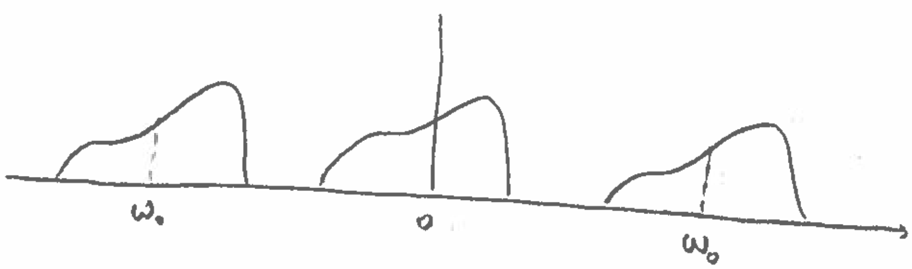
(a) Sketch of a representation of $x_p(t)$, is given below:



b) For us to sketch $P(\omega)$, we do the following:



c) For us to sketch $X_p(\omega)$, we do the following:

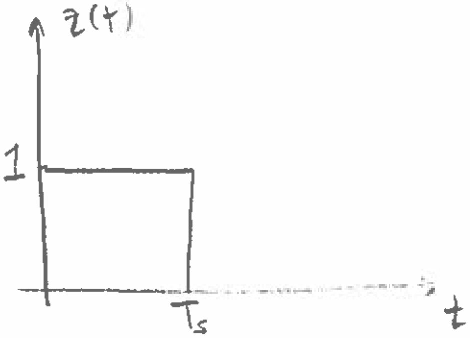


, $\omega_0 = \frac{2\pi}{T_s}$

d) To ensure that $X_p(\omega)$ contains all the information present in $X(\omega)$, we need to have that $\frac{2\pi}{T_s} < \frac{\omega_M}{2}$.

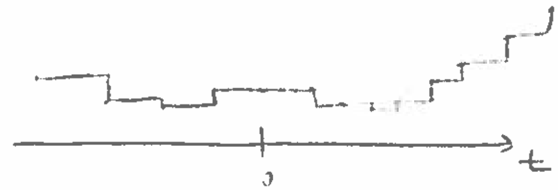
) In order to recover $x(t)$ from $x_p(t)$ exactly, we can use a bandpass with frequency $[-\omega_M, \omega_M]$, which in the time domain is a pure sinc function

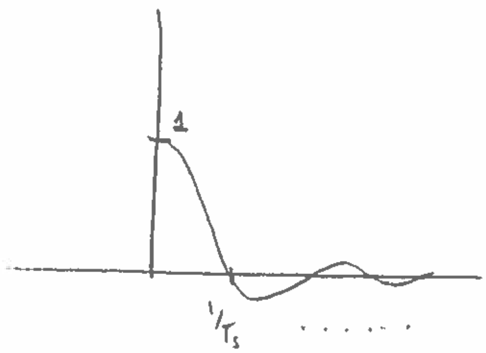
(f)



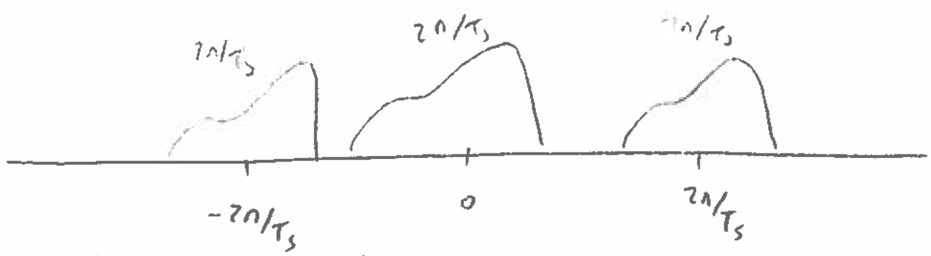
$\xrightarrow{x_z(t) = x_p \cdot z(t)}$

(g) It is in essence the zero hold of $x_p(t)$ and it gives us the following result

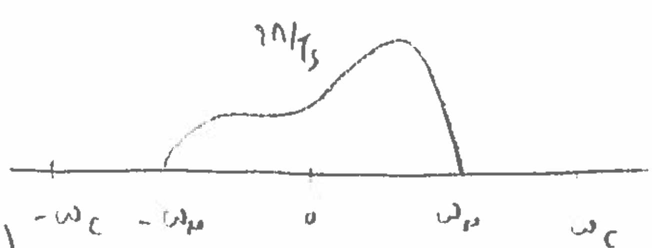


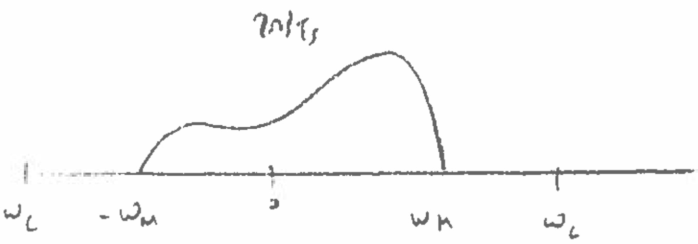


(h) Now we need to sketch $X_z(\omega)$. It is known that $Z(\omega) = \dfrac{2\sin(\omega \frac{T_s}{2}) e^{-j\omega}}{\omega}$ since it is a sinc function. Hence,



i) $\bar{X}(\omega) = X_z(\omega) H(\omega)$ $\qquad$ $\hat{X}(\omega) = X_p(\omega) H(\omega)$

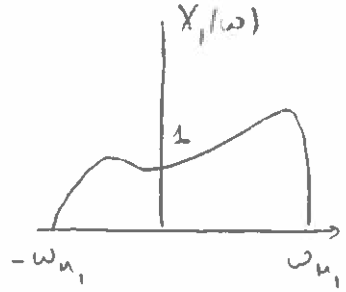


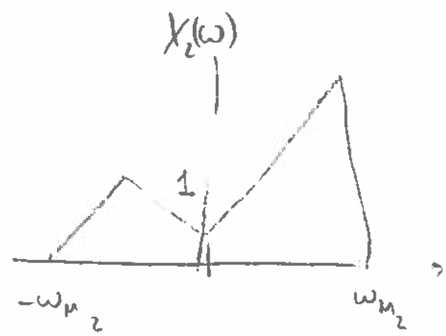


j) $\bar{X}(\omega)$ is going to decay since it contains a sinc function in its fourier transform, while they are both the same at 0, $2\pi/T_s$.

k) The ratio is 1, as pre-stated they are the same at that point $\overline{X}(\omega) = 1$

2. $y(t) = x_1(t)\cos(\omega_1 t) + x_2(t)\cos(\omega_2 t)$, if $X_1(\omega) = 0$, $X_2(\omega) = 0$, if $|\omega| > \omega_M$



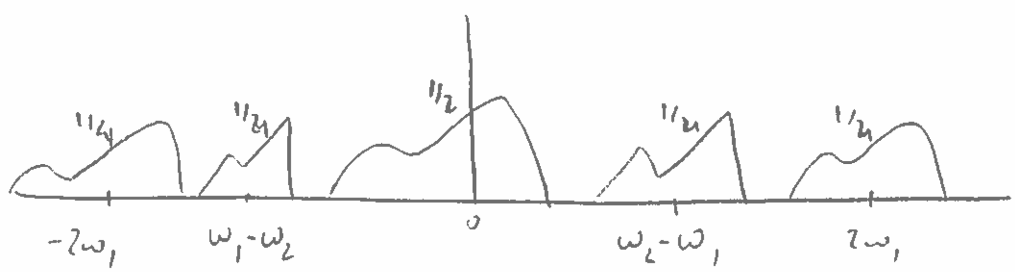


(a)

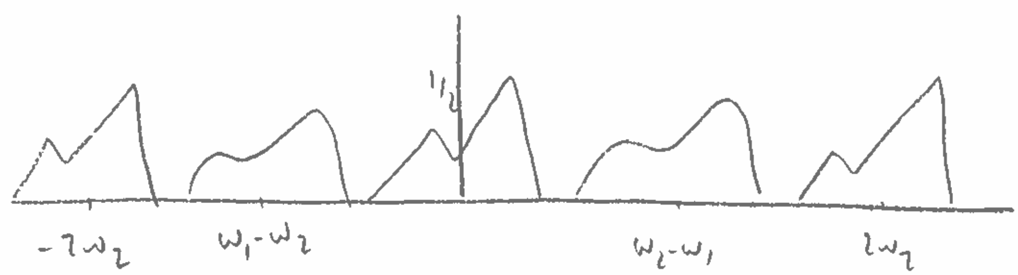
(b) $y(t)\cos(\omega_1 t) = x_1(t)\cos^2(\omega_1 t) + x_2(t)\cos(\omega_1 t)\cos(\omega_2 t)$ (1)

$y(t)\cos(\omega_2 t) = x_1(t)\cos(\omega_1 t)\cos(\omega_2 t) + x_2(t)\cos^2(\omega_2 t)$ (2)

(1)



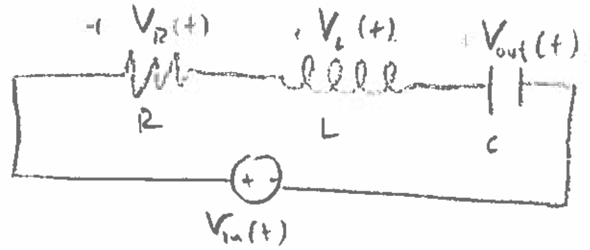
(2)



(c) We could multiply $y(t)$ with $\cos(\omega_1 t)$, use a bandpass filter of $[-\omega_M, \omega_M]$ and get back $x_1(t)$. The same process can be followed to deduce $x_2(t)$, multiplying by $\cos(\omega_2 t)$. It is important to mention that this will leave the signal scaled at $1/2$, so multiply by

3. (a) $i(t) = C\dfrac{d}{dt}V_{out}(t)$

$V_L(t) = L\dfrac{d}{dt}i(t)$ $\Bigg\} \Rightarrow$



$\Rightarrow V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t) \Rightarrow$

$$\Rightarrow \boxed{V_{in}(t) = RC\frac{d}{dt}V_{out}(t) + CL\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t)}$$

(b) $V_{in}(t) = e^{j\omega_1 t}$

$V_{out}(t) = H(\omega_1)e^{j\omega_1 t}$ $\Bigg\| \rightarrow e^{j\omega_1 t} = \left[RCj\omega_1 e^{j\omega_1 t} - LC\omega^2 e^{j\omega_1 t} + e^{j\omega_1 t}\right]H($

$$\Rightarrow \boxed{H(\omega) = \frac{1}{RCj\omega - LC\omega^2 + 1}}$$

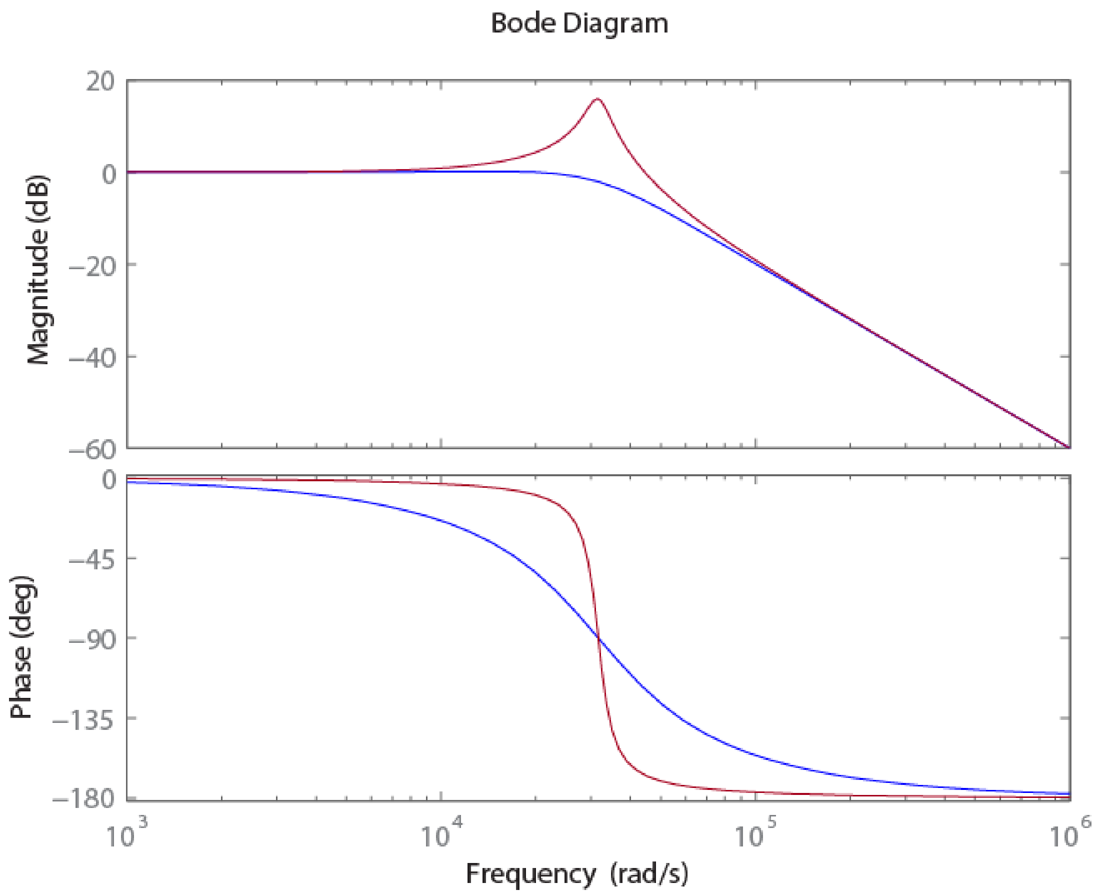
(c) $$|H(\omega)| = \frac{1}{\sqrt{R^2C^2\omega^2 - (LC\omega^2+1)^2}}$$

d) $$\frac{d}{d\omega}\left(|H(\omega)|\right) = \frac{1}{2\sqrt{R^2C^2\omega^2 - (LC\omega^2+1)^2}}\left(2\omega R^2C^2 - 2(LC\omega^2+1)\cdot 2LC\omega\right) =$$

$$= 2\omega R^2C^2 - 4LC\omega(LC\omega^2+1) = 2\omega R^2C^2 - 4L^2C^2\omega^3 + 4LC\omega = 0$$

$$= 2R^2C^2 - 4L^2C^2\omega^2 + 4LC = 0 \Rightarrow \omega = \pm\sqrt{\frac{4LC + 2R^2C^2}{4L^2C^2}} \Rightarrow$$

$$\Rightarrow \boxed{\omega = \pm\sqrt{\frac{1}{LC} + \frac{R^2}{2L^2}}} \text{ or } \boxed{\omega = \frac{\sqrt{2}}{2LC}\cdot\sqrt{2LC + R^2C^2}}$$ maximizes


Bode Diagram
Magnitude (dB)
20
0
−20
−40
−60
Phase (deg)
0
−45
−90
−135
−180
10^3
10^4
10^5
10^6
Frequency (rad/s)