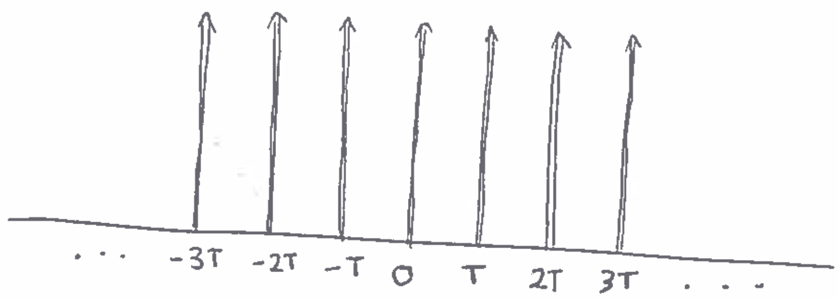
FILIPPOS LYMPEROPOULOS

(a) $p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT) \Rightarrow$ we can sketch the representation of $p(t)$



The representation above is also known as a Dirac comb, where T is the given period.

(b) Since the Dirac Comb ($p(t)$) is a periodic distribution, a periodic function, in other words, it can be represented as a Fourier Series. Given that it is periodic, we can state that:

$$p(t+T) = p(t) \quad , \quad \text{for all } t \in \mathbb{R}$$

And, therefore,

$$p(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{\frac{2\pi k t}{T} j} dt$$

We can also compute C_k, the fourier coefficients of $p(t)$, as follows

$$C_k = \frac{1}{T} \int_{-T/2}^{T/2} p(t) \cdot e^{-\frac{2\pi k t}{T} j} dt \Rightarrow$$

$$\Rightarrow C_k = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) \, e^{-2\pi k \frac{t}{T} j} dt \Rightarrow$$

$$\Rightarrow \boxed{C_k = \frac{1}{T}} \quad , \text{hence} \quad \boxed{p(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{\frac{2\pi k t}{T} j}}$$

(c) Given that $x(t)$ is a function that can be represented as a Fourier series,

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jw_0 kt} \quad , \quad w_0 = \frac{2\pi}{T}$$

We can find $X(\omega)$, in terms of C_k, as follows:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Hence, we also know that $C_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jw_0 t} dt \Rightarrow$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k e^{jw_0 kt} e^{-j\omega t} dt \Rightarrow$$

$$\Rightarrow X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k e^{jt(w_0 k - \omega)} dt \Rightarrow$$

$$\Rightarrow X(\omega) = \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} e^{j(w_0 k - \omega)t} dt \Rightarrow$$

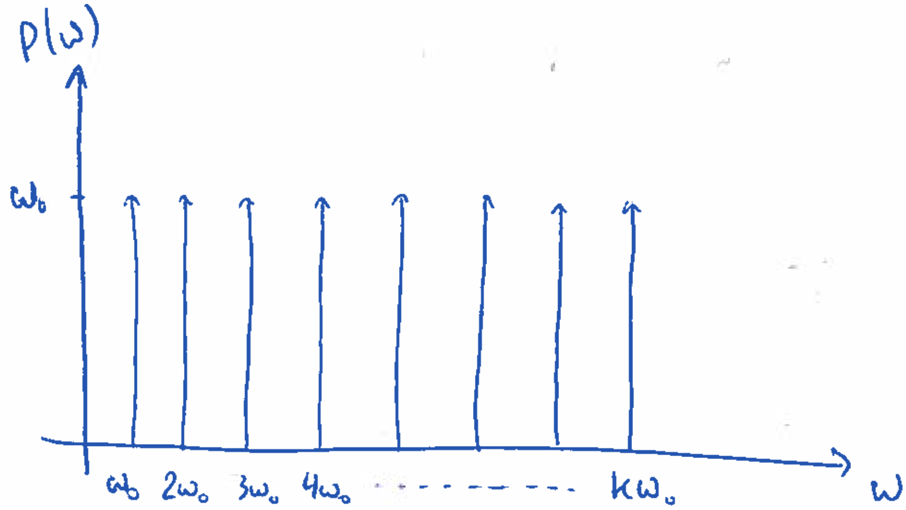
$$\Rightarrow \boxed{X(\omega) = \sum_{k=-\infty}^{\infty} C_k 2\pi \delta(\omega - k\omega_0)}$$

d) Here, we can determine $P(\omega)$, as follows:

$$P(\omega) = \sum_{k=\infty}^{\infty} \frac{1}{T} 2\pi \delta(\omega - k\omega_0)$$

$$\Rightarrow \boxed{P(\omega) = \sum_{k=-\infty}^{\infty} \omega \delta(\omega - k\omega_0)}$$

(e)



A change in T, is going to alter $p(t)$ as well as $P(\omega)$, as an increase in T will result in a drop in both $p(t)$ and $P(\omega)$. When we change T, in the time domain the impulses are further apart. In the frequency domain they are scaled and $\frac{2\pi}{T}$ factor closer.

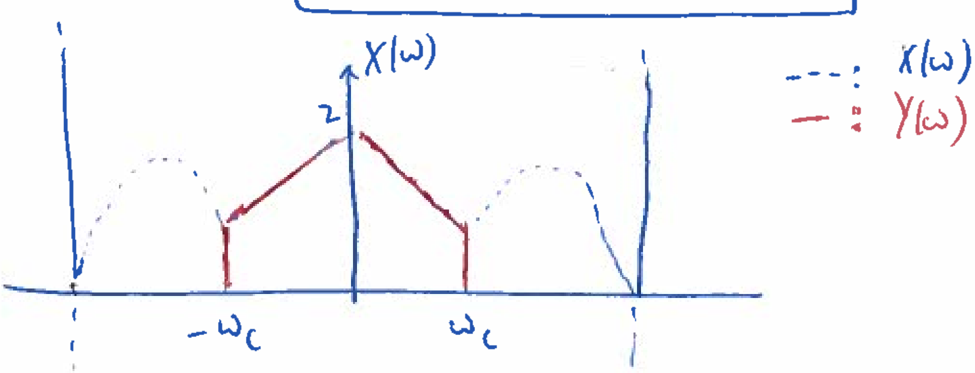
2 (a) Considering an LTI system with $h(t), x(t)$ and $y(t)$ and $H(\omega)$ we can determine $h(t)$ as:

$$h(t) = \int_{-\omega_c}^{\omega_c} e^{\frac{2\pi}{T} t j} d\omega \Rightarrow h(t) = \int_{-\omega_c}^{\omega_c} e^{2\pi f t j} d\omega \Rightarrow \boxed{h(t) = \frac{\sin(2\omega_c t)}{\pi t}}$$

Or we can use the sinc function as giving $h(t)$ as:

$$\boxed{h(t) = 2\omega_c \operatorname{sinc}(2\omega_c t)}$$

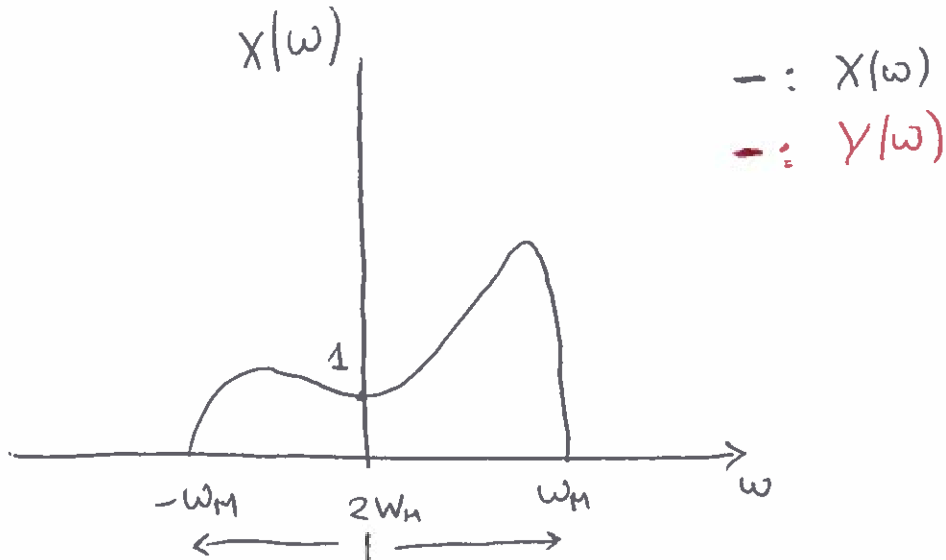
(b)

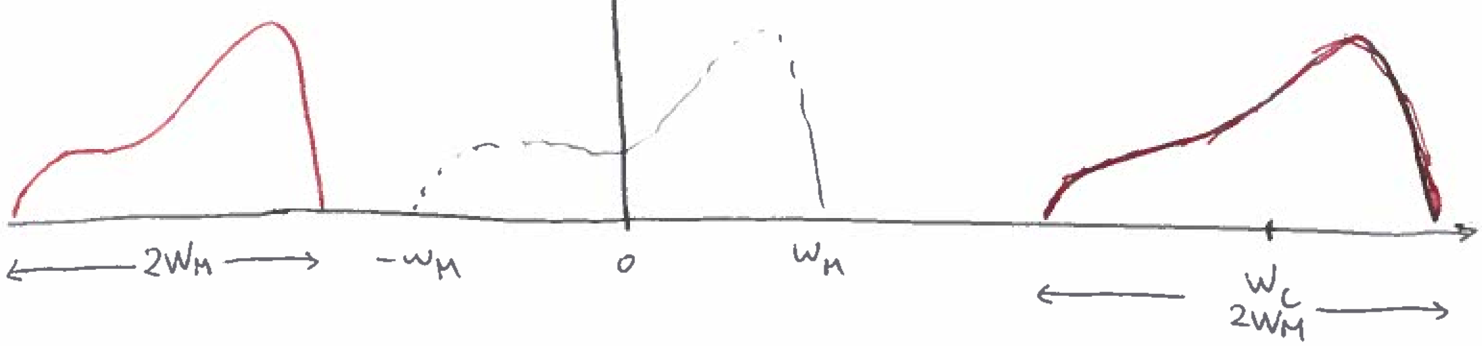


c) It is an ideal low pass filter because any frequency values not in the range $[-\omega_c, \omega_c]$ have zero amplitude and are cut off.

(d) figures shown in the pages that follow.

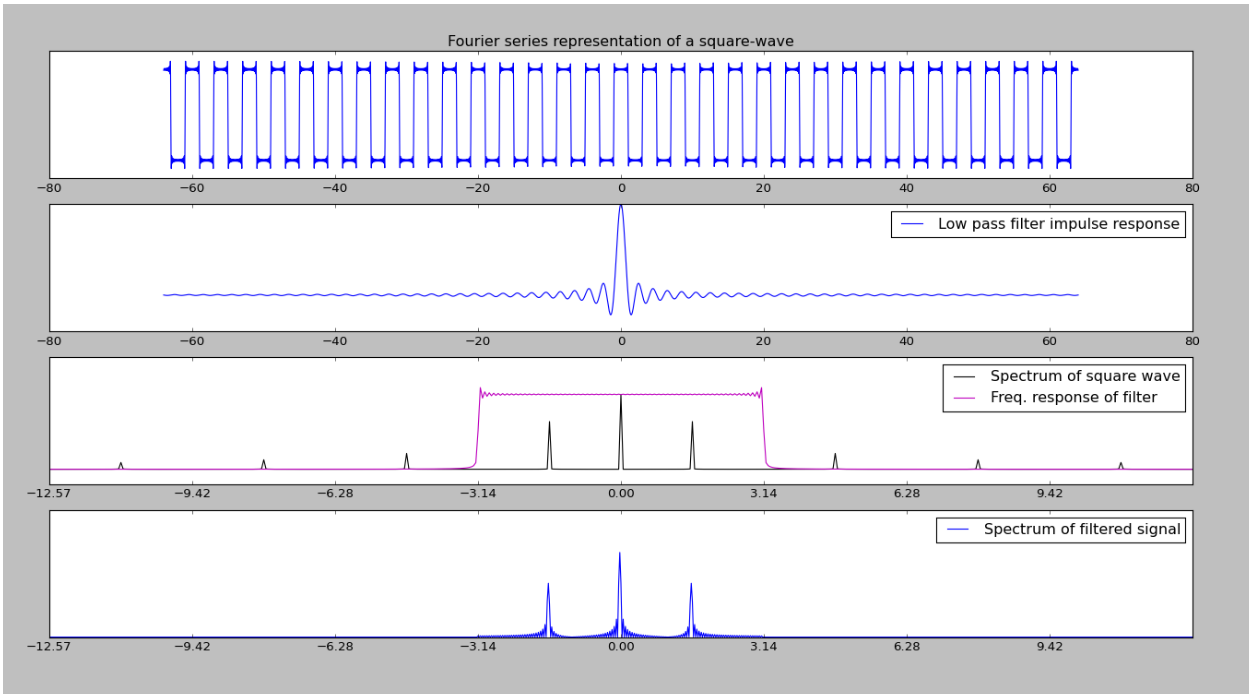
3. signal $x(t)$, with range $[-w_M, w_M]$. Given that $X(\omega)=0$, for $\omega < -\omega_M$ and $\omega > \omega_M$. Let $\boxed{y(t)=x(t)\cos(\omega_c t)}$, $\omega_c >> \omega_M$.





Looking above, we can point out that the carrier frequency has to be larger than twice the band width ($\omega_c \geq 2W$).

Wc = 0.75*np.pi



Wc = 1.75*np.pi