FILIPPOS LYMPEROPOULOS

$\boxed{1}$ (a)

$L = \alpha^{2^n}$, to prove whether or not it is regular.

We assume that L is regular. Let "m" be the number of states, from the pumping lemma, to describe the language. Hence,

$$s' = \alpha^{2^m}$$

where

$s' \in L$,

$|s'| \geq m$,

$s' = xyz$, with $|y| > 0$ and $|xy| \leq m$.

With $|xy| \leq m$, we have:

$x = \alpha^{2^q}$, $\quad 0 \leq q < m$

$y = a^{2^k}$, $\quad 0 < k \leq m$

$z = \alpha^{2^{(m-k-q)}}$

Now, the P.L. says that $xyyz$ is also in the language $(xyyz \in L)$.

$$xyyz = \alpha^{2^q} \cdot \alpha^{2^k} \cdot \alpha^{2^k} \cdot \alpha^{2^{(m-k-q)}} = \alpha^{2^q} \cdot \alpha^{2^{2k}} \cdot \alpha^{2^{(m-k-q)}} = \underline{\underline{\alpha^{2^{(m+k)}}}}$$

Thus, L ~~xyyz~~ is not regular, since $[xyyz \notin L]$, as $m+k \neq m$, with $k \in (0, m]$.

(b) $L = \{w \mid w$ has an equal number of occurrences of 01 and 10$\}$

↳ L is regular, described by the following regex:

It has to end and start with the same digit. Hence,

$$(1(0|1)^*1) | (0(0|1)^*0) | 1 | 0 | \varepsilon \quad (*)$$

(c) $L = \{(w)^n \cdot (w^R)^n, \; n \geq 0\}$

Let p be the number of states by the pumping lemma.

$$s' = w^p \cdot w^{R^p}$$

where $s' = xyz$ and, therefore, $s' = w_a^x w_b^y w_c^z w_{inv}^p \Rightarrow \boxed{x+y+z=p}$ (1)

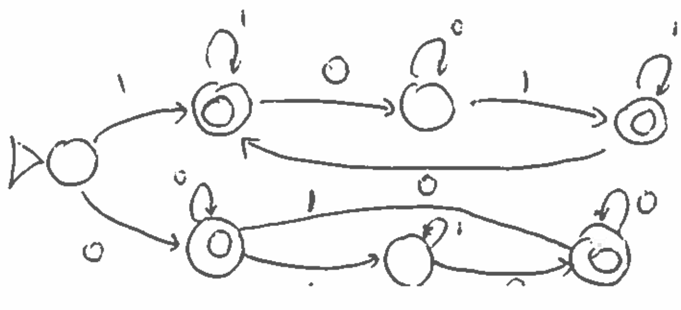
For L to be a regular, then $xyyz$ should be in the language

$$s'' = xyyz = w_a^x w_b^y w_c^z w_{inv}^p \Rightarrow \boxed{x+2y+z=p} \quad (2)$$

(1), (2) cannot both be true, so the language L is <u>not</u> regular.

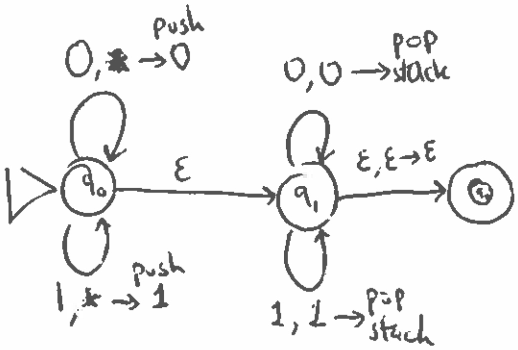
Also we can construct a FSM out of the regex:

(*)



2 Inline in the hw8.md file

3



non-deterministic PDA for 1c.