Tarea Unidad 1 MM-411

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1.Resuelva las siguientes ecuaciones diferenciales:

a. $\boldsymbol{\left(1+x^2\right)\frac{dy}{dx}=(1+y)^2}$

$$
\begin{aligned}
(1+x^2)\frac{dy}{dx} &= (1+y)^2 && (1)\\
[(1+x^2)\frac{dy}{dx} &= (1+y)^2]dx(1+x^2)(1+y)^2 && (2)\\
\frac{dy}{(1+y)^2} &= \frac{dx}{1+x^2} && (3)\\
\int\frac{dy}{(1+y)^2} &= \int\frac{dx}{1+x^2} && (4)\\
-\frac{1}{1+y}+c_1 &= \tan^{-1}x+c_2 && (5)\\
-\frac{1}{1+y} &= \tan^{-1}x+c_3, \quad (c_3=c_2-c_1) && (6)\\
-1-\tan^{-1}x-c_3 &= y(\tan^{-1}x+c_3) && (7)
\end{aligned}
$$

$$\boxed{y=-1-\frac{1}{tan^{-1}x+c}}$$

b. $\boldsymbol{\frac{dy}{dx}=6e^{2x-y}, y(0)=0}$

$$
\begin{aligned}
[\frac{dy}{dx} &= 6\frac{e^{2x}}{e^y}](e^y dx) && (1)\\
e^y dy &= 6e^{2x}dx && (2)\\
\int e^y dy &= \int 6e^{2x}dx && (3)\\
e^y &= 3e^{2x}+c && (4)\\
y &= Ln(3e^{2x}+c) && (5)
\end{aligned}
$$

$$
\begin{aligned}
y(0) &= 0 && (6.1)\\
0 &= Ln(3e^{2(0)}+c) && (6.2)\\
1 &= 3+c && (6.3)\\
c &= -2 && (6.4)
\end{aligned}
$$

$$\boxed{y=Ln(3e^{2x}-2)}$$

c. $\boldsymbol{xy' + y = 3xy, y(1) = 0}$

$$xy' = 3xy - y \tag{1}$$
$$y' = 3y - \frac{y}{x} \tag{2}$$
$$\frac{dy}{dx} = 3y - \frac{y}{x} \tag{3}$$
$$[\frac{dy}{dx} = 3y - \frac{y}{x}](\frac{dx}{y}) \tag{4}$$
$$\frac{dy}{y} = (3 - \frac{1}{x})dx \tag{5}$$
$$\int \frac{dy}{y} = \int (3 - \frac{1}{x})dx \tag{6}$$
$$ln(y) = 3x - ln(x) + c \tag{7}$$
$$\boxed{y = \frac{e^{3x+c}}{x}}$$

d. $\boldsymbol{(1+x)y' + y = cosx, y(0) = 1}$

$$[(1+x)y' + y = cosx](\frac{1}{1+x}) \tag{1}$$
$$y' + \frac{y}{1+x} = \frac{cosx}{1+x} \tag{2}$$

$$\mu(x) = e^{\int \frac{1}{1+x}dx} \tag{3.1}$$
$$\mu(x) = e^{ln(1+x)} \tag{3.2}$$
$$\mu(x) = 1 + x \tag{3.3}$$

$$[y' + \frac{y}{1+x} = \frac{cosx}{1+x}](1+x) \tag{4}$$
$$(1+x)y' + y = cosx \tag{5}$$
$$\int \frac{d}{dx}[(1+x)y] = \int cosxdx \tag{6}$$
$$(1+x)y = senx + c \tag{7}$$
$$y = \frac{senx + c}{1+x} \tag{8}$$

$$y(0) = 1 \tag{9.1}$$
$$1 = \frac{sen(0) + c}{1 + (0)} \tag{9.2}$$
$$1 = \frac{c}{1} \tag{9.3}$$
$$c = 1 \tag{9.4}$$

$$\boxed{y = \frac{senx + 1}{1 + x}}$$

e. $\boldsymbol{xyy' = y^2 + x\sqrt{4x^2 + y^2}}$

$$[xy\frac{dy}{dx} = y^2x\sqrt{4x^2 + y^2}](dx) \tag{1}$$

$$xydy = (y^2x\sqrt{4x^2 + y^2})dx \tag{2}$$

$$xydy - (y^2x\sqrt{4x^2 + y^2})dx = 0 \tag{3}$$

$$u = \frac{y}{x} \tag{4.1}$$

$$y = ux \tag{4.2}$$

$$dy = udx + xdu \tag{4.3}$$

$$ux^2(udx + xdu) - (u^2x^2 + x\sqrt{4x^2 + u^2x^2})dx = 0 \tag{5}$$

$$x^2(u^2)dx + x^2(-u^2 - \sqrt{4 - u^2})dx + x^3du = 0 \tag{6}$$

$$x^2(-\sqrt{4 - u^2})dx = -x^3du \tag{7}$$

$$x^2(-\sqrt{4 - u^2})dx = -x^3du](\frac{1}{(-x^3)(-\sqrt{4 - u^2})}) \tag{8}$$

$$\frac{dx}{x} = \frac{du}{\sqrt{4 - u^2}} \tag{9}$$

$$\int \frac{dx}{x} = \int \frac{du}{\sqrt{4 - u^2}} \tag{10}$$

$$Ln(x) = \int \frac{du}{\sqrt{4 - u^2}} \tag{11.1}$$

$$2tan\theta = u = tan\theta = \frac{u}{2} \tag{11.1.a}$$

$$\sqrt{4tan^2\theta + 4} = 2sec\theta \tag{11.1.b}$$

$$2sec^2\theta d\theta = du \tag{11.1.c}$$

$$\int \frac{du}{sqrt4 - u^2} = \int \frac{2sec^2\theta}{2sec\theta} \tag{11.1.d}$$

$$\int \frac{2sec^2\theta}{2sec\theta} = \int sec\theta d\theta \tag{11.1.e}$$

$$\int sec\theta d\theta = ln(tan\theta + sec\theta) + c_1 \tag{11.1.f}$$

$$\int sec\theta d\theta = ln(\frac{u}{2} + \frac{\sqrt{4 - u^2}}{2}) + c_1 \tag{11.1.g}$$

$$Ln(x) = ln(\frac{u + \sqrt{4 - u^2}}{2}) + c_1 \tag{ 12 }$$

$$x = c\left(u + \sqrt{4 - u^2}\right), c = \frac{e^{c_1}}{2} \tag{ 13 }$$

$$x = c\left((\frac{y}{x}) + \sqrt{4 - \frac{y^2}{x^2}} \right) \tag{14}$$

$$x = c\left(\frac{y}{x} + \frac{\sqrt{4x^2 + y^2}}{x} \right) \tag{15}$$

$$x = c\left(\frac{y + \sqrt{4x^2 + y^2}}{x} \right) \tag{16}$$

$$\boxed{\frac{x^2}{y + \sqrt{4x^2 + y^2}} = c}$$