Tarea Unidad 1 MM-411

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1.Resuelva las siguientes ecuaciones diferenciales:

a. $\boldsymbol{\left(1+x^2\right)\frac{dy}{dx}=(1+y)^2}$

$$(1+x^2)\frac{dy}{dx}=(1+y)^2 \tag{1}$$
$$[(1+x^2)\frac{dy}{dx}=(1+y)^2]dx(1+x^2)(1+y)^2 \tag{2}$$
$$\frac{dy}{(1+y)^2}=\frac{dx}{1+x^2} \tag{3}$$
$$\int\frac{dy}{(1+y)^2}=\int\frac{dx}{1+x^2} \tag{4}$$
$$-\frac{1}{1+y}+c_1=\tan^{-1}x+c_2 \tag{5}$$
$$-\frac{1}{1+y}=\tan^{-1}x+c_3,\quad (c_3=c_2-c_1) \tag{6}$$
$$-1-\tan^{-1}x-c_3=y(\tan^{-1}x+c_3) \tag{7}$$
$$\boxed{y=-1-\frac{1}{tan^{-1}x+c}}$$

b. $\boldsymbol{\frac{dy}{dx}=6e^{2x-y}, y(0)=0}$

$$[\frac{dy}{dx}=6\frac{e^{2x}}{e^y}](e^ydx) \tag{1}$$
$$e^ydy=6e^{2x}dx \tag{2}$$
$$\int e^ydy=\int 6e^{2x}dx \tag{3}$$
$$e^y=3e^{2x}+c \tag{4}$$
$$y=Ln(3e^{2x}+c) \tag{5}$$

$$y(0)=0 \tag{6.1}$$
$$0=Ln(3e^{2(0)}+c) \tag{6.2}$$
$$1=3+c \tag{6.3}$$
$$c=-2 \tag{6.4}$$

$$\boxed{y=Ln(3e^{2x}-2)}$$

c. $\boldsymbol{xy' + y = 3xy, y(1) = 0}$

$$xy' = 3xy - y \tag{1}$$
$$y' = 3y - \frac{y}{x} \tag{2}$$
$$\frac{dy}{dx} = 3y - \frac{y}{x} \tag{3}$$
$$[\frac{dy}{dx} = 3y - \frac{y}{x}](\frac{dx}{y}) \tag{4}$$
$$\frac{dy}{y} = (3 - \frac{1}{x})dx \tag{5}$$
$$\int \frac{dy}{y} = \int (3 - \frac{1}{x})dx \tag{6}$$
$$ln(y) = 3x - ln(x) + c \tag{7}$$
$$\boxed{y = \frac{e^{3x+c}}{x}}$$

d. $\boldsymbol{(1+x)y' + y = cosx, y(0) = 1}$

$$[(1+x)y' + y = cosx](\frac{1}{1+x}) \tag{1}$$
$$y' + \frac{y}{1+x} = \frac{cosx}{1+x} \tag{2}$$

$$\mu(x) = e^{\int \frac{1}{1+x} dx} \tag{3.1}$$
$$\mu(x) = e^{ln(1+x)} \tag{3.2}$$
$$\mu(x) = 1 + x \tag{3.3}$$

$$[y' + \frac{y}{1+x} = \frac{cosx}{1+x}](1+x) \tag{4}$$
$$(1+x)y' + y = cosx \tag{5}$$
$$\int \frac{d}{dx}[(1+x)y] = \int cosx dx \tag{6}$$
$$(1+x)y = senx + c \tag{7}$$
$$y = \frac{senx + c}{1+x} \tag{8}$$

$$y(0) = 1 \tag{9.1}$$
$$1 = \frac{sen(0) + c}{1 + (0)} \tag{9.2}$$
$$1 = \frac{c}{1} \tag{9.3}$$
$$c = 1 \tag{9.4}$$

$$\boxed{y = \frac{senx+1}{1+x}}$$

e. $\boldsymbol{xyy' = y^2 + x\sqrt{4x^2+y^2}}$

$$[xy\frac{dy}{dx} = y^2x\sqrt{4x^2+y^2}](dx) \tag{1}$$

$$xydy = (y^2x\sqrt{4x^2+y^2})dx \tag{2}$$

$$xydy - (y^2x\sqrt{4x^2+y^2})dx = 0 \tag{3}$$

$$u = \frac{y}{x} \tag{4.1}$$

$$y = ux \tag{4.2}$$

$$dy = udx + xdu \tag{4.3}$$

$$ux^2(udx+xdu) - (u^2x^2 + x\sqrt{4x^2+u^2x^2})dx = 0 \tag{5}$$

$$x^2(u^2)dx + x^2(-u^2 - \sqrt{4-u^2})dx + x^3du = 0 \tag{6}$$

$$x^2(-\sqrt{4-u^2})dx = -x^3du \tag{7}$$

$$x^2(-\sqrt{4-u^2})dx = -x^3du](\frac{1}{(-x^3)(-\sqrt{4-u^2})}) \tag{8}$$

$$\frac{dx}{x} = \frac{du}{\sqrt{4-u^2}} \tag{9}$$

$$\int\frac{dx}{x} = \int\frac{du}{\sqrt{4-u^2}} \tag{10}$$

$$Ln(x) = \int\frac{du}{\sqrt{4-u^2}} \tag{11.1}$$

$$2tan\theta = u = tan\theta = \frac{u}{2} \tag{11.1.a}$$

$$\sqrt{4tan^2\theta+4} = 2sec\theta \tag{11.1.b}$$

$$2sec^2\theta d\theta = du \tag{11.1.c}$$

$$\int\frac{du}{sqrt4-u^2} = \int\frac{2sec^2\theta}{2sec\theta} \tag{11.1.d}$$

$$\int\frac{2sec^2\theta}{2sec\theta} = \int sec\theta d\theta \tag{11.1.e}$$

$$\int sec\theta d\theta = ln(tan\theta + sec\theta) + c_1 \tag{11.1.f}$$

$$\int sec\theta d\theta = ln(\frac{u}{2} + \frac{\sqrt{4-u^2}}{2}) + c_1 \tag{11.1.g}$$

$$Ln(x) = ln(\frac{u+\sqrt{4-u^2}}{2}) + c_1 \tag{ 12 }$$

$$x = c\left(u+\sqrt{4-u^2}\right), c = \frac{e^{c_1}}{2} \tag{ 13 }$$

$$x = c\left((\frac{y}{x}) + \sqrt{4 - \frac{y^2}{x^2}} \right) \quad (14)$$
$$x = c\left(\frac{y}{x} + \frac{\sqrt{4x^2 + y^2}}{x} \right) \quad (15)$$
$$x = c\left(\frac{y + \sqrt{4x^2 + y^2}}{x} \right) \quad (16)$$
$$\boxed{\frac{x^2}{y + \sqrt{4x^2 + y^2}} = c}$$

f. $\boldsymbol{x^2 y' = xy + x^2 e^{\frac{y}{x}}}$

$$[x^2 y' = xy + x^2 e^{\frac{y}{x}}] \left(\frac{1}{x^2} \right) \quad (1)$$
$$\frac{dy}{dx} = \frac{y}{x} + e^{\frac{y}{x}} \quad (2)$$
$$dy = (\frac{y}{x} + e^{\frac{y}{x}})dx \quad (3)$$

$$u = \frac{y}{x}$$
$$y = ux$$
$$dy = udx + xdu$$

$$udx + xdu = (u + e^u)dx \quad (4)$$
$$udx - (u + e^u)dx = xdu \quad (5)$$
$$e^u dx = xdu \quad (6)$$
$$\int \frac{dx}{x} = \int e^{-u} \quad (7)$$
$$ln(x) = -e^{-u} + c \quad (8)$$
$$ln(x) = -e^{-\frac{y}{x}} + c \quad (9)$$
$$\boxed{ln(x) + e^{-\frac{y}{x}} = c}$$

g . $\boldsymbol{y' = \sqrt{y' = x + y + 1}}$

$$dy = \sqrt{x + y + 1}dx \quad (1)$$

$$w = x + y + 1 \quad (2.1)$$
$$dw = dy \quad (2.2)$$

$$dw = \sqrt{w}dx \quad (3)$$
$$\int \frac{dw}{\sqrt{w}} = \int dx \quad (4)$$
$$\frac{1}{2}\sqrt{w} = x + c \quad (5)$$
$$\frac{1}{2}\sqrt{x + y + 1} = x + c \quad (6)$$

$$x + y + 1 = (x + c)^2 \quad (7)$$

$$\boxed{y = (x + c)^2 - x - 1}$$

h . $\boldsymbol{y' = y + y^3}$

$$y' - y = y^3 \quad (1)$$

$$u = y^{1-3} \quad (2.1)$$

$$y = u^{-\frac{1}{2}} \quad (2.2)$$

$$y' = -\frac{1}{2}u^{-\frac{3}{2}}u' \quad (2.3)$$

$$-\frac{1}{2}u^{-\frac{3}{2}}u' - u^{-\frac{1}{2}} = u^{-\frac{3}{2}} \quad (3)$$

$$u' + 2u = -1 \quad (4)$$

$$\mu(x) = e^{\int 2dx} \quad (5.1)$$

$$\mu(x) = e^{2x} \quad (5.2)$$

$$[u' + 2u = -1]e^{2x} \quad (6)$$

$$u'e^{2x} + 2ue^{2x} = -e^{2x} \quad (7)$$

$$\frac{d}{dx}[ue^{2x}] = -e^{2x} \quad (8)$$

$$ue^{2x} = \int -e^{2x}dx \quad (9)$$

$$ue^{2x} = -\frac{1}{2}e^{2x} + c \quad (10)$$

$$u = ce^{-2x} - \frac{1}{2} \quad (11)$$

$$\frac{y}{x} = ce^{-2x} - \frac{1}{2} \quad (12)$$

$$\boxed{y = \frac{c}{e^{2x}x} - \frac{1}{2x}}$$

i . $\boldsymbol{xy' + 6y = 3xy^{\frac{4}{3}}}$

$$y' + 6\frac{y}{x} = 3y^{\frac{4}{3}} \quad (1)$$

$$u = y^{-\frac{1}{3}} \quad (2.1)$$

$$y = u^{-3} \quad (2.2)$$

$$y' = -3u^{-4}u' \quad (2.3)$$

$$-3u^{-4}u' + 6u^{-3}x^{-1} = 3u^{-4} \quad (3)$$

$$u' + 2ux^{-1} = -1 \quad (4)$$

$$\mu(x) = e^{2\int \frac{dx}{x}} \quad (5.1)$$
$$\mu(x) = 2e^{ln(x)} \quad (5.2)$$
$$\mu(x) = x^2 \quad (5.3)$$

$$u'x^2 + 2xu = -x^2 \quad (6)$$
$$\frac{d}{dx}[ux^2] = -x^2 \quad (7)$$
$$ux^2 = \int -x^2 dx \quad (8)$$
$$ux^2 = -\frac{1}{3}x^3 + c \quad (9)$$
$$yx = -\frac{1}{3}x^3 + c \quad (10)$$
$$\boxed{y = -\frac{1}{3}x^2 + \frac{c}{x}}$$

j. $\boldsymbol{\frac{dy}{dx} = \frac{2y-x+7}{4x-y-18}}$

$$(4x - y - 18)dy = (2y - x + 7)dx \quad (1)$$

$$\begin{cases} 4x - y = 18 \\ 2y - x = -7 \end{cases}$$

$$x = 3 \qquad y = -2$$
$$x = u + 3 \qquad y = v - 2$$
$$dx = du \qquad dy = dv$$

$$(4u - 3v)dv = (2v - u)du \quad (2)$$
$$(2v - u)du - (4u - 3v)dv = 0 \quad (3)$$

$$w = \frac{v}{u} \quad (4.1)$$
$$v = wu \quad (4.2)$$
$$dv = wdu + udw \quad (4.3)$$

$$(2wu - u)du - (4u - 3wu)(wdu + udw) = 0 \quad (5)$$
$$u(3w^2 - 2w - 1)du = u^2(4 - 3w)dw \quad (6)$$
$$\int \frac{du}{u} = \int \frac{4 - 3w}{3w^2 - 2w - 1}dw \quad (7)$$

$$\int \frac{4 - 3w}{3w^2 - 2w - 1}dw = \frac{1}{4}\int \frac{1}{w - 1}dw - \frac{15}{4}\int \frac{1}{3w + 1}dw \quad (8.1)$$
$$\int \frac{4 - 3w}{3w^2 - 2w - 1}dw = \frac{1}{4}ln(w - 1) - \frac{3}{4}ln(3w + 1) \quad (8.2)$$

$$ln(u) = \frac{1}{4}ln(w - 1) - \frac{3}{4}ln(3w + 1) + c \quad (9)$$

$$u = C\left(w-1\right)^{\frac{1}{4}}\left(3w+1\right)^{-\frac{3}{4}} \tag{10}$$

$$x-3 = C\left(\frac{y+2}{x-3}-1\right)^{\frac{1}{4}}\left(3\left(\frac{y+2}{x-3}\right)+1\right)^{-\frac{3}{4}} \tag{11}$$

$$\boxed{(x-3)\left(\frac{y+2}{x-3}-1\right)^{-\frac{1}{4}}\left(3\left(\frac{y+2}{x-3}\right)+1\right)^{\frac{3}{4}} = C}$$

k. $\boldsymbol{\frac{dy}{dx} = \frac{x-y-1}{x+y+3}}$

$$(x+y+3)dy = (x-y-1)dx \tag{1}$$

$$\begin{cases} x+y=-3 \\ x-y=1 \end{cases}$$

$$x = -1 \qquad y = -2$$

$$x = u-1 \qquad y = v-2$$

$$dx = du \qquad dy = dv$$

$$(v+u)dv = (u-v)du \tag{2}$$

$$(u-v)du-(v+u)dv = 0 \tag{3}$$

$$w = \frac{v}{u} \tag{4.1}$$

$$v = wu \tag{4.2}$$

$$dv = wdu + udw \tag{4.3}$$

$$(wu-u)du-(wu+u)(wdu+udw) = 0 \tag{5}$$

$$u(1-2w-w^2)du = u^2(w+1)dw \tag{6}$$

$$\int \frac{du}{u} = \int \frac{w+1}{1-2w-w^2}dw \tag{7}$$

$$t = 1-2w-w^2 \tag{8.1}$$

$$dt = -2(1+w)dw \tag{8.2}$$

$$\frac{dt}{-2(1+w)} = dw \tag{8.3}$$

$$\int \frac{w+1}{1-2w-w^2}dw = -\frac{1}{2}\int \frac{1}{t}dt \tag{8.4}$$

$$-\frac{1}{2}\int \frac{1}{t}dt = -\frac{1}{2}ln(t)+c \tag{8.5}$$

$$\int \frac{w+1}{1-2w-w^2}dw = -\frac{1}{2}ln(1-2w-w^2)+c \tag{8.6}$$

$$ln(u) = -\frac{1}{2}ln(1-2w-w^2)+c \tag{9}$$

$$u = C\left(1 - 2w - w^2\right)^{-\frac{1}{2}} \quad (10)$$

$$x + 1 = C\left(1 - 2\left(\frac{y+2}{x+1}\right) - \left(\frac{y+2}{x+1}\right)\right)^{-\frac{1}{2}} \quad (11)$$

$$\boxed{(x+1)\left(1 - 2\left(\frac{y+2}{x+1}\right) - \left(\frac{y+2}{x+1}\right)\right)^{\frac{1}{2}} = C}$$

l. $\boldsymbol{(e^x seny + tany)dx + (e^x cosy + xsec^2y)dy = 0}$

$$\frac{\partial M}{\partial y} = e^x cosy + sec^2 y = \frac{\partial N}{\partial x} = e^x cosy + sec^2 y \quad (1)$$

$$F(x,y) = \int \left(e^x seny + tany\right) dx + g(y) \quad (2.1)$$

$$F(x,y) = e^x seny + xtany + g(y) \quad (2.2)$$

$$\frac{\partial F}{\partial y} = N(x,y) \quad (3.1)$$

$$e^x cosy + xsec^2 y + g'(y) = e^x cosy + xsec^2 y \quad (3.2)$$

$$g'(y) = 0 \quad (3.3)$$

$$g(y) = \int 0dy \quad (3.4)$$

$$g(y) = C \quad (3.5)$$

$$F(x,y) = e^x seny + xtany + C \quad (4)$$

$$\boxed{e^x seny + xtany = C}$$

m. $\boldsymbol{\left(x + tan^{-1}y\right) dx + \left(\frac{x+y}{1+y^2}\right) dy = 0}$

$$\frac{\partial M}{\partial y} = \frac{1}{1+y^2} = \frac{\partial N}{\partial x} = \frac{1}{1+y^2} \quad (1)$$

$$F(x,y) = \int \left(x + tan^{-1}y\right) dx + g(y) \quad (2.1)$$

$$F(x,y) = \frac{1}{2}x^2 + xtan^{-1}y + g(y) \quad (2.2)$$

$$\frac{\partial F}{\partial y} = N(x,y) \quad (3.1)$$

$$\frac{x}{1+y^2} + g'(y) = \frac{x}{1+y^2} + \frac{y}{1+y^2} \quad (3.2)$$

$$g'(y) = \frac{y}{1+y^2} \quad (3.3)$$

$$g(y) = \int \frac{y}{1+y^2} dy \quad (3.3)$$

$$g(y) = \frac{1}{2} ln\left(1 + y^2\right) \quad (3.4)$$

$$F(x,y)=\frac{1}{2}x^2+xtan^{-1}y+\frac{1}{2}ln\left(1+y^2\right) \tag{4}$$

$$\boxed{\frac{1}{2}x^2+xtan^{-1}y+\frac{1}{2}ln\left(1+y^2\right)=C}$$

n . $\boldsymbol{2xydx+\left(y^2-x^2\right)dy=0}$

$$u=\frac{y}{x}$$
$$y=ux$$
$$dy=udx+xdu$$

$$(2x^2u)dx+(x^2y^2-x^2)\,(udx+xdu)=0 \tag{1}$$
$$x^2\left(u+u^3\right)dx=-x^3\left(u^2-1\right)du \tag{2}$$
$$-\int\frac{dx}{x}=\int\frac{u^2-1}{u(1+u^2)}du \tag{3}$$

$$\int\frac{u^2-1}{u(1+u^2)}du=-\int\frac{1}{u}du+2\int\frac{1}{1+u^2}du \tag{4.1}$$
$$\int\frac{u^2-1}{u(1+u^2)}du=-ln(u)+2tan^{-1}u+c \tag{4.2}$$

$$-ln(x)=-ln(u)+2tan^{-1}u+c \tag{5}$$
$$x^{-1}=Cu^{-1}e^{2tan^{-1}u} \tag{6}$$
$$\boxed{x^{-1}=C\left(\frac{y}{x}\right)^{-1}e^{2tan^{-1}\left(\frac{y}{x}\right)}}$$

o. $\boldsymbol{(cosxcosy-cotx)\,dx-(senxseny)\,dy=0}$

$$(cosxcosy-cotx)\,dx+(-senxseny)\,dy=0 \tag{1}$$
$$\frac{\partial M}{\partial y}=-cosxseny=\frac{\partial N}{\partial x}=-cosxseny \tag{2}$$

$$F(x,y)=\int(cosxcosy-cotx)\,dx+g(y) \tag{3.1}$$
$$F(x,y)=senxcosy-ln(senx)+g(y) \tag{3.2}$$

$$\frac{\partial F}{\partial y}=N(x,y) \tag{4.1}$$
$$-senxseny+g'(y)=senxseny \tag{4.2}$$
$$g'(y)=0 \tag{4.3}$$
$$g(y)=C \tag{4.4}$$

$$F(x,y)=senxcosy-ln(senx)+C \tag{5}$$
$$\boxed{senxcosy-ln(senx)=C}$$

p. $\left(\boldsymbol{2y^2+3x}\right)\boldsymbol{dx+2xydy=0}$

$$\frac{\partial M}{\partial y}=4y\neq\frac{\partial N}{\partial x}=2y \qquad (1)$$

$$\mu(x)=e^{\int\frac{4y-2y}{2xy}dx} \qquad (2.1)$$
$$\mu(x)=e^{\int\frac{2y}{2xy}dx} \qquad (2.2)$$
$$\mu(x)=e^{\int\frac{1}{x}dx} \qquad (2.3)$$
$$\mu(x)=e^{ln(x)} \qquad (2.4)$$

$$\left(2y^2x+3x^2\right)dx+\left(2x^2y\right)dy=0 \qquad (3)$$

$$F(x,y)=\int\left(2y^2x+3x^2\right)dx+g(y) \qquad (4.1)$$
$$F(x,y)=y^2x^2+x^3+g(y) \qquad (4.2)$$

$$\frac{\partial F}{\partial y}=N(x,y) \qquad (5.1)$$
$$2xy^2+g'(y)=2xy^2 \qquad (5.2)$$
$$g'(y)=0 \qquad (5.3)$$
$$g(y)=C \qquad (5.4)$$

$$F(x,y)=y^2x^2+x^3+C \qquad (6)$$
$$\boxed{y^2x^2+x^3=C}$$

2.Resuelva las siguientes aplicaciones:

- Una medicina se inyecta en el torrente sanguíneo de un paciente a un flujo constante de $r\frac{g}{s}$. Al mismo tiempo, esa medicina desaparece con una razón proporcional a la cantidad x(t) presente en cualquier momento t. Formule la ecuación diferencial que describa la cantidad x(t) y resuélvala.

$$\frac{dx}{dt}=r-kx \qquad (1)$$
$$\dot{x}+kx=r \qquad (2)$$

$$\mu(x)=e^{\int kdt} \qquad (3.1)$$
$$\mu(x)=e^{kt} \qquad (3.2)$$

$$\dot{x}e^{kt}+kxe^{kt}=re^{kt} \qquad (4)$$
$$\frac{d}{dt}\left[xe^{kt}\right]=re^{kt} \qquad (5)$$
$$xe^{kt}=\int re^{kt}dt \qquad (6)$$
$$xe^{kt}=\frac{r}{k}e^{kt}+c \qquad (7)$$

$$\boxed{x(t) = \frac{r}{k} + \frac{c}{e^{kt}}}$$

- Un tanque tiene 500 gal de agua pura y le entra salmuera con 2 lb de sal por galón a un flujo de 5 gal/min. El tanque está bien mezclado, y sale de él el mismo flujo de solución. Calcule la cantidad A(t) de libras de sal que hay en el tanque en cualquier momento t.

$$R_in = (2)(5) = 10 \tag{1.1}$$
$$R_out = (\frac{A}{500})(5) = \frac{A}{100} \tag{1.2}$$
$$\dot{A} = 10 - \frac{A}{100} \tag{2}$$
$$\dot{A} + \frac{1}{100}A = 10 \tag{3}$$

$$\mu(x) = e^{\int \frac{1}{100}dt} \tag{4.1}$$
$$\mu(x) = e^{\frac{t}{100}} \tag{4.2}$$

$$\frac{d}{dt}\left[Ae^{\frac{t}{100}}\right] = 10e^{\frac{t}{100}} \tag{5}$$
$$Ae^{\frac{t}{100}} = \int 100e^{\frac{t}{100}}dt \tag{6}$$
$$Ae^{\frac{t}{100}} = e^{\frac{t}{100}} + c \tag{7}$$
$$\boxed{A(t) = 1 + \frac{c}{e^{\frac{t}{100}}}}$$

- $\boldsymbol{q(t) e i(t)}$

$$Ri(t) + \frac{1}{c}q = E(t) \tag{1}$$
$$200\dot{q} + \frac{1}{10^{-4}}q = 100 \tag{2}$$
$$\dot{q} + 50q = \frac{1}{2} \tag{3}$$

$$\mu(x) = e^{\int 50dt} \tag{4.1}$$
$$\mu(x) = e^{50t} \tag{4.2}$$

$$\frac{d}{dt}\left[Ae^{50t}\right] = \frac{1}{2}e^{50t} \tag{5}$$
$$qe^{50t} = \int \frac{1}{2}e^{50t}dt \tag{6}$$
$$qe^{50t} = \frac{1}{100}e^{50t} + c \tag{7}$$
$$\boxed{q(t) = \frac{1}{100} - \frac{1}{100}\frac{c}{e^{50t}}}$$