Unproductive by choice: substitution and the slowdown in aggregate productivity growth in the United States

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Abstract

I decompose aggregate total factor productivity using a model where all output is produced by the use of some factor. Measured productivity is then an index of unmeasured factors. This model introduces a new avenue through which productivity can grow or decline: changes in the effective price of labor and capital cause *substitution* to or from productivity. I study whether changing relative factor prices explain the slowdown in US productivity growth from 2005 to 2016. I find that if not for the declining effective price of labor and capital, productivity growth would be accelerating.

1. Introduction

Total factor productivity growth has slowed in the United States in the past decade. The causes and consequences of its decline matter for long run economic welfare. The two leading explanations for the decline in measured productivity growth are that technological growth has truly slowed or that productivity is increasingly mismeasured and is not capturing the full economic value of new innovations. If the fall in the growth rate of productivity is the result of slowing technological growth, welfare-per-person is improving less rapidly now than it used to. If the declining growth rate is illusory, the result of some error in how we measure productivity, it may not matter at all[1]. In this paper, I propose a model of productivity that offers a different explanation than these two leading theories.

I model productivity as a function of factors of production that we do not observe. With this view of productivity, there is a new avenue through which productivity can grow or decline. As the effective prices[2] of labor and capital rise or fall, the economy will substitute towards or away from productivity. Substitution from or to productivity has a very different economic consequence than either slowing technological growth or the mismeasurement hypothesis. In the mismeasurement hypothesis, there is no slowdown. In the slowing technological growth hypothesis, there is a slowdown and it is the result of the economy no longer innovating as rapidly as it once did (Gordon,

[1] See Syverson (2017) and Byrne, Fernald, and Reinsdorf (2016) for a comparison of these two competing explanations for falling productivity growth.

[2] Effective price is the ratio of price to the input's output elasticity, the "buck-per-bang".

2016) or of it inefficiently allocating resources (Baqaee and Farhi 2019 propose a measurement of productivity for this case). In the substitution hypothesis proposed in this paper, there *is* a slowdown but it is an efficient slowdown: the factors of production that determine productivity have a higher effective price relative to capital and labor than they did previously. The economy is efficiently substituting away from productivity and towards capital and labor.

This view of productivity underlies Griliches and Jorgenson (1967) who argue that there would be little left of the productivity residual if we could accurately and fully account for all the various factors of production in the economy. They memorably call productivity, "the measure of our ignorance". The growth literature on human capital accumulation is effectively pulling human capital, a factor of production produced at a cost, out of the Solow residual. The literature on intangible capital and factor utilization (Basu and Kimball, 1997; Fernald, 2014) also tries to pull factors of production out of the Solow residual (see, for example, Fernald and Wang 2016). What I add to the literature in this paper is a simple framework for modeling how the totality of the unmeasured factors of production change in response to changes in the effective price of labor and capital. Instead of trying to pull individual factors out of productivity, I assume productivity is determined by many unobserved factors and model how an index of those factors changes in response to other changes in the economy.

My framework is entirely within the standard neoclassical model. The economy is modeled as a representative firm. Returns to scale are constant. Prices are equal to marginal cost. The firm has a Cobb Douglas production function. As a consequence, my measure of productivity itself is no different than Solow (1957)'s measure. The difference is in what changes in the economy influence this measure of productivity.

I develop a new decomposition of productivity based on its conditional factor demand equation. The decomposition has five terms: changes in the effective price of labor, changes in the effective price of capital, changes in the effective price of productivity, changes in demand, and changes in the overall technology of using labor and capital. I use this decomposition to study the recent productivity slowdown in the United States and measure the extent to which substitution from productivity and to labor and capital are behind the slowdown. I find that the growth rate of the effective price of labor and capital has declined over the past decade. If that decline had not occurred, I find productivity growth would have *accelerated*. The decrease in productivity growth is not, within this framework, primarily a sign of a slowdown in technological progress but a sign of changes in relative factor prices.

Section 2 introduces my model of production and productivity, deriving the factor demand equation for productivity and the decomposition of productivity that forms the basis for my empirical analysis.

Section 3 uses this decomposition to study the economy of the United States from 1987 to 2016 and to decompose the causes of the recent decline in total factor productivity from 2005 to 2016.

Section 4 concludes with the implications of this understanding of the recent decline in productivity and some ideas for future research.

2. The factor model of productivity

2.1. What is productivity?

In the neoclassical growth model, productivity is an exogenous process. Productivity in period t is A_t and in period $(t+1)$ productivity is $A_{t+1} = (1+g_t) A_t$ where g_t is an exogenous growth rate. This model of productivity is also used in the endogenous growth models following Romer (1986).

I model productivity differently. I suppose that were we to accurately measure all factors of production in the economy that there would not be a residual in the production relationship,

$$\text{Output} = F\left(\text{Inputs}\right). \tag{1}$$

In this setting, productivity is the part of output that we cannot explain with the set of inputs we can measure. But, in reality, this part of output is also produced using factors of production. The main implication is that shocks that affect the wage of labor or the rental rate of capital can result in changes to productivity. This cannot happen if we treat productivity as an exogenous sequence.

I show that we can measure the cost of productivity even though we do not observe the individual inputs that make up productivity. I use the conditional factor demand function for productivity to understand how changes in wages or the price of capital affect productivity. This function decomposes productivity growth into growth resulting from changes in the price of labor and capital relative to their output elasticity, changes in aggregate demand, and changes in the cost of building productivity.

2.2. The pure factor production function

Let Q be output, L be labor, K be capital, and $Z \in \mathbb{R}^M$ be a vector of unobserved factors of production. The aggregate production function is Cobb Douglas,

$$Q = F_0 \times L^{\theta_L} K^{\theta_K} \times \prod_{m=1}^{M} Z_m^{\gamma_m}. \tag{2}$$

Suppose the economy minimizes its costs like a representative firm would,

$$\min_{L,K,Z} W_L L + W_K K + W_Z^\top Z \quad \text{ST:} \quad F_0 \times L^{\theta_L} K^{\theta_K} \times \prod_{m=1}^{M} Z_m^{\gamma_m} \geq Q. \tag{3}$$

I model productivity (A) as the contribution of unobserved factors of production to output,

$$A = \prod_{m=1}^{M} Z_m^{\gamma_m}. \tag{4}$$

The economy will minimize the cost of producing A from the Z inputs given whatever decision it makes about labor and capital. So,

$$C(A) = \min W_Z^\top Z \quad \text{ST:} \quad \prod_{m=1}^{M} Z_m^{\gamma_m} \geq A. \tag{5}$$

From textbook algebra on the cost function of the Cobb Douglas production function, $C(A)$ has the following functional form,

$$C(A) = C_0(W_Z, \gamma) A^{\frac{1}{\sum_m \gamma_m}}. \tag{6}$$

Define $\theta_A = \sum_m \gamma_m$. We can think of the sum of the Z output elasticities as the output elasticity of productivity.

The economy's cost minimization problem in terms of (L, K, A) is then,

$$\min_{L,K,A} W_L L + W_K K + C_0(W_Z, \gamma) A^{\frac{1}{\theta_A}} \quad \text{ST:} \quad F_0 \times L^{\theta_L} K^{\theta_K} A \geq Q. \tag{7}$$

Equivalently, we can define $\widetilde{A} = A^{\frac{1}{\theta_A}}$ and treat $\widetilde{A}$ symmetrically to (L, K),

$$\min_{L,K,\widetilde{A}} W_L L + W_K K + C_0(W_Z, \gamma) \widetilde{A} \quad \text{ST:} \quad F_0 \times L^{\theta_L} K^{\theta_K} \widetilde{A}^{\theta_A} \geq Q. \tag{8}$$

2.3. Measuring productivity

I make two assumptions that are firmly in the tradition of the neoclassical model.
Assumption 1. The price of the economy's output is equal to its marginal cost,

$$P = \text{MC}. \tag{9}$$

Assumption 2. The production function exhibits constant returns to scale in both its observed and unobserved inputs,

$$\theta_L + \theta_K + \sum_{m=1}^{M} \gamma_m = \theta_L + \theta_K + \theta_A = 1. \tag{10}$$

I choose F_0 to normalize productivity so that its units remain constant even as θ_L and θ_K change. I set $F_0 = L_{2012}^{-\theta_L} K_{2012}^{-\theta_K}$ where L_{2012} is labor use in 2012 and K_{2012} is capital use in 2012. As F_0 *decreases* the firm has a better technology in labor and capital.

Let λ be the Lagrange multiplier on the economy's output constraint in its cost minimization problem. The first order conditions with respect to L and K give,

$$\frac{W_L L}{\lambda \times Q} = \theta_L, \quad \frac{W_K K}{\lambda \times Q} = \theta_K \tag{11}$$

From the envelope theorem, λ is marginal cost. Therefore, by Assumption 1, $\lambda = P$. So,

$$\frac{W_L L}{PQ} = \theta_L, \quad \frac{W_K K}{PQ} = \theta_K. \tag{12}$$

(θ_L, θ_K) can then be measured using data on nominal output and spending on labor and capital. (θ_L, θ_K) would be measured in the same way regardless of whether we allowed productivity to be selected by the economy or assumed it was an exogenous sequence. Because $A = QF_0^{-1}L^{-\theta_L}K^{-\theta_K}$, the measure of productivity does not change with this model only the interpretation of it. Productivity is exactly the classic Solow residual.

From Assumption 2, I recover θ_A,

$$\theta_A = 1 - \theta_L - \theta_K. \tag{13}$$

The first order condition with respect to A then allows us to recover the last parameter of the economy's cost minimization problem, C_0.

$$\frac{C_0}{\theta_A} A^{\frac{1-\theta_A}{\theta_A}} = \lambda F_0 L^{\theta_L} K^{\theta_K} \tag{14}$$

Because we can recover $A = QF_0^{-1}L^{-\theta_L}K^{-\theta_K}$ given our measure of (θ_L, θ_K)[3] and because $\lambda = P$, I have the following measure of C_0,

$$C_0 = P \times F_0 \theta_A L^{\theta_L} K^{\theta_K} A^{\frac{\theta_A - 1}{\theta_A}} = P \times \theta_A \times \frac{Q}{A^{\frac{1}{\theta_A}}}. \tag{15}$$

So all parameters of the economy's cost minimization problem can be measured.

2.4. Conditional factor demand for productivity

My goal is to understand the role of labor and capital prices in driving productivity growth. To do so, I decompose productivity using its conditional factor demand function. The decomposition allows us to write productivity as the product of cross-price effects (labor and capital prices), own-price effects (productivity prices), demand effects, and labor-capital techology effects.

It is useful to write the problem replacing $\widetilde{A} = A^{\frac{1}{\theta_A}}$ in the economy's cost minimization problem as mentioned above because the problem is then symmetric in three inputs,

$$\min_{L,K,\widetilde{A}} W_L L + W_K K + C_0 \widetilde{A} \quad \text{ST:} \quad F_0 L^{\theta_L} K^{\theta_K} \widetilde{A}^{\theta_A} \geq Q \tag{16}$$

From the first order conditions, the conditional factor demand for $\widetilde{A}$ is,

$$\widetilde{A} = \left(\frac{\theta_L}{W_L}\right)^{-\theta_L} \left(\frac{\theta_K}{W_K}\right)^{-\theta_K} \left(\frac{\theta_A}{C_0}\right)^{-\theta_A} \times \frac{\theta_A Q}{C_0 F_0} \tag{17}$$

[3]My measure of θ_L and θ_K does not depend on how or whether productivity is chosen.

Exponentiating both sides with respect to θ_A, I then have the conditional factor demand for A,

$$A = \left(\frac{\theta_L}{W_L}\right)^{-\theta_A\theta_L}\left(\frac{\theta_K}{W_K}\right)^{-\theta_A\theta_K}\left(\frac{\theta_A}{C_0}\right)^{-\theta_A^2} \times \left(\frac{\theta_A Q}{C_0 F_0}\right)^{\theta_A} \tag{18}$$

$$= \left(\frac{\theta_L}{W_L}\right)^{-\theta_A\theta_L}\left(\frac{\theta_K}{W_K}\right)^{-\theta_A\theta_K}\left(\frac{\theta_A}{C_0}\right)^{\theta_A(1-\theta_A)} \times Q^{\theta_A} \times F_0^{-\theta_A} \tag{19}$$

Taking logs, the conditional factor demand decomposes productivity into terms corresponding to the effective price of the labor input, the effective price of the capital input, the effective price of productivity, output, and the baseline technology for labor and capital,

$$\log A = \underbrace{\theta_A\theta_L \log\left(\frac{W_L}{\theta_L}\right)}_{\text{Effective labor price}} + \underbrace{\theta_A\theta_K \log\left(\frac{W_K}{\theta_K}\right)}_{\text{Effective capital price}} - \underbrace{\theta_A\left(1-\theta_A\right)\log\left(\frac{C_0}{\theta_A}\right)}_{\text{Effective productivity price}} + \underbrace{\theta_A \log Q}_{\text{Output}} - \underbrace{\theta_A \log F_0}_{\text{Baseline technology}} \tag{20}$$

The first two terms capture the substitution effect. Increases in the effective price of labor and capital relative to productivity price increase productivity. The third term captures own price effects. Increases in the effective productivity price will lower productivity all else equal. The fourth term captures demand effects because this equation is the conditional factor demand so supply determinants of output are already incorporated. The model says larger economies are more productive for fixed effective prices. The fifth term captures pure technological progress in using labor and capital. Naturally, improvements in technology (F_0 decreasing) increase productivity.

3. Productivity and its cost in the United States

3.1. Data

I use annual data on the US economy from 1987 to 2016. All of the data is publicly available. I downloaded it from FRED at the St Louis Federal Reserve. In Table 1, I tabulate exactly how each piece of data I use translates into the variables used in my model. The original data comes primarily from the Bureau of Labor Statistics.

All of the data, its source series, and my R code are available on my Github at `https://github.com/flynnzac/factprod-data/`.

3.2. Productivity growth from 1987-2016

Total factor productivity grew sharply starting after the 2001 recession, before growth decelerated around 2005 (see Figure 1). The annual growth rate of productivity from 1987 to 2005 was 0.33%. From 2005 to 2016, the annual growth rate of productivity was only 0.22%. The annual growth rate dropped by a third. I decompose productivity using the model in Section 2 to understand the role of labor and capital prices in slowing productivity growth.

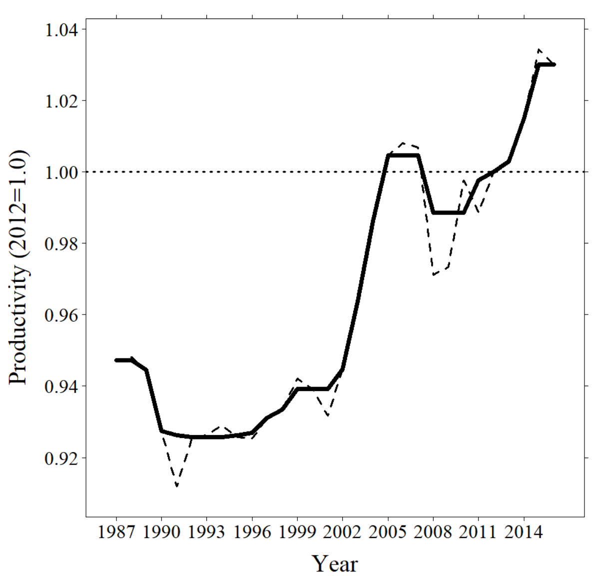
Table 1: Data and parameters

Parameter	Description	Series Name
P, output price	Consumer Price Index/100	CPIAUCNS
Q, output	Private Nonfarm Business Sector Output / P	MPU4910101
L, labor	Private Nonfarm Hours Worked (by salary workers)	PRSCA
W_L, labor price	Total Private Nonfarm Business Sector Labor Compensation / L	MPU4910121
K, capital	Private Nonfarm Business Sector Capital Services	MPU4910042
W_K, capital price	Private Nonfarm Business Sector Capital Income / K	MPU4910111

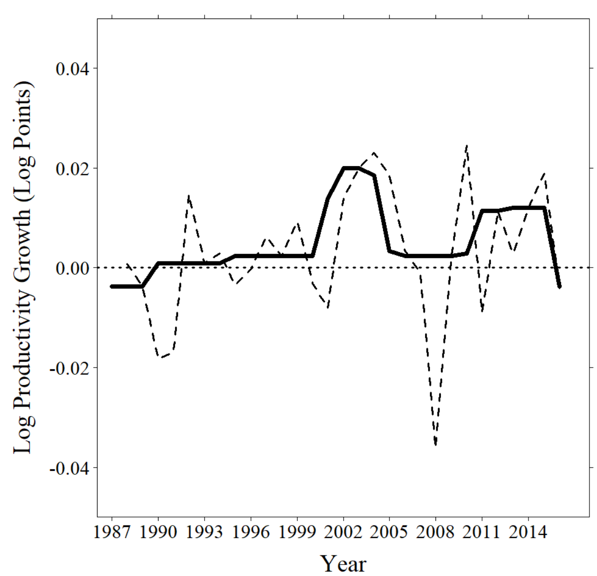
The data I used can be fetched by visiting `https://alfred.stlouis.org/series?seid=SERIESNAME` where SERIESNAME is what appears in the Series Name column above.

Figure 1: Productivity in the United States (1987-2016) (solid lines are smoothed by Tukey's smoothing and dashed lines are original series)

(a) Productivity



(b) Productivity Growth



3.3. Decomposition of productivity growth from 1987-2016

I use the decomposition of productivity's conditional factor demand to study how effective capital, labor, and productivity prices affected productivity growth from 1987 to 2016 in the United States.

The growth rates of the output, effective price of labor, and effective price of capital components of productivity have fallen over time. In recent years, almost none of the productivity growth there has been has come from growth in these terms. See Figure 3. But these three were substantial drivers of productivity growth in the past. The slowdown in productivity growth is primarily a result of the slowdown in these factors.

Almost all recent productivity growth is a consequence of the falling effective price of productivity, see Table 2, consistent with either technological progress in using the unobserved factors of production or more effective technology for supplying them. From 1987 to 2005, the price of productivity actually rose, rising by 0.24% on average annual. Between 2005 and 2016, the price of productivity term fell by 0.64% on average annually. So productivity is becoming increasingly cheap to produce. If everything else had remained constant, we would be seeing accelerating productivity growth instead of declining growth.

The primary cause of the productivity slowdown in this decomposition is falling wages and capital prices relative to the labor and capital elasticity. The economy substituted towards labor and capital and away from productivity. If the effective price of labor and capital grew at the same rate after 2005 as they did before (holding all else constant), productivity would have grown 0.46% annually, more than 40% greater than it did from 1987 and 2005. So productivity growth would be accelerating were it not for the declining growth rate of the effective price of labor and capital.

I also decompose what *part* of the effective cost of productivity term is responsible for recent productivity growth. There are two underlying components of the cost of productivity term, and they have different economic consequences. The first component is θ_A, the technological capability of the underlying inputs Z, and the second component is W_Z, the price of the Z inputs. I find θ_A has been remarkably constant over time at around 0.06. See Figure 2. So variation in the cost of productivity term over time is driven primarily by changes in the price of non-labor and non-capital inputs, W_Z.

To summarize, the deceleration in productivity growth is due primarily to the falling growth rate of effective capital and labor price encouraging the economy to substitute from productivity and towards capital and labor. The decomposition argues that the falling growth rate of productivity is not a sign of slowing technical progress but a change in factor prices causing substitution between observed and unobserved factors of production.

Figure 2: θ_A over time

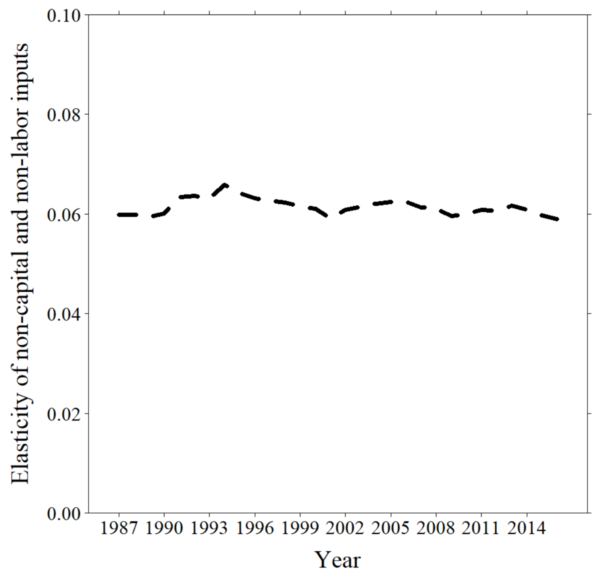


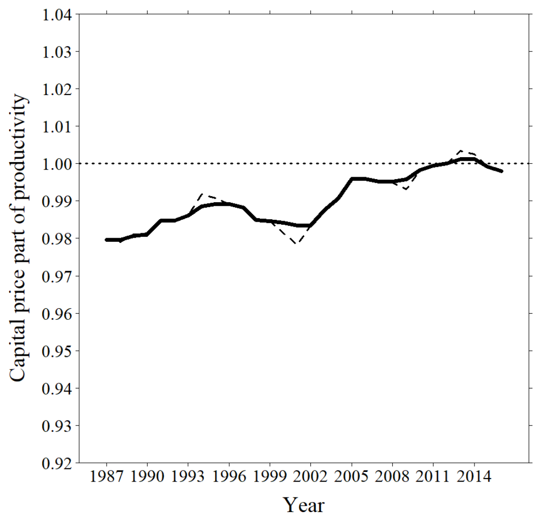
Table 2: Decomposition of productivity growth

Part of productivity	Annual Growth Rate (1987-2005)	Annual Growth Rate (2006-2016)
All	0.33%	0.22%
Labor Price	0.15%	−0.03%
Capital Price	0.09%	0.02%
Output	0.28%	−0.25%
Technical Progress	0.06%	−0.16%
Productivity Price	0.24%	−0.64%

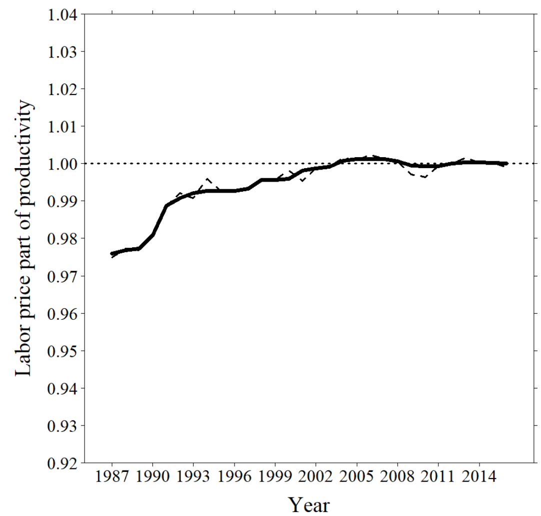
Each row corresponds to a different term in the productivity decomposition. So the row for Technical Progress is $-\theta_A \times \log F_0$ (recall that decreases in F_0 reflect increases in labor-capital technology).

Figure 3: Non-cost of productivity components of productivity (solid lines are smoothed by Tukey's method, dashed lines are original series, and dotted line is at 1.0)

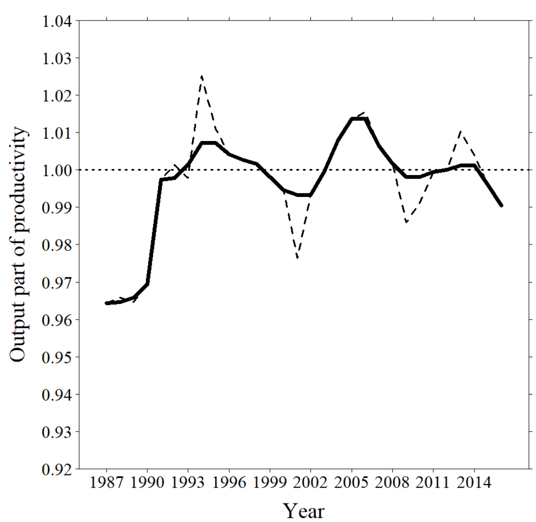
(a) Effective price of capital component of productivity (Index: 2012 = 1.0)



(b) Effective price of labor component of productivity (Index: 2012 = 1.0)



(c) Output component of productivity (Index: 2012 = 1.0)



(d) Technical progress in labor and capital component of productivity (Index: 2012 = 1.0)

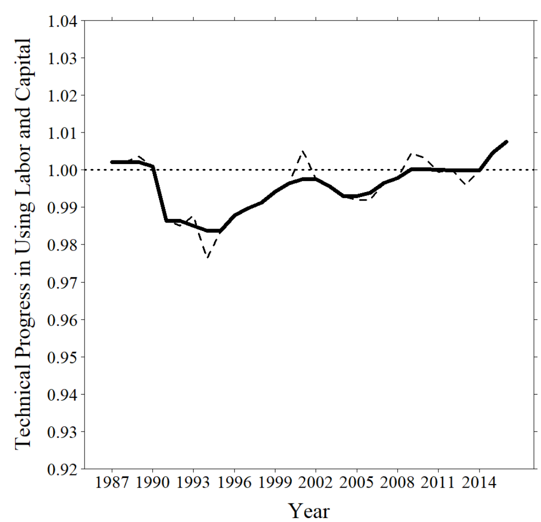
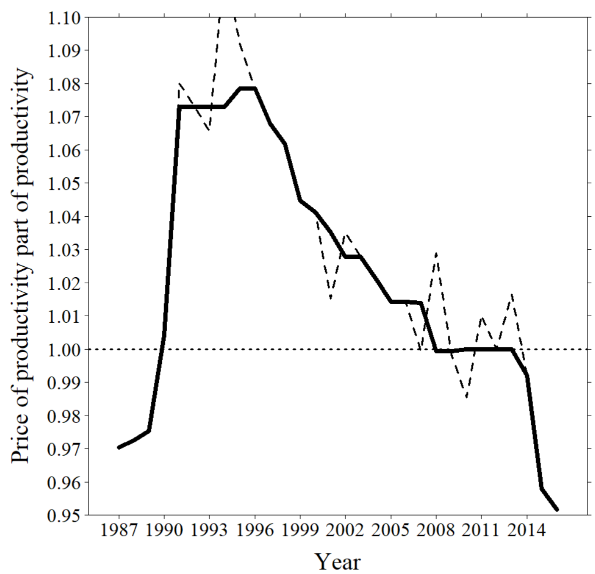


Figure 4: The falling effective price of productivity (solid lines are smoothed by Tukey's smoothing, dashed are original series, dotted line is at 1.0)



4. What does all of this mean for productivity growth?

A prominent theory for the decline in productivity is that technology is improving more slowly than it used to (Gordon, 2016). But from the perspective of the model in this paper, this is not the whole story. The price of becoming more productive has fallen dramatically and has continued to decline without slowing down. While growth in the baseline technology for using labor and capital has decelerated, the primary cause behind falling productivity is that the growth rate of the effective price of labor and capital have declined, encouraging relatively more substitution towards those factors of production and away from productivity. The decline in the rate of productivity growth is not then a sign of technological stagnation, but a sign that labor and capital are relatively cheaper than they used to be. There is less benefit to the economy of being more productive even as the cost of being so falls.

In this paper, I developed a method of decomposing productivity supposing that it is produced by unobserved factors of production. I applied this decomposition to study productivity growth in the United States over the past thirty years. The analysis offered a new interpretation of part of the slowdown in productivity. One useful direction for future research is to uncover what factors of production lie in productivity and are behind the recent slowdown. The methodology could also be applied to micro data on individual firms to provide a new measure of productivity based on the factor interpretation of productivity.

References

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–351.

Baqaee, D. and E. Farhi (2019). A short note on aggregating productivity. *NBER Working Papers* (25688).

Basu, S. and M. Kimball (1997). Cyclical productivity with unobserved input variation. *NBER Working Papers* (5915).

Byrne, D., J. Fernald, and M. Reinsdorf (2016). Does the united states have a productivity slowdown or a measurement problem? *Brookings Papers on Economic Activity*, 109–182.

Fernald, J. (2014). A quarterly, utilization-adjusted series on total factor productivity. *Federal Reserve Bank of San Francisco Working Paper* (2012-19).

Fernald, J. and C. Wang (2016). Why has the cyclicality of productivity changed? what does it mean? *Federal Reserve Bank of San Francisco Working Paper Series* (2016-07).

Gordon, R. (2016). *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*. Princeton University Press.

Griliches, Z. and D. Jorgenson (1967). The explanation of productivity change. *The Review of Economic Studies 34*(3), 249–283.

Romer, P. (1986). Increasing returns and long-run growth. *The Journal of Political Economy 94*(5), 1002–1037.

Solow, R. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics*, 312–320.

Syverson, C. (2017). Challenges to mismeaurement explanations for the us productivity slowdown. *Journal of Economic Perspectives 31*(2), 165–186.