Unproductive by choice: substitution and the slowdown in aggregate productivity growth in the United States

Zach Flynn

April 9, 2019

Abstract

I develop a new decomposition of aggregate total factor productivity. The decomposition is based on the idea that all output produced in the economy is the result of the use of some factor of production, but we cannot measure all factors of production. I model productivity as an index of these unobserved factors of production. This view of productivity introduces a new avenue through which productivity can either grow or decline. Changes in the effective price of labor and capital will cause *substitution* to or from productivity. I decompose productivity using its conditional factor demand equation to measure the extent to which changes in the effective price of labor and capital are behind the slowdown in productivity growth in the United States from 2006 to 2016. I find fully two-thirds of the slowdown can be explained by relatively more substitution to labor and capital and away from productivity.

1. Introduction

Total factor productivity growth has slowed in the United States in the past decade. The causes and consequences of its decline matter for long run economic welfare. The two leading explanations for the decline in measured productivity growth are that technological growth has truly slowed or that productivity is increasingly mismeasured and is not capturing the full economic value of new innovations. If the fall in the growth rate of productivity is the result of slowing technological growth, growth in welfare-per-person has been anemic over the past decade. If the declining growth rate is illusory, the result of some error in how we measure productivity, it may not matter at all[1]. In this paper, I propose a model of productivity that offers a different explanation than these two leading theories.

I model productivity as a function of factors of production that we do not observe. With this view of productivity, there is a new avenue through which productivity can grow or decline. As the effective prices of labor and capital rise or fall, the economy will substitute towards or away from productivity. Substition from or to productivity has a very different economic consequence than either slowing technological growth or the mismeasurement hypothesis. In the mismeasurement hypothesis, there is no slowdown. In the slowing technological growth hypothesis, there is a

[1] See Syverson (2017) for a comparison of these two competing explanations for falling productivity growth.

slowdown and it is the result of the economy no longer innovating as rapidly as it once did (Gordon, 2016) or of it inefficiently allocating resources (Baqaee and Farhi 2019 propose a measurement of productivity for this case). In the substitution hypothesis proposed in this paper, there *is* a slowdown but it is an efficient slowdown: the factors of production that determine productivity have a higher effective price relative to capital and labor than they did previously. The economy is efficiently substituting away from productivity and towards capital and labor.

This view of productivity underlies Griliches and Jorgenson (1967) who argue that there would be little left of the productivity residual if we could accurately and fully account for all the various factors of production in the economy. They memorably call productivity, "the measure of our ignorance". The growth literature on human capital accumulation is effectively trying to pull human capital, a factor of production produced at a cost, out of the Solow residual. The literature on intangible capital and factor utilization also tries to pull factors of production out of the Solow residual (see, for example, Fernald and Wang 2016). What I add to the literature in this paper is a simple framework for modeling how the totality of the unmeasured factors of production change in response to changes in the effective price of labor and capital. Instead of trying to pull individual factors out of productivity, I assume productivity is determined by many unobserved factors and model how those factors change in response to other changes in the economy.

My framework is entirely within the standard neoclassical growth model. The economy is modeled as a representative firm. Returns to scale are constant. Prices are equal to marginal cost. The firm has a Cobb Douglas production function. The one difference is that all output is produced by some factor of production and we do not observe all of these factors. As a consequence, my measure of productivity itself is no different than Solow (1957)'s measure. The difference is in what changes in the economy influence this measure of productivity.

I develop a new decomposition of productivity based on its conditional factor demand equation. The decomposition has four terms: changes in the effective price of labor, changes in the effective price of capital, changes in the effective price of productivity, and changes in the size of the economy. I use this decomposition to study the recent productivity slowdown in the United States and measure the extent to which substitution from productivity and to labor and capital are behind the slowdown. I find two-thirds of the fall in productivity growth rates can be attributed to the relative decrease in the effective price of labor and capital, encouraging relatively more substitution from productivity and towards these factors of production.

Section 2 introduces my model of production and productivity, deriving the factor demand equation for productivity and the decomposition of productivity that forms the basis for my empirical analysis.

Section 3 applies my method to study the economy of the United States from 1987 to 2016 and to decompose the causes of the recent decline in total factor productivity from 2006 to 2016.

Section 4 concludes with the implications of this understanding of the recent decline in productivity and some ideas for future research.

2. The factor model of productivity

2.1. What is productivity?

In the neoclassical growth model, productivity is an exogenous process. Productivity in period t is A_t and in period $(t + 1)$ productivity is $A_{t+1} = (1 + g_t) A_t$ where g_t is an exogenous growth rate. This model of productivity is also used in the endogenous growth models following Romer (1986). In the Schumpeterian growth models following Aghion and Howitt (1992), changes in productivity are determined by changes in the supply of workers who decide to research improvements in production versus those who choose to produce with the current technology.

I model productivity differently. I suppose that were we to accurately measure all factors of production in the economy that there would not be a residual in the production relationship,

$$\text{Output} = F(\text{Inputs}). \tag{1}$$

I take seriously Griliches and Jorgenson (1967)'s description of productivity as "the measure of our ignorance". It is the part of output that we cannot explain with the set of inputs we can measure. But, in reality, this part of output is also produced using factors of production and those factors substitute for observed factors of production like labor and capital. An implication is that shocks that affect the wage of labor or the rental rate of capital can result in changes to productivity. This cannot happen if we treat productivity as an exogenous sequence.

I develop a method of decomposing productivity growth into growth resulting from changes in the price of labor and capital relative to their output elasticity, changes in aggregate demand, and changes in the cost of building productiviy. I do so within a model where productivity is a measure of the contribution of the unobserved factors of production to output. The model is a standard neoclassical model of the supply side of the economy. Output is produced by a representative firm. The production function is Cobb Douglas. Returns to scale are constant. Markets are competitive. But some factors of production are not observed. Productivity is a function of these unobserved factors. I show that we can measure the cost of productivity and its overall elasticity even though we do not observe the inputs that make up productivity. I then use the conditional factor demand function for productivity to understand how changes in wages or the price of capital affect productivity.

2.2. The pure factor production function

Let Q be output, L be labor, K be capital, and let $Z \in \mathbb{R}^M$ be a vector of unobserved factors of production. The aggregate production function is Cobb Douglas,

$$Q = L^{\theta_L} K^{\theta_K} \times \prod_{m=1}^{M} Z_m^{\gamma_m}. \tag{2}$$

I assume the economy minimizes its costs like a representative firm would,

$$\min_{L,K,Z} W_L L + W_K K + W_Z^\top Z \quad \text{ST:} \quad L^{\theta_L} K^{\theta_K} \times \prod_{m=1}^{M} Z_m^{\gamma_m} \geq Q. \tag{3}$$

Productivity is the contribution of unobserved factors of production to output. Let A be productivity, or,

$$A = \prod_{m=1}^{M} Z_m^{\gamma_m}. \tag{4}$$

From the separability of the problem, I write the cost minimization problem in terms of A instead of Z. The economy will minimize the cost of producing A from the Z inputs so,

$$C(A) = \min W_Z^\top Z \quad \text{ST:} \quad \prod_{m=1}^{M} Z_m^{\gamma_m} \geq A. \tag{5}$$

From textbook algebra on the cost function of the Cobb Douglas production function, I have that $C(A)$ has the following functional form,

$$C(A) = C_0 (W_Z, \gamma) A^{\frac{1}{\sum_m \gamma_m}}. \tag{6}$$

Write $\theta_A = \sum_m \gamma_m$ because we can think of the sum of the Z output elasticities as the output elasticity of productivity.

The economy's cost minimization problem in terms of (L, K, A) is then,

$$\min_{L,K,A} W_L L + W_K K + C_0 (W_Z, \gamma) A^{\frac{1}{\theta_A}} \quad \text{ST:} \quad L^{\theta_L} K^{\theta_K} A \geq Q. \tag{7}$$

Equivalently, we can define $\widetilde{A} = A^{\frac{1}{\theta_A}}$ and treat $\widetilde{A}$ symmetrically to (L, K),

$$\min_{L,K,\widetilde{A}} W_L L + W_K K + C_0 (W_Z, \gamma) \widetilde{A} \quad \text{ST:} \quad L^{\theta_L} K^{\theta_K} \widetilde{A}^{\theta_A} \geq Q. \tag{8}$$

2.3. Measuring productivity

I make two assumptions that are firmly in the tradition of the neoclassical model.
Assumption 1. The price of the economy's output is equal to its marginal cost,

$$P = \text{MC}. \tag{9}$$

Assumption 2. The production function exhibits constant returns to scale in both its observed and unobserved inputs,

$$\theta_L + \theta_K + \sum_{m=1}^{M} \gamma_m = \theta_L + \theta_K + \theta_A = 1. \tag{10}$$

Let λ be the Lagrange multiplier on the economy's output constraint in its cost minimization problem. The first order conditions with respect to L and K give,

$$\frac{W_L L}{\lambda \times Q} = \theta_L \tag{11}$$
$$\frac{W_K K}{\lambda \times Q} = \theta_K \tag{12}$$

From the envelope theorem, λ is marginal cost. Therefore, by Assumption 1, $\lambda = P$. So,

$$\frac{W_L L}{PQ} = \theta_L \tag{13}$$
$$\frac{W_K K}{PQ} = \theta_K. \tag{14}$$

(θ_L, θ_K) can then be measured using data on nominal output and spending on labor and capital. (θ_L, θ_K) would be measured in the same way regardless of whether we allowed productivity to be selected by the economy or assumed it was an exogenous sequence. Because $A = QL^{-\theta_L}K^{-\theta_K}$, the measure of productivity does not change with this model only the interpretation of it.

From Assumption 2, I recover θ_A,

$$\theta_A = 1 - \theta_L - \theta_K. \tag{15}$$

The first order condition with respect to A then allows us to recover the last parameter of the economy's cost minimization problem, C_0.

$$\frac{C_0}{\theta_A} A^{\frac{1-\theta_A}{\theta_A}} = \lambda L^{\theta_L} K^{\theta_K} \tag{16}$$

Because we can recover $A = QL^{-\theta_L}K^{-\theta_K}$ given our measure of (θ_L, θ_K)[2] and because $\lambda = P$, I have the following measure of C_0,

$$C_0 = P \times \theta_A L^{\theta_L} K^{\theta_K} A^{\frac{\theta_A - 1}{\theta_A}} = P \times \theta_A \times \frac{Q}{A^{\frac{1}{\theta_A}}}. \tag{17}$$

So all parameters of the economy's cost minimization problem can be measured.

2.4. Conditional factor demand for productivity

My goal in this paper is to understand the role of labor and capital prices in driving productivity growth. To measure how much they affect productivity for a given size of the economy, I decompose productivity using its conditional factor demand function. The decomposition will allow us to

[2] My measure of θ_L and θ_K does not depend on how or whether productivity is chosen.

write productivity as the product of cross-price effects (labor and capital prices), own-price effects (productivity prices), and size effects (economy-wide output).

It is useful to write the problem replacing $\widetilde{A} = A^{\frac{1}{\theta_A}}$ in the economy's cost minimization problem as mentioned above because the problem is then symmetric in three inputs,

$$\min_{L,K,\widetilde{A}} W_L L + W_K K + C_0 \widetilde{A} \quad \text{ST:} \quad L^{\theta_L} K^{\theta_K} \widetilde{A}^{\theta_A} \geq Q. \tag{18}$$

The conditional factor demand for $\widetilde{A}$ is[3],

$$\widetilde{A} = \left(\frac{\theta_L}{W_L}\right)^{-\theta_L} \left(\frac{\theta_K}{W_K}\right)^{-\theta_K} \left(\frac{\theta_A}{C_0}\right)^{-\theta_A} \times \frac{\theta_A Q}{C_0} \tag{19}$$

Exponentiating both sides with respect to θ_A, I then have the conditional factor demand for A,

$$A = \left(\frac{\theta_L}{W_L}\right)^{-\theta_A\theta_L} \left(\frac{\theta_K}{W_K}\right)^{-\theta_A\theta_K} \left(\frac{\theta_A}{C_0}\right)^{-\theta_A^2} \times \left(\frac{\theta_A Q}{C_0}\right)^{\theta_A} \tag{20}$$

$$= \left(\frac{\theta_L}{W_L}\right)^{-\theta_A\theta_L} \left(\frac{\theta_K}{W_K}\right)^{-\theta_A\theta_K} \left(\frac{\theta_A}{C_0}\right)^{\theta_A(1-\theta_A)} \times Q^{\theta_A} \tag{21}$$

Taking logs, the conditional factor demand decomposes productivity into terms corresponding to the effective price of the labor input, the effective price of the capital input, the effective price of productivity, and output,

$$\log A = \underbrace{\theta_A\theta_L \log\left(\frac{W_L}{\theta_L}\right)}_{\text{Effective labor price}} + \underbrace{\theta_A\theta_K \log\left(\frac{W_K}{\theta_K}\right)}_{\text{Effective capital price}} - \underbrace{\theta_A\left(1-\theta_A\right)\log\left(\frac{C_0}{\theta_A}\right)}_{\text{Effective productivity price}} + \underbrace{\theta_A \log Q}_{\text{Output}} \tag{22}$$

The first two terms capture the substitution effect or the cross-price effect. Increases in the effective price of labor and capital relative to productivity price will increase productivity for an economy of fixed size. The third term captures own price effects. Increases in the effective productivity price will lower productivity all else fixed. The fourth term captures size effects. The model says larger economies are more productive for fixed effective prices.

3. Productivity and its cost in the United States

3.1. Data

I use data on the US economy from 1987 to 2016. All of the data is publicly available. I downloaded it from FRED at the St Louis Federal Reserve. In Table 1, I tabulate exactly how each piece

[3]This problem is a textbook Cobb Douglas cost minimization problem so I will skip the derivation.

Table 1: Data and parameters

Parameter	Description	Series Name
P, output price	Consumer Price Index/100	CPIAUCNS
Q, output	Private Nonfarm Business Sector Output / P	MPU4910101
L, labor	Private Nonfarm Hours Worked (by salary workers)	PRSCA
W_L, labor price	Total Private Nonfarm Business Sector Labor Compensation / L	MPU4910121
K, capital	Private Nonfarm Business Sector Capital Services	MPU4910042
W_K, capital price	Private Nonfarm Business Sector Capital Income / K	MPU4910111

The data I used can be fetched by visiting https://alfred.stlouis.org/series?seid=SERIESNAME where SERIESNAME is what appears in the Series Name column above.

of data I use translates into the variables used in my model. The original data comes primarily from the Bureau of Labor Statistics.

All of the data, its source series, and my R code are available on my Github at https://github.com/flynnzac/factprod-data/.

3.2. Productivity growth from 1987-2016

Total factor productivity grew sharply starting after the 2001 recession, before growth decelerated around 2005 (see Figure 1). The annual growth rate of productivity from 1987 to 2005 was 0.50%. From 2006 to 2016, the annual growth rate of productivity was only 0.15%. The annual growth dropped by more than two-thirds. My empirical goal in this paper is to decompose productivity using the model in Section 2 to understand the role of labor and capital prices in slowing productivity growth.

3.3. Decomposition of productivity growth from 1987-2016

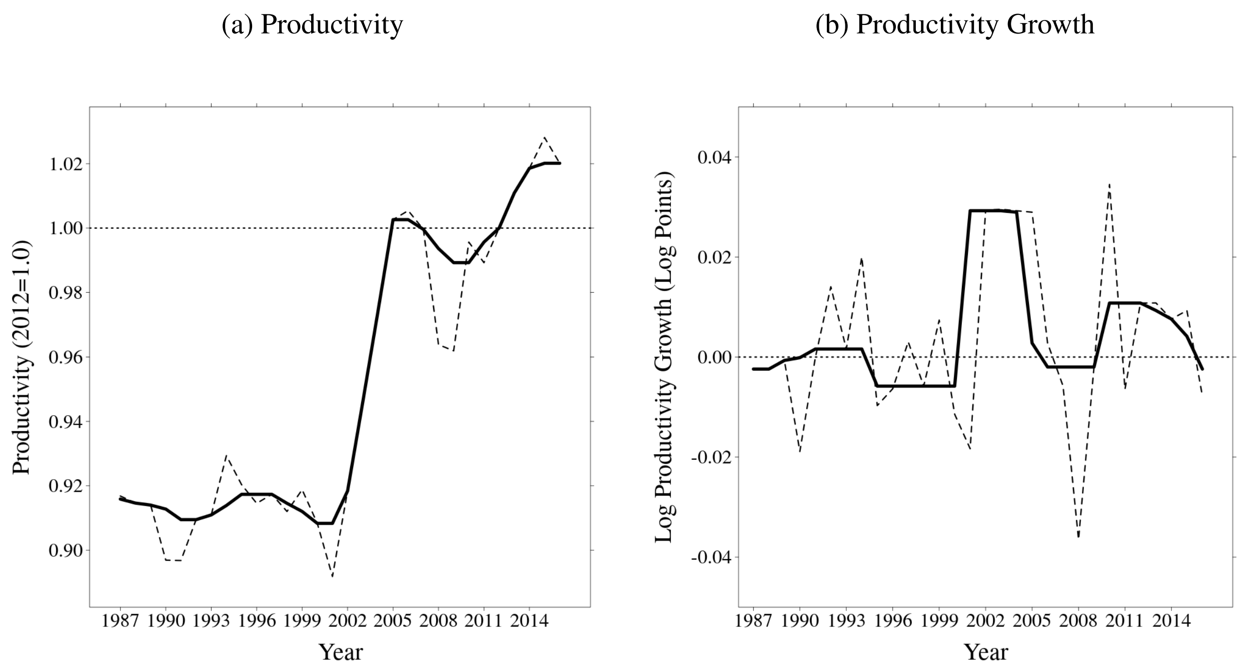
I use the decomposition of productivity's conditional factor demand to study how effective capital, labor, and productivity prices affected productivity growth from 1987 to 2016 in the United States.

$$\log A = \underbrace{\theta_A \theta_L \log\left(\frac{W_L}{\theta_L}\right)}_{\text{Effective price of labor}} + \underbrace{\theta_A \theta_K \log\left(\frac{W_K}{\theta_K}\right)}_{\text{Effective price of capital}} - \underbrace{\theta_A (1-\theta_A) \log\left(\frac{C_0}{\theta_A}\right)}_{\text{Effective price of productivity}} + \underbrace{\theta_A \log Q}_{\text{Output}}. \quad (23)$$

The growth rate of the output, effective price of labor, and effective price of capital components of productivity has fallen over time. In recent years, almost none of mutltifactor productivity growth has come from growth in these terms. See Figure 3. But these three were substantial drivers of productivity growth in the past. The slowdown in productivity growth is primarily a result of the slowdown in these factors.

In recent years, almost all productivity growth has been a consequence of the falling price of productivity, see Table 2. Between 2006 and 2016, the price of productivity term fell by 0.41% on

Figure 1: Productivity in the United States (1987-2016) (solid lines are smoothed by Tukey's smoothing and dashed lines are original series)



average annually. From 1987 to 2005, the price of productivity was nearly flat, rising by 0.02%. So the effective price of productivity by itself is accelerating productivity growth.

Between 2006 and 2016, productivity itself grew at about one-third the rate it grew between 1987 and 2005. The decomposition demonstrates that the reason behind the productivity slowdown is, in part, falling wages and capital prices relative to the labor and capital elasticity. The economy substituted towards labor and capital and away from productivity. If labor and capital grew at the same rate after 2005 as they did before (holding all else constant), productivity would have grown 0.40%, only 20% less than it did beween 1987 and 2005. So a full two-thirds of the decline in the the productivity growth rate can be explained by a decrease in the rate of decline in labor and capital prices (relative to θ_L and θ_K).

I also decompose what part of the effective cost of productivity term is responsible for recent productivity growth. There are two underlying components of the cost of productivity term, and they have different economic consequencs. The first component is θ_A, the technological capability of the underlying inputs Z, and the second component is W_Z, the price of the Z inputs. I find θ_A has been remarkably constant over time at around 0.06. See Figure 2. So variation in the cost of productivity term over time is driven primarily by changes in the price of non-labor and non-capital inputs, W_Z. In this model, productivity growth in recent years is a consequence of falling non-labor and non-capital prices.

Figure 2: θ_A over time

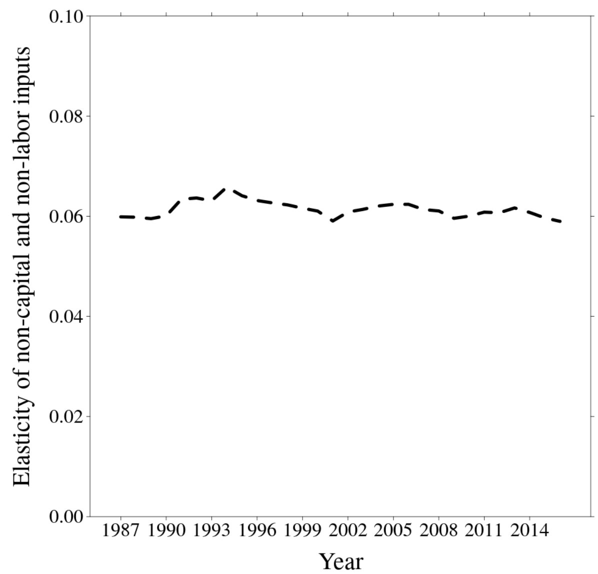
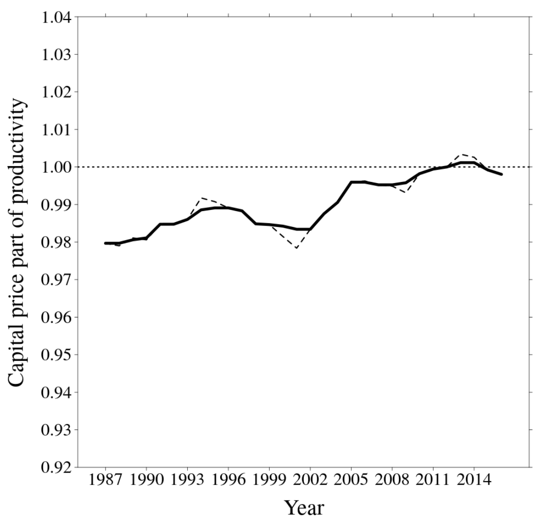


Table 2: Decomposition of productivity growth

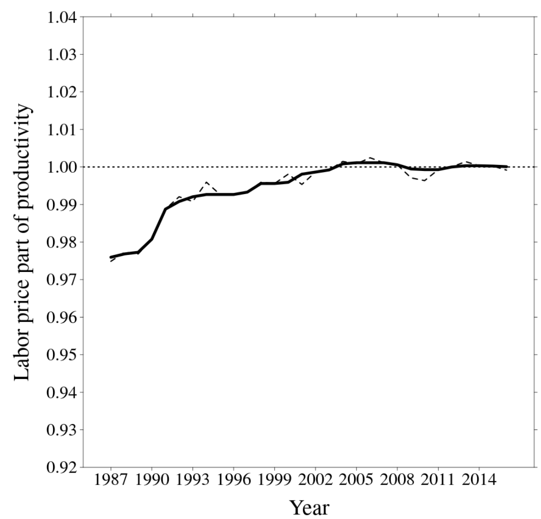
Part of productivity	Annual Growth Rate (1987-2005)	Annual Growth Rate (2006-2016)
All	0.50%	0.15%
Labor Price	0.15%	−0.03%
Capital Price	0.09%	0.02%
Output	0.28%	−0.25%
Productivity Price	0.02%	−0.41%

Figure 3: Non-cost of productivity components of productivity (solid lines are smoothed by Tukey's method, dashed lines are original series, and dotted line is at 1.0)

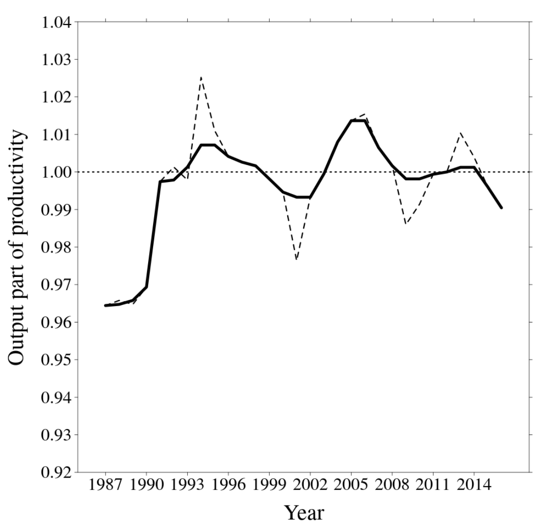
(a) Effective price of capital component of productivity (Index: 2012 = 1.0)



(b) Effective price of labor component of productivity (Index: 2012 = 1.0)



(c) Output component of productivity (Index: 2012 = 1.0)



(d) Total non-effective price of productivity components of productivity (Index: 2012 = 1.0)

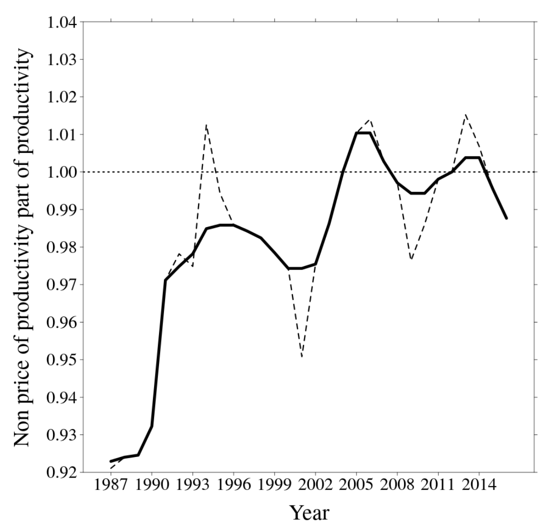
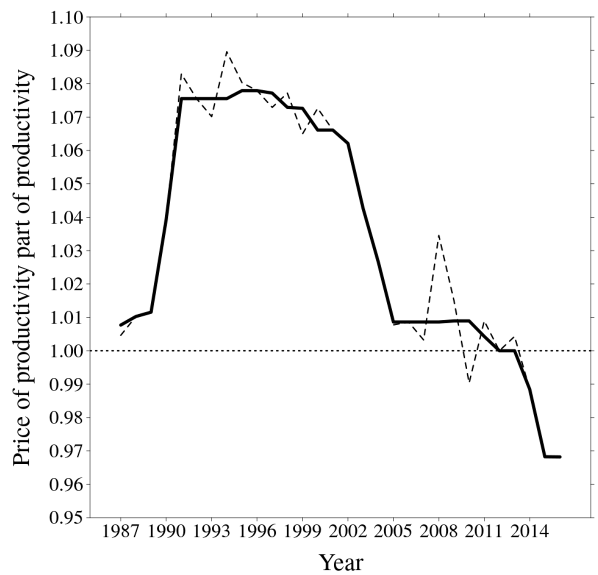


Figure 4: The falling effective price of productivity (solid lines are smoothed by Tukey's smoothing, dashed are original series, dotted line is at 1.0)



3.4. What part of the growth rate of effective capital and labor prices has declined?

Having established that the model predicts that the slowdown in the growth rate of the effective price of labor and capital is an important component of the slowdown in productivity growth, I now turn to exactly how effective prices have changed over time. Effectice prices increase both when prices increase and when output elasticities fall.

For capital, both elasticity rising and falling prices explain the substitution away from productivity. This indicates that capital has both technologically improved and its price has fallen. In recent years, both the capital elasticity and capital prices appear to have stabilized, but in the past they both moved substantially. The price of capital was roughly 20% higher in 1987 than in 2016, but it is roughly unchanged since 2005.

For labor, while the labor elasticity has fallen, increasing the effective price of labor, wages have increased, increasing the price of labor. The magnitude of the wage increase has been greater than the elasticity decrease historically, but wage growth has leveled out in recent years and the labor elasticity has as well causing the rate of growth for the effective price of labor to decrease.

Figure 5: Elasticities of labor and capital

(a) Labor Elasticity over time (b) Capital Elasticity over time

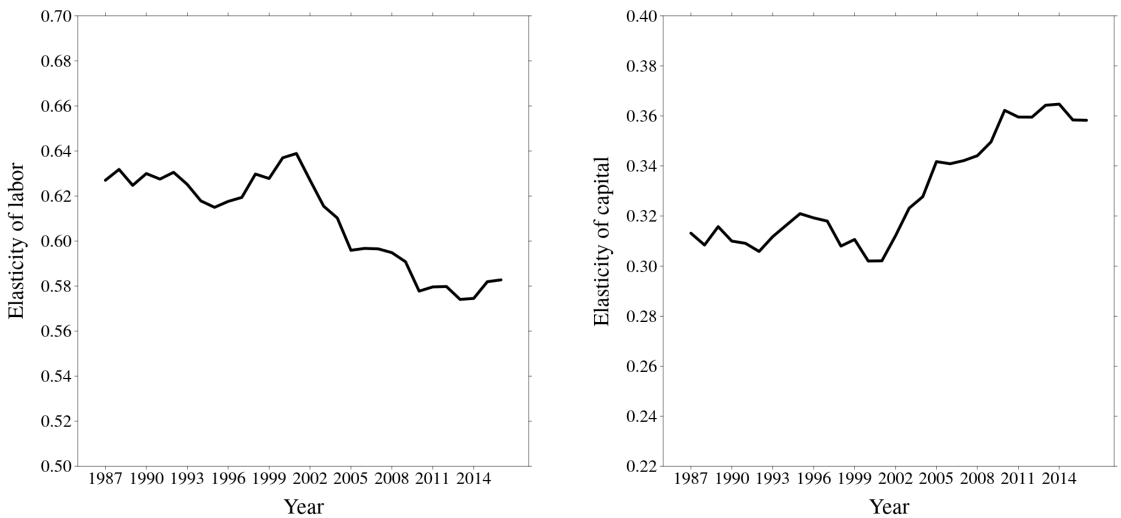
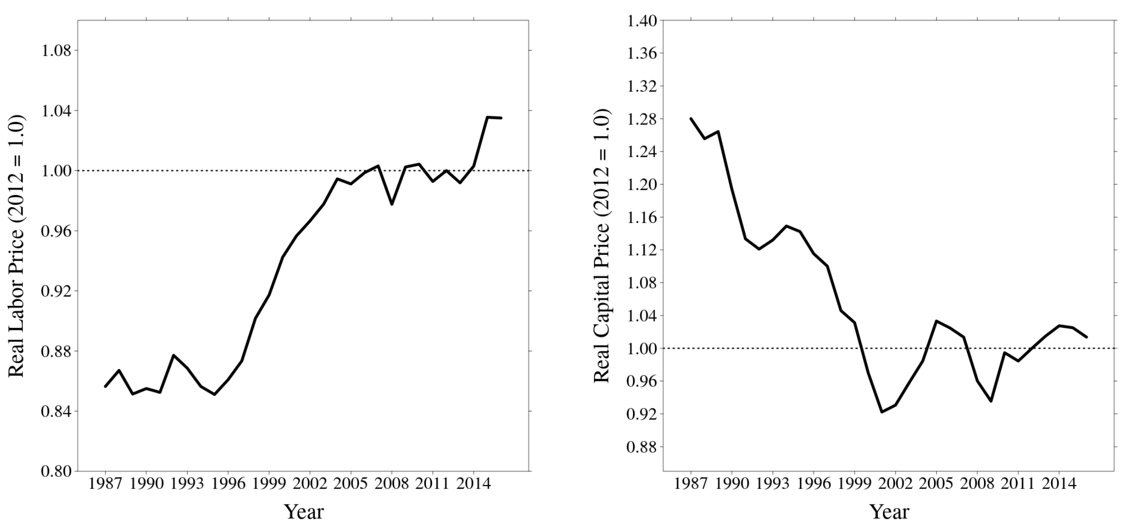


Figure 6: Prices of labor and capital

(a) Labor price over time (b) Capital price over time



4. What does all of this mean for productivity growth?

A prominent theory for the decline in productivity is that technology is improving more slowly than it used to (Gordon, 2016). But from the perspective of the model in this paper, this is not the case. The price of becoming more productive has fallen dramatically and has continued to decline without slowing down. What has happened instead is that the growth rate of the effective price of labor and capital have declined, encouraging relatively more substitution towards those factors of production and away from productivity. The decline in the rate of productivity growth is not then a sign of technological stagnation, but a sign that labor and capital are relatively cheaper than they used to be. There is less benefit to the economy of being more productive even as the cost of being so falls.

In this paper, I developed a method of decomposing productivity supposing that it is produced by unobserved factors of production. I applied this decomposition to study productivity growth in the United States over the past thirty years. The analysis offered a new interpretation of part of the slowdown in productivity. One useful direction for future research is to uncover what factors of production lie in productivity and are behind the recent slowdown. The methodology could also be applied to micro data on individual firms to provide a new measure of productivity based on the factor interpretation of productivity. Another useful direction for future research would be to change assumptions 1 and/or 2 or extend the general framework to the heterogenous firms case.

References

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–351.

Baqaee, D. and E. Farhi (2019). A short note on aggregating productivity. *NBER Working Papers* (25688).

Fernald, J. and C. Wang (2016). Why has the cyclicality of productivity changed? what does it mean? *Federal Reserve Bank of San Francisco Working Paper Series* (2016-07).

Gordon, R. (2016). *The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War*. Princeton University Press.

Griliches, Z. and D. Jorgenson (1967). The explanation of productivity change. *The Review of Economic Studies 34*(3), 249–283.

Romer, P. (1986). Increasing returns and long-run growth. *The Journal of Political Economy 94*(5), 1002–1037.

Solow, R. (1957). Technical change and the aggregate production function. *The Review of Economics and Statistics*, 312–320.

Syverson, C. (2017). Challenges to mismeaurement explanations for the us productivity slowdown. *Journal of Economic Perspectives 31*(2), 165–186.