Journal of Physics:
Conference Series

PAPER • OPEN ACCESS

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To cite this article: Farah Hayati Mustapa and Mohd Tahir Ismail 2019 *J. Phys.: Conf. Ser.* **1366** 012130

Modelling and forecasting S&P 500 stock prices using hybrid Arima-Garch Model

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Abstract. The S&P 500 is a bellwether and leading indicator for the economy as well as the default vehicle for passive investors who want exposure to the U.S. economy via index funds. Since 1957, the S&P 500 has performed amazingly, outpacing other leading asset classes such as bonds or commodities. This study seeks to develop an appropriate ARIMA model that best fit the monthly stock price of S&P 500 for a period of 17 years, 2001–2017, thus make a short-term forecast in a way to give an overview and help the investor or portfolio manager in decision making. EViews software was used to run the analysis of the data. Our analysis involved 2-step procedure, which were identifying ARIMA model then fitting GARCH (1,1) into the model. As a result, ARIMA (2,1,2)-GARCH (1,1) model was found to be the best model for forecasting the S&P500 stock prices. The research findings indicate that a dynamic forecast gave a better result compared to a static forecast.

1. Introduction

Stock prices' analysis of forecasting always be an interesting area of researchers in finance and economy fields. Stock market is very volatile due to various economic indicators. Such indicators include but not limited to Consumer Price Index, CPI, and rate of inflation. Volatility in the market can be beneficial for traders, as it offers an opportunity to profit. However, it also increases the risk of loss. Forecasting can provide an important benchmark for market researchers, traders or investor. Since stock prices recorded over a specified period, therefore we can analyse and model the series using time series analysis. Time series analysis is a useful statistical method that allows the user to find meaningful information in data collected over time. Time series analysis, among other method such as artificial neural network [1,2], demonstrates the power of forecasting stock prices based on historical prices. In this study, we will conduct a time series analysis to identify the best model to predict future stock values. We are using S&P 500 stock price to demonstrate the powerful technique of Autoregressive Integrated Moving Average, ARIMA modelling in prediction of future prices. The S&P 500 is an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the New York Stock Exchange, NYSE or Nasdaq Stock Market, NASDAQ.

In 1976, two statisticians, Box and Jenkins [3] developed systematic procedures for identifying and estimating ARIMA(p,d,q) model. ARIMA is a class of statistical models for analysing and forecasting time series data. The procedures also include statistical tests for residuals diagnostic and short-term forecasting. This model considers changing trends, seasonality, and random noise in time series. An

ARIMA process can be divided into three fragments: AR(auto regressive), I(order of Integration) and MA(moving average).

In general, an ARMA(p,q) process [4] can be written as

$$Y_t = \emptyset_1 Y_{t-1} + \cdots + \emptyset_p Y_{t-p} + \varepsilon_t - \theta_1 \epsilon_{t-1} - \cdots - \theta_q \epsilon_{t-q} \tag{1}$$

where $\boldsymbol{Y_t}$ is the observed value and $\boldsymbol{\varepsilon_t}$ is the random error at t, $\emptyset_i$ and $\boldsymbol{\theta_i}$ are the AR and MA coefficients respectively, while p and q are the number of time lags of AR and MA models, respectively. ARIMA modelling, as distinct from ARMA model, has the additional 'I' in the acronym, standing for 'integrated'. An ARMA(p,q) model in the variable differenced d times is equivalent to an ARIMA(p,d,q) model.

Whilst, Autoregressive Conditional Heteroskedasticity, ARCH model was first proposed by Engle[5] for which he won the 2003 Nobel Memorial Prize in Economic Sciences. A useful generalization of this model is GARCH(m,n) model introduced by Bollerslev[6]. ARCH and GARCH models treat heteroskedasticity as a variance to be modelled. The GARCH model allows the conditional variance to be dependent upon previous own lags [7], so that the equation will be as simple as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

Equation (2) is the GARCH(1,1) model. It was found that GARCH(1,1) is superior among other GARCH models [8]. Various studies includes [9], [10] and [11] have shown that a simple GARCH(1,1) is usually sufficient to model heteroscedasticity of variance.

The present paper has two-fold objectives, (i) develop an ARIMA(p,d,q) model that best fit the S&P 500 stock prices movement based on statistically significant coefficients, fulfil the assumption of normally identical independently distributed residuals and lowest value of selected information criterion, and (ii) fitting GARCH(1,1) into the model to model heteroscedasticity in variance of the series. Then, we do forecast to illustrate how good is the fitted model and, we make comparison between a static forecast and a dynamic forecast.

In addition to the introduction, the rest of this paper is organised as follows: Section 2 presents the previous researches in the literature. Section 3 elaborates the research methodology used in this study. The findings of the study will be demonstrated in Section 4 and the final section provides the conclusion on the outcome.

2. Literature review

There are many related researches in modelling and forecasting stock prices using ARIMA model. Adebiyi et.al[12] demonstrated extensive process to obtain the best ARIMA model (based on the smallest value of Schwarz Information Criterion, SIC and the smallest standard error of regression, S.E of Regression) to predict stock prices. Meanwhile Kamruzzaman et.al[13] calculated returns by using Relative Difference method and chose ARMA(2,2) model (based on the smallest Akaike Information Criterion, AIC value) as the parsimonious model for forecasting the monthly market returns of Dhaka Stock Exchange. Also, Abbasi et.al[14] applied ARIMA model in their case study of flying cement industry. They took first log differencing on their data series to achieve stationarity. To determine the order of ARMA model, they referred to the ACF or PACF patterns. Since all points were relatively small and lie within the confidence interval, they took second order lagged difference and obtained a result of both the ACF and PACF plots cut off at lag one suggesting that ARIMA(1,2,1) was an appropriate model for forecasting cement stock prices in their study.

So far, the ARCH effect in the models had been ignored by the researchers [12], [13] and [14]. In extension to Abbasi et.al[14] work, Almarashi et.al[11] used GARCH(1,1) to model volatility in the cement stock prices. The authors found that the ARIMA(1,2,1)-GARCH(1,1) model performed better than the ARIMA(1,2,1) model because the values of AIC and SIC using the ARIMA-GARCH model is smaller than that in the ARIMA model. Earlier in 2015, Xu et.al[15] used the ARIMA-GJR-GARCH model in forecasting exchange rate of Renminbi against the Hong Kong dollar. The Glosten-Jagannathan-Runkle GARCH, GJR-GARCH model can capture asymmetry in the conditional variance equation. The authors presented a model of ARIMA(1,1,1)-GJR-GARCH(1,1) to be the best fitting model of the exchange rates and do forecasting.

3. Methodology
There are 3 major steps involved in building an ARIMA model, namely model identification, parameter estimation and residuals diagnostics. The data used was retrieved from Yahoo Finance[16], on a period of 17 years of the S&P 500 stock price data, ranging from January 2001 to December 2018. There is a total of 216 monthly observations. We chose to use the adjusted closing price for this analysis because it is an unbiased and accurate representation of the firm's equity value beyond the simple market price. The tool used for conducting the time series analysis was EViews software 10th version.

3.1. Model identification
In modelling an ARIMA model, also well known as the Box & Jenkin's model, firstly, we check for the stationarity of the data series by plotting the graph of the raw data to get a general idea of the data and to see its pattern. The main assumption is that the mean and variance of a stationary process are constant over time. However, in economy and finance, the series usually exhibit time-varying conditional variance. In addition, non-stationarity in variance usually occurs in high frequency data such as hourly, daily and weekly data.

Tools used for checking the stationarity of the data series are the Autocorrelation Function, ACF plot and the Augmented Dicker Fuller Test (Unit Root Test). The ACF plots autocorrelation of the time series against lags. Lag refers to the time difference between one observation and a previous observation in a dataset. If the data series is not stationary, transformation of the data will be carried out to achieve stationarity. In this study, we do differencing to achieve stationarity [1]. Doing differencing means we compute the difference between consecutive observations to remove the trend pattern from data. Normally, first differencing or d =1 is enough to stabilize the mean, thereby making the time series data stationary. Over-differencing tends to introduce unnecessary correlation in the model. Stationarity is a main condition in employing ARIMA model. We observe the pattern of correlogram i.e. plots of Autocorrelation Function, ACF and Partial Autocorrelation Function, PACF to identify the numbers of AR or MA terms in ARIMA model.

3.2. Parameter estimation
In this step, we split the data series into 2 parts; data from January 2001 to December 2017 are used for parameter estimation, while data from January to December 2018 are used for evaluating the forecasting performance. We employ 2-steps procedure i.e. first, we find the optimal ARIMA model for mean then we add GARCH(1,1) to model the variability. We estimate the parameter of the ARIMA model by using least squares estimator.

We use the trial and error method to identify order of p and q in ARIMA (p, 1, q) model. We employ the "simplest to best" approach until we get the best parsimonious model. The parsimonious model gives a better forecast than the over-parameterised model. The best fitted model here is based on the statistically significant coefficients. In selection of the best model, we use Akaike-Information-Criterion, AIC and Bayesian or Schwarz Information Criterion, SIC scores. The model with the lowest values of AIC and SIC is preferred.

3.3. Residuals diagnostics
Once a model with statistically significant coefficients identified, we do residuals diagnostics to check whether the residuals from the fitted model is white noise. If the fitted model is an adequate model for the data then the residuals should satisfy the assumption of non-autocorrelated random stationary process, $\varepsilon_t \sim$ NID (normal, and independently distributed). These can be done by observing the ACF plot of the residuals, interpreting Jarque-Bera statistic for normality test, conducting the Ljung-Box Q-

statistic test and the Breusch-Godfrey Serial Correlation LM test to check for autocorrelation in the ARIMA model. If the assumptions are not met, we need to identify a better model by overfitting. Overfitting is conducted by adding number of parameters in AR or MA model. Then, we test for the ARCH effect using ARCH Lagrange Multiplier (LM) Test. If there exists ARCH effect, it means the variance is heteroscedastic, therefore a better fitting of the model can be obtained by using a GARCH(1,1) model to encounter non-stationarity in variance.

3.4. Forecasting

For forecasting purpose, we use out-of-sample data from January – December 2018. We compare between a static forecast and a dynamic forecast. Static forecast is a serial of one-step ahead forecasts, rolling the sample forwards one observation after each forecast, which use data up to the last month of the out-sample data series (in case of this study). In contrast, dynamic forecast is a multi-step forecasts, use information available at the last period of in-sample data series. Dynamic forecast assumes that we do not have any information about the out-sample data series even when we do have the relevant data.

To measure the forecast accuracy, we use Mean Absolute Error, MAE and Root Mean Squared Error, RMSE. The MAE and RMSE are calculated for the data set as follows:

$$\mathrm{MAE} = \frac{1}{n}\sum_{j=1}^{n}\left|y_j - \hat{y}_j\right| \tag{3}$$

$$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(y_j - \hat{y}_j\right)^2} \tag{4}$$

MAE measures the average absolute values of the differences between forecast and actual value, while RMSE is the square root of the average sum of squares error. Both MAE and RMSE measure the average magnitude of errors of the predictions without considering their direction. The better prediction is the one with the lowest MAE and RMSE scores. Figure 1 summarized our research methodology.

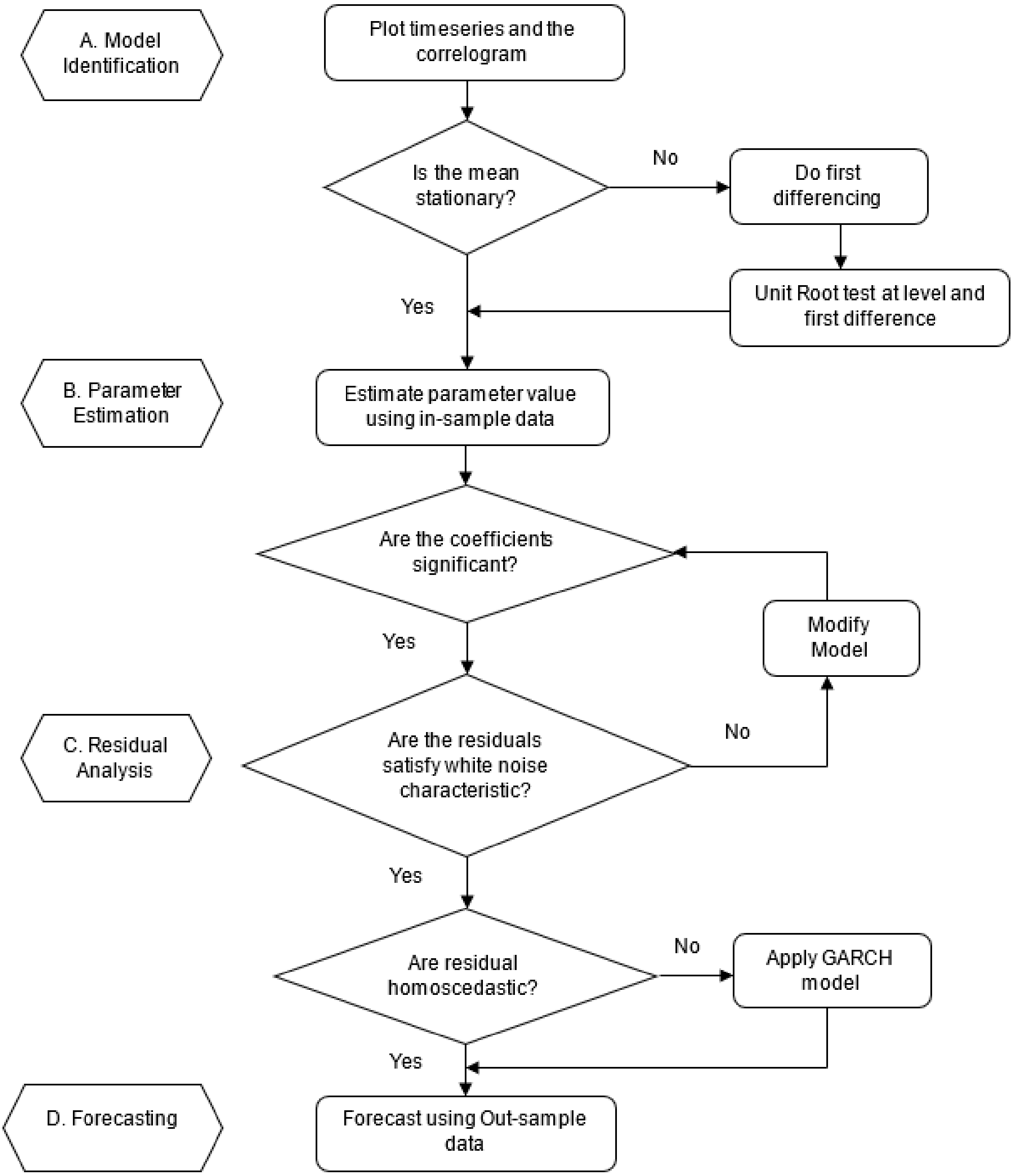

Figure 1. Research methodology flow chart.

4. Results and discussion

4.1. Model identification

Figure 2 shows that the series illustrates long-term increasing and decreasing trends. It suggests a non-stationary series. We can see that the mean and variance are not constant over time. There is structural break in 2008 due to world global financial crisis.

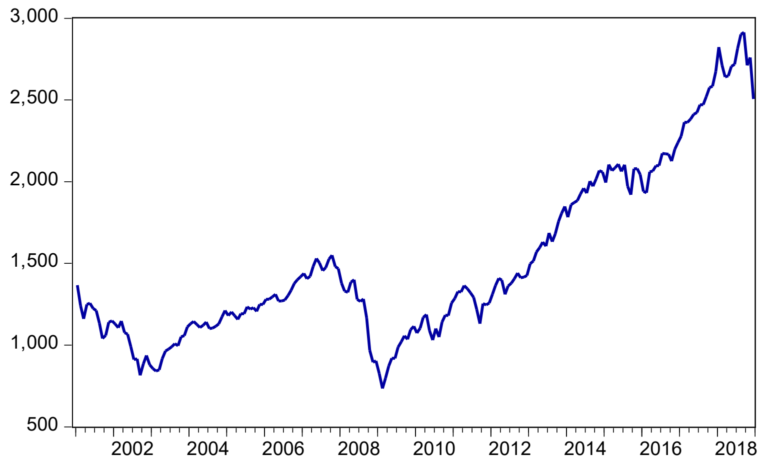

Figure 2. The S&P 500 Adjusted Closing Price.

Figure 3 illustrates that the ACF plot tailing off extremely slowly in linear fashion, indicating the series is trended and non-stationary. We do first differencing to tackle of the non-stationarity in the series.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.974	0.974	196.37	0.000
		2	0.949	0.005	383.68	0.000
		3	0.924	-0.000	562.32	0.000
		4	0.900	-0.001	732.70	0.000
		5	0.877	-0.011	894.98	0.000
		6	0.851	-0.049	1048.7	0.000
		7	0.827	0.018	1194.5	0.000
		8	0.803	-0.016	1332.7	0.000
		9	0.777	-0.043	1462.8	0.000
		10	0.753	0.010	1585.5	0.000
		11	0.729	-0.001	1701.1	0.000
		12	0.706	0.008	1810.2	0.000
		13	0.684	-0.004	1913.1	0.000
		14	0.662	0.005	2010.1	0.000
		15	0.644	0.041	2102.2	0.000
		16	0.623	-0.056	2188.8	0.000
		17	0.600	-0.040	2269.7	0.000
		18	0.578	-0.003	2345.3	0.000
		19	0.560	0.044	2416.4	0.000
		20	0.538	-0.064	2482.6	0.000

Figure 3. The correlogram of S&P 500 Adjusted Closing Price.

Figure 4 displays the fluctuation above and below 0 line, indicating the series achieve stationarity in mean after the first differencing. However, it is obvious that the series is not stationary in variance.

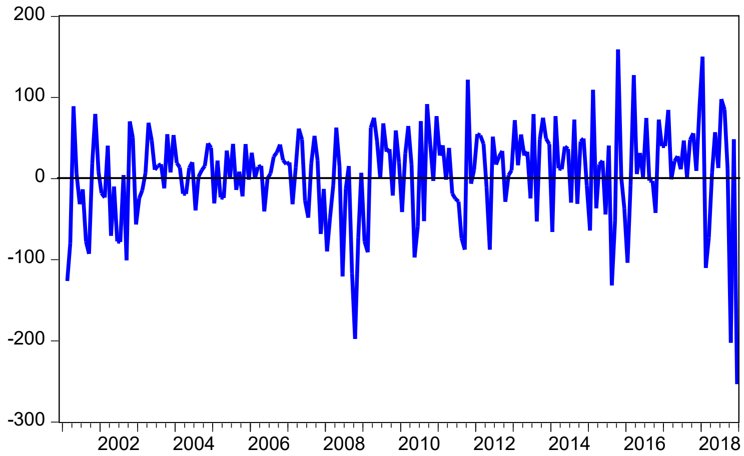

Figure 4. S&P 500 series after first differencing.

We also tested for stationarity using the Augmented Dickey-Fuller Test. The null hypothesis; data is not stationary against the alternative hypothesis; data is stationary.

Table 1. Augmented Dickey-Fuller test.

Model	Test for Unit Root in	Test Statistic	Probability
Trend and intercept	Level	-1.241988	0.8984
	First difference	-13.26276	0.0000

Based on table 1, the p-value = 0.8984, greater than alpha = 0.05, we do not reject null hypothesis, indicates that the data is not stationary at level. Then, we repeat the test at the first difference, and we get p-value = 0, which is less than alpha = 0.05, we reject the null hypothesis, then we can conclude that after the first differencing, the data is stationary. Therefore, we proceed to estimate a model.

4.2. Parameter estimation

Based on Figure 2, the ACF remain large in values for a long time and the PACF cuts off at lag 1, therefore we start with the simplest models: AR(1), MA(1) and ARIMA(1,1,1) until we got a model with significant coefficients. We increase the order of AR and MA alternately to avoid redundancy. Table 2 summarized our steps to find the model with significant coefficients. Note that we used the differenced series to fit the ARIMA model.

Table 2. Summary of the derived model.

Model	Coefficient(s)	White noise	AIC	BIC
AR(1)	Not significant	-	-	-
MA(1)	Not significant	-	-	-
ARMA(1,1)	Not significant	-	-	-
AR(2)	Not significant	-	-	-
MA(2)	Not significant	-	-	-
ARMA(2,1)	Not significant	-	-	-
ARMA(1,2)	Not significant	-	-	-
AR(3)	Not significant	-	-	-
ARMA(3,1)	Not significant	-	-	-
ARMA(2,2)	Significant	yes	10.6993	10.7815

After the trial and error processes using EViews, we gained an ARIMA model with significant coefficients which is shown in Table 3.

Table 3. The ARIMA(2,1,2) model for S&P 500.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.437638	3.722384	1.998084	0.0471
AR(1)	-0.218663	0.040113	-5.451156	0.0000
AR(2)	-0.924514	0.037760	-24.48426	0.0000
MA(1)	0.313445	0.037930	8.263734	0.0000
MA(2)	0.934931	0.037147	25.16822	0.0000

R-squared	0.070569	Mean dependent var	7.528757
Adjusted R-squared	0.051601	S.D. dependent var	51.67242
S.E. of regression	50.32160	Akaike info criterion	10.69931
Sum squared resid	496323.6	Schwarz criterion	10.78148
Log likelihood	-1070.280	Hannan-Quinn criter.	10.73256
F-statistic	3.720410	Durbin-Watson stat	1.946932
Prob(F-statistic)	0.006078		

All the p-values of the ARIMA(2,1,2) model are less than alpha = 0.05, therefore the coefficients are all significant at alpha = 0.05. The Akaike-Information-Criterion, AIC value is 10.69931 and Bayesian or Schwarz Information Criterion, SIC value is 10.78148.

4.3. Residuals diagnostics

Figure 5 shows that the Jarque-Bera statistic is equal to 6.20173 and the p-value is 0.045, which is less than alpha=0.05, therefore we reject H_0, meaning that the residuals of our model do not obey the normal distribution. Thus, instead of using the GARCH model based on Normal(Gaussian) distribution to fit the differenced series, we use the GARCH model based on Generalized Error Distribution, GED.

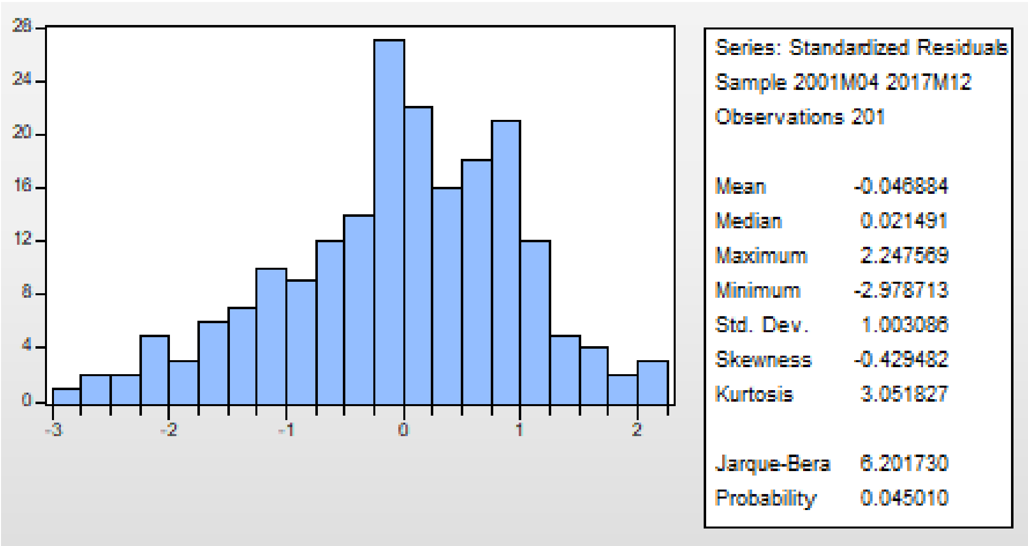

Figure 5. Histogram of the residuals.

Figure 6 demonstrates that the correlogram of the residuals are mostly small in magnitude, falling inside the 95% confidence interval, suggesting that the residuals are independently distributed (no autocorrelation in the residuals), implying the fitted ARIMA(2,1,2) model is adequate. Moreover, the probability of the Ljung-Box Q-statistic are greater than alpha=0.05, therefore we are certain that the error terms of the selected model are white noise.

	AC	PAC	Q-Stat	Prob
1	0.017	0.017	0.0601	
2	0.036	0.036	0.3306	
3	0.131	0.130	3.8574	
4	-0.014	-0.019	3.8979	
5	0.095	0.088	5.7811	0.016
6	0.007	-0.012	5.7917	0.055
7	-0.010	-0.011	5.8126	0.121
8	0.095	0.074	7.7195	0.102
9	-0.048	-0.048	8.2137	0.145
10	0.027	0.020	8.3676	0.212
11	0.066	0.049	9.3113	0.231
12	-0.029	-0.017	9.4943	0.302
13	0.051	0.029	10.056	0.346
14	-0.018	-0.025	10.129	0.429
15	0.000	0.004	10.129	0.519
16	0.002	-0.025	10.130	0.605
17	-0.044	-0.026	10.551	0.648
18	-0.072	-0.085	11.704	0.630
19	0.128	0.139	15.392	0.424
20	-0.132	-0.126	19.303	0.253

Figure 6. Ljung-Box Test for ARIMA(2,1,2).

Table 4 demonstrates the result of the Breusch-Godfrey Serial Correlation LM test. This test used to check for serial correlation among the residuals. The probability(F-statistic) = 0.4585 while the probability(Obs*R-squared) = 0.4377. Since these probabilities are greater than alpha = 0.05, we do not reject the null hypothesis. Once again indicating that there is no serial correlation in the residuals, i.e. the residuals are independent, implying the fitted ARIMA(2,1,2) model is perfect.

Table 4. Breusch-Godfrey Serial Correlation LM test of the residuals.

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.952975	Prob. F(6,190)	0.4585
Obs*R-squared	5.872002	Prob. Chi-Square(6)	0.4377

We conducted overfitting to find the second or the third best model. However, Table 5 shows that both the ARIMA(3,1,2) and ARIMA(2,1,3) are not significant in term of model coefficients.

Table 5. Overfit with ARIMA(2,1,3) and ARIMA(3,1,2).

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.120751	3.749274	1.899235	0.0590
AR(1)	0.002500	0.930552	0.002687	0.9979
AR(2)	0.002500	0.633875	0.003944	0.9969
AR(3)	0.002500	0.071309	0.035059	0.9721
MA(1)	0.002500	0.933038	0.002679	0.9979
MA(2)	0.002500	0.642035	0.003894	0.9969

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.403838	3.971876	1.864066	0.0638
AR(1)	0.954895	0.020873	45.74866	0.0000
AR(2)	-0.944883	0.019053	-49.59199	0.0000
MA(1)	-0.883688	0.074844	-11.80706	0.0000
MA(2)	0.895462	0.072117	12.41678	0.0000
MA(3)	0.084538	0.072394	1.167743	0.2443

The ARCH-LM test is conducted to see whether there is a presence of heteroscedasticity in variance. As can be seen on Table 6, before fitting GARCH(1,1) into ARIMA(2,1,2) model, the p-values are equal to 0.0340 and 0.0346 respectively, less than alpha = 0.05, therefore we reject the null hypothesis, indicating that the heteroscedasticity is present in the residuals. Therefore, we model the ARCH effect with GARCH(1,1). After fitting GARCH(1,1) into ARIMA(2,1,2) model, we rerun the ARCH-LM test of the residuals. The p-values now are greater than the significance level of 0.05. Therefore, we failed to reject the null hypothesis and conclude that there is no more ARCH effect, implying that the residuals are now homoscedastic.

Table 6. ARCH-LM test of the residuals.

Heteroskedasticity Test: ARCH				
Before fitting GARCH(1,1) into ARIMA(2,1,2)	F-statistic	2.947486	Prob. F(3,194)	0.0340
	Obs*R-squared	8.631361	Prob. Chi-Square(3)	**0.0346**
After fitting GARCH(1,1) into ARIMA(2,1,2)	F-statistic	0.265227	Prob. F(3,194)	0.8504
	Obs*R-squared	0.808768	Prob. Chi-Square(3)	**0.8474**

Table 7 displays the ARIMA(2,1,2)-GARCH(1,1) model. It is a better model with statistically significant coefficients, fulfilled the assumption of NID residuals and the AIC = 10.63743 and SIC = 10.78534 respectively, are smaller than that for the ARIMA(2,1,2). Therefore, we ensure that the ARIMA(2,1,2)-GARCH(1,1) is our best fit model for S&P 500 stock prices.

Table 7. The ARIMA(2,1,2)-GARCH(1,1) model for S&P500.

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	12.06184	3.584423	3.365073	0.0008
AR(1)	-0.454690	0.043844	-10.37068	0.0000
AR(2)	-0.861970	0.049284	-17.48998	0.0000
MA(1)	0.495702	0.018691	26.52038	0.0000
MA(2)	0.967254	0.019289	50.14632	0.0000
		Variance Equation		
C	265.0988	217.6057	1.218253	0.2231
RESID(-1)^2	0.246199	0.115484	2.131899	0.0330
GARCH(-1)	0.665725	0.148400	4.486029	0.0000
GED PARAMETER	1.632215	0.284890	5.729290	0.0000

R-squared	0.023910	Mean dependent var	7.528757
Adjusted R-squared	0.003990	S.D. dependent var	51.67242
S.E. of regression	51.56924	Akaike info criterion	10.63743
Sum squared resid	521239.7	Schwarz criterion	10.78534
Log likelihood	-1060.062	Hannan-Quinn criter.	10.69728
Durbin-Watson stat	1.860990		

4.4. Forecasting S&P 500 stock prices using the ARIMA(2,1,2)-GARCH(1,1) model

The ARIMA(2,1,2)-GARCH(1,1) for S&P 500 closing stock prices is estimated using the first 204 observations (or observations January 2001 – December 2017), leaving 12 remaining observations to construct forecasts and to test the forecast accuracy. From Table 8, we conclude that the dynamic forecast result gives a better prediction of the future S&P 500 stock prices when compared to static forecast. This is because the Mean Absolute Error and Root Mean Squared Error values of the dynamic forecast are significantly lower than that of the static forecast.

In other words, suppose we are going to forecast the stock prices over the next 12 months, then we will have to do a dynamic forecast. Table 9 presents the actual value of S&P 500 monthly stock prices of 2018, the forecast values and the differences between actual values and both forecast values. Please note that both forecast methods will always yield identical results in the first period of a forecast. Thus, two forecast series, one dynamic and the other static, should be identical for the first observation in the forecast sample.

Table 8. The forecast results based on dynamic forecast and static forecast.

Forecast: DYNAMICFORECAST Actual: DSNP500 Forecast sample: 2018M01 2018M12 Included observations: 12		Forecast: STATICFORECAST Actual: DSNP500 Forecast sample: 2018M01 2018M12 Included observations: 12	
Root Mean Squared Error	120.5778	Root Mean Squared Error	125.0665
Mean Absolute Error	89.96143	Mean Absolute Error	102.4464
Mean Absolute Percentage Error	80.17978	Mean Absolute Percentage Error	141.1880
Theil Inequality Coefficient	0.925860	Theil Inequality Coefficient	0.888211
Bias Proportion	0.045146	Bias Proportion	0.034282
Variance Proportion	0.912299	Variance Proportion	0.600778
Covariance Proportion	0.042555	Covariance Proportion	0.364940
Theil U2 Coefficient	0.984424	Theil U2 Coefficient	1.045156
Symmetric MAPE	135.2198	Symmetric MAPE	175.4872

Table 9. Comparison between the observed stock prices and the forecast values.

Date	Observation, O	Dynamic Forecast value, D	Static Forecast value, S	Difference O - D	Difference O - S
1/1/2018	2823.81	2833.15	2833.15	-9.34	-9.34
1/2/2018	2713.83	2723.03	2728.81	-9.20	-14.98
1/3/2018	2640.87	2656.58	2663.66	-15.71	-22.79
1/4/2018	2648.05	2660.92	2635.63	-12.87	12.42
1/5/2018	2705.27	2713.82	2709.94	-8.55	-4.67
1/6/2018	2718.37	2731.33	2759.11	-12.96	-40.74
1/7/2018	2816.29	2830.97	2826.08	-14.68	-9.79
1/8/2018	2901.52	2911.62	2890.60	-10.10	10.92
1/9/2018	2913.98	2924.68	2951.67	-10.70	-37.69
1/10/2018	2711.74	2726.12	2741.05	-14.38	-29.31
1/11/2018	2760.17	2772.35	2730.15	-12.18	30.02
1/12/2018	2506.85	2516.86	2502.02	-10.01	4.83

5. Conclusions

This paper presents the estimation process in developing the best fitted ARIMA-GARCH model to generate forecast values of the S&P 500 stock prices. We have shown the method we used to solve the non-stationarity in the mean and variance of the data series before the further analysis could be done. The experimental results revealed that the dynamic forecast gave a better prediction compared to the static forecast. However, this study was limited to short-term forecast only. This should be a great overview for the investors or portfolio manager to make a profitable decision.

Acknowledgments

The authors would like to express sincere gratitude to Universiti Teknologi MARA, Cawangan Pulau Pinang for the opportunity to carry out this study by supporting financial aid.

References

[1] Laboissiere L A, Fernandes R A S and Lage G G 2015 *Applied Soft Computing Journal* vol **35** pp 66-74
[2] Henrique B M, Sobreiro V A and Kimura H 2019 *Elsevier*: *Expert Systems with Applications* Vol **124** pp 226-251
[3] Box G E P and Jenkins G M 1976 *Time series analysis: Forecasting and control 2nd edition* (Holden-Day, San Francisco)
[4] Gujarati D N 1995 *Basic Econometrics* 3rd ed (New York: McGraw-Hill)
[5] Engle R F 1982 *Econometrica*, **50**:4 pp 987-1007
[6] Bollerslev T 1986 *Journal of Econometrics* vol **31** pp 307-326
[7] Brooks C 2002 *Introductory econometrics for finance* (United Kingdom: Cambridge University Press)
[8] Engle R F and Patton A J 2001 *Quantitative Finance*, **1**(2), pp. 237-245
[9] Maana I, Mwita P N and Odhiambo R 2010 *African Journal of Business Management*, **4**(7), pp 1401–1408
[10] Zhang X Q, Zhao K and Xu, S N 2015 *Applied Engineering Sciences: Proceedings of the AASRI International Conference on Applied Engineering Sciences, 2014*; Boca Raton, Vol. 1
[11] Almarashi A M, Abbasi U, Saman H, Alzahrani M R and Khan K 2018 *Sci. Int. (Lahore)*, **30**(1), pp 89-94

[12] Adebiyi A A, Adewumi A O and Ayo C K 2014 *2014 UK Sim-AMSS 16th International Conference on Computer Modelling and Simulation* pp 105-111
[13] Kamruzzaman M, Khudri M and Rahman M 2017 *Dhaka University Journal of Science* **65**(2), pp 97-101
[14] Abbasi U, Almarashi A M, Khan K and Hanif S 2017 *Sci. Int. (Lahore)*, **29**(5), p 1039-1043
[15] Xu S N, Zhao K and Sun J J 2015 *Proceedings of the AASRI International Conference on Applied Engineering Sciences* Vol. **1**, pp 83-86
[16] https://finance.yahoo.com/quote/%5EGSPC/history?period1=978278400&period2=1546185600&interval=1mo&filter=history&frequency=1mo