A User's Guide to $\mathbb{Z}$ot

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1 Overview

$\mathbb{Z}$ot is an agile and easily extendible bounded model checker, which can be downloaded at http://home.dei.polimi.it/pradella/.

The tool supports different logic languages through a multi-layered approach: its core uses CLTLB(DL)[1] and on top of it a decidable predicative fragment of TRIO [9] is defined. An interesting feature of $\mathbb{Z}$ot is its ability to support different encodings of temporal logic as SMT problems by means of plug-ins. This approach encourages experimentation, as plug-ins are expected to be quite simple, compact (usually around 500 lines of code), easily modifiable, and extendible. At the moment, a variant of the eventuality encoding presented in [3] is supported, (approximated) dense-time MTL [6], and a bi-infinite encoding [13], [14].

$\mathbb{Z}$ot offers three basic usage modalities:

1. *Bounded satisfiability checking (BSC)*: given as input a specification formula, the tool returns a (possibly empty) history (i.e., an execution trace of the specified system) which satisfies the specification. An empty history means that it is impossible to satisfy the specification.

2. *Bounded model checking (BMC)*: given as input an operational model of the system, the tool returns a (possibly empty) history (i.e., an execution trace of the specified system) which satisfies it.

3. *History checking and completion (HCC)*: The input file can also contain a partial (or complete) history H. In this case, if H complies with the specification, then a completed version of H is returned as output, otherwise the output is empty.

The provided output histories have temporal length $\leq k$, the bound given by the user, but may represent infinite behaviors thanks to the loop selector variables, marking the start of the periodic sections of the history. The BSC/BMC modalities can be used to check if a property *prop* of the given specification *spec* holds over every periodic behavior with period $\leq k$. In this case, the input file contains spec $\wedge \neg$prop, and, if prop indeed holds, then the output history is empty. If this is not the case, the output history is a counterexample, explaining why prop does not hold.

2 Installation

ℤot's core is written in Common Lisp (with ASDF packaging http://www.-cliki.net/asdf). It can be used under Linux, Windows, or MacOS X, but has been tested only under Linux and Windows XP, using the following Common Lisps[1]:

- SBCL (http://www.sbcl.org),
- CLISP (http://clisp.cons.org),
- CMUCL (http://www.cons.org/cmucl/),
- ABCL (http://common-lisp.net/project/armedbear/),
- Clozure CL (http://www.clozure.com/clozurecl.html),

This approach makes ℤot an open system, as it uses Common Lisp also as internal scripting language of the tool, both to define complex verification activities, and to add new constructs and languages on top of the existing ones.

Typically, to install ℤot in a Debian system (or Ubuntu), the user must install a Common Lisp (e.g. one of the packages `clisp`, `sbcl`, `cmucl`, ...), and the `common-lisp-controller` package. To perform a system-wide install of the ℤot packages, just put symbolic links to its .ads files in the /usr/share/common-lisp/systems/ directory. Note that it is possible to avoid a system-wide installation, but in this case the user has to work inside the main ℤot directory.

ℤot works with external SAT-solvers. The supported SAT-solvers are MiniSat (default) [4], MiraXT [10], PicoSAT [2], and zChaff [11]. ℤot assumes that executable files called `minisat`, `MiraXTSimp` (optional), `picosat` (optional), `zchaff` (optional), are system-wide installed.

A pre-packaged all-inclusive version for Windows (*WinZot*, based on Cygwin-compiled binaries and SBCL) is available from the author.

All ℤot's components are available as open source software (GPL v2).

2.1 Hello world

Here, the user can find short examples of the functionality of ℤot which can be used to test the installation and to understand the basic notion supporting the tool (knowledge about verification and bounded model checking are hardly recommended).

ℤot is tailored to solve the Bounded Model Checking problem (BMC) and Bounded Satisfiability Checking (BSC) which are both reduced to a satisfiability problem. The aim of the satisfiability process is to represent

[1]SBCL and CMUCL are usually the fastest implementations, for running Zot.

infinite model of CLTLB/LTL formulae by means of a finite representation of length k (i.e., k instant of time). A model is an infinite word of propositional atoms (and counters' values when CLTLB is considered). In the following sections, the term model and behavior (of a system), in particular when the satisfiability checking is performed, have the same meaning and they can be equivalently used.

The first example shows how to solve the BSC for a LTL formula by means of a finite representation of its model of length 5.

```
(asdf:operate 'asdf:load-op 'eezot)
(use-package :trio-utils)

(eezot:zot 5
  (&& (-P- Hello)
      (next (-P- world))))
```

The first two lines load the plugin *eezot* for LTL and the basic definitions of operators available from `trio-utils.lisp`. The Common Lisp function `zot` of the package `eezot` is invoked with the length of the (finite) model and the LTL formula to be satisfied. The output reported is the model satisfying the LTL formula $hello \wedge \mathbf{X} world$ (`hello` holds in the origin instant and `world` in the next one).

```
------ time 0 ------
  WORLD

------ time 1 ------
  HELLO
  WORLD

------ time 2 ------
  HELLO
  WORLD

------ time 3 ------

------ time 4 ------

------ time 5 ------

------ end ------
```

The formula is evaluated in 1, which is the origin of the model; the instant 0 is defined for technical reasons. It is worth noticing that predicate `hello` holds in 1 and `world` holds in 2. If no formula defines the truth value of generic subset of the atomic propositions (or formulae) for a generic time instant, then, in that instant, it may, or may not, holds; this is the case of `world` which holds in 1 and `hello` in 2. The resulting finite model is a prefix of *all* infinite models satisfying the formula since the proposition `*LOOP*`, denoting the position of the periodicity, does not appear. This is not the case of the following example in which the LTL formula $\mathbf{G}(hello \wedge \mathbf{X} world)$ (for all instant from the origin towards the future, `hello` holds and `world` in the following one) is considered:

```
(asdf:operate 'asdf:load-op 'eezot)
(use-package :trio-utils)

(eezot:zot 5
  (alwf
    (&&
      (-P- hello)
      (next (-P- world)))))
```

the resulting model is:

```
------ time 0 ------
  HELLO
  WORLD

------ time 1 ------
  **LOOP**
  HELLO
  WORLD

------ time 2 ------
  HELLO
  WORLD

------ time 3 ------
  HELLO
  WORLD

------ time 4 ------
  HELLO
  WORLD

------ time 5 ------
  HELLO
  WORLD

------ end ------
```

The LTL operator $\mathbf{G}$ is written as `alwf` (always in the future) and the symbol `&&` is the usual $\wedge$ boolean connective. The finite model represents *the* infinite periodic model given by the ω-regular expression

$$(\{h, w\}, \{h, w\}, \{h, w\}, \{h, w\}, \{h, w\})^{\omega},$$

where h, w are shorthands for *hello* and *world*, respectively.

Let consider the following example in which it is shown an unsatisfiable LTL formula $\mathbf{G}(hello \wedge \mathbf{X}world) \wedge \mathbf{F}(\neg hello)$:

```
(asdf:operate 'asdf:load-op 'eezot)
(use-package :trio-utils)

(eezot:zot 5
  (&&
    (alwf
      (&&
        (-P- hello)
        (next (-P- world))))
    (somf (!! (-P- hello)))))
```

The output is, clearly:

```
---------UNSAT---------
```

The operator `somf` corresponds to the LTL operator $\mathbf{F}$ and means "eventually in the future".

3 Languages

Being an open system, $\mathbb{Z}$ot supports different languages. At present, the main native language is CLTLB[1] (linear temporal logic with future and past operators over constraint systems) and its subclasses. The other main layer based on PLTL is the metric temporal logic TRIO.

$\mathbb{Z}$ot scripts are interpreted as a usual Lisp code[2]. Beside the CLTLB temporal operators and relations/functions available from the constraints system used (Difference Logic, Linear Integer Arithmetic), the user can define own constructs and embeds them in the CLTLB formulae. These constructs are defined accordingly to Common Lisp language. They are firstly interpreted by the Common Lisp interpreter and the results is used to define the CLTLB formulae. Here, a short list of useful constructs of the Common Lisp language is provided.

Constants: a Lisp constant c of value v is declared in the following way:

```
(defconstant c v)
```

Variables: a Lisp variable v is declared in the following way:

```
(defvar v 1)
(defvar v '(One Two))
```

Note that the value of a constant or a variable can be any Common Lisp object including CLTLB formula (since they are Common Lisp list), as shown later in the section 3.3.

Functions: an n-ary Lisp function fun of domain is defined as:

```
(defun fun (x_1 ... x_n) axioms )
```

where `axioms` describe the properties of fun. The user should consider these functions as procedure and should not confuse them with a CLTLB unintepreted function/relation, defined in the following section. At the end of the section, after the definition of of the syntax of CLTLB, a simple example is provided.

Valuation of a function: if fun is an n-ary Lisp function, the value $fun(x_1, \ldots, x_n)$ is written as `(fun x_1 ... x_n)`.

Predefined operation and relations: a small number of binary operation/relations between integer are predefined in Common Lisp, namely `=`, `\=`, `<`, `<=`, `>`, `>=`, `+`, `-`, with their usual meanings:

```
(op x_1 x_2);
```

[2]A basic knowledge of the language is required. It is very easy to find online a lot of tutorials and short presentations.g. http://gigamonkeys.com/book/

3.1 CLTLB

CLTLB (Counters LTL)[3] is, essentially, Propositional LTL with both future and past operators (PLTLB), with in addition terms that are arithmetic constraints in Integer Difference Logic, CLTLB(DL), or in Linear Integer Arithmetic, CLTLB(LIA).

Propositional operators: are written as `!!` (not), `&&` (and), `||` (or), `->` (implies) and `<->` (equivalent).

Propositional letters: a proposition Q is written `(-P- Q)`.

Relations: there exist two possible way to define a relation.

1. *Uninterpreted time-variant relations:* an uninterpreted (i.e. satisfying no axioms) time-variant relations (i.e. varying in time) *rel* on the set $type_1 \times \cdots \times type_n$ is defined as

```
(define-tvar 'rel *type_1* ... *type_n*)
```

2. *Uninterpreted time-invariant relations:* an uninterpreted (i.e. satisfying no axioms) time-invariant relations (i.e. constant in time) *rel* on the set $type_1 \times \cdots \times type_n$ is defined as

```
(define-var 'rel *type_1* ... *type_n*)
```

Time-variant/unvariant variables are defined as 0-ary relations. The domains specified as `*type_i*` are, actually, the ones supported by the used SMT-solver. Available keywords are: one to define Integers, `*int*`, one to define Reals, `*real*` and `*bool*` to define Boolean (supported in Z3).

Valuation of a predicate: if R is an n-ary relation, the boolean $R(x_1, \ldots, x_n)$ is written as `(-V- R x1 ... xn)`.

Valuation of a variable: a variable is evaluated differently depending on if it is a Lisp variable or a 0-ary function:

- if *var* is a Lisp variable, its value is written as a usual Lisp variable;
- if *var* is a logic variable, its value is written as `(-V- var)`.

Predefined operation and relations: a small number of binary operation/relations between integers are predefined in the SMT-library, namely

[3] only available in the plugin *ae2zot*

`=`, `<`, `<=`, `>`, `>=`, `+`, `-`, `%` with their usual meanings (`%` is the remainder operator). As CLTLB operations (which map to SMT-LIB operations) they are written:

```
([op] x_1 x_2).
```

Observe that some operations are not defined in all constraint systems: in order to use $+$ and $-$ the logic `QF_UFLIA` has to be declared.

Arithmetic temporal terms: if α is an arithmetic temporal term, $\mathbf{X}\alpha$ and $\mathbf{Y}\alpha$ are written respectively as `(next (-V- alpha))` and `(yesterday (-V- alpha))`.

Temporal operators: the following temporal operators are supported: `until`, `since`, `release`, `trigger`, `next`, `yesterday`, `zeta`. The last one is the dual of `yesterday`, and is used only in the mono-infinite semantics.

For the semantics of these operators, see e.g. [1]. TRIO operator are also supported, see section 3.5

3.2 Quantifiers

Quantification is possible over finite and infinite domains (in the second case undecidability problems can arise). Finite domains can be declared as Lisp list:

```
(defvar Set '(One Two))
```

The formula $\exists t \in Set : Formula(t)$ is written

```
(-E- t Set (Formula t)).
```

The definition of the Lisp variable `Set` is optional, and the same formula can be written as:

```
(-E- t '(One Two) (Formula t)).
```

-A- is the universal quantifier.

Infinite domains are implicitly declared by the definition of the constraint system. Available domains from SMT-LIB are Integers and Reals. By the definition of satisfiability, all logic variables (as well as values of uninterpreted functions and predicates) are existentially quantified. The formula $\exists t \in \mathbb{Z} : Formula(t)$ requires the definition of a logic variable by `(define-var 'h *int*)` or `(define-var 'h *real*)` and it is written:

```
(Formula (-V- h)).
```

by simply substituting the instance of t with the evaluation of `(-V- h)`.

It is worth noticing that the Naturals can be defined by constraining the value of the existentially quantified variable. The formula $\exists t \in \mathbb{N} : Formula(t)$ can be written:

```
(&& ([>=] (-V- h) 0) (Formula (-V- h))).
```

and `Formula` should be defined by the construct `defun`. The first form, which uses the Lisp definition of sets as lists, can be equivalently written by using logic variables; values of a variable over a finite set are defined by an explicit list:

```
(&& (|| ([=] (-V- h) c_0) ([=] (-V- h) c_1) ...)
        (Formula (-V- h))).
```

3.3 Short examples

Some simple examples of compound syntax of CLTLB and Common Lisp are here shown.

```
(defconstant p
             (<->
               (until (-P- A) ([>] (-V- x) 0))
               (!! (-P- B))))"
```

The following function is useful to define the transition from the value 1 to 0 of an arithmetic temporal term.

```
(defun g (z)
  (&& ([=] z 0) (yesterday ([=] z 1))))
```

The definition of `g` is realized by combining CLTLB language and Common Lisp language. During the process of evaluation of the function, the variable `z` is substituted with its evaluation. The function has to be used accordingly to the syntax defined by the declared formula. In this case, the variable `z` is an arithmetic temporal term $\mathbf{X}^j x$; a correct use is, for example:

```
(<-> (-P- A) (g (next (-V- y)))))
```

where the variable `y` is a CLTLB variable defined by using the construct `define-tvar`.
The following example makes use of two uninterpreted function `A` and `B` and quantification over finite domains `(-A- ...)` and `(-E-...)`".

```
(define-tvar 'A *int* *int* *int*)
(define-tvar 'B *int* *int*)

(defconstant f
```

```
(-A- x '(1 2 3)
  (-A- y '(1 2 3)
    (alwf
      (->
        (&& ([!=] x y) ([>=] (-V- A x y) 0))
        ([=] (next (-V- A x y)) ([+] (-V- A x y) (-V- B x)))))))
```

...

3.4 PLTL

PLTL is a subclass of CLTLB in which no relations, functions and variables appear. Nevertheless variables over finite domains can still be used (actually this is only a useful shorthand). If the problem is expressed in PLTL there is no need to declare a costraint system.

The following temporal operators are supported: `until`, `since`, `release`, `trigger`, `next`, `yesterday`, `zeta`. The last one is the dual of `yesterday`, and is used only in the mono-infinite semantics.

For the semantics of these operators, see e.g. [1] or any other note about LTL.

3.5 TRIO

ℤot was originally born as a satisfiability checker for the TRIO metric temporal logic [9].

The list of supported operators (and their correct "ℤot spelling") is the following:

```
dist
futr
past
lasts    lasts_ee    lasts_ie    lasts_ei    lasts_ii
lasted   lasted_ee   lasted_ie   lasted_ei   lasted_ii
withinf  withinf_ee  withinf_ie  withinf_ei  withinf_ii
withinp  withinp_ee  withinp_ie  withinp_ei  withinp_ii
lasttime lasttime_ee lasttime_ie lasttime_ei lasttime_ii
nexttime nexttime_ee nexttime_ie nexttime_ei nexttime_ii
somf     somf_e      somf_i      som
somp     somp_e      somp_i
alwf     alwf_e      alwf_i      alw
alwp     alwp_e      alwp_i
until    until_ie    until_ee    until_ii    until_ei
since    since_ie    since_ee    since_ii    since_ei
```

Bounded version of since and until are written as:

```
(until_ie_<=_<= t1 t2 A B)
B will be true at t instants in the future with t1<=t<=t2
(until_ie_>= t1 A B)
B will be true at t instants in the future with t>=t1
since_ie_<=_<=
since_ie_>=
```

Caveat emptor! The default `until` is PLTL's (which is usually called `until_ie` in TRIO). For example, the following model satisfies `(until A B)` at 0:

```
0                    B
----------------------
AAAAAAAAAAAAAAAAAAAAA
```

B may appear at 0.

For MTL users:

1. $\Diamond_{=t} A$ (or $\Box_{=t} A$)) is written `(futr (-P- A) t)`;
2. $\Box_{\leq t} A$ is written `(lasts (-P- A) t)`;
3. $\Diamond_{\leq t} A$ is written `(withinf (-P- A) t)`;
4. $\blacklozenge_{=t} A$ (or $\blacksquare_{=t} A$)) is written `(past (-P- A) t)`;
5. $\blacksquare_{\leq t} A$ is written `(lasted (-P- A) t)`;
6. $\blacklozenge_{\leq t} A$ is written `(withinp (-P- A) t)`;

with $t > 0$.

3.6 Operational constructs

$\mathbb{Z}$ot offers some simple facilities to describe operational systems. The following constructs are represented by means of a propositional encoding and, therefore, they are available both for SAT-based plug-in, e.g., *eezot*, and for SMT-based plug-in, *ae2zot*.

```
(define-item  <varname> <domain>)
```

is used to define variables à la Von Neumann over finite domains (e.g. counters).

```
(define-array <varname> <index-domain> <domain>)
```

is used to define mono-dimensional arrays.

Example usage:

```
(define-item  cont (loop for i from 0 to 9 collect i))
(define-array arr  (loop for i from 0 to 9 collect i)
                   '(on off unknown))
```

In the spec, the user can e.g. write `(cont= 6)`; `(arr= 6 'off)`.

Caveat: both define-item and define-array have side effects. It is therefore wrong to "define-items" after a zot main procedure call, since successive calls may work with spurious constraints. It is therefore recommended to perform `(clean-up)` before defining items or arrays.

Typically, to define an operational model means to constraint operational variables and arrays. This can be done either by using simple next-time formulae, i.e. containing only the `next` temporal operator, or by using the two dual constructs `and-case` and `or-case` [15].

To give the reader an idea of their semantics, here is an automatic translation made by $\mathbb{Z}$ot on two simple examples.

```
(and-case (x '(1 2) y '(3 4))
          ((-P- P x) (-P- Q x))
          ((-P- R y) (-P- R1 y))
          (else (-P- R2 x)))
```

expands to

```
(-A- X '(1 2)
    (-A- Y '(3 4)
     (&& (-> (-P- R Y) (-P- R1 Y)) (-> (-P- P X) (-P- Q X))
      (-> (&& (!! (-P- R Y)) (!! (-P- P X))) (-P- R2 X)))))
```

and

```
(or-case (x '(1 2) y '(3 4))
          ((-P- P x) (-P- Q x))
          ((-P- R y) (-P- R1 y))
          (else (-P- R2 x)))
```

expands to

```
(-E- X '(1 2)
    (-E- Y '(3 4)
     (|| (&& (-P- R Y) (-P- R1 Y)) (&& (-P- P X) (-P- Q X))
      (&& (!! (-P- R Y)) (!! (-P- P X)) (-P- R2 X)))))
```

3.7 MTL

There is an experimental plug-in (called *ap-zot* for using a variant of dense-time MTL through approximation (see [6], and [5]). Observe that MTL is a pure propositional temporal logic, so relations, functions and variables over possible infinite domains are not supported.

Here is a list of the time operator defined in ap-zot.

```
until-b   until-b-v   until-b-^
since-b   since-b-v   since-b-^
release-b release-b-^ release-b-v
trigger-b trigger-b-^ trigger-b-v

until-b-inf   until-b-v-inf   until-b-^-inf
since-b-inf   since-b-v-inf   since-b-^-inf
release-b-inf release-b-^-inf release-b-v-inf
trigger-b-inf trigger-b-^-inf trigger-b-v-inf

diamond   diamond-inf
diamond-p diamond-inf-p
box       box-inf
box-p     box-inf-p
```

The plug-in offers the following operations

```
normalize
basicize
compute-granularity
over-approximation
under-approximation
nth-divisor
```

To compute over- and under-approximations, an axiom must be prepared through the two functions *basicize* and *normalize*
(e.g. with `(setf ax1 (normalize (basicize ax1)))`).

The two functions *over-approximation* and *under-approximation* are used to compute the approximated formulae, while *compute-granularity* is used to set the ρ parameter (see [6] for details).

The interested reader may find a complete example in `coffee.lisp`.

3.8 Timed Automata

Timed Automata (TA) are supported through a *very* experimental plug-in called *ta-zot* (see [7], [8]), which is based on the approximations offered by *ap-zot*. As for *ap-zot* only the pure propositional version is supported.

First, here is a list of the added operators, and approximations procedures:

```
white-tri
white-tri/3
black-tri
black-tri/3

timed-automaton-under-formula
timed-automaton-over-formula

timed-automata-under-formula
timed-automata-over-formula
```

Here is the main data structure used to represent TA's, together with its interface:

```
(defstruct timed-automaton
  alphabet
  states
  initial-states
  clocks)

(defgeneric add-trans (autom from to lamb constr))
(defgeneric add-label (autom state list-of-symbols))
(defgeneric alpha (autom state))
(defgeneric get-trans-from-states (autom from to))
(defgeneric all-connected-pairs (autom))
(defgeneric all-unconnected-pairs (autom))
(defgeneric get-all-trans (autom))
(defgeneric get-trans-from-clock-reset (autom clock))
```

The interested reader may find a complete example in

`trans_prot.lisp`.

4 Usage

4.1 SAT-solvers

The supported SAT-solvers are MiniSat [4] (which is used by default), MiraXT [10], and zChaff [11].

To use the zChaff SAT-solver, the user has to set the *zot-solver* parameter. For example:

```
(setq sat-interface:*zot-solver* :zchaff)
```

MiraXT is a multi-threaded solver, so to use it we also have to choose the maximum number of threads that it will use:

```
(setf sat-interface:*zot-solver* :miraxt)
(setf sat-interface:*n-threads* 3)
```

4.2 SMT-solvers

SMT-solver are used in the decision procedure for the plug-in *ae2zot*. The supported SAT-solvers are Z3 [?] (which is used by default), Yices [?], CVC3 [?], MathSat [?], VeriT [?].

To change the SMT-solver, the user has to set the Lisp keyword `:solver` when invoking the function `zot`. For example:

```
(zot 10
     formula
     :solver :yices)
```

Available keyword are: `:z3`,`:yices` and `:mathsat`.

ae2zot is written to agree with the SMT-LIB v.1 [?] whose major goal is to establish common standards and library of benchmarks for Satisfiability Modulo Theories, that is, satisfiability of formulas with respect to background theories for which specialized reasoning procedures exist. Every other SMT-solvers, respecting the standard of the SMT-LIB, can be easily added to the set of available SMT-solver.

4.3 Model Checking

To perform Bounded Model Checking, the user must provide the model as transition system through as argument `:transitions`. In this section, only the `next` operator is admitted. Important: every variable used must be declared implicitly by e.g. an initialization formula as the second argument of $\mathbb{Z}$ot.

Here we start with a simple example: mutex3 (a simple mutual exclusion protocol with three processes).

As mutex3 is expressible in LTL, the first part is used to load the propositional mono-infinite plug-in, and defines the used variables. The first line loads the propositional mono-infinite plug-in, called *eezot*. (*bezot* is the bi-infinite one.)

```
(asdf:operate 'asdf:load-op 'eezot)
(use-package  :trio-utils)

(defvar state-d '(N T C))
(defvar turn-d  '(1 2 3))


(define-array state turn-d state-d)
(define-item  turn  turn-d)

(defconstant decl ; optional declarations, just for checking usage
  (append
   (loop for x in state-d append
         (loop for y in turn-d collect (state= y x)))
   (loop for x in turn-d collect (turn= x))))
```

Then, we define the system initialization and transitions:

```
(defvar init   ; system initialization (at 0)
  (&& (-A- x turn-d (state= x 'N))
      (turn= 1)))

(defvar trans  ; list of model constraints
  (list
   (-A- p turn-d
        (or-case (x state-d)
                 ((state= p 'N)
                  (next (state= p 'T)))

                 ((&& (state= p 'T)
                      (|| (-A- p1 turn-d (-> (not (equal p p1))
                                             (state= p1 'N)))
                       (turn= p)))
                  (next (state= p 'C)))

                 ((state= p 'C)
                  (next (state= p 'N)))

                 (else
                  (&& (state= p x)
```

```
                    (next (state= p x))))))

   (or-case (x turn-d)  ; -- schedule --

             ((&& (state= 1 'N) (state= 2 'T) (state= 3 'N))
              (next (turn= 2)))
             ((&& (state= 1 'T) (state= 1 'N) (state= 3 'N))
              (next (turn= 1)))
             ((&& (state= 1 'N) (state= 1 'N) (state= 3 'T))
              (next (turn= 3)))


           ; --- random choice policy ---
             ((&& (state= 1 'T)(state= 2 'T))
              (next (|| (turn= 1)(turn= 2))))
             ((&& (state= 1 'T)(state= 3 'T))
              (next (|| (turn= 1)(turn= 3))))
             ((&& (state= 2 'T)(state= 3 'T))
              (next (|| (turn= 2)(turn= 3))))

             (else
              (&& (turn= x) (next (turn= x)))))))
```

As the reader may see, the transitions are defined as a list of constraints, which must hold on every instant of the time domain.

We then write a simple property we wish to check on the system:

```
(defvar spec
    (alw
      (&&
        (-> (turn= 1) (somf (|| (turn= 2)(turn= 3))))
        (-> (turn= 2) (somf (|| (turn= 1)(turn= 3))))
        (-> (turn= 3) (somf (|| (turn= 1)(turn= 2)))))))
```

The main procedure is called *zot*, and has two arguments: the time bound and the formula to be satisfied (plus some optional switches, e.g. :transitions, :declarations, :loop-free).

To check if spec-0 holds for a time bound of 30, we perform:

```
(eezot:zot 30               ; time bound
      (&& (yesterday init) ; initialization (init must hold at 0)
          (!! spec))       ; (negated) property
      :transitions trans   ; list of model constraints
      :declarations decl   ; (optional) declarations
      )
```

UNSAT means that the desired property holds. If the output is SAT, then *spec* does not hold and $\mathbb{Z}$ot returns a counter-example.

Now we show an example that require the full expressiveness of CLTLB(LIA). It represents a time varying variable taking two possible values (0 and 1) in an hysteresis like way, led by an independent variable `x`. The two parameters of the hysteresis cycle are the thresholds `x0` and `x1`. The system is defined by a set of CLTLB(LIA) formulae (descriptive specification) and, therefore it is not represented by a transition system.

The first line loads the arithmetic mono-infinite plug-in, called *ae2zot*.

```
(asdf:operate 'asdf:load-op 'ae2zot)
(use-package :trio-utils)

(define-tvar 'x *int*)
(define-tvar 'y *int*)
(define-tvar 'd *int*)
(define-var 'x0 *int*)
(define-var 'x1 *int*)
```

Observe that `x0` and `x1` are time-invariant variables, so they are actually undefined constant.

We define some high-level construct, following the suggested constructs explained in Section 3.1, in order to have a coincise description of formulae defining the system:

```
(defun up-down (z)
  (&& ([=] z 0) (yesterday ([=] z 1))))

(defun down-up (z)
  (&& ([=] z 1) (yesterday ([=] z 0))))

(defun low (z)
  ([=] z 0))

(defmacro high (z)
  ([=] z 1))
```

As previously anticipated, it is worth noticing that the definition of these function is realized by merging CLTLB language and Common Lisp language. During the process of evaluation of the function, the variable `z` will be substituted with its evaluation.

Then, we define the system initialization and formulae defining the behavior of the system:

```
(defvar init
  (&&
```

```
    ([=] (-V- x) 3)
    ([=] (-V- y) 0)))


(defvar tr-up-down
  (alwf
    (->
      (&&
        (yesterday (&& ([<] (-V- x) (-V- x1)) (high (-V- y))))
        ([>=] (-V- x) (-V- x1)))
      (up-down (-V- y)))))


(defvar tr-down-up
  (alwf
    (->
      (&&
        (yesterday (&& ([>] (-V- x) (-V- x0)) (low (-V- y))))
        ([<=] (-V- x) (-V- x0)))
      (down-up (-V- y)))))


(defvar s-up-down
  (alwf
    (->
      (up-down (-V- y))
      (&&
        (yesterday ([<] (-V- x) (-V- x1)))
        ([>=] (-V- x) (-V- x1))))))


(defvar s-down-up
  (alwf
    (->
      (down-up (-V- y))
      (&&
        (yesterday ([>] (-V- x) (-V- x0)))
        ([<=] (-V- x) (-V- x0))))))


(defvar low-state
  (alwf
    (->
      (low (-V- y))
```

```
      (since
        (until (low (-V- y)) (down-up (-V- y)))
        (up-down (-V- y))))))


(defvar high-state
  (alwf
    (->
      (high (-V- y))
      (since
        (until (high (-V- y)) (up-down (-V- y)))
        (down-up (-V- y))))))


(defvar high-low-state-is
  (alwf (|| (low (-V- y)) (high (-V- y)))))


(defvar continuous-x
  (alwf (||
          ([=] (next (-V- x)) ([+] (-V- x) 1))
          ([=] (next (-V- x)) ([-] (-V- x) 1)))))


(defvar Ncontinuous-x
  (alwf ([=] (next (-V- x)) ([+] (-V- x) (-V- d)))))


(defvar safe-state
  (alwf
    (&&
      (-> ([=] (-V- (-V- y)) 1) ([<=] (-V- x) (-V- x1)))
      (-> ([=] (-V- (-V- y)) 0) ([>=] (-V- x) (-V- x0))))))


(defvar thresholds
      ([>] (-V- x1) (-V- x0)))


(defvar syst
  (&&
    high-low-state-is
    high-state
    low-state
```

```
    tr-up-down
    tr-down-up
    s-up-down
    s-down-up
    continuous-x
    thresholds))
```

As already anticipated, the main procedure is called *zot*, and has two arguments: the time bound and the formula to be satisfied (plus some optional switches, e.g. :transitions, :declarations, :logic).

To check if the system is satisfiable, i.e., it admits at least one behavior, for a time bound of 15, we perform:

```
(ae2zot:zot 15
      (&&
         init
         syst
      :logic :QF_UFLIA)
```

If it is the case, then the tool provides two values for the two thresholds `x0` and `x1`. The satisfiability of the formula represents an instance of synthesis problem, since the two thresholds' values are not previously defined. Being variables of the problem, they will be given a value which fulfills the axioms.

To check if the property `safe-state` holds for a time bound of 15, we perform:

```
(ae2zot:zot 15
      (&&
         init
         syst
         (!! safe-state)
      :logic :QF_UFLIA)
```

If the `safe-state` property holds, then the tool answers UNSAT.

Available keyword for `:logic` are `QF_UFIDL` and `QF_UFLIA`. Observe that the formula uses explicitly sum and difference, and so the constraints system `QF_UFLIA` must be loaded. If not the default constraint system is `QF_UFIDL`.

4.4 Completeness

A switch of the *zot* procedure (:loop-free, `nil` by default) of the propositional plug-in (*eezot*, *beezot*, ...) is used to check completeness. In the first example above, we can check completeness by performing:

```
(eezot:zot 30            ; time bound
     (yesterday init)   ; initialization (init must hold at 0)
```

```
  :transitions trans ; list of model constraints
  :declarations decl ; (optional) declarations
  :loop-free t       ; check completeness
  )
```

UNSAT means that the completeness bound is reached.

The *zot* procedure returns `t` if the spec is satisfiable, `nil` otherwise. So, it is possible to write a loop to actually find the completeness bound, e.g.:

```
(format t "Found: ~s~%"
        (loop for bound from 2 unless
             (eezot:zot bound
                       (yesterday init)
                       :transitions trans
                       :declarations decl
                       :loop-free t
                       )
           return bound))
```

4.5 Satisfiability Checking

Let us now consider a simple example to show how satisfiability checking can be performed with ℤot.

The first line loads the bi-inifinite plug-in.

```
(asdf:operate 'asdf:load-op 'bezot)
(use-package  :trio-utils)
```

We then define the timed lamp spec:

```
(defconstant delta 5)

; Alphabet
; on:  the "on"  button is pressed
; off: the "off" button is pressed
; L:   the light is on

(defconstant init
  (&& (!! (|| (-P- on)(-P- off)(-P- L)))))

(defconstant the-lamp
  (alw (&&
         (<->
          (-P- L)
          (|| (yesterday (-P- on))
              (-E- x (loop for i from 2 to delta collect i)
                   (&& (past (-P- on) x)
                       (!! (withinP_ee (-P- off) x))))))
         (!! (&& (-P- on) (-P- off))))))
```

To obtain a history compatible with the spec, we perform:

```
(bezot:zot  10
    (&& init the-lamp))
```

This is an example history generated by $\mathbb{Z}$ot, where **LOOP**, and **POOL** are the loop selector variables (**POOL** towards the past, **LOOP** towards the future):

```
------ time 0 ------

------ time 1 ------
  **LOOP**
  ON

------ time 2 ------
  ON
  L

------ time 3 ------
  ON
  L

------ time 4 ------
  OFF
  L

------ time 5 ------
  OFF

------ time 6 ------
  OFF

------ time 7 ------
  OFF

------ time 8 ------
  OFF

------ time 9 ------
  **POOL**
  OFF

------ time 10 ------

------ end ------
```

4.6 Temporary data

ℤot uses files to save temporary data during the verification activity. They differ depending on the used plug-in. The propositional plug-in, like *eezot*, creates

1) `output.cnf.txt`
2) `output.sat.txt`
3) `output.hist.txt`

(1) contains the resulting boolean formula of the system (in the standard DIMACS CNF format); (2) is the output of the SAT-solver; (3) is the resulting trace of the system (e.g. a TRIO history), if it exists.

SMT-based *ae2zot* creates different files:

1) `output.smt.txt`
2) `output1.txt`
3) `output.hist.txt`

(1) contains the resulting SMT-LIB description of the system (the standard is SMT-LIB v.1); (2) is the output of the SMT-solver, whose syntax and structure vary with respect to the used solver; (3) is the resulting trace of the system, if it exists.

5 Architecture

ℤot's architecture is based on a PLTL-to-SAT core, which interacts with the "outside world" through a TRIO-based interface and different plug-ins. The core itself is structured as a plug-in, so that different encodings can be defined and used.

More recently (May 2009), we added two plugins to ℤot, natively supporting metric operators (like *lasts, withinf*). These native metric plugins are called *meezot* (mono-infinite), and *mbezot*. Their usage is exactly the same as *eezot* and *bezot* [12].

5.1 PLTL-to-SAT encodings

As said before, ℤot's core is based on encoding PLTL into SAT and CLTLB(IDL) or CLTLB(LIA) into SMT. At present two main propositional encodings are available in the standard distribution: *eezot*, which is a standard eventuality-based encoding on a mono-infinite time domain (ℕ, see e.g. [3]), and the bi-infinite one, *bezot* [13] on ℤ. The arithmetic encoding (see [1]) is implemented in *ae2zot*.

The two propositional encodings are packaged (as asdf systems) in the following files:

```
eezot.lisp   eezot.asd
bezot.lisp  bezot.asd
```

The arithmetic encoding is packaged in:

```
ae2zot.lisp   ae2zot.asd
```

The file `kripke.lisp` contains the basic data structure and the definition of the generics.[4]

```
(defclass kripke ()
  (; time bound i.e. [0..k]
   (the-k       :accessor kripke-k)

   ; number of used prop. variables
   (numvar      :accessor kripke-numvar)

   ; formula -> integer data structure (hash-table)
   (the-list    :accessor kripke-list)

   ; integer -> formula data structure (hash-table)
   (the-back    :accessor kripke-back)
```

[4] *kripke* does not actually contain a Kripke structure - names of data structures and generics come from previous, forsaken incarnations of the tool-set.

```
; list of propositional letters
(sf-prop     :accessor kripke-prop)

; list of used boolean subformulae
(sf-bool     :accessor kripke-bool)

; list of used future-tense subf.
(sf-futr     :accessor kripke-futr)

; list of used past-tense subf.
(sf-past     :accessor kripke-past)

; n. of props used in the encoding
(max-prop    :accessor kripke-maximum)

; resulting SAT formula
(the-formula :accessor kripke-formula)))
```

There is also an old variant of *eezot*, called *ezot*, which supports virtual unrollings (as presented in [3], usually called δ), so its data structure is extended through inheritance. The user may change the default behavior (i.e. $\delta = 0$), by setting ezot:*FIXED-DELTA* to nil, which tells eezot to actually compute δ, or (s)he may change to set it to a fixed meaningful value.

The *call* generic translates a formula/proposition and a time instant into an integer (the SAT-solver proposition); *self* must be an instance of kripke (or of a subclass).

```
(defgeneric call (self obj the-time &rest other-stuff))
```

The *back-call* generic is used to translate an integer in $[0..k]$ into the corresponding subformula; self must be an instance of kripke (or of a subclass).

```
(defgeneric back-call (self x))
(defgeneric back-call-time (self x))
```

5.2 Main Interface

There are two interfaces:

```
sat-interface.lisp
```

the first one is with the SAT/SMT-solver, and it is used to send the output of the PLTL/CLTLB encoding to it; then, to parse its output and get a counter-example, if any.

The other one,

`trio-utils.lisp`

is the basic interface with the user, and is based on TRIO (see Section 3.5) augmented with the operational constructs covered in Section 3.6.

5.3 Other modules and plug-ins

At present just *ap-zot* and *ta-zot* are available. Please refer to Sections 3.7, 3.8, and the related papers.

The two plug-ins are implemented and packaged (as asdf systems) in

```
ap-zot.lisp  ap-zot.asd
ta-zot.lisp  ta-zot.asd
```

ta-zot is based on *ap-zot*, which uses TRIO as underlying language (through the *trio-utils* interface).

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