In general, a vector field is a function whose domain is a set of points in $\mathbb{R}^2$ or $\mathbb{R}^3$ and whose range is a set of vectors in V_2 or V_3.

Definition 1. Let D be a set in $\mathbb{R}^2$. A **vector field** on $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns to each point (x, y) in D a two-dimensional vector $\mathbf{F}(x, y)$.

Since $\mathbf{F}(x, y)$ is a two-dimensional vector, we can write it in terms of its **component functions** P and Q as follows:

$$\mathbf{F}(x, y) = P(x, y)\ \mathbf{i} + Q(x, y)\ \mathbf{j}$$

P and Q are scalar functions and sometimes called **scalar fields** to distinguish them from vector fields.

Definition 2. A vector field $\mathbf{F}$ is called a **conservative vector field** if it is the gradient of some scalar function, that is, if $\mathbf{F} = \nabla f$. f is called a **potential function** for $\mathbf{F}$.