10.3 Representing Graphs and Graph Isomorphism

Representing Graphs

Adjacency Matrices

Given an undirected graph G. An adjacency Matrix representing G is a Matrix M s.t.

$$a_{ij} = \begin{cases} 1 & \text{if nodes } i \,\&\, j \text{ are neighbors} \\ 0 & \text{otherwise} \end{cases}$$

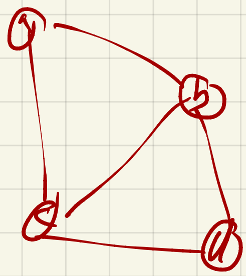
$G = (V, E)$

$V = \{ \cdot \; - - - - \}$

$E = \{ (a,b) \mid a \text{ connect to } b \}$

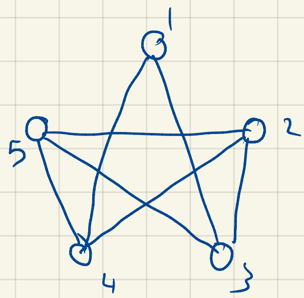
$\{ (a,b) \mid a \in V \wedge b \in V \} \subseteq V \times V$

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

G 

adjacency Matrix of G

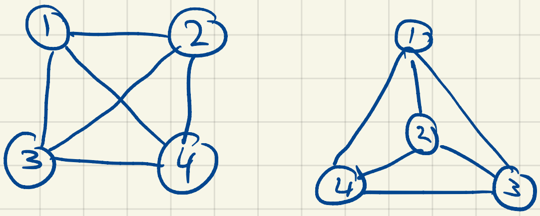
$$\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 & \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

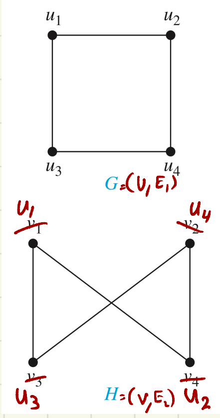


Isomorphism of Graphs

*The word *isomorphism* comes from the Greek roots *isos* for "equal" and *morphe* for "form."

DEFINITION 1 The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an *isomorphism*.* Two simple graphs that are not isomorphic are called *nonisomorphic*.



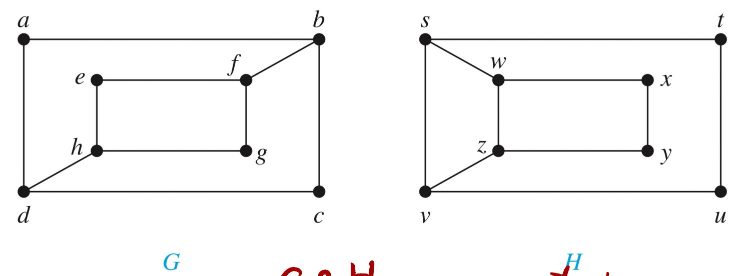


$f(u_1) = v_1$
$f(u_2) = v_4$
$f(u_3) = v_3$
$f(u_4) = v_2$
$f: U \to V$

Two garphs $G=(V,E)$ & $H=(W,F)$ are <u>not</u> isomophic if $|V| \neq |W|$ or $|E| \neq |F|$.

Example

Are the below graphs isomorphic?



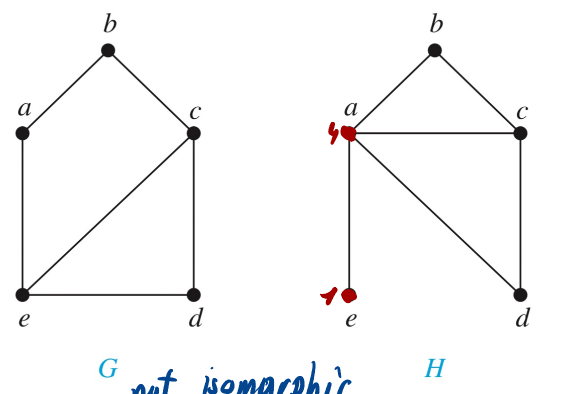
G & H are not isomophic

FIGURE 10 **The Graphs G and H.**

$G=(V,E)$
$H=(W,E_2)$

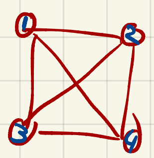
$f: V \to W$

$E_2 = \{(f(a), f(b)) \mid a, b \in V\}$

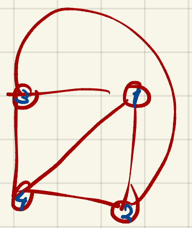


not isomorphic

FIGURE 9 **The Graphs G and H.**



K_4



K_4

Two graphs G & H are isomorphic

then,

1- They have same structure.

2- They have the same # of nodes # of edges

3- They have same node degree distribution

(3, 3, 2, ———)

4- I can redraw H to match G.