Introduction to Aeroelastic Instabilities with Jupyter Notebooks

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Universidad Carlos III de Madrid

Politecnico di Bari

First session – February 2nd 2026

About me

Poor sailor

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Poor climber

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Poor tennis player

About me

Poor sailor

Poor climber

Poor tennis player

Aerospace engineer

– Politecnico di Torino 🇮🇹

Source: Polito Sailing Team Facebook

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– TU Delft 🇳🇱

Source: Yachting World

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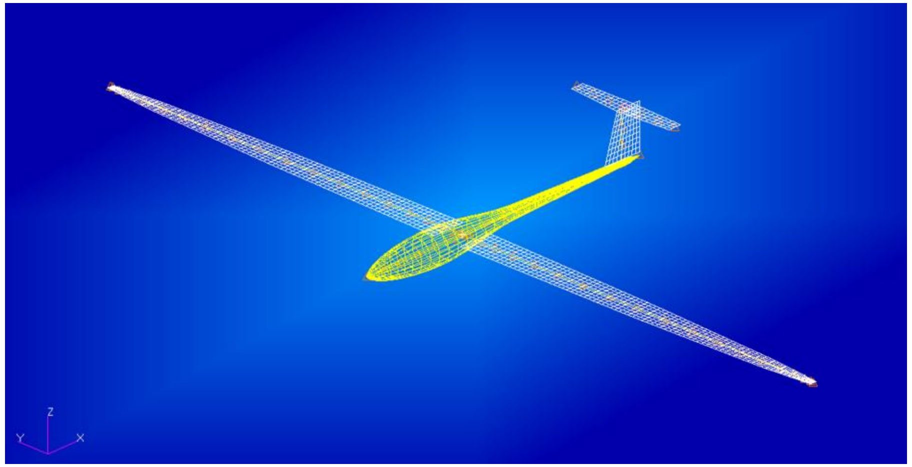

Research internship at

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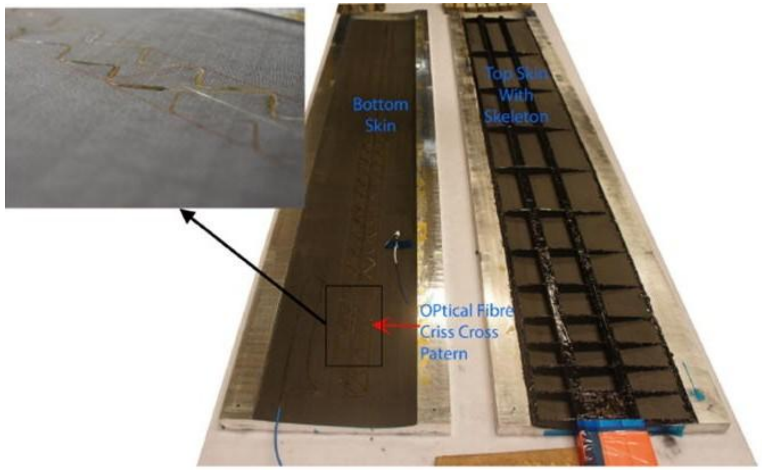

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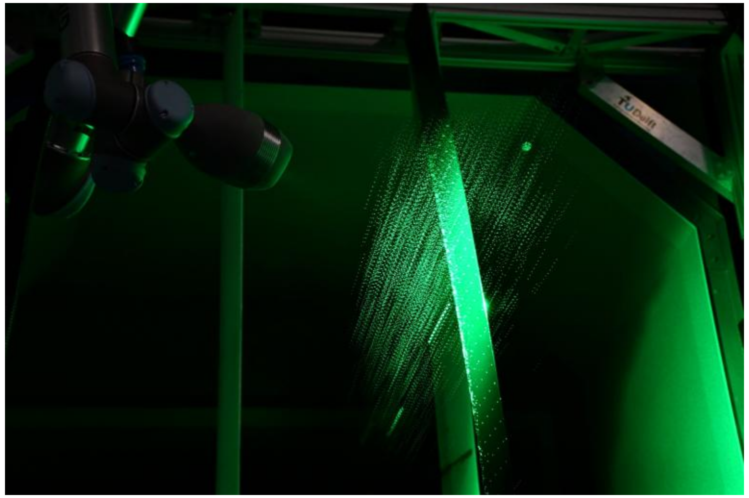

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– University of Bristol

PhD co-funded by

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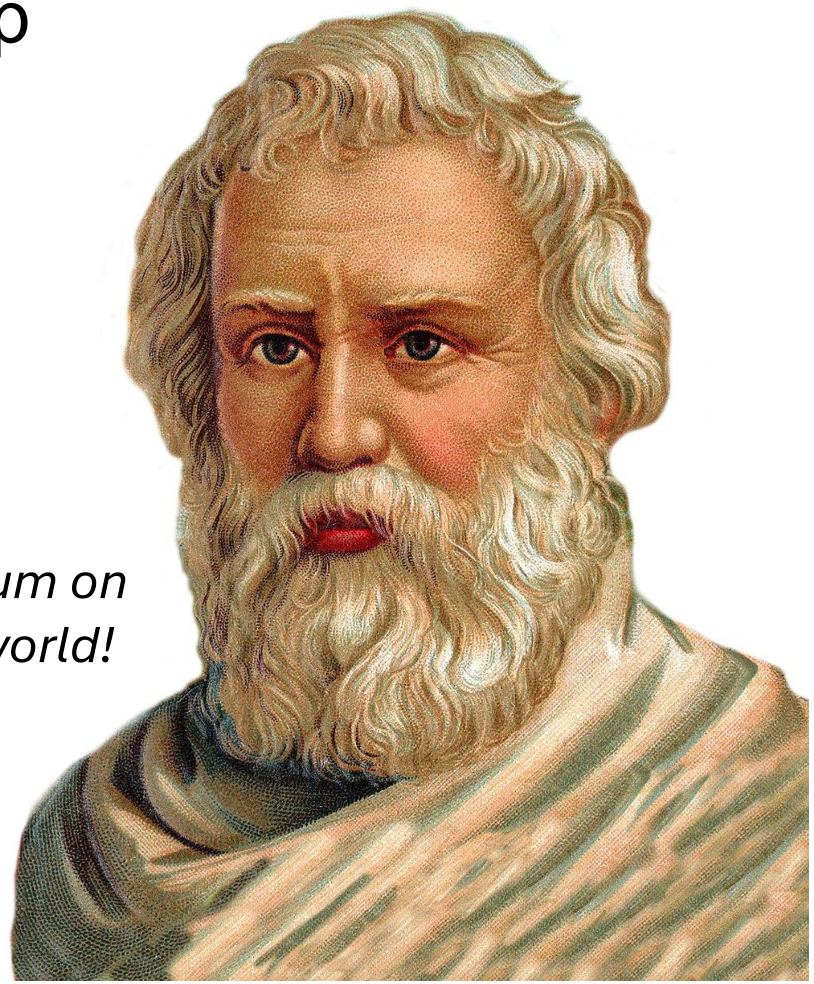
– TU Delft 🇳🇱
– University of Bristol 🇬🇧
– UC3M 🇪🇸

National research project in collaboration with AIRBUS DEFENCE & SPACE

What is Aeroelasticity?

We are going to ask an old friend for help



https://www.meisterdrucke.uk/fine-art-prints/Unknown-artist/917155/Portrait-of-Archimedes-of-Syracuse---Greek-Mathematician,-Physicist,-Engineer,-Inventor.html

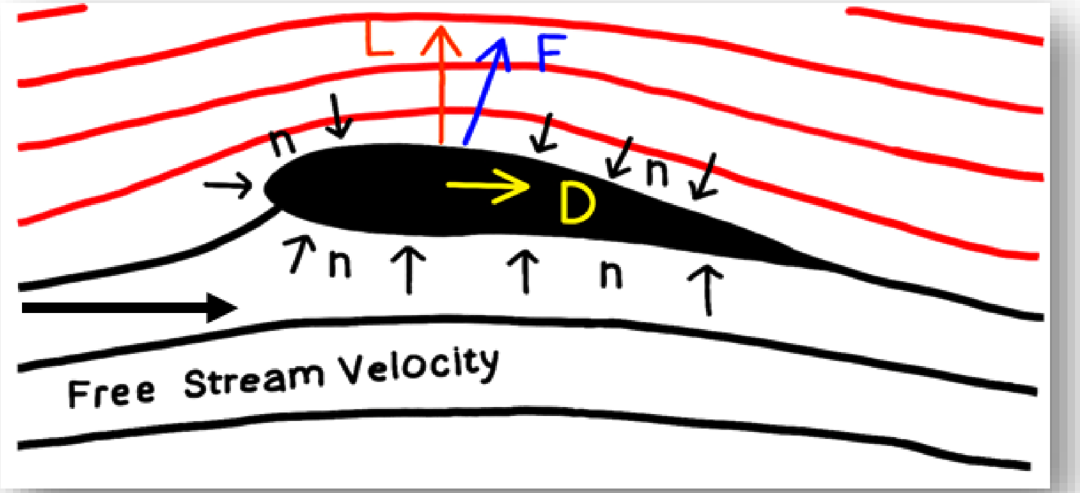
Archimedes of Syracuse

Give me a lever long enough and a fulcrum on which to place it, and I shall move the world!

What is Aeroelasticity?

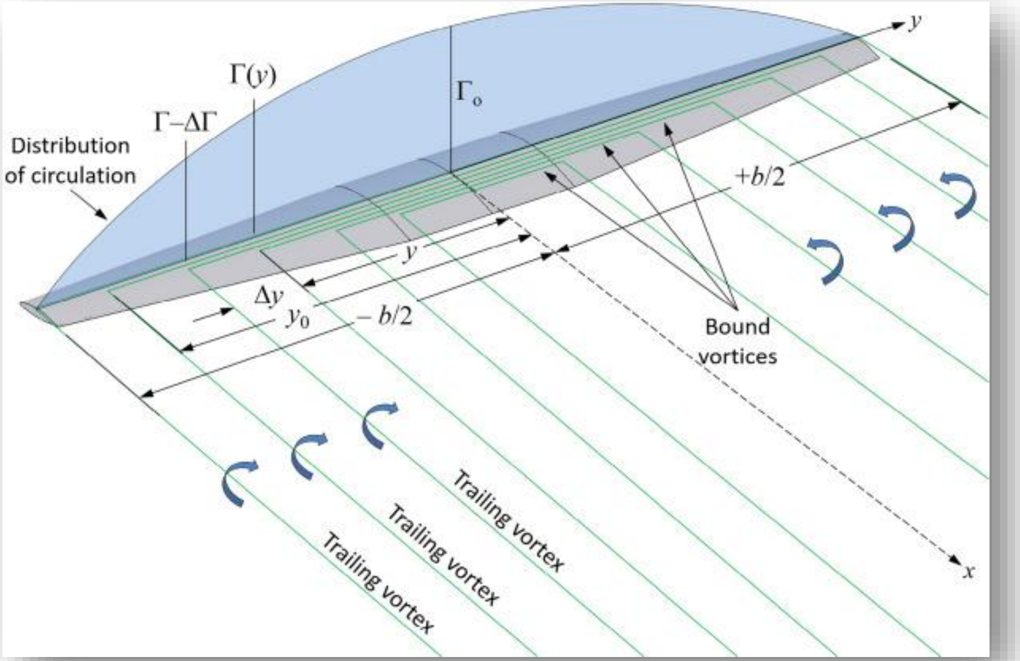
What you were taught in your aerodynamics courses:

For an airfoil



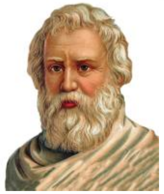
https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/aerodynamic-forces/

For a wing



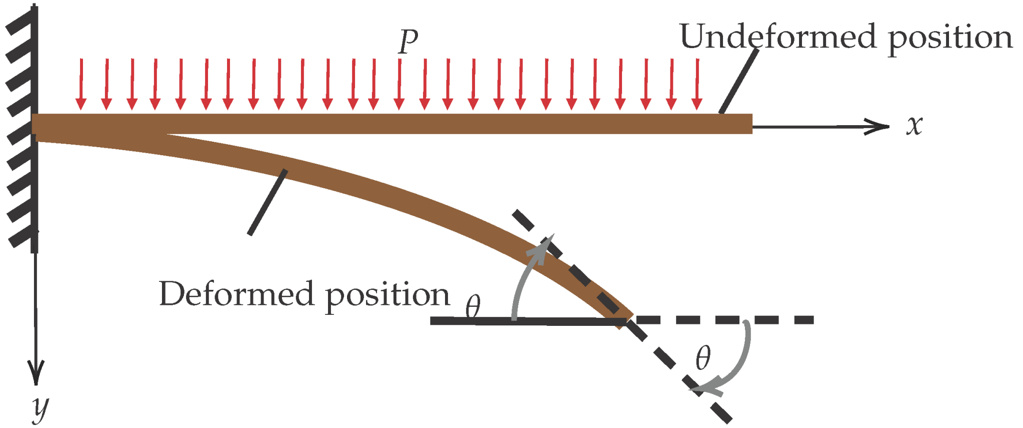
https://www.sciencedirect.com/science/chapter/monograph/pii/B9780128184653000094

Saying it like Archimedes: *give me a geometry and a velocity, and I shall calculate the aerodynamic force (distribution)!*



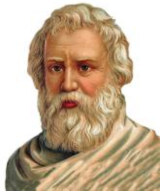
What is Aeroelasticity?

What were taught in your structures courses:



https://www.mdpi.com/2504-3110/7/2/141

Saying it like Archimedes: *give me a geometry and a force, and I shall calculate the structural displacement (and strain, and stresses, etc.)!*



What is Aeroelasticity?

But wait a second Archi, what happens to the flow and to the aerodynamic forces when the structure deforms?

And what happens to the structure when the aerodynamic forces change?

And what happens to the aerodynamic forces when the structural deformation changes?

…

What is Aeroelasticity?

Aerodynamics + Structures (**Elasticity**)

On the ground

In the air

Go check out the video *Aeroelasticity: why aircraft are elastic* by DLR:
https://www.youtube.com/watch?v=fdMlepVlpR4

What is Aeroelasticity?

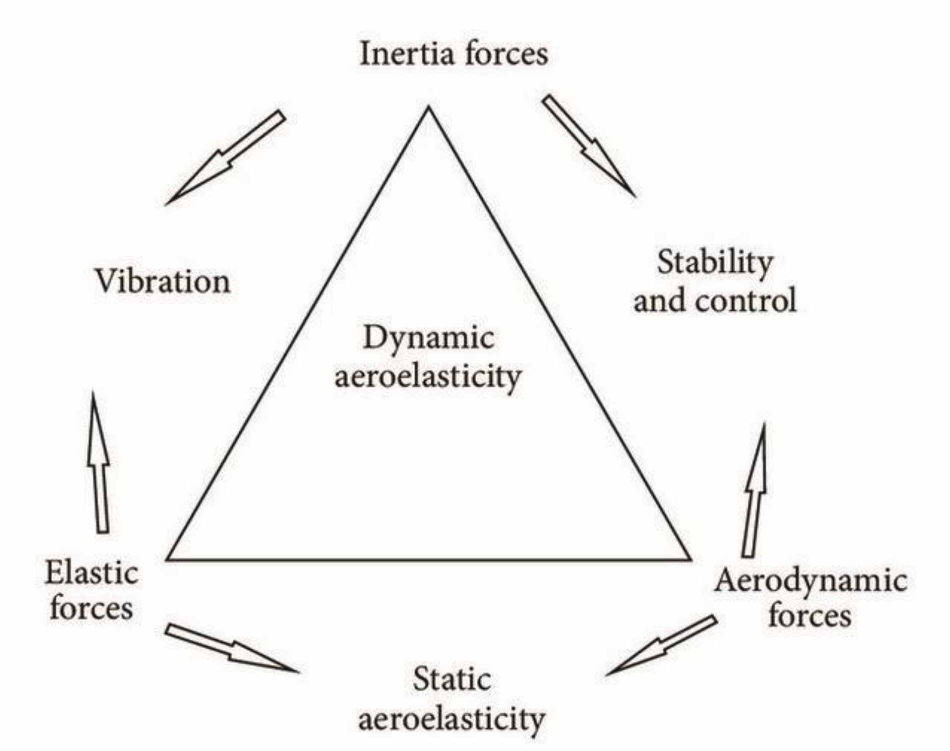
Aerodynamics + Structures (**Elasticity**) + Mass (Inertia)

https://www.youtube.com/watch?v=r-v9H2uKD9k

What is Aeroelasticity?

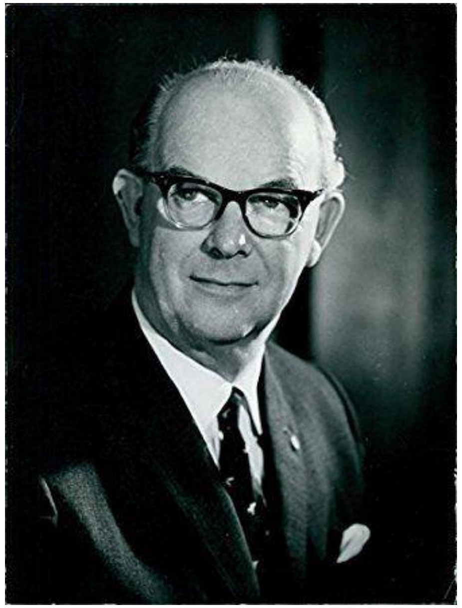
Aerodynamics + Structures (**Elasticity**) + Mass (Inertia)

Collar's triangle



Wright, J. R., & Cooper, J. E. (2014). *Introduction to aircraft aeroelasticity and loads* (2nd edition). Wiley.

Arthur Roderick Collar



https://alchetron.com/Arthur-Roderick-Collar

Aeroelasticity – not only aircraft wings

Mil Mi-26 helicopter

https://www.reddit.com/r/indianaviation/comments/1h3dhhh/sorry_for_foreign_post_i_have_a_question_that_why/

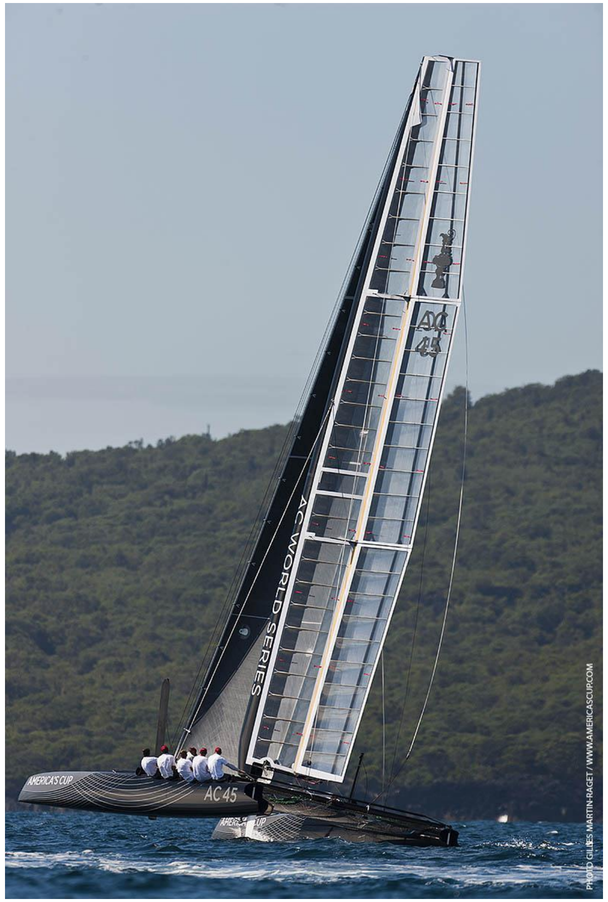
https://live.warthunder.com/post/860495/en/?comment=3288223

Aeroelasticity – not only aeronautics

Wind turbines

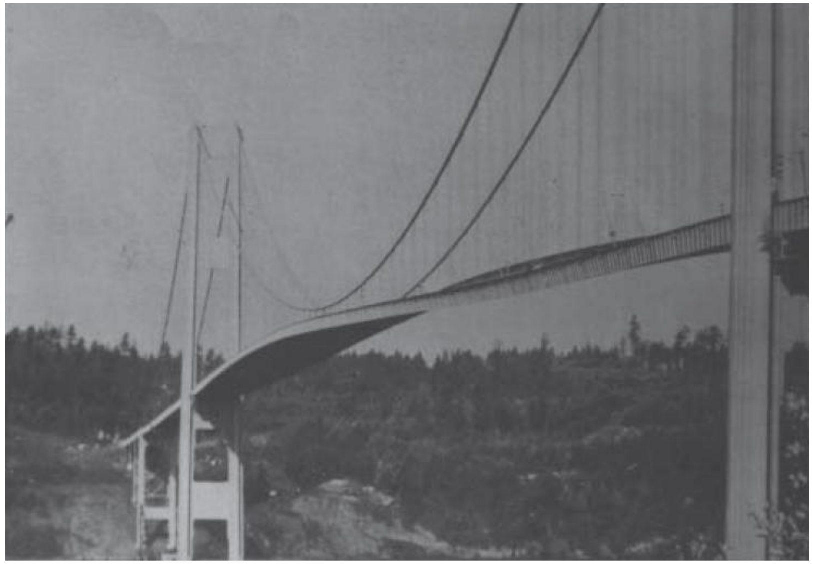
https://www.instagram.com/p/DGQSNrOt6Ge/

Sail boats



https://www.yachtsandyachting.com/photo/58388

Bridges



https://doi.org/10.5604/12314005.1137397

Classification of Aeroelastic Problems

	Stability problems	Response problems
Static aeroelasticity	E + A = 0	**E + A = Q**
Dynamic aeroelasticity	E + A + I = 0	E + A + I = Q

What's the force distribution and the deformation of my wing given a steady flight condition?

Classification of Aeroelastic Problems

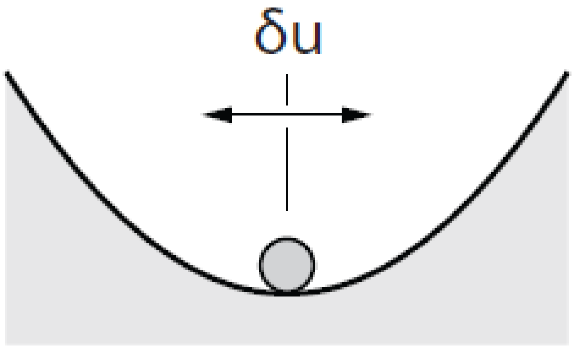
	Stability problems	Response problems
Static aeroelasticity	E + A = 0	E + A = Q
Dynamic aeroelasticity	E + A + I = 0	**E + A + I = Q**

What’s the dynamic response of my wing to an unsteady flight condition?

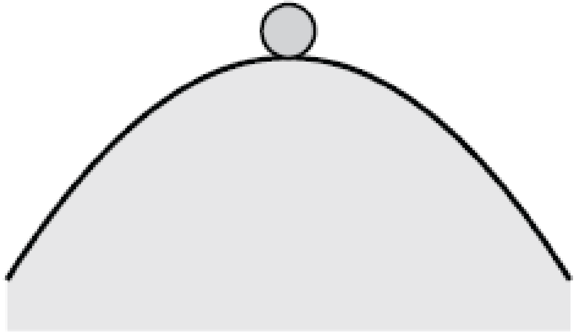
Classification of Aeroelastic Problems

	Stability problems	Response problems
Static aeroelasticity	**E + A = 0**	E + A = Q
Dynamic aeroelasticity	**E + A + I = 0**	E + A + I = Q

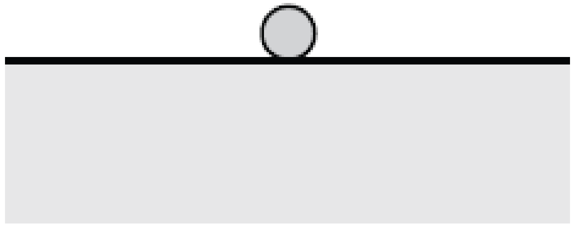
What about **stability**?



stable



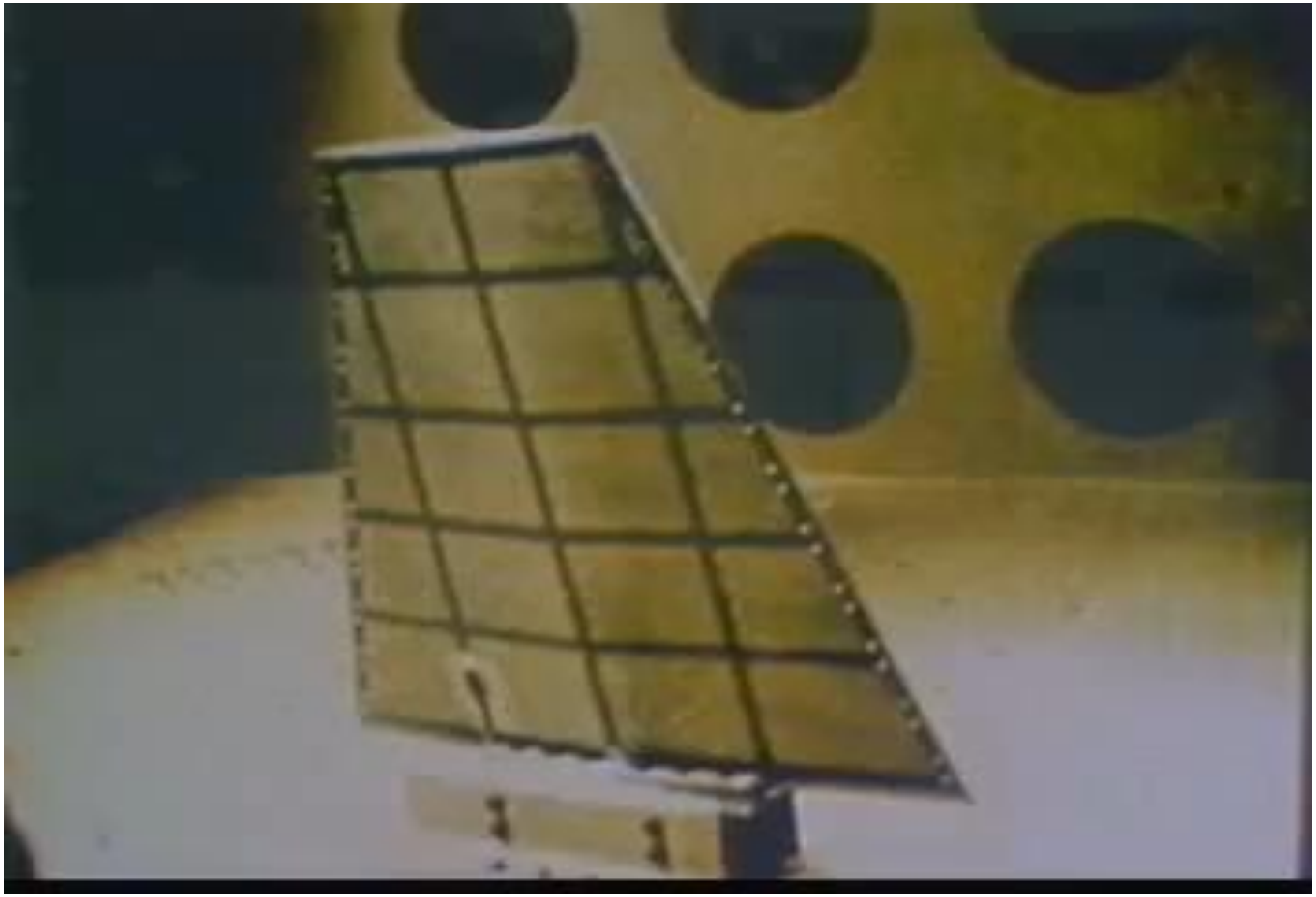
unstable



neutral

Image credits: Mark Schenk, University of Bristol, Nonlinear Structures and Structural Stability

Static Aeroelastic Instability – Divergence



https://www.youtube.com/watch?v=k0bGuq_evPQ

Non-oscillatory instability!

Dynamic Aeroelastic Instability – Flutter

https://www.youtube.com/watch?v=k0bGuq_evPQ

Oscillatory instability!

Dynamic Aeroelastic Instability – Flutter

https://www.youtube.com/watch?v=k0bGuq_evPQ

Oscillatory instability!

Dynamic Aeroelastic Instability – Flutter

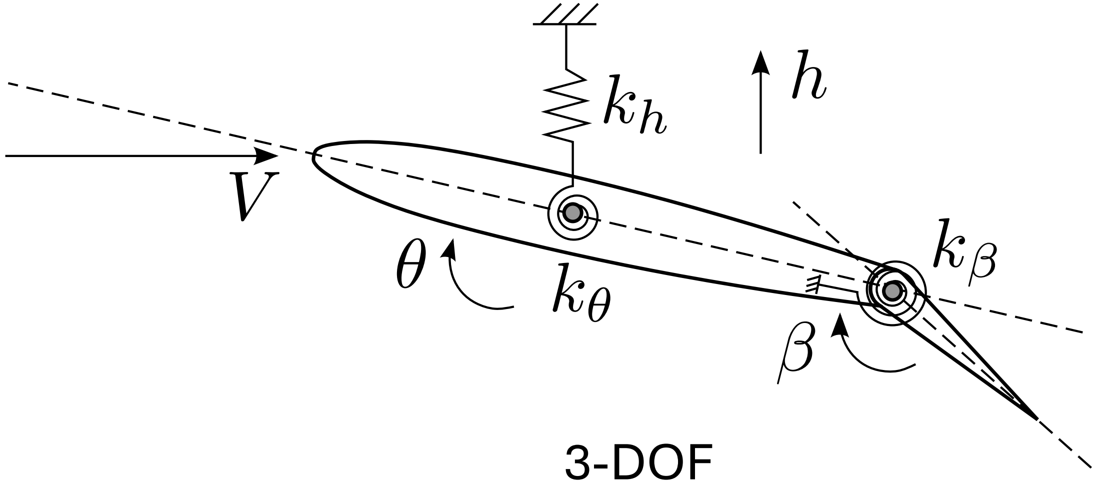
https://www.youtube.com/watch?v=egDWh7jnNic

Oscillatory instability!

Course Objective

Introduce you to the analysis of aeroelastic instabilities (divergence & flutter) using the **typical section model**

https://calaero.edu/aeronautics/airplane-parts/wing-flaps-function-and-purpose/



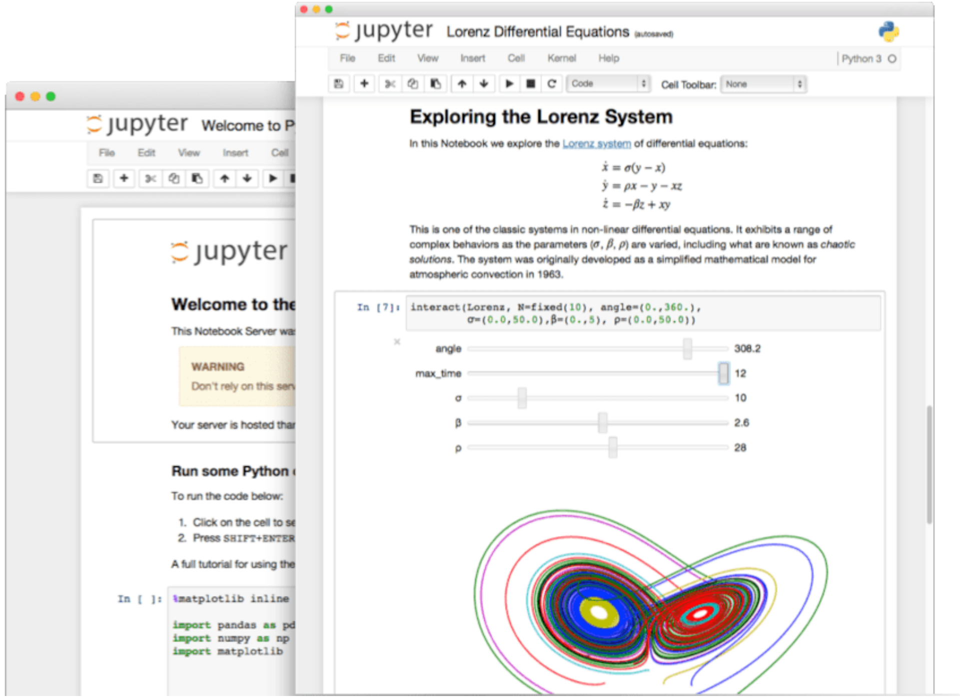
3-DOF

Wing bending → k_h spring → heave DOF (h)
Wing torsion → k_θ spring → pitch DOF (θ)
Aileron → k_β spring → control surface DOF (β)

Using **jupyter notebooks!**

Jupyter Notebooks

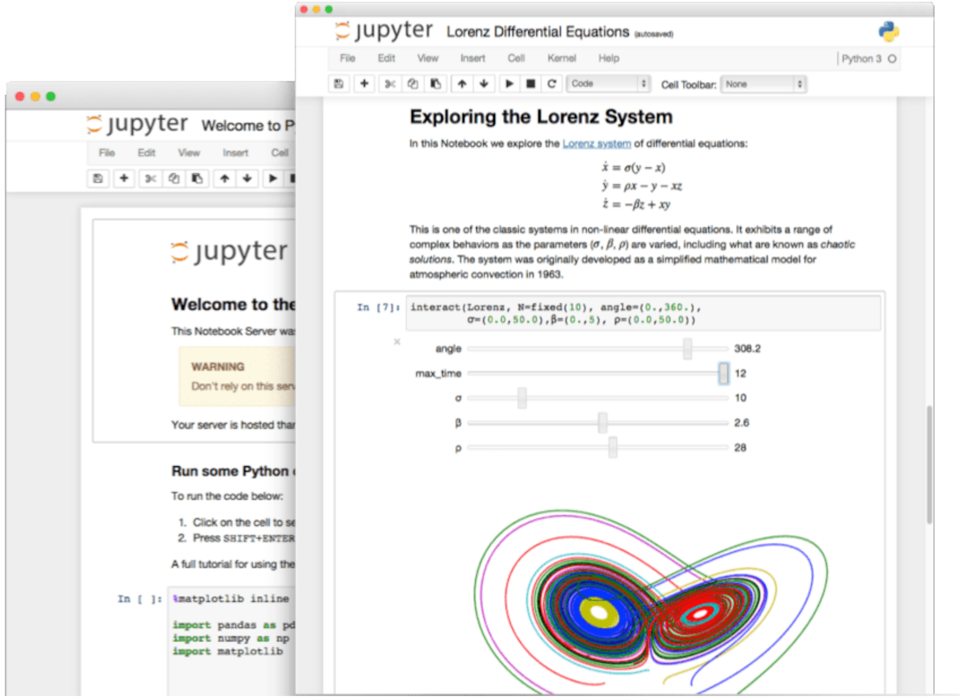
- **What is it?** An open-source web application that allows you to create and share documents containing live code, equations, visualizations, and narrative text.
- **The Name:** A reference to the core supported languages: **Ju**lia, **Pyth**on, and **R**
- **Why for this course?** Aeroelasticity is mathematically intensive.
  - Standard scripts separate the *derivation* (paper) from the *solution* (code).
  - Jupyter keeps them together in a single **Computational Story**



https://jupyter.org/

Jupyter Notebooks

- **Beyond Scripting:** We aren't just writing "code." We are building a document that explains *why* we are writing that code
- "Scaffolded" Coding
  1. **Understand:** We read the theory in the Markdown cell.
  2. **Implement:** You fill in the "skeleton" code
  3. **Verify:** You run the cell and compare the numerical result with the physical expectation

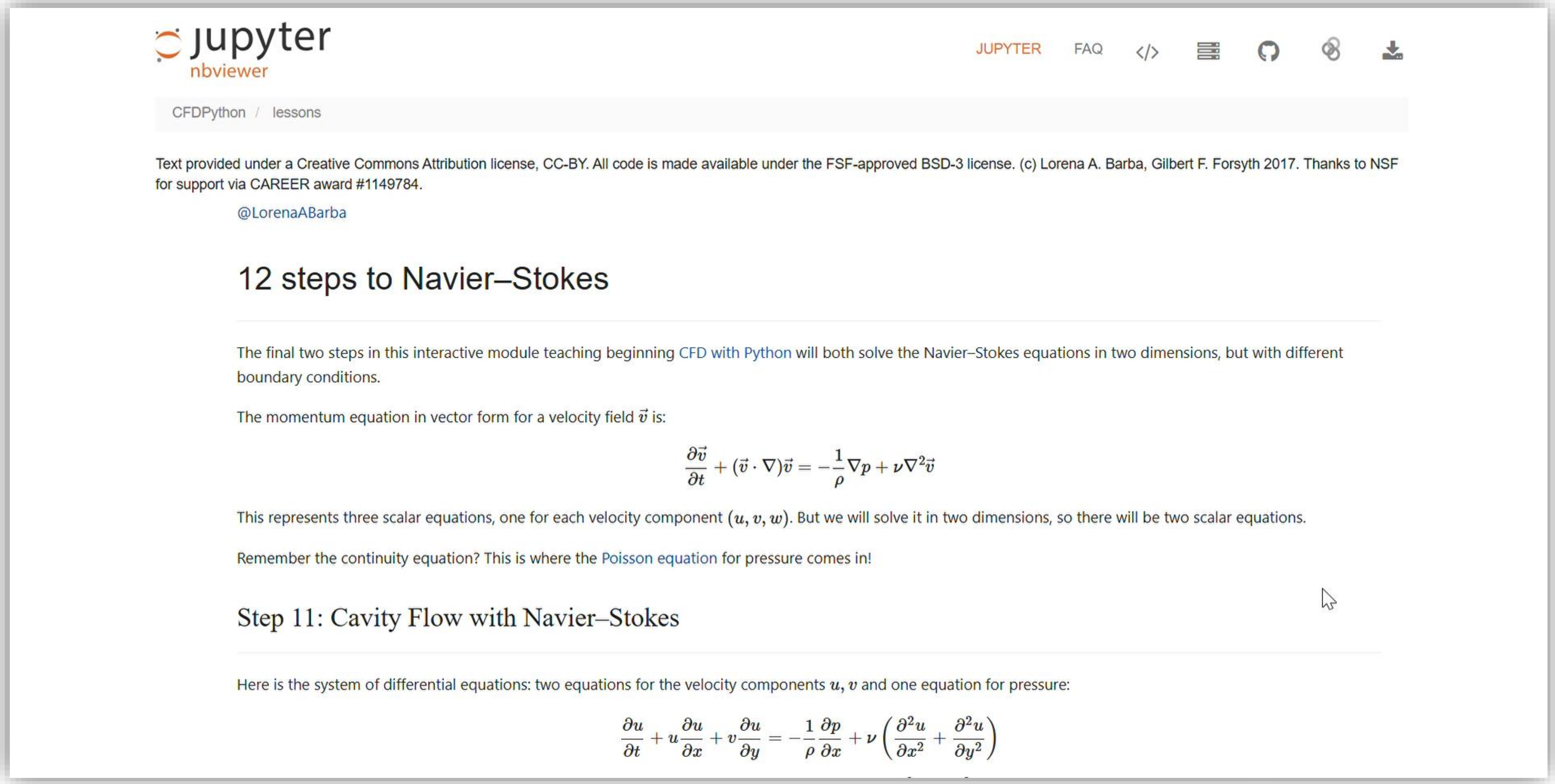


https://jupyter.org/

Notable Examples

CFD Python: 12 Steps to Navier-Stokes (Prof. Lorena Barba)

- *The Gold Standard:* Our course's "parent." It demonstrates how to build a complex CFD solver from the simplest 1D wave equation.



jupyter nbviewer

JUPYTER FAQ

CFDPython / lessons

Text provided under a Creative Commons Attribution license, CC-BY. All code is made available under the FSF-approved BSD-3 license. (c) Lorena A. Barba, Gilbert F. Forsyth 2017. Thanks to NSF for support via CAREER award #1149784.

@LorenaABarba

12 steps to Navier–Stokes

The final two steps in this interactive module teaching beginning CFD with Python will both solve the Navier–Stokes equations in two dimensions, but with different boundary conditions.

The momentum equation in vector form for a velocity field $\vec{v}$ is:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \vec{v}$$

This represents three scalar equations, one for each velocity component (u, v, w). But we will solve it in two dimensions, so there will be two scalar equations.

Remember the continuity equation? This is where the Poisson equation for pressure comes in!

Step 11: Cavity Flow with Navier–Stokes

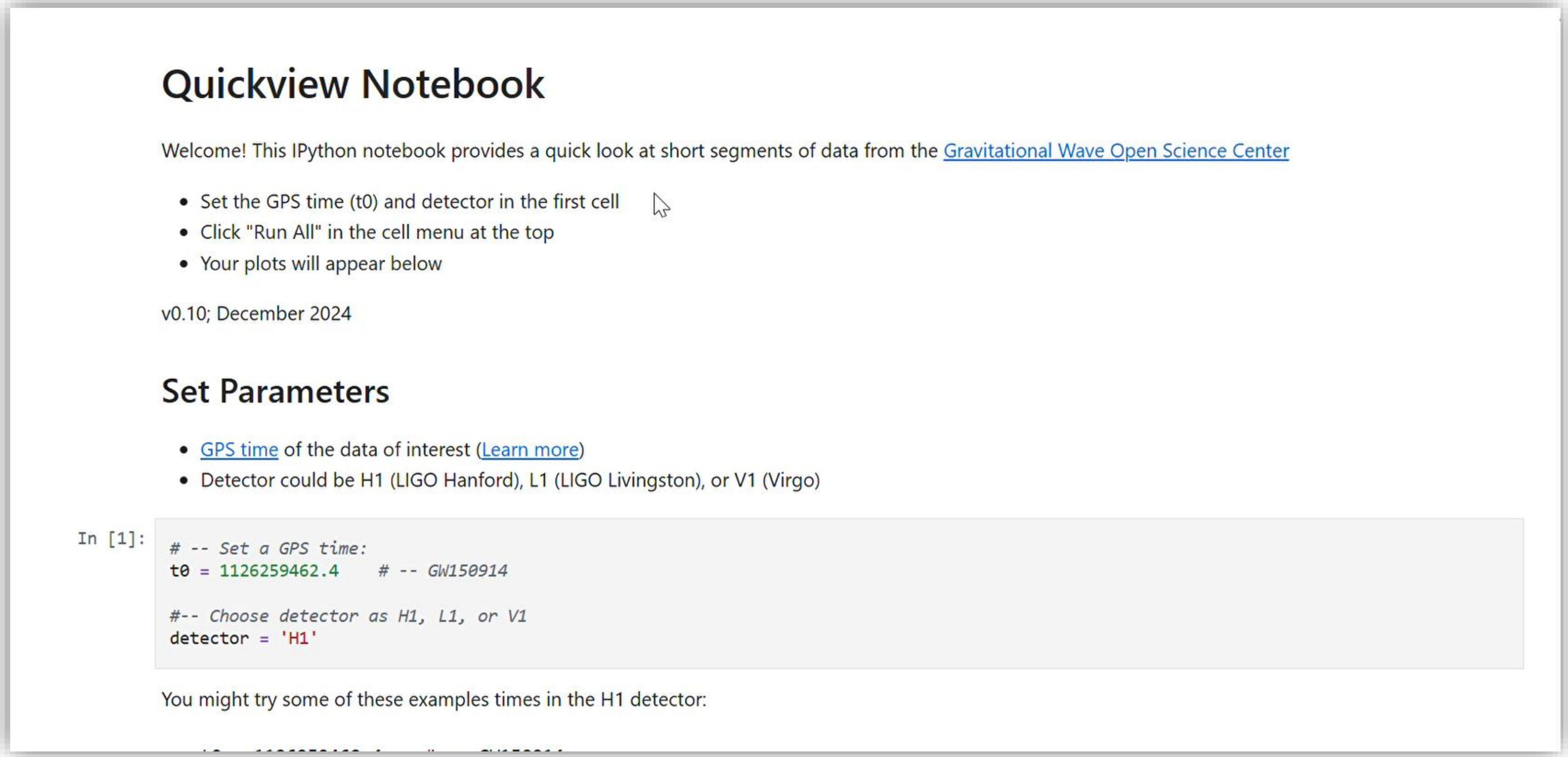
Here is the system of differential equations: two equations for the velocity components u, v and one equation for pressure:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Notable Examples

LIGO: Binary Black Hole Merger

- *Scientific History:* The actual notebook used by the LIGO collaboration to process the data of the first gravitational wave detection. It shows how Nobel-prize-winning science is documented today.



Quickview Notebook

Welcome! This IPython notebook provides a quick look at short segments of data from the Gravitational Wave Open Science Center

- Set the GPS time (t0) and detector in the first cell
- Click "Run All" in the cell menu at the top
- Your plots will appear below

v0.10; December 2024

Set Parameters

- GPS time of the data of interest (Learn more)
- Detector could be H1 (LIGO Hanford), L1 (LIGO Livingston), or V1 (Virgo)

In [1]:

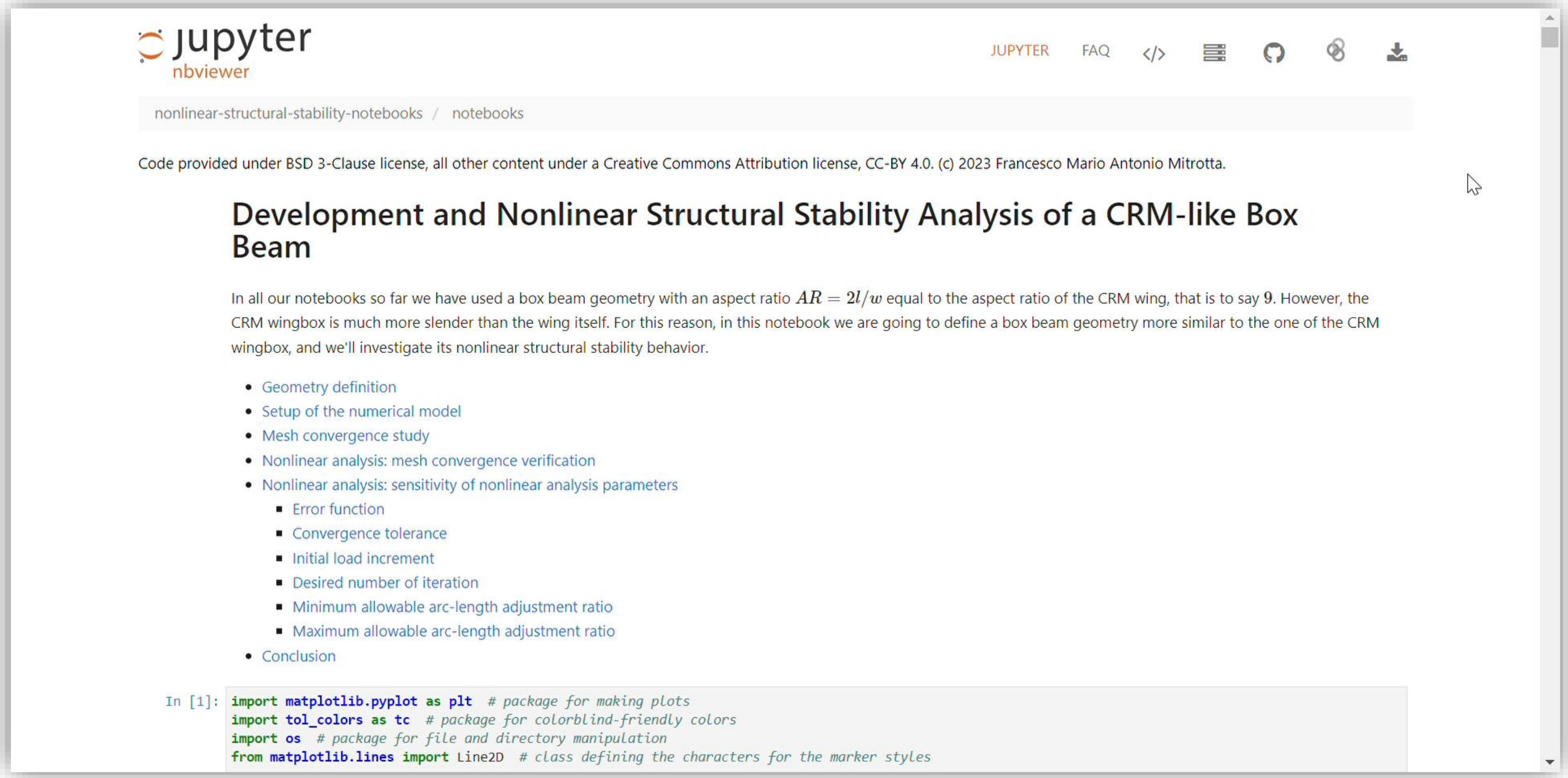
```
# -- Set a GPS time:
t0 = 1126259462.4    # -- GW150914

#-- Choose detector as H1, L1, or V1
detector = 'H1'
```

You might try some of these examples times in the H1 detector:

Notable Examples (maybe less notable)

Nonlinear Structural Stability Notebooks (me!)

- Computational story of my PhD on wingbox optimization with nonlinear structural stability constraints with the aim of facilitating reproducibility and knowledge transfer



Course Outline

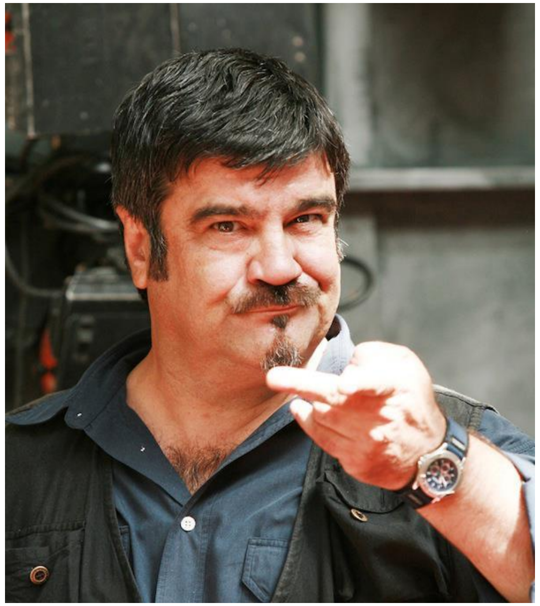
- Today – Monday February 2nd (3h)
  - Notebook 0: Python and Jupyter Introduction
  - Notebook 1: Aeroelastic Polar and Torsional Divergence
- Tomorrow – Tuesday February 3rd (3h)
  - Notebook 2: Quasi-Steady Aerodynamics and the Heave-Only Model
  - Notebook 3: Quasi-Steady Flutter of the Heave-Pitch Typical Section
- Take-home assignment (4h)

This is an Experiment!

https://praxilabs.com/en/blog/2022/09/14/chemistry-experiments-virtual-labs/

Paraphrasing the famous director René Ferretti:

Another way of teaching engineering is possible!



Source: Facebook

To the notebooks!

Let’s code!