Problem 1

c) Given the code from the lecture, the function $RB\text{-}insert(T, z)$ always initially sets the colour of the new node to red before calling the $RB\text{-}fixup(T, z)$ function. The first node is changed to black in $RB\text{-}fixup(T, z)$ as it becomes the root.
Since we always initially insert red nodes we can go through the cases of $RB\text{-}fixup(T, z)$. Assume current size of n where $n \geq 1$:

Option 1 : Red Node$(n+1)$ is inserted as child of a black node. Trivial case. We have at least one red node.

Option 2 : Red Node$(n+1)$ is inserted as a child of a red node. Go to $RB\text{-}fixup(T, z)$ and check.

Case 1 The node remains red after recolouring the parent, grandparent and uncle. Thus we have at least one red node.

Case 2 The parent of the node remains red after rotation of node with parent and grandparent. The inserted node is made black. Thus we have at least one red node.

Case 3 The node remains red after rotating the parent with the grandparent. The parent and grandparent are recoloured to black. Thus we have at least on red node.

Q.E.D.