A Systems Engineering Approach to Mathematical Properties of System Readiness Levels

by Eileen B. McConkie

B.S. in Mechanical Engineering, June 1982, Virginia Polytechnic Institute and State University
M.S. in Systems Engineering, May 1997, Virginia Polytechnic Institute and State University

A Dissertation submitted to

The Faculty of
The School of Engineering and Applied Science
of The George Washington University
in partial satisfaction of the requirements
for the degree of Doctor of Philosophy

January 31, 2013

Dissertation directed by

Thomas A. Mazzuchi
Professor of Operations Research and Engineering Management
Shahram Sarkani
Professor of Engineering Management and Systems Engineering

The School of Engineering and Applied Science of The George Washington University certifies that Eileen Brogan McConkie has passed the Final Examination for the degree of Doctor of Philosophy as of Aug 22, 2012. This is the final and approved form of the dissertation.

A SYSTEM ENGINEERING APPROACH TO MATHEMATICAL PROPERTIES OF SYSTEM READINESS LEVELS

Eileen Brogan McConkie

Dissertation Research Committee:

Thomas Andrew Mazzuchi, Professor of Operations Research and of Engineering Management, Dissertation Co-Director

Shahram Sarkani, Professor of Engineering Management and Systems Engineering, Dissertation Co-Director

Edward Lile Murphree, Professor of Engineering Management and Systems, Committee Chair

Thomas H. Holzer, Professorial Lecturer in Engineering Management and Systems Engineering, Committee Member

Dedication

This dissertation is dedicated in memory of my father, Mr. Raymond F. Brogan, the best dad a girl could have, and to my loving mother, Rita C. Brogan. Their love and support in all that I attempted instilled in me that I could accomplish all.

This dissertation is also dedicated to my loving husband, James F. McConkie. His love, support and care through these long years have truly been a blessing.

This dissertation is also dedicated to my three children, J. Colin, Bridget and Sean Patrick McConkie. Each, in their special way, gave me encouragement to persevere and to finish the course.

Finally, this dissertation is dedicated to the numerous family members, friends and colleagues who have offered words of encouragement and support throughout this endeavor.

Acknowledgement

A dissertation is a collection of work that cannot be done alone. I would like to express a sincere word of thanks to the numerous people who have helped in so many ways.

First I would thank my advisors, Dr. Mazzuchi and Dr. Sarkani, for their help and guidance during the research phase of my doctorial program. Their foresight in setting up the doctoral cohorts, setting goals and their advice during the research phase has made the road to success doable.

I would like to acknowledge the time, energy and mathematical direction that Dr. David Marchette gave. Dave's instruction and explanations helped to formulate this research. He encouraged me to present and get feedback to improve the contents.

Much thanks goes to my dear friends in EMSE Southern Maryland Cohort 2 for their continuous encouragement and support through the course work and research. It was a pleasure studying with you, receiving encouragement and feedback during research and sharing in your research, especially Lynwood Townsend and John Seel.

I have many friends and family to thank. First I would like to thank my family, especially my sisters, Mary Ann, Kathy, Clare, Betty and Rose, they were my loudest cheerleaders. Also my brothers, Raymond, Patrick, William, Kevin, Marty and Joseph, all my brothers- and sisters-in-laws, and my many nieces and nephews who would never let me give up, they were always willing to give an encouraging word, to listen and to review my work. Second, I would like to give thanks to my many friends who through their prayers and words kept me going. I would especially like to thank Teri and Carey Priebe who have kept this goal in front of me since we first met in early 1990s, they have offered numerous hours of listening to my frustrations and in reviewing my work. Third, I would like to say thank you to my church family who were willing to pray, listen and ask for updates throughout the years. Without this community of support, my road to success would have been a lonely one.

Abstract

A System Engineering Approach to Mathematical Properties of System Readiness Level

System engineers use qualitative and quantitative measurements in their work. The use of quantitative measurements is growing but the mathematical rigor behind these measurements is not always done and presented in the literature. Without the mathematical rigor misleading results can occur. System readiness level (SRL), is a new quantitative measurement being developed as a decision-making aid for system engineers and program managers. The SRL was first developed by Sauser et al [2006] and is continuing to be developed and refined, by Sauser and others. The SRL is defined as a function of technology readiness level (TRL) and integration readiness level (IRL), both of which are ordinal ranking numbers. Currently matrix algebra, with and without graph theory, is being used to define this function. Concern has been voiced in the system engineering community about the continuous development of the SRL without the proper mathematical rigor being conducted.

This dissertation examines the development of the SRL, the intended use of the SRL, the mathematical operations that have been proposed for calculating SRL and the inherent mathematical properties of these operations. This dissertation develops and defines desired mathematical properties that a quantitative measurement used to define the readiness level of a system should possess. These mathematical properties are introduced to increase the validity and rigor of SRL calculations, increasing the

confidence system engineers and program managers will have in the calculated SRL for their system and the decisions made based on these calculations.

Through the development of the mathematical properties a new method for calculating SRL was formulated. This new method uses tropical algebra as the mathematical operation for calculating SRL. Tropical algebra, an area of mathematics, combines ordinal numbers using addition, versus multiplication like matrix algebra. Addition of ordinal numbers is more meaningful than the multiplication of ordinal numbers; therefore the resulting SRL will be more meaningful to the system engineer and program manager when making resource allocation decisions and technology selections.

Table of Contents

List of Figures

List of Tables

Acronyms

AD^2	Advancement Degree of Difficulty
A	Assess
C	Control
CDR	Critical Design Review
CAE	Component Acquisition Executive
CI	Configuration Item
D	Detect
E	Engage
DoD	Department of Defense
DoD I	Department of Defense Instruction
DT&E	Developmental test and evaluation
DUSD(S&T)	Deputy Under Secretary of Defense (Science and Technology)
EMD	Engineering and Manufacturing Development
GAO	Government Accounting Office
GT	Graph theory
GTTA	Graph theory tropical algebra
LRIP	Low-rate initial production

IRL	Integration Readiness Level
IMM	Integration maturity metric
ITI	Integration Technology Index
ITRL	Subsystem Readiness Level
MDA	Management Decision Authority
NASA	National Aeronautics and Space Agency
OT&E	Operational Test and Evaluation
PEO	Program Executive Office
PDR	Preliminary Design Review
R&D	Research and Development
$R\&D^3$	Research and Development Degree of Difficulty
RFID	Radio Frequency Identification
SoS	Systems of System
SRL	System Readiness Level
S-SRL	SRL model developed by Dr. Sauser and his co-authors
SWOT	Strength, Weakness, Opportunity and Threat
TRL	Technology Readiness Level
TRA	Technology Readiness Assessment

USD (AT&L)	Under Secretary of Defense for Acquisition, Technology and Logistics

Chapter 1 INTRODUCTION

According to the U.S. Government Accountability Office (GAO) [1998:76] report, "demonstrating a high level of maturity before new technologies are incorporated into product development programs puts these programs in a better position to succeed". GAO report [2011] states that a lack of mature technology is one of a four characteristics that leads to increase in cost and slipped schedules. Of the programs studied, those that started with technology that had been demonstrated in a relevant environment had 4% less growth in costs than did the programs that entered development with technology nearing maturity and 33% less growth than those with at least one technology at a low readiness level [GAO, 2011]. The system engineer is responsible for determining how mature each critical subsystem, as defined by the Technology Readiness Assessment Handbook [2009], of a system is, how it is association with all other components in the system and how to relate these two characteristics to determine the overall readiness of the system [Bilbro, 2006; Malone and Wolfarth, 2012]. System engineers rely on measurements to accomplish this task [Ramirez-Marquez and Sauser, 2009; Sauser, 2010].

In 1979 Stan Sadin an employee of National Aeronautics Space Administration (NASA) developed a seven level metric that defined the technology readiness, known as technology readiness level (TRL). In 1995, John Mankins wrote a whitepaper adding definition to the TRL scale, which had grown to a nine-level scale [Mankins, 1995]. This scale defined the state of development and testing the technology was at and assigned a ranking number and was designed to inform the program mangers of space programs where a particular technology was in its development cycle [Mankins, 2009]. TRL was adopted by the Department of Defense (DoD) in the late 1990s and expanded to include more general wording and to include software [Garrett et al., 2011; Mankins, 2009; Sauser et al., 2010]. However, the world of today has moved beyond simple technologies to System of Systems (SoS). Program managers need a tool to assist with decision-making in this more complex environment [Baron et al., 2011; Garrett et al., 2011; Sauser et al., 2006; Sauser and Boardman, 2008]. Several efforts have been made to expand the TRL to make it effective for system-level readiness assessments; to understand the association of one technology with every other technology within the system [Azizian et al., 2011; Fernandez, 2010; Garrett et al., 2011; Long, 2011; Ramirez-Marquez and Sauser, 2009;Sauser et al., 2006; Sauser and Boardman, 2008]. Two new measurements, Integration Readiness Level (IRL) and System Readiness Level (SRL) were introduced by Sauser et al [2006] to address this need [Azizian et al., 2011; Baron et al., 2011; Garrett et al., 2011; Sauser et al., 2006; Sauser and Boardman, 2008].

The IRL was designed as a scale similar to the TRL that characterizes the degree to which the integration between two or more technologies has been proven [Sauser et al., 2006]. The SRL, a function of the TRL and IRL, was designed to give a holistic approach to the maturity of a system [Sauser et al., 2006].

Since its introduction in 2006, the SRL is growing in popularity. Program managers recognize that the earlier in the development process a system risk is identified, the easier and less costly it is to eliminate or mitigate [Maier and Rechtin, 2000; Sage and Rouse, 1999]. The SRL is designed to help program managers quantitatively identify the developmental risk of a system. Since 2006, Sauser and others have been expanding the SRL development to assist program managers with resource allocation and trade studies to determine the optimal method for achieving their SoS performance [Baron et al, 2011; Garrett et al, 2011; Ramirez-Sauser, 2009; Sauser and Broadman, 2008; 2011]. Garrett et al [2011] include the use of SRL for assessing end-to-end performance for mission thread analysis-SoS having specific military functions.

1.1 Problem Statement

As systems become increasingly complex, program managers rely on measurements for decision-making. As Stephen Anderson stated in his keynote speech at the Quantitative Methods for Defense and National Security on April 20, 2012, system engineers need these measurements to be more quantitative. As measurements move to quantitative in nature the mathematics needs to be understood (McConkie et al, 2012). The SRL, as discussed above, is being developed as such a measurement for decision-making teams to be used for resource allocations and in performing trade studies. As decision-making teams continue to use SRLs without understanding the mathematics behind the calculations, bad decisions can result such as misallocation of resources and increased cost and schedule slippage. Kujawski (2010), in his presentation at the 13th Annual National Defense Industrial Association systems engineering conference in San Diego, California, raised the concern about the mathematics involved in calculating SRL as defined by Sauser et al. The literature cautions about the mathematical operations used for calculating SRL [Bowles, 2003; Cox, 2008; Garrett et al., 2011; Kujawski, 2010; Ramirez-Marquez and Sauser, 2009; Sauser and Broadman, 2008; Sauser et al., 2011]. Although there is caution stated no research has been performed on the mathematics of the SRL calculations. This dissertation addresses this short fall; it adds the mathematical rigor needed for SRL calculations.

1.2 Scope of Dissertation

There are two types of properties for measurements, mathematical and utility. This dissertation develops mathematical properties that are necessary for a SRL to possess. It examines the mathematical operation, matrix algebra, currently used to calculate SRL and a proposed mathematical operation, tropical algebra, and their inherent mathematical properties. These inherent properties are combined with an understanding of the purpose of SRL to develop properties that are mathematically rigorous and logical. These properties are the bases for a mathematical framework to help ensure that the decision-making metric, SRL, results will be mathematically sound.

This dissertation does not evaluate the utility properties of SRL values, how helpful are SRL, nor does it determine if there is a correlation between SRL values and system cost and schedule. Such a study would be used to determine how helpful SRL values are and whether they inform the system engineer and program manager of programmatic risk. This dissertation accepted that SRL are being used and are believed to be helpful to decision-making teams. A statistical comparison of the three SRL models discussed in this dissertation would be part of a utility study and is not performed in this dissertation.

This dissertation does introduce a SRL model developed by combining graph theory and tropical algebra. Analyzing the literature and the mathematics behind the SRL and applying the current SRL models to a notional fire control loop revealed some interesting consequences of multiplying TRL and IRL, two ordinal numbers. Since tropical algebra combines TRL and IRL through addition, the resulting SRL model avoids some of these consequences. Garrett et al [2011] and Baron et al [2011] introduced using graph theory, a mature area of discrete mathematics, to analyze systems and system of systems. Combining these two areas of mathematics, graph theory and tropical algebra, a new SRL model, GTTA, is developed to give the program manager and systems engineers a quantitative measurement for evaluating the readiness of a system that has mathematical rigor behind it.

1.3 Organization of the Dissertation

Chapter 2 presents a review of the literature that was used to develop the problem statement and the mathematical properties. It presents background information on the Department of Defense acquisition cycle as it relates to SRL development and use, the underlying need for an SRL and short a introduction to the mathematics used in the SRL development. Chapter 3 presents this dissertation's contribution to the systems engineering body of knowledge; the logical mathematic properties, an explanation of their importance and an introduction to a new SRL model, GTTA. Chapter 4 presents the research method used to identify the mathematical properties and the new method for calculating SRLs. Chapter 5 presents mathematical calculations for a notional fire control system using the current SRL models, identifies some shortfalls associated with multiplication of two ordinal numbers as they relate to the calculated SRLs and the results of the analysis of which mathematical operations used for SRL calculations meet the desired properties. Chapter 5 demonstrates how to calculate SRL using GTTA and compares the result using this new model with the results from the current models.

Chapter 6 presents the conclusion along with areas of future research.

Chapter 2 LITERATURE REVIEW

Much of the literature, points to the need for robust tools and methods to measure the readiness of a system, or SoS, to continue toward development and acquisition [Azizian et al., 2011; GAO, 2006; Ramirez-Marquez and Sauser, 2009; Sauser et al., 2006; 2008]. The system readiness level, SRL, is a relatively new metric developed to answer this need. The SRL, as currently defined, is the product of two ordinal numbers. This has raised concern within the community of interest to its validity [Kujawski, 2010]. This dissertation will present the literature that defines the need for mathematical properties for SRLs and an alternative definition for the function used to define SRL, which does not involve multiplication of ordinal numbers. This section will also present background information on the difference between readiness and maturity, the DoD acquisition lifecycle phases, technology readiness level, TRL, and integration readiness level, IRL, and present an introduction to two areas of mathematics, graph theory and tropical algebra, found in the SRL literature.

2.1. Maturity versus Readiness

The American Heritage Dictionary of the English Language defines readiness as "the state of being ready or prepared, as for use or action." Sauser *et al* [2011:26] states "readiness of a system, technology or integration, implies how ready it is to be deployed on a numerical scale."

The word maturity is defined by *The American Heritage Dictionary of the English Language* as a "the state or quality of being mature; full development," while Sauser *et* *al*, [2011] states that "maturity is the characterization of the physical development that is quantified by the readiness."

These definitions show that readiness and maturity are similar but different; however, in the literature discussing systems and SoS they are used synonymously [Azizian, 2009; Smith, 2005]. Valerdi and Kohl [2004:3] gives a perfect example of how these words are interchangeably used: … "the lower the maturity, or readiness, of an incoming technology…." Tetley and John [2010] contend that there should be a clear distinction between readiness and maturity but within literature this is not the case leading to confusion. Smith [2005] also states that readiness and maturity are often used synonymously, but he contends there is a difference. A mature system and/or technology may not be ready for a particular environment. The Technology Readiness Assessment Deskbook (TRAD) [2009] gives an example of this—Radio Frequency Identification (RFID) tags for material assets management. "RFID tags provide automatic identification of tagged assets as they pass through locations equipped with interrogators." Although the military has used radio frequency tagging on a limited basis, using commercially available technology such as that used by Wal-Mart Stores, Inc., to manage its supply chain still has several problem areas: the need for real time data, i.e., where it is right now not where it was tagged; multilevel tagging, i.e., elements, boxes, crates; loss of contact, such as advance notice of deployment destination, is needed to ensure an interrogator is available. These difficulties demonstrate that the present mature technology, used in the real-world commercial environment, is not ready for military operations. This illustrates the difference between mature technology and technology readiness. Although there is a distinction between the two the literature does use them interchangeably; however, this dissertation will discuss the readiness and not the maturity level.

2.2. Department of Defense Acquisition Lifecycle

The Department of Defense Acquisition Life Cycle is a five-phase cycle with entrance and exit criteria at each phase. The five phases are presented in Figure 2-1.

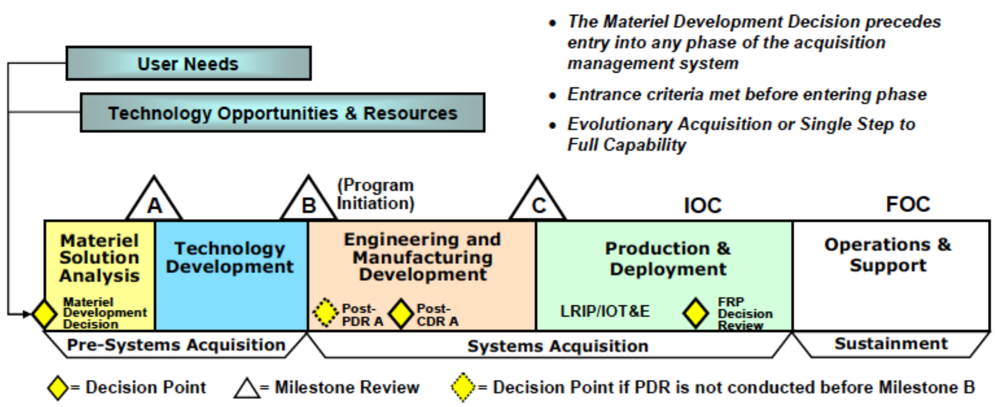


Figure 2-1 DoD Acquisition Lifecycle Phases (DoDI 5000.02)

The first phase, Materiel Solution Analysis Phase, assess potential materiel solutions. Analysis of Alternatives are conducted in which the technologies associated with the proposed materiel solutions are assessed for their maturity level, integration risk, and manufacturing feasibility along with other attributes. This phase is mandatory for all programs and ends when the Management Decision Authority (MDA) determines that the entrance criteria for the next phase are met.

The second phase, Technology Development Phase, plans for the development of the critical technologies to the appropriate maturity level. This development plan breaks down the materiel solution into incremental capabilities acquisition cycles. Each incremental military useful capability will define the set of technologies that will be integrated into the system and demonstrated on prototypes of the system. This plan reduces the risk associated with incorporating non-mature technologies and gets a capability to the user in a short amount of time. The exit criteria for this phase is defined by DoDI 5000.02 as

> an affordable program or increment of militarily useful capability has been identified; the technology and manufacturing processes for that program or increment have been assessed and demonstrated in a relevant environment; manufacturing risks have been identified; a system or increment can be developed for production within a short timeframe (normally less than 5 years for weapon systems); or, when the MDA decides to terminate the effort. [DoDI 5000.02 pg 19]

The third phase, Engineering and Manufacturing Development (EMD) Phase, begins at Milestone B and is only entered after all critical technologies have been demonstrated in a relevant environment. The functionality and interface requirements are established and the hardware and software designs are completed during this phase. A successful demonstration of the system and the manufacturing process must occur before exiting this phase.

The fourth phase, Production and Development Phase, demonstrates the effectiveness and suitability of the materiel solution so the MDA can commit the DoD to production. First a low-rate initial production is established and produced to demonstrate production feasibility and to produce articles for testing. After successful testing and manufacturing is demonstrated, the system moves into Full-Rate Production with MDA approval.

The fifth phase, Operations and Support Phase, develops a plan to sustain the system over its entire life cycle and a plan for disposal at the end of its usefulness. Disposal plan must include demilitarization in accordance with laws, regulation and environmental concerns.

These five phases are conducted in an evolutionary acquisition strategy; delivering capabilities in small manageable increments. This strategy ensures deliverance of a military useful capability in a short time frame with the knowledge that future capabilities will need to be developed [DoD I 5000.02]. Figure 2-2 shows the acquisition process flow used for evolutionary acquisition strategy. Each increment takes its set of technologies through the five-phase acquisition lifecycle.

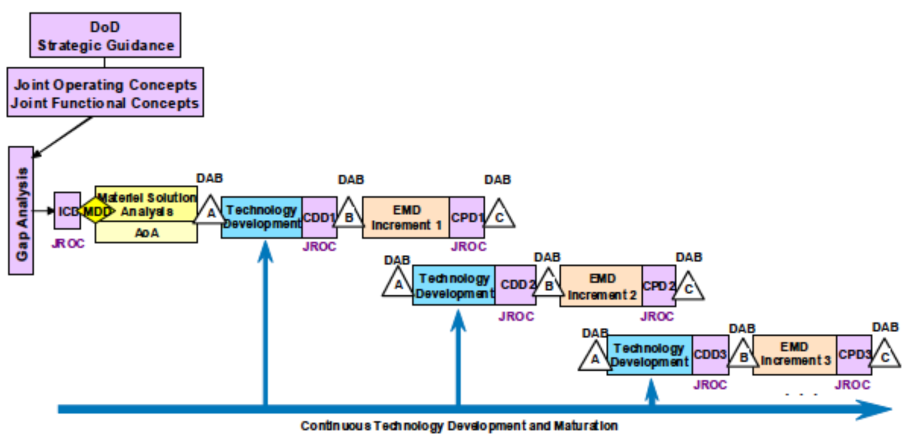


Figure 2-2 Requirements and Acquisition Process Flow [DoD I 5000.02]

Along with the evolution acquisition strategy, a knowledge-based approach is now embedded in law and policy [GAO, 2011]. A knowledge-based approach produces product knowledge at three key phases of the acquisition cycle; a high level of technology maturity at the start of development, design is stable by design review, and product can be manufactured within cost at the start of production [GAO, 2011]. Adding a reliable SRL value to this mix would inform the acquisition community of not only the critical technology maturity level but also how ready the overall system is to operate in the desired environment. This in turn would increase the confidence, the decision-making team has of meeting schedule and cost objectives.

2.3. Technology Readiness Level

The DoD Acquisition Lifecycle above discussed the technological risk. This section will review the development of the technology readiness levels (TRL), a systematic scale used to evaluate the maturity of the technology [Mankins, 1995; Dion-Schwarz, 2008]. The TRL was first developed and used by NASA in the 1980s as a seven-level scale to define the readiness of a technology for use in the space program [Azizian et al., 2011; Mankins, 2009; Garrett et al., 2011; Sauser et al., 2006; 2008; 2010]. John Mankins [1995] wrote a white paper in which he added definition and example for each of the now nine-level scale. The levels start at 1, a paper study, and advance to 9, a "successful mission operation" [Mankins, 2009:1221]. The DoD adopted and modified the TRLs [Azizian et al., 2011; Sauser et al., 2006; 2008]. These TRLs are ranking numbers, as the level of readiness increases so does the number. However, the level of effort to move between the levels is not equal; to move from TRL5 to TRL6 is a major effort, as noted in TRL6 definition; goes from testing in a simulated environment to a relevant environment. Table 2-1 and Table 2-2 are the current definitions and descriptions from the 2011 version of the DoD Technology Readiness Assessment (TRA) Guidance [DUSD(S&T),2011]. The DoD defines readiness levels for both software and hardware.

Table 2-1 DoD Hardware TRL Definition and Description

Level	TRL Definition	TRL Description
1	Basic principles observed and reported.	Lowest level of technology readiness. Scientific research begins to be translated into applied research and development (R&D). Examples might include paper studies of a technology's basic properties.
2	Technology concept and/or application formulated.	Invention begins. Once basic principles are observed, practical applications can be invented. Applications are speculative, and there may be no proof or detailed analysis to support the assumptions. Examples are limited to analytic studies.
3	Analytical and experimental critical function and/or characteristic proof of concept.	Active R&D is initiated. This includes analytical studies and laboratory studies to physically validate the analytical predictions of separate elements of the technology. Examples include components that are not yet integrated or representative.
4	Component and/or breadboard validation in laboratory environment.	Basic technological components are integrated to establish that they will work together. This is relatively "low fidelity" compared with the eventual system. Examples include integration of "ad hoc" hardware in the laboratory.
5	Component and/or breadboard validation in relevant environment.	Fidelity of breadboard technology increases significantly. The basic technological components are integrated with reasonably realistic supporting elements so they can be tested in a simulated environment. Examples include "high-fidelity" laboratory integration of components.
6	System/subsystem model demonstration in relevant environment.	Representative model or prototype system, which is well beyond that of TRL 5, is tested in a relevant environment. Represents a major step up in a technology's demonstrated readiness. Examples include testing a prototype in a high-fidelity laboratory environment or in a simulated operational environment.
7	System prototype demonstration in relevant environment.	Prototype near or at planned operational system. Represents a major step up from TRL 6 by requiring demonstration of an actual system prototype in an operational environment (e.g., in an aircraft, in a vehicle, or in space).
8	Actual system completed and qualified through test and demonstration.	Technology has been proven to work in its final form and under expected conditions. In almost all cases, this TRL represents the end of true system development. Examples include developmental test and evaluation (DT&E) of the system in its intended weapon system to determine if it meets design specifications.
9	Actual system proven through successful mission operations.	Actual application of the technology in its final form and under mission conditions, such as those encountered in operational test and evaluation (OT&E). Examples include using the system under operational mission conditions.

Table 2-2 DoD Software TRL Definition and Description

TRL	Definition	Description
1	Basic principles observed and reported.	Lowest level of software technology readiness. A new software domain is being investigated by the basic research community. This level extends to the development of basic use, basic properties of software architecture, mathematical formulations, and general algorithms.
2	Technology concept and/or application formulated.	Once basic principles are observed, practical applications can be invented. Applications are speculative, and there may be no proof or detailed analysis to support the assumptions. Examples are limited to analytic studies using synthetic data.
3	Analytical and experimental critical function and/or characteristic proof of concept	Active R&D is initiated. The level at which scientific feasibility is demonstrated through analytical and laboratory studies. This level extends to the development of limited functionality environments to validate critical properties and analytical predictions using non-integrated software components and partially representative data.
4	Module and/or subsystem validation in a laboratory environment, i.e. software prototype development environment	Basic software components are integrated to establish that they will work together. They are relatively primitive with regard to efficiency and robustness compared with the eventual system. Architecture development initiated to include interoperability, reliability, maintainability, extensibility, scalability, and security issues. Emulation with current/legacy elements as appropriate. Prototypes developed to demonstrate different aspects of eventual system.
5	Module and/or subsystem validation in a relevant environment	Level at which software technology is ready to start integration with existing systems. The prototype implementations conform to target environment/interfaces. Experiments with realistic problems. Simulated interfaces to existing systems. System software architecture established. Algorithms run on a processor(s) with characteristics expected in the operational environment.
6	Module and/or subsystem validation in a relevant end-to-end environment	Level at which the engineering feasibility of a software technology is demonstrated. This level extends to laboratory prototype implementations on full-scale realistic problems in which the software technology is partially integrated with existing hardware/software systems.
7	System prototype demonstration in an operational high fidelity environment	Level at which the program feasibility of a software technology is demonstrated. This level extends to operational environment prototype implementations, where critical technical risk functionality is available for demonstration and a test in which the software technology is well integrated with operational hardware/soft- ware systems.
8	Actual system completed and mission qualified through test and demonstration in an operational environment	Level at which a software technology is fully integrated with operational hardware and software systems. Software development documentation is complete. All functionality tested in simulated and operational scenarios.
9	Actual system proven through successful mission proven operational capabilities	Level at which a software technology is readily repeatable and reusable. The software based on the technology is fully integrated with operational hardware/ software systems. All software documentation verified. Successful operational experience. Sustaining software engineering sup-port in place. Actual system.

The TRLs are used in the DoD to assist program managers with decision-making in relationship to the risk of development and transition of a particular technology [Dion-Schwarz, 2008]. Reviewing Table 2-1 and Table 2-2 gives one the impression that assigning TRL to technology is very straightforward. Unfortunately it is not, TRL assignment relies on judgment call, qualified subject matter experts and knowledge of intended use [Dion-Schwarz, 2008; DUSD(S&T), 2011]. The correct TRL helps the acquisition team to assess the uncertainty of developing the critical technology. If a TRL is to high, schedule slips and cost overruns may occur; to low and technology could be overlooked as a possible alternative. The DoD, recognizing the difficulty in assessing technology and the importance of assigning the correct TRL, has developed the DoD Technology Readiness Assessment (TRA) Guide [DUSD(S&T), 2011; Malone and Wolfarth, 2012]. This guide lays out a process for evaluating technology and should be followed by an independent team. The first step is to establish a plan; this plan should identify the critical technology,

> A technology element is "critical" if the system being acquired depends on this technology element to meet operational requirements (within acceptable cost and schedule limits) and if the technology element or its application is either new or novel or in an area that poses major technological risk during detailed design or demonstration. [DUSD(S&T), 2011].

Once the program manager along with the system engineer has listed all the critical technology, a team of subject matter experts should be established. This team consists of members who not only are experts in the technology but also are independent of the program. This team reviews the program manager's list and makes additions or subtractions. This list is then evaluated for each technologies level of readiness. All previous planning, development or demonstration of technology should be examined and compared to the TRL tables, to determine the level of readiness. After collection of all available data, a TRA should be written by the program manager and reviewed by the acquisition management; for the DoD this would be the Program Executive Office and Component Acquisition Executive. The approved TRA should be delivered to the office of the Director of Defense Research and Engineering.

Bilbro [2006] states that determining the TRL of a technology is a straightforward process; determine the level of demonstration and one has the TRL. He says the problem is in the terminology, how judgment calls are conducted, and the team assembled to determine TRL. He contends that if a project manager assembles the correct team, defines the terminology used for that particular project, and develops a process for deciding on the judgment calls, then defining the TRL for each critical technology is just a matter of asking the correct questions [Bilbro, 2006]. Figure 2-3 is the flow chart associated with asking the correct questions, from the space industry's perspective, to determine the right TRL.

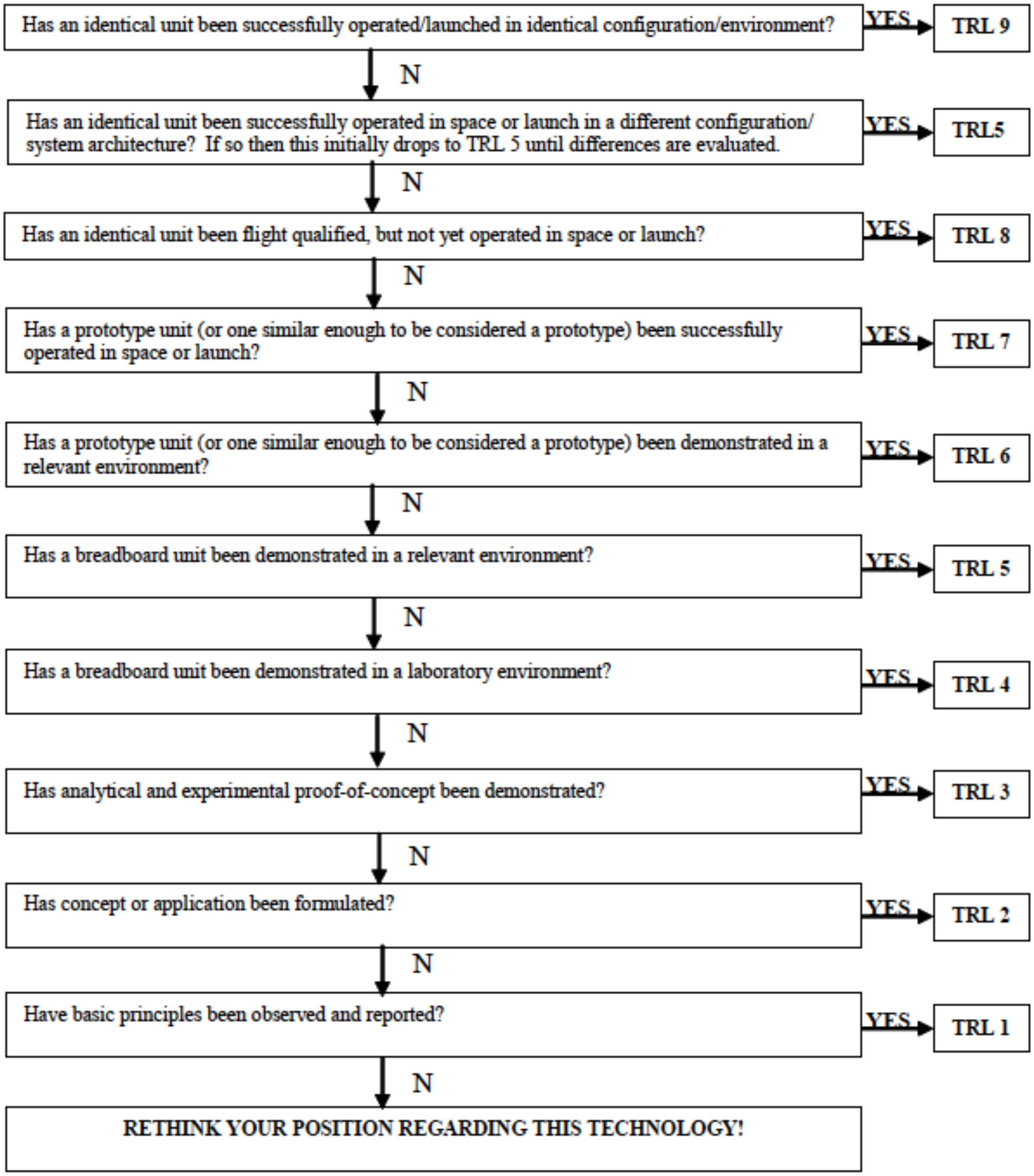


Figure 2-3 Flow Chart of Questions for TRL Determination (Bilbro, 2006)

2.4. Integration Readiness Level

TRLs are adequate for technology—defined by Webster's Dictionary as "the practical application of knowledge especially in a particular area; a capability given by the practical application of knowledge" [Webster Dictionary online accessed 2011;

Majumdar, 2007]. However, today's technologies are being used in systems and SoS [Azizian et al., 2011; Long, 2011; Sauser et al., 2006]. Maier and Rechtin [2002:23] define a system as "a collection of different things that together produce results unachievable by themselves alone."

The Office of the Deputy Under Secretary of Defense for Acquisition and Technology's *Systems Engineering Guide to System of Systems* [2008:3], defines systems as "a functionally, physically and/or behaviorally related groups of regularly interacting or independent elements; that group of elements forming a unified whole." This same document defines a SoS as "a set or arrangement of systems that results when independent and useful systems are integrated into a larger system that delivers unique capabilities" [2008:3].

All three definitions identify that there is an interaction or integration between technologies when technologies are formed into a system or SoS. It is this integration component that is missing from TRLs, requiring an additional measure of readiness [Long, 2011; Sauser et al., 2006; 2008; 2010]. TRLs were designed to assess the maturity of an individual technology [Azizian et al., 2011; Sauser et al., 2006]. Dr. Ashton Carter, former Under Secretary of Defense for Acquisition, Technology, and Logistics (USD (AT&L)) emphasized in his Memorandum for Acquisition Professionals [2010] that TRL's intent was to assess the "technologies maturity and risk, as opposed to engineering and integration risk." Malone and Wolfarth [2012] emphasizes that while the TRA Deskbook guides in the evaluation of technology it does not direct how to evaluate the readiness of the overall system nor the integration of technology into a system or system of system. Garrett et al., [2011] emphasized the importance of managing the integration of technology when inserting technology into systems or SoS by managing the interstitial space [Baron et al., 2011].

Many efforts have been conducted to address the shortcomings of TRLs. As systems increase in complexity the TRL is no longer sufficient for measuring the readiness of a system; they do not take into account the difficulty of integrating technologies into a system nor testing the integration [Azizian et al., 2009; Bilbro, 2006]. Mankins [1998] developed a Research and Development Degree of Difficulty ($R\&D^3$) metric. He proposed this metric to complement the original TRL and to capture the amount of effort needed to develop a technology through the various levels. Each technology is different and each level of readiness takes different effort in accomplishing. Mankins' definitions and description for each $R\&D^3$ have been assembled in Table 2-3.

In 2002, Mankins expanded this effort by introducing the Integration Technology Index (ITI) and developing a methodology to incorporate the uncertainty aspect of incorporating technologies into systems, the Integrated Technology Analysis Methodology. The ITI is a function of the difference between the current TRL and the desired TRL of a technology, the $R\&D^3$, and the technology need value [Mankins, 2002].

Table 2-3 Research and Development Degree of Difficulty (Mankins, 1998)

$R\&D^3$	Definition	Description	Probability of Success in "Normal" R&D effort
I	A very low degree of difficulty is anticipated in achieving research and development objectives for this technology.	Only a single, short-duration technological approach needed to be assured of a high probability of success in achieving technical objectives in later system applications.	99%
II	A moderate degree of difficulty should be anticipated in achieving R&D objectives for this technology.	A single technological approach will probably be sufficient; however, this R&D should be conducted early to allow an alternate approach to be pursued, if needed in, order to be assured of a high probability of success in achieving technical objectives in later systems applications.	90%
III	A high degree of difficulty anticipated in achieving R&D objectives for this technology.	At least two technological approaches will probably be needed and these efforts should be conducted early enough to allow an alternate subsystem approach to be pursued to be assured of a high probability of success in achieving technical objectives in later systems applications.	80%
IV	A very high degree of difficulty anticipated in achieving R&D objectives for this technology	Multiple technological approaches need to be pursued. These activities should be conducted early enough to allow an alternate system concept to be pursued in order to allow managers to be assured of a high probability of success in achieving technical objectives in later systems applications.	50%
V	The degree of difficulty anticipated in achieving R&D objectives for this technology is so high that a fundamental breakthrough is required	Basic research in key areas needed before feasible system concepts can be refined.	20%

Bilbro [2006] introduced a systems engineering approach to determine the effort required to advance the technology and modify Mankins $R\&D^3$ levels to reflect this level of effort. This approach called the Advancement Degree of Difficulty (AD^2) combines several processes such as Manufacturing Readiness Level, Software Readiness

Level and Integration Readiness Level and several others into one process. His modified Degree of Difficulty is presented in Table 2-4; they are an expansion of Mankins R&D[3] and are modeled after the TRL and IRL measurements [Bilbro, 2006].

Table 2-4 Advancement Degree of Difficulty (Bilbro, 2006)

Degree of Difficulty	Description
9	0% Developmental Risk-Exist with no or only minor modifications being required. A single development approach is adequate.
8	10% Development Risk-Exist but requires major modifications. A single development approach is adequate.
7	20% Development Risk-Requires new development well within the experience base. A single development approach is adequate.
6	30% Development Risk- Requires new development but similarity to existing experience is sufficient to warrant comparison across the board. A single development approach can be taken with a high degree of confidence for success.
5	40% Development Risk- Requires new development but similarity to existing experience is sufficient to warrant comparison in all critical areas. Dual development approaches should be pursued to provide a high degree of confidence for success.
4	50% Development Risk- Requires new development but similarity to existing experience is sufficient to warrant comparison in only a subset of critical areas. Dual development approaches should be pursued in order to achieve a moderate degree of confidence for success. (Desired performance can be achieved in subsequent block upgrades with high degree of confidence.)
3	60% Development Risk- Requires new development but similarity to existing experience is sufficient to warrant comparison in only a subset of critical areas. Multiple development routes must be pursued.
2	80% Development Risk- Requires new development where similarity to existing experience base can be defined only in the broadest sense. Multiple development routes must be pursued.
1	100% Development Risk-Requires new development outside of any existing experience base. No viable approaches exist that can be pursued with any degree of confidence. Basic research in key areas needed before feasible approaches can be defined.

Others have developed automated methods for adopting or modifying the TRL metric for systems and SoS. Azizian et al [2009] discusses the various methods and presents a strength, weakness, opportunity and threat (SWOT) analysis on the methods available as of 2009.

Ms. Jennifer Long, Naval Air Systems Command, [2011] contends that all systems will eventually need new technology, configuration item (CI), inserted into the baseline; whether because of old technology becoming obsolete or scheduled upgrades. She defines an integration readiness level, IRL, to capture the integration characteristic of the configuration item being inserted into the system. She states that the maturity of the technology is not the concern; the concern is how hard is it to insert the new technology. What is involved with the interfaces and how much work needs to be done for the system or SoS to accept this new or updated subsystem? She rates this work both for software and for hardware; software looks at the amount of code to be developed and the experience of the supplier with this type of software development, hardware looks at the experience the supplier has with the particular CI and the application. This IRL is modeled after the TRL; each IRL has a level associated with it, 1-10, and a risk level, low medium high. The definition and risk level are presented in Table 2-5 and Table 2-6.

Table 2-5 Software IRL as defined by Long (2011)

Risk Level	IRL	Definition	Characteristics
High	1	Configuration item (CI) is a software-intensive system with no previous baseline to build on. The interface with the host system is not defined.	The CI is a software-intensive system, no re-use of code is planned, the supplier has little or no experience with the current application, and interfaces with the host system are not defined.
High	2	CI is a software-intensive system with some re-use possible, and interface requirements are not defined.	The CI is a software-intensive system, some re-use of code is planned, the supplier has experience with a similar application, and interfaces with the host system are not defined.
High	3	CI is a software-intensive system with a high degree of re-use possible, but interface requirements are incomplete.	The CI is a software-intensive system, a high degree of re-use of code is planned, the supplier has experience with a similar application, and interface documentation is in work.
High	4	CI is a software-intensive system with a high degree of re-use possible, and interface requirements are defined.	The CI is a software-intensive system, a high degree of re-use of code is planned, the supplier has experience with a similar application, and interface documentation is base lined.
High	5	CI software is mature, but needs to be ported to new hardware architecture.	The CI is being updated to comply with new hardware architecture, but the basic functionality of the software is unchanged. A high degree of integration testing will be required, and the interface with the host system needs to be redesigned.
High	6	CI software is mature, but needs to work with modified hardware architecture.	The CI is being updated to comply with modified hardware architecture, and a significant degree of integration testing will be required. The interface with the host system needs significant changes.
Medium	7	CI software is mature, but changes are needed for the modified hardware architecture.	The CI is being updated to slightly modified hardware architecture, and a moderate degree of integration testing will be required. The interface with the host system needs minor changes.
Medium	8	CI software is mature and compatible with the modified hardware architecture.	The CI is being updated to slightly modified hardware architecture, and only minor integration testing will be required. The interface with the host system will work "as is".
Low	9	CI software and hardware work "as is" in a very similar system.	The CI software does not require any changes, and the interface with the host system will work "as is".
Low	10	CI software is embedded, and the system is fielded.	The CI contains only embedded software that is already integrated. There is no software interface with the host system.

Table 2-6 Hardware IRL as defined by Long (2011)

Risk Level	IRL	Definition	Characteristics
High	1	Supplier has access to the necessary technology.	The CI supplier owns or has ready access to the necessary technology, but no experience with building or testing items.
High	2	Supplier has some experience with the technology for different applications.	The CI supplier has developed items that are not similar to the current CI, for programs with different applications.
High	3	Supplier has developed similar items for different applications.	The CI supplier has developed similar CIs for programs with different applications.
High	4	Supplier has developed a similar item for a related application.	The CI supplier has developed a similar item that has been used in a program with a related application, but major changes are needed. (Example: all circuit cards need to be re-spun)
High	5	Supplier has developed a vey similar item for a closely related application.	The CI supplier has developed a very similar item that has been used in a program with a closely related application, such that only moderate changes are needed. (Example: multiple components, other than CCAs, need to be changed).
High	6	Supplier has developed a vey similar item for a very similar application.	The CI supplier has developed a very similar item that has been used in a program with a very similar application, and only minor changes are needed. (Example: only a few components need to be changed, mostly for obsolescence rather than performance).
Medium	7	Supplier has developed a representative prototype.	The CI supplier has already developed a prototype of the CI, but it hasn't undergone any electromagnetic or environmental testing.
Medium	8	Supplier has developed a representative prototype with some testing.	The CI supplier has already developed a prototype of the CI, and it has passed testing to environments that are expected to cause the highest performance risk.
Low	9	Supplier is using the CI in a similar system.	The CI supplier has already integrated the item into another system with similar requirements, and that system is in test with some positive results.
Low	10	Supplier is providing the CI to a fielded system.	The item is integrated into a mature system with similar requirements.

Sauser et al [2010:21], discuss and compare several "measurements that can be used to evaluate integration," but argue that none measure the "integration maturity".

Further, Sauser et al [2006:2] state "a comprehensive set of concerns becomes relevant when TRL is abstracted from the level of an individual technology to a system context." Sauser et al [2010] developed four requirements for a metric for measuring the integration, Integration Maturity Metric (IMM), these requirements are:

1. IMM shall provide an integration specific metric, to determine the integration maturity between two configuration items, components, and/or subsystems.
2. IMM shall provide a means to reduce the risk involved in maturing and integrating a technology into a system.
3. IMM shall provide the ability to consider the meeting of system requirements in the integration assessment so as to reduce the integration of obsolete technology over less mature technology.
4. IMM shall provide a common platform for both new system development and technology insertion maturity assessment. [Sauser et al, 2010, pg 20]

Sauser et al [2006] introduce the concept of a seven-level IRL and then expand on their original IRL ratings to a nine-level rating scale in 2010, to ensure the IRL satisfies all four requirements discussed above [Sauser et al., 2010]. These definitions of IRL are presented in Table 2-7 [Sauser et al., 2010].

Table 2-7 IRL Values, Definitions and Descriptions [Sauser et al., 2010]

Level	IRL Definition	IRL Description
1	An interface between technologies has been identified with sufficient detail to allow characterization of the relationship.	This is the lowest level of integration readiness and describes the selection of a medium for integration
2	There is some level of specificity to characterize the interaction between technologies through their interface.	Once a medium has been defined, a “signaling” method must be selected such that two integrating technologies are able to influence each other over that medium. Since IRL 2 represents the ability of two technologies to influence each other over a given medium, this represents integration proof-of-concept.
3	There is compatibility between technologies to orderly and efficiently integrate and interact.	There is compatibility (i.e. common language) between technologies to orderly and efficiently integrate and interact.
4	There is sufficient detail in the quality and assurance of the integration between technologies.	Many technology integration failures never progress past IRL 3, due to the assumption that if two technologies can exchange information successfully, then they are fully integrated. IRL 4 goes beyond simple data exchange and requires that the data sent is the data received and there exists a mechanism for checking it.
5	There is sufficient control between technologies necessary to establish, manage, and terminate the integration.	IRL 5 simply denotes the ability of one or more of the integrating technologies to control the integration itself; this includes establishing, maintaining, and terminating.
6	The integrating technologies can accept, translate, and structure information for its intended application.	IRL 6 is the highest technical level to be achieved, it includes the ability to not only control integration, but also specify what information to exchange, unit labels to specify what the information is, and the ability to translate from a foreign data structure to a local one.
7	The integration of technologies has been verified and validated with sufficient detail to be actionable.	IRL 7 represents a significant step beyond IRL 6; the integration has to work from a technical perspective, but also from a requirements perspective. IRL 7 represents the integration meeting requirements such as performance, throughput, and reliability.
8	Actual integration completed and mission qualified through test and demonstration in the system environment.	Mission qualified through test and demonstration, in the system environment. IRL 8 represents not only the integration meeting requirements, but also a system-level demonstration in the relevant environment. This will reveal any unknown bugs/defect that could not be discovered until the interaction of the two integrating technologies was observed in the system environment.
9	Integration is mission proven through successful mission operations.	IRL 9 represents the integrated technologies being used in the system environment successfully. In order for a technology to move to TRL 9 it must first be integrated into the system, and then proven in the relevant environment, so attempting to move to IRL 9 also implies maturing the component technology to TRL 9.

2.5. System Readiness Level

Having a TRL and IRL for each ‘critical’ subsystem, as defined by TRA Handbook [2009], in a system or SoS is not sufficient to define the readiness of a system [Garrett et al., 2011; Majumdar, 2007; Sauser et al., 2006; 2008, 2011]. Sauser et al [2006; 2008] defined the SRL as a function of the TRL and IRL of the critical subsystems that make up the system. The IRL mentioned here is the IRL first introduced in Sauser et al [2006] and later expanded in Sauser et al [2010]. A conceptual diagram of the SRL is shown in Figure 2-4. This diagram denotes each critical subsystem as circles and their integration as lines. The numbers inside the circles are the TRL for that subsystem and the numbers in boxes are the IRL for the integration between two subsystems. The outer oval represents the technology and integration captured in the overall SRL [Sauser et al, 2006]. In 2011, Sauser et al [2011:9] defines SRL as “a descriptive model that characterizes the effects of technology and integration maturity on a systems engineering effort, particularly with respect to integrating discrete functional systems into a coherent mission capability.” In order to distinguish the overall system’s SRL from the individual subsystem’s SRL, Sauser et al [2011] introduced the term $ITRL_i$, shown in Figure 2-4 as the small oval surrounding one TRL and two IRLs, which defines the technology and integration readiness level of technology *i* with respect to all other technologies.

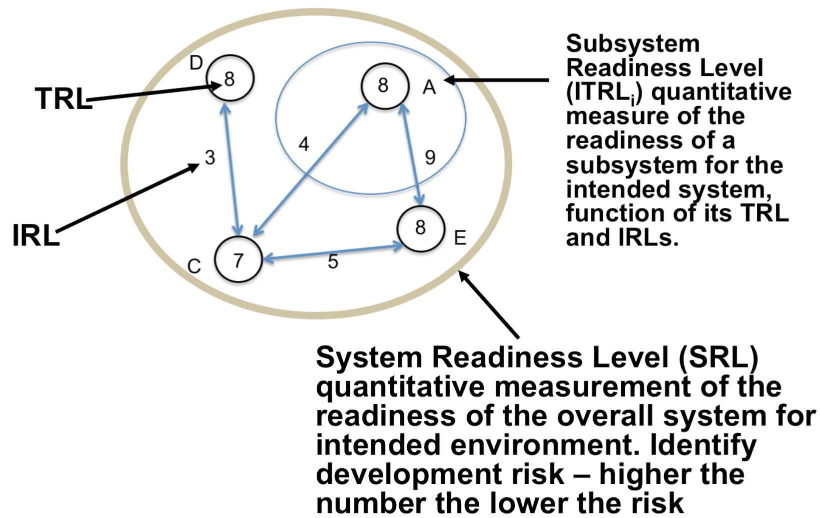


Figure 2-4 SRL Concept (from Sauser et al, 2008)

Table 2-8 presents the SRL definitions and their value as defined by Sauser et al [2011].

Table 2-8 SRL Values, Definition and Acquisition Phases [Sauser et al., 2011]

SRL Value	SRL	Acquisition Phase
0.10 to 0.19	Refine initial concept. Develop system/technology development strategy	Materiel Solution Analysis
0.20 to 0.49	Reduce technology risks and determine appropriate set of technologies to integrate into a full system.	Technology Development
0.50 to 0.79	Develop a system or increment of capability; reduce integration and manufacturing risk; ensure operational supportability; reduce logistics footprint; implement human systems integration; design for producibility; ensure affordability and protection of critical program information; and demonstrate system integration, interoperability, safety, and utility.	Engineering and Manufacturing Development
0.80 to 0.89	Achieve operational capability that satisfies mission needs	Production and Development
0.90 to 1.00	Execute a support program that meets operational support performance requirements and sustains the system in the most cost-effective manor over its total life cycle.	Operations and Support

Figure 2-5 graphically maps the individual subsystems $ITRL_i$ and the system or SoS SRL to the DoD acquisition cycle. Sauser et al [2011] emphasize that the SRL/$ITRL_i$ value at the various milestones are not concrete; there should be a ±10% tolerance. This mapping of SRL to DoD acquisition life cycle phases is at the preliminary stage of research; research is continuing to verify and validate this mapping [Ramirez and Sauser, 2009; Sauser et al., 2006; 2008; 2010; 2011]

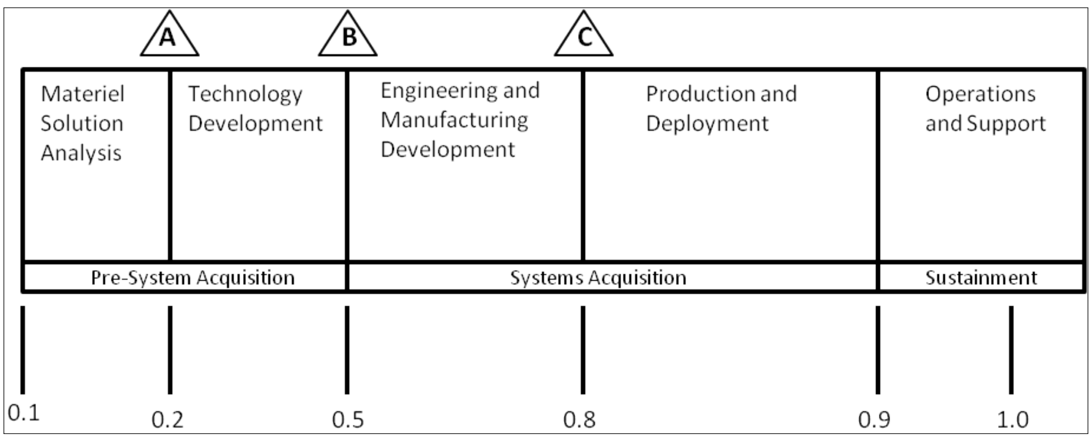


Figure 2-5 Mapping of ITRL and SRL to DoD Acquisition Cycle (Sauser et al, 2011)

The Sauser et al SRL, henceforth referred to as the S-SRL model in this dissertation, model uses matrix algebra [Sauser, 2006]. The S-SRL ITRL is the product of the IRL (nxn) matrix and the TRL (nx1) matrix. This ITRL results in a vector in which each subsystem $ITRL_i$, "quantifies the readiness level of a specific technology with respect to every other technology in the system" [Ramirez-Marquez and Sauser, 2009:536]. The SRL is the average of the ITRL elements. A sample calculation using S-SRL model will be presented in Chapter 5 of this dissertation. For a complete explanation of S-SRL calculations the reader is directed to Sauser et al [2008].

The use of SRLs has been expanding over the years by program managers to help with cost, scheduling and to prioritize resources [Baron et al., 2011; Malone and Wolfarth, 2012; Tan et al., 2010]. Baron et al [2011] present using SRL in conjunction with an assessment framework to determine which subsystems are best used against certain threats. Tan et al [2010] propose developing an importance measurement (IM) to determine which technology or integration will affect the SRL value the most on three levels: capability, function and system. They plan to future their research to incorporate the IM to assist program managers throughout the system development cycle. Malone and Wolfarth [2012] are developing a cost model using SRL, AD^2, McCabe's cyclomatic complexity number and graph theory.

2.6. Graph Theory

Garrett et al [2011] proposes that systems and SoS are increasing in complexity. To capture this complexity and to improve the S-SRL model they proposed using graph theory as a method for representing systems and SoS. Graph theory, a well-developed area of mathematics used for discrete problems, is useful for complex systems and for understanding the interaction between the subsystems (Garrett et al., 2011). To use graph theory a system in question is depicted as a series of circles connected by rays, see Figure 2-6. This is similar to S-SRL model depiction, except with GT SRL model there are certain rules, associated with graph theory, that are applied to the circles and lines.

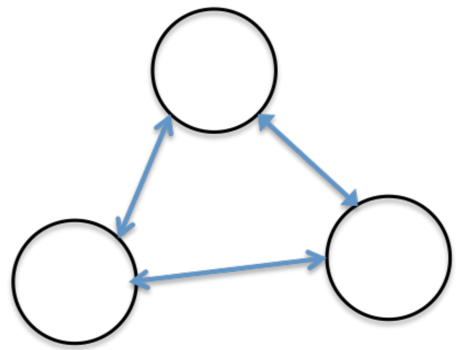


Figure 2-6 A Simple System Represented as a Graph

The circles represent the subsystems and are referred to as nodes. The nodes form a vector. The rays represent the integration between the nodes and are know as edges in graph theory. If the edges are bidirectional then the graph is known as a symmetrical graph. If at least one of the edges are unidirectional, i.e. the information from one subsystem (node) to another subsystem (node) flows only in one direction, then the graph is a directed graph. Graphs have an adjacency matrix associated with them. This matrix is symmetrical if the graph is undirected and non-symmetrical if the graph is directed. The adjacency matrix is formed from elements denoted as a_{ij}, where each a_{ij} is equal to one if there is a path, i.e. information flow, between node i and node j and zero otherwise. Paths are useful in analysis of SoS; path analysis can be used for example to compute the amount of bandwidth available within a system [Baron et al., 2011]. An example of an adjacency matrix is shown in Figure 2-7. The 1 in the first row, second column and in the second row, first column indicates that there is an edge between A and B and that information flows back and forth from A to B. The one in the third row, second column indicates that there is an edge between C and B but that information only flows from C to B since there is a zero in the second row, third column. For a complete explanation of graph theory, the reader is directed to Graduate Texts in Mathematics: Graph Theory by J. A. Bondy and U. S. R. Murty.

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{ccc} A & B & C \\ \end{array}$$

$$\begin{array}{c|ccc|} & A & B & C \\ A & 0 & 1 & 0 \\ B & 1 & 0 & 0 \\ C & 0 & 1 & 0 \end{array}$$

Figure 2-7 Sample Adjacency Matrix

Garrett et al [2011] uses graph theory to analyze a SoS. The functions performed by military systems are referred to as mission threads [Baron et al., 2011; Garrett et al, 2011]. Baron et al [2011:3] states 'Mission thread is a sequence of successive warfighting functions collectively executed to achieve a desired warfighting outcome'. A mission thread might be made up of several capabilities. For example a military mission thread to defend an asset might have a capability that consist of detecting a target, tracking the target, engaging the target and then assessing the engagement of the target. This capability is defined as a kill chain [Baron et al., 2011; Garrett et al., 2011].

Garrett et al [2011] uses graph theory and matrix algebra, applied to a kill chain, to calculate the SRL of a system used within a mission thread to "relatively compare the maturity (risk) of multiple capability threads the system is expected to perform" [Garrett et al., 2011:101]. Baron et al [2011] further developed this idea creating what this dissertation refers to as the Graph Theory (GT) SRL model. The GT SRL model relates the IRL matrix, from the S-SRL model, with a weighted adjacency matrix obtained from the graph of a system. The GT SRL model populates the graph with information from the system architecture, flow of information, resulting in a directed graph. Each a_{ij}

within the adjacency matrix, is weighted by the IRL value of the integration between each subsystem. The nodes are weighted by the TRL values of the subsystems [Garrett et al., 2011; Baron et al., 2011]. A sample calculation using GT SRL model will be presented in Chapter 5 of this dissertation. A complete explanation of GT SRL calculations can be found in Baron et al [2011].

2.7. Tropical Algebra

The previous two sections discuss the current methods for calcuclating SRL. Both these methods rely on multiplying ordinal numbers. According to Bowles [2003] the product of ordinal numbers is meaningless and can result in misleading results. Kujawski in his presentation at the National Defense and Industry Associations annual System Engineering conference in October 2010 cautioned the system engineering community about using the product of ordinal numbers as a decision-making tool.

Another problem with the current methods is that the product of qualitative rating systems, which both TRL and IRL are, will always have at least one set of outputs that will rate two systems in the wrong order [Cox et al., 2005]. Cox et al [2005] proved the following Theorem:

> Theorem 1: No direct qualitative rating system satisfying monotonicity is sound for arbitrary quantitative risk functions, or even for those functions of greatest practical interest, such as the product function $r_p(x)=x_1x_2...x_n$ = product of components of x. In other words, given any direct qualitative rating system f(x) and a quantitative risk function such as $r_p(x)$, it is always possible to choose two points, say x and w, such that x is assigned a higher qualitative risk rating than w, even though x has a lower quantitative risk than w.

Since SRL is the product of two qualitative ratings, TRL and IRL, it is possible for the calculated SRL value for one system, A, to be lower than another SRL value for another system, B, even though the system A is actually more mature than system B, i.e. system A's TRL and IRLs are higher rated. This could lead to misallocation of resources or ineffective trade studies.

Garrett et al [2011] and Baron et al [2011] mention that research is still needed in finding the correct mathematics for evaluating the maturity of SoS and in light of the complication of possible misleading results from the product of two ordinal numbers, tropical algebra, a mathematical area gaining popularity since the early 1990's, was an area this dissertation researched as a possible function for relating the IRL and TRL, to calculate the SRL. Tropical algebra was attractive because it combines numbers using addition and the minimum value rather than multiplication. Addition of ordinal numbers is more meaningful than the product [Bowles, 2003]. Also taking the minimum of ranking numbers is logical. Tropical algebra, also know as the min-plus algebra, gets its name from french mathematicians who wanted to honor the Brazilian mathematician, Imre Simon, who pioneered the min-plus algebra [Maclagan and Sturmfels, 2009; Mikhali, 2006]. Tropical algebra defines the mathematical operation 'product of a and b' as the sum of a and b and the 'sum of a and b' as the minimum value of a or b, see equations 2.1 and 2.2.

$$\mathrm{a} \otimes \mathrm{b} = \mathrm{a} + \mathrm{b} \tag{2.1}$$

$$\mathrm{a} \oplus \mathrm{b} = min(\mathrm{a}, \mathrm{b}) \tag{2.2}$$

Sauser et al. [2011:20] state "This algebra is an attractive tool for computing SRL when a fundamental promise is that a system cannot be 'more ready' than the 'less ready' of its subsystems." Tropical algebra will be compared to the desired properties, in section 5, because some of the literature suggest that the SRL should not be higher than the lowest subsystem’s technology and integration readiness level [(DUSD(S&T)), 2009; Engle et al., 2009; Kujawski, 2010; Sauser et al., 2011].

Chapter 3 CONTRIBUTION

The primary contribution of this research to the advancement of systems engineering is applying the mathematical rigor to the newly developing SRL. SRL are being used by system engineers and program managers to make resource management decisions. They are used to quantitatively define the program risk and to relate the system to the DoD acquisition lifecycle [Sauser et al, 2011]. It is important that the decision-making team understands the mathematics behind and the mathematical limitations of SRLs . This dissertation developed mathematical properties that are associated with SRLs. This mathmatical look at the properties instills confidence in the decision-making team, when using SRL, to assist in acquisition decisions on systems or SoS. The mathematical properties presented here help to establish a mathematical framework for developing a readiness level for a system [McConkie et al, 2012]. The properties' definitions and a justification from the literature, for each property, are presented in the following paragraphs. The mathematical rigor behind these properties is presented in Chapter 5. The limitations of SRL is presented in Chapter 6.

A secondary contribution of this research is the introduction of a new SRL model using graph theory and tropical algebra, GTTA. This new model advances the systems engineering body of knowledge by incorporating two fields of mathematics to enhance a quantitative system engineering measurement. As systems engineers rely on measurements, preferably quantitative, for improving the efficientcy and effectiveness of systems; the mathematics need to be studied [Azizian et al., 2011; Sauser et al., 2010]. A complete description of this proposed model is presented in Chapter 5.

3.1. Property 1:

Closure Property: The *$ITRL_i$* and *SRL* of a system or SoS cannot exceed the maximum available value for TRL or IRL.

Since the SRL is a measure of the system or SoS readiness level and a function of TRL and IRL, it should fall within the same range as the TRL and IRLs [Sauser and Boardman, 2008]. This is the closure property.

3.2. Property 2: Increase TRL

Increasing TRL_i will not decrease the $ITRL_i$ and SRL values.

If the TRL value of one subsystem, t_i ,is increased without changing the IRL_{ij} defined between that subsystem i and any other subsystem j or any other TRL values, then the system readiness level for that subsystem ITRL and the overall system readiness level, SRL, needs to increase or remain the same. One of the possible uses for SRL models is to help in resource management. Program managers will use SRL models to analyze which sub-system or integration or combination of sub-sytem and integration to improve to gain the best payoff. The program manager expects that improving one sub-system or integration will either improve the overall system or SoS or have no effect on the overall system [Baron et al., 2011; Kujawski, 2010; Tan et al., 2010].

3.3. Property 3: New Technology

Introducing new technology with TRL grater than the average value of the subsystems' readiness levels will not decrease SRL of the system or SoS.

If a new technology with a TRL and IRL equal to or greater than the average TRL/IRL of the system is introduced, then the new SRL will not be less than the original SRL. Many systems or SoS will go through various design changes after their initial design. As these design changes come about and technologies are added to existing systems, whether because of new desired capabilities or technology obsolescence, if the new technology is at a sufficient readiness level, the overall system readiness level should not decrease [Long, 2011].

3.4. Property 4: Maximum $ITRL_i$

The ITRL of a component or subsystem cannot exceed the maximum TRL or IRL of that component or subsystem.

Property 4 does have some controversy around it because some would argue that a system or SoS is greater than its parts. This may be true for performance or capability, but not necessarily true for readiness levels, which measure how ready the system is to perform in the desired environment. A system may perform better than individual subsystems, but logically a system cannot have a greater readiness level than the maximum TRL or IRL of any subsystem [(DUSD(S&T)), 2009; Engle et al., 2009; Kujawski, 2010; Sauser et al., 2011].

3.5. Property 5: Constant Value

If all TRLs and IRLs of a system are equal to a constant, then the SRL should be equal to that constant.

It is logical that the overall system cannot be more ready than the maximum subsystem readiness, which is analogous to property 4, maximum $ITRL_i$. From this it is reasonable to constrain the SRL of a system in which every TRL and IRL equals a constant value z, to be equal to the value z.

These properties were developed from the intended use of SRL and are necessary for a valid and mathematically rigorous quantitative measurement used for decision-making. If these properties were met by the mathematical operations used to define the SRL value, then program managers, system engineers and the decision-making team would have increased confidence in using SRL as part of their toolbox. Research has not been done, before this dissertation, to determine if in fact the SRL calculations do meet these properties. The next chapter will describe the research methodology for making this determination.

Chapter 4 RESEARCH METHOD

A system analysis waterfall process as shown in Figure 4-1 was followed for the research phase of this dissertation. This five-step process starts with defining the problem and ends with evaluating the proposed solution.

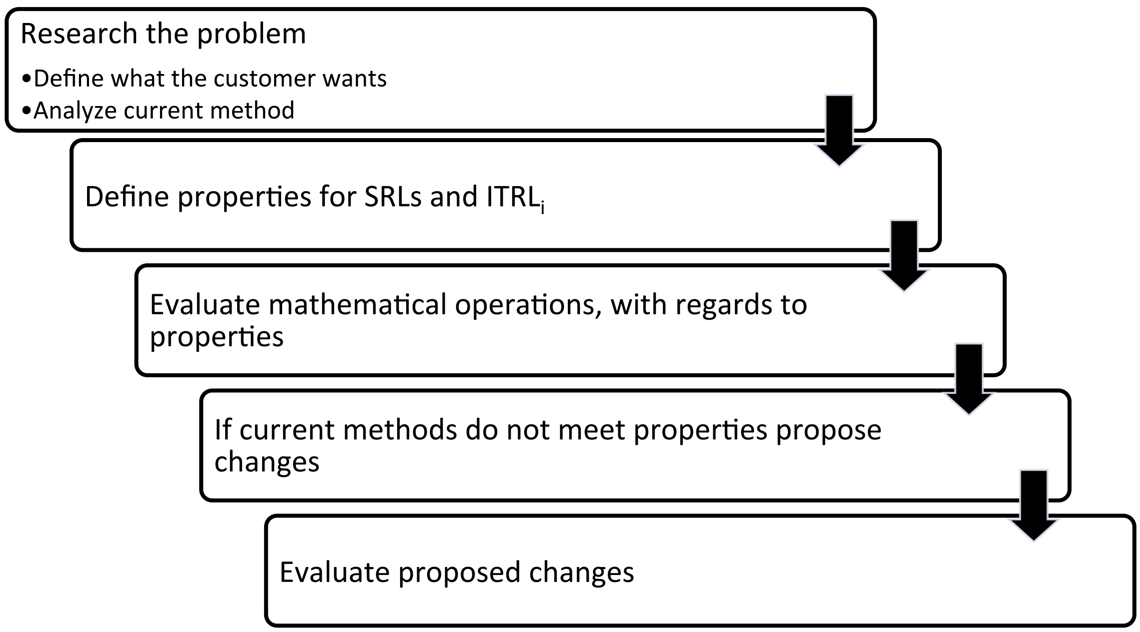


Figure 4-1 Modified Waterfall Method

4.1. First step: Research the problem

This step was accomplished by reviewing the literature. The literature review was conducted to determine the need for a system readiness level, SRL, the uses of the SRL and what potential problems exist with the current SRL method. This helped to define the customer's wants and to determine the mathematical requirements needed for SRL.

A simple system, based on a fire control system, introduce in Garrett et al [2011] and presented in Chapter 5, was constructed for use with MATLAB® and Excel® to determine all possible values of SRL for this fire control kill chain. Appendix A has the MATLAB® and Excel® programs used. The TRL for each technology was varied between 3-9 since TRL of less than 3 would not be technically developed enough to be integrated with another technology. The IRL was varied between 1-9. The definition of the IRL led to all values being valid for this analysis. The SRL for this simple system was calculated using the S-SRL model and GT SRL model. The SRL values were studied to determine whether the models produced holes in the range of SRL values, what combinations of TRL and IRL gave the same SRL result and whether the calculated values made logical sense in a system environment. The results are shown in Chapter 5 of this dissertation along with an explanation of how each model calculates SRL values.

4.2. The second step: Define the properties for SRLs and ITRLs

This step was accomplished by taking the customer's wants and the mathematical operations inherent properties to establish desired mathematical properties. These properties are logical and have mathematical rigor. They are described in Chapter 3 and mathematically defined in Chapter 5.

4.3. The third step: Evaluate mathematical operations, with regards to properties

The mathematics was reviewed to determine if the operations discussed in the literature, matrix algebra and tropical algebra, would meet the desired properties. For each property a mathematical analysis was conducted to determine if matrix algebra (product of ordinal numbers) and/or tropical algebra (min-plus of ordinal numbers) meet the desired properties. Also the two existing SRL models were evaluated to how well they met these properties. The current tools use matrix algebra but then introduce a weighting factor, which is designed to ensure the SRL tools meet at least one of these properties, the closure property. The results of this evaluation are presented in Chapter 5, Data Analysis.

4.4. The fourth step: If current methods do not meet properties propose changes

Evaluated the possibility of using graph theory and tropical algebra to calculate the SRL. Sauser et al [2011] discussed the possibility of using tropical algebra to define the function between TRL and IRL but opted to use matrix algebra. Kujawski [2010], Bowles [2003] and Cox et al [2005] caution about the using the product of ordinal numbers. Garrett et al [2011] introduces the benefits of evaluating SoS using Graph Theory. Developing a SRL model that combines graph theory and tropical algebra has the potential to give the system engineer and program manager a quantitative measurement for assessing the readiness level of the overall system with the mathematical rigor to increase the confidence of the decision-making team using SRL.

4.5. The fifth step: Evaluate the proposed change

This last step subjected the new model to the same sensitivity analysis performed in step 2. A discussion of how the new model meets all the desired properties and produces logical results is presented in Chapter 6.

Chapter 5 DATA ANALYSIS

This chapter presents the results of the research. The first section of this chapter describes how to use the two current SRL models and discusses the results of a sensitivity analysis performed. The next section, presents the mathematical evaluation of matrix algebra and tropical algebra to the defined properties. This is followed by a discussion on how the two current models use a scaling factor with matrix algebra to meet the desired properties and the problems this scaling factor introduces. The last section of this chapter discusses the new SRL model, giving a detailed explanation on how to calculate SRL using graph theory and tropical algebra and the results of the sensitivity analysis.

For this chapter a Fire Control Kill Chain example consisting of four subsytems that work together to produce the capability to detect a target, to track the target, engage the target and then assess the results of the engagement is introduced and used for all explanations and analysis. Figure 5-1 shows this simple fire control kill chain; this figure is a modification of a kill chain presented in Garrett et al [2011] denoting the Fire Control Loop. Each subsystem in the kill chain has a TRL and the interaction between the systems has an associated IRL. The TRL and IRL values would normally come from a team of subject matter experts who would evaluate each critical subsystem; however, for this dissertation they were assigned by the author. The detect system (D) could be a radar, the track system would be a control (C) system, the engage system (E) would be a weapon system and the assess system (A) could be a radar. Each system is represented by a circle with its TRL stated within the circle, t_i, and the interaction between the various systems represented by lines with the IRL stated on the line, a_{ij}.

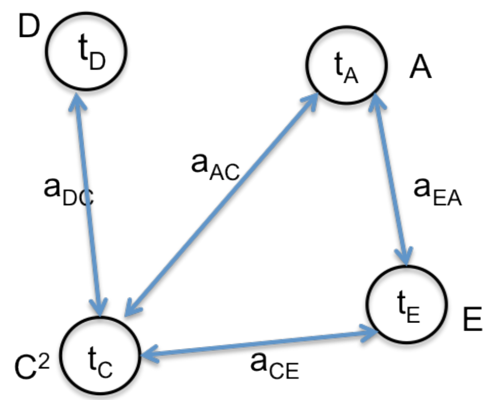


Figure 5-1 Fire Control Kill Chain

5.1. Calculating SRL with SysDML and GT

5.1.1. S-SRL Model

This section gives a brief description of how Sauser and his fellow co-authors calculate the SRL value for a system, referred to as the S-SRL model for this dissertation. The operations to develop the model and calculate SRL, within this section, are defined by Sauser et al [2006]. For a complete description of S-SRL calculations the reader is directed to Sauser et al [2006]. S-SRL uses matrix algebra. Figure 5-2 illustrates the fire control kill chain discussed above as seen from SysDML SRL model perspective. Sauser et al [2006] depict the lines as joint integration between any two subsystems with the integration going both directions and with the same IRL; the IRL values are shown next to the lines, e.g. IRL_{CE}=5. The circles again represent the various subsystems, and their TRL values are included within the circle, e.g. TRL_D=8. TRL are formed into a vector for the system (see Eq 5-1), and the IRL are arranged into a matrix (see Eq 5-2) per Sauser et al [2006]. The TRL vector and the IRL matrix are normalized, dividing by 9, to bring the values of TRL and IRL into the interval 0 to 1. Note the IRL matrix is a symmetrical matrix and that the integration of one subsystem with itself is defined as total by the S-SRL model; a normalized value of 1 is given to a_{ii} [Sauser et al., 2006].

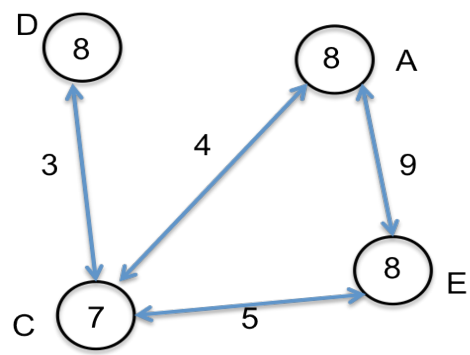


Figure 5-2 Fire Control Kill Chain from the S-SRL Perspective

$$TRL = \begin{bmatrix} t_D \\ t_C \\ t_E \\ t_A \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 8 \end{bmatrix}_{normalized} = \begin{bmatrix} .89 \\ .78 \\ .89 \\ .89 \end{bmatrix} \qquad \text{Eq 5- 1}$$

$$IRL = \begin{bmatrix} a_{DD} & a_{DC} & a_{DE} & a_{DA} \\ a_{CD} & a_{CC} & a_{CE} & a_{CA} \\ a_{ED} & a_{EC} & a_{EE} & a_{EA} \\ a_{AD} & a_{AC} & a_{AE} & a_{AA} \end{bmatrix} =$$

$$\begin{bmatrix} 9 & 3 & 0 & 0 \\ 3 & 9 & 5 & 4 \\ 0 & 5 & 9 & 9 \\ 0 & 4 & 9 & 9 \end{bmatrix}_{normalized} = \begin{bmatrix} 1 & .33 & 0 & 0 \\ .33 & 1 & .56 & .44 \\ 0 & .56 & 1 & 1 \\ 0 & .44 & 1 & 1 \end{bmatrix} \qquad \text{Eq 5- 2}$$

The subsystem's readiness level with respect to all other technologies, $ITRL_i$, is obtained by multiplying the normalized IRL matrix, Eq 5-2, by the normalized TRL vector, Eq 5-1. In order to have the resulting $ITRL_i$ be in the interval (0, 1), each $ITRL_i$ is divided by m_i a weighting factor equal to the number of integrations a subsystem has within the system or SoS, including the integration with itself [Sauser et al., 2006]. The resulting ITRL matrix is shown in Eq 5-3.

$$ITRL = \begin{bmatrix} 1 & .33 & 0 & 0 \\ .33 & 1 & .56 & .44 \\ 0 & .56 & 1 & 1 \\ 0 & .44 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} .89 \\ .78 \\ .89 \\ .89 \end{bmatrix} =$$

$$\begin{bmatrix} \dfrac{1*.89+.33*.78+0+0}{2} \\ \dfrac{.33*.89+1*.78+.56*.89+.44*.78}{4} \\ \dfrac{0+.56*.78+1*.89+1*.89}{3} \\ \dfrac{0+.44*.78+1*.89+1*.89}{3} \end{bmatrix} = \begin{bmatrix} .57 \\ .48 \\ .74 \\ .71 \end{bmatrix} \qquad \text{Eq 5- 3}$$

The overall system's SRL is the average of the elements in the ITRL vector (see Eq 5-4).

$$SRL = \frac{1}{n}\sum_{i=1}^{n} ITRL_i = \frac{1}{4}\sum_{i=1}^{4}\begin{bmatrix} .57 \\ .48 \\ .74 \\ .71 \end{bmatrix} = .63 \qquad \text{Eq 5- 4}$$

5.1.2. GT

The GT SRL model uses matrix algebra and graph theory to compute SRL values; graph theory is discussed in section 2.6. Figure 5-3 illustrates the fire control kill chain discussed above as seen from GT SRL model perspective; this is different from S-SRL model because of rules associated with graph theory. Graph theory has certain properties

1. Nodes, i.e. circles, represent the subsystems and if weighted form a nx1 vector.
2. Edges, i.e. lines, represent the integration between subsystems, have direction and form a nxn matrix in which the elements are the 1 or 0 depending on if there is and edge or not.
3. Because there is no edge between a subsystem and itself, a_{ii}=0 by definition not 1 as defined by S-SRL.

The GT SRL model weights the nodes with the TRL for each subsystem, see Eq 5-5, and weights the a_{ij} elements within the adjacency matrix with the IRL, for each integration, see Eq 5-6.

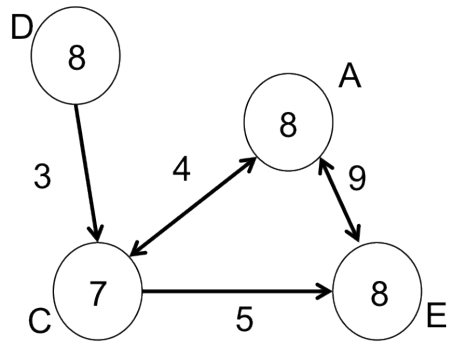


Figure 5-3 Fire Control Kill Chain from the GT Perspective

$$TRL = \begin{bmatrix} t_D \\ t_C \\ t_E \\ t_A \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 8 \end{bmatrix} normalized = \begin{bmatrix} .89 \\ .78 \\ .89 \\ .89 \end{bmatrix} \qquad \text{Eq 5- 5}$$

$$IRL = \begin{bmatrix} a_{DD} & a_{DC} & a_{DE} & a_{DA} \\ a_{CD} & a_{CC} & a_{CE} & a_{CA} \\ a_{ED} & a_{EC} & a_{EE} & a_{EA} \\ a_{AD} & a_{AC} & a_{AE} & a_{AA} \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 3 & 0 & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 9 \\ 0 & 4 & 0 & 0 \end{bmatrix} normalized = \begin{bmatrix} 0 & .33 & 0 & 0 \\ 0 & 0 & .56 & .44 \\ 0 & 0 & 0 & 1 \\ 0 & .44 & 1 & 0 \end{bmatrix} \qquad \text{Eq 5- 6}$$

The ITRL vector is the product of the IRL matrix with the TRL vector, as it is with S-SRL model, and is shown in Eq 5-7. Each $ITRL_i$ is divided by a scalar, m_i. This scalar is defined, similarly to Sauser et al [2006], as the number of integrations a subsystem i

has within the system; however, unlike Sauser et al [2006] it does not include an integration with itself.

$$ITRL = \begin{bmatrix} \dfrac{(a_{DD} \times t_D + a_{DC} \times t_C + a_{DE} \times t_E + a_{DA} \times t_A)}{m_D} \\ \dfrac{(a_{CD} \times t_D + a_{CC} \times t_C + a_{CE} \times t_E + a_{CA} \times t_A)}{m_C} \\ \dfrac{(a_{ED} \times t_D + a_{EC} \times t_C + a_{EE} \times t_E + a_{EA} \times t_A)}{m_E} \\ \dfrac{(a_{AD} \times t_D + a_{AC} \times t_C + a_{AE} \times t_E + a_{AA} \times t_A)}{m_A} \end{bmatrix} = \begin{bmatrix} \dfrac{(0(.89) + .33(.78) + 0(.89) + 0(.89))}{1} \\ \dfrac{(0(.89) + 0(.78) + .56(.89) + .44(.89))}{2} \\ \dfrac{(0(.89) + 0(.78) + 0(.89) + 1(.89))}{1} \\ \dfrac{(0(.89) + .44(.78) + 1(.89) + 0(.89))}{2} \end{bmatrix} = \begin{bmatrix} .26 \\ .45 \\ .89 \\ .64 \end{bmatrix}$$

Eq 5- 7

To calculate the overall system SRL the average of the ITRL$_i$ is found, see Eq 5-8, where n equals the number of subsystems.

$$SRL = \frac{1}{n} \sum_{i=1}^{n} ITRL_i = \frac{1}{4} \sum_{i=1}^{4} \begin{bmatrix} .26 \\ .45 \\ .89 \\ .64 \end{bmatrix} = .56$$

Eq 5- 8

5.2. MatLab® Calculations

5.2.1. All Possible SRL values

A MatLab® program, see appendix A, was written to calculate all the possible SRL values, for the fire control kill chain example, using both SysDML and GT models. The TRLs for the kill chain varied between 3 and 9 since TRL 3 is where active research and development begins and before that you don't have a system just a group of subsystems [Sauser et al, 2006]. The IRLs for the fire control kill chain varied between 1 and 9 since by definition integration starts at IRL1. These values are shown in Figure 5-4 and Figure 5-5. The x-axis is the possible SRL values mapped to the DoD acquisition lifecycle and the y-axis is the frequency of that SRL value. The S-SRL model is bell shaped with a mean of approximately 0.45; therefore most systems will be rated at pre-milestone B in the DoD acquisition lifecycle. The GT SRL model is more conservative than the S-SRL model and has a mean of approximately 0.375. Again most systems will fall in the pre-milestone B phase of the DoD acquisition lifecycle.

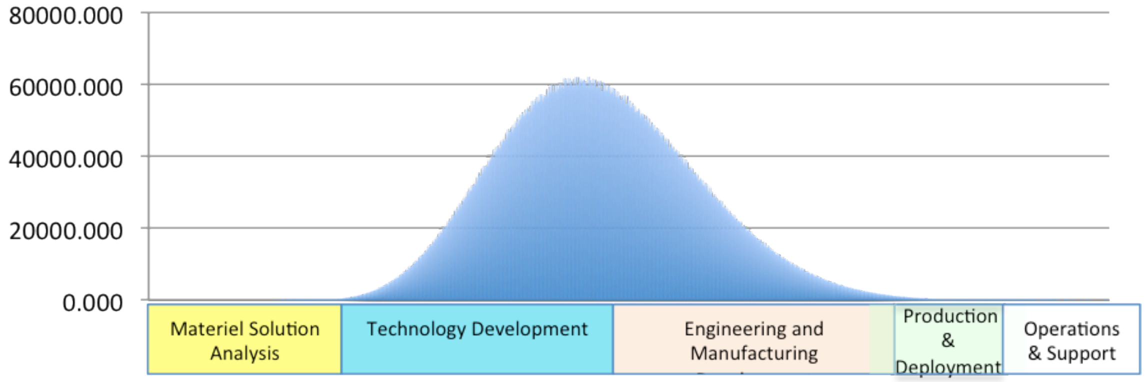


Figure 5-4 All Possible SRL for Fire Control Kill Chain with SysDML

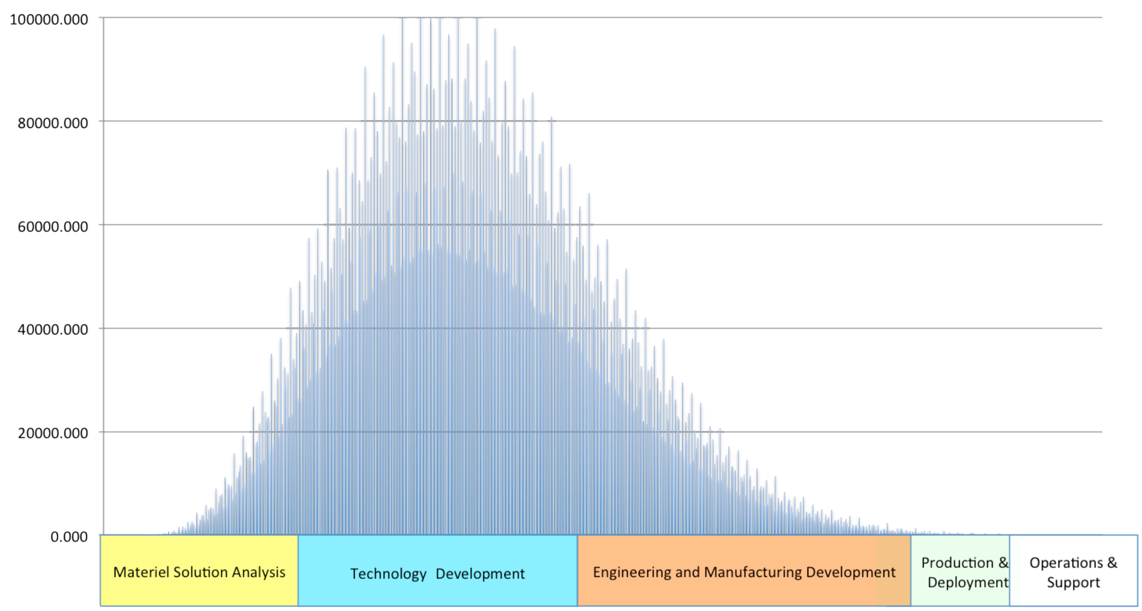


Figure 5-5 All Possible SRL for Fire Control Kill Chain with GT

5.2.2. SRL with constant TRL and IRL

Table 5-1 shows the calculated value of SRL, for both SysDML and GT, when all technologies and integrations are at the same level, varying between 3 and 9. The top row shows the constant value for both TRL and IRL and is the expected value for a SRL with these constant TRL and IRLs, i.e. property 5 discussed in Chapter 3. Figure 5-6 and Figure 5-7 show the mapping of the calculated SRLs for S-SRL and GT SRL models, respectively. These figures show that for S-SRL model as the TRL and IRL constant value increases the calculated SRL value approaches the expected SRL value and that the GT SRL model lacks behind more significantly from the expected value than the S-SRL model.

Table 5-1 SRL Values for Constant TRL and IRL

TRL and IRL Level	3 (0.333)*	4 (0.444)	5 (0.556)	6 (0.667)	7 (0.778)	8 (0.889)	9 (1.000)
GT SRL	0.111	0.197	0.309	0.444	0.605	0.790	1.000
SysDML SRL	0.189	0.285	0.396	0.523	0.666	0.825	1.000

*Numbers in parenthesis are the normalized value

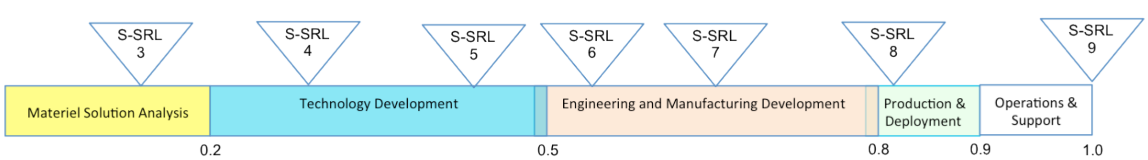


Figure 5-6 Mapping of SRL for Constant TRL and IRL for S-SRL

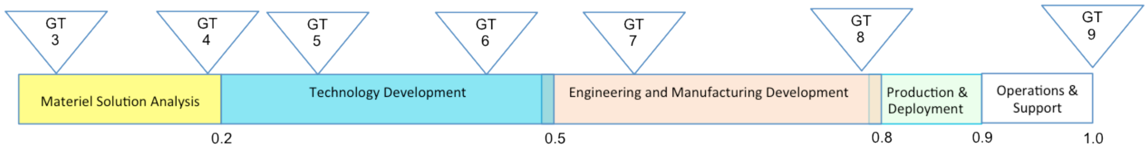


Figure 5-7 Mapping of SRL for Constant TRL and IRL for GT SRL Model

5.2.3. Sensitivity Analysis

A sensitivity analysis was performed to determine what, if any effect, increasing the TRL of a subsystem by one while leaving the remaining TRLs and IRLs the same would have on the ITRLs and the SRL. First TRL_D was increased by one and those results are shown in Table 5-1 column one, next TRL_D was reset to its original value and TRL_C was increased by one, those results are shown in Table 5-1 column two. This process was repeated until all the TRLs and IRLs were increased by one and results are shown in respective columns in Table 5-1. The percent change in the SRL for each change in TRL and IRL is plotted in Figure 5-8. As can be seen in Table 5-1 and Figure 5-8 the SRL as calculated by S-SRL is increased from 2.23% to 3.5% with increasing TRL_C and TRL_E having the most effect; the SRL as calculated by GT SRL model is increased from 0% to 7.5% with increasing IRL_E having the most effect.

Table 5-2 Sensitivity Analysis

	GT Percentage Increase When TRL and IRL are Increased by 1 Individually							
	TRL_D+1	TRL_C+1	TRL_E+1	TRL_A+1	IRL_D+1	IRL_C+1	IRL_E+1	IRL_A+1
$ITRL_D$	0.00%	14.29%	0.00%	0.00%	33.59%	0.00%	0.00%	0.00%
$ITRL_C$	0.00%	0.00%	12.55%	0.00%	0.00%	20.04%	0.00%	0.00%
$ITRL_E$	0.00%	0.00%	0.00%	12.59%	0.00%	0.00%	0.00%	0.00%
$ITRL_A$	0.00%	4.69%	8.29%	0.00%	0.00%	0.00%	0.00%	7.13%
SRL	0.00%	3.11%	5.35%	4.43%	4.43%	5.04%	7.54%	2.19%
	SysDML P Percentage Increase When TRL and IRL are Increased by 1 Individually							
$ITRL_D$	9.76%	3.31%	0.00%	0.00%	7.49%	0.00%	0.00%	0.00%
$ITRL_C$	1.83%	5.70%	3.05%	2.44%	4.89%	4.89%	0.00%	4.89%
$ITRL_E$	0.00%	2.98%	5.51%	4.32%	0.00%	4.32%	4.92%	0.00%
$ITRL_A$	0.00%	2.49%	4.52%	5.76%	0.00%	0.00%	5.14%	4.52%
SRL	2.73%	3.49%	3.41%	3.28%	2.82%	2.23%	2.78%	2.23%

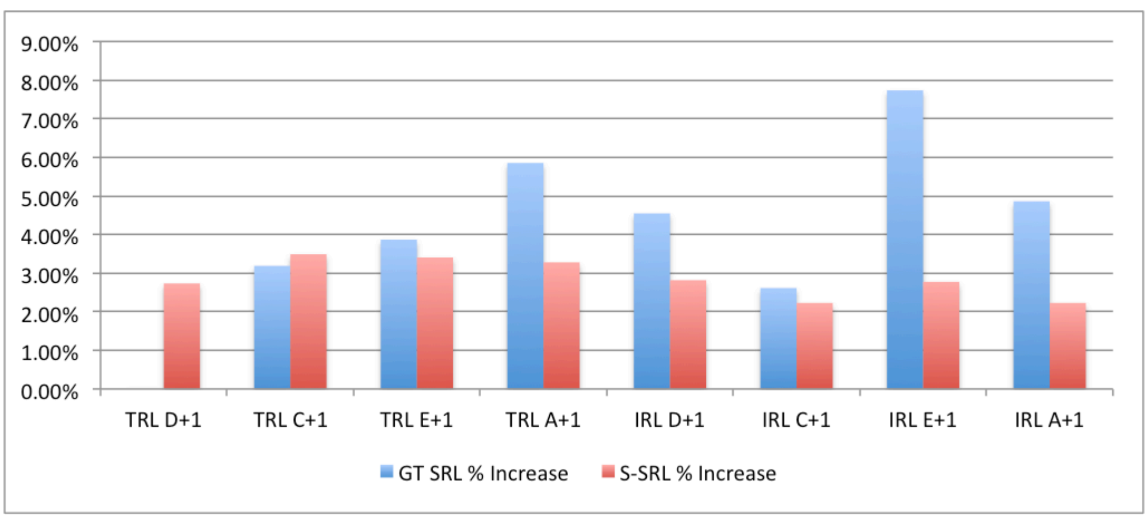


Figure 5-8 Percent Change in SRL with Increase in TRL and IRL

5.2.4. Added Interface

A connection between Assess and Detect was added to the Fire Control Kill Chain example to determine the effect of adding additional connections. The new interface was inserted at an IRL9. Figure 5-9 shows the original values of the $ITRL_i$ and SRL compared to the new values of $ITRL_i$ and SRL with the added interface. Adding this connection increases $ITRL_D$ by 18.3% and increases $ITRL_A$ by 9.61% for S-SRL and increases only $ITRL_D$ by 4.0% for GT. The SRL for both models were increased by 6.99% and 4.01% respectively. These results show that it is possible to game the SRL. An independent team of subject matter experts should be used to evaluate TRLs, IRLs and SRLs.

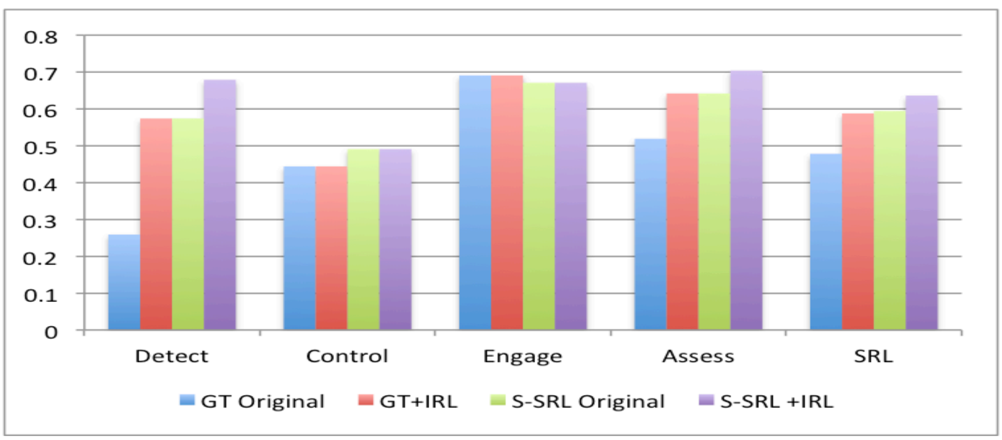


Figure 5-9 ITRL Values with and without IRL_{DA}

5.3. Evaluating mathematical operations in relationship to the desired properties.

This section looks at each property and presents the mathematical analysis to determine whether the property is met. Each property will be stated in words and then mathematically [McConkie et al., 2012]. The improtance and desire for each property is stated in Chapter 3 and will not be repeated here. A simple generic system, Figure 5-10 below, is used to illustrate the mathematics rather than the Fire Control Kill Chain example. This was done for simplicity and to show that the results hold for any system not just a specific kill chain.

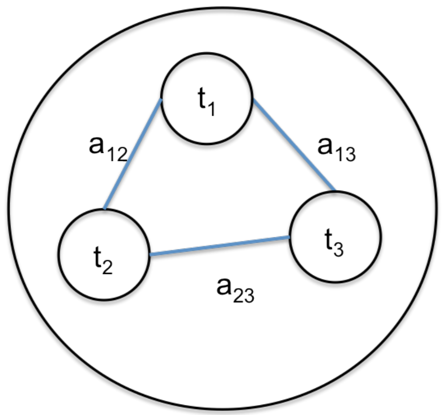


Figure 5-10 General Systems

Let

$$TRL = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} \quad \text{Eq 5- 9}$$

$$IRL = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{Eq 5- 10}$$

The SRL is calculated using either matrix algebra or tropical algebra by first calculating the subsystem's readiness level, $ITRL_i$. The $ITRL_i$ is formed from the product of IRL and TRL, resulting in a vector, ITRL. The ITRL is a function of the t_i and the a_{ij}. For brevity sake the function is represented in Eq 5-13 and Eq 5-14 using the symbols ⊕ and ⊗. For matrix algebra ⊕ equals the sum and ⊗equals the product and the SRL is the average of ITRL. For tropical algebra ⊕ equals the minimum and ⊗ equals the sum and the SRL is the min of ITRL.

$$ITRL = \oplus_{j=1}^{n}(a_{ij} \otimes t_i) \qquad \text{Eq 5- 11}$$

$$SRL = f(ITRL_i) = k(\oplus_{i=1}^{n} \oplus_{j=1}^{n} (a_{ij} \otimes t_i)) \qquad \text{Eq 5- 12}$$

5.3.1. Property 1

Closure Property: The *$ITRL_i$* and *SRL* of a system or SoS cannot exceed the maximum available value for TRL or IRL.

Specific Case

(i) Matrix algebra: $\oplus = sum, \otimes = product, k = \frac{1}{n}$ *where n=number of subsystems*

Matrix algebra does not meet this property. Using Eq 5-13 and Eq 5-14 and n=2;

$t_1=1,\ a_{11}=1, t_2=1, a_{12}=1, a_{22}=1,\ a_{21}=1 \Rightarrow ITRL_1 \geq 2\ and\ SRL = 1/2(2+2)=2$

(ii) Tropical algebra: *⊕=min, ⊗=sum, k=1, n=number of subsystems*

Tropical algebra meets this property under the following conditions:

$$ITRL_j = \min(a_{ij} + t_j) \leq 1 \qquad \text{Eq 5- 13}$$

$$SRL = \min_{i=1}^{n}(\min_{j=1}^{n}(a_{ij} + t_j)) \le 1 \qquad \text{Eq 5- 14}$$

a) For each i, there exists a j such that $a_{ij}=0$ (i.e. there is at least one subsystem that is connected to at least one other subsystem and not connected to at least one other subsystem in the system.)

b) For each i, there exists a j *such that* $a_{ij} \le 1 - t_j$ (i.e. there exist at least one subsystem with an IRL that is less than one minus the TRL.)

5.3.2. Property 2

Increasing one TRL_i without changing the other TRLs or IRLs will not decrease the $ITRL_i$ nor the SRL values.

Eq 5-17 and Eq 5-18 state this property mathematically and is used to show that matrix algebra and tropical algebra meets this property.

$$ITRL_i \rightarrow ITRL_i^{'} = ITRL_i \oplus (a_{ij} \otimes \Delta) \qquad \text{Eq 5- 15}$$

$$SRL \rightarrow SRL^{'} = k(\oplus_{i=1}^{n}(\oplus_{j=1}^{n}(a_{ij} \otimes t_j) \oplus (a_{ij} \otimes \Delta))) \qquad \text{Eq 5- 16}$$

Specific case:

(i) Matrix algebra: *$\oplus$=sum, $\otimes$=product, k=1/n where n=number of subsystems*

Matrix algebra meets this property. The new subsystem readiness level is the original readiness level plus the change in TRL times the IRL and the new SRL is equal to or greater than the original SRL as long as there is at least one interface within the system, $a_{ij} \neq o$. The definition of a system states that there has to be at least one interface so matrix algebra meets this property for all systems.

$$ITRL_j^{'} = ITRL_j + \Delta \times a_{ij}$$

and $SRL^{'} \geq SRL \leftrightarrow a_{ij} \neq 0$

(ii) Tropical algebra: $\oplus$=*min*, $\otimes$=*sum*, *k*=*1*, *n*=*number of subsystems*

Tropical algebra meets this property. Increasing a number will always be greater than or equal to the original number.

$$\begin{aligned} &ITRL_j^{'} \geq ITRL_j \\ &\min(t_1 + a_{1j}, \ldots t_i + \Delta + a_{ij}, \ldots t_n + a_{nj}) \geq \min(t_1 + a_{1j}, \ldots t_i + a_{ij}, \ldots t_n + a_{nj}) \end{aligned} \quad \text{Eq 5- 17}$$

5.3.3. Property 3

Introducing new technology with sufficient readiness levels, both the TRL and IRL, will not decrease the SRL of the system or SoS.

The mathematics is represented in Eq 5-20 through Eq 5-25, which shows that the new SRL is, for matrix algebra, the original SRL plus the product of the new $IRL_{(an+1,n+1)}$ and the new $TRL_{(tn+1\ )}$, Eq 5-24 and for tropical algebra, the original SRL, Eq 5-25.

$SRL' \geq SRL$ if new technology's TRL and IRL are ≥ SRL Eq 5- 18

$$ITRL_j = \oplus_{j=1}^{n}(a_{ij} \otimes t_j) \quad \text{Eq 5- 19}$$

$$SRL = f(ITRL_j) = \frac{1}{k}(\oplus_{i=1}^{n}(\oplus_{j=1}^{n}(a_{ij} \otimes t_j) \quad \text{Eq 5- 20}$$

$$SRL^{'} = \frac{1}{l}\oplus_{i=1}^{n+1}(\oplus_{j=1}^{n+1}(a_{ij} \otimes t_j)$$

$$= \frac{1}{l}(\oplus_{i=1}^{n+1}(\oplus_{j=1}^{n+1}(a_{ij} \otimes t_j) \oplus (a_{i,n+1} \otimes t_{n+1})))$$

$$= \frac{1}{l}(\oplus_{i=1}^{n}(\oplus_{j=1}^{n}(a_{ij} \otimes t_j) \oplus (a_{n+1,n+1} \otimes t_{n+1}))) \oplus (\oplus_{j=1}^{n}(a_{n+1,j} \otimes t_j))$$

$$= \frac{k}{l} SRL \oplus \frac{1}{l}(\oplus_{j=1}^{n}(a_{n+1,j} \otimes t_j) \oplus (a_{n+1,n+1} \otimes t_{n+1}))$$

$$= \frac{l}{l} SRL \oplus a_{n+1,n+1} \otimes t_{n+1} \qquad \text{Eq 5- 21}$$

Specific case:

(i) Matrix algebra: *⊕=sum, ⊗=product, k=1/n,l=n+1,where n=number of subsystems*

Matrix algebra meets this property.

$$SRL' \geq \frac{l}{l} SRL + a_{n+1,n+1} t_{n+1} \geq SRL \qquad \text{Eq 5- 22}$$

(ii) Tropical algebra: *⊕=min, ⊗=sum, k=1, l=1,n=number of subsystems*

Tropical algebra meets this property.

$$SRL' \geq \min(ITRL_i(a_{n+1,n+1} + t_{n+1}) \qquad \text{Eq 5- 23}$$

5.3.4. Property 4

The ITRL of a component or subsystem cannot exceed the maximum TRL or IRL of that component or subsystem.

Eq 5-26 and Eq 5-27 state this property in mathematical terms and are used to show that matrix algebra does not meet this property but tropical algebra does.

$$ITRL_i \le \max(TRL_i, IRL_{ij}) \qquad \text{Eq 5- 24}$$

$$ITRL_i = t_1 \otimes a_{i1} \oplus \ldots \oplus t_i \otimes a_{ij} \oplus \ldots \oplus t_n \otimes a_{in} \qquad \text{Eq 5- 25}$$

Specific Case:

(i) Matrix algebra: $\oplus = sum,\ \otimes = product,\ \ n = number\ of\ subsystems$

Matrix algebra does not meet this property.

$$t_1=2,\ a_{1j}=1, t_2=1, a_{ij}=1 \Longrightarrow ITRL_j \ge 3 \qquad \text{Eq 5- 26}$$

(ii) Tropical algebra: $\oplus = sum,\ \otimes = product,\ n = number\ of\ subsystems$

Tropical algebra meets this property for the following condition:

$$\min(t_i + a_{i1}, \ldots, t_i + a_{ij}, t_i + a_{in}) \le \max(t_i, a_{ij}) \qquad \text{Eq 5- 27}$$

a) For each i, there exists a j such that $a_{ij}=0$ i.e. there is at least one subsystem that is not connected to at least one other subsystem in the system.

5.3.5. **Property 5**

If all TRLs and IRLs of a system are equal to a constant, then the SRL should be equal to that constant.

$$f(cons) = cons \qquad \text{Eq 5- 28}$$

Specific Case:

(i) Matrix algebra: $\oplus = sum,\ \otimes = product,\ \ n = number\ of\ subsystems$

Matrix algebra does not meet this property.

$$t_1=2,\ a_{1j}=2, t_2=2, a_{ij}=2 \Longrightarrow SRL=2.44 \qquad \text{Eq 5- 29}$$

(ii) Tropical algebra: $\oplus = sum,\ \otimes = product,\ n = number\ of\ subsystems$

Tropical algebra meets this property for the following condition:

$$\min(t_i + a_{i1}, \ldots, t_i + a_{ij}, \ldots, t_n + a_{nn}) \geq \min(t_i, a_{ij}) \qquad \textbf{Eq 5- 30}$$

a) For each i, there exists a j such that $a_{ij}=0$

b) For each i, there exists a j such that $a_{ij} \geq t_i$

5.4. Results of Property Discussion

The math shows that matrix algebra only meets two of the five desired properties, property 2 and 3, see Table 5-3, and that tropical algebra meets all five desired properties, with conditions. Broadman and Sauser (2008) and Garrett et al (2011) used a scaling factor to ensure that their SRL calculations met the closure property, property 1. With this scaling factor their calculations also satisfies property 4 but not property 5.

Table 5-3 Summary of Properties Met

	Property 1- Closure Property	Property 2- Increasing TRL	Property 3- New technology	Property 4- ITRL not greater than TRL or IRL	Property 5- SRL equals constant TRL and IRL
Matrix Algebra		X	X		
With Scaling Factor	X	X	X	X	
Tropical Algebra	X	X	X	X	X

The scaling factor, m_i, used by both SysDML and GT SRL model, to bring the SRL value within the range of 0 to 1, possibly introduces biases. The effect of this scaling factor is that it levels the field with respect to the risk associated with the integration of a subsystem into a system. If a technology integrates with several subsystems with each interface at a different IRL then the scaling factor evenly distributes the integration influence across the subsystems, thus in essence lowering the high IRL and raising the low IRL. Also by using this scaling factor the subsystems readiness level with respect to the other subsystems with in the system, $ITRL_i$, takes into account the readiness of the interface twice, once through the IRL and then a second time by dividing the product of TRL and IRL by the number of subsystems a subsystem, i, interacts with. The more interaction a subsystem has within a system the more its readiness level with respect to other subsystems will be influence by this scaling factor. Using tropical algebra eliminates the need for this scaling factor.

5.5. New SRL model using Graph Theory and Tropical Algebra

This research proposes a third SRL model that builds on the other two models. This research combined the graph theory concept as presented by Garrett et al [2011] and Baron et al [2011] and tropical algebra to calculate the SRL because tropical algebra met all five desired properties.

This new model is defined as the graph theory tropical algebra (GTTA) model. Tropical algebra uses ⊗ and ⊕ operations. The ⊗ is defined as the algebraic sum of 2 or more numbers and ⊕ is the minimal value of the expression. For example $2 \otimes 5=2+5=7$ and $2 \oplus 5=\min(2,5)=2$.

Using the same example presented for the GT model $ITRL_i$ and SRL are calculated. Again referring to Figure 5-3**,** the normalized TRL and IRL are shown in Eq 5-5 and Eq 5-6.

The $ITRL_i$ is calculated using the ⊗ and ⊕ operations to obtain Eq 5-33 and Eq 5-34.

$$ITRL_i = a_{i1} \otimes t_1 \oplus \cdots \oplus a_{i4} \otimes t_4 = \min\left[(a_{i1}+t_1), \cdots, (a_{i4}+t_4)\right] \quad \text{Eq 5- 31}$$

$$ITRL = \begin{vmatrix} min[(0+0.89),(0.33+0.67),(0+1),(0+0.89)] \\ min[(0+0.89),(0+0.67),(0.56+1),(0.44+0.89)] \\ min[(0+0.89),(0+0.67),(0+1),(1+0.89)] \\ min[(0+0.89),(0.44+0.67),(1+1),(0+0.89)] \end{vmatrix} = \begin{bmatrix} .89 \\ .67 \\ .67 \\ .89 \end{bmatrix} \quad \text{Eq 5- 32}$$

The overall SRL is calculated by taking the minimum value of the ITRL vector, Eq 5-35, rather than the arithmetic average as with SysDML and GT. The minimum was chosen to stay consistent with tropical algebra.

$$SRL = min(ITRL_i) = .67 \qquad \text{Eq 5- 33}$$

5.5.1. MATLAB® Calculations

Just as in section 5.2 with the S-SRL and GT SRL models a MATLAB® program was used to calculate all possible SRL values for the Fire Control Kill Chain example are shown in Figure 5-11. The x-axis shows the values of the SRL calculated as they relate to the DoD acquisition lifecycle. The y-axis shows the number of times each SRL was calculated. The mean value for the GTTA SRL falls in the Engineering and Manufacturing Development phase of the DoD acquisition lifecycle and is approximately 0.59. This figure shows that the majority of systems will fall in the post milestone B phase of the DoD acquisition lifecycle. This model's mean SRL value is higher than the S-SRL and GT SRL mean values.

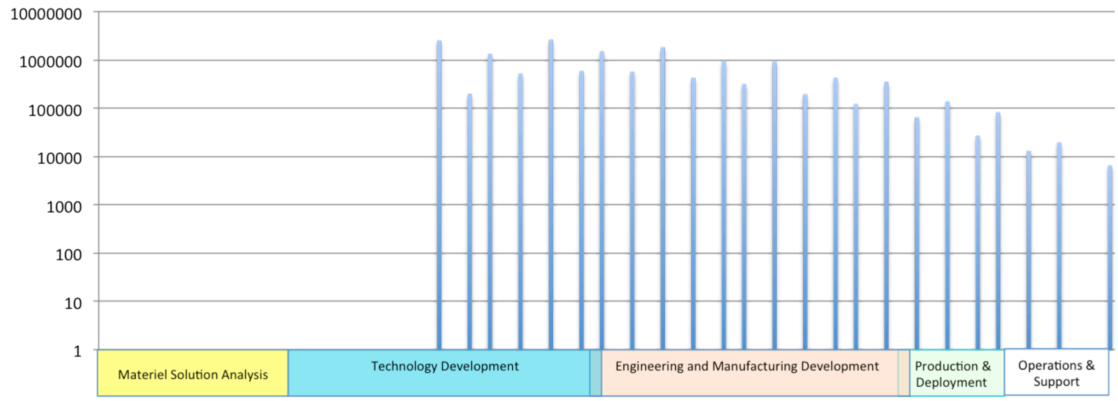


Figure 5-11 All Possible SRL Values for Fire Control Kill Chain with GTTA

5.5.2. SRL with constant TRL and IRL

As in section 5.2.2 the SRL was calculated with all the TRLs and IRLs equaling a constant. This constant was varied between 3 and 9. The results are shown in Table 5-4. The top row shows the constant used for the TRL and IRL and is the expected value for SRL. The second row shows the SRL calculated using GTTA. These values are plotted in Figure 5-12 and show again that the GTTA SRL is the expected value, unlike S-SRL and GR SRL values, see section 5.2.2.

Table 5-4 GTTA SRL Values for Constant TRL and IRL

TRL and IRL Level	3 (0.333)*	4 (0.444)	5 (0.556)	6 (0.667)	7 (0.778)	8 (0.889)	9 (1.000)
GTTA SRL	0.333	.444	.556	.667	.778	.889	1.000

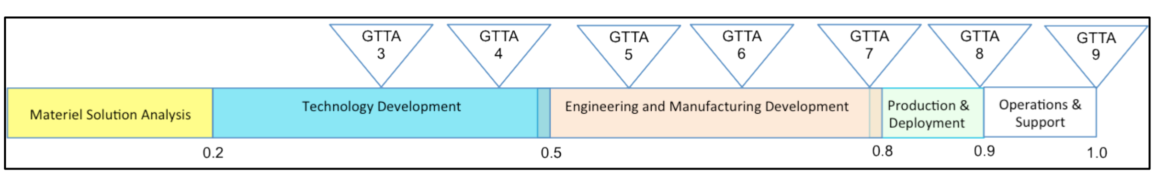


Figure 5-12 Mapping of SRL for Constant TRL and IRL for GTTA

5.5.3. Sensitivity Analysis

A sensitivity analysis, as done with the other two models, section 5.2.3, which increased a TRL by one while leaving all other TRL and IRL the same revealed that the GTTA SRL model returns a SRL equal to the lowest TRL. There was no difference in SRL value except when TRL_C was increased by one, and then there was a 14% increase in SRL, see Table 5-5 and Figure 5-13. Adding an additional interface between Assess and Detect, as we did with Sy-SRL and GT, had no effect on the SRL value for GTTA, see Figure 5-14. The y-axis is the ITRL and SRL calculated value and the x-axis tells which subsystem the value is associated with.

Table 5-5 Increasing TRL and IRL Percent Change in SRL for GTTA SRL Model

	GTTA Percentage Increase When TRL and IRL are Increased by 1 Individually							
	TRL_D+1	TRL_C+1	TRL_E+1	TRL_A+1	IRL_D+1	IRL_C+1	IRL_E+1	IRL_A+1
$ITRL_D$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$ITRL_C$	0.00%	14.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$ITRL_E$	0.00%	14.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
$ITRL_A$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
SRL	0.00%	14.27%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

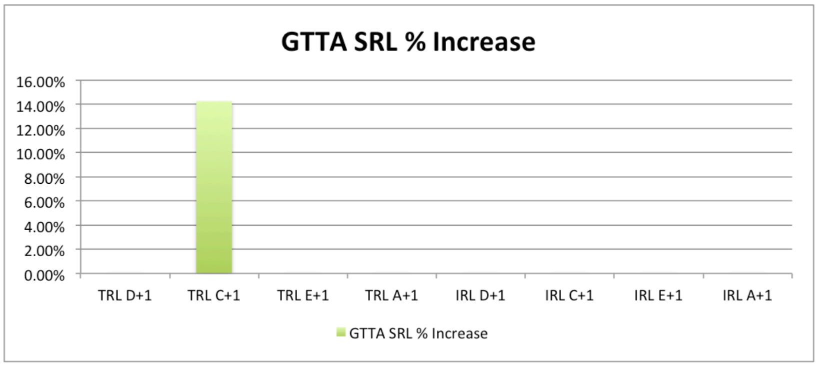


Figure 5-13 Percentage Change in SRL with Increase TRL and IRL

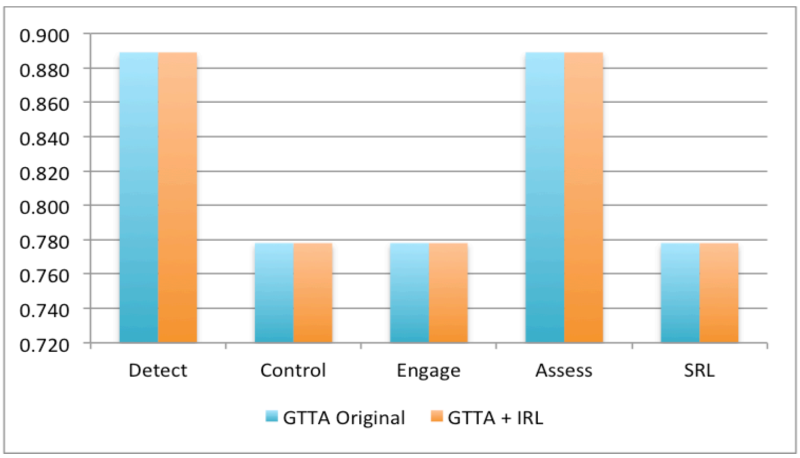


Figure 5-14 Calculated ITRL and SRL for GTTA SRL Model with IRL Added

5.6. Results Discussion

The mathematical results showed that the GT SRL model was the most conservative of the three models. Determining which model was the most sensitive to small changes depended on how you defined sensitive. If the desired result was the largest percent increase in SRL with the least change than the GTTA model did that but only if the lowest TRL was increased. A small change in any IRL for the GTTA model did not have an effect on the SRL values, when looking at the Fire Control Kill Chain. If the desired result was to have any effect on the SRL by increasing the TRL or IRL of one of the subsystems than the GT model did that. The GT model had SRL percent increase between 0 and 7.5%, depending on which TRL and IRL was increased. An increase of IRL_E had the largest percent increase of SRL. Increasing TRL_D did not have any effect on SRL value when calculated using GT or GTTA; it increased the SRL by 2.7% when using S-SRL.

The SRL value was increased by adding an additional interface between two subsystems, that were not originally connected, except when using the GTTA model. The S-SRL model was the most sensitive to this addition with a percent increase in SRL of 7% for the Fire Control Kill Chain used for this dissertation. This percent increase could be higher if a more complex system was used.

Another phenomenon that was observed from the MatLab® calculations was the possibility of readiness reversal. This can be demonstrated with the Fire Control Kill Chain example used to illustrate the models. If the TRLs and IRLs are all at a level 6, or .67 normalized, except for a_{AC} which is at a level 7, or .78 normalized, then $ITRL_C$=.52 is less than the $ITRL_D$= .56 even though the IRLs associated with C on average are larger than the IRLs associated with D, when using the S-SRL model. This phenomenon can also be found in a system that Tan et al [2010] used to demonstrate how an importance factor can be calculate using SRLs. This phenomenon is of interest to system engineers and program managers espcially if SRLs will be used to allocate resources. This phenomenon can happen with other sets of numbers in larger systems. According to Theorem 1 [Bowles, 2003] anytime you multiply qualitative ranking numbers, i.e. ordinal numbers, there is a set of numbers that will cause rank reversal.

This chapter demonstrated the differences in the three SRL models, S-SRL, GT SRL and GTTA model. It was found that only GTTA met all five properties. Sensitivity analysis were performed for all models and results varied according to model. The next chapter will present the conclusions drawn from the data analysis and future work that should be considered.

Chapter 6 CONCLUSION AND FUTURE WORK

Mathematical rigor was applied to a quantitative measurement, SRL, for use by decision-making teams. This research defined five necessary mathematical properties for SRL. These properties are desirable by decision-making teams to decrease the uncertainty in selection of subsystems for a system and for resource allocation. These properties are mathematically and logically sound and increase the program manager's and systems engineer's confidence in decisions when using SRL as a part of their toolbox. They are the building block for a mathematical framework for defining the readiness level of a system.

The research showed that the current SRL models, S-SRL and GT, do not meet all the desired properties and could produce questionable results. They can produce SRL values that are higher than at least one of their subsystem's readiness level, leading a decision maker to believe their system is more ready than it actually is. This increases the likelihood of cost overrides and schedule slips. The current models calculate SRL values that can be the result of several combinations of TRLs and IRLs for the system's subsystems. This results in calculated SRL values high enough to enter the Engineering and Manufacturing Development phase of the DoD acquisition lifecycle without having all TRLs at a level 6 or higher, which according to the 2011 GAO report will increase the programs change of cost and schedule over runs. However, with the new SRL model, GTTA, the SRL value is never higher than its minimal TRL/IRL combination, therefore it cannot enter the Engineering and Manufacturing Development phase without having all its TRLs at a level of 6 or higher.

By defining the five mathematical properties of a SRL model this dissertation increases the validity and rigor of the SRL calculations, especially for the newly developed SRL model, GTTA. The research shows that matrix algebra, which the current models use meets only properties 2 , increasing TRL does not degrade SRL values and property 3, introducing new technology that is at least as ready as the average existing subsystems does not degrade the SRL value. Matrix algebra does not meet the closure property and it is possible to calculate a subsystem readiness level that exceeds its maximum TRL or IRL value or lower than a constant value for TRL/IRL. Tropical algebra, which the new SRL model, GTTA, uses, meets all five properties. The current models introduce a scaling factor to calculate the SRL. With this scaling factor the current models meet the closure property and restricts the subsystem readiness level to be less than the maximum TRL/IRL. However the scaling factor introduces new problems, such as readiness reversal, as discussed in section 5.6. This readiness reversal could lead to ineffective resource allocation. This would happen when resources are allocated to improve a subsystem's readiness, which is calculated to be lower than a second subsystem's readiness level when in fact the second subsystem has higher TRLs and IRLs than the first subsystem.

The new SRL model, GTTA SRL model, meets all five mathematical properties by defining the function of TRL and IRL used to calculate the SRL as the min-plus of ordinal numbers rather than the product. As the research demonstrated the sum of ordinal numbers is more meaningful than the product of ordinal numbers, which the current SRL models use [Bowles, 2003]. GTTA SRL model does not rely on a scaling factor and does not have readiness reversal. Since the GTTA SRL model is sensitive to the minimal TRL value a system cannot be more ready than its lowest subsystem. Also the GTTA SRL model does not have readiness reversal, as discussed above with the current SRL models. The GTTA SRL model calculates all subsystem's readiness to be equal to their minimal TRL/IRL combination, so the subsystem with the higher TRLs and IRLs will have a higher subsystem readiness value.

The research also demonstrated the importance of the validity of the TRL and IRL values used to calculate the SRL. If a TRL or IRL is inflated the SRL can be inflated. Also SRL are calculated using only the critical technologies as determined by the program manager and system engineers. This selection of critical technologies can influence the calculated SRL, known as gaming the system. To prevent the gaming of the system an independent team of subject matter experts should be used to evaluate the critical subsystems and the TRLs and IRLs of the subsystems. The new GTTA SRL model is not as easily gamed as the current models. In the research conducted the GTTA SRL value was not influenced by adding an additional interface between two subsystems that were not originally connected where as the current SRL models' SRL was increased by adding an additional interface.

Although the example used for this research was based on a military scenario/kill chain the results of this research can be expanded to many systems engineering fields. The example did not have anything uniquely DoD/militarily and can easily be adapted for other system environments.

6.1. Future Research

Malone and Wolfarth [2012] proposed to use SRLs as a component of a cost model. Before SRLs are used for costing, future research should be done to evaluate the various combinations of TRL and IRL values to get a particular SRL value and what effect these combinations have on the cost to a system. For example, a system similar to the fire control kill chain presented earlier can be used to show that an SRL value of 0.53, stating that the system is ready to enter the EMD Phase of the acquisition life cycle, can be found at least six different ways, using non-normalized values for TRL and IRL:

a) all TRL =6 and all IRL =6 except IRLCA =7,
b) all TRL=6 and all IRL=6 except IRLCE =7,
c) all TRL=6 except TRLD =7 and all IRL=6 except IRLCA = IRLCE =7,
d) all TRL = 6 except TRLC = 7 and TRLD =5 and all IRL =6,
e) all TRL=6 except TRLD =4 and TRLA =7 and all IRL=6 except IRLCE = IRLEA =7,
f) all TRL=6 except TRLC =7 and all IRL =6.

Each of these cases would have a different schedule and cost associated with them, especially cases d and e where at least one of the TRL is less than 6.

In the calculation of SRL, for all three models discussed in this dissertation, the IRL between two subsystems that are not connected is defined as zero. It is artificially defined; there is no logic or rating scale supporting this definition. What affect does zero have on the SRL value is of interest to program managers and system engineers who rely on SRLs for decision-making. Future research should be conducted be to better define the math so that the zeros are ignored and then compare against the properties presented here.

Systems are about the three Ps, people, process and product [Clark and Kenney, 2006]. The TRL and IRL as defined only take into account the last two Ps; it leaves out the people. Therefore the SRL as a function of TRL and IRL also only take into account the last two Ps, product and process. The readiness of the people working on the team developing the system is just as important as the readiness of the product and process. A technology with a high readiness level if turned over to a young team may take longer to integrate into a system. For the SRL to truly capture the readiness of the system it must rate the team readiness as well. More research needs to be conducted to determine the optimal way to rate the team readiness and then how to combine this rating with the TRL and IRL to obtain a SRL that incorporates the three P's of system engineering.

References

Azizian, N., Mazzuchi, T., Sarkani, S., & Rico, D. (2011). A framework for evaluating technology readiness, system quality, and program performance of U.S. DoD acquisition. *Journal of System Engineering,* doi: 10.1002/sys.20186

Azizian, N., Sarkani, S., & Mazzuchi, T. (2009). A comprehensive review and analysis of maturity assessment approaches for improved decision support to achieve efficient defense acquisition. *Proceedings of the World Congress on Engineering and Computer Science,* San Francisco, California. *, II*(WCECS 2009)

Bahill, A. T., & Gissing, B. (1998). Re-evaluating systems engineering concepts using systems thinking. *Systems, Man, and Cybernetics, Part C: Applications and Reviews, IEEE Transactions on, 28*(4), 516-527.

Baron, N., Holland, O. T., Marchette, D., & Wallace, S. (2011). Integrating through the fire control loop. Paper presented at the *National Fire Control Symposium,* Lake Buena Vista, Florida. 1.

Bilbro, J. W. (2006). *Systematic assessment of the Program/Project impacts of technological advancement and insertion.* ().George C. Marshall Space Flight Center.

Blackburn, C., & Valerdi, R. (2009). Navigating the metric landscape; an introductory literature guide to metric selection, implementation and decision-making.

Bondy, J. A., & Murty, U. S. R. (2008). In Axler S., Ribert K. A. (Eds.), *Graduate texts in mathematics: Graph theory* (First ed.). Springer.com: Springer. doi: 10.1007/978-1-84628-970-5

Bowles, J. B. (2003). An assessment of RPN prioritization in a failure modes effects and criticality analysis. Paper presented at the *Reliability and Maintainability Symposium, 2003. Annual,* 380-386.

Bowles, J. B. (2004). An assessment of RPN prioritization in a failure modes effects and criticality analysis. *JOURNAL- IEST, 47*, 51-56. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=edsbl&AN=RN156830653&site=eds-live

Carter, A. (2010). ***USD_ATL_Guidance_Memo_September_14_2010_FINAL***. http://www.acq.osd.mil/docs/USD_:

Clark, J. O., & Kenney, W. (2006). *Systems engineering and software engineering processes, products, and people from a standards perspective (subtitle: Stand on the standards).* Unpublished manuscript. Retrieved July 12, 2012, from http://sstc-online.org/2006/pdfs/JC1244.pdf

Cox, L. A. (. (2008). What's wrong with risk matrices? *Risk Analysis, 28*(2) doi: 10.1111/j.1539-6924.2008.01030.x

Cox, L. A. (., Babayev, D., & Huber, W. (2005). Some limitations of qualitative risk rating systems. *Risk Analysis, 25*(3), 651-662. doi: 10.1111/j.1539-6924.2005.00615.x

Dion-Schwarz, C. (2008). *How the department of defense uses technology readiness levels.* Unpublished manuscript.

Engle, M., Sarkani, S., & Mazzuchi, T. (2009). Technical maturity evaluations for sensor fusion technologies. Paper presented at the *Applied Imagery Pattern Recognition Workshop (AIPRW), 2009 IEEE,* 1-6.

Fernandez, J. A. (2010). *Contextual role of TRLs and MRLs in technology management.* Unpublished manuscript. Retrieved June 10, 2011, from http://prod.sandia.gov/techlib/access-control.cgi/2010/107595.pdf

Garrett, R., Anderson, S., Baron, N., & Moreland, J. (2011). Managing the interstitials, a system of systems framework suited for the ballistic missile defense system. *Systems Engineering, 14*(1), 87-109. doi: 10.1002/sys.20173

John, L., Boardman, J., & Sauser, B. (2009). Leveraging paradox in systems engineering: Discovering wisdom. *Information Knowledge Systems Management, 7*(4), 357-376. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=buh&AN=45726697&site=ehost-live

Kalagnanam, J., Simon, H. A., & Iwasaki, Y. (1991). The mathematical bases for qualitative reasoning. *IEEE Expert, 6*(2), 11-19.

Kujawski, E. (2010). The trouble with the system readiness level (SRL) index for managing the acquisition of defense systems. *13th Annual Systems Engineering Conference,* San Diego, California.

Long, J. M. (2011). Integration readiness levels. Paper presented at the *Aerospace Conference, 2011 IEEE,* 1-9.

Maclagan, D., & Sturmfels, B. (2009). *Introduction to tropical geometry.* Unpublished manuscript.

Maier, M., & Rechtin, E. (2000). *The art of systems architecting* (2nd ed.). Boca Raton: CRC Press.

Majumdar, W. S.*System of systems technology readiness assessment.* (Unpublished Master's Thesis). Naval Postgraduate School, 2007,

Malone, P., & Wolfarth, L. (2012). System-of-systems: An architectural framework to support development cost strategies. Paper presented at the *Aerospace Conference, 2012 IEEE,* 1-13.

Mankins, J. (2002). Approaches to strategic research and technology (R&T) analysis and road mapping. *ACTA ASTRONAUTICA, 51 1-9*, p3-p21.

Mankins, J. C. (1998). *Research and development degree of difficulty (R&D3) A white pape.* Unpublished manuscript.

Mankins, J. C. (2009). Technology readiness assessments: A retrospective. *Acta Astronautica, 65*(9-10), 1216-1223. doi: 10.1016/j.actaastro.2009.03.058

McConkie, E. (2012) Mathematical Properties of System Readiness Levels, *Quantitative Methods for Defense and National Security,* Fairfax, Virginia, American Statistical Association, unpublished presentation.

McConkie, E., Mazzuchi, T., Sarkani, S., & Marchette, D. (2012). Mathematical Properties of System Readiness Levels. *Journal of Systems Engineering*, in press.

Mikhalkin, G. (2006). Tropical geometry and its application. *Institut Henri Poincare, , 2*(060104)

Nobuhara, H. (2008). Max-plus algebraic agent description and its application to various social systems. Paper presented at the *Automation Congress, 2008. WAC 2008. World,* 1-6.

Office of the Deputy Under Secretary of Defense for Acquisition and Technology, *Systems and Software Engineering. Systems Engineering Guide for Systems of Systems Version 1.0*. http://www.acq.osd.mil/se/docs/SE-Guide-for-SoS.pdf 2008

OUSD(AT&L) Systems and Software Engineering/Enterprise Development. (2006). *Risk management guide for DoD acquisition, sixth edition.* (). Washington, DC: Department of Defense.

Pachter, L., & Sturmfels, B. (2005). Computation. In L. Pachter, & B. Sturmfels (Eds.), *Algebraic statistics for computational biology* (First ed., pp. 43-47). The Pitt Building, Trumpington Street, Cambridge, United Kingdom: Cambridge University Press.

Pin, J.*Tropical semirings.* Unpublished manuscript.

Ramirez-Marquez, J. E., & Sauser, B. J. (2009). System development planning via system maturity optimization. *Engineering Management, IEEE Transactions on, 56*(3), 533-548.

Sage, A. P., & Rouse, W. B. (1999). In Sage A. P., Rouse W. B. (Eds.), *Handbook of systems engineering and management*. United States of America: John Wiley & Sons, Inc.

Sauser, B., & Boardman, J. (2008). Taking hold of system of systems management. *Engineering Management Journal, 20*(4), 3. Retrieved from http://proxygw.wrlc.org/login?url=http://proquest.umi.com/pqdweb?did=1653077441&Fmt=7&clientId=31812&RQT=309&VName=PQD

Sauser, B., Verma, D., Ramirez-Marquez, J., & Gove, R. (2006). *From TRL to SRL: The concept of systems readiness levels* Aerospace Corporation. Retrieved from http://search.ebscohost.com/login.aspx?direct=true&db=edsbl&AN=CN061414494&site=eds-live

Sauser, B. J., Ramirez-Marquez, J., Nowicki, D., Deshmukh, A., & Sarfaraz, M. (2011). *Development of systems engineering* *maturity models and management tools; final technical report 2011-TR-014.* Unpublished manuscript.

Sauser, B., Gove, R., Forbes, E., & Ramirez-Marquez, J. (2010). Integration maturity metrics: Development of an integration readiness level. *Information Knowledge Systems Management, 9*(1), 17-46. doi: 10.3233/IKS-2010-0133

Schwenn, R. E., Chitikila, R. C., Laufer, D. R., Rozzi, A. D., Smythe, W. P., Best, D. B., . . . Graveline, W. R. (2011). *Defense aquisitions: Assessments of selected weapon programs.* (No. GAO-11-233SP Assessments of Selected Weapon Programs). Washington DC: Government Accountability Office.

Smith, J. D.,II. (2005). An alternative to technology readiness levels for non-developmental item (NDI) software. Paper presented at the *System Sciences, 2005. HICSS '05. Proceedings of the 38th Annual Hawaii International Conference on,* 315a-315a.

Speyer, D., & Sturmfels, B. (2004). *Tropical mathematics.* Unpublished manuscript.

Tan, Weiping, Sauser, B. J., & Ramirez-Marquez, J. E. (2011). Analyzing component importance in multifunction multicapability systems developmental maturity assessment. *Engineering Management, IEEE Transactions on, 58*(2), 275-294.

Tetlay, A., & John, P. (2009). Determining the lines of system maturity, system readiness and capability readiness in the system development lifecycle. Paper presented at the *7th Annual Conference on Systems Engineering Research* Loughborough University (UK). Retrieved from http://cser.lboro.ac.uk/papers/S01-01.pdf

United States. General Accounting Office. (1999). *Best practices*. Washington, D.C.; Gaithersburg, MD P.O. Box 6015, Gaithersburg 20884-6015: The Office; The Office, distributor. Retrieved from http://magik.gmu.edu/cgi-bin/Pwebrecon.cgi?DB=local&FT=%22%28OCoLC%2943518522%22&CNT=25+records+per+page

United States. Government Accountability Office. (2006). *Best practices*. Washington, D.C.: U.S. Government Accountability Office. Retrieved from http://magik.gmu.edu/cgi-bin/Pwebrecon.cgi?DB=local&FT=%22%28OCoLC%29ocm77079706%22&CNT=25+records+per+page; http://purl.access.gpo.gov/GPO/LPS76591

Valerdi, R., & Kohl, R. J. (2004). An approach to technology risk management. Paper presented at the *Engineering Systems Division Symposium,* 1. Retrieved from http://web.mit.edu/rvalerdi/www/TRL%20paper%20ESD%20Valerdi%20Kohl.pdf

Volkert, R., & Jackson, C. (2010). *Proposed methodology for performance prediction and monitoring for an acknowledged systems of systems.* Retrieved Oct 15, 2011, from http://www.acquisitionresearch.net/_beta/files/FY2011/NPS-AM-11-C8P09Z03-111.pdf

Appendix A – MATLAB® and Excel® Programs

S-SRL all possible (repeated for dc and ce to equal 1-9)

```
clear
clc
close
n=2400;

IRL=zeros(4,n,'single');

TRL=zeros(1,4,'single');

SRL=zeros(1,n,'single');
i=0;
dd=1; cc=1; ee=1;aa=1;
de=0; da=0;

dc=9/9;ce=9/9;

for irlca=1:9;
    ca=irlca/9;

for irlea=1:9;
    ea=irlea/9;


IRL=[dd dc de da; dc cc ce ca; de ce ee ea; da ca ea aa];


for h=3:9;
d=h/9;
for g=3:9;
    c=g/9;

for f=3:9;
    e=f/9;
for b=3:9;
    a=b/9;
    i=i+1;
    TRL=[d c e a];m=[.5 .25 .33 .33];

    ITRL1=IRL*TRL';
    ITRL=ITRL1.*m';
    SRL(i)=mean(ITRL);
end
end
end
```

```
end
end
end

SUCCESS =
xlswrite('/Users/eileenmcconkie/Documents/MATLAB/MIdterm/SR
L99.xls',SRL')
```

GT-SRL all possible (repeated for dc and ce to equal 1-9)

```
clear
clc
close
n=2400;

IRL=zeros(4,n,'single');

TRL=zeros(1,4,'single');

SRL=zeros(1,n,'single');
i=0;
dd=0; cc=0; ee=0;aa=0;
de=0; da=0; cd=0; ed=0; ec=0; ad=0;
de=0; da=0;
dc=9/9;ce=9/9;

for irlca=1:9;
    ca=irlca/9;
    ac=ca;
for irlea=1:9;
    ea=irlea/9;
    ae=ea;

IRL=[dd dc de da; cd cc ce ca; ed ec ee ea; ad ac ae aa];

for h=3:9;
d=h/9;
for g=3:9;
    c=g/9;

for f=3:9;
    e=f/9;
for b=3:9;
    a=b/9;
    i=i+1;
    TRL=[d c e a];m=[1 .5 1 .5];

    ITRL1=IRL*TRL';
    ITRL= ITRL1.*m';
```

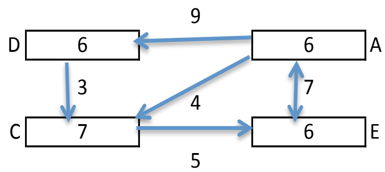
```
    SRL(i)=mean(ITRL);
end
end
end
end
end
end
SUCCESS =
xlswrite('/Users/eileenmcconkie/Documents/MATLAB/SRLGT99.xl
s',SRL')
```

GTTA-SRL all possible

```
clear
clc
close


i=0;
dd=0; cc=0; ee=0;aa=0;de=0;da=0;cd=0;ed=0;ec=0;ad=0;


 dc=1; ce=6;
for irlca=1:9;
    ca=irlca/9;
for irlea=1:9;
    ea=irlea/9;



for trld=3:9;
    d=trld/9;
for trlc=3:9;
    c=trlc/9;
for trle=3:9;
    e=trle/9;
for trla=3:9;
    a=trla/9;
    i=i+1;

    h=dd+d;j=dc+c;k=de+e;l=da+a;
    m=cd+d;n=cc+c;o=ce+e;p=ca+a;
    q=ed+d;r=ec+c;s=ee+e;t=ea+a;
    u=ad+d;v=ca+c;w=ea+e;x=aa+a;
    ITRL= [h, m, q, u; j, n, r, v; k, o, s, w; l, p, t, x];

    Z=min(ITRL);
```

```
    SRL(i)=mean(Z);
end
end
end
end
end
end

SUCCESS =
xlswrite('/Users/eileenmcconkie/Documents/MATLAB/SRLGTTA16.
xls',SRL')
%HIST(SRL',100)
```

S-SRL EXCEL



TRL		
	6.000	0.667
	7.000	0.778
	6.000	0.667
	6.000	0.667

IRL								
Detect	0	3	0	0	0.000	0.333	0.000	0.000
Control	0	0	5	4	0.000	0.000	0.556	0.444
Engage	0	0	0	7	0.000	0.000	0.000	0.778
Assess	9	4	7	0	1.000	0.444	0.778	0.000
SRL								
					1	0.333	0.000	1.000
					0.333	1	0.556	0.444
					0.000	0.556	1	0.778
					1.000	0.444	0.778	1

	ITRL GT		SRL GT	ITRL S-SRL		S-SRL	ITRL GTTA
ITRL(D)	0.259	0.259		1.593	0.531		0.667
ITRL(C)	0.667	0.333	0.405	1.667	0.417	0.509	0.667
ITRL(E)	0.519	0.519		1.617	0.539		0.667
ITRL(A)	1.531	0.510		2.198	0.549		0.667
SRL		0.405			0.509		0.667
		0.259			0.309		0.667
		0.333	0.34975		0.417	0.412	0.667
		0.519			0.539		0.667
		0.288			0.383		0.667
		0.34975			0.412		0.667