Prelab Assignment 4

Felix Marrar - abomrar2 - 676763531

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Problem 1

The equation for the first critical load is given by:

$$P_{crit} = \frac{\pi^2 EI}{L_k^2} \tag{1}$$

where L_k is the buckling length. Case 1 is when the rod is pinned at both ends (knife-edge/knife-edge). Case 2 is when the rod is free at one end and fixed at the other end. Case 3 is when the rod is fixed at both ends (clamped/clamped). Case 4 is when one end is fixed, and the other is pinned (clamped/knife-edge). According to the lab manual, S4, S6, S7 will be mounted to Case 1, Case 4, and Case 3 respectively. Assume for the purposes of this exercise that S2 is mounted by Case 2. The moment of inertia is given by:

$$I = \frac{1}{12} t h^3 \tag{2}$$

where t is the thickness of the rod, and h is the width of the rod. S2, S4, S6 and S7 have the same cross sectional area and lengths. So $I = 2.782 \times 10^{-9}$ m^4

$$S4: \qquad P_{crit} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (210 \times 10^9)(2.782 \times 10^{-9})}{(650 \times 10^{-3})^2} = 13.65 \text{ kN} \tag{3}$$

$$S6: \qquad P_{crit} = \frac{\pi^2 EI}{0.49L^2} = \frac{\pi^2 (210 \times 10^9)(2.782 \times 10^{-9})}{0.49(650 \times 10^{-3})^2} = 27.85 \text{ kN} \tag{4}$$

$$S7: \qquad P_{crit} = \frac{4\pi^2 EI}{L^2} = \frac{4\pi^2 (210 \times 10^9)(2.782 \times 10^{-9})}{(650 \times 10^{-3})^2} = 54.59 \text{ kN} \tag{5}$$

$$S2: \qquad P_{crit} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 (210 \times 10^9)(2.782 \times 10^{-9})}{4(500 \times 10^{-3})^2} = 5.77 \text{ kN} \tag{6}$$

Problem 2

The maximum deflection at the center of the rod is given by:

$$w\left(\frac{L}{2}\right) = e\left(\frac{1}{\cos\frac{\kappa L}{2}} - 1\right) \tag{7}$$

where

$$\kappa = \sqrt{\frac{F}{EI}} \tag{8}$$

Calculate κ for each rod:

$$\kappa_4 = \sqrt{\frac{0.5(13.08 \times 10^3)}{(210 \times 10^9)(2.782 \times 10^{-9})}} = 3.418 \tag{9}$$

$$\kappa_6 = \sqrt{\frac{0.5(26.70 \times 10^3)}{(210 \times 10^9)(2.782 \times 10^{-9})}} = 4.882 \tag{10}$$

$$\kappa_7 = \sqrt{\frac{0.5(53.33 \times 10^3)}{(210 \times 10^9)(2.782 \times 10^{-9})}} = 6.835 \tag{11}$$

$$\kappa_2 = \sqrt{\frac{0.5(5.53 \times 10^3)}{(210 \times 10^9)(2.782 \times 10^{-9})}} = 2.221 \tag{12}$$

Then the eccentricity for each rod

$$e_4 = e_6 = e_7 = 0.3(650 \times 10^{-3}) = 0.0195 \tag{13}$$

$$e_2 = 0.3(500 \times 10^{-3}) = 0.015 \tag{14}$$

Finally, the maximum deflection at the center for each rod:

$$\begin{aligned} w_4 &= 0.0244 \text{ m} \\ w_6 &= -1.242 \text{ m} \\ w_7 &= -0.0517 \text{ m} \\ w_2 &= 0.0027 \text{ m} \end{aligned} \tag{15}$$

Problem 3

The elastic modulus appears in the first critical load equation as directly proportional to the critical load. The higher the modulus, the higher the critical load, which is the point at which the material buckles. For instance, S8 (steel) has an elastic modulus of 70 GPa, and S9 (brass) has an elastic modulus of 104 GPa, and S10 (copper) has an elastic modulus of 125 GPa. So we'd expect copper to have the highest buckling critical load since it has the highest elastic modulus, followed by brass, and finally steel. For S11, the elastic modulus is given by:

$$E = \frac{P_{crit} L^2}{\pi^2 I} \tag{16}$$

The moment of inertia is $I = (1/12)(9.9 \times 10^{-3})(25 \times 10^{-3})^3 = 1.289 \times 10^{-8}$ m^4. Plugging that into the above equation yields $E = 3.96$ GPa