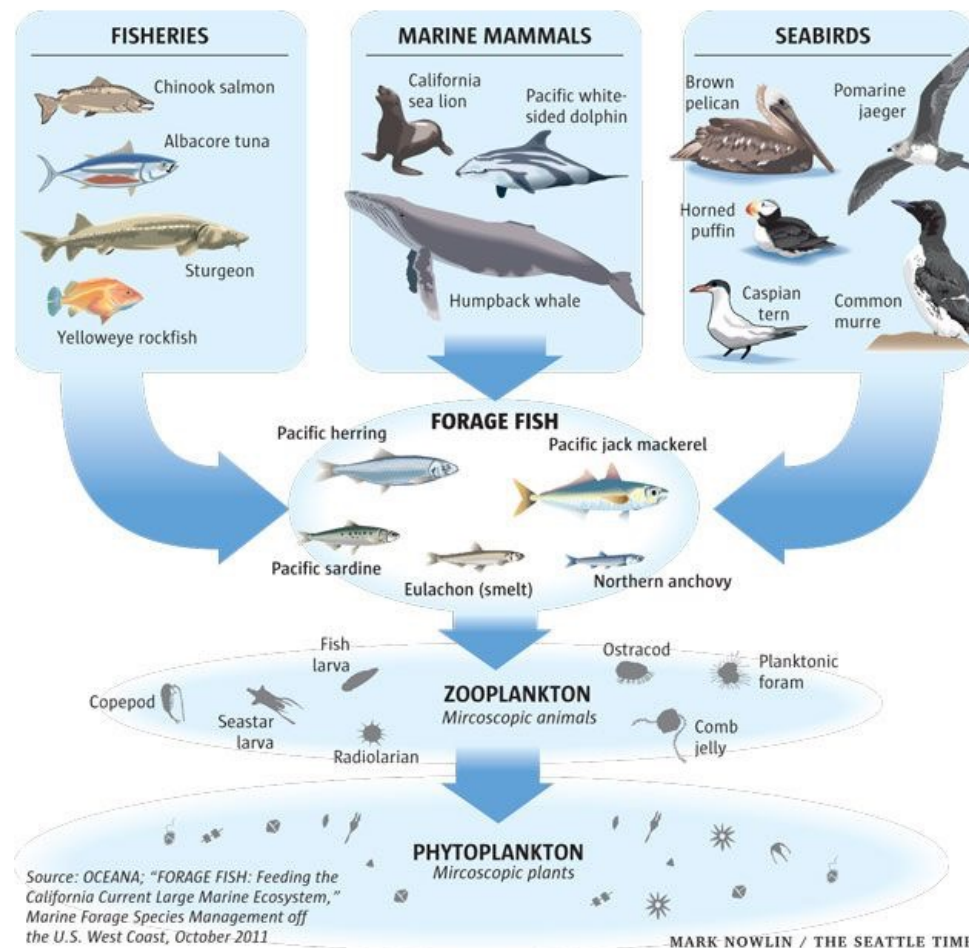


Complexity and stability of empirical food webs

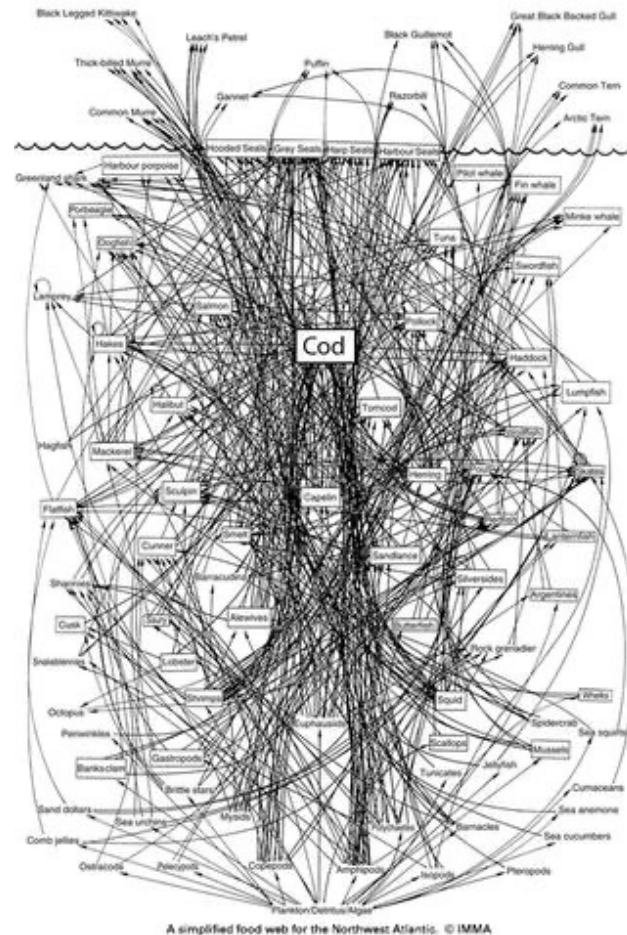
Food webs

Describe energy and material flows among species



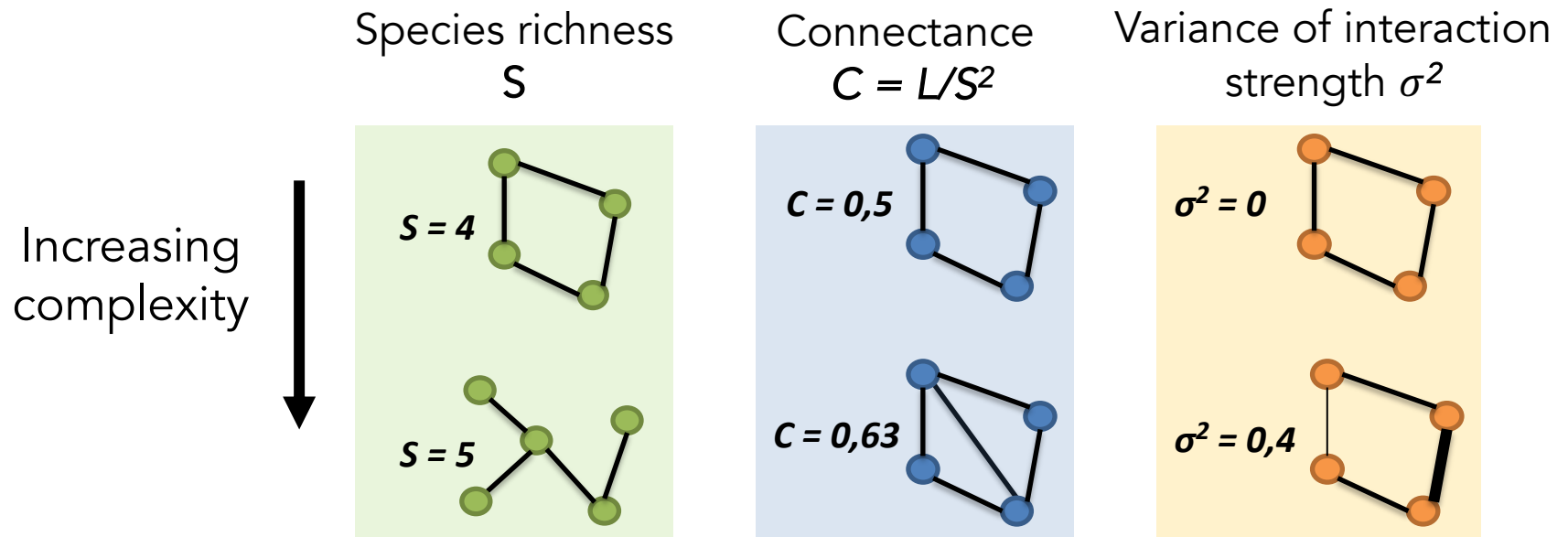
Food webs

Complex systems



Will the ecosystem
buffer or amplify a
perturbation ?

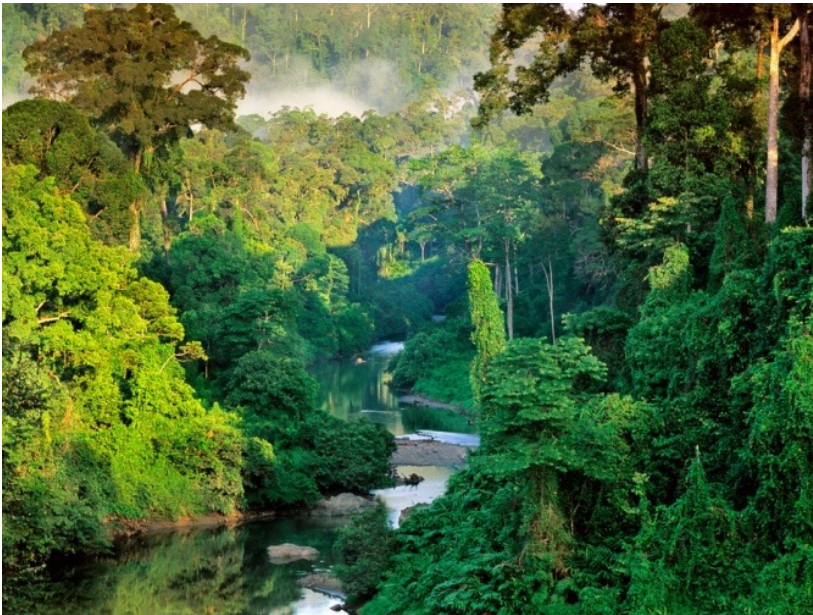
Ecosystem complexity



The complexity stability debate

Until the 70's:

Diversity stabilizes ecosystems (Odum 1953, Elton 1958, MacArthur 1955)



Guyane, tropical forest.



Alaska, boreal forest.

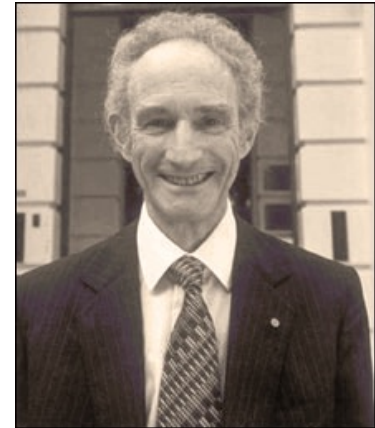
The complexity stability debate

Complexity decreases ecosystem stability (May 1972)

Complexity:

- Species richness S
- Connectance $C = L/S^2$
- Variance of interaction strengths σ^2

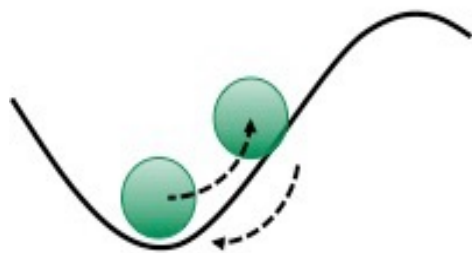
Stability criterion: $\sigma\sqrt{SC} < 1$



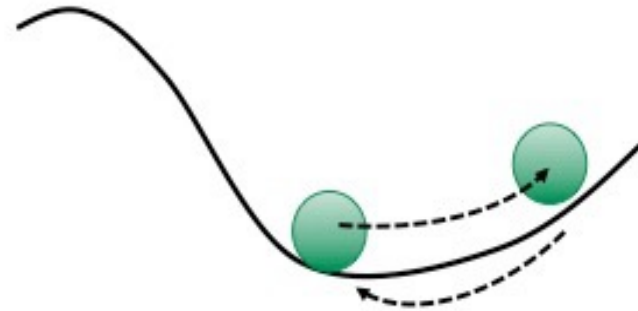
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Local stability analysis

Asymptotic stability: rate to which species populations go back to their initial densities after a pulse disturbance



High stability



Low stability

Local stability analysis

Population dynamic: $\frac{dX_i(t)}{dt} = f_i(\mathbf{X}(t)) \quad (i = 1, \dots, S)$

Equilibrium point \mathbf{X}^* : $\left. \frac{dX_i(t)}{dt} \right|_{\mathbf{X}^*} = f_i(\mathbf{X}^*) = 0$

Coefficients of the community matrix $M_{ij} = J_{ij}|_{\mathbf{X}^*} = \left. \frac{\partial f_i(\mathbf{X}(t))}{\partial X_j} \right|_{\mathbf{X}^*}$

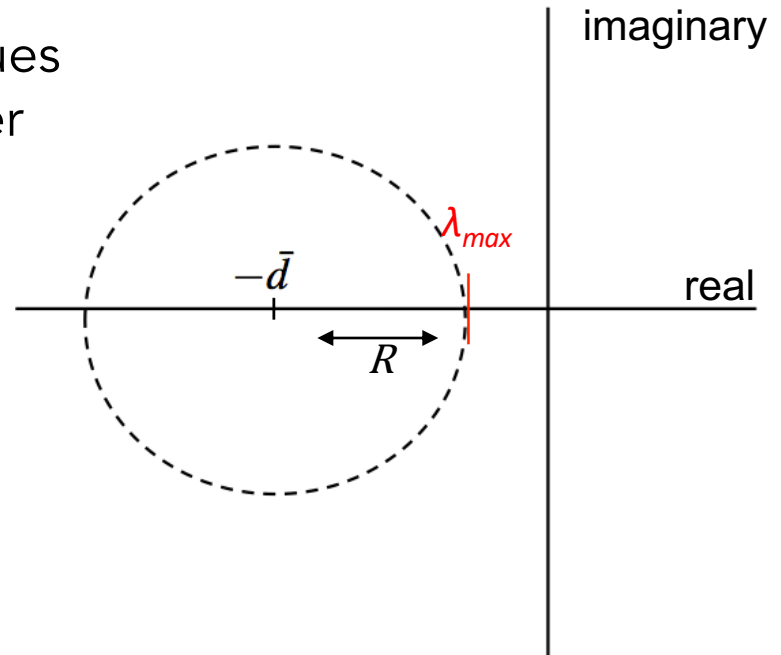
Eigenvalues of \mathbf{M} : real ($\lambda=a$) or complex ($\lambda = a \pm ib$)

The circular law (*Girko 1985, Tao 2010*)

On a complex plan: all the eigenvalues are contained in a circle of center $(-d, 0)$ and radius R .

The system is stable if $\text{Re}(\lambda_{\max}) < 0$.

Stability criterion?

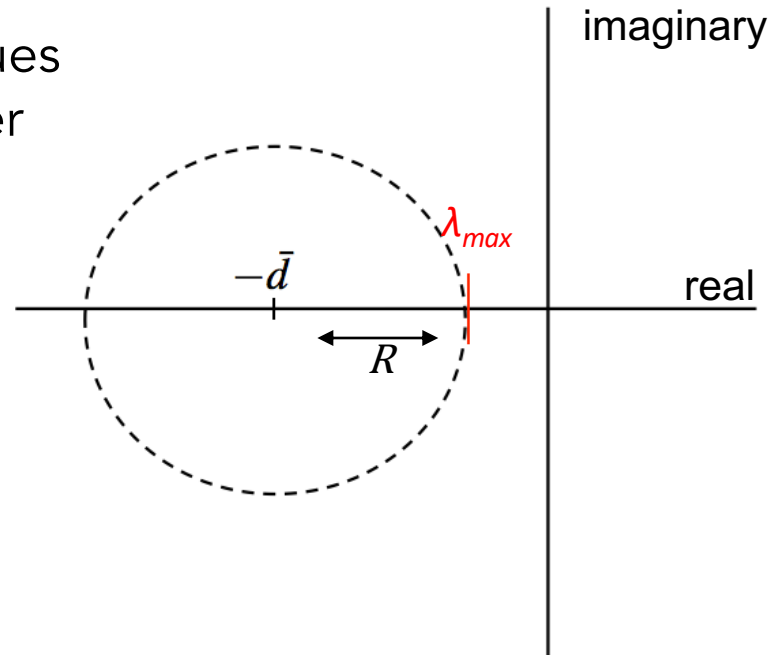


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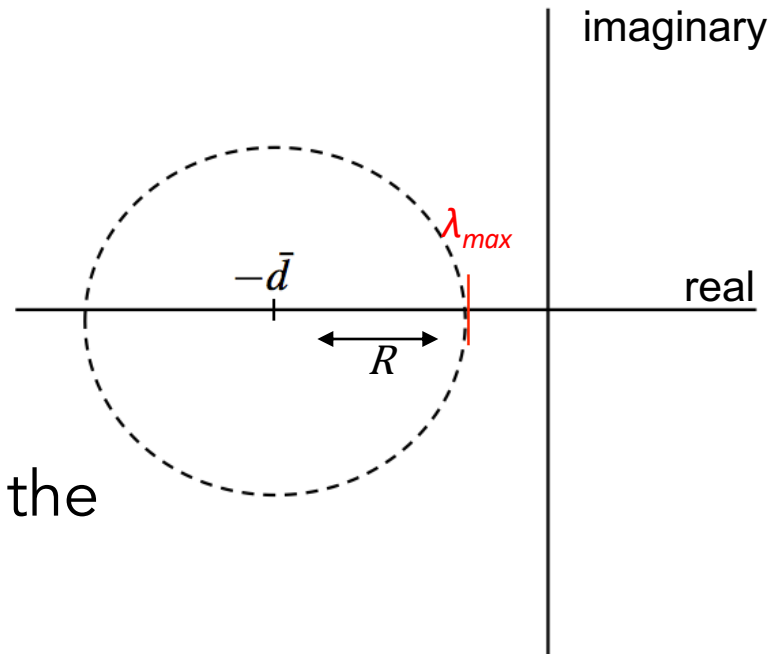
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Stability criterion: $R < d$



The circular law (*Girko 1985, Tao 2010*)

$$\mathbf{C} = \begin{pmatrix} -d_1 & +c_{2,1} & +c_{3,1} & 0 \\ -c_{1,2} & -d_2 & 0 & +c_{4,2} \\ -c_{1,3} & 0 & -d_3 & +c_{4,3} \\ 0 & -c_{2,4} & -c_{3,4} & -d_4 \end{pmatrix}$$



\bar{d} corresponds to the mean of the diagonal terms of \mathbf{C}

In random communities: $R = \sigma\sqrt{SC}$

σ^2 : variance of c_{ij}

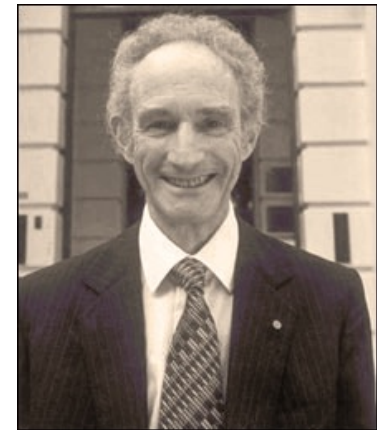
S : size of the matrix

C : proportion of non-zero terms

Application to ecological networks

May's stability criterion: $\sigma\sqrt{SC} < 1$

- Species richness S
- Connectance $C = L/S^2$
- Variance of interaction strengths σ^2
- \bar{d} : Magnitude of intraspecific competition (diagonal terms of the matrix) \rightarrow set to 1

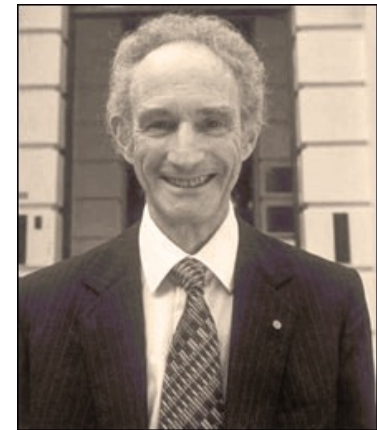


Professor Lord Robert May

Application to ecological networks

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Professor Lord Robert May

Theoretical communities:

- species interact at random
- interaction strengths are picked from a normal distribution

« In short, there is no comfortable theorem assuring that increasing diversity and complexity beget community stability; rather, as a mathematical generality the opposite is true.

The task, therefore, is to elucidate the devious strategies which make for stability in enduring natural systems. » (May 2001).

1. What is the actual complexity-stability relationship in empirical communities?

1. What is the actual complexity-stability relationship in empirical communities?
2. What are the « *devious strategies* » of real communities that allow them to persist despite their complexity?

Food-web dataset

116 quantitative food webs from Ecopath models
(Christensen 1992)

For each species i :

- biomass B_i (tons/km²)
- *production* $(P/B)_i$ (year⁻¹)
- consumption $(Q/B)_i$ (year⁻¹)
- diet composition DC_{ji}

Derivation of interaction strengths from data

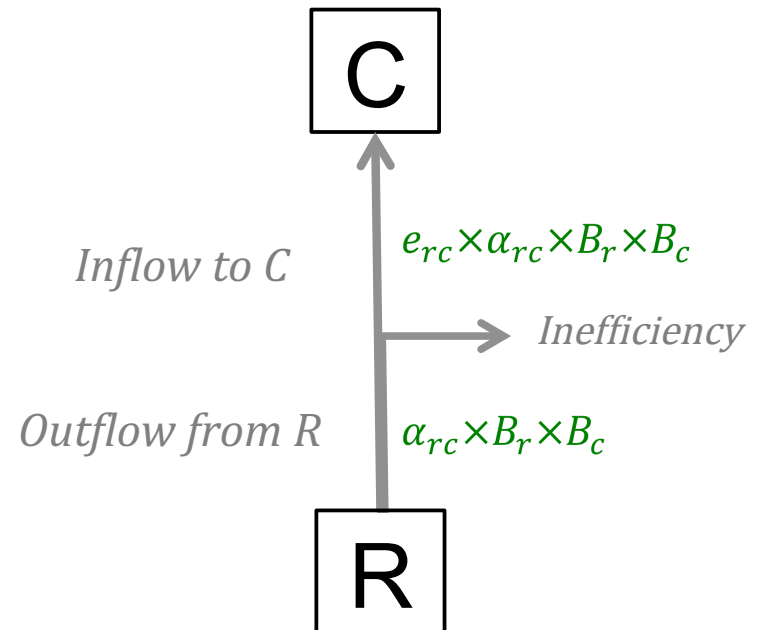
α_{ij} : per capita effect of species j on species i ($\text{mass}^{-1}\text{year}^{-1}$)

e_{ij} : efficiency of j to convert i into its own biomass

$$\alpha_{ji} = e_{ij} \times \alpha_{ij}$$

Data for each species:

- biomass B_i (tons/km^2)
- production $(P/B)_i$ (year^{-1})
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Derivation of interaction strengths from data

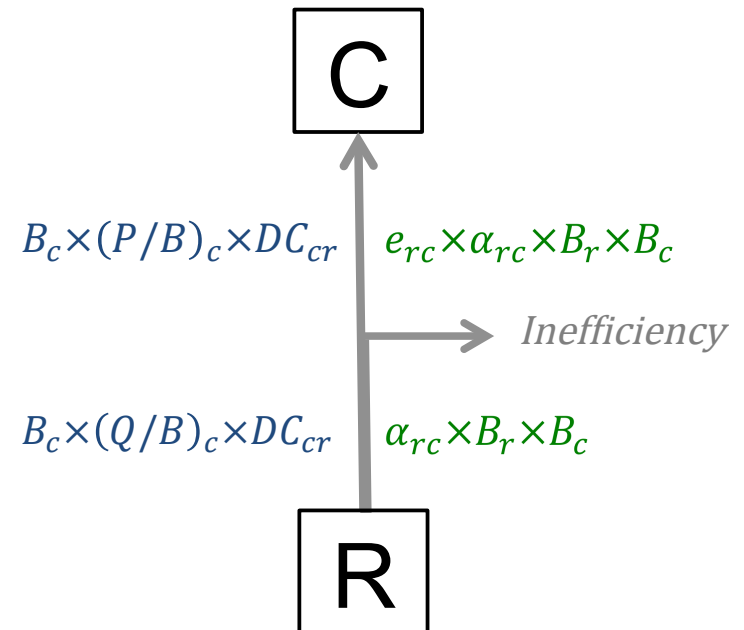
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Practice

1. computing food-web complexity and stability in R
2. analysing the relationship between complexity and stability in empirical food webs
3. comparing the complexity-stability relationship of empirical and « randomized » food webs