ICTEAM Seminars
11 February 2025

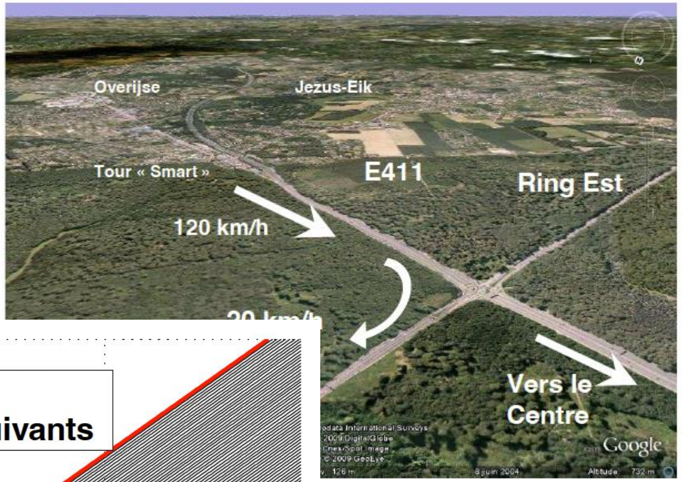
Climate science

A goldmine for applied mathematicians

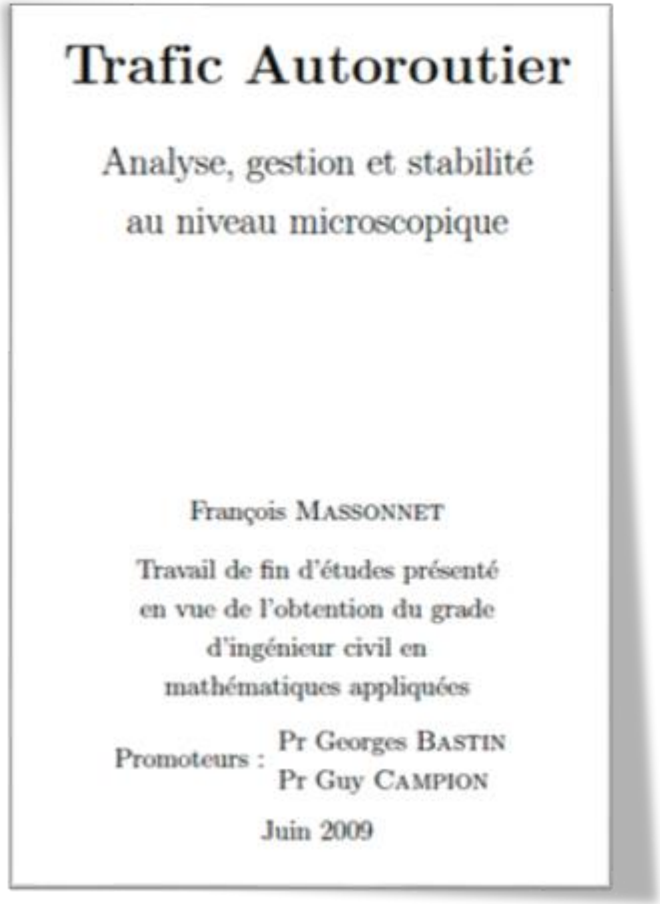
François Massonnet
francois.massonnet@uclouvain.be
www.elic.ucl.ac.be/u/fmasson

Long time no see...

A visionary master thesis 🤣



oche du Carrefour Léonard (Bruxelles).



Trafic Autoroutier

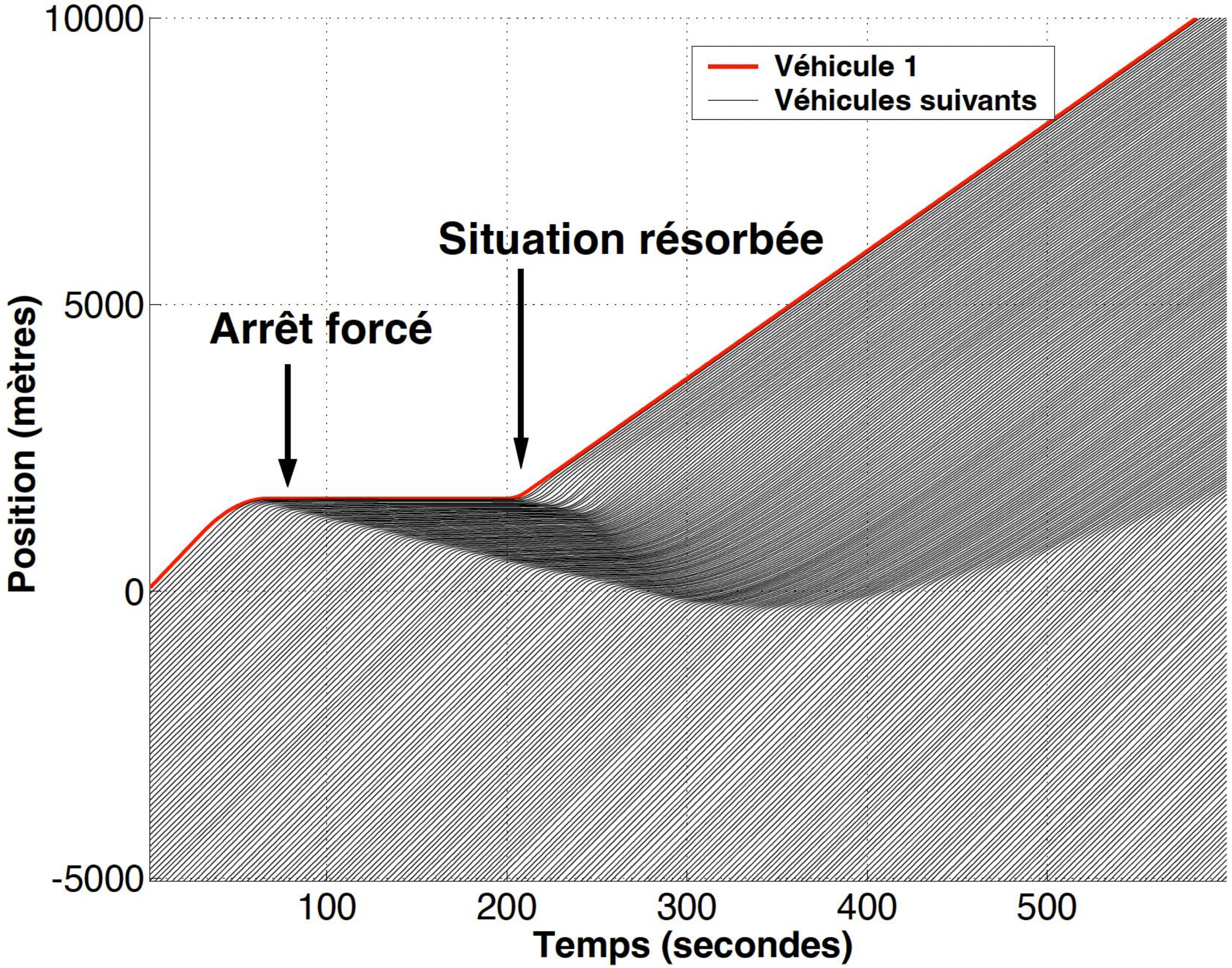
Analyse, gestion et stabilité au niveau microscopique

François Massonnet

Travail de fin d'études présenté en vue de l'obtention du grade d'ingénieur civil en mathématiques appliquées

Promoteurs : Pr Georges Bastin
Pr Guy Campion

Juin 2009



« Climate »: what does this word make you think of?

Climate science: goldmine for applied mathematicians

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Joseph Fourier (1768-1830)

MÉMOIRE

SUR

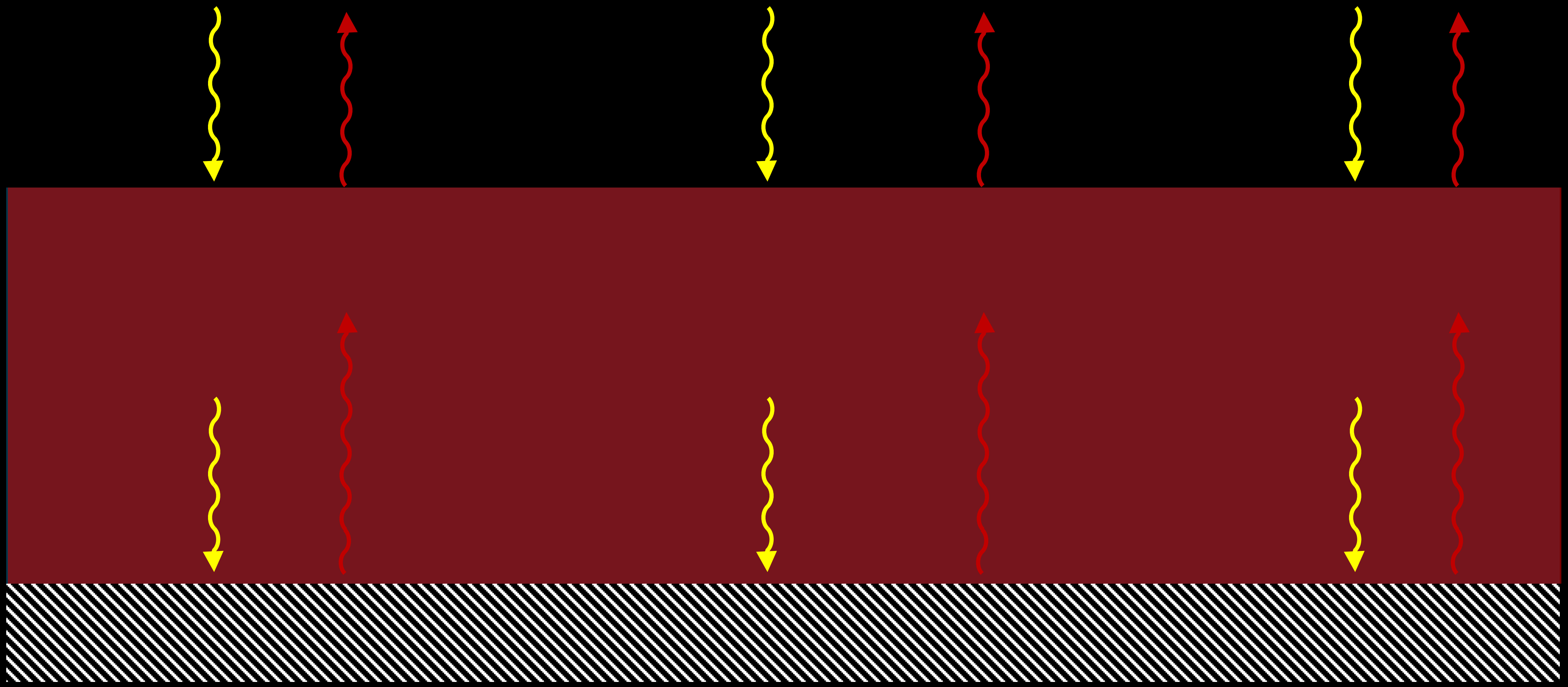
LES TEMPÉRATURES DU GLOBE TERRESTRE ET DES ESPACES PLANÉTAIRES.

PAR M. FOURIER.

LA question des températures terrestres, l'une des plus importantes et des plus difficiles de toute la philosophie naturelle, se compose d'éléments assez divers qui doivent être considérés sous un point de vue général. J'ai pensé qu'il serait utile de réunir dans un seul écrit les conséquences principales de cette théorie; les détails analytiques que l'on

Fourier, J. (1827). Mémoire sur les températures du globe terrestre et des espaces planétaires.
https://www.academie-sciences.fr/pdf/dossiers/Fourier/Fourier_pdf/Mem1827_p569_604.pdf

« This is how the [air] temperature is increased by the **interposition of the atmosphere**: because heat encounters less resistance when penetrating the air in the form of light than it does when attempting to return to the air after being converted into invisible heat» -- J. Fourier, 1824



The simplest model for Earth's radiative balance

Incoming shortwave radiation (from the Sun):

Top of the atmosphere

Radiative equilibrium: $\Delta R = 0$

Earth's albedo, 0.31

Solar constant: 1361 W/m^2

$$(1-\alpha)\frac{S_0}{4} - \sigma T^4 = 0$$

Due to Earth's geometry

Stefan-Boltzman constant: 5.67 × 10^{-8} W/m^2/K^4

→ T = -19°C

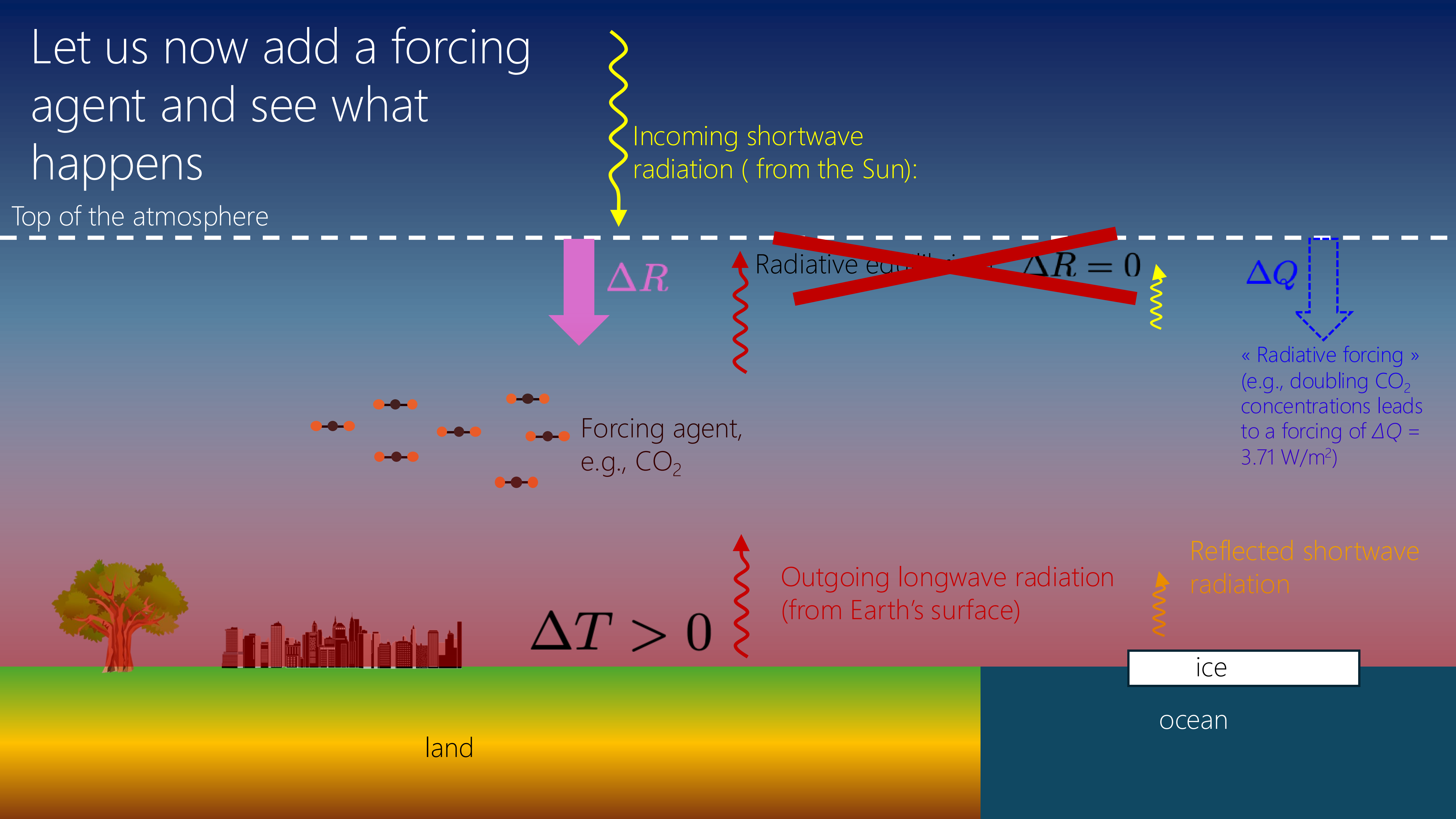
Outgoing longwave radiation (from Earth's surface)

Reflected shortwave radiation

ice

land

ocean



Let us now add a forcing agent and see what happens
Top of the atmosphere
Incoming shortwave radiation (from the Sun):
ΔR
~~Radiative eq~~ ~~$\Delta R = 0$~~
ΔQ
« Radiative forcing » (e.g., doubling CO_2 concentrations leads to a forcing of ΔQ = 3.71 W/m^2)
Forcing agent, e.g., CO_2
Reflected shortwave radiation
Outgoing longwave radiation (from Earth's surface)
$\Delta T > 0$
ice
ocean
land

Let us now add a forcing agent and see what happens

Incoming shortwave radiation (from the Sun): 0.25-2 μm

Top of the atmosphere

Basic model for the response of the climate system to a perturbation:

$$\Delta R = \Delta Q + \boxed{\lambda} \Delta T$$

« climate feedback parameter » (W/m^2/K)

The Earth system's radiative response ΔR to a perturbation ΔQ is enhanced (λ>0) or damped (λ<0) proportionally to the Earth's temperature response to the perturbation ΔT

ΔQ

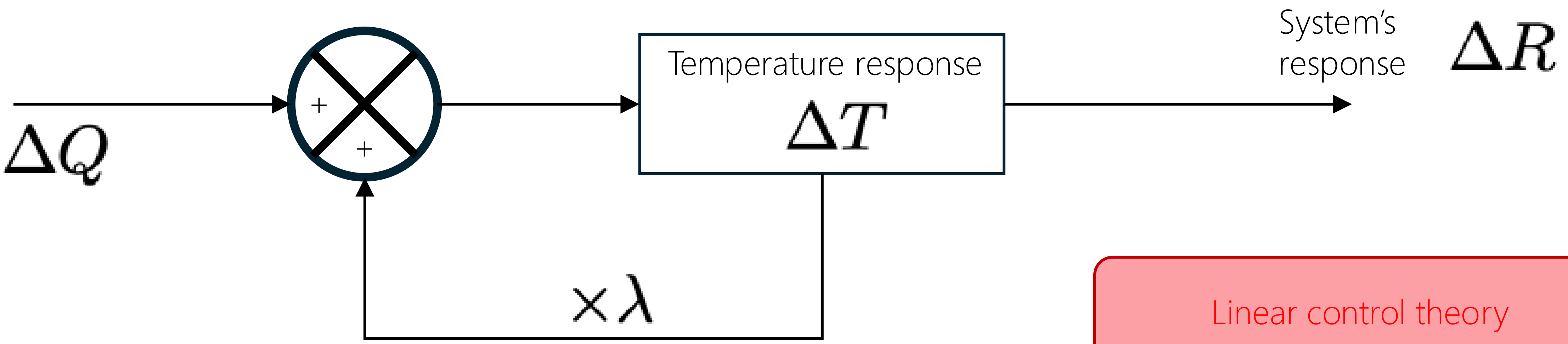
« Radiative forcing » (e.g., doubling CO_2 concentrations leads to a forcing of ΔQ = 3.71 W/m^2)

ice

ocean

land

A linear control theory view



What is the value of the Earth's climate feedback parameter λ ?

Through which procedures can we reliably estimate λ?

What is the most appropriate block diagram representation for the Earth's system?

Linear control theory

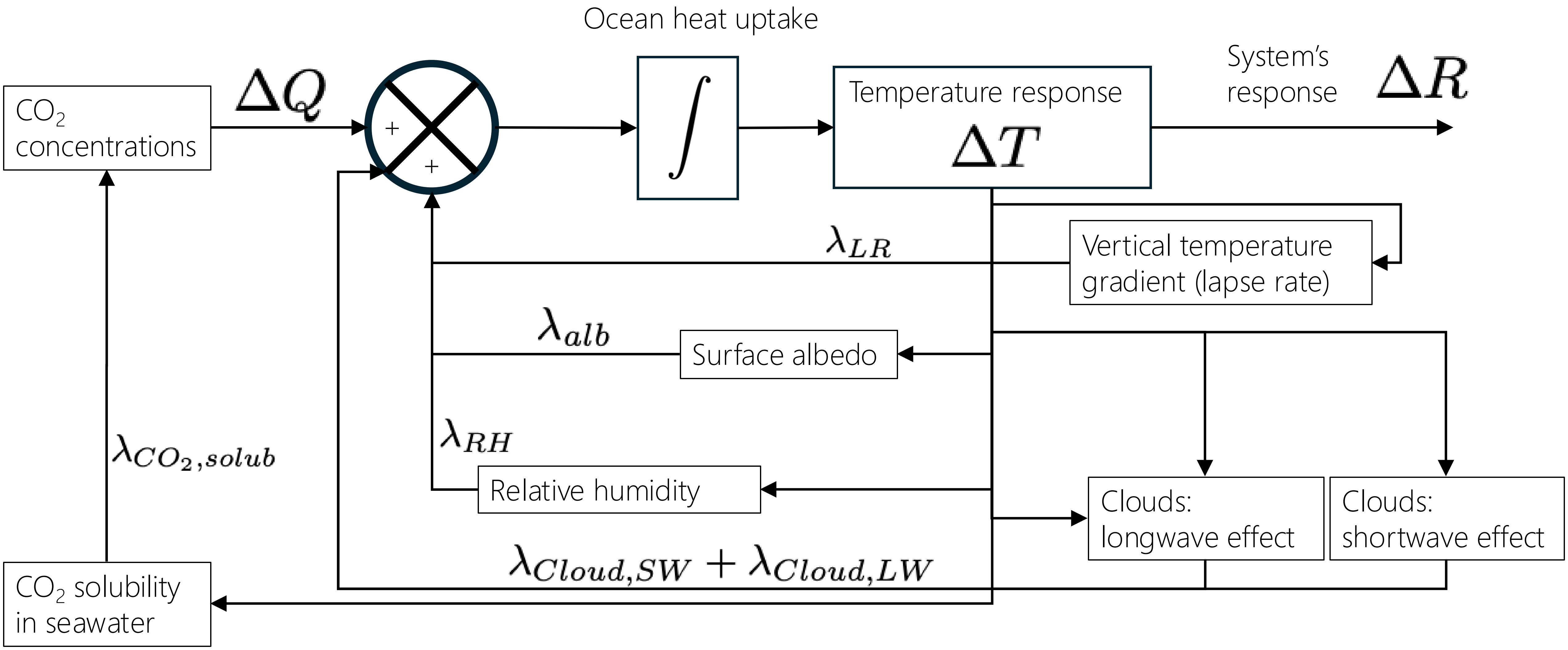
Green's function, impulse response

Laplace's transform theory, transfer function, gain, closed/open loops

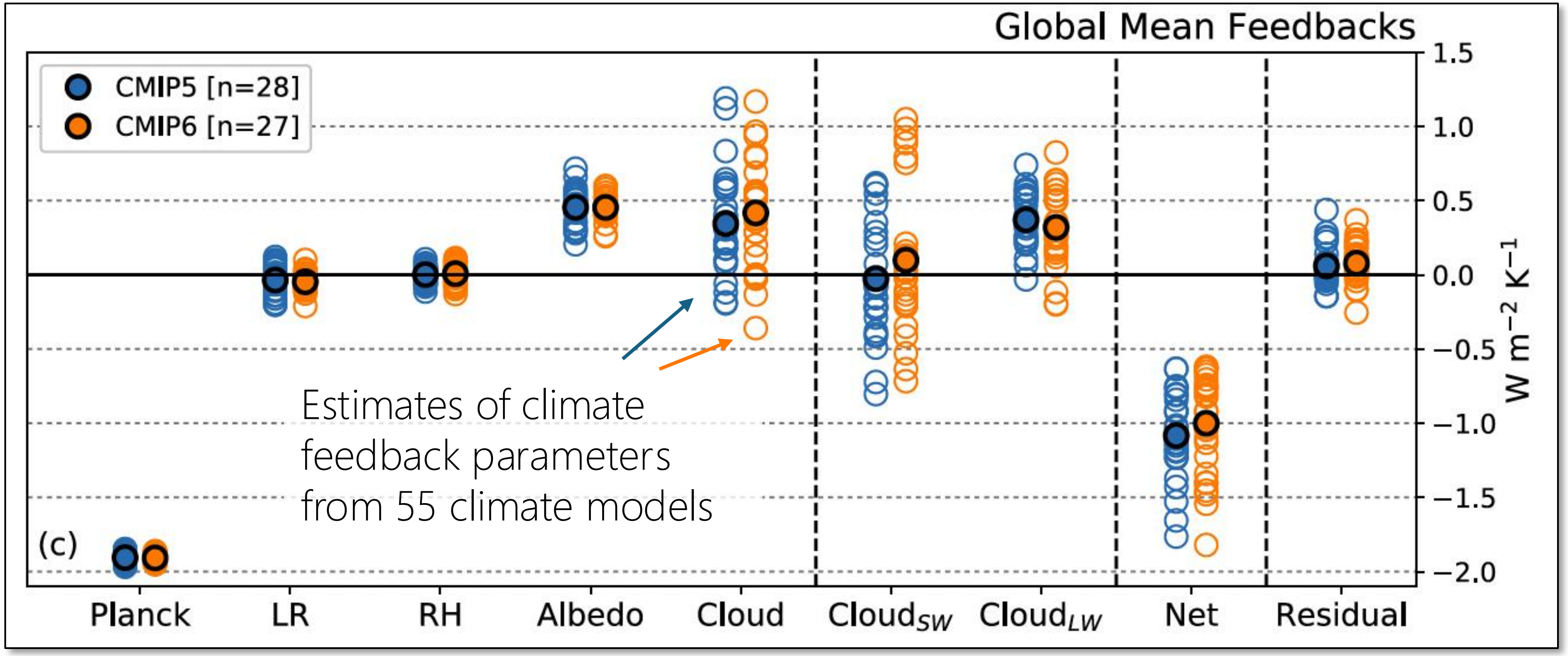
Controllability, observability

A slightly more realistic block diagram

$$\lambda = \frac{\partial R}{\partial T} = \sum_{i=1}^{n} \frac{\partial R}{\partial x_i} \frac{\partial x_i}{\partial T}$$



Climate feedback uncertainty in contemporary climate models



Radiative forcing associated with a doubling of CO_2 concentrations

$$\Delta T_{2\times CO_2} = \frac{-3.71\ \mathrm{W/m^2}}{\lambda}$$

« Climate sensitivity »

Current estimates of Earth's climate sensitivity: 2.5-6.5°C!

Zelinka, M. D., Myers, T. A., McCoy, D. T., Po-Chedley, S., Caldwell, P. M., Ceppi, P., Klein, S. A., & Taylor, K. E. (2020). Causes of Higher Climate Sensitivity in CMIP6 Models. Geophysical Research Letters, 47(1), e2019GL085782. https://doi.org/10.1029/2019GL085782

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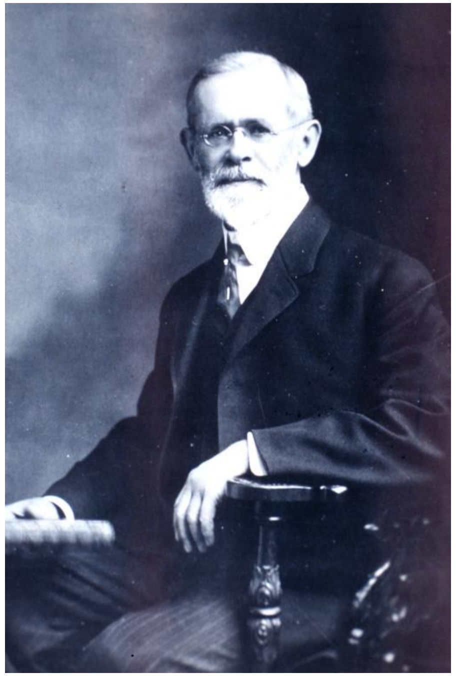
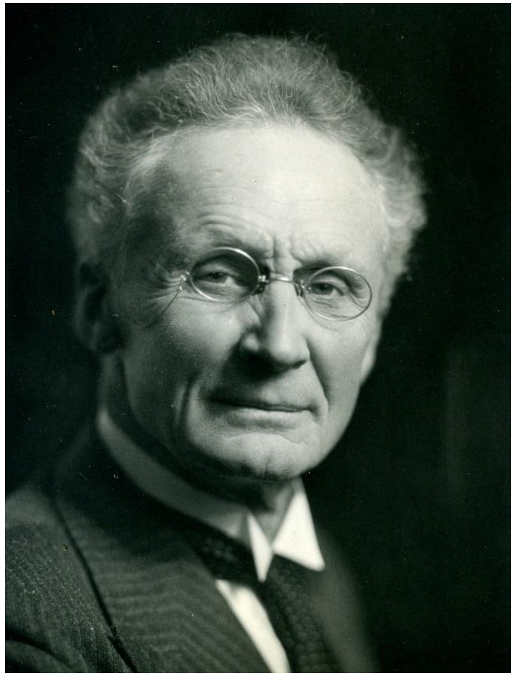


Cleveland Abbe
(1838-1916)



Vilhelm Bjerknes
(1893-1928)

Acceleration

Momentum advection

$$-\frac{\partial P}{\partial x}-\frac{1}{\sigma}\frac{\partial p}{\partial x}=\boxed{\frac{\partial u}{\partial \tau}}+\boxed{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}}-\mu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right]$$

$$\boxed{-\frac{\partial P}{\partial y}-\frac{1}{\sigma}\frac{\partial p}{\partial y}}=\frac{\partial v}{\partial \tau}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}-\mu\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right]$$

External and pressure forces

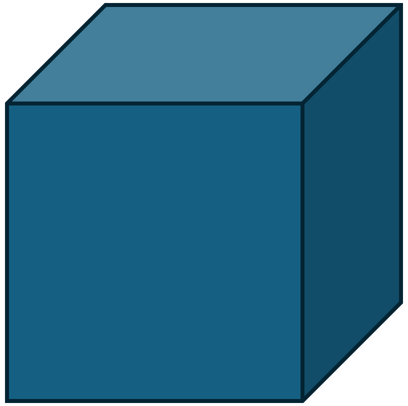
$$-\frac{\partial P}{\partial z}-\frac{1}{\sigma}\frac{\partial p}{\partial z}=\frac{\partial w}{\partial \tau}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}-\mu\boxed{\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right]}$$

Internal forces

Coupled nonlinear partial differential equations

Abbe, Cleveland. "The Physical Basis of Long-Range Weather Forecasts." Monthly Weather Review 29, no. 12 (December 1, 1901): 551–61. https://doi.org/10.1175/1520-0493(1901)29[551c:TPBOLW]2.0.CO;2.

Corollary: under assumptions, certain quantities are conserved following the motion



Absolute vorticity is conserved following the motion for a barotropic fluid of constant depth under the Boussinesq approximation

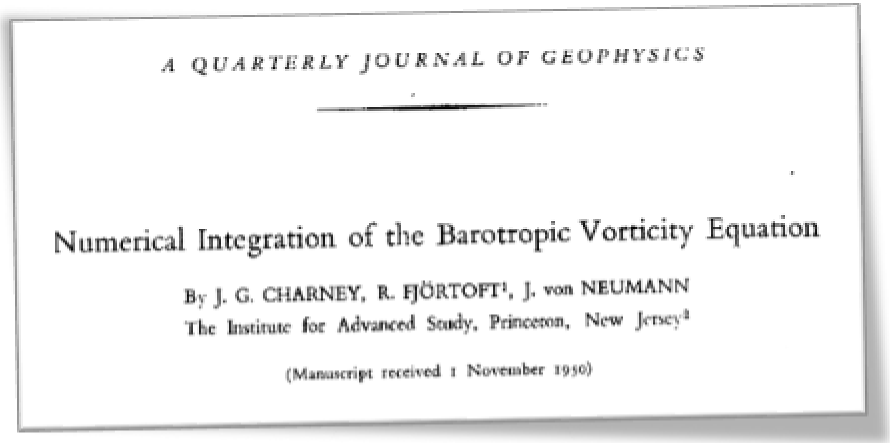
Vertical component of the fluid's vorticity

Coriolis parameter proportional to latitude

$$\frac{\mathrm{D}(\zeta + f)}{\mathrm{D}t} \overset{\text{DEF}}{=} \frac{\partial \zeta}{\partial t} + \mathbf{u} \cdot \nabla(\zeta + f) = 0$$

Fluid's velocity

Jules Gregory Charney (1917-1981)

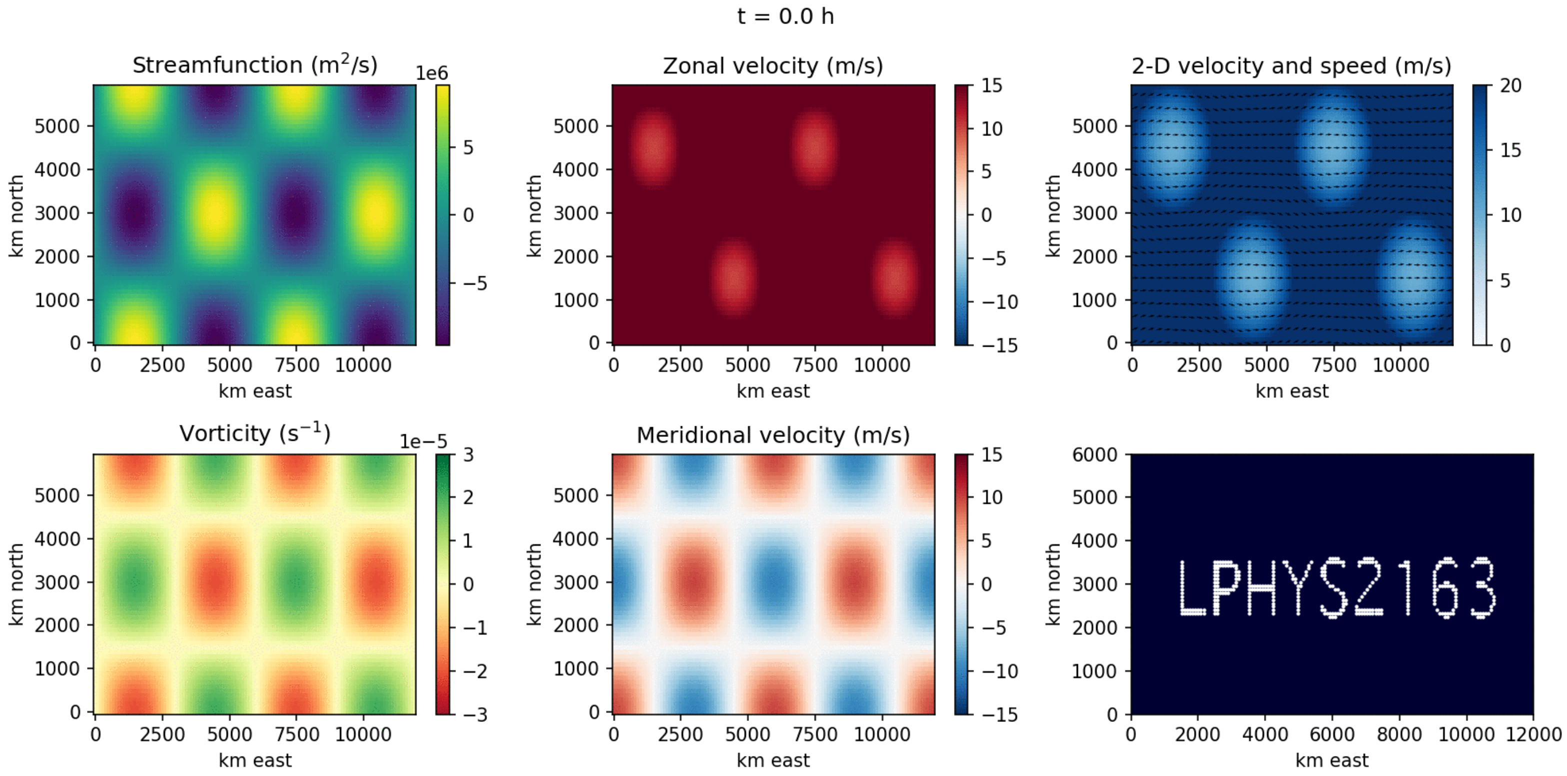


A QUARTERLY JOURNAL OF GEOPHYSICS

Numerical Integration of the Barotropic Vorticity Equation

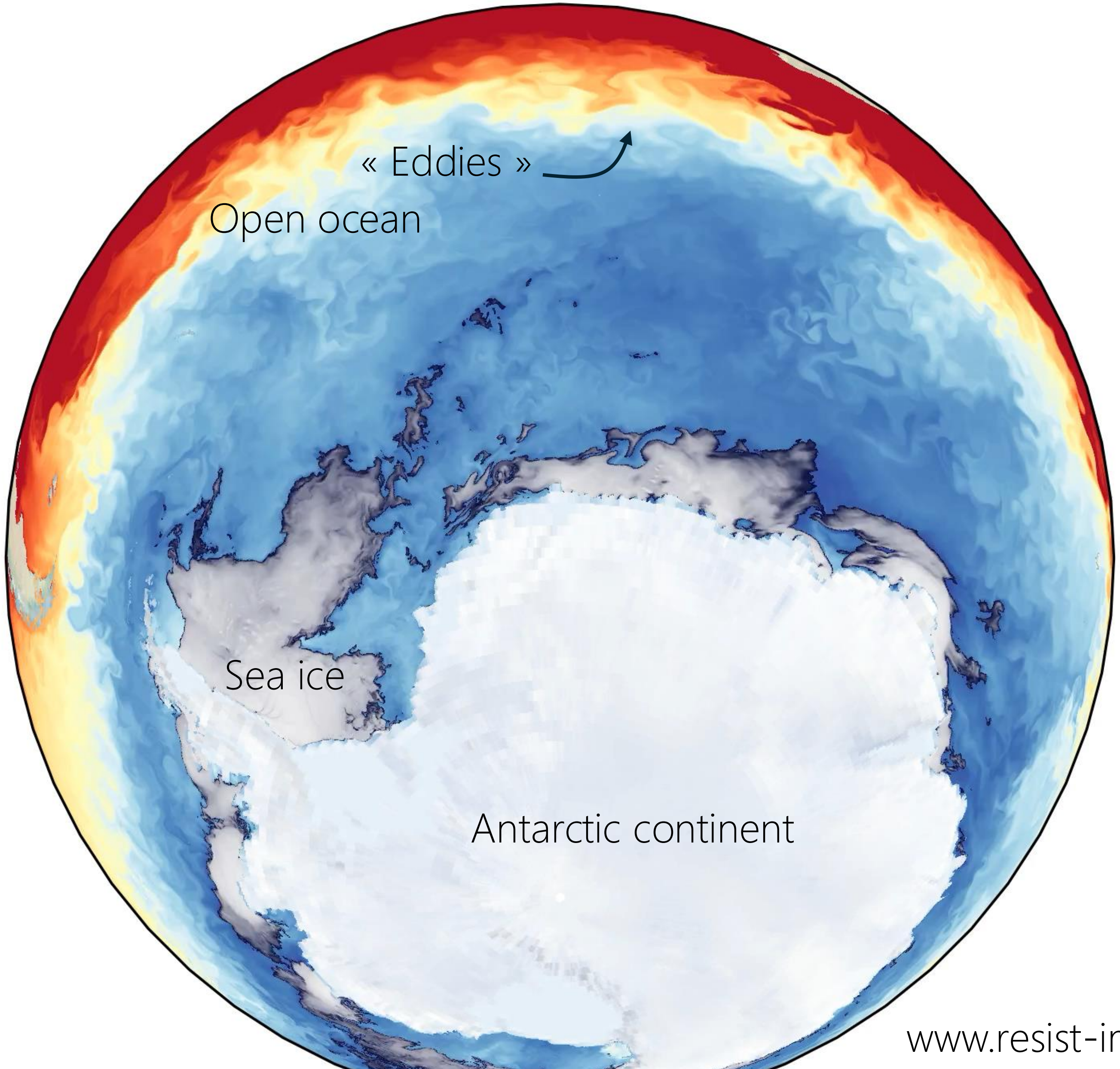
By J. G. CHARNEY, R. FJÖRTOFT[1], J. von NEUMANN
The Institute for Advanced Study, Princeton, New Jersey[2]

(Manuscript received 1 November 1950)



Barotropic Rossby waves solved by students of LPHYS2163: « Atmosphere and Ocean: Physics and Dynamics »

Numerical simulation of the ocean and sea ice with the NEMO4.2-SI3 ocean—sea ice model (1/12°)



How does kinetic energy cascade through spatial and temporal scales?

How does the general atmospheric and oceanic circulations work?

How are mass, energy, momentum redistributed over the planet?

Navier-Stokes equations, Reynolds averaging

Turbulent diffusion, eddy transport

Planetary waves, resonant behaviours

Compressible and incompressible flows

Numerical analysis and methods

High performance computing

www.resist-impuls.github.io

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A revolution in weather numerical prediction

Edward Lorenz (1917-2008)

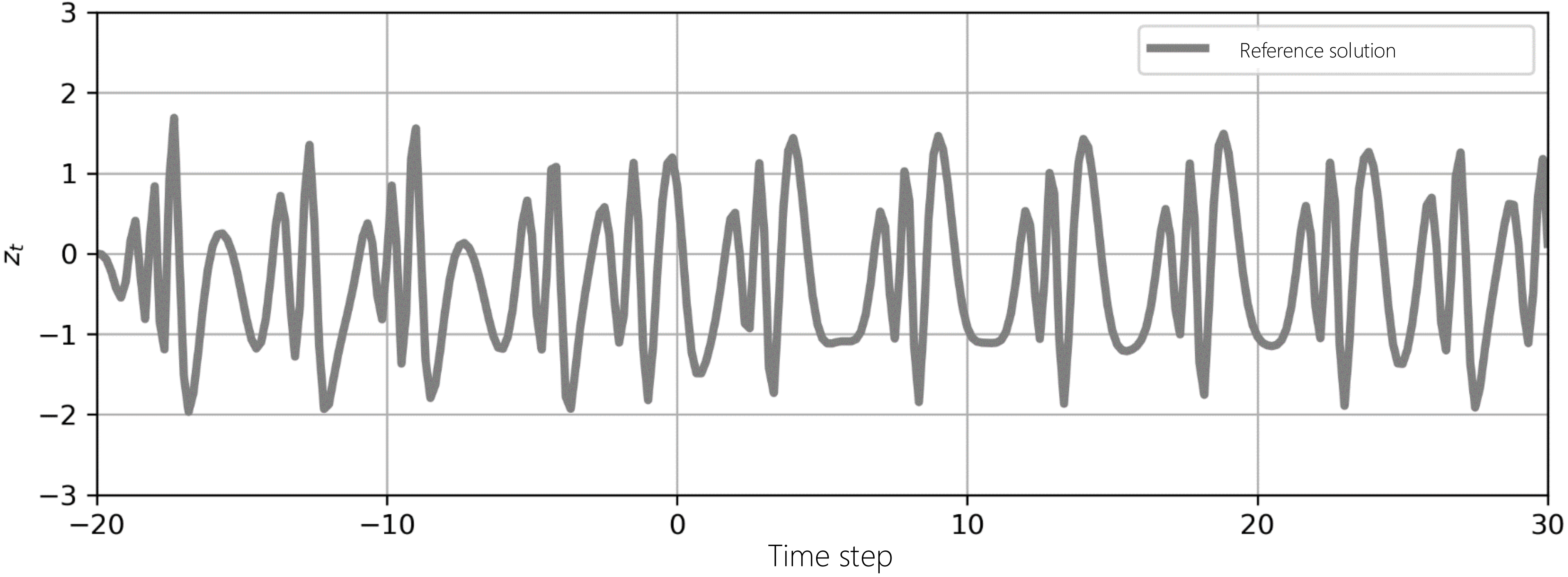
By studying simplified model for atmospheric convection, Lorenz showed in the 1960s that these equations admit solutions that exhibit high **sensitivity to initial conditions, and display irregular, quasi-periodic behaviour**

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$

Deterministic….

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$

Plot of *z(t)*



Deterministic….

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
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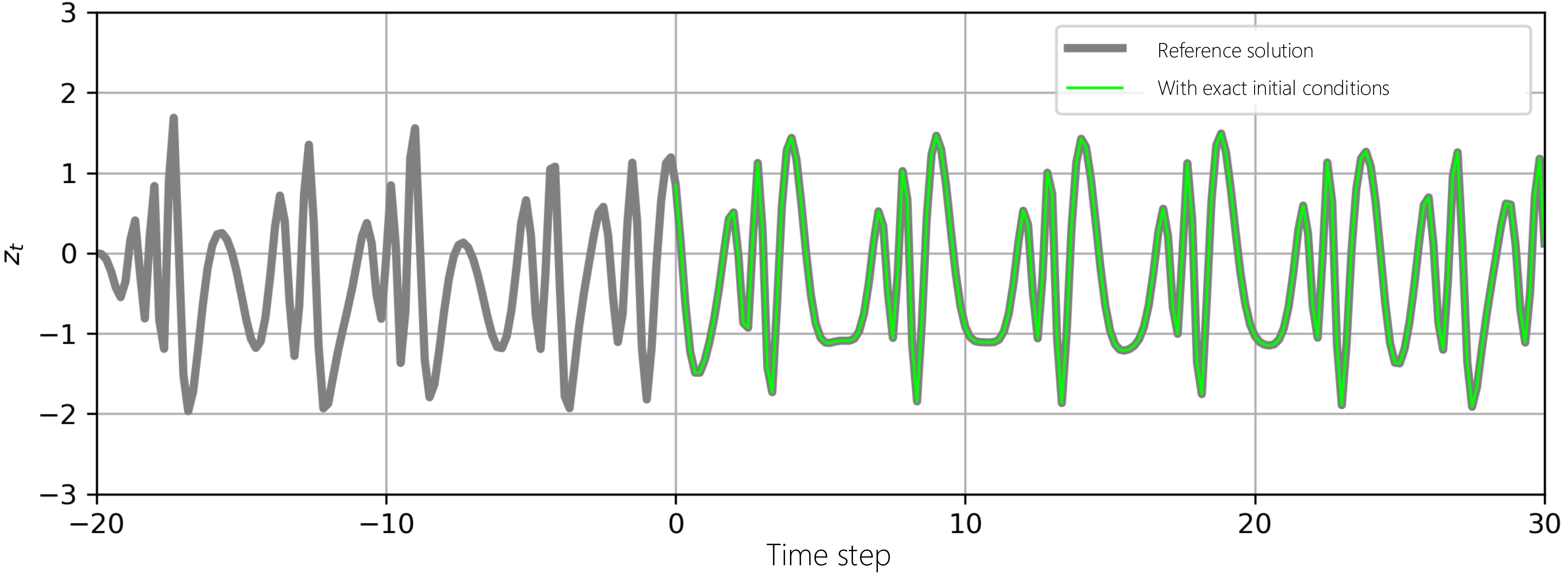
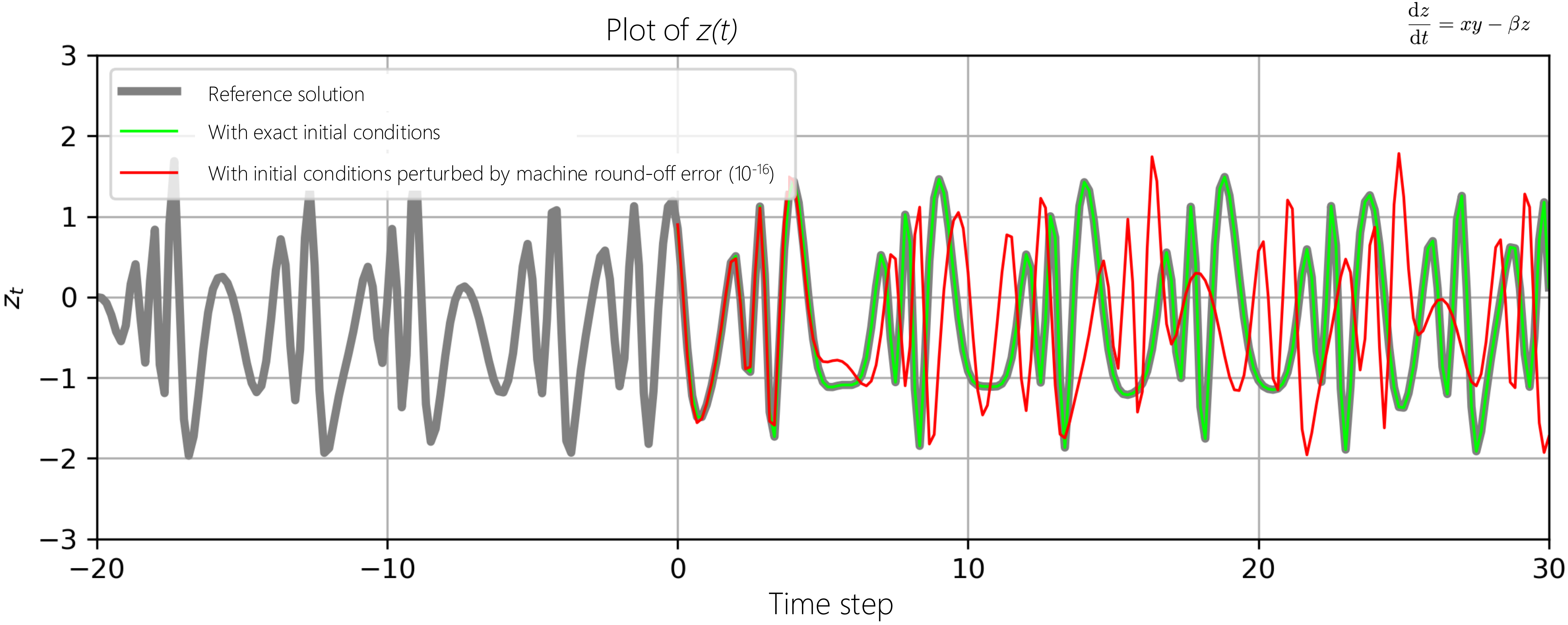
Plot of *z(t)*



… but highly sensitive to initial conditions

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \sigma(y - x)$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = x(\rho - z) - y$$
$$\frac{\mathrm{d}z}{\mathrm{d}t} = xy - \beta z$$



How a fortuitous discovery shook up weather and climate science

1963: Discovery of the « butterfly effect »

1968: Quantification of predictability cascade through scales

1980s: First ensemble predictions

1997: Evidence for predictability limits in ocean (Griffies & Bryan, 1997)

How can one predict climate if weather is unpredictable beyond 15 days?

What are upper limits of predictability in the climate system?

How to reliably sample initial-condition uncertainty?

Linear stability theory, eigenvalue/vector analysis

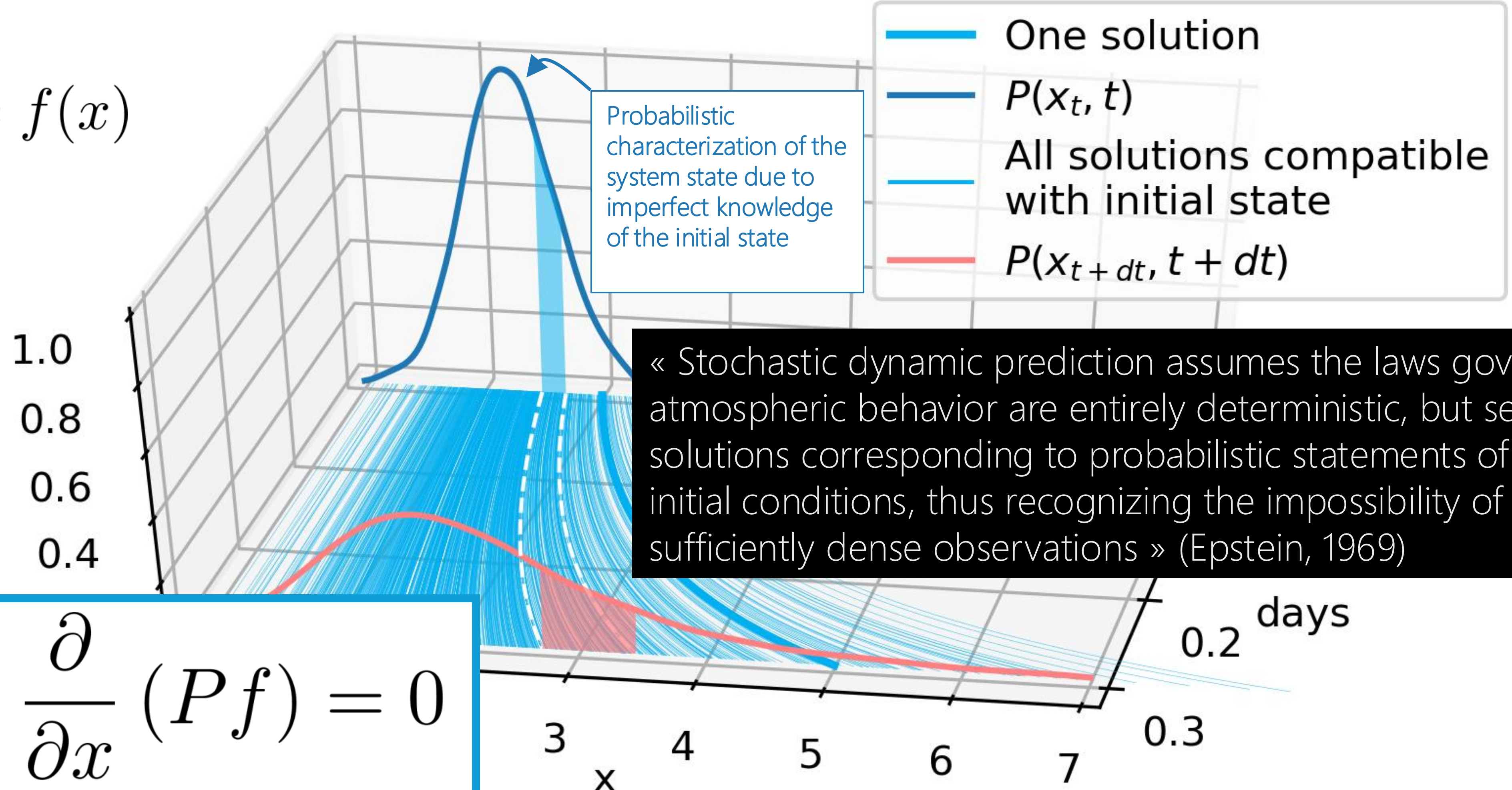
Lyapunov exponents, error growth

Optimal ensemble generation

Stochastic modeling and prediction

Stochastic dynamic prediction: uncertainty does not imply unpredictability

$$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$$



« Stochastic dynamic prediction assumes the laws governing atmospheric behavior are entirely deterministic, but seeks solutions corresponding to probabilistic statements of the initial conditions, thus recognizing the impossibility of exact or sufficiently dense observations » (Epstein, 1969)

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x}(Pf) = 0$$

The (1-D) Liouville Equation

Consequences of the butterfly effect: internal climate variability

- Since the 1960s, we know that despite the deterministic nature of laws that govern the climate system, **any attempt to model it deterministically is a vain hope**.
- Weather and climate model simulations **always** come as finite-size ensembles to sample the climate's **internal variability**
- Any observed climate time series should be thought as **one realization of a stochastic process**, which could have generated many other time series compatible with the underlying dynamics

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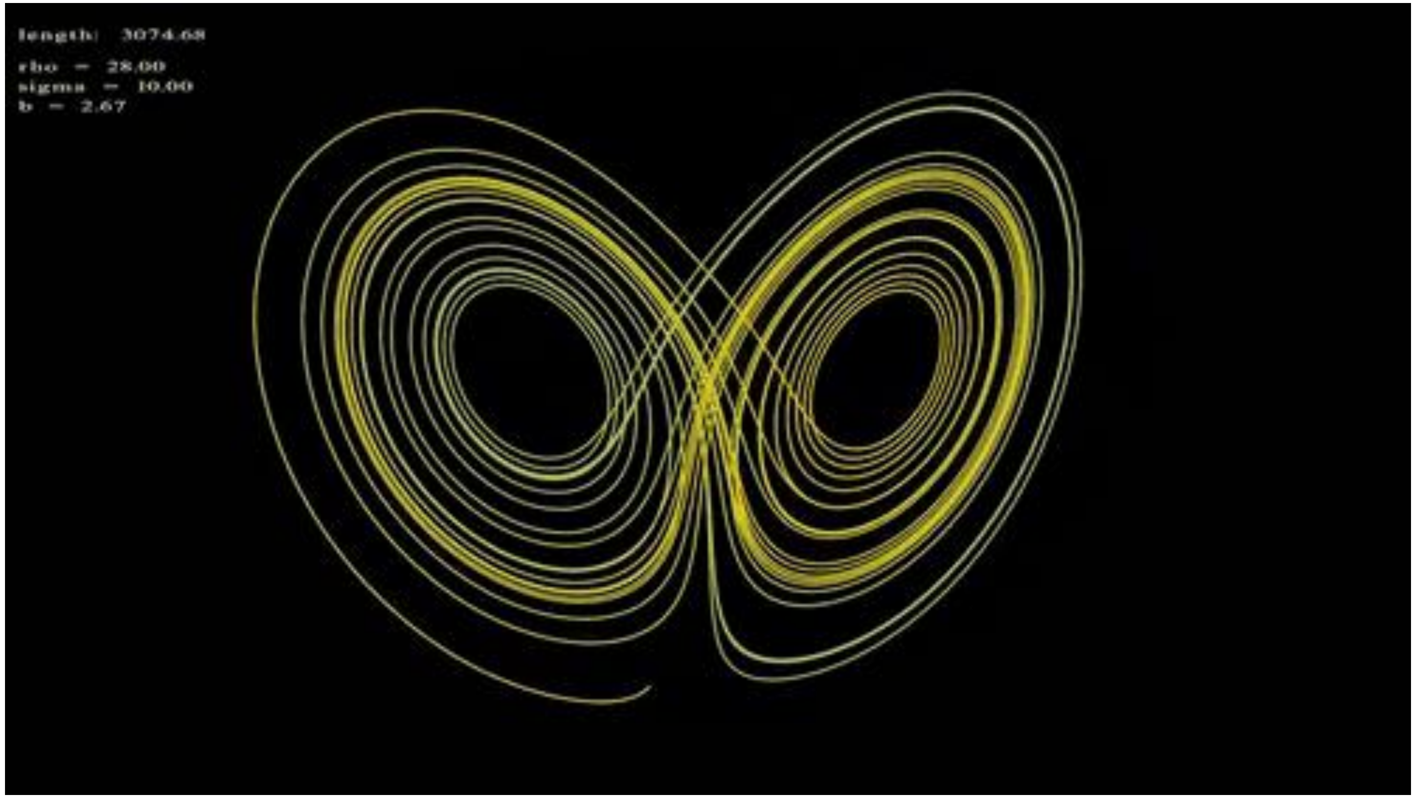


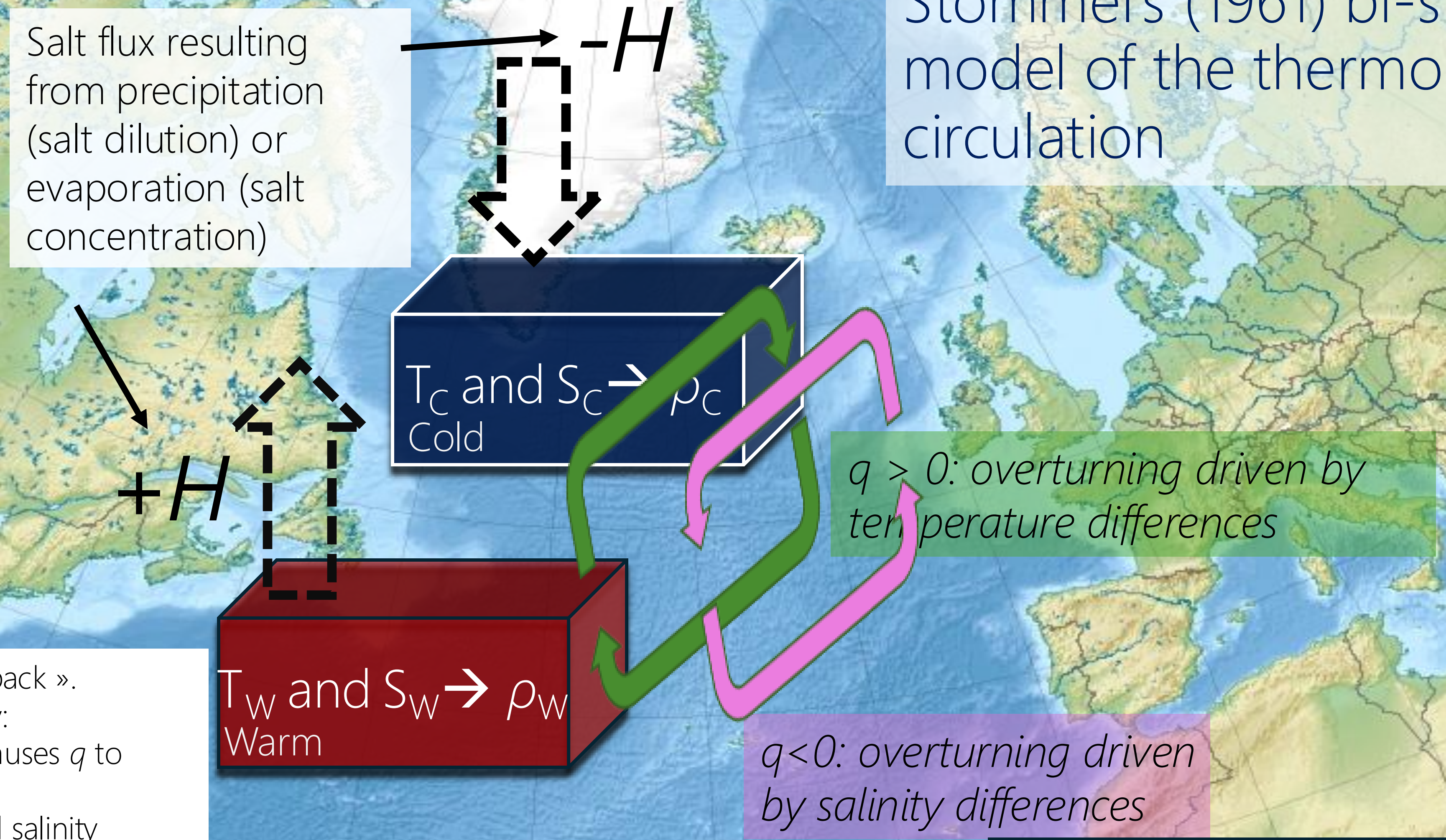
What are the equilibrum states of the Earth's sub-components?

What are the Earth's tipping points, and have we crossed some of them?

Limit cycles, fixed points, bifurcations, hysteresis

Potentials, stability, exponential growth of perturbations

Stommel's (1961) bi-stable two-box model of the thermohaline circulation



« Salt advection feedback ».
Suppose $q > 0$ initially:

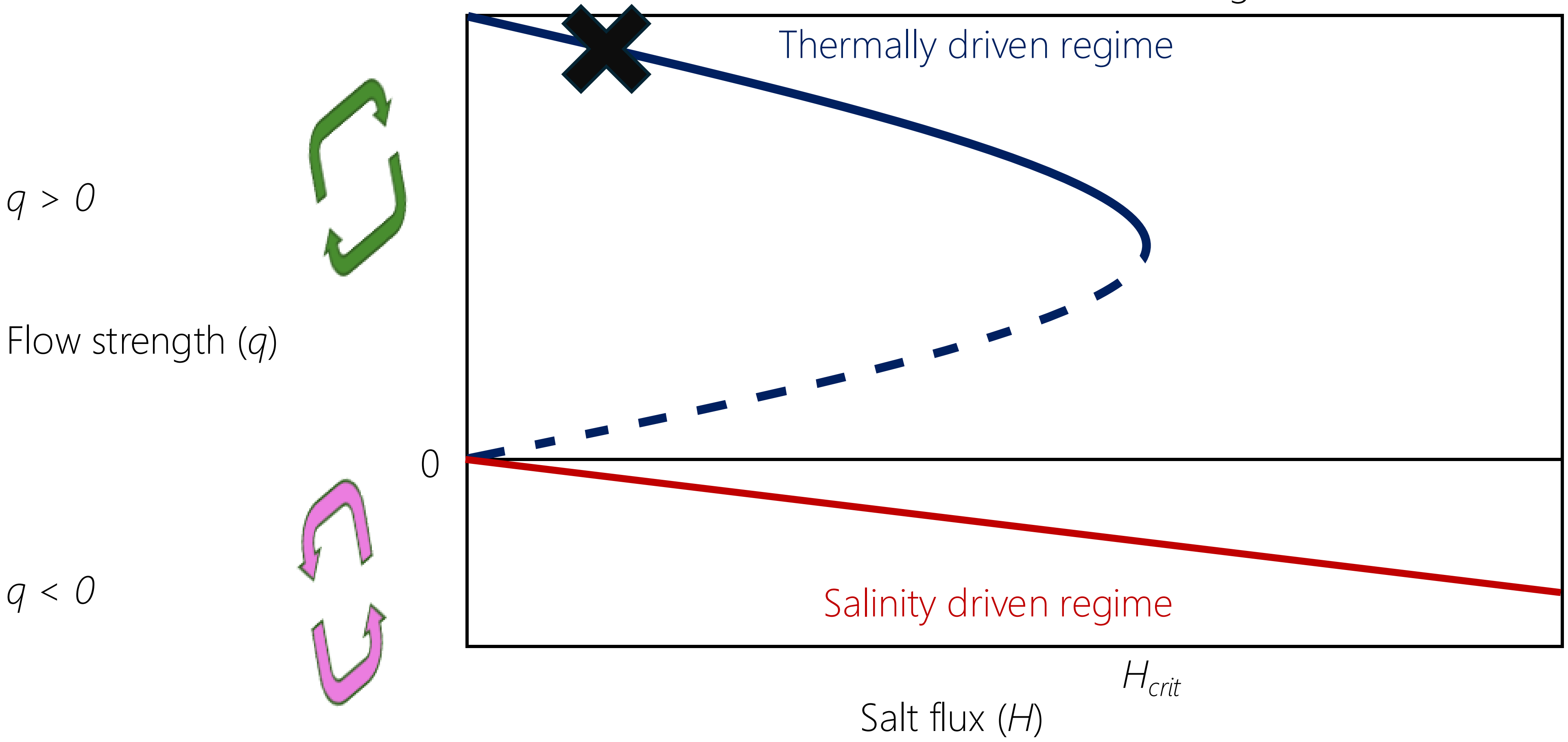
1. A perturbation causes q to increase: $|q| \nearrow$
2. The net poleward salinity transport increases: $S_C \nearrow$, $S_W \searrow$
3. The poleward box's density increases even more
4. The overturning q is further enhanced

$$\frac{\mathrm{d}S_C}{\mathrm{d}t} = + |q|(S_W - S_C)$$

$$\frac{\mathrm{d}S_W}{\mathrm{d}t} = - |q|(S_W - S_C)$$

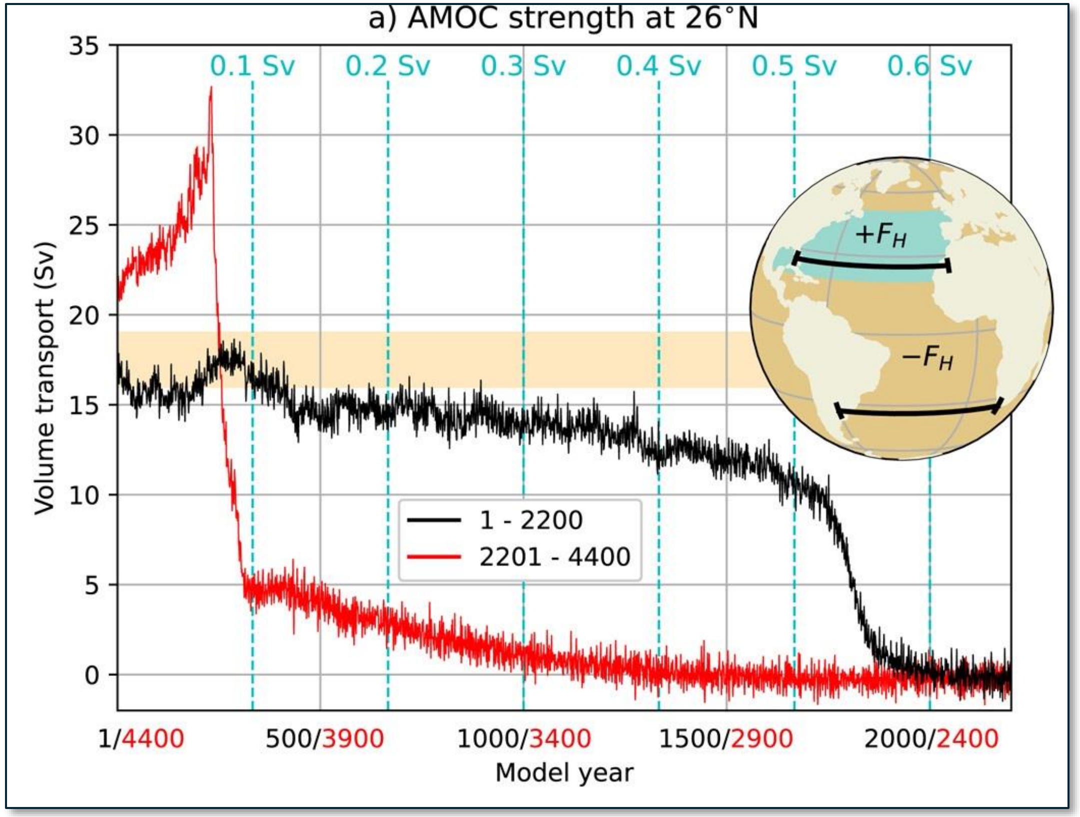
Stommel's (1961) bi-stable two-box model of the thermohaline convection

Bifurcation diagram of the Stommel dynamical model: steady state solution as a function of the forcing H



A stable equilibrium can be sustained until a critical salt flux H_{crit}, beyond which an abrupt transition to another stable equilibrium occurs

This hysteresis behaviour is captured by more complex climate models (and subject to much mediatic fantasies…)



van Westen, R. M., & Dijkstra, H. A. (2023). Asymmetry of AMOC Hysteresis in a State-Of-The-Art Global Climate Model. Geophysical Research Letters, 50(22), e2023GL106088. https://doi.org/10.1029/2023GL106088

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Detection: Has a signal emerged from the background climate noise?

THE ARTIFICIAL PRODUCTION OF CARBON DIOXIDE AND ITS INFLUENCE ON TEMPERATURE

By G. S. CALLENDAR

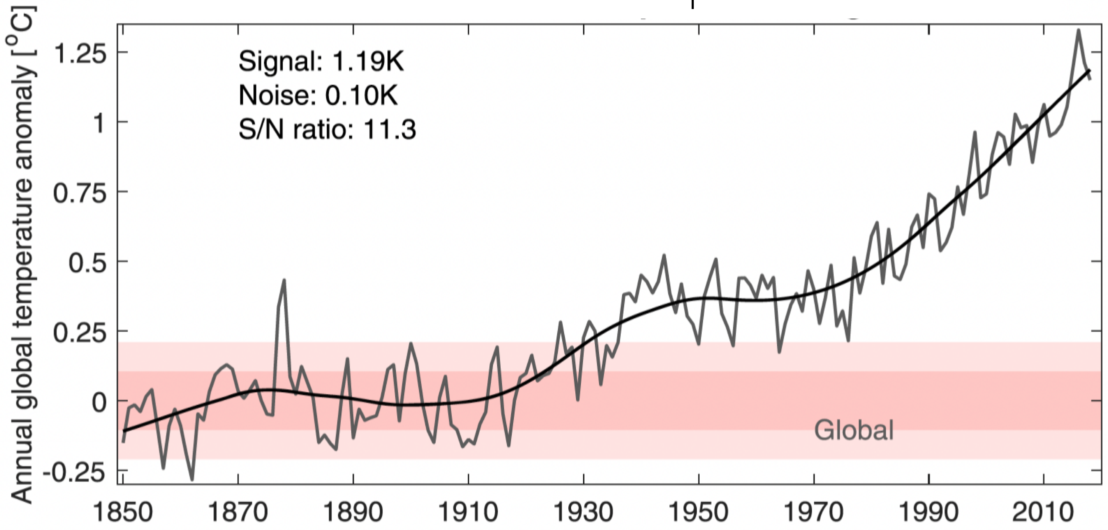
(Steam technologist to the British Electrical and Allied Industries Research Association.)

(Communicated by Dr. G. M. B. Dobson, F.R.S.)

[Manuscript received May 19, 1937—read February 16, 1938.]

1938

Global mean temperature



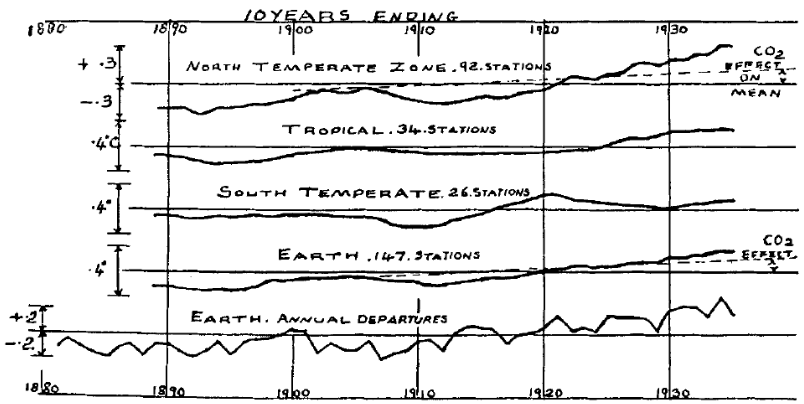


Fig. 4.—Temperature variations of the zones and of the earth. Ten-year moving departures from the mean, 1901-1930, °C.

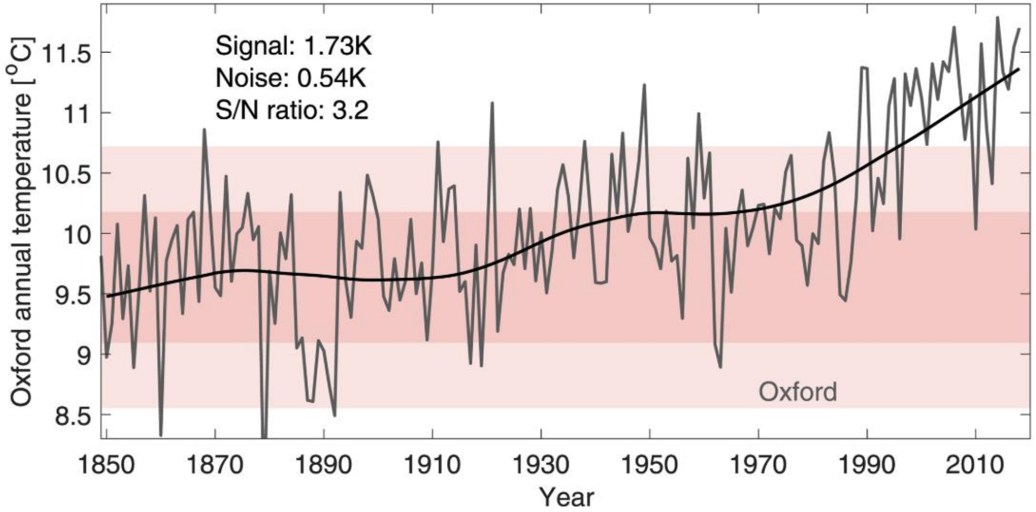
Background climate variability due to natural drivers

Globally, a temperature change has been detectable since the 1930s

Hawkins, E., Frame, D., Harrington, L., Joshi, M., King, A., Rojas, M., & Sutton, R. (2020). Observed Emergence of the Climate Change Signal: From the Familiar to the Unknown. Geophysical Research Letters, 47(6), e2019GL086259. https://doi.org/10.1029/2019GL086259

Detection: Has a signal emerged from the background climate noise?

Oxford temperature

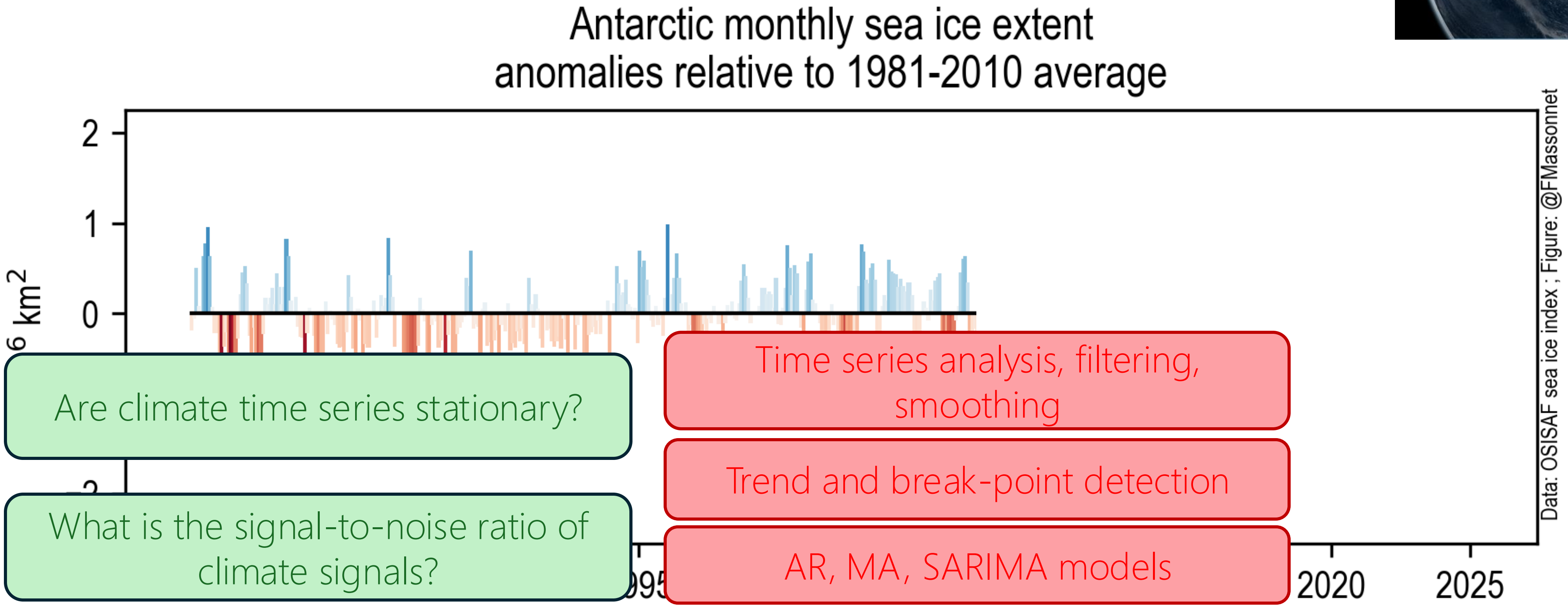


Background climate variability due to natural drivers

Regionally (here, Oxford), temperature changes are less easy to detect

Hawkins, E., Frame, D., Harrington, L., Joshi, M., King, A., Rojas, M., & Sutton, R. (2020). Observed Emergence of the Climate Change Signal: From the Familiar to the Unknown. Geophysical Research Letters, 47(6), e2019GL086259. https://doi.org/10.1029/2019GL086259

Detection: is Antarctic sea ice experiencing a regime shift?



Are climate time series stationary?

What is the signal-to-noise ratio of climate signals?

What is the frequency content of climate signals?

Time series analysis, filtering, smoothing

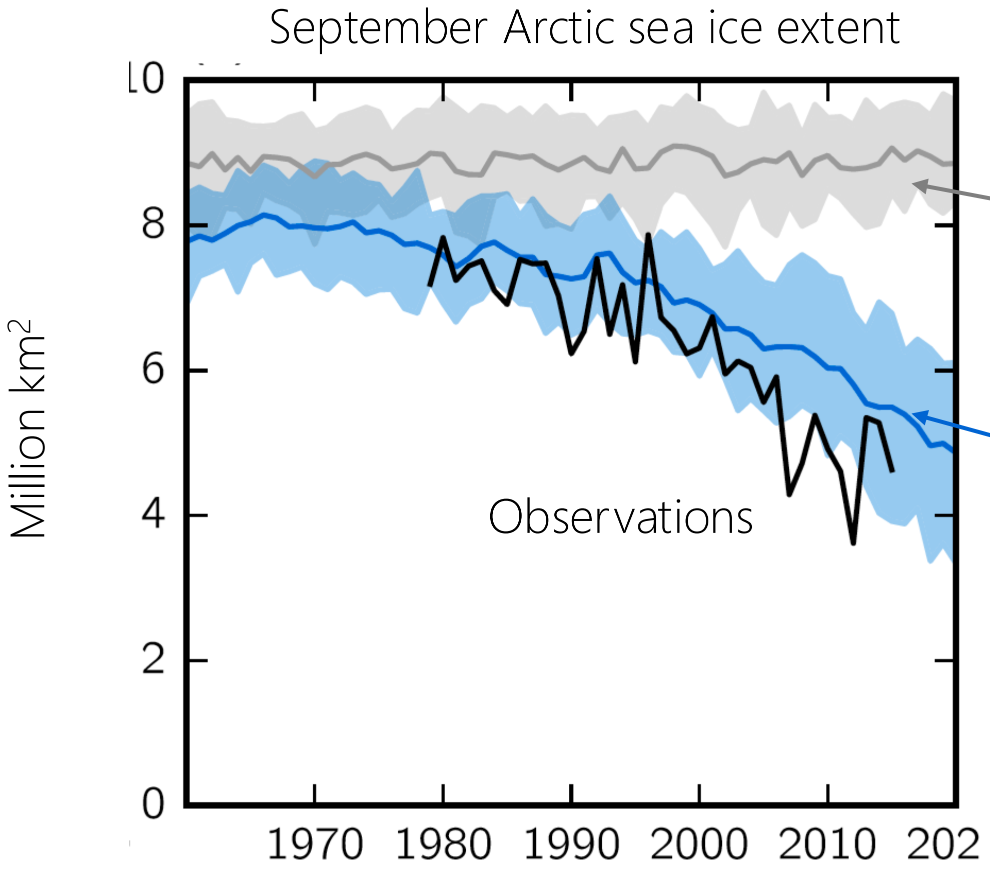
Trend and break-point detection

AR, MA, SARIMA models

Spectral, wavelet analyses

Noise estimation

Attribution: what causal factors are driving a detected change in the time series?



Mean and 90% confidence interval of climate model ensemble with **pre-industrial forcings only**

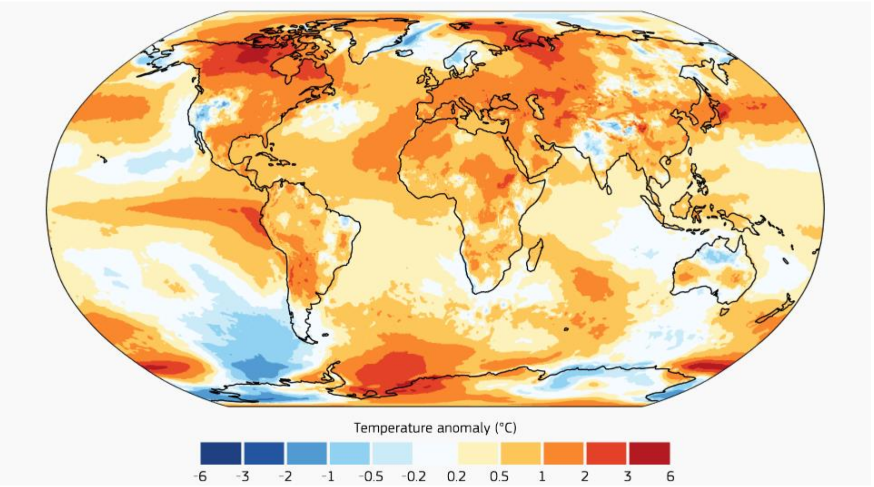
« counter-factual world »

Mean and 90% confidence interval of climate model ensemble with **all forcings (incl. anthropogenic)**

« Factual world »

Kirchmeier-Young, M. C., Zwiers, F. W., & Gillett, N. P. (2017). Attribution of Extreme Events in Arctic Sea Ice Extent. Journal of Climate, 30(2), 553–571. https://doi.org/10.1175/JCLI-D-16-0412.1

Disentangling drivers of observed changes



Scaling factor

$$\mathbf{Y}(\mathbf{x},t) = \sum_{i=1}^{n} \beta_i \underbrace{\mathbf{X}_i(\mathbf{x},t)}_{\text{« Fingerprint »}} + \mathbf{U}(\mathbf{x},t)$$

Observed field

Candidate drivers

Residual (internal climate variability, unexplained variability)

- Which causal factors are necessary / sufficient to explain the data?
- What are the relative contributions of causal factors to the observed change?
- Is the hypothesis of additivity / linearity valid?
- How much did climate change increase the probability of occurrence a given extreme event?

- Optimal fingerprinting
- Multilinear regression, least squares
- Hypothesis testing, maximum likelihood
- Model selection, BIC, AIC
- Extreme value theory, extremal distributions

Climate science: goldmine for applied mathematicians

1. What sets Earth's average temperature?	Radiative balance, feedback theory
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Data assimilation: estimate the complete state of a system given noisy observations and imperfect dynamical models

$\mathbf{x}_f$

First guess (same dimension as the state), i.e., an estimate of x obtained from a numerical climate model with errors

Error statistics known:

$$\mathbf{x}_f - \mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{P}_f)$$

m x 1

$\mathbf{x}$

State of the system (O(10^{10}))
All climate variables everywhere at any time

m x 1

$\mathbf{d}$

p x 1

Observations (O(10^5))
i.e. weather station data, satellite, ice cores, ... Close to reality but very sparse

Error statistics known:

$$\mathbf{d} - \mathbf{H}\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$$

Projection of x in observation space

$$\mathbf{x}_a = \mathbf{L}\mathbf{x}_f + \mathbf{K}\mathbf{d}$$

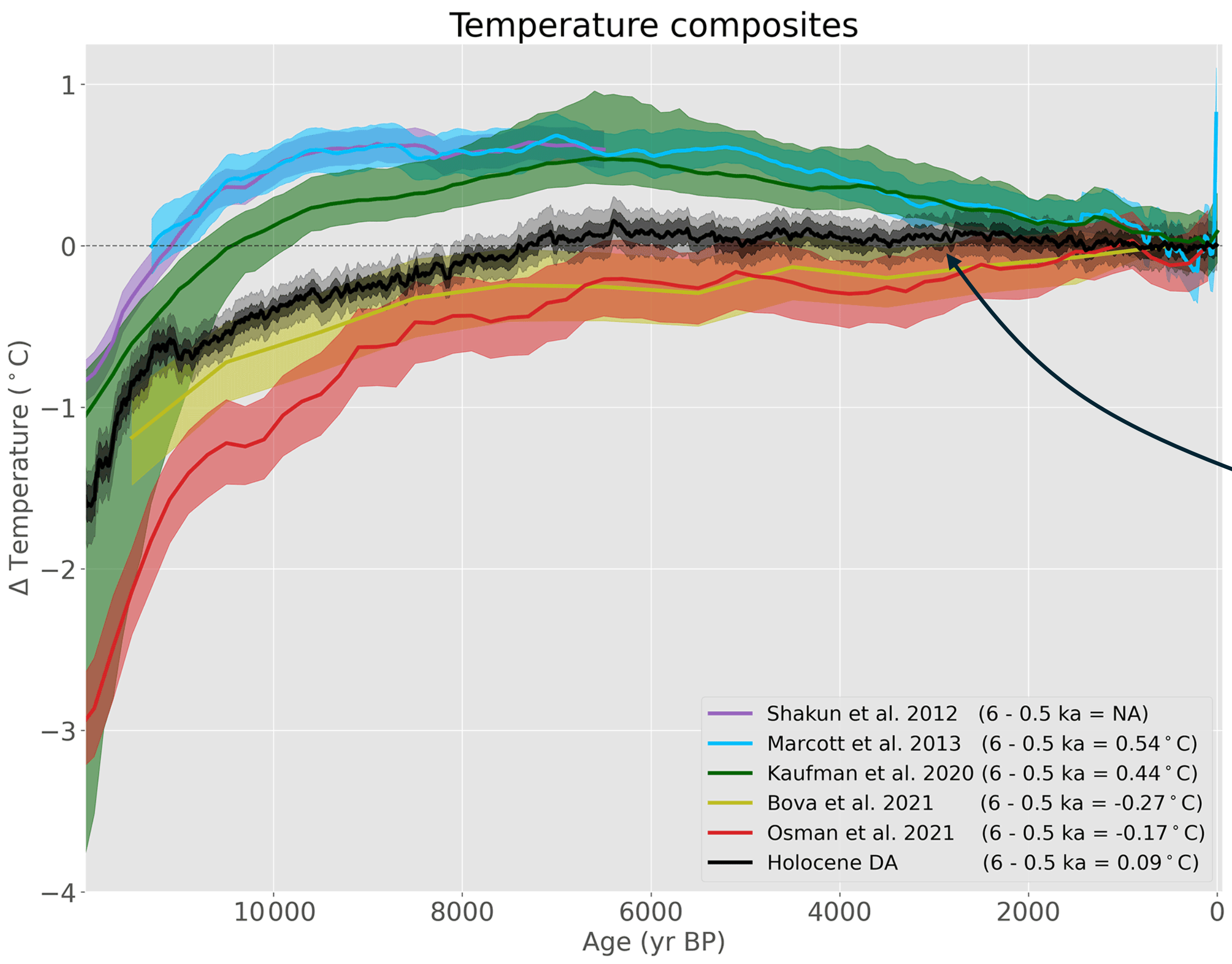
Optimization problem: Find L (*m* x *m*) and K (*m* x *p*) so that the error associated with x_a is minimal in a least square sense (usually: minimize the trace of its error covariance matrix)

Kalman filtering

Variational data assimilation (3D, 4D Var)

Particle filtering

How data assimilation helps to reconstruct past climate changes (and realize where we are…)



We are here (2025)

Temperature reconstruction with data assimilation of Earth's temperature since the end of the last glaciation

Erb, M. P., McKay, N. P., Steiger, N., Dee, S., Hancock, C., Ivanovic, R. F., Gregoire, L. J., & Valdes, P. (2022). Reconstructing Holocene temperatures in time and space using paleoclimate data assimilation. Climate of the Past, 18(12), 2599–2629. https://doi.org/10.5194/cp-18-2599-2022

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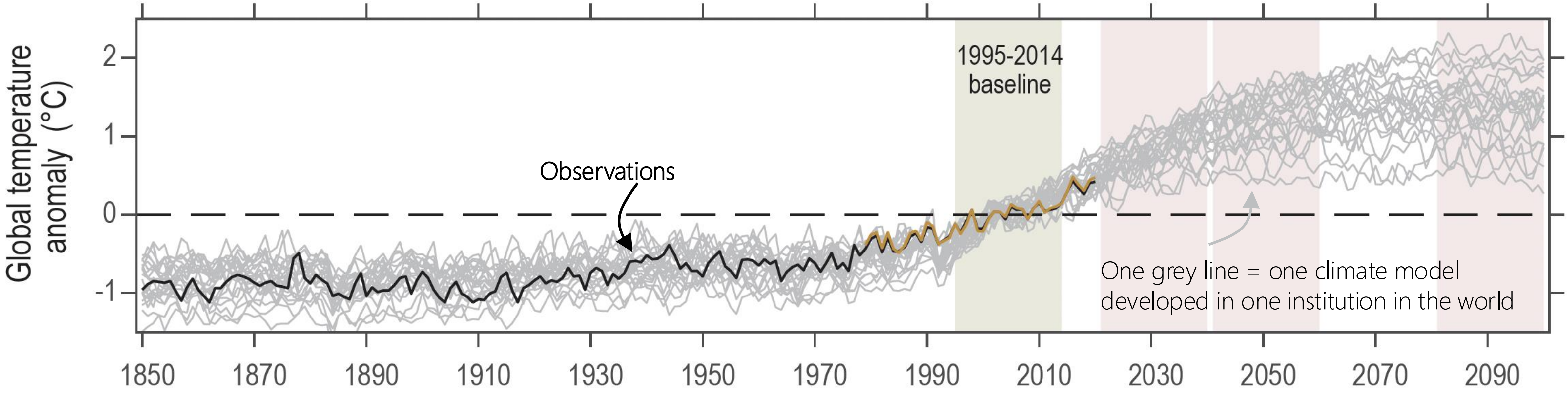
What to do with the output from 50 climate models?

« Spaghetti plot » of projections of global mean temperature from models assessed in IPCC reports



- « Model democracy »? (one model = one vote)
- Model selection?: discard poorly performing models? On what basis?
- Non-uniform model weighting?
  - Based on performance? Based on code proximity?

Chen, D., et al, 2021: Framing, Context, and Methods. In Climate Change 2021: The Physica Science Basis. Contribution of Working Group the Sixth Assessment Report of the Intergovernmental Panel on Climate Change, doi:10.1017/9781009157896.003.

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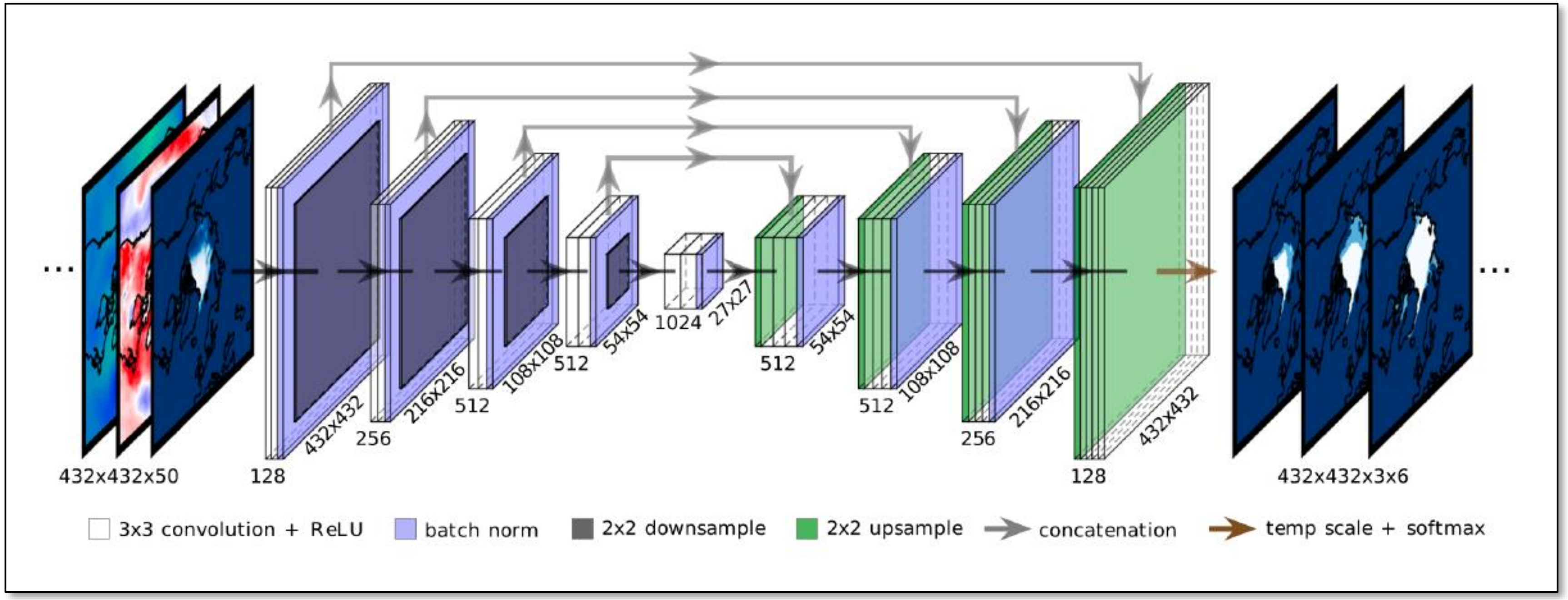
Seasonal forecasting with deep learning

U-net architecture for a seasonal prediction system of sea ice in the Antarctic
(Master thesis of Justin Lalieu, co-supervised by J.-C. Delvenne and F. Massonnet)



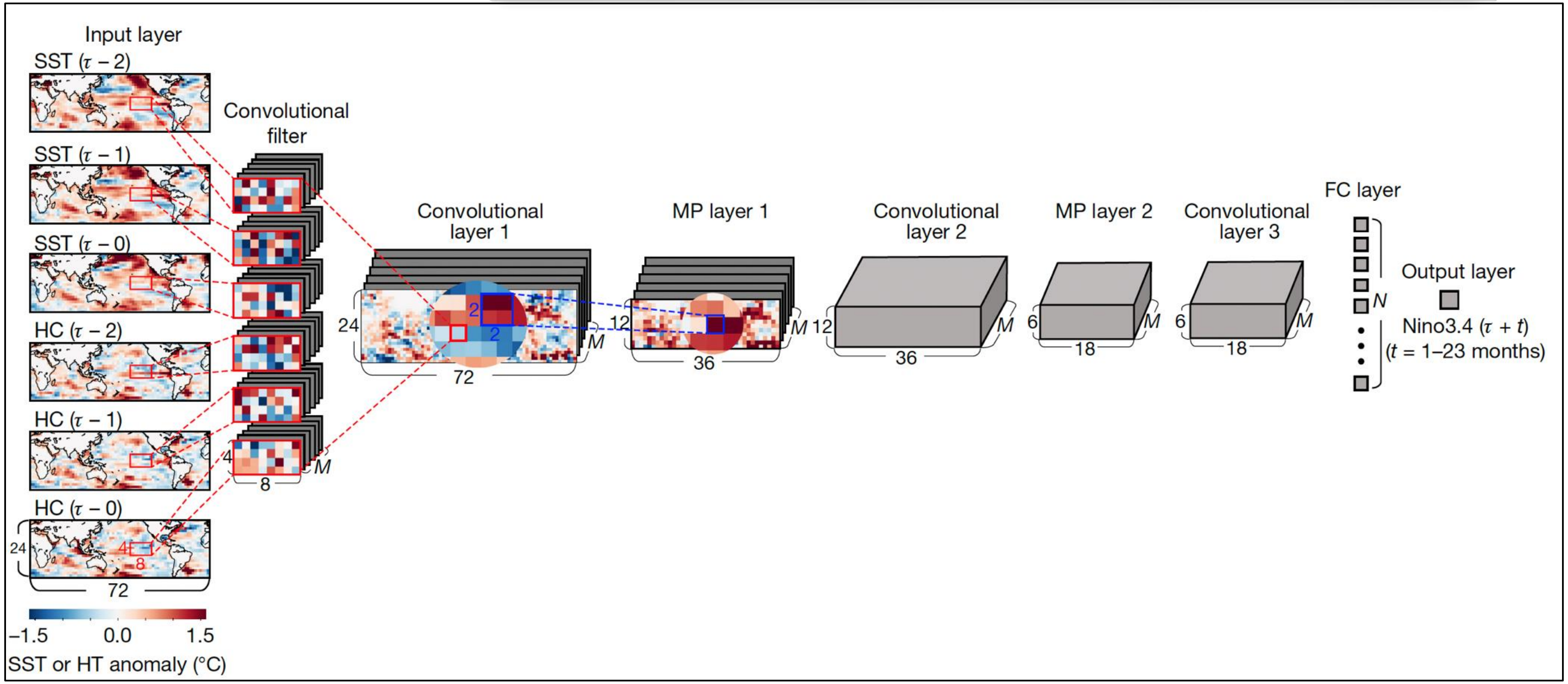
Seasonal forecasting with deep learning


LETTER

https://doi.org/10.1038/s41586-019-1559-7

Deep learning for multi-year ENSO forecasts

Yoo-Geun Ham[1*], Jeong-Hwan Kim[1] & Jing-Jia Luo[2,3]



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The atmospheric circulation features:

- Auto-correlation in time
- Auto-correlation in space
- High dimensionality
- Complex variability but also recurrent patterns

Is there a way to decorrelate the data and make it 'speak' through only a few main modes?

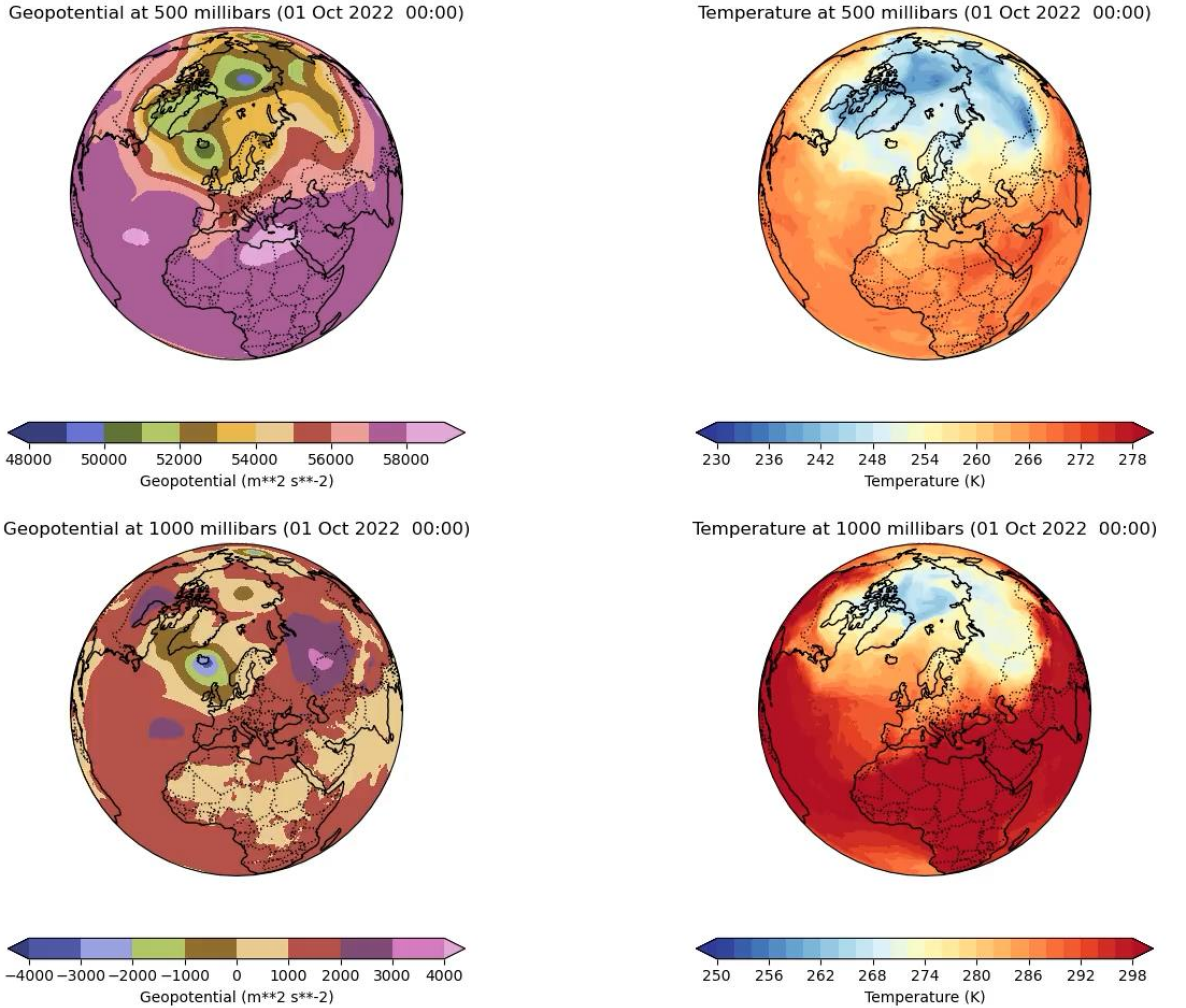
What are the canonical patterns of atmospheric variability?

Principal component analysis (in climate science: Empirical Orthogonal Functions)

K-means clustering

Self-organizing maps

Agglomerative/divisive hierarchical clustering



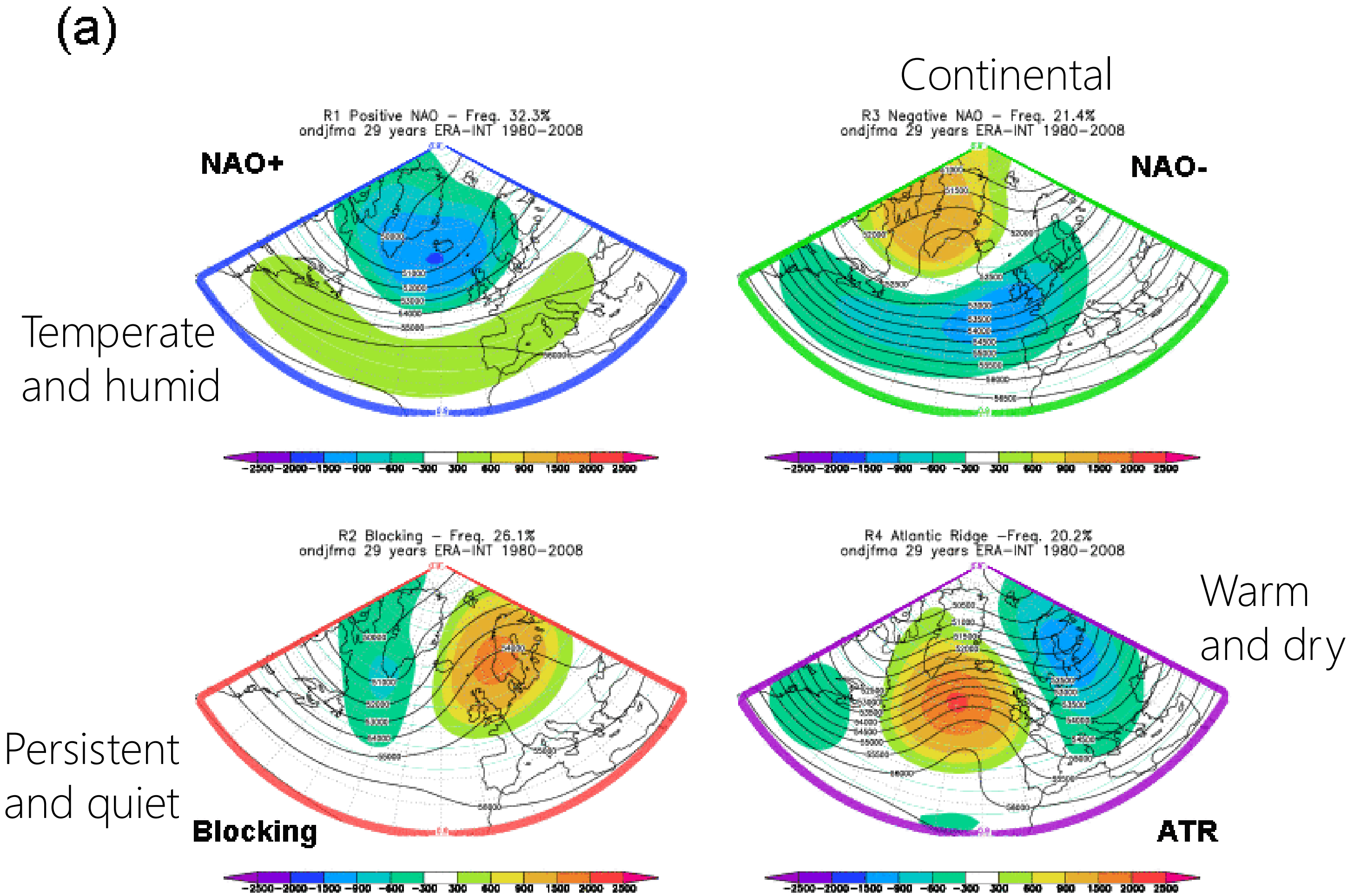
Weather regime classification

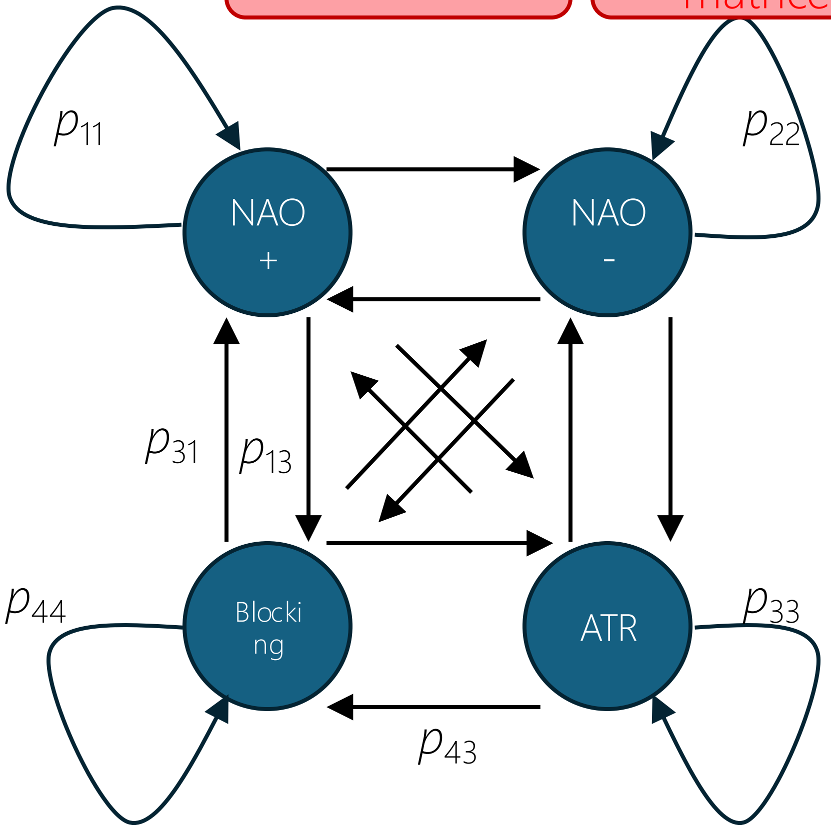
What is the marginal probability of being in state *X*?

What is the conditional probability of being in state *X* tomorrow given we are in state *Y* today ?

Markov chains

Transition matrices





https://charts.ecmwf.int/products/extended-regime-probabilities?forecast_from=latest

Master thesis A. Loiseau

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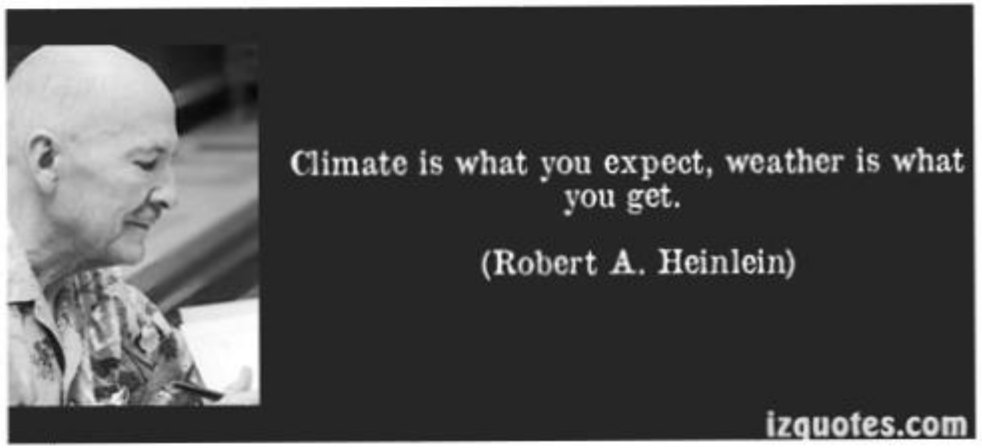
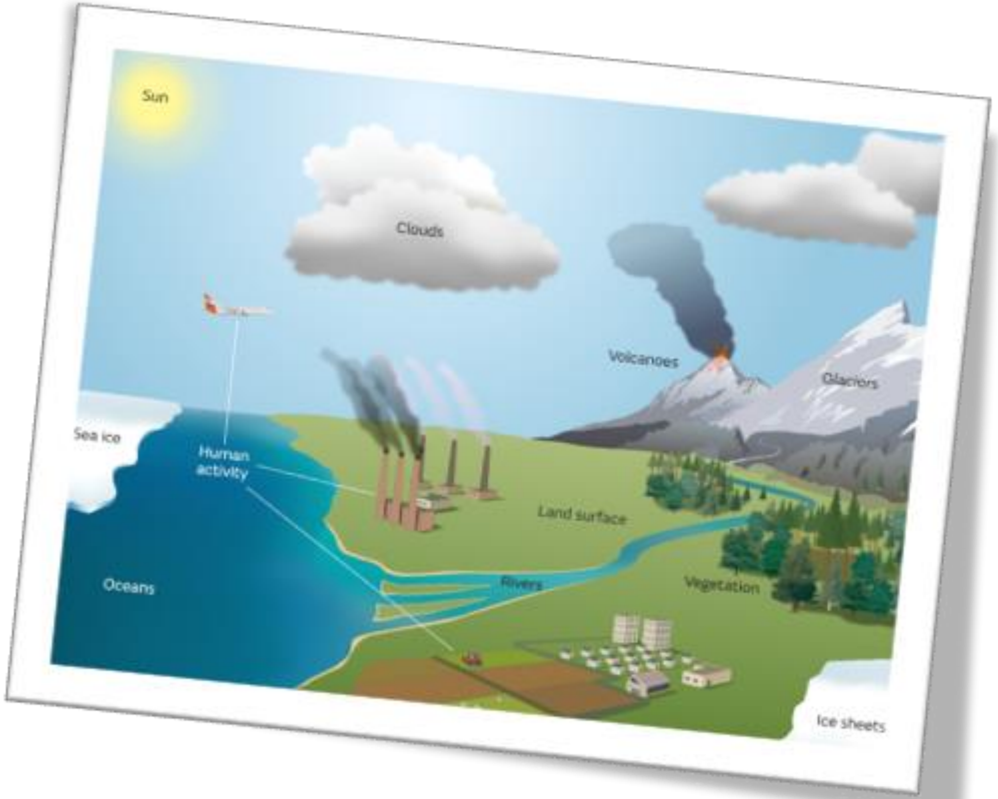
What is climate?

Climate is the general weather over a long period. This can include rainfall, temperature, snow or any other weather condition. We usually define a region's climate over a period of 30 years.





Climate: a system? A state? Statistics? Both? Can we be a bit more precise?

Climate: a definition for applied mathematicians

Climate science deals with a **high-dimensional** (state dimension $O(10^{10})$) **multi-scale**, highly **coupled non-linear system**, whose dynamics obeys **deterministic laws** but whose solutions exhibit **highly non-linear** behaviour, and thus are treated non-deterministically.

In view of that, climate science seeks to:

- Quantify the system's response to external forcings
- Understand the system's own dynamics and predict the system's evolution given known forcings
- Disentangle causal mechanisms in the presence of background noise
- Describe the entire state of the system with incomplete observations
- Develop data mining methods to reduce the system to its bare essentials
- Quantify uncertainty for all these questions

We are just next door (Mercator building)

thierry.fichefet
hugues.goosse
michel.crucifix
qiuzhen.yin
francois.massonnet
@uclouvain.be

Hugues Goosse
FNRS Director

Thierry Fichefet
Full Professor

Michel Crucifix
Full Professor

Physical climate modeling

Data assimilation

Nonlinear dynamical systems

Detection and attribution

Paleoclimate

Tipping points

Data exploration & reduction

Exreme value theory

Forecasting

Uncertainty quantification

François Massonnet
FNRS Research Associate

Qiuzhen Yin
FNRS Research Associate

~35 PhD and Post-docs, ITs

Courses linked to today’s presentation

LGEO1242	Projections cartographiques et géodésie (M. Crucifix)
LPHY1365:	Météorologie (M. Crucifix, T. Fichefet)
LPHYS2162:	Introduction to the physics of the climate system and its modelling (H. Goosse)
LPHYS2163:	Atmosphere and ocean : physics and dynamics (T. Fichefet, F. Massonnet)
LPHYS2264:	Oscillations and instabilities in the climate system (M. Crucifix)
LPHYS2265:	Sea ice-ocean-atmosphere interactions in polar regions (T. Fichefet)
LPHYS2267:	Paleoclimate dynamics and modelling (Q. Yin)
LPHYS2268:	Forecast, prediction and projection in climate science (F. Massonnet)

Thank you!

Climate Science: A Goldmine for Applied Mathematicians

Climate science is an untapped source of mathematical challenges, many of which remain underexplored by the applied mathematics community. From linear feedback theory to nonlinear dynamical systems, from optimization in decision-making to inverse problems in detection and attribution. Numerical methods are the backbone of climate models, while machine learning is revolutionizing predictions, regime classification, and feature tracking of extreme events like hurricanes and ocean eddies. Applied mathematics underpins our understanding of the climate system. In this talk, I will showcase 20 concrete examples where applied mathematics plays a crucial role in climate science, highlighting opportunities for deeper collaboration between our institutes.

François Massonnet holds a MSc thesis of civil engineering in applied mathematics from Université catholique de Louvain (2009) and a PhD in Sciences from UCLouvain. He is now a F.R.S.-FNRS Research Associate at the Earth and Life Institute and works as a climate scientists on aspects of climate predictability with a focus on high latitudes. He is author or co-author of 84 articles from the peer-reviewed literature and is the PI or co-PI of three research projects, including an ERC Starting Grant that develops early warning systems for future rapid Arctic sea ice loss.