LES LOGARITHMES

1. $4^x = 0,0625 \Rightarrow x \log 4 = \log 0,0625 \Rightarrow x = \dfrac{\log 0,0625}{\log 4} = -2$

2. $4 \cdot 2^x = 0,25 \Rightarrow \log\left(4 \cdot 2^x\right) = \log 0,25 \Rightarrow \log 4 + x \log 2 = \log 0,25 \Rightarrow x = \dfrac{\log 0,25 - \log 4}{\log 2} = -4$

3. $144^x = 2\sqrt{3} \Rightarrow \log 144^x = \log\left(2\sqrt{3}\right) \Rightarrow x \log 144 = \left(\log 2 + \dfrac{1}{2}\log 3\right) \Rightarrow x = \dfrac{2\log 2 + \log 3}{2\log 144} = 0,25$

4. $\sqrt{8^x} = 0,125 \Rightarrow \log\sqrt{8^x} = \log 0,125 \Rightarrow \dfrac{x}{2}\log 8 = \log 0,125 \Rightarrow x = \dfrac{2\log 0,125}{\log 8} = -2$

5. $\left(\dfrac{1}{64}\right)^x = 4'096 \Rightarrow \log\left(\dfrac{1}{64}\right)^x = \log 4'096 \Rightarrow x\left(\log 1 - \log 64\right) = \log 4'096$

   $\Rightarrow x\left(-\log 64\right) = \log 4'096 \Rightarrow x = -\dfrac{\log 4'096}{\log 64} = -2$

6. $2\cdot\left(1,07\right)^x = 3\cdot\left(1,05\right)^x \Rightarrow \log\left(2\cdot\left(1,07\right)^x\right) = \log\left(3\cdot\left(1,05\right)^x\right) \Rightarrow \log 2 + x\log 1,07 = \log 3 + x\log 1,05$

   $\Rightarrow x\left(\log 1,07 - \log 1,05\right) = \log 3 - \log 2 \Rightarrow x = \dfrac{\log 3 - \log 2}{\log 1,07 - \log 1,05} \cong 21,489$

7. $54^{x+1} = \dfrac{\sqrt[3]{4}}{6} \Rightarrow \log 54^{x+1} = \log\dfrac{\sqrt[3]{4}}{6} \Rightarrow \left(x+1\right)\log 54 = \dfrac{1}{3}\log 4 - \log 6$

   $\Rightarrow x\log 54 + \log 54 = \dfrac{\log 4 - 3\log 6}{3} \Rightarrow x = \dfrac{\log 4 - 3\log 6 - 3\log 54}{3\log 54} = -\dfrac{4}{3}$

8. $\sqrt[x+1]{4} = \sqrt[3x-1]{16} \Rightarrow \log\sqrt[x+1]{4} = \log\sqrt[3x-1]{16} \Rightarrow \dfrac{\log 4}{x+1} = \dfrac{\log 16}{3x-1} \Rightarrow 3x-1 = \dfrac{\log 16}{\log 4}\left(x+1\right)$

   $\Rightarrow 3x-1 = 2\left(x+1\right) \Rightarrow x = 3$

9. $\dfrac{10'206}{3^{x+3}} = 14 \Rightarrow \log\dfrac{10'206}{3^{x+3}} = \log 14 \Rightarrow \log 10'206 - (x+3)\log 3 = \log 14$

$\Rightarrow (x+3)\log 3 = \log 10'206 - \log 14$

$\Rightarrow x + 3 = \dfrac{\log 10'206 - \log 14}{\log 3} \Rightarrow x = \dfrac{\log 10'206 - \log 14 - 3\log 3}{\log 3} = 3$

10. $\sqrt{3^{3x+2}} = 59'049 = \log\sqrt{3^{3x+2}} = \log 59'049 \Rightarrow \dfrac{3x+2}{2}\log 3 = \log 59'049 \Rightarrow 3x + 2 = \dfrac{2\log 59'049}{\log 3}$

$x = \dfrac{2\log 59'049 - 2\log 3}{3\log 3} = 6$

11. $\sqrt[5]{4^{2x+1}} = 2,3 \Rightarrow \log\sqrt[5]{4^{2x+1}} = \log 2,3 \Rightarrow \dfrac{2x+1}{5}\log 4 = \log 2,3 \Rightarrow 2x + 1 = \dfrac{5\log 2,3}{\log 4}$

$\Rightarrow x = \dfrac{5\log 2,3 - \log 4}{2\log 4} \cong 1.002$

12. $3^{\sqrt{x}} = 243 \Rightarrow \log 3^{\sqrt{x}} = \log 243 \Rightarrow \sqrt{x}\log 3 = \log 243 \Rightarrow \sqrt{x} = \dfrac{\log 243}{\log 3} \Rightarrow x = \left(\dfrac{\log 243}{\log 3}\right)^2 = 25$

ÉQUATIONS LOGARITHMIQUES

1. $\log(3x - 5) = 0$

$\text{Df} = \left\{x \mid x > \dfrac{5}{3}\right\} \Leftrightarrow 3x - 5 = 1 \Leftrightarrow x = 2$

2. $\log(6 - x) = 1$

$\text{Df} = \{x \mid x < 6\} \Leftrightarrow 6 - x = 10 \Leftrightarrow x = -4$

3. $\log(3x + 7) = 2\log(2)$

$\text{Df} = \left\{x \mid x > -\dfrac{7}{3}\right\} \Leftrightarrow 3x + 7 = 2^2 \Leftrightarrow x = -1$

4. $\log(x+3)+\log(x+5)=\log(15)$

    $Df=\{x|x>-3\}\Leftrightarrow(x+3)(x+5)=15\Leftrightarrow x^2+8x=0\Leftrightarrow x'=0$ et $x''=-8$ refusée

5. $\log(x-2)+\log(x+2)=\log(45)$

    $Df=\{x|x>2\}\Leftrightarrow(x-2)(x+2)=45\Leftrightarrow x^2-49=0\Leftrightarrow x'=7$ et $x''=-7$ refusée

6. $2\log(x-4)=\log(x)-2\log(2)$

    $Df=\{x|x>4\}\Leftrightarrow(x-4)^2=\dfrac{x}{2^2}\Leftrightarrow 4x^2-33x+64=0$

    $\Leftrightarrow x'=\dfrac{33+\sqrt{65}}{8}$ et $x''=\dfrac{33-\sqrt{65}}{8}$ refusée

7. $\log(4x+1)+\log(x+2)-2\log(3x)=0$

    $Df=\{x|x>0\}\Leftrightarrow\dfrac{(4x+1)(x+2)}{9x^2}=10^0\Leftrightarrow 5x^2-9x-2=0$

    $\Leftrightarrow x'=2$ et $x''=-\dfrac{1}{5}$ refusée

8. $\log(x^2+2x-3)-2\log(x-1)=2$

    $Df=\{x|x>1\}\Leftrightarrow\dfrac{x^2+2x-3}{(x-1)^2}=10^2\Leftrightarrow 99x-103=0\Leftrightarrow x=\dfrac{103}{99}$

9. $\log(-x-2)=\log\left(\dfrac{-x-11}{x+3}\right)$

    $Df=\{x|-11<x<-3\}\Leftrightarrow -x-2=\dfrac{-x-11}{x+3}\Leftrightarrow x^2+4x-5=0$

    $\Leftrightarrow x'=-5$ et $x''=1$ refusée

10. $\log(x+2)=\log(-x-11)-\log(x+3)$ $\qquad$ $Df=\varnothing$