The complexity of soundness in workflow nets

Filip Mazowiecki

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FI_t 2022

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Plan

1. Petri nets and reachability

2. Workflow nets and soundness

3. Some proofs

4. Implementation

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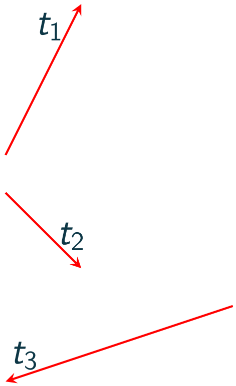

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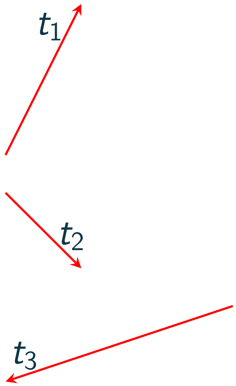

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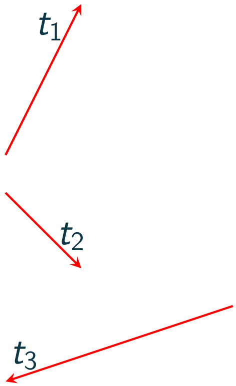

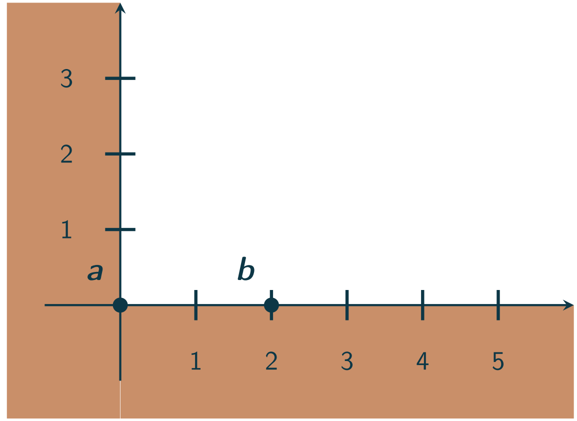

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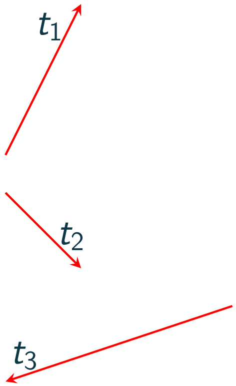

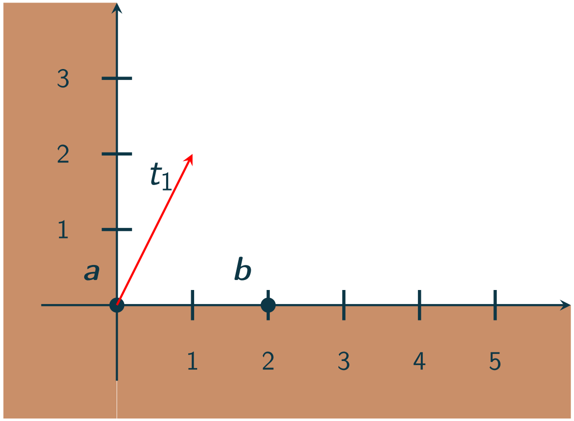

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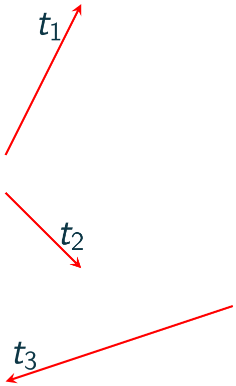

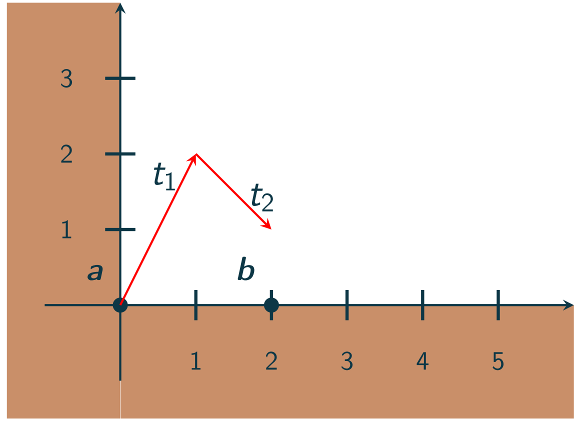

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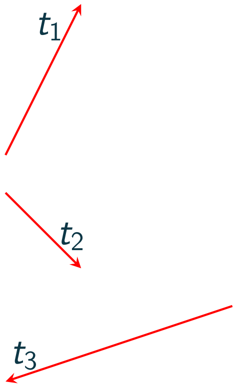

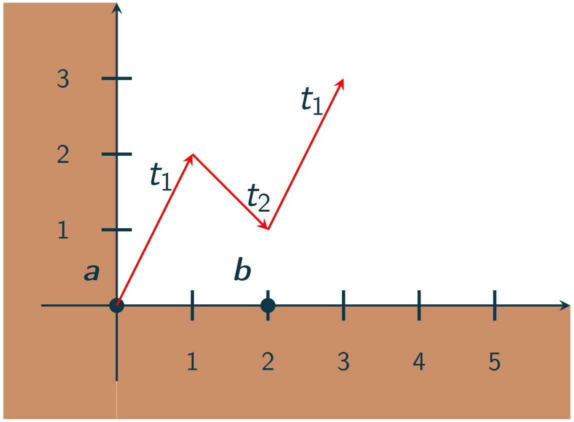

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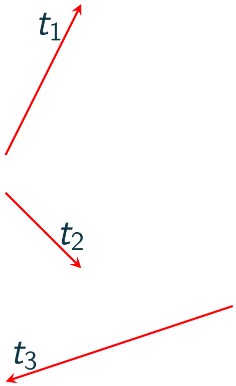

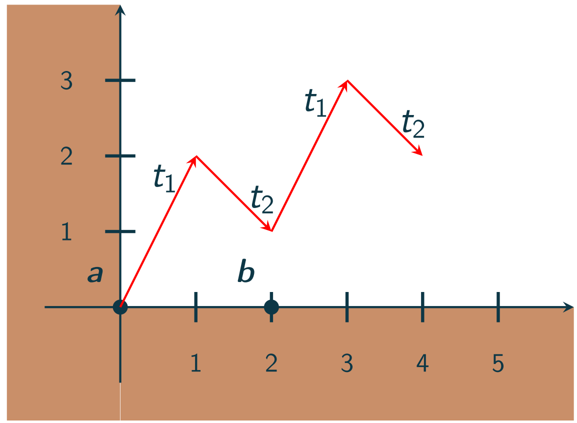

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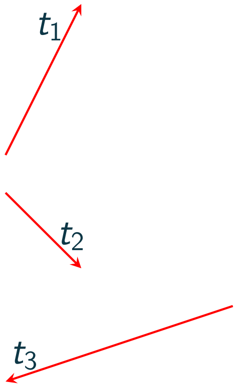

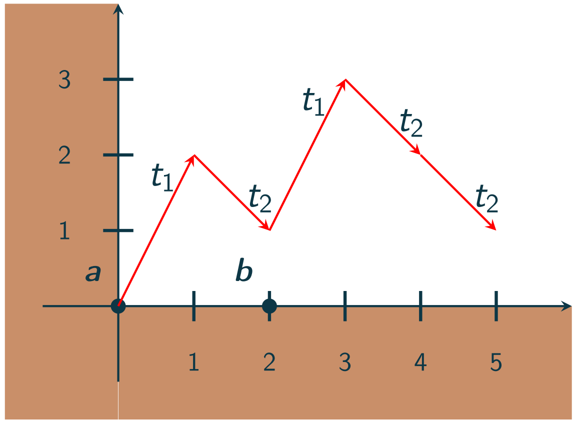

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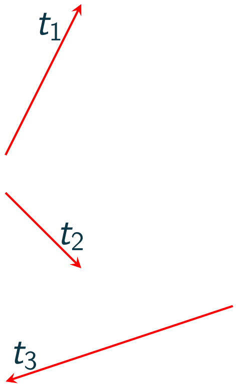

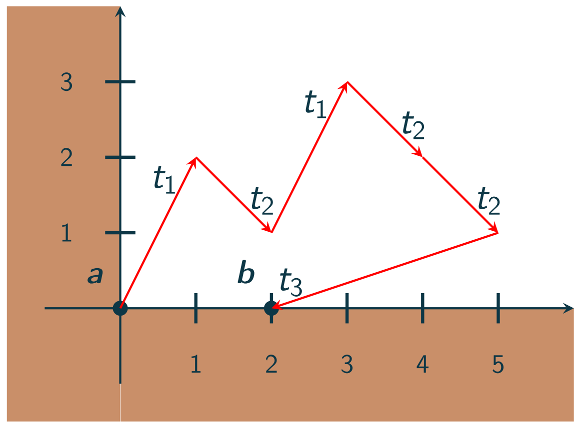

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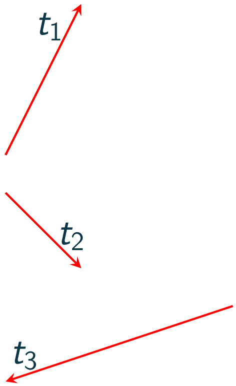

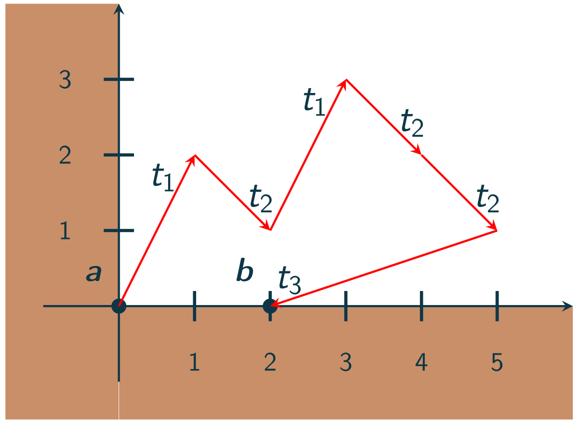

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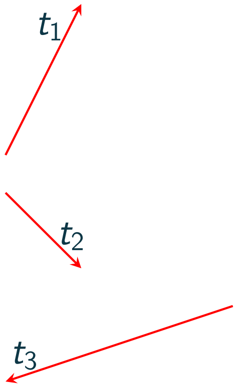

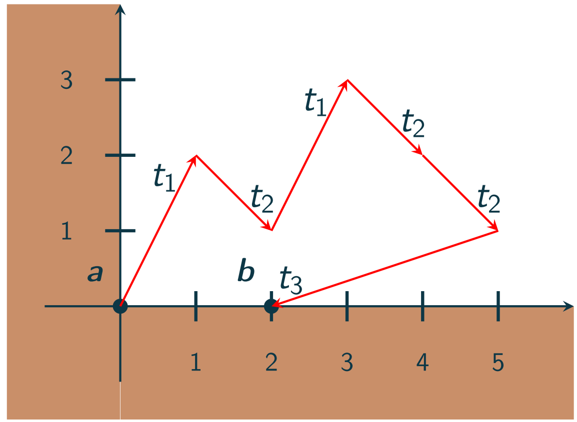

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Notation: $\boldsymbol{a} \rightarrow^* \boldsymbol{b}, \quad \boldsymbol{a} \not\rightarrow^* \boldsymbol{b}$

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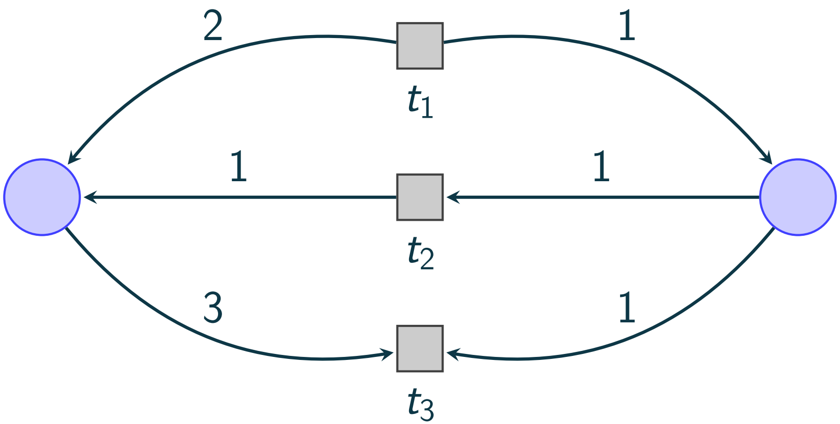

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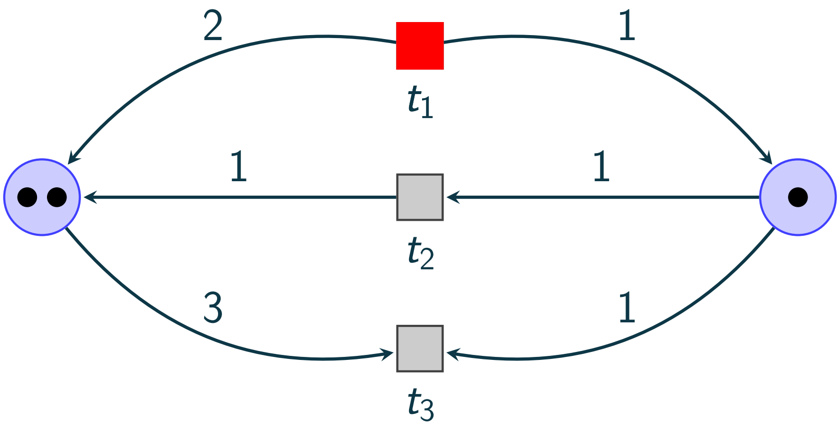

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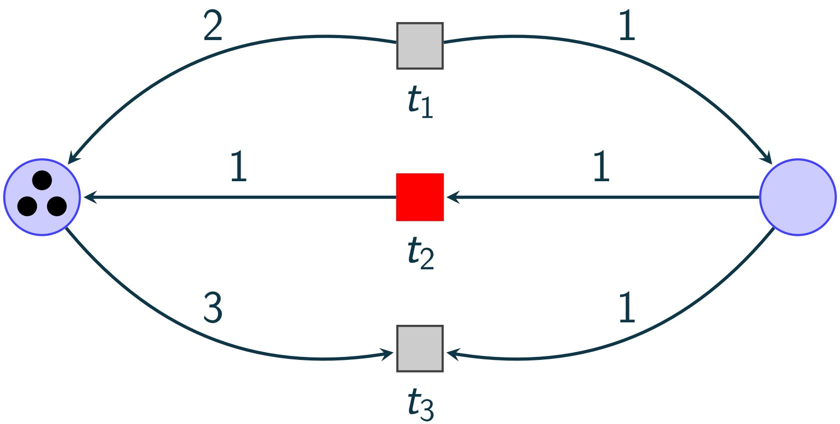

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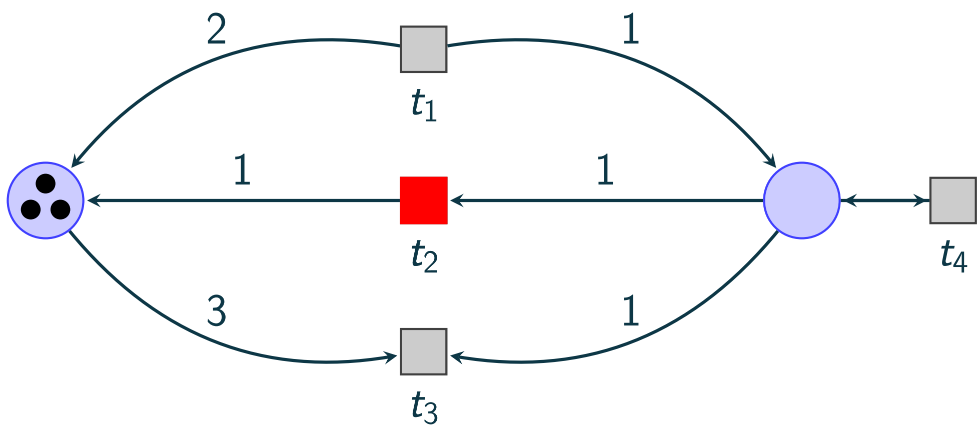

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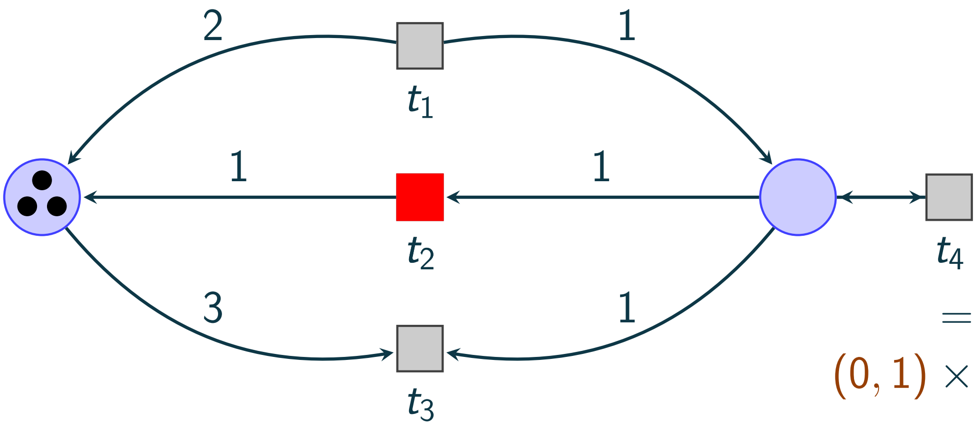
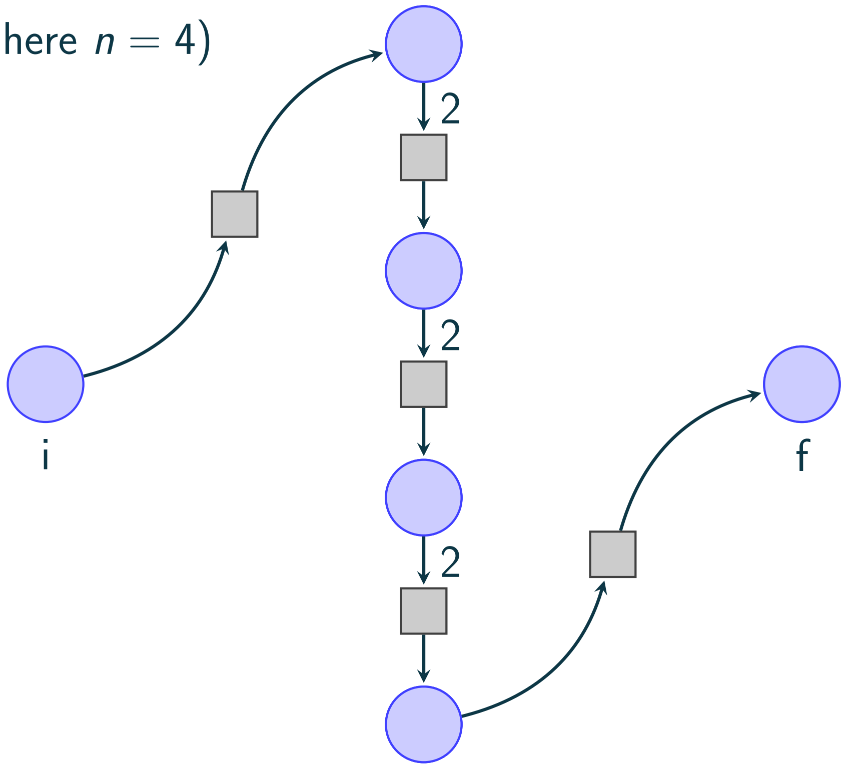


$t_4 = (0, 1) \times (0, 1)$

A more interesting example

$d = 2 + n \quad$ (here $n = 4$)



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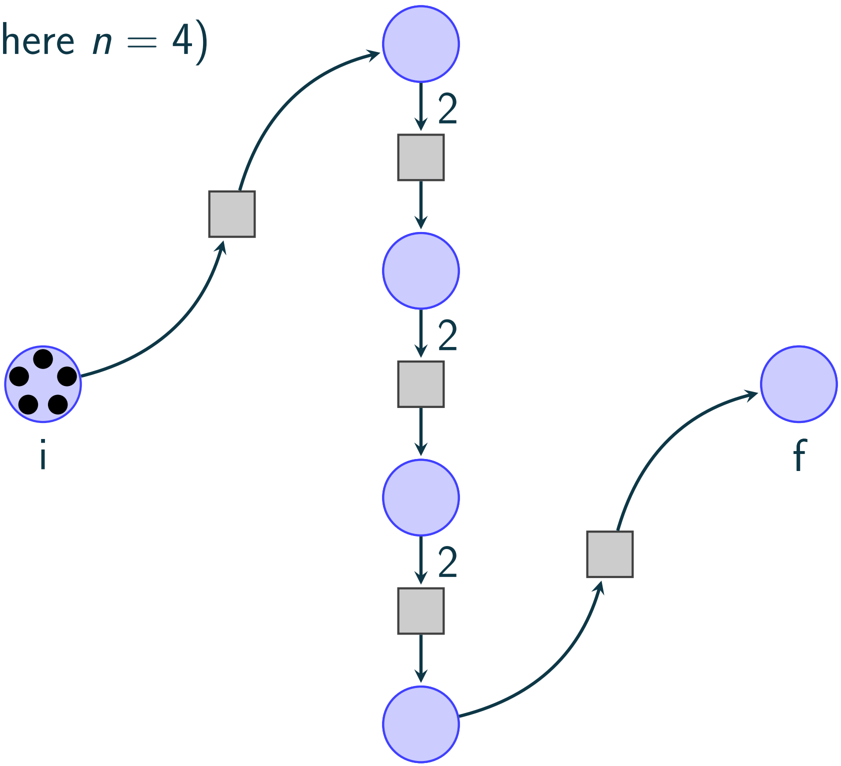

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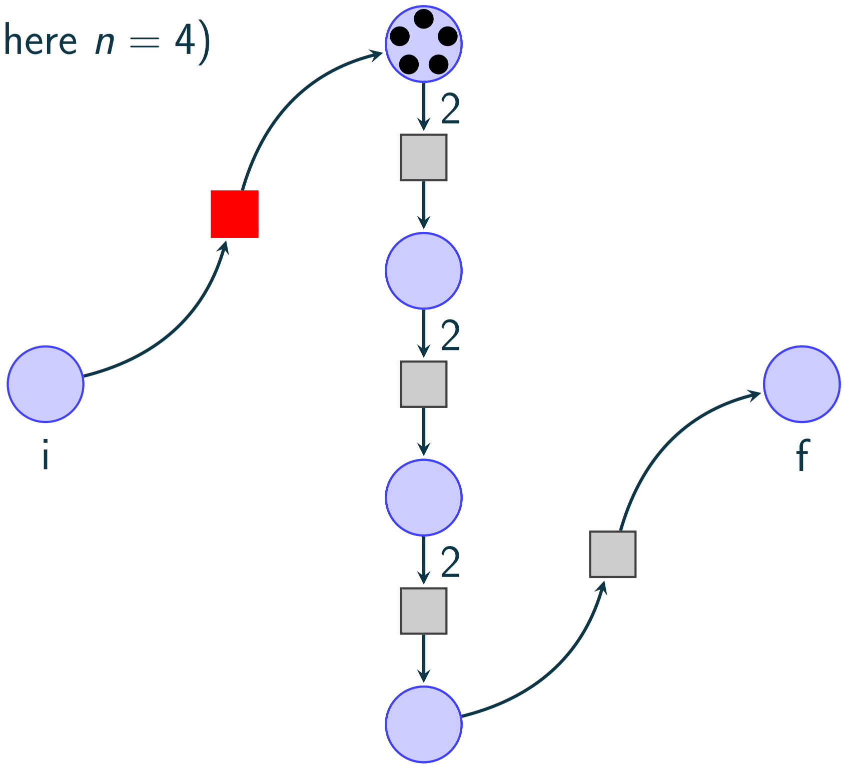

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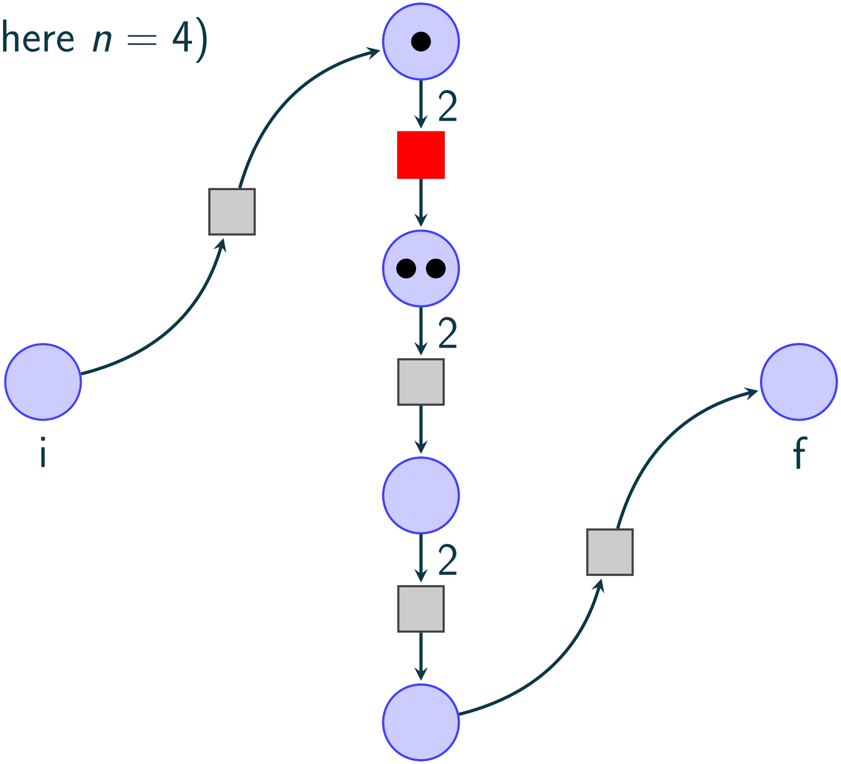

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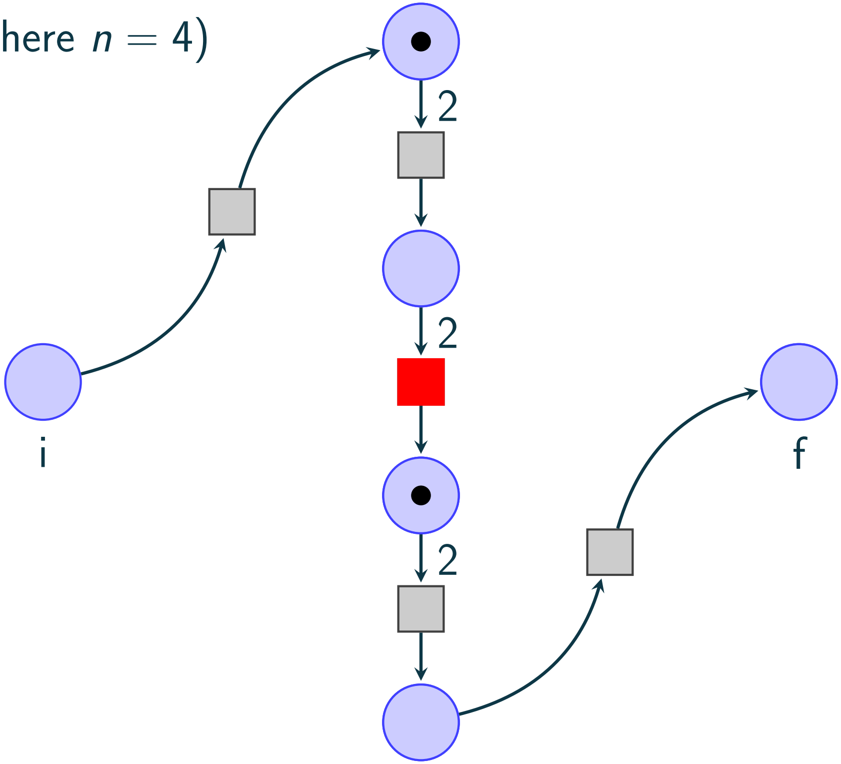

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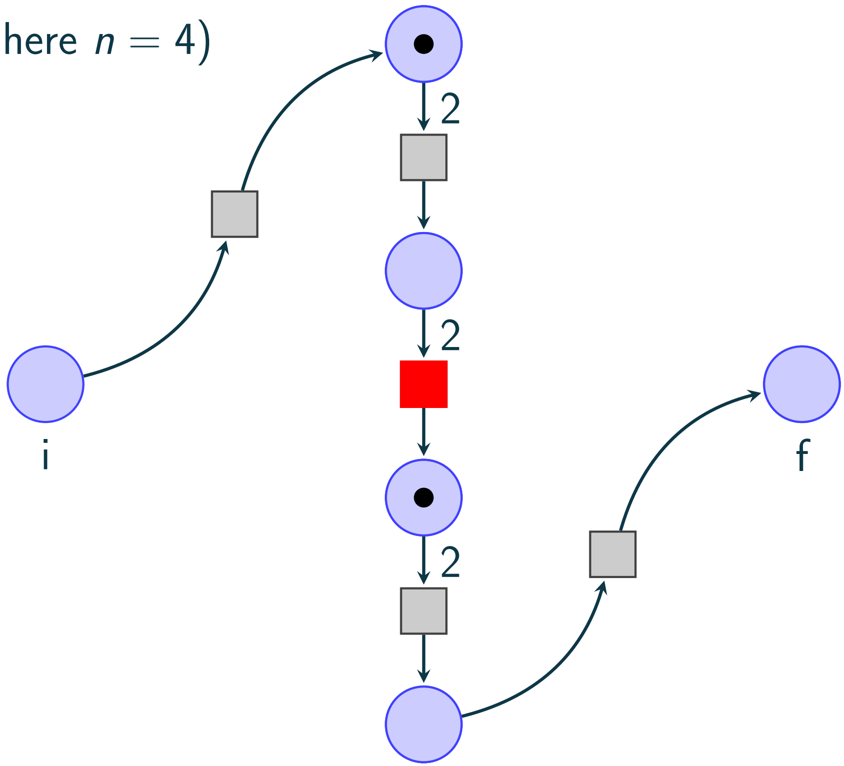

If we start with k tokens in i

then we reach f only if $k \geq 2^{n-1}$

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- Reachability is in Ackermann [Leroux and Schmitz, 2019]

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- Both problems are EXPSPACE-hard [Lipton, 1976]
- Coverability is in EXPSPACE [Rackoff, 1978]
- Reachability is decidable [Mayr, 1981]
- Reachability is in Ackermann [Leroux and Schmitz, 2019]
- Reachability is Ackermann-hard [Czerwiński and Orlikowski 2021], [Leroux, 2021]

Plan

1. Petri nets and reachability

2. Workflow nets and soundness

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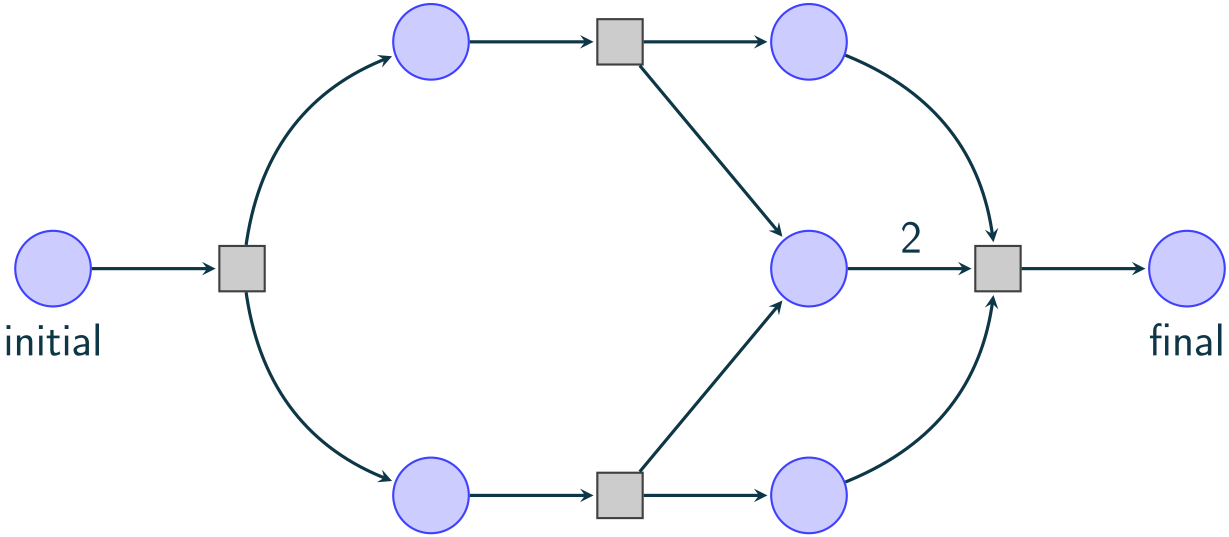
Workflow nets

Workflow nets



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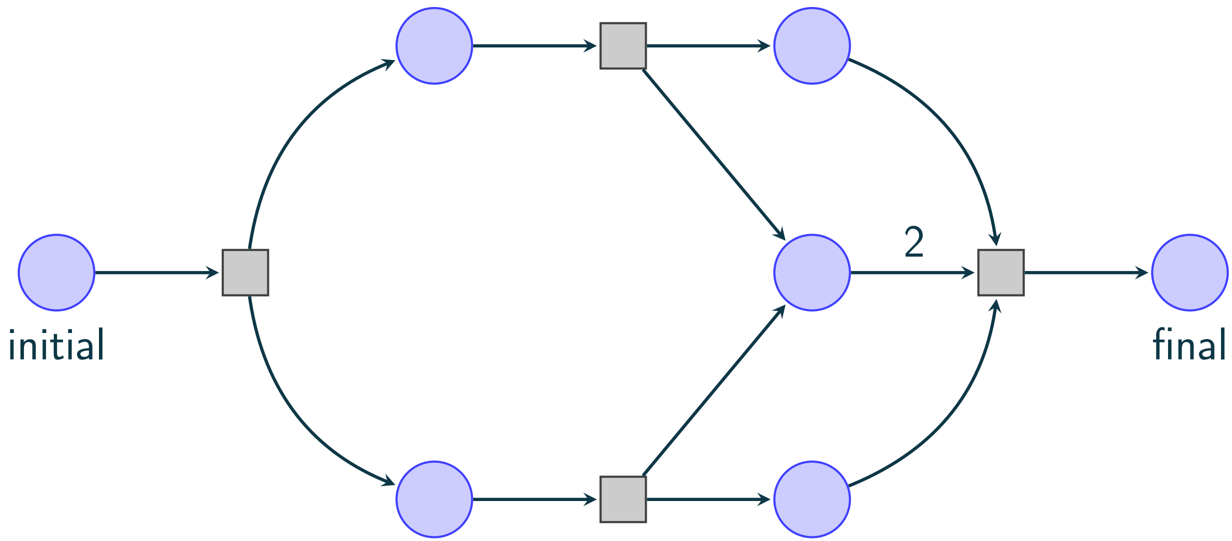

Workflow nets are Petri nets such that:

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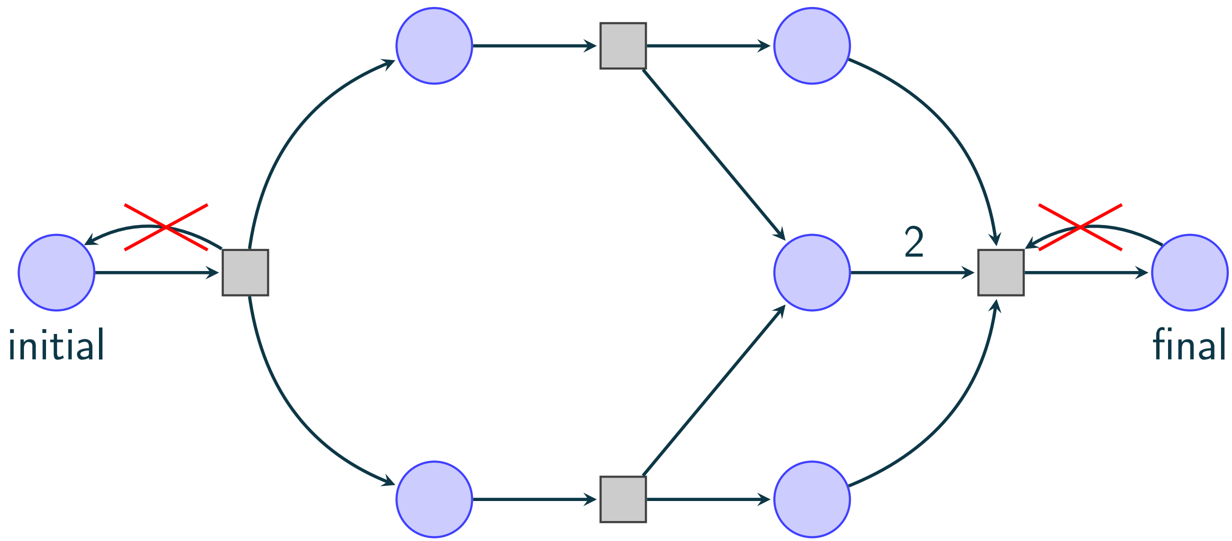

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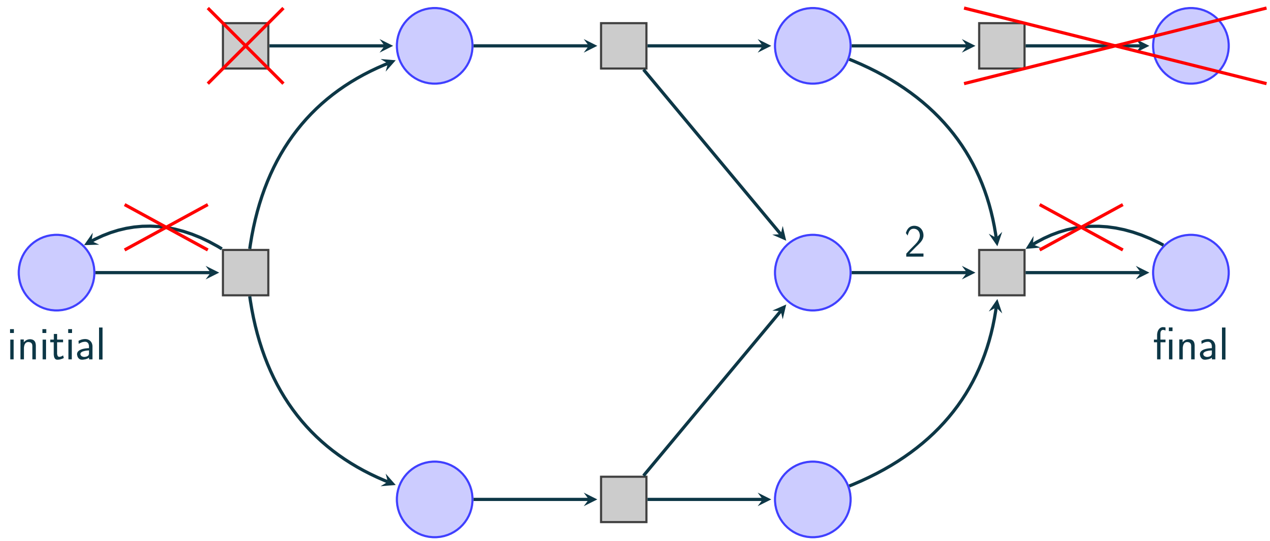

Workflow nets are Petri nets such that:

- Two places are distinguished: initial and final
- No ingoing edges to initial and no outgoing edges from final
- All places and transitions are on a path from initial to final

Business processes

Suppose a professor wants to hire a student

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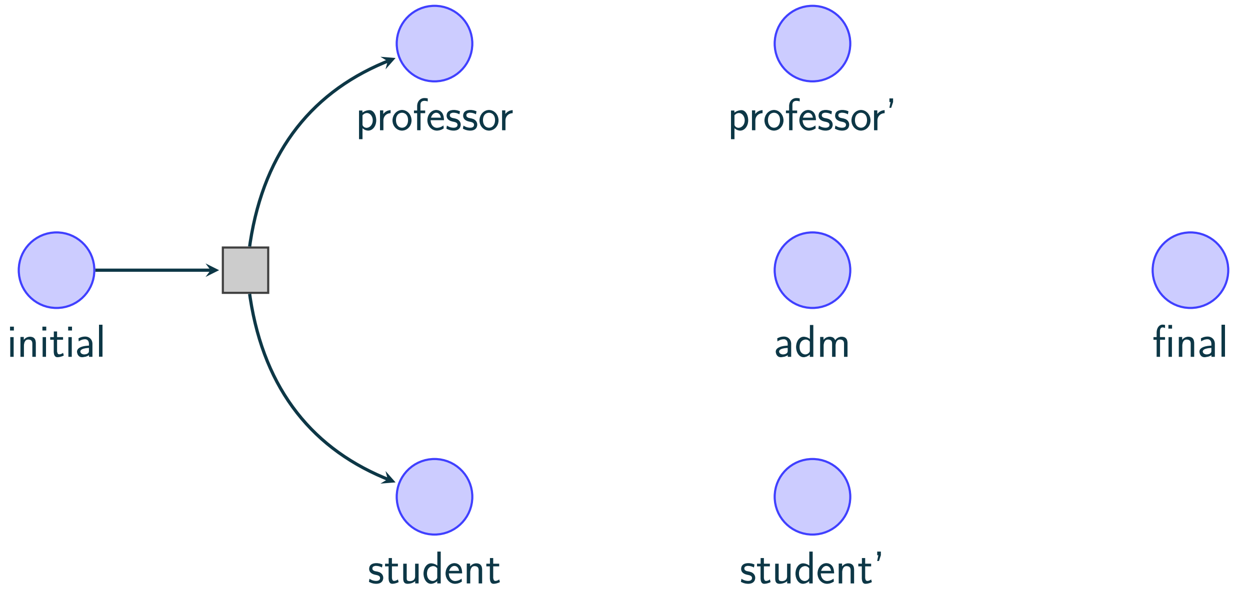

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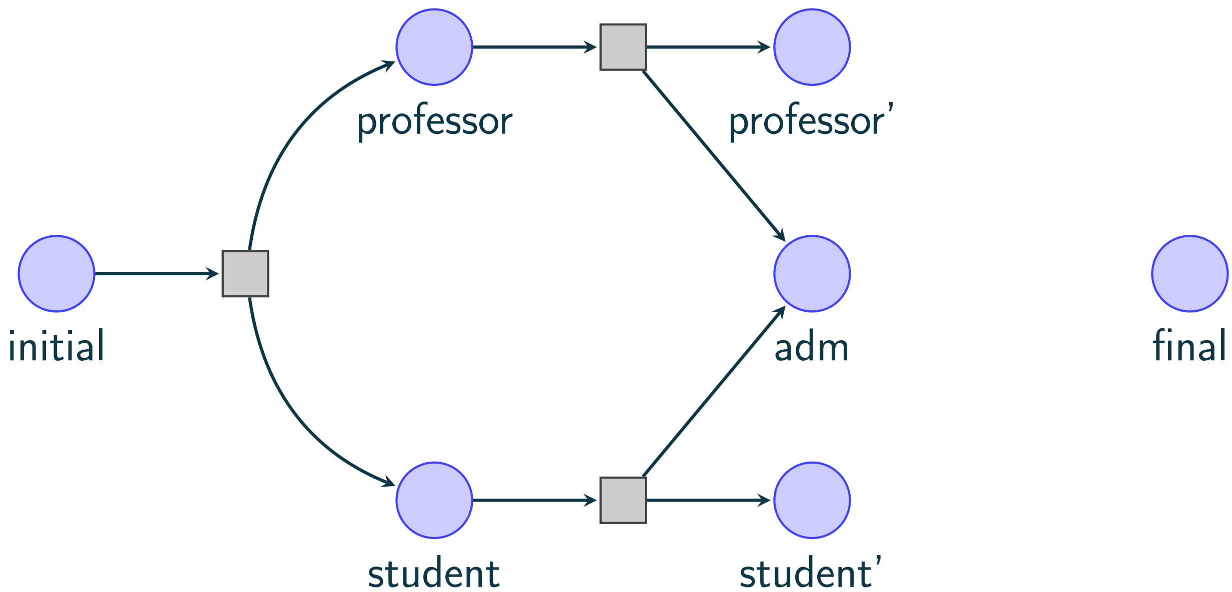

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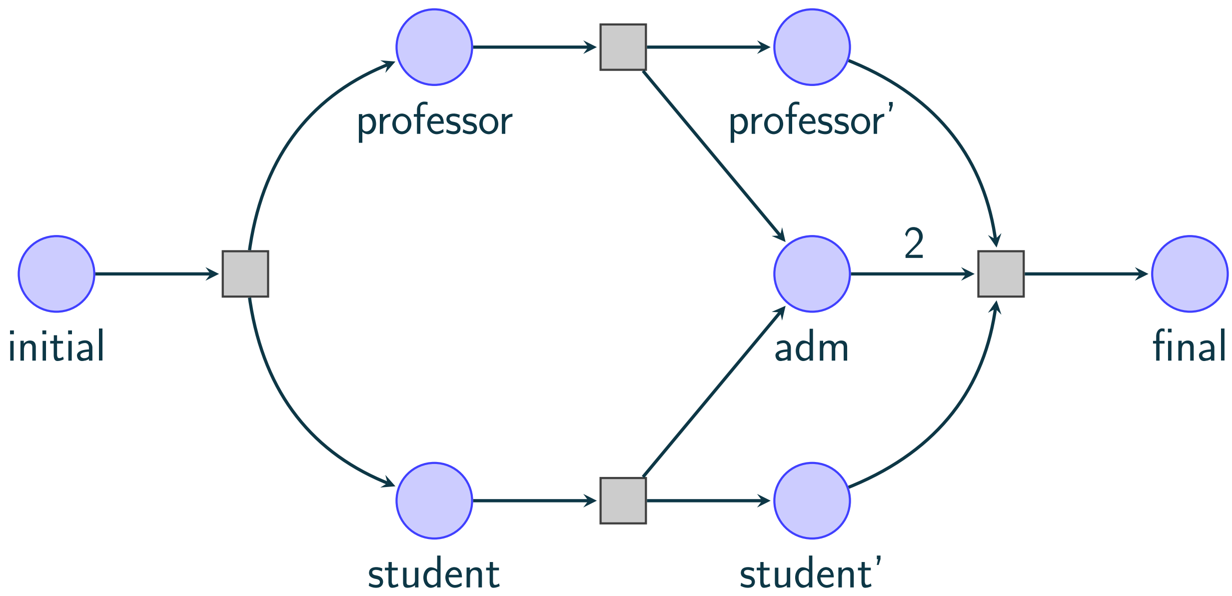

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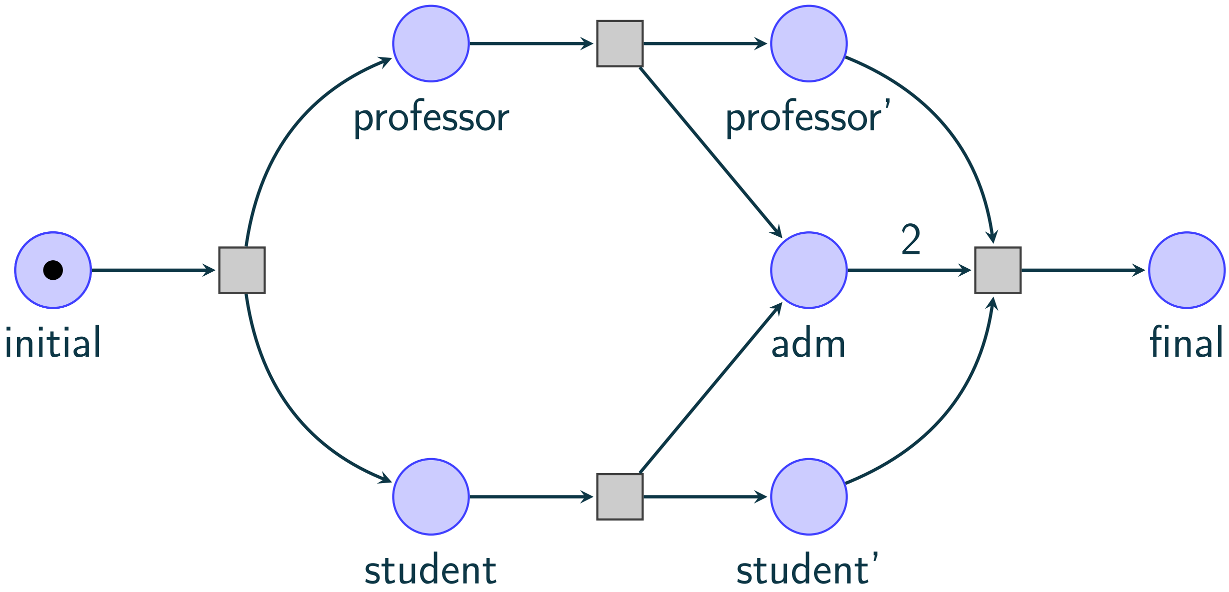

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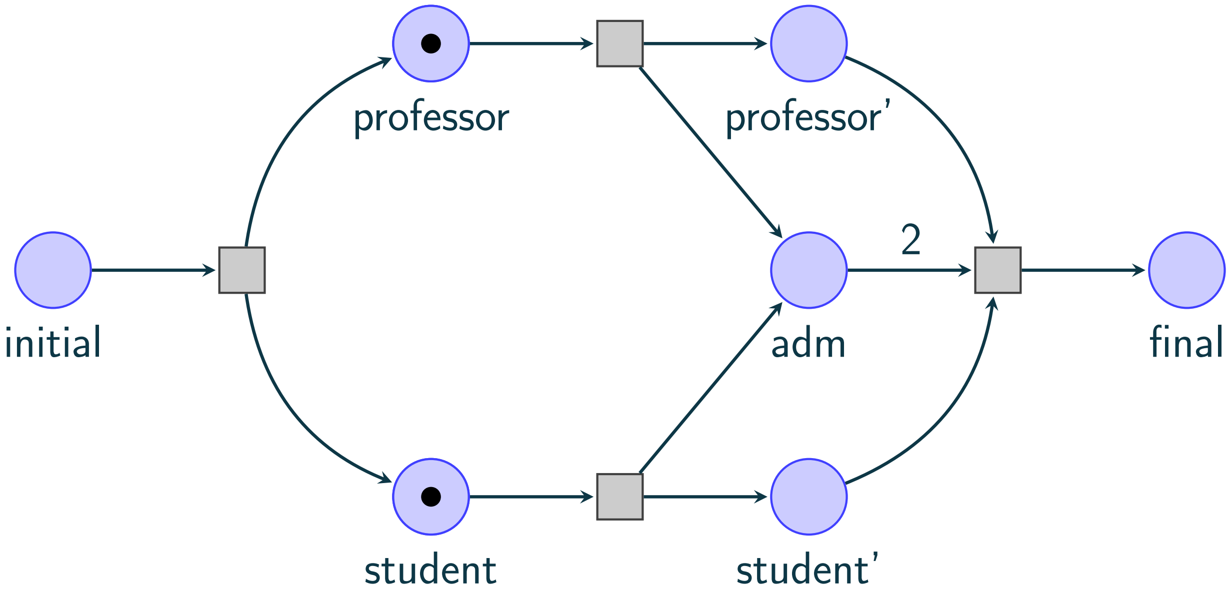

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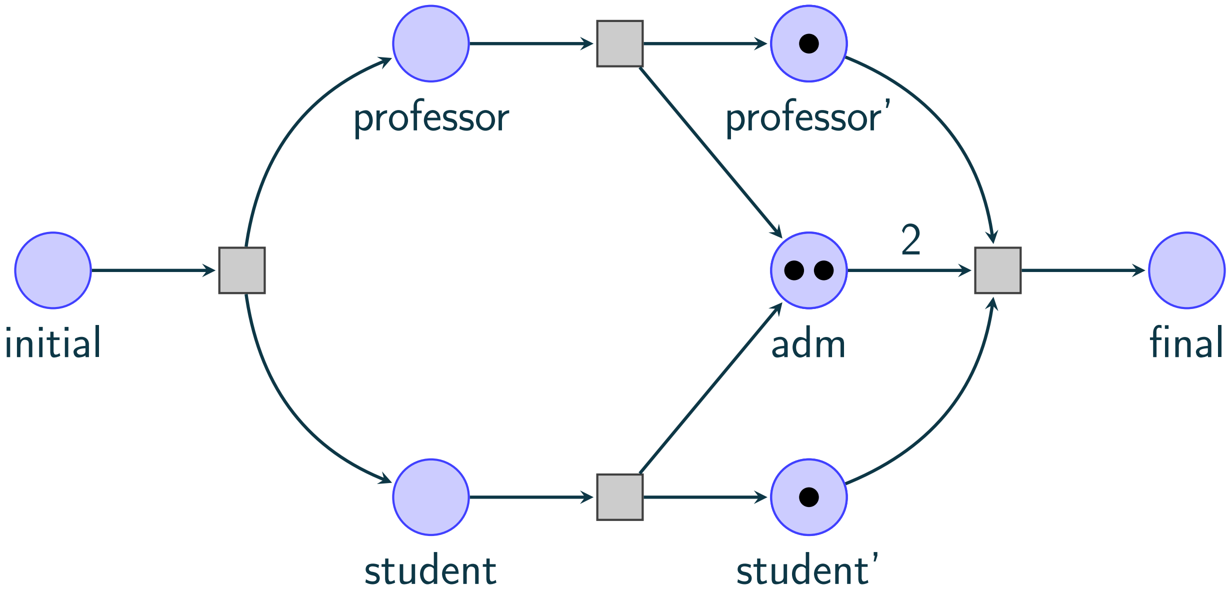

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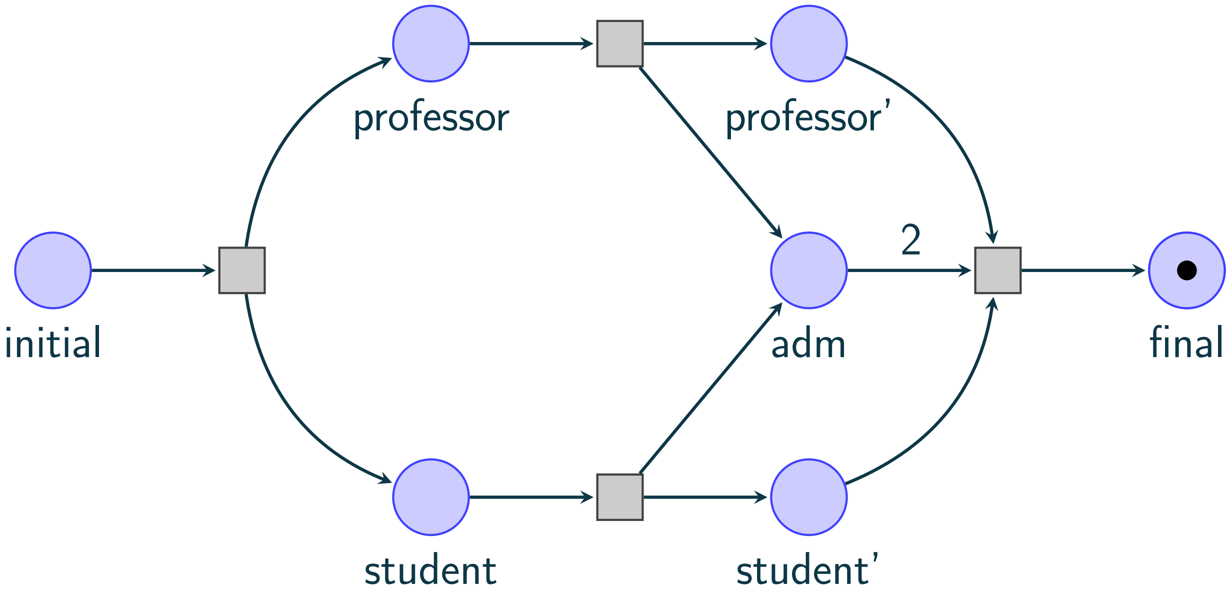

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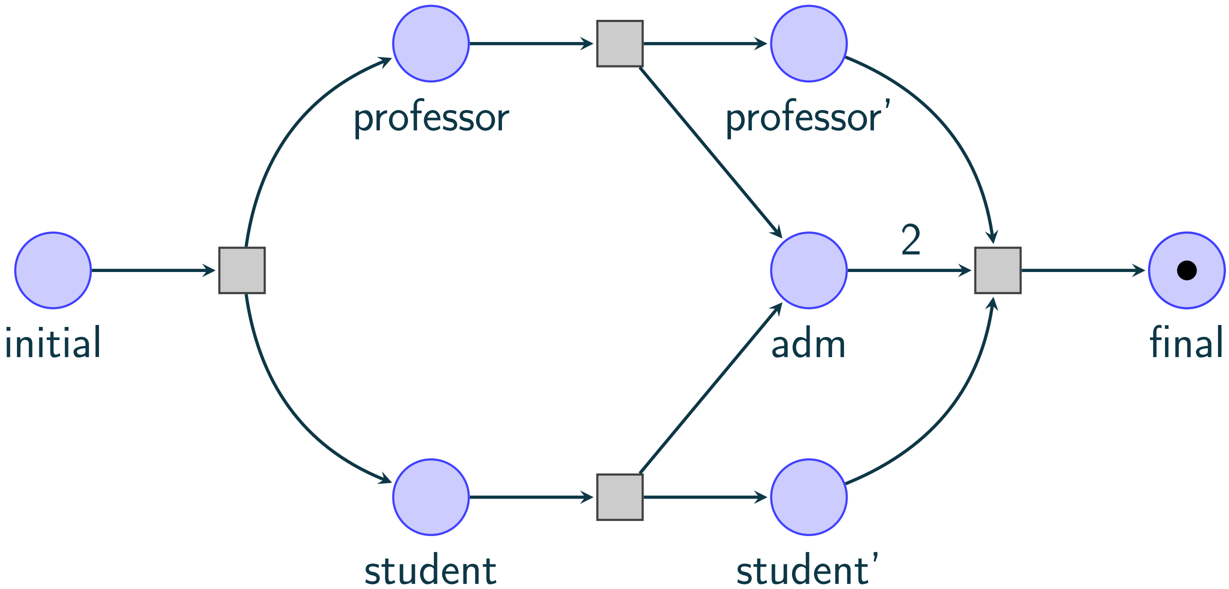

Behaves good even with many students at once

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What would we like to verify in the previous example?

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The previous example is sound and even k-sound for every $k > 0$

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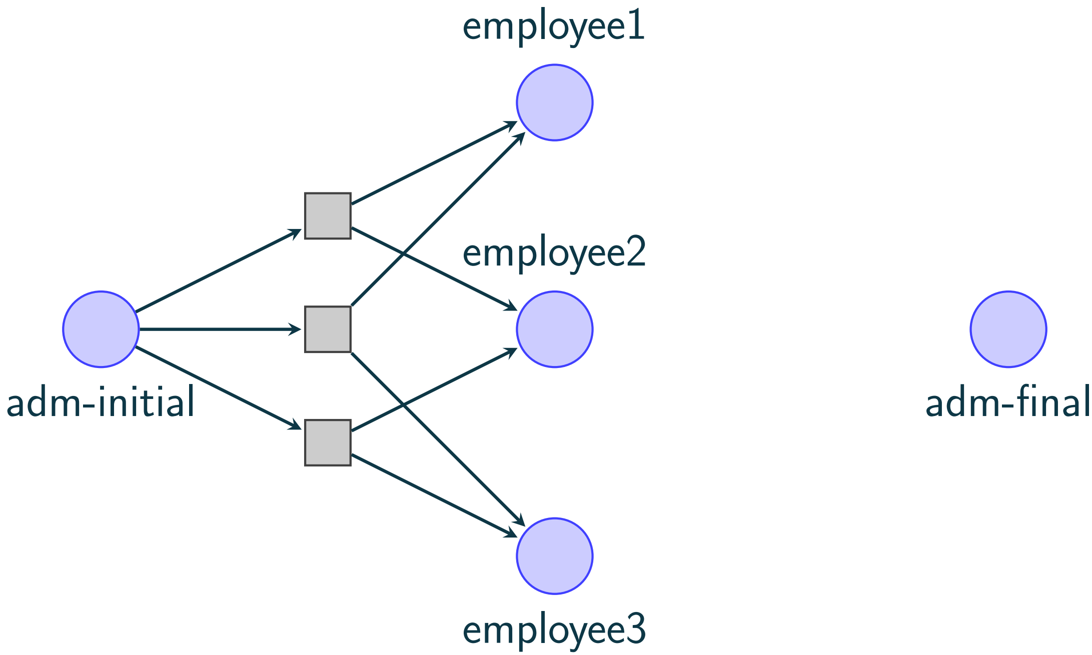

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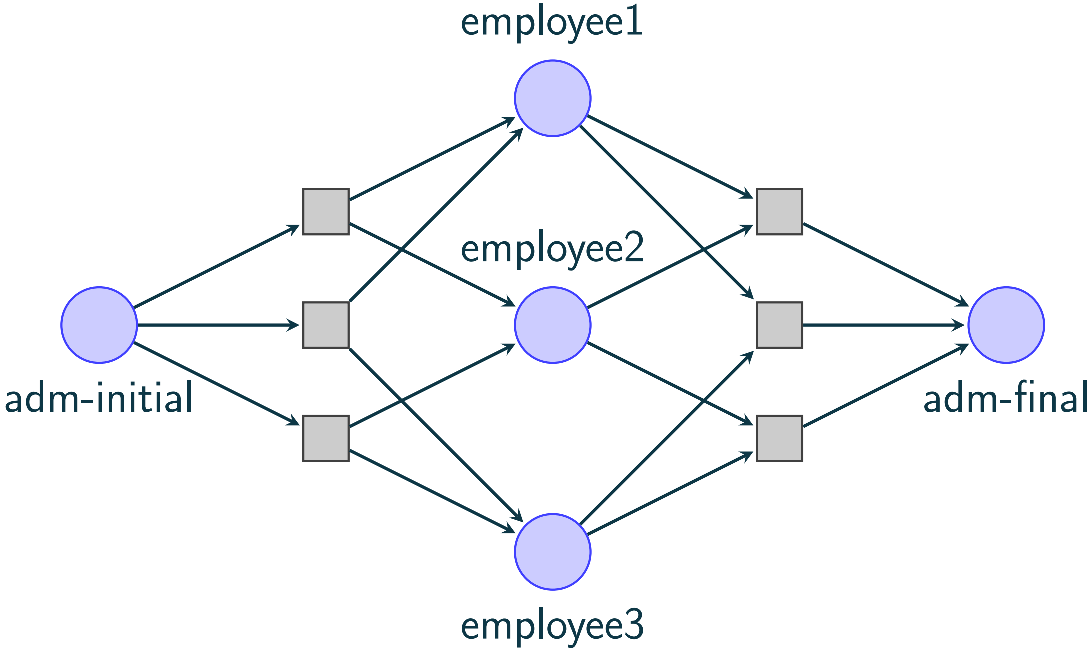

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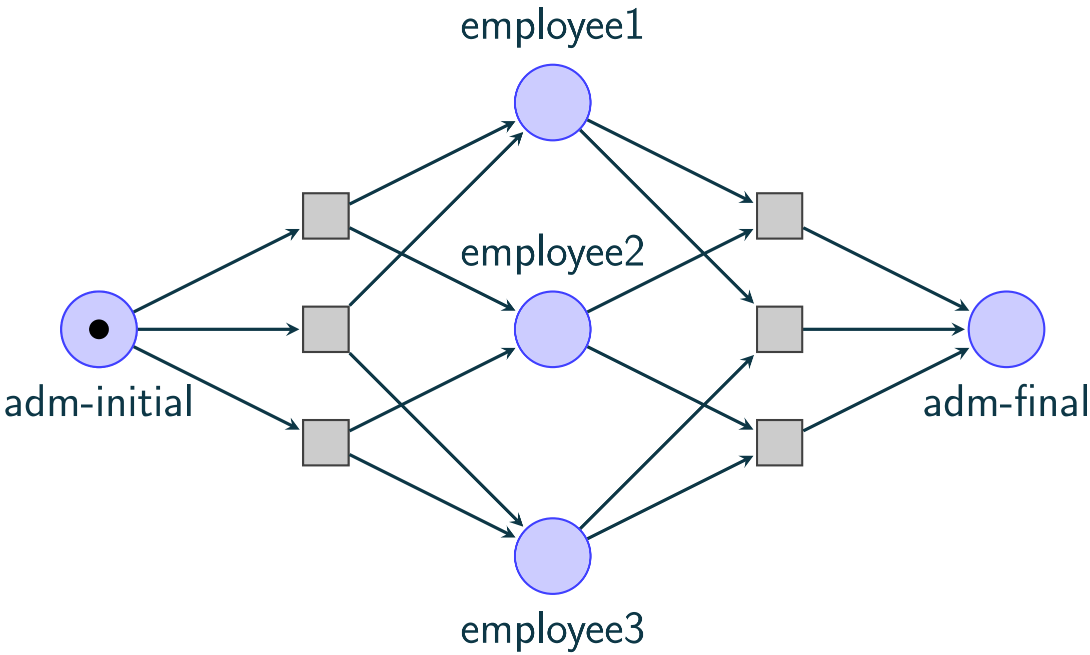

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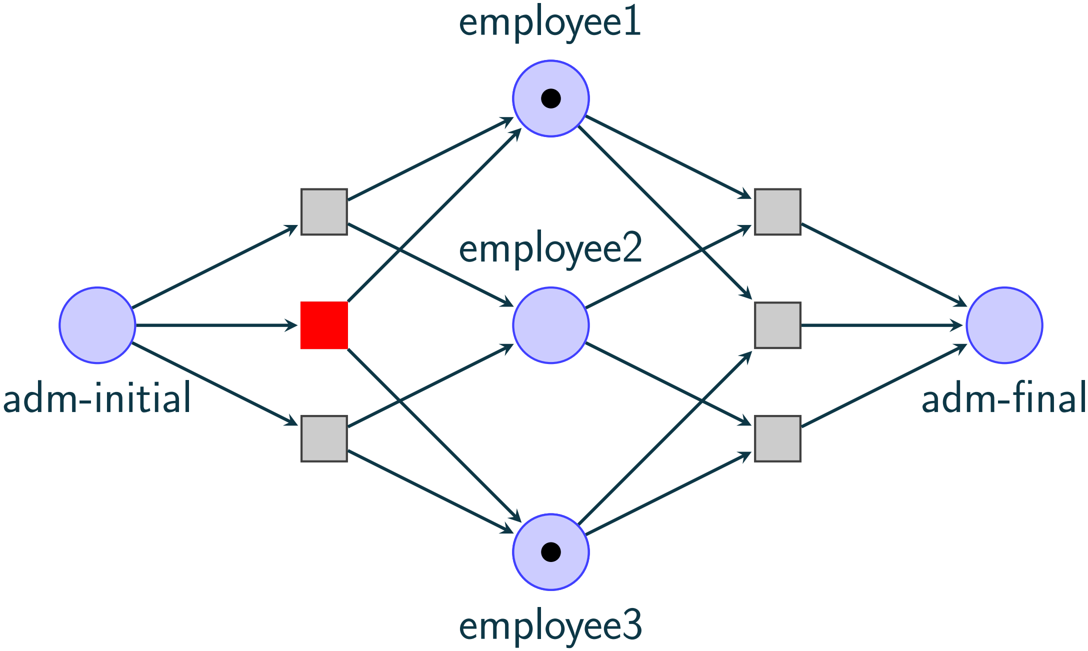

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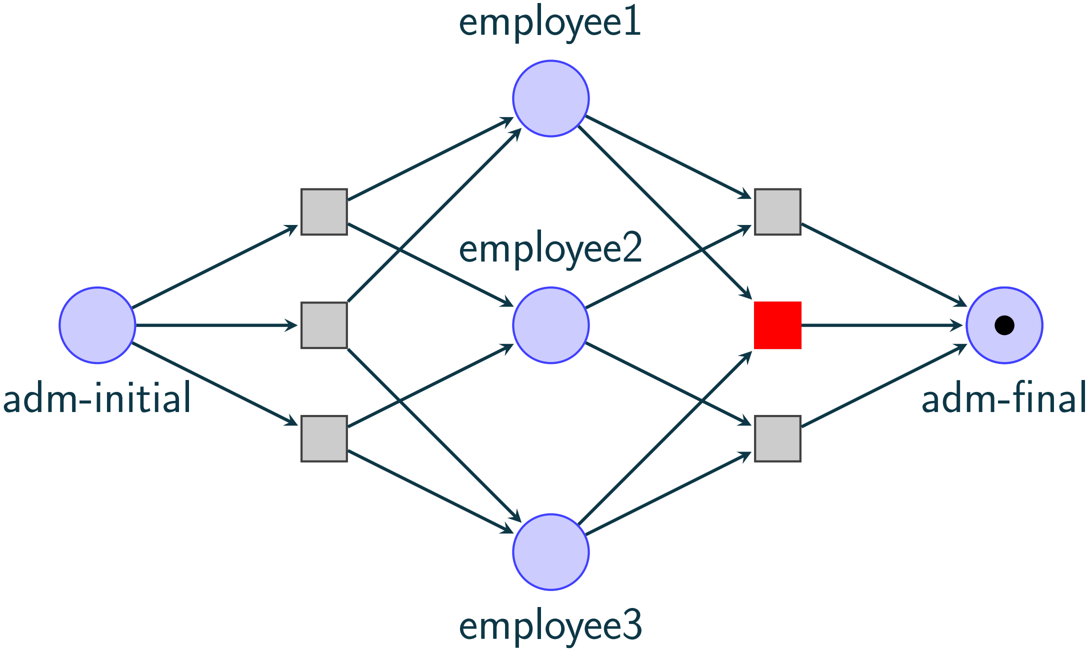

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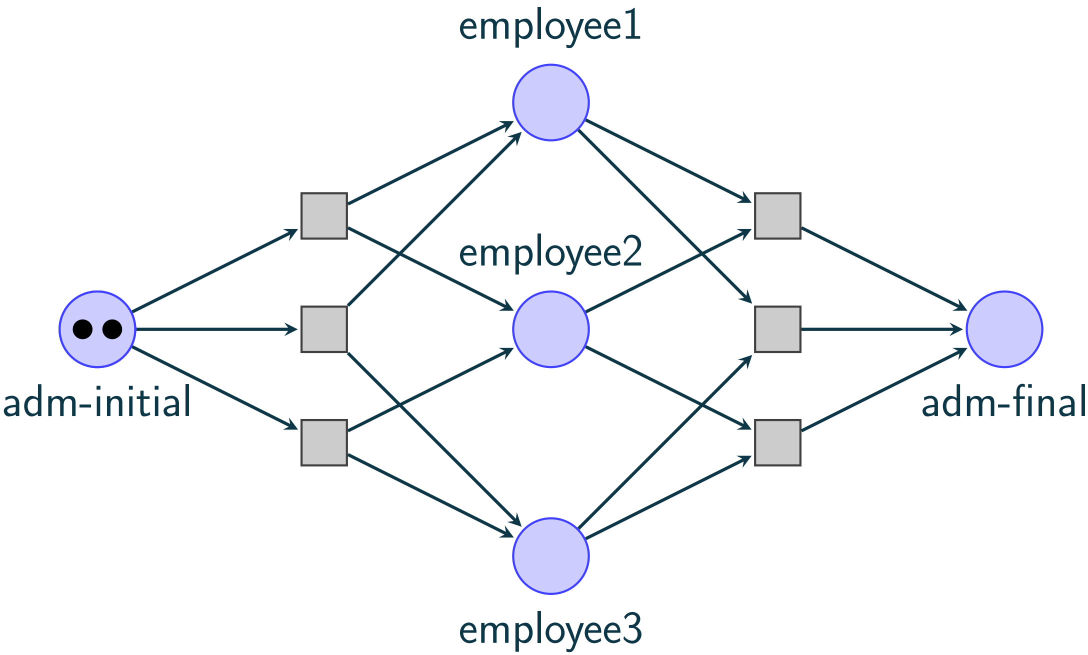

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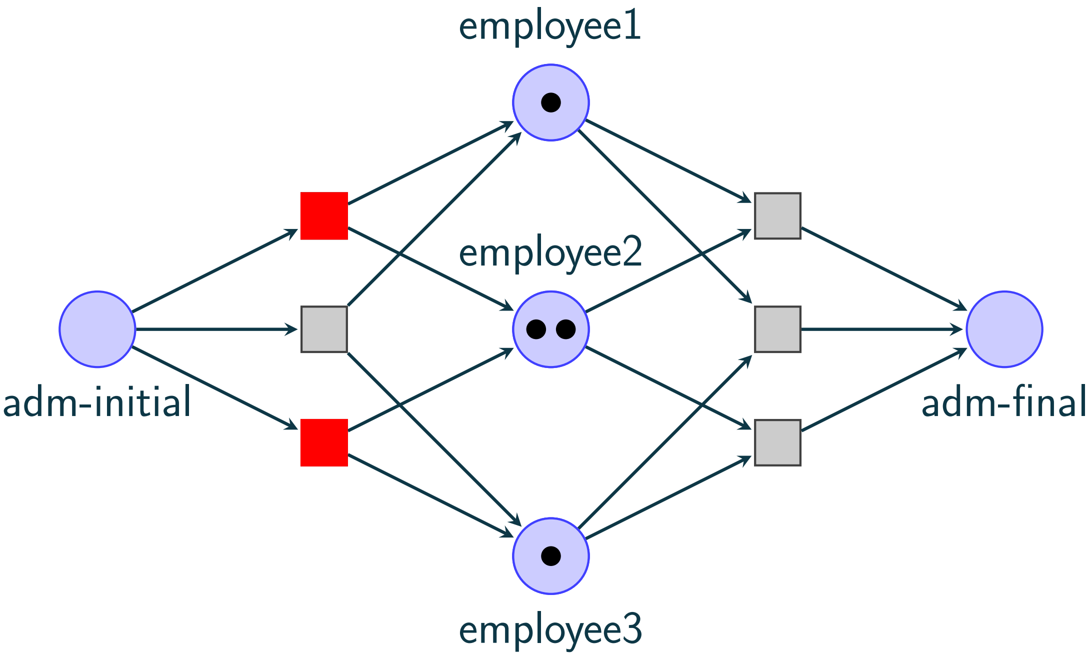

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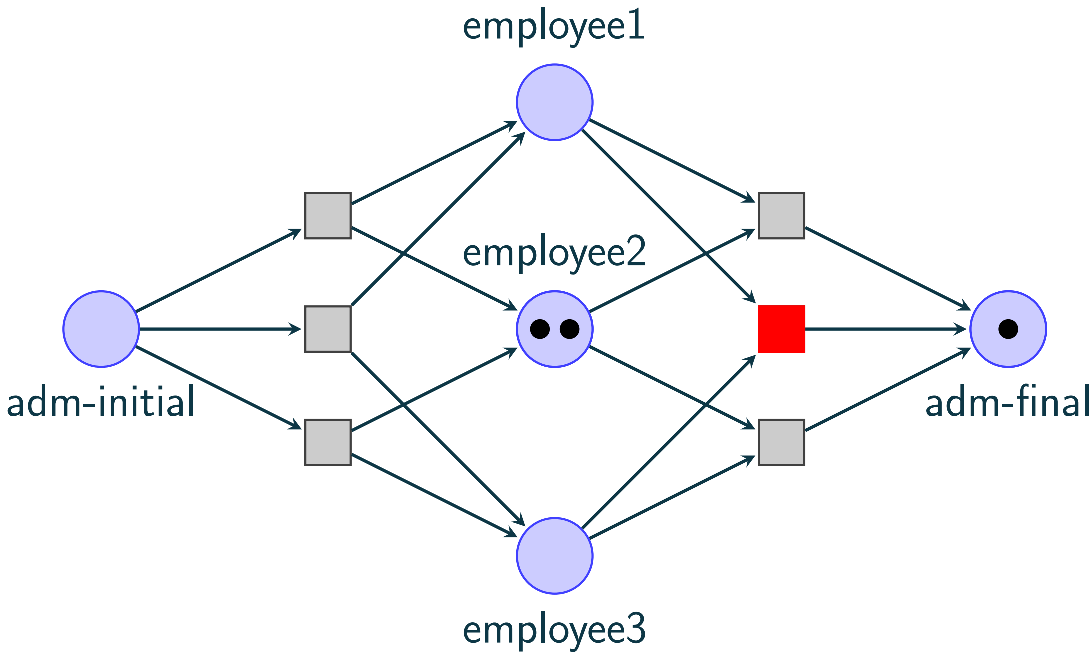

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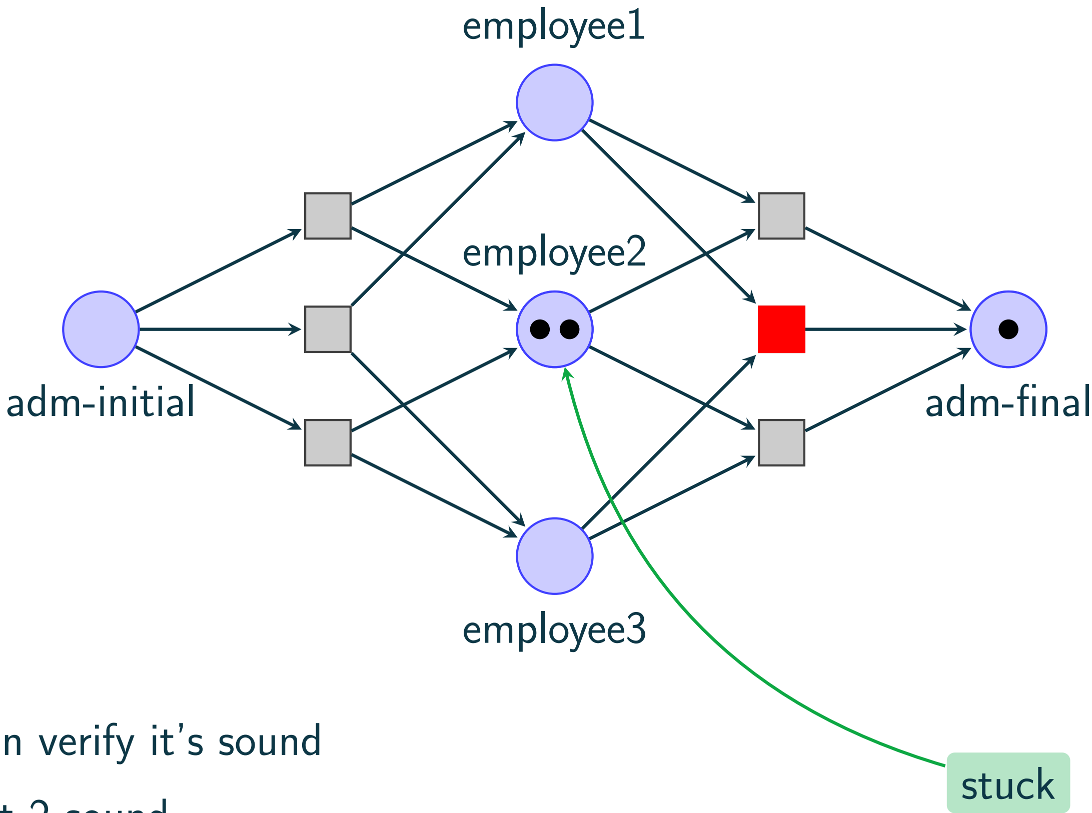

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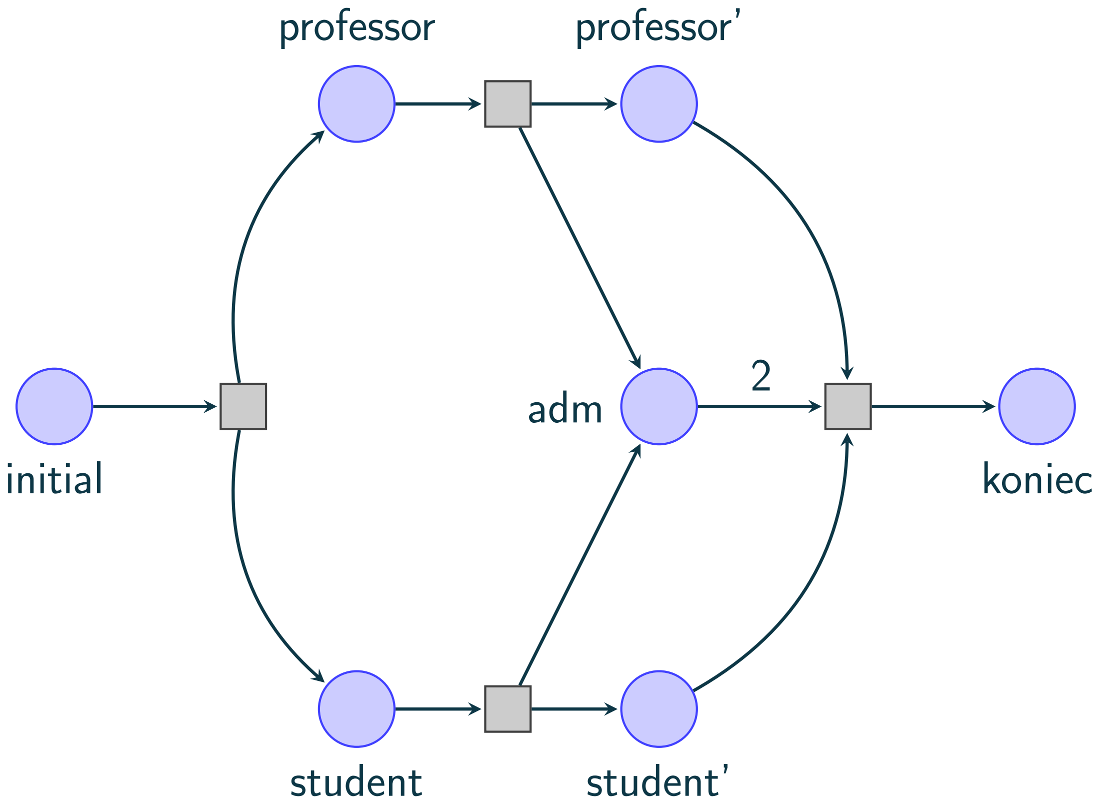

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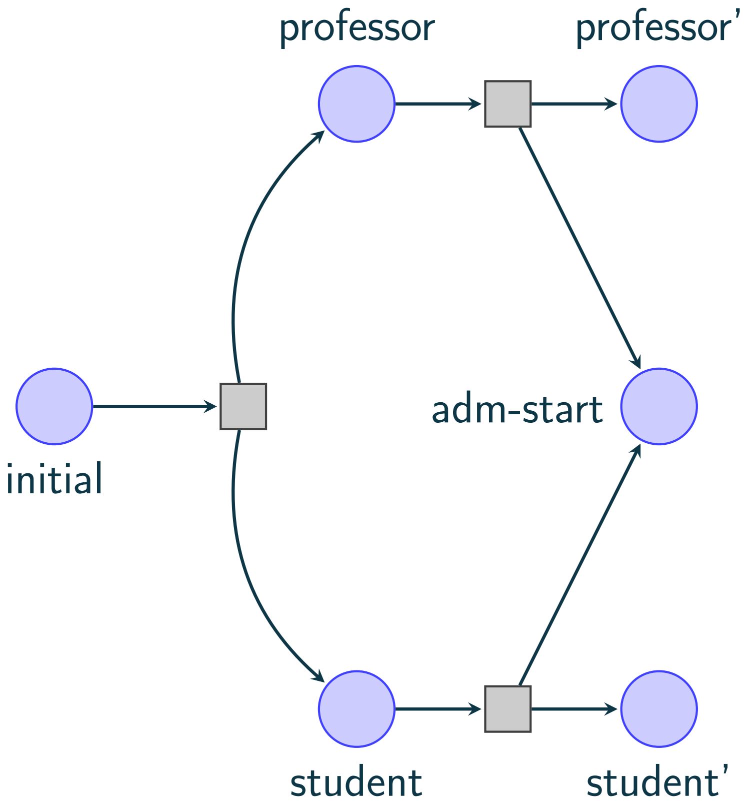

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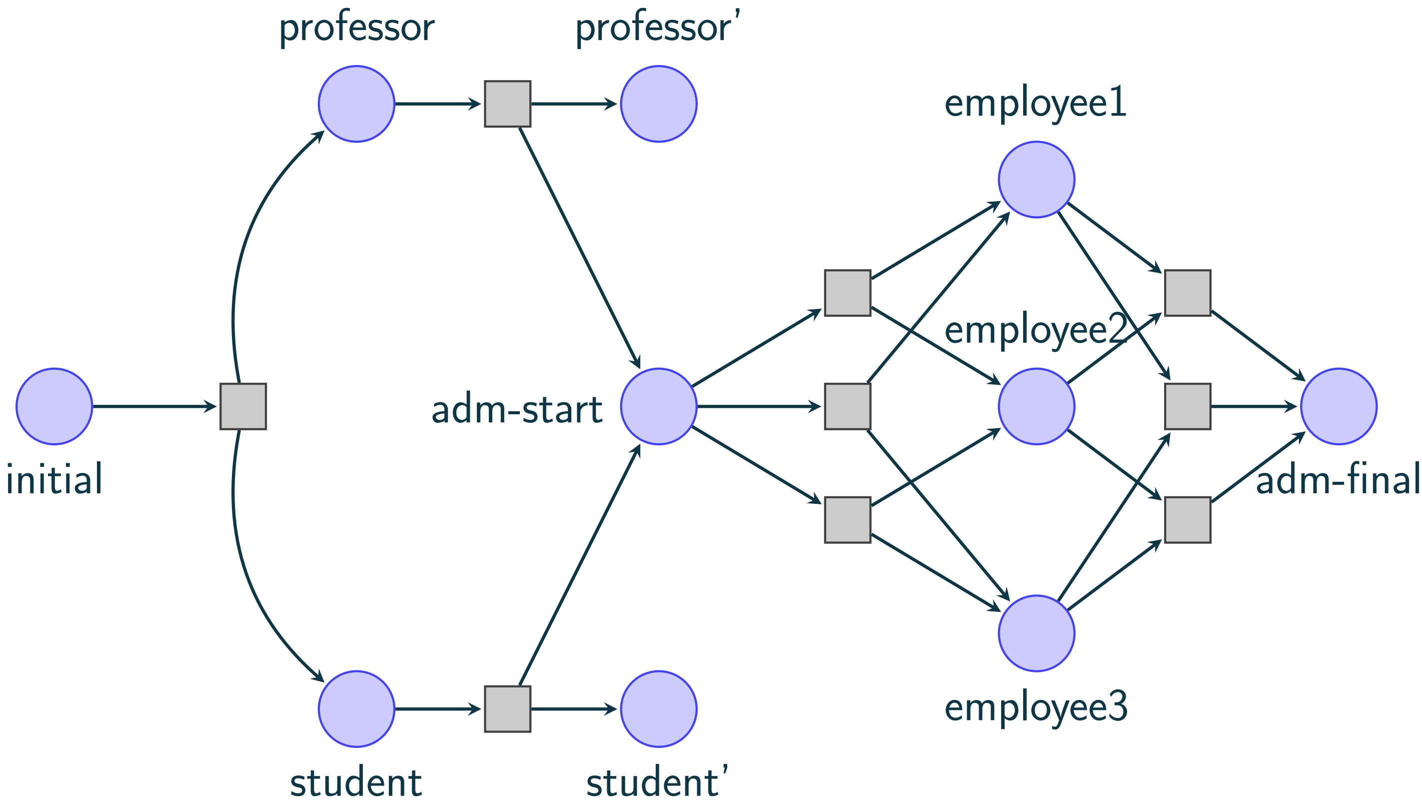

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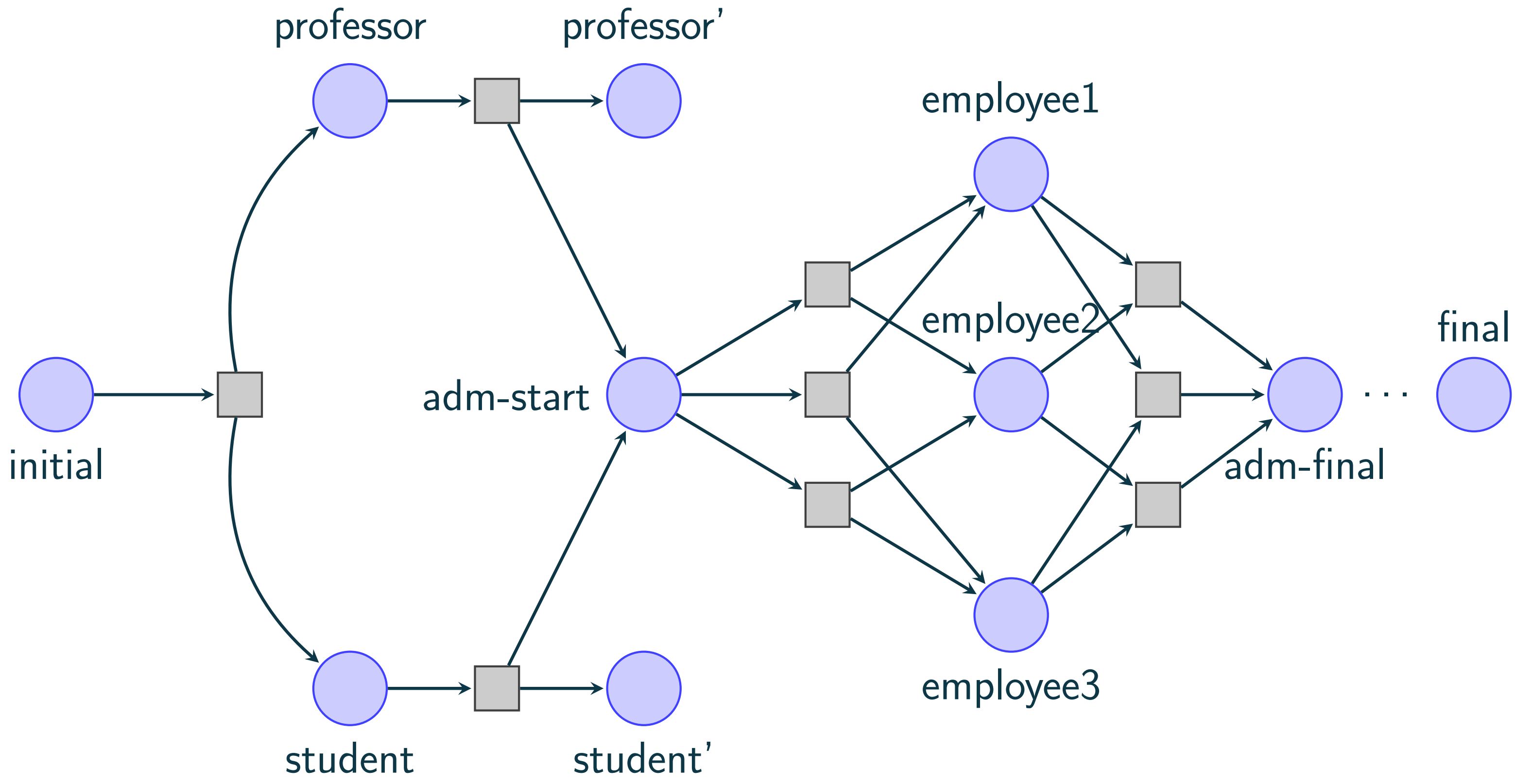

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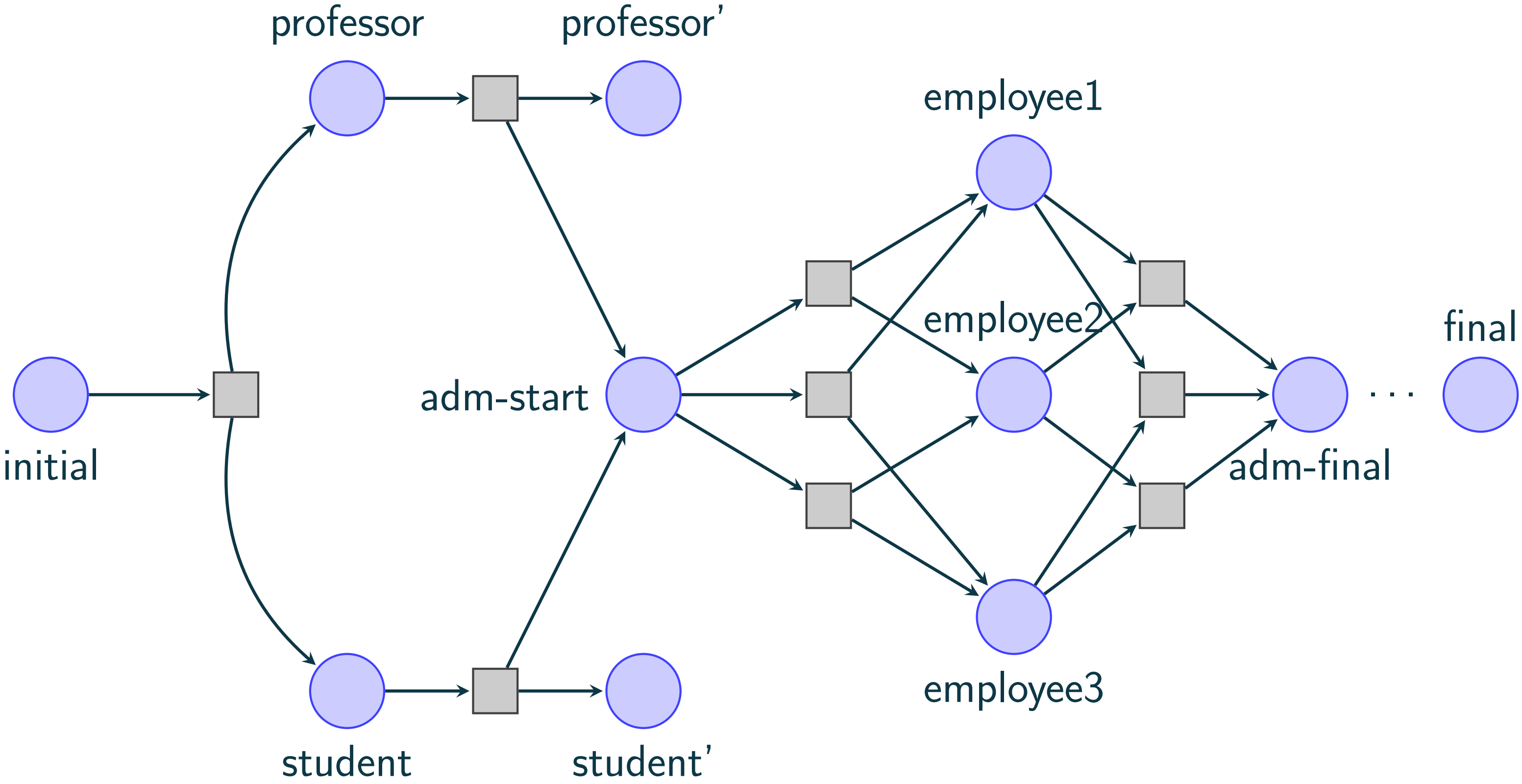

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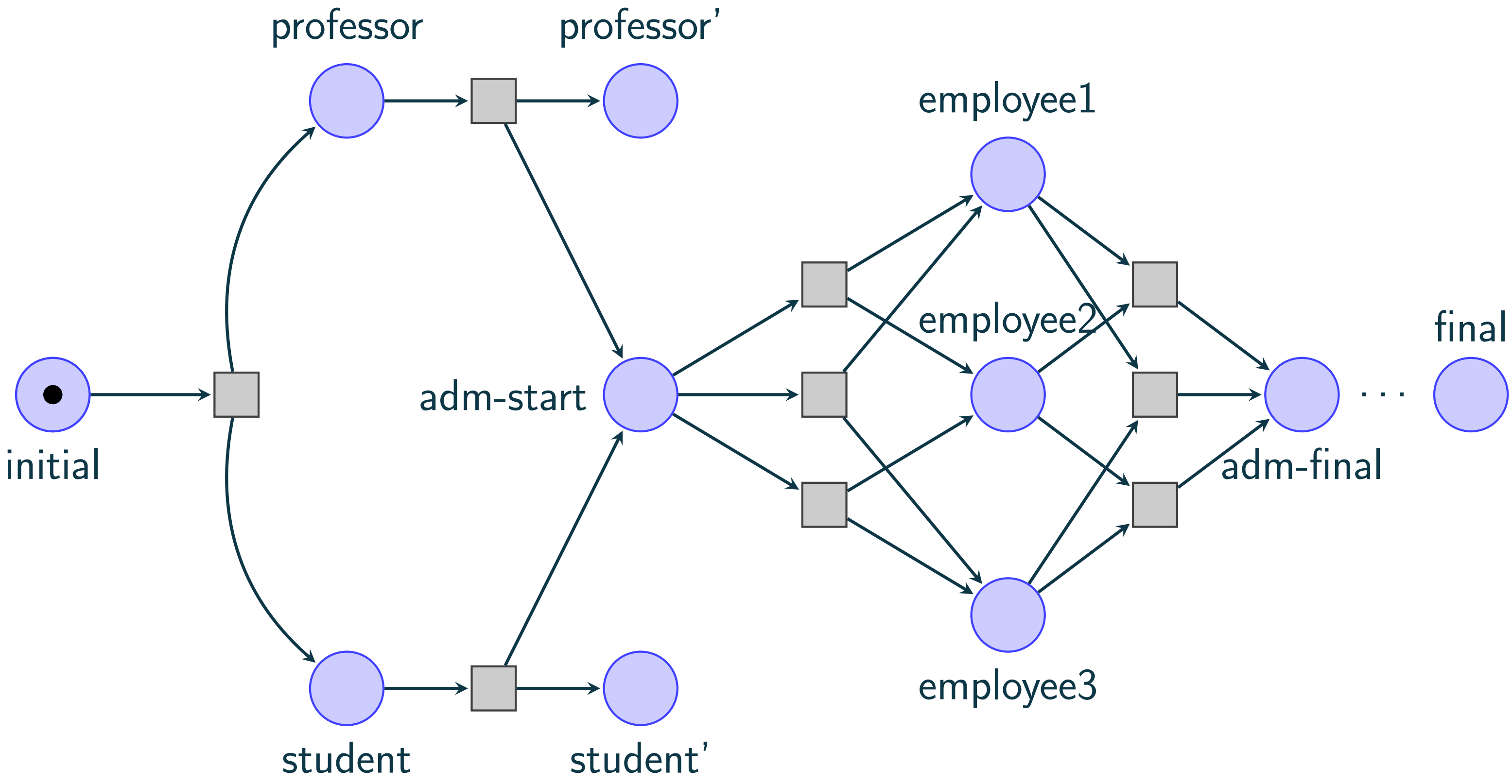

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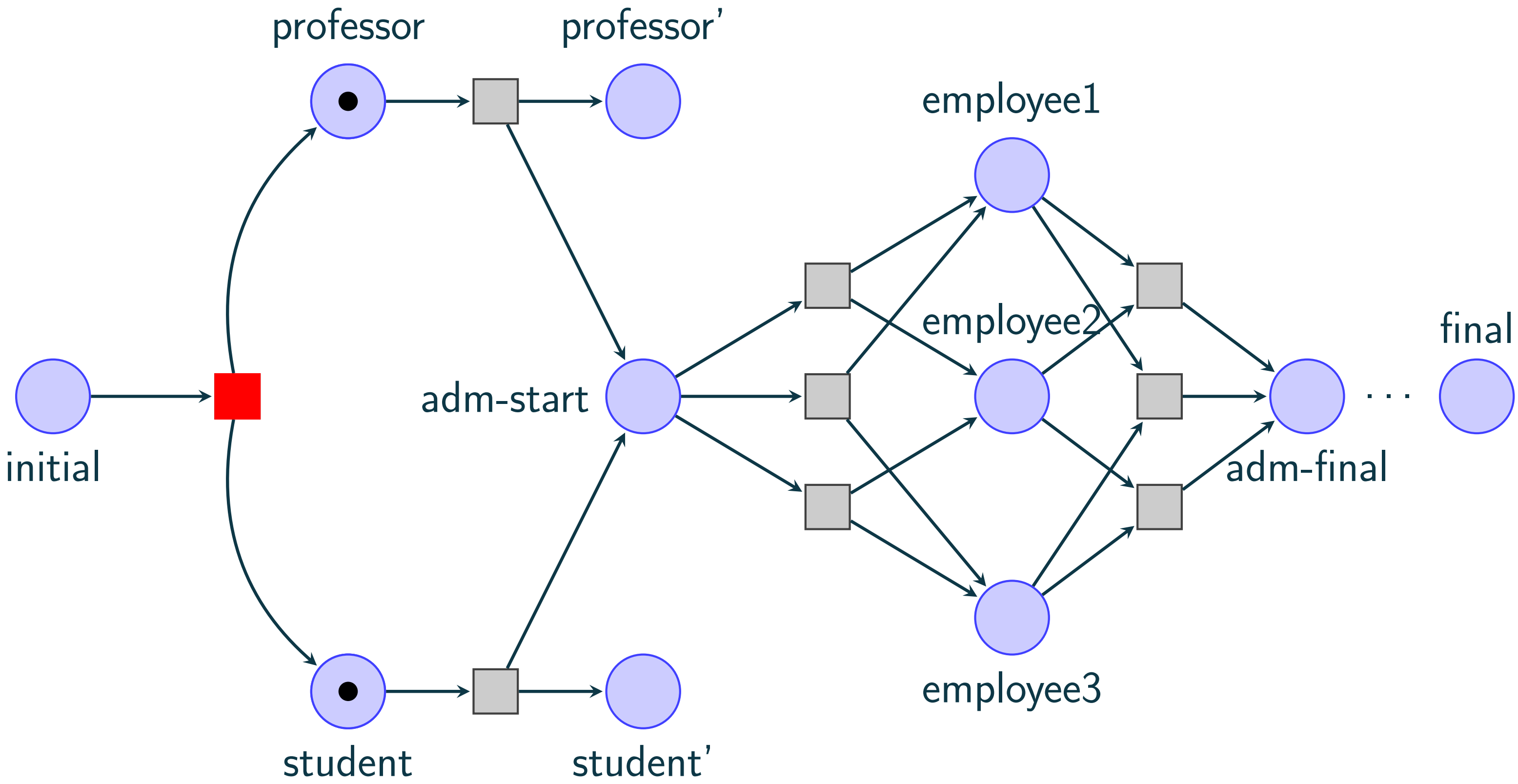

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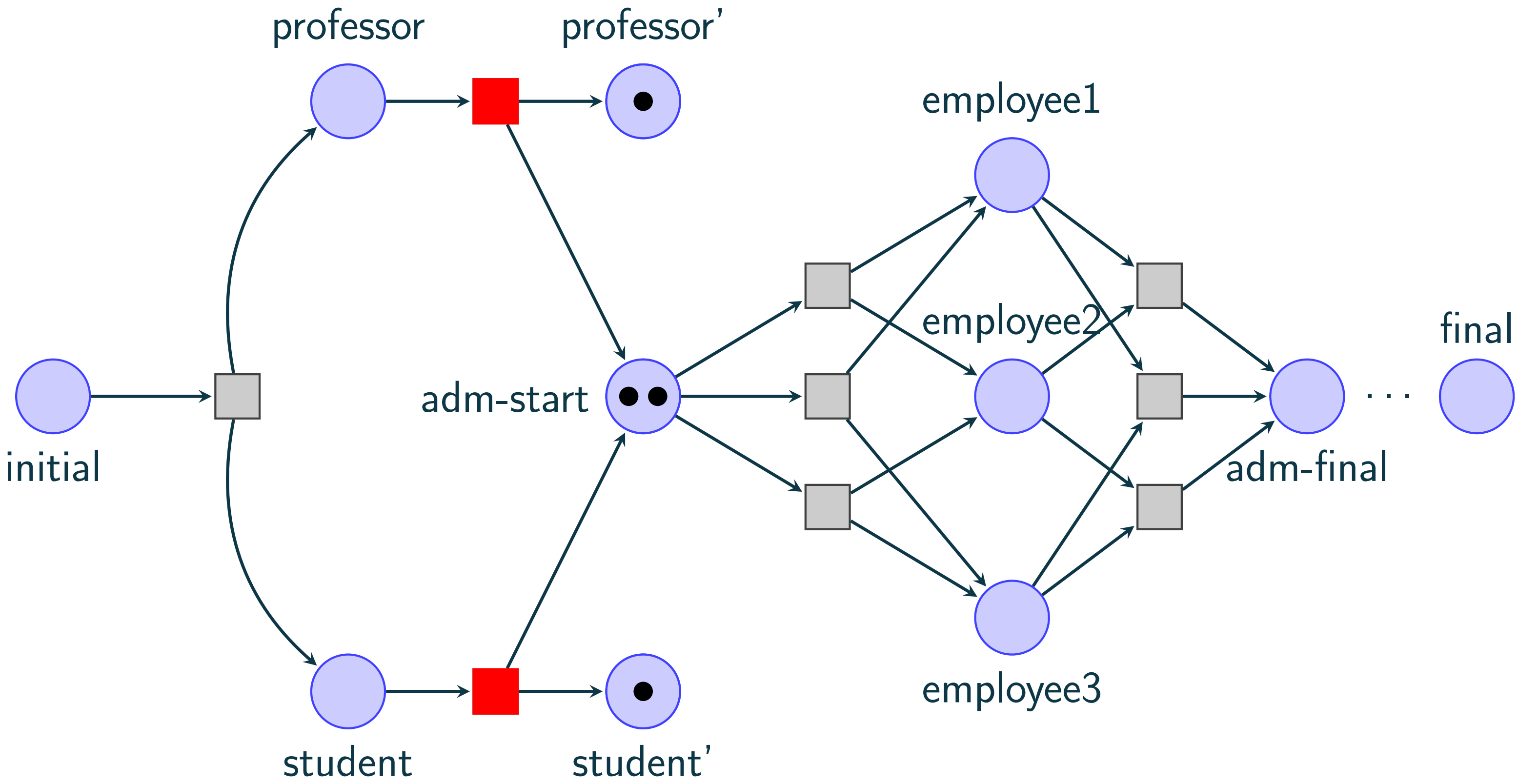

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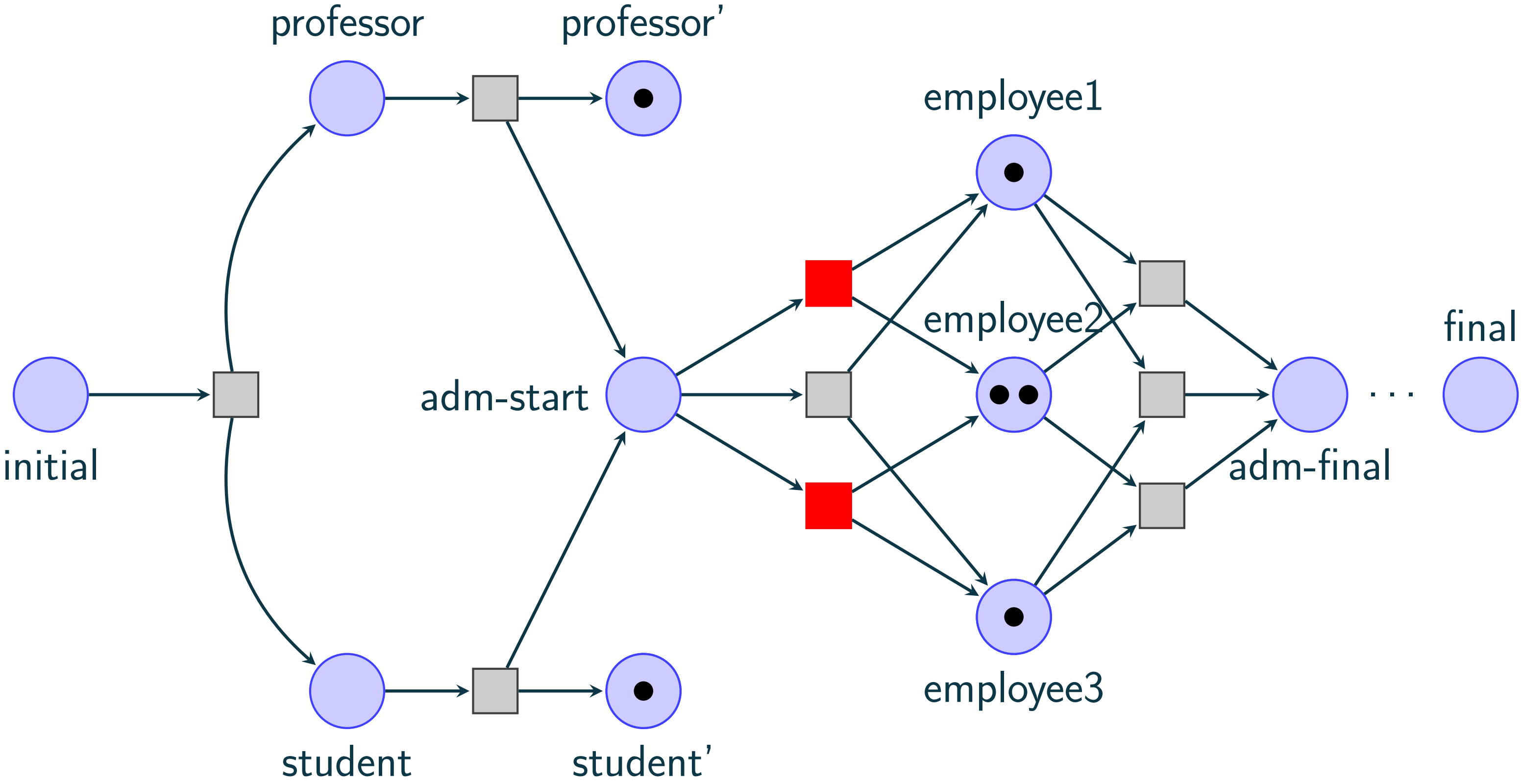

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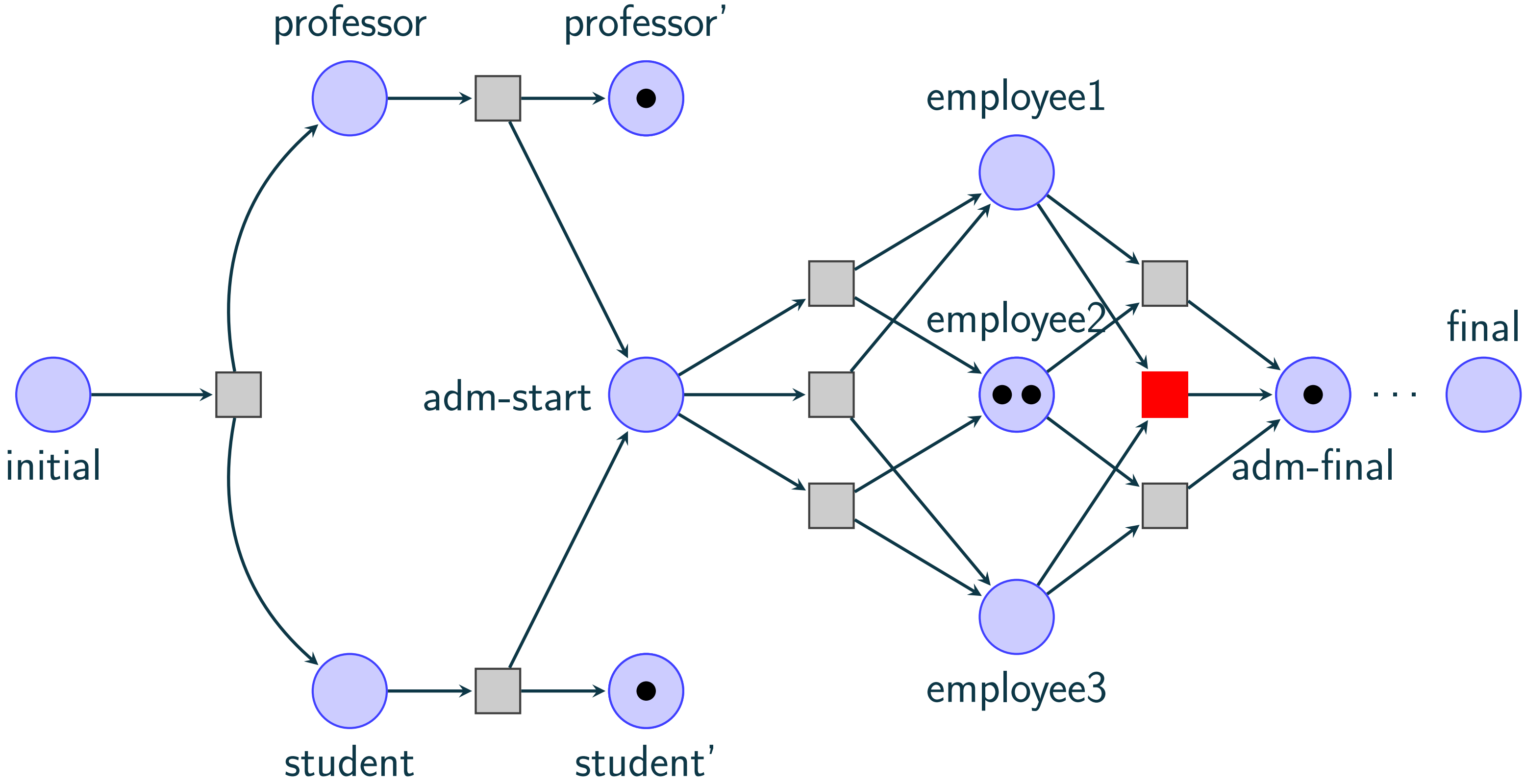

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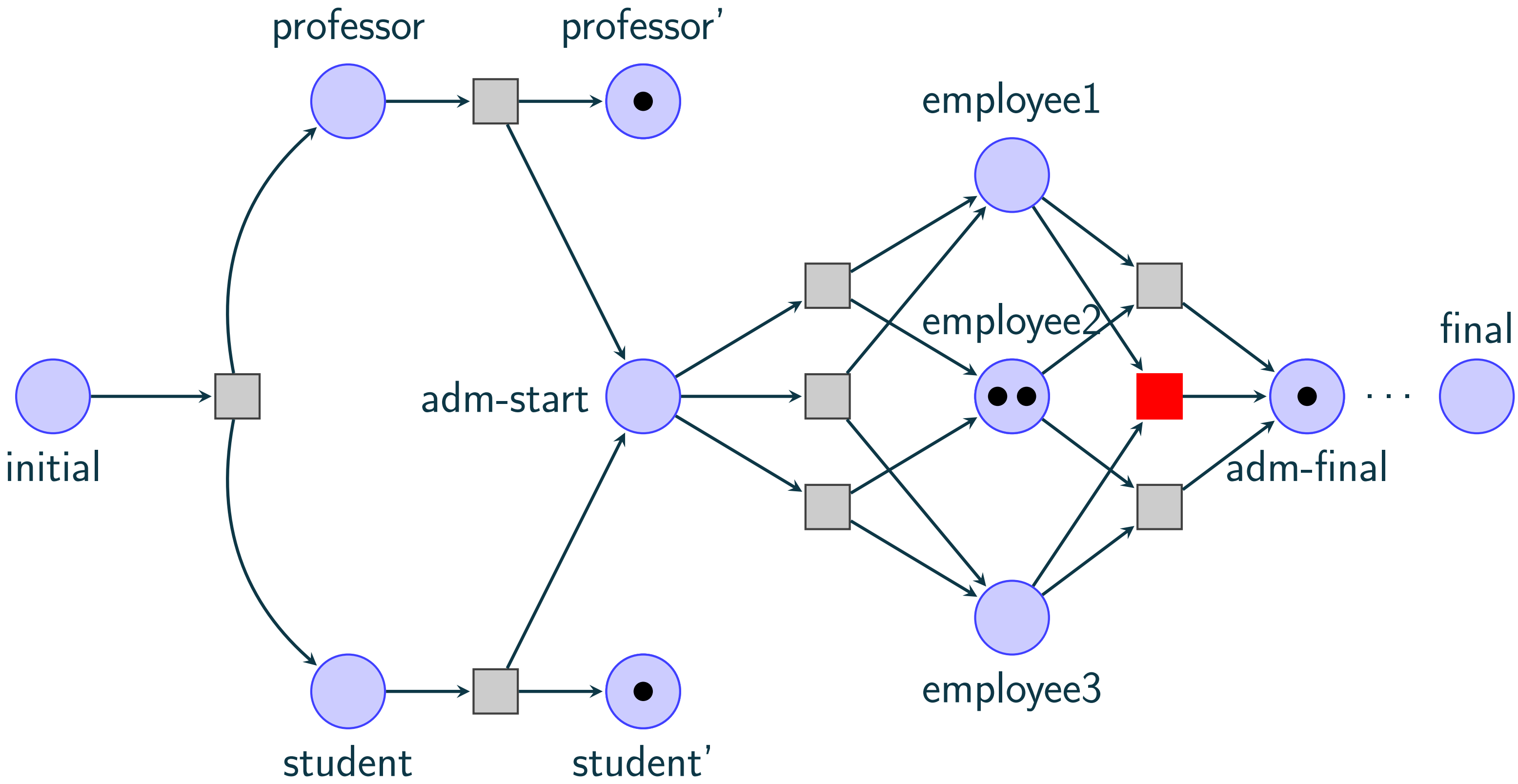

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  Some papers vaguely claim it's EXPSPACE-hard (see later)

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Our results

Theorem (Blondin, M., Offtermatt 2022)

1. Classical soundness is EXPSPACE-complete
2. Generalised soundness is PSPACE-complete
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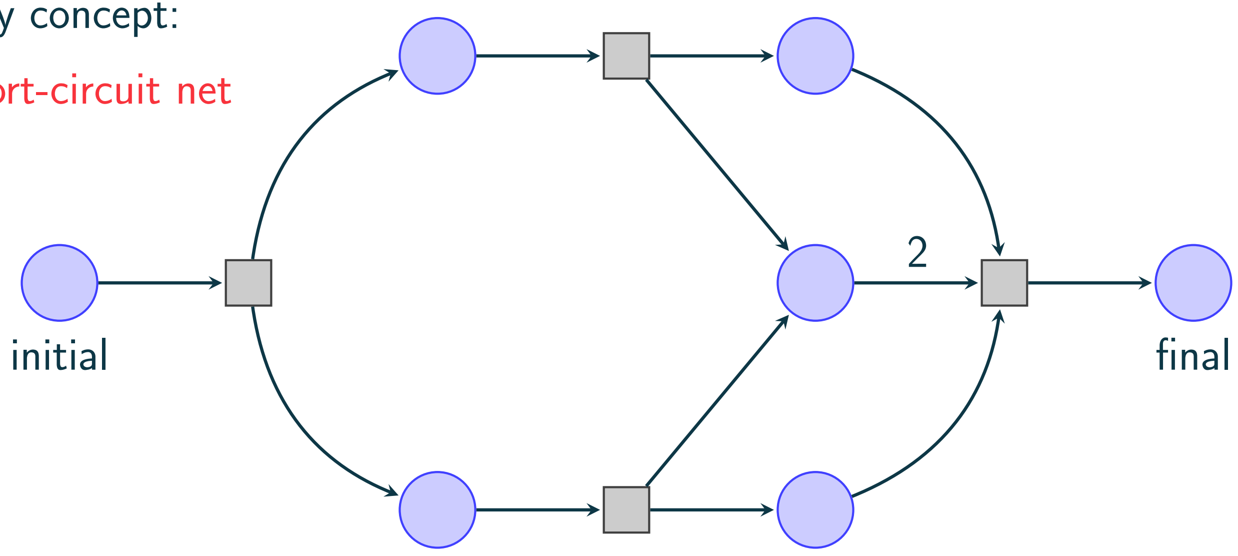
Soundness decidability

Key concept:

short-circuit net



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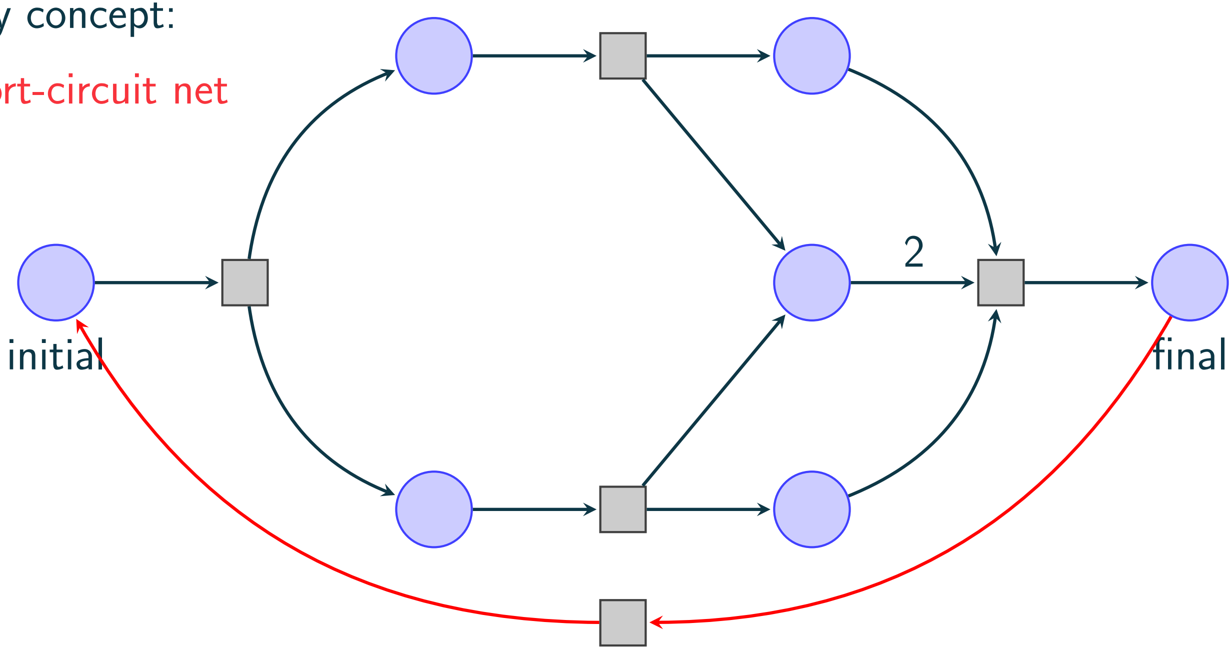

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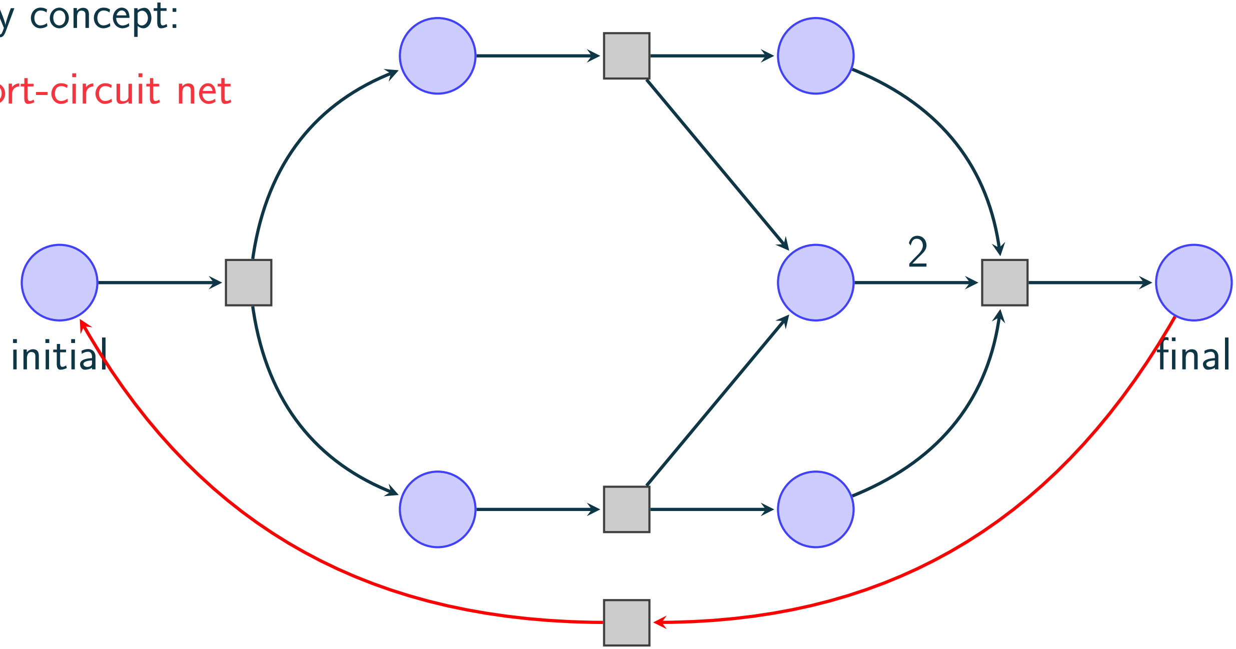

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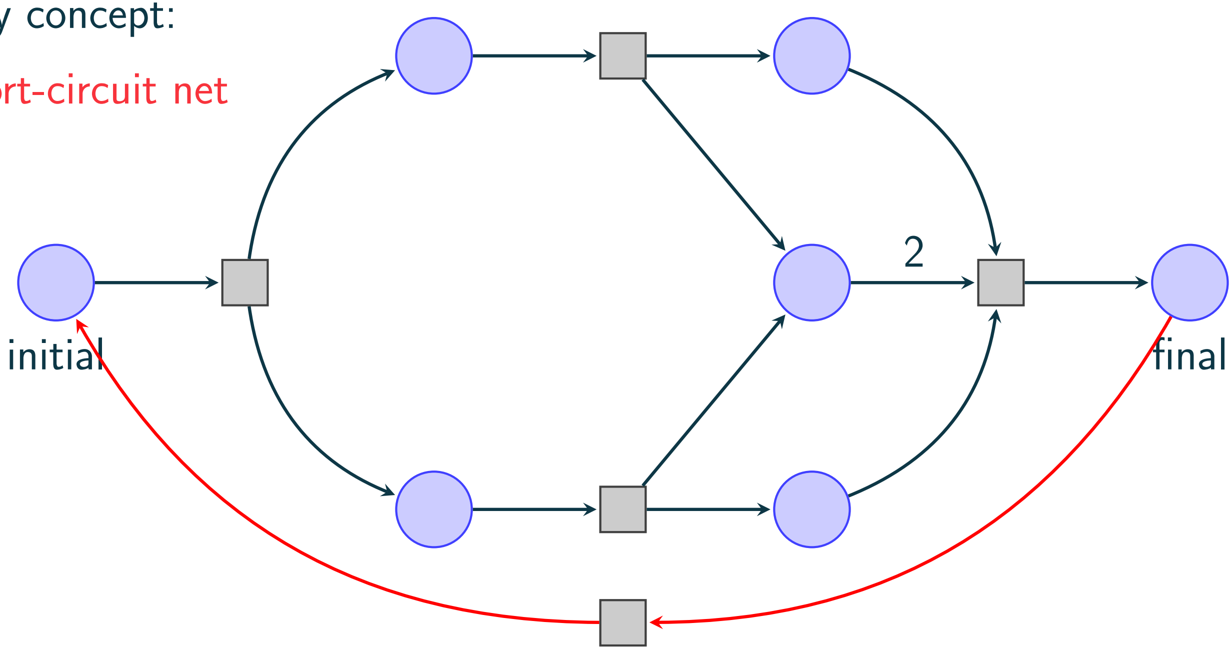

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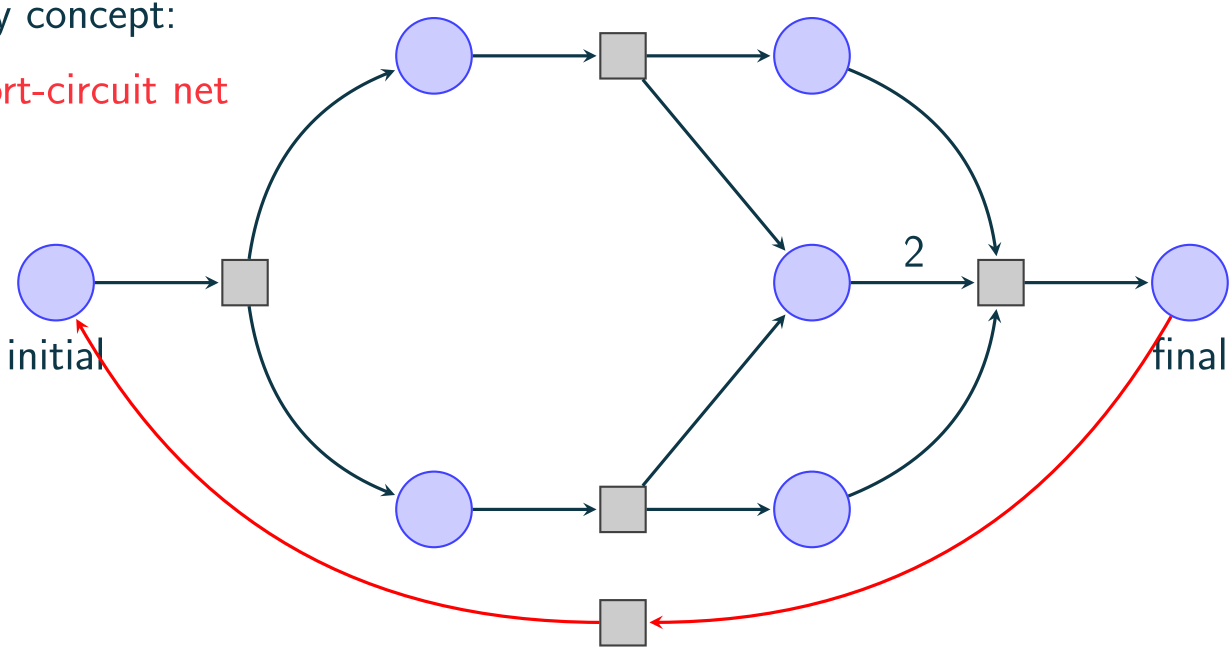

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If 1-sound then $\boldsymbol{m}' \rightarrow^* \{f : 1\} + \boldsymbol{m}' - \boldsymbol{m}$ ∎

Our improvement to EXPSPACE

Lemma (Aalst 1997) quasi-live + cyclic

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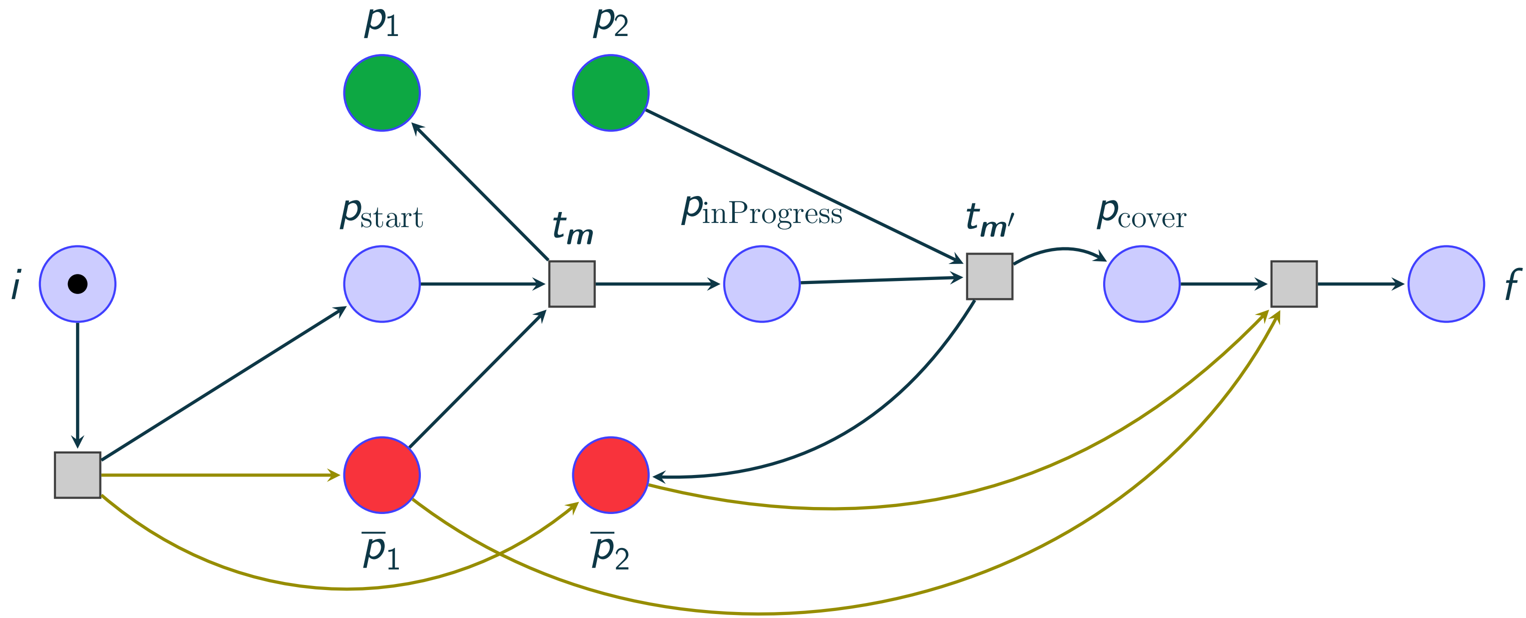
Lemma (Blondin, M., Offtermatt 2022)

Classical soundness is EXPSPACE-hard

via a technical reduction from reachability for reversible Petri nets

EXPSPACE-hardness

Encoding reachability $m \to^* m'$ in a reversible Petri net



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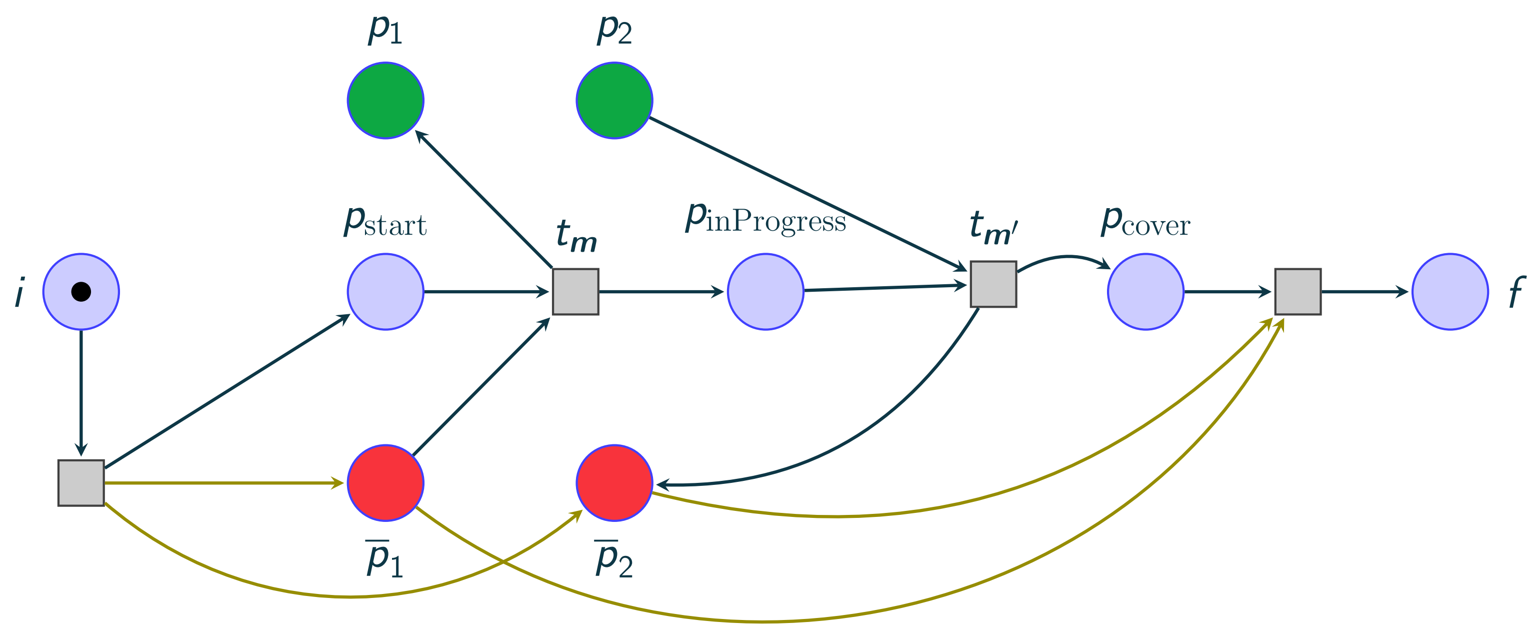

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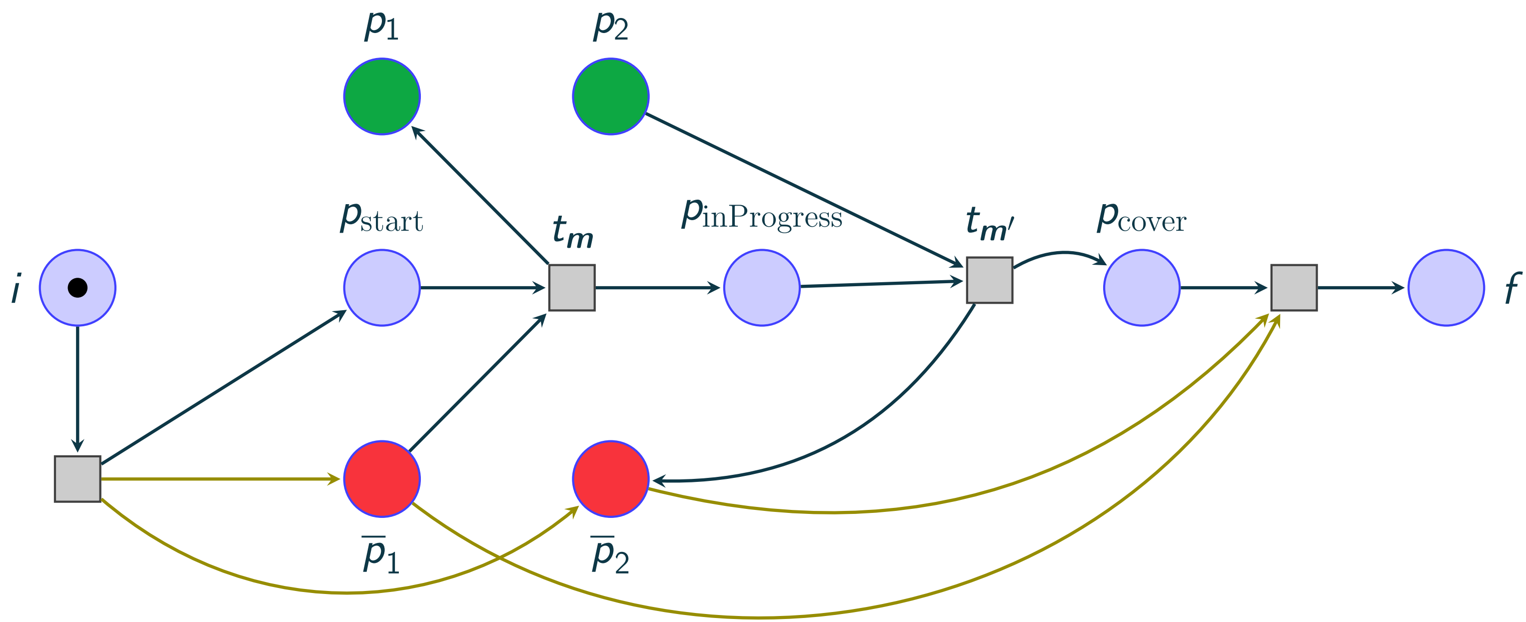

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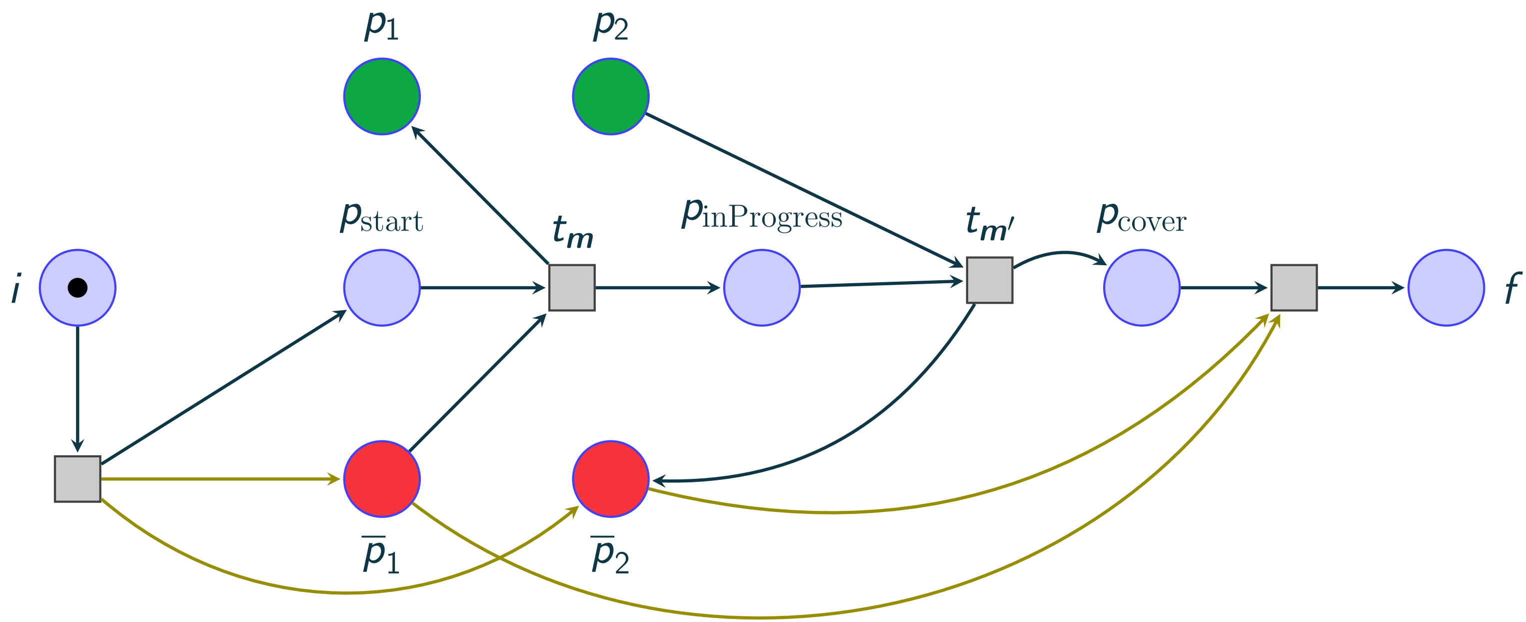

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- Keep as an invariant that $p + \overline{p} = 2^{2^n}$

Generalised soundness

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"small" $=$ exponential (for 1-soudness (2) was double exponential)

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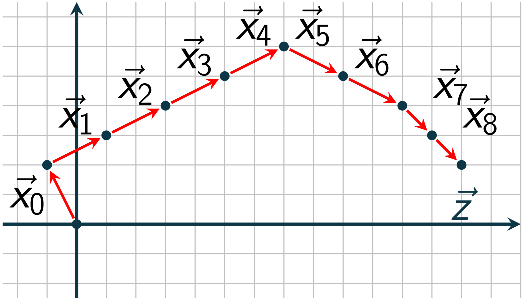
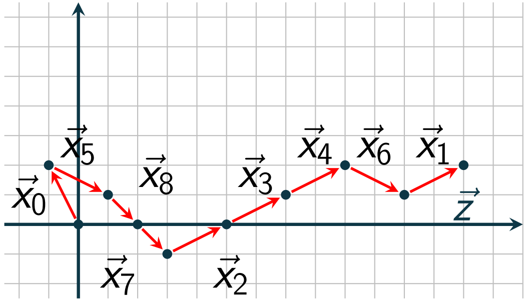
  Then we can verify generalised soundness in PSPACE

  (go through all reachable configurations)

Reachable m are small





Steinitz Lemma: if $\boldsymbol{m} \rightarrow^*_{\mathbb{Z}} \boldsymbol{m}'$ then one can reorder vectors to be "close" to the line $\boldsymbol{m}' - \boldsymbol{m}$

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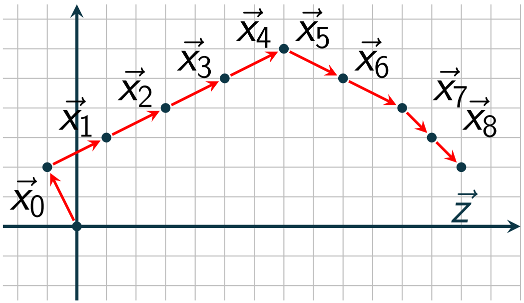

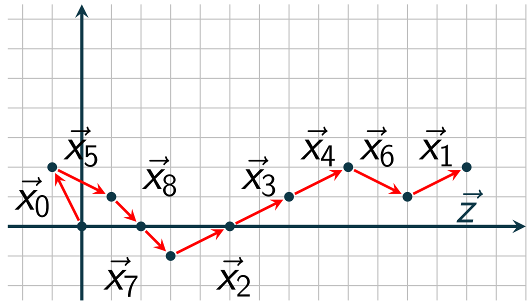

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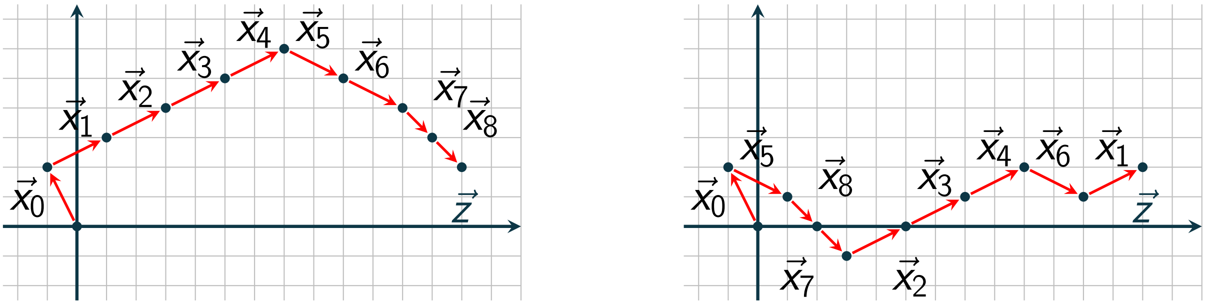

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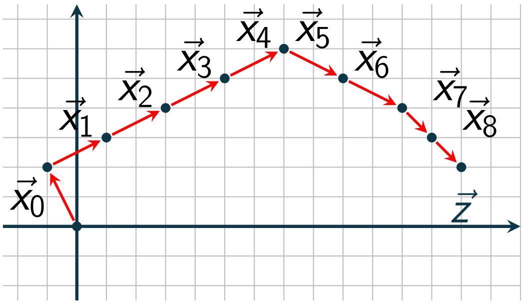

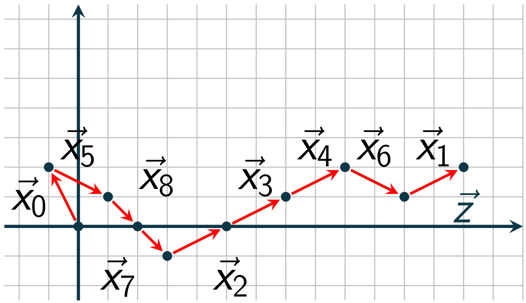

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- By previous slide $\mathbb{Z}$-unboundedness implies not generalised soundness

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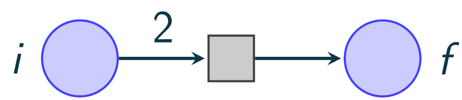

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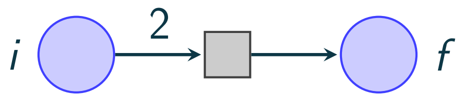

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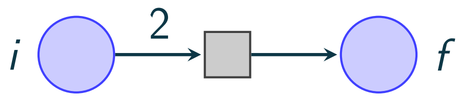

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  Essentially, because S is closed under subtraction

Plan

1. Petri nets and reachability

2. Workflow nets and soundness

3. Some proofs

4. Implementation

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By relaxing problems.

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- Used in many implementations [Esparza et al. 2014], [Blondin et al., 2016], ...

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This and some other observations give us a nice implementation
(most benchmarks are free-choice nets)

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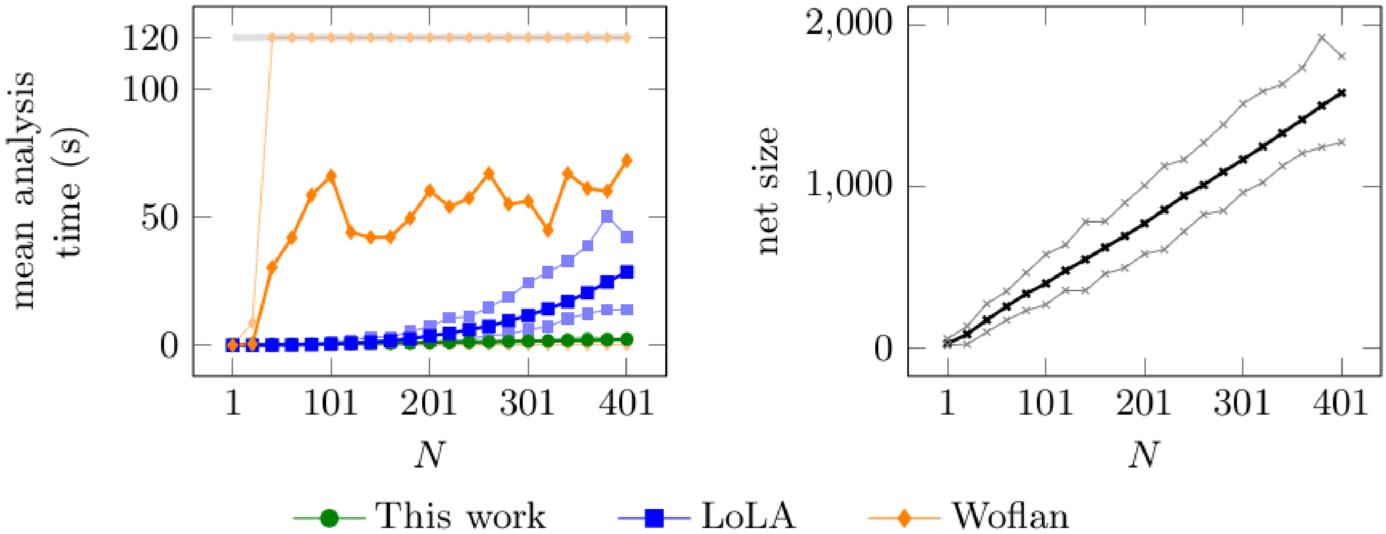
mean analysis time (s)
120
100
50
0
1
101
201
301
401
N
net size
2,000
1,000
0
1
101
201
301
401
N
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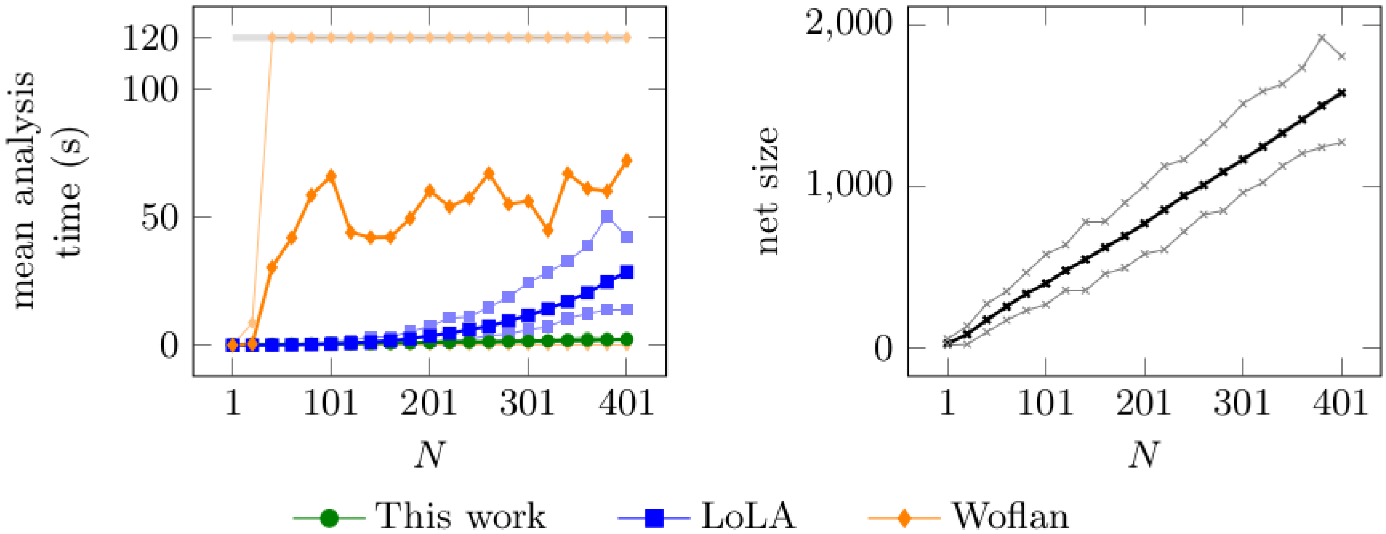

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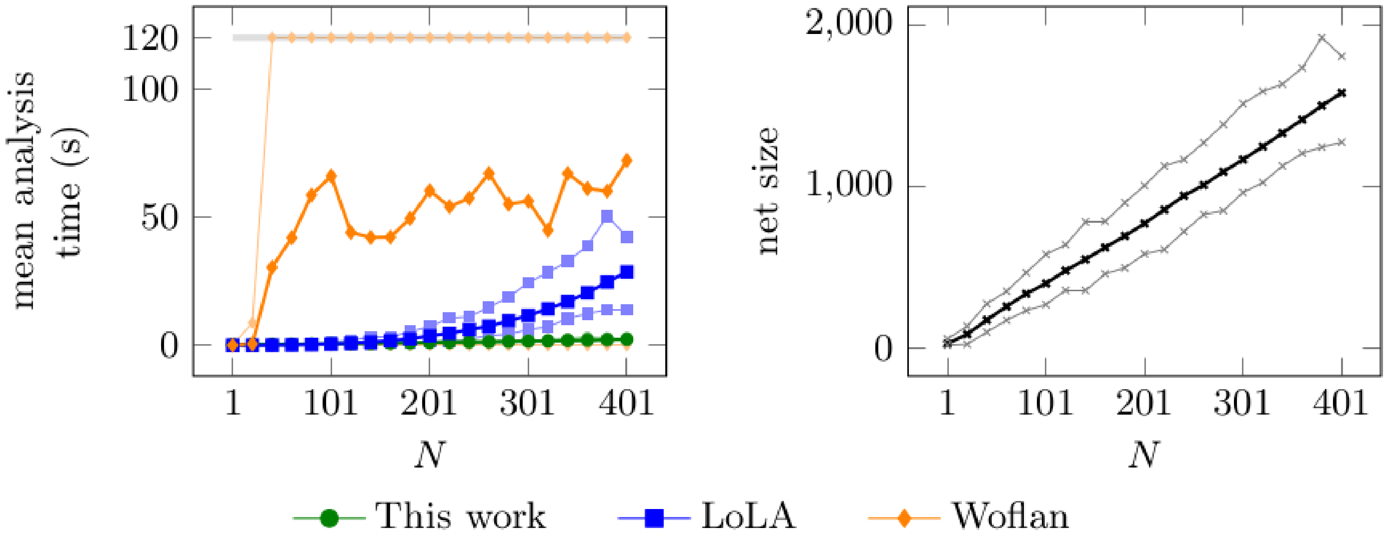

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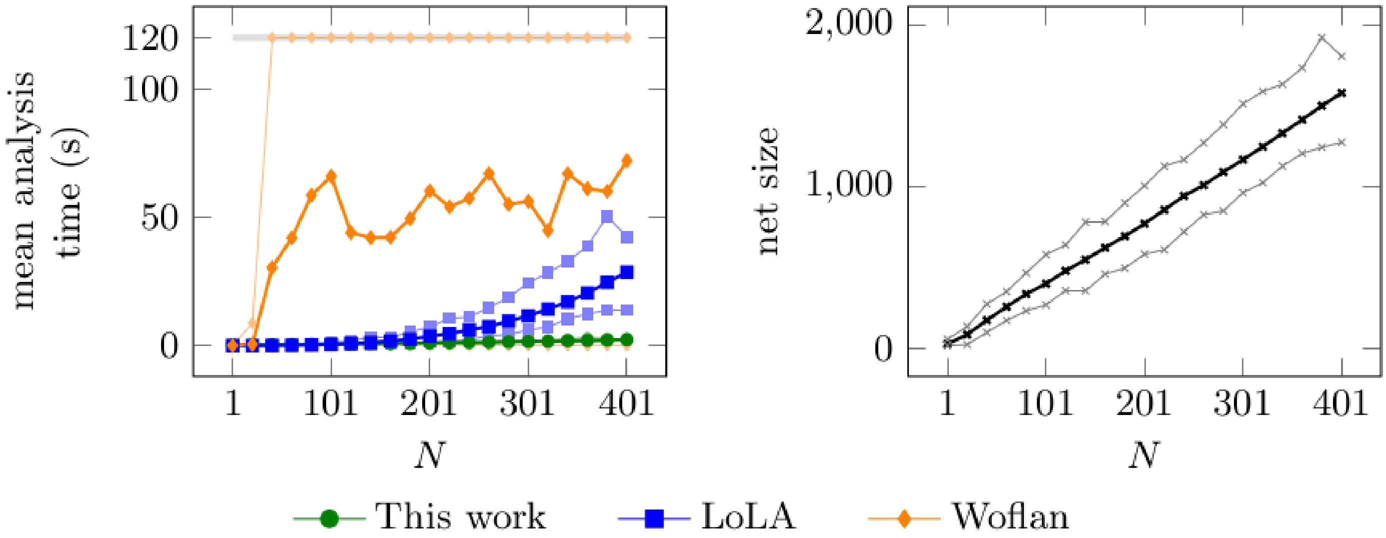

- On “small” benchmarks our tool is slower (but all three are very fast)
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- For big N our tool scales much better

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- Soundness can be seen as a containment problem

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- The talk is based on two papers with Michael Blondin and Philip Offtermatt

  1. "The complexity of soundness in workflow nets". LICS 2022.

  2. "Verifying Generalised and Structural Soundness of Workflow Nets via Relaxations". CAV 2022.