The monitoring problem for timed automata

Filip Mazowiecki

Max Planck Institute for Software Systems,
Saarbrücken Germany

based on joint work with A. Grez, M. Pilipczuk, G. Puppis and C. Riveros

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  finite alphabet: $\Sigma = \{a, b\}$

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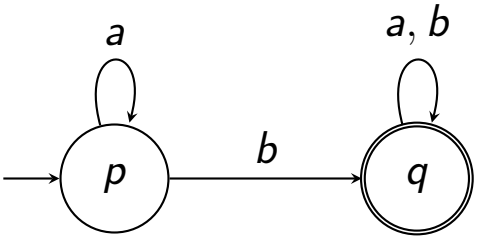

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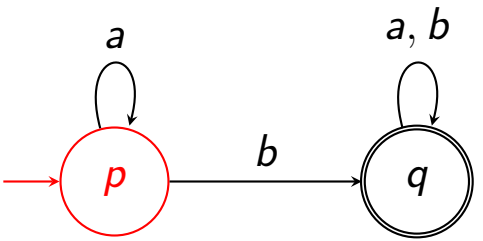

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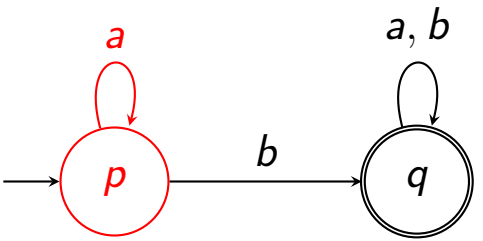

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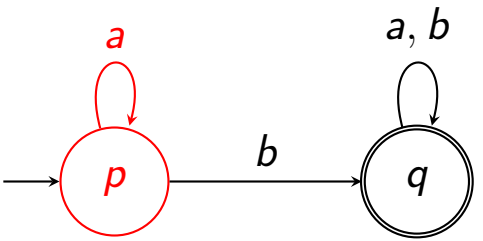

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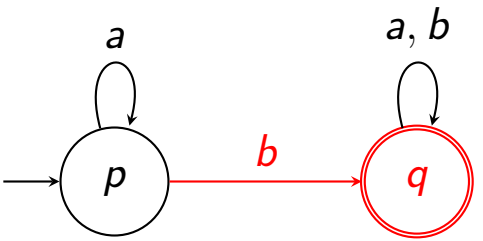

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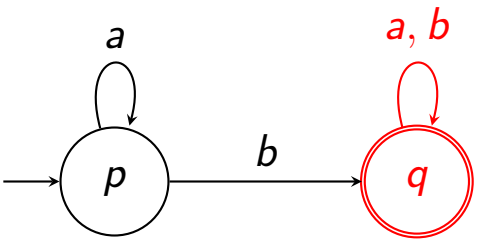

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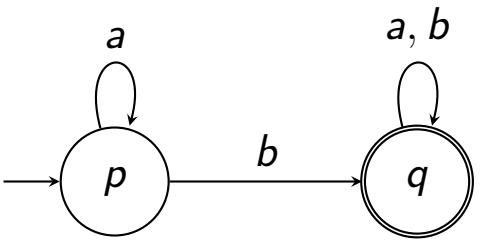

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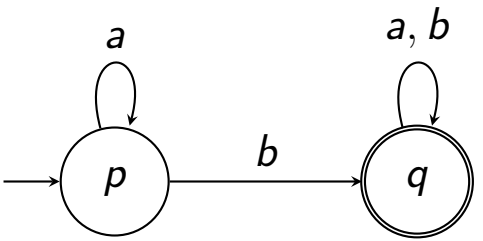

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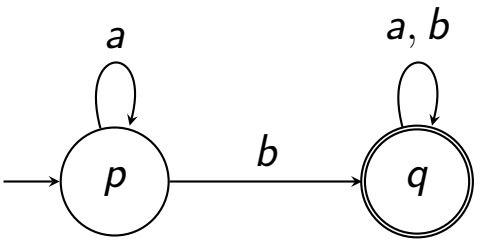

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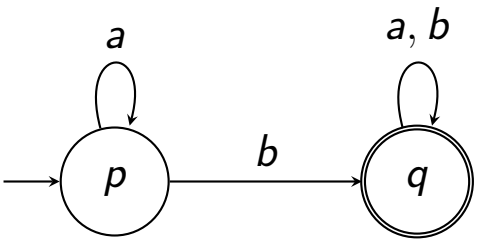

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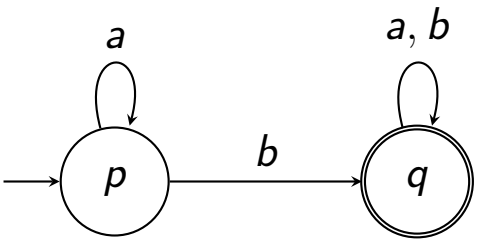

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  - Preprocessing time — constant
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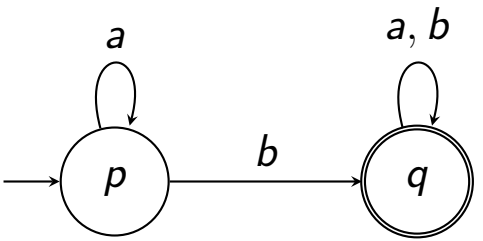

Challenges

- Output for P
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$|P|$ usually means constant

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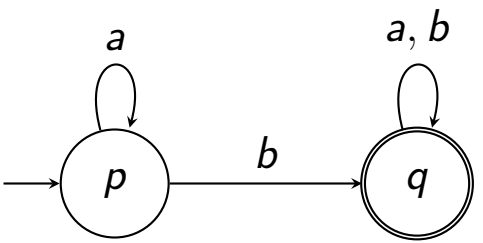

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Does the property P hold within a window frame?

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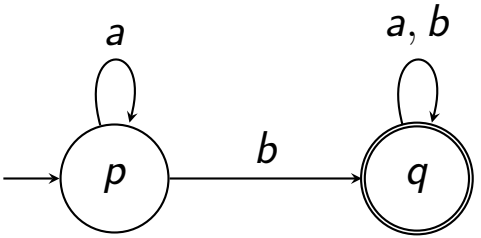
Timed automata

An extension of finite automata



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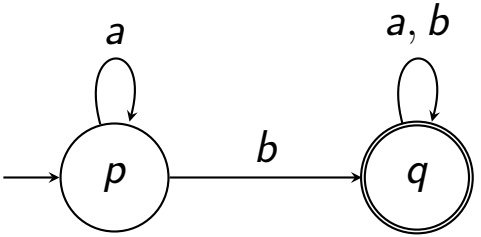

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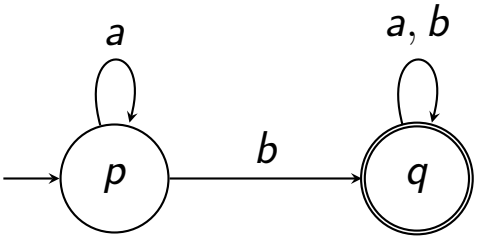

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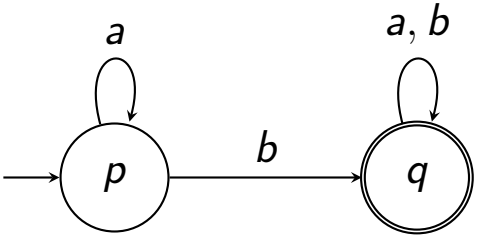

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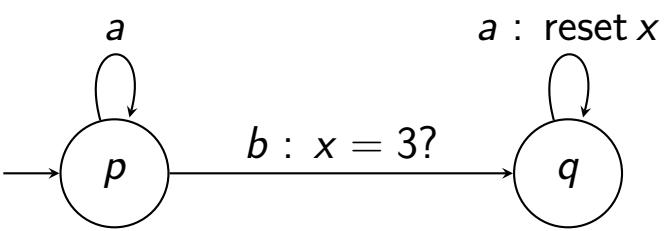

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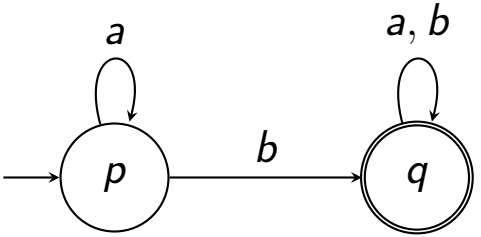

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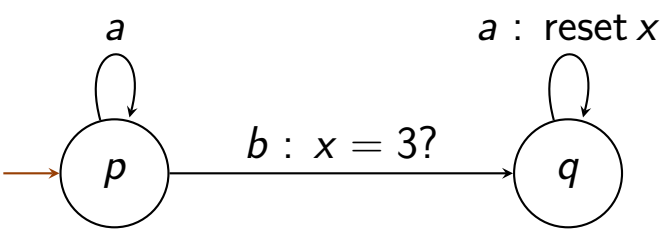

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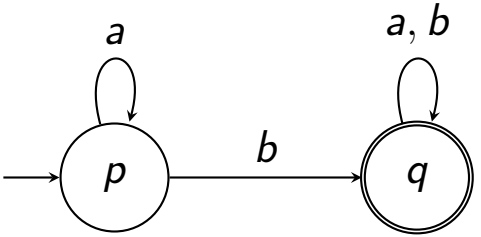

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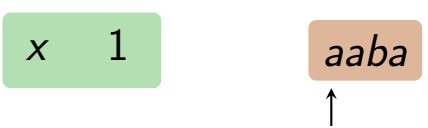

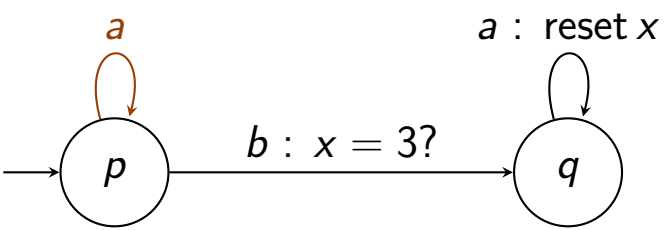

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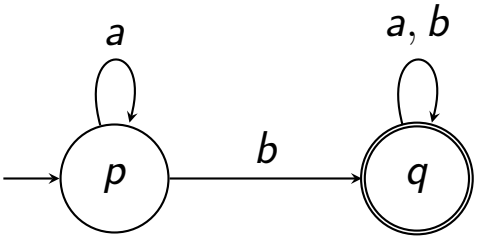

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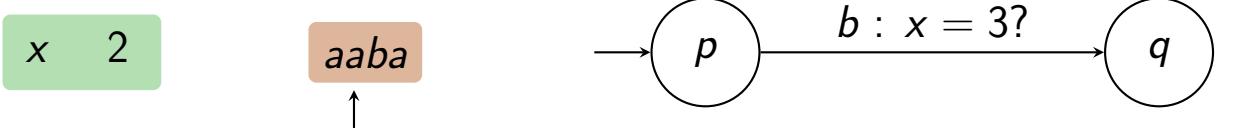

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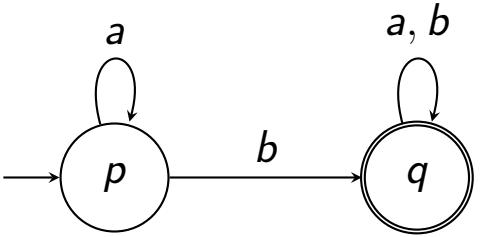

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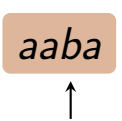

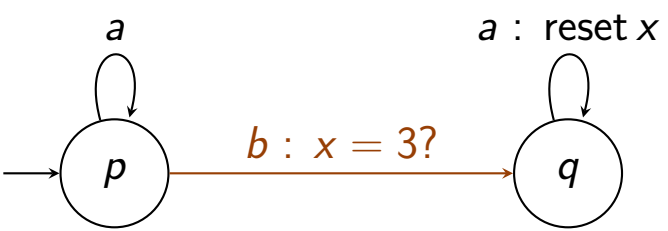

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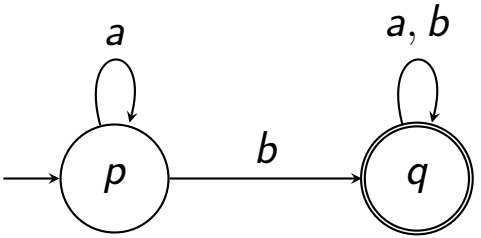

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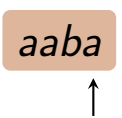

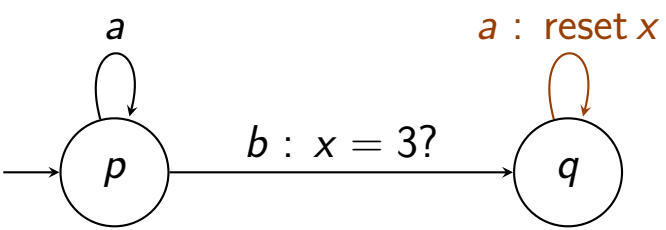

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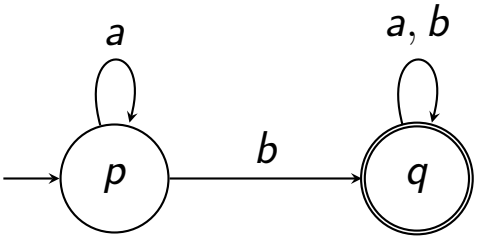

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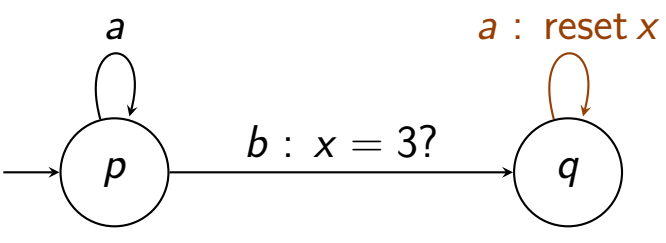
$\text{reset}\, x,\ x \leq N,\ x = N,\ x \geq N$

$x \quad 0$

$aaba$



“Real” example

Professor Joe

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Angie

Marco

Dan

“Real” example

Professor Joe

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Marco

Dan

Thor

“Real” example

Professor Joe Angie Marco Dan Thor

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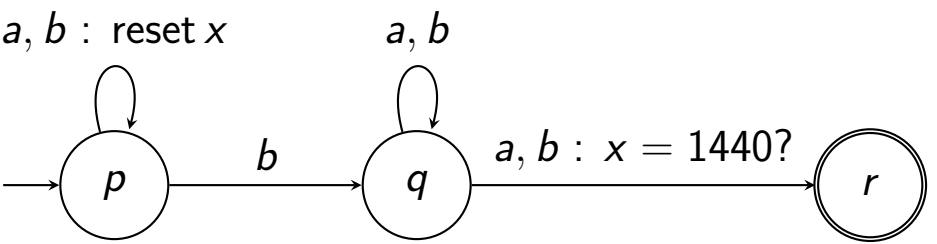

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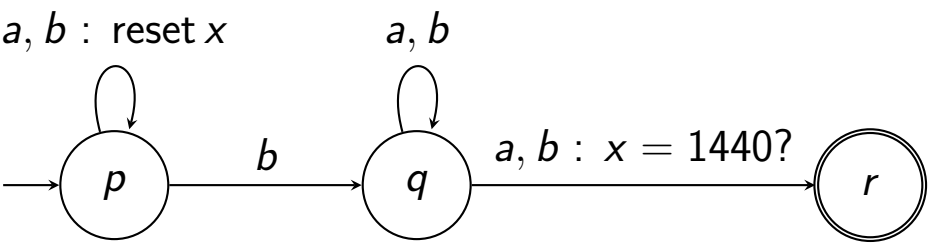
$a\ a\ b\ a\ a\ b\ a \ldots$



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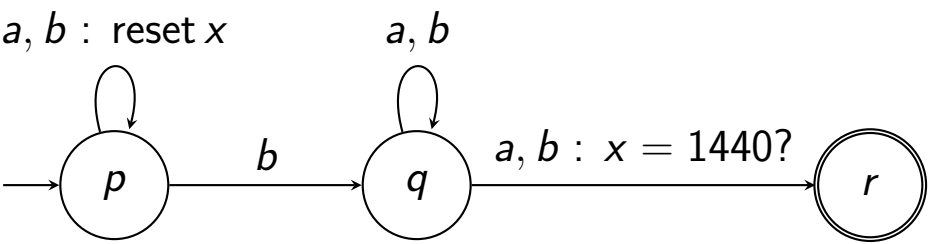
	a	a	b	a	a	b	a ...
x	0	0	1	2	3	4	5 ...



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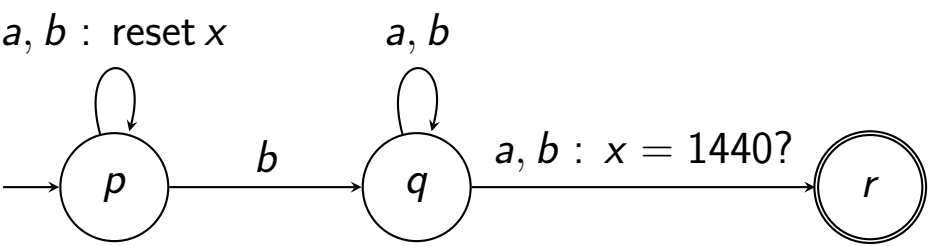
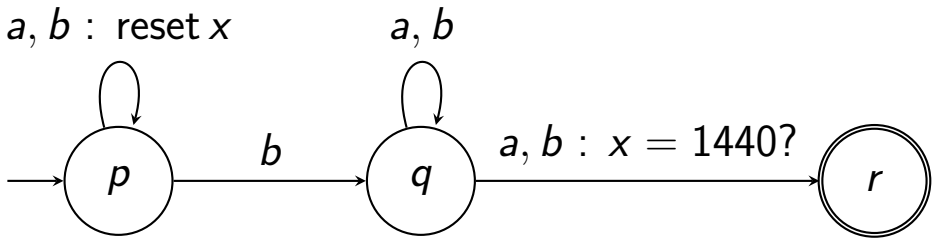
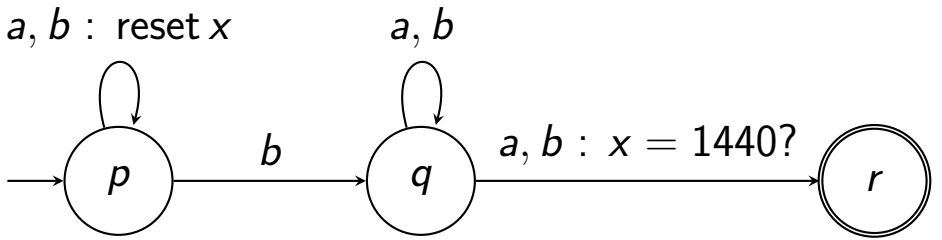
x	0	0	0	0	0	1	2	$\ldots$



Structure for the example (not depending on 1440)



Structure for the example (not depending on 1440)

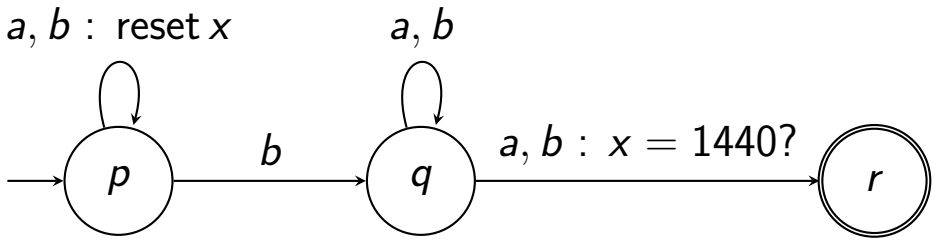


Output NO

Global clock 0 List

Input

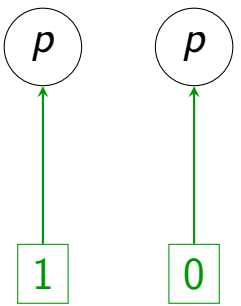
Structure for the example (not depending on 1440)



Output NO

Global clock 1

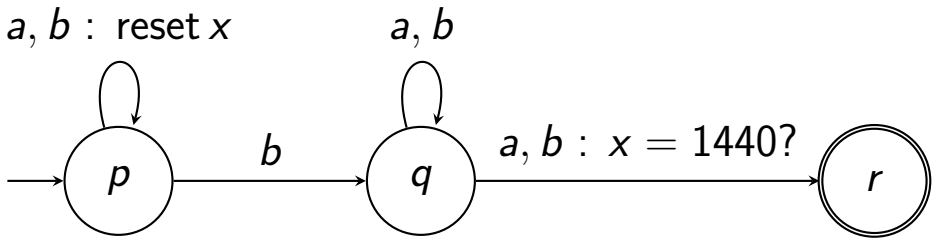
List



(reset)

Input b

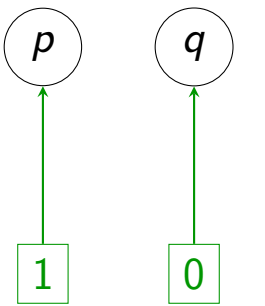
Structure for the example (not depending on 1440)



Output NO

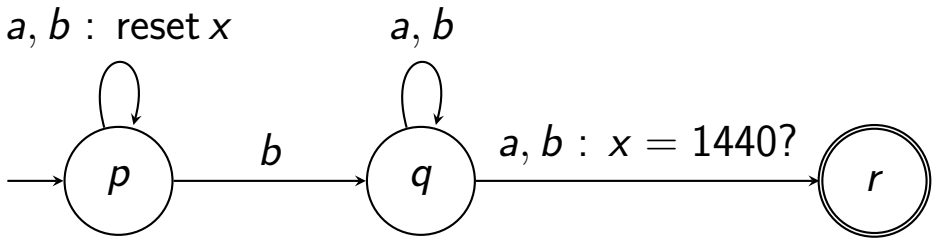
Global clock 1

List

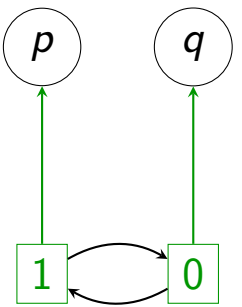


Input b

Structure for the example (not depending on 1440)



Output NO

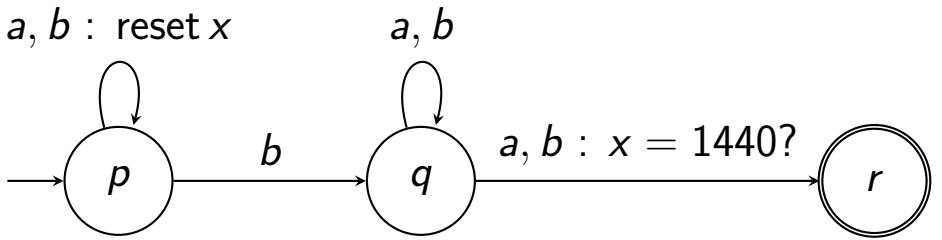


Global clock 1 List

(reset) (other)

Input b

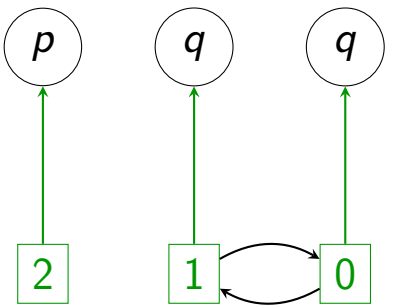
Structure for the example (not depending on 1440)



Output NO

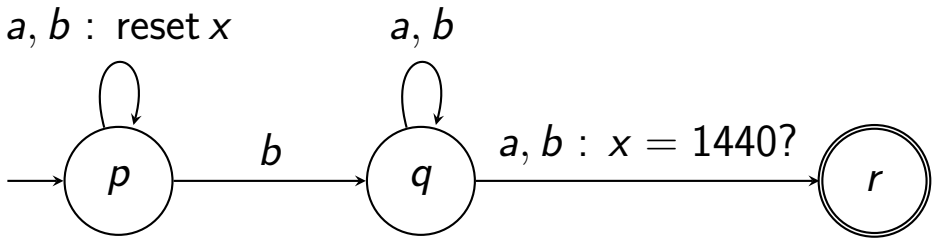
Global clock 2

List

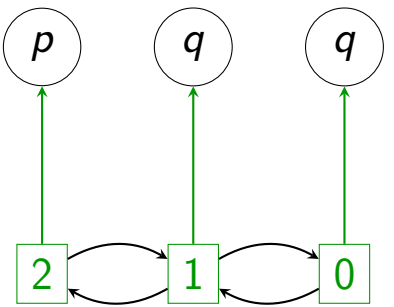


Input *bb*

Structure for the example (not depending on 1440)



Output NO

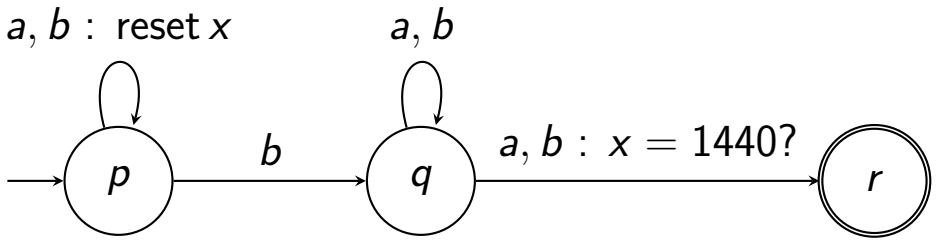


Global clock 2 List

(reset) (other)

Input *bb*

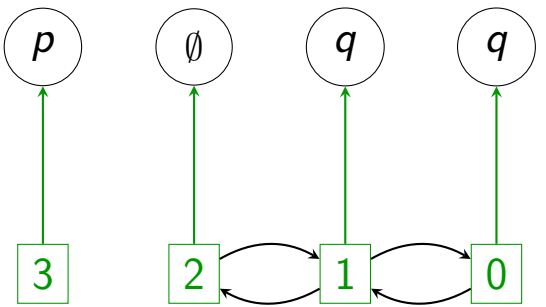
Structure for the example (not depending on 1440)



Output NO

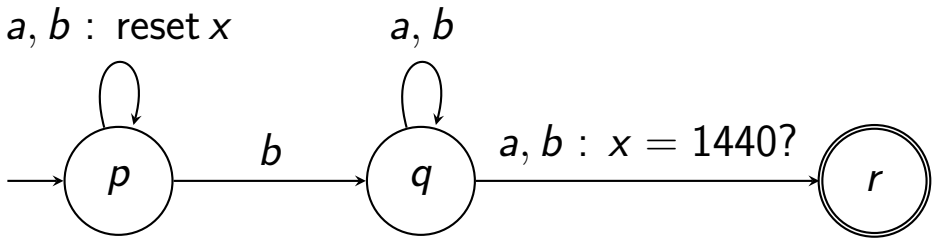
Global clock 3

List

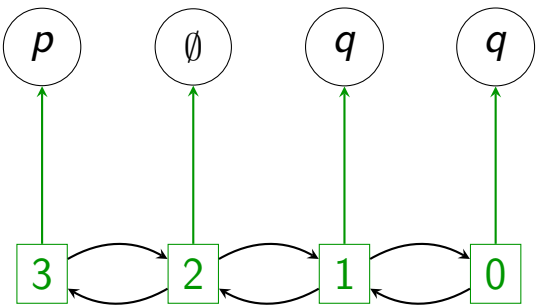


Input *bba*

Structure for the example (not depending on 1440)



Output NO

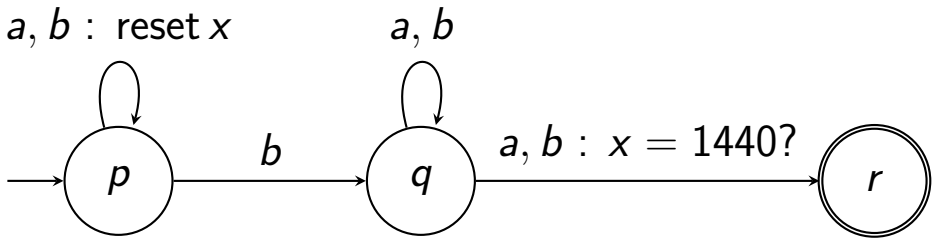


Global clock 3 List

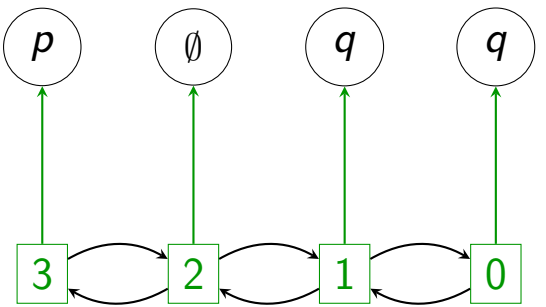
(reset) (other)

Input *bba*

Structure for the example (not depending on 1440)



Output YES

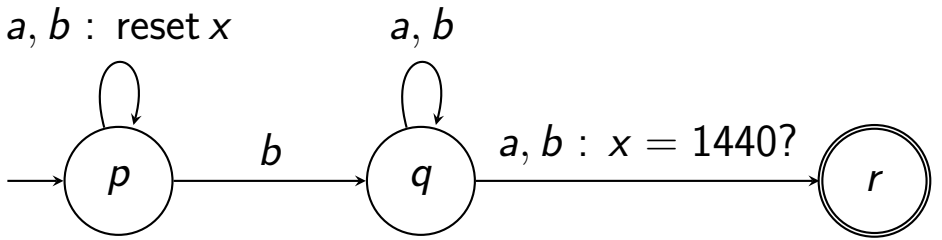


Global clock 1440 List ...

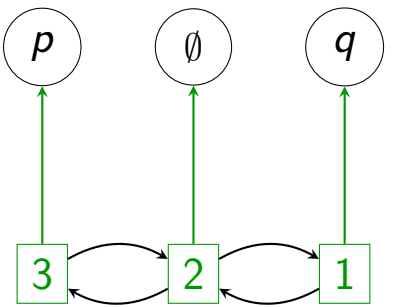
(reset) (other)

Input *bba* . . .

Structure for the example (not depending on 1440)



Output YES



Global clock 1440 List ...

(reset) (other)

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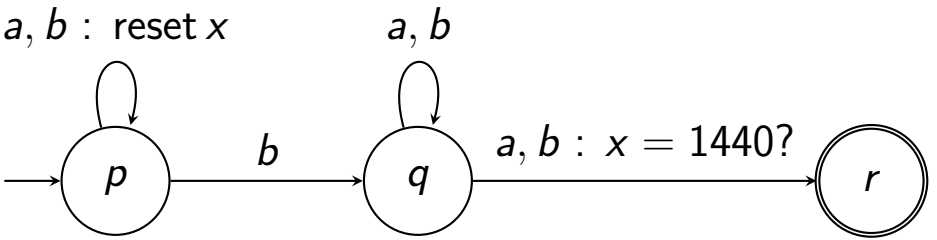
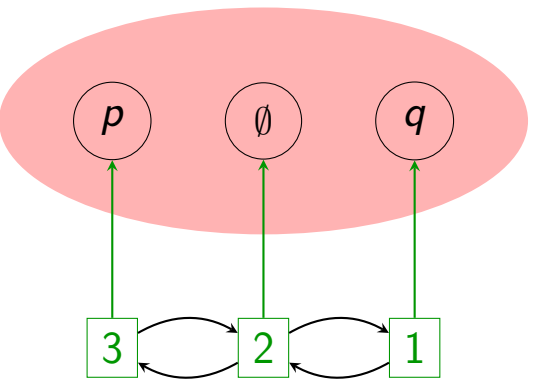


Output YES



Global clock 1440 List ...

(reset) (other)

Input *bba* . . .

- Issue: updating states

Current work

Theorem (Positive)

For 1 clock there is always a structure supporting amortised constants time.

When time is discrete $(0, 1, 2, \ldots)$ it supports (non-amortised) constant time.

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- Can we restrict timed automata?

Other monitoring problems

- How much space is needed? (in terms of window size N)

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  Amortised constant time using two stacks

Remove *Add* (val, max)

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  2

  $(2, 2)$

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  Amortised constant time using two stacks

  25

  $(5, 5)$

  $(2, 2)$

  Remove *Add* (val, max)

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  Amortised constant time using two stacks

  $(3,5)$
  $(5,5)$
  $(2,2)$ 253

  Remove *Add* (val, max)

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  $(3,5)$
  $(5,5)$
  $(2,2)$

  2 5 3

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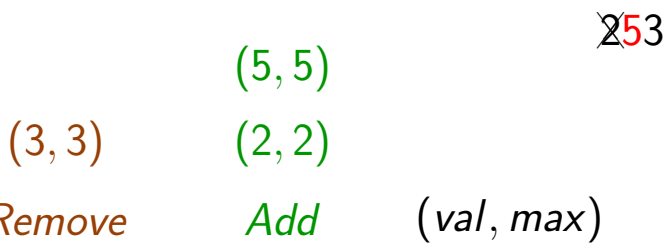

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  ~~2~~53

  $(5, 5)$

  $(3, 3)$ $(2, 2)$

  Remove *Add* (val, max)

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  (Georg's future postdoc)

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  FIFO using insert() and evict()

  Amortised constant time using two stacks

  $(2,5)$

  $(5,5)$

  $(3,3)$

  $\not{2}53$

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Other monitoring problems

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  53

  $(5, 5)$

  $(3, 3)$

  Remove *Add* (val, max)

Other monitoring problems

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  (Georg's future postdoc)

- Maximum of an integer

  FIFO using insert() and evict()

  Amortised constant time using two stacks

  534

  $(5, 5)$

  $(3, 3)$ $(4, 4)$

  Remove *Add* (val, max)

Other monitoring problems

- How much space is needed? (in terms of window size N)

  $a\Sigma^*$ $\Theta(N)$ [Moses Ganardi et. al]

  (Georg's future postdoc)

- Maximum of an integer

  FIFO using insert() and evict()

  Amortised constant time using two stacks

  34

  $(3,3)$ $(4,4)$

  Remove *Add* (val, max)

Other monitoring problems

- How much space is needed? (in terms of window size N)

  $a\Sigma^*$ $\Theta(N)$ [Moses Ganardi et. al]

  (Georg's future postdoc)

- Maximum of an integer Works for any monoid

  FIFO using insert() and evict()

  Amortised constant time using two stacks

  34

  $(3, 3)$ $(4, 4)$

  Remove *Add* (val, max)

  Can be even (non-amortised) constant [Tangwongsan, Hirzel, Schneider 2017]