Affine Extensions of Integer Vector Addition Systems with States

Michael Blondin[1], Christoph Haase[2] and Filip Mazowiecki[3]

[1]Technische Universität München

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Infinity 2018

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Vector Addition Systems with States (VASS)

Automata with counters

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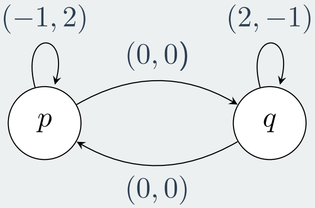
VASS example



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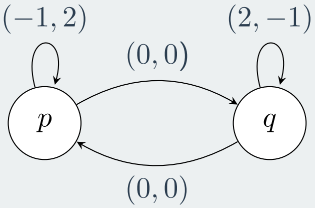

example run:

$$p(1,0) \to p(0,2) \to q(0,2) \to q(2,1) \to q(4,0) \to p(4,0)$$

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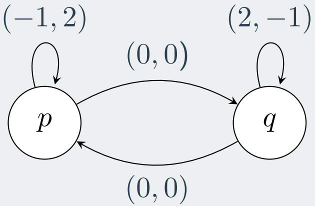

example run:

$p(1, 0) \to p(0, 2) \to q(0, 2) \to q(2, 1) \to q(4, 0) \to p(4, 0)$

Important restriction: no negative values

Affine VASS

Interaction between counters

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Transitions updates before: $p(\boldsymbol{v}) \to q(\boldsymbol{v} + \boldsymbol{w})$

Transitions updates now: $p(\boldsymbol{v}) \to q(\mathbf{A}\boldsymbol{v} + \boldsymbol{w})$

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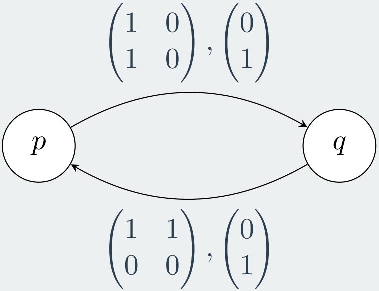

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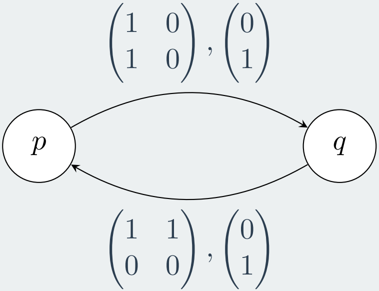

$p(x, y) \rightarrow q(x, x + 1)$
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Affine VASS example

$$\begin{tikzcd}
p \arrow[rr, bend left, "{\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}}"] & & q \arrow[ll, bend left, "{\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}}"]
\end{tikzcd}$$

$p(x, y) \to q(x, x + 1)$
(copy)

$q(x, y) \to p(x + y, 1)$
(transfer)

Affine VASS subclasses

What matrices are allowed?

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- $\mathbf{A}$ has exactly one 1 in each row and each column

(permutation VASS)

Decision problems

For affine VASS

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Reachability problem:

GIVEN: an affine VASS $\mathcal{V}$ and $p(\boldsymbol{u}), q(\boldsymbol{v})$

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- Usually affine VASS $\rightarrow$ some specific class
- In this talk mostly affine $\mathbb{Z}$-VASS

(counters can be negative)

State of art

Over $\mathbb{N}$

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VASS:

- Coverability: EXPSPACE-complete
- Reachability: decidable, EXPSPACE-hard

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Over $\mathbb{Z}$

- Reachability and Coverability are inter-reducible
- VASS and reset VASS NP-complete [Haase and Halfon, 2014]

$\mathbb{Z}$-VASS instead of VASS

This simplifies the problem (?)

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But for $\mathbb{Z}$-VASS reachability and coverability are inter-reducible

So undecidability for coverability of affine $\mathbb{Z}$-VASS (in dimension 4)

Motivation

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Used $\mathbb{Z}$-VASS or continuous VASS

Quite successful [Esparza et al., 2014], [Blondin et al., 2016] [Geffroy, Leroux and Sutre, 2016]

Matrix monoid of the affine VASS $\mathcal{V}$

$\mathcal{M}_{\mathcal{V}}$ – the matrix monoid generated by matrices in $\mathcal{V}$

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(even in dimension 3)

Results overview

Finite monoids

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$\mathbb{Z}$-VASS (NP)

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Finite monoids

$$\begin{tikzcd} & \mathbb{Z}\text{-VASS (NP)} \arrow[dl, bend right] \arrow[dr, bend left] & \\ \text{Reset } \mathbb{Z}\text{-VASS (NP)} & & \text{Permutation } \mathbb{Z}\text{-VASS (NP/PSPACE)} \end{tikzcd}$$

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Infinite monoids

Transfer+copy $\mathbb{Z}$-VASS (UNDEC)

Affine $\mathbb{Z}$-VASS (UNDEC)

States and semilinearity for affine $\mathbb{Z}$-VASS

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- For $|Q| = 1$ and $\mathcal{M}_{\mathcal{V}}$ with one generator: semilinearity iff $\mathcal{M}_{\mathcal{V}}$ finite

[Boigelot, 1998], [Finkel and Leroux, 2002]

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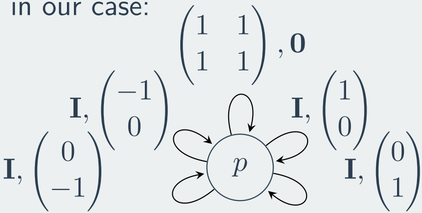
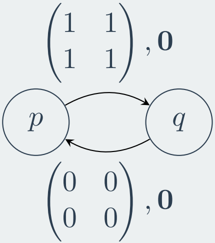
Not in our case:





From affine $\mathbb{Z}$-VASS to $\mathbb{Z}$-VASS

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so $Q' = Q \times \mathcal{M}_{\mathcal{V}}$

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Two counters (x, y) with occasional transfer $x \to y$

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$(0, 0) \xrightarrow{+(5,1)} (5, 1) \xrightarrow{+(-3,1)} (2, 2) \xrightarrow{x \to y} (0, 4)$

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$$(0,0) \xrightarrow{+(0,6)} (0,6) \xrightarrow{+(0,-2)} (0,4) \xrightarrow{\text{update } \mathbf{M}} (0,4)$$

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Given affine $\mathbb{Z}$-VASS $\mathcal{V}$ construct $\mathbb{Z}$-VASS $\mathcal{V}'$

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so $Q' = Q \times \mathcal{M}_{\mathcal{V}}$

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Two counters (x, y) with occasional transfer $x \to y$

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for reset VASS we get NP (already known)

Permutation + reset VASS is PSPACE-hard

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- Tools for transfer VASS?