Bilkent University

Electrical and Electronics Department

EE202-03 Lab 4 Report:

“The Design of Two Linear Circuits to Transfer Maximum Power Using Impedance Matching”

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1- Software Implementation

1.1- Introduction:

In this lab, we were assigned to design at least two passive linear circuits to transfer maximum power to a 180Ω load from a voltage source with a serial output impedance 50Ω at a frequency between 5 and 10Mhz **(Figure 1.1.1)**.

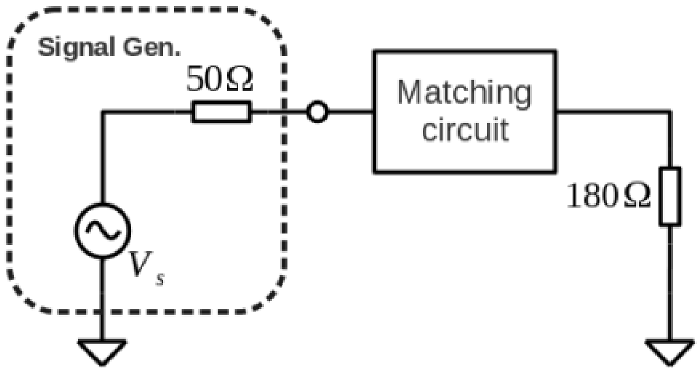

Figure 1.1.1: The Lab Assignment in a Brief Schematic

There were many methods I could use in order to achieve maximum power transfer such as a narrowband filter, a wideband filter, an L-section, a T-section, a π-section... These were all valid options to choose from. I finally decided to use a T-section and a π-section in my circuits. I also decided to choose my operating frequency as 10MHz.

1.2- Analysis:

For maximum power transfer from the source, load resistance must be equal to the source resistance. Here you can see the relevant figure and the necessary equations regarding the relationship between source and load resistances **(Figure 1.2.1 & Equations 1.1 to 1.5)**:

Equation 1.1 shows us the mathematical relation between power drawn from the source and the source and load resistances. In **Equation 1.5**, if source and load resistances are equal, the derivative goes to zero. This means we achieve maximum power transfer if $R_{load} = |R_{source}|$.

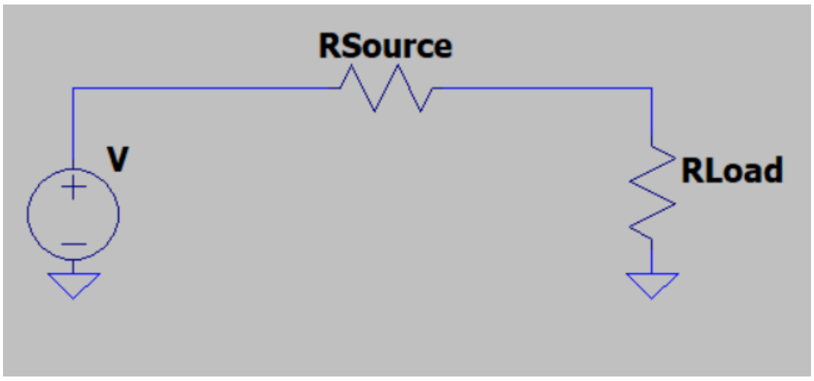

Figure 1.2.1: The relationship between source and load resistances.

$$P_{load} = \frac{V_{load}{}^{2}}{R_{load}} = \frac{\left(\frac{V \cdot R_{load}}{R_{load}+R_{source}}\right)^{2}}{R_{load}} = V^{2}\frac{R_{load}{}^{2}}{R_{load}\cdot\left(R_{load}+R_{source}\right)^{2}} \quad \textbf{(Equation 1.1)}$$

$$\frac{d\left(P_{R_{load}}\right)}{d(R_{load})} = V^{2}\,\frac{2R_{load}^{2}\,(R_{load}+R_{source})^{2} - R_{load}^{2}\,[(R_{load}+R_{source})^{2} + 2R_{load}\cdot(R_{load}+R_{source})^{2}]}{[R_{load}\cdot(R_{load}+R_{source})^{2}]^{2}}$$

(Equation 1.2)

$$\frac{d\left(P_{R_{load}}\right)}{d(R_{load})} \approx R_{load}^{2}[2\cdot(R_{load}+R_{source})^{2} - (R_{load}+R_{source})^{2} - 2\cdot R_{load}\cdot(R_{load}+R_{source})]$$

(Equation 1.3)

$$\frac{d\left(P_{R_{load}}\right)}{d(R_{load})} \approx R_{load}^{2}[(R_{load}+R_{source})^{2} - 2\cdot R_{load}\cdot(R_{load}+R_{source})] \qquad \textbf{(Equation 1.4)}$$

$$\frac{d\left(P_{R_{load}}\right)}{d(R_{load})} \approx R_{load}^{2}\cdot(R_{load}+R_{source})\cdot(R_{load}-R_{source}) \qquad \textbf{(Equation 1.5)}$$

Now, let's take a closer look to two different impedance matching methods.

1.2.1- T-Section Impedance Matching Circuit

Here you can see a schematic for a sample T-section circuit and the relevant equations **(Figure 1.2.2 & Equations 2.1 to 2.4)**:

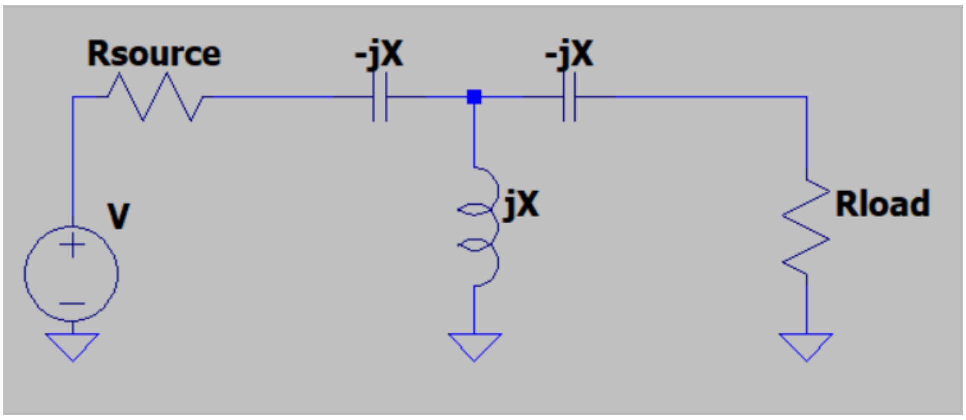

Figure 1.2.2: A Sample T-Section Circuit

Let the impedance seen by the source resistance be Z.

$$Z = -jX + \frac{1}{\frac{1}{jX} + \frac{1}{-jX + R_{load}}} \quad \textbf{(Equation 2.1)}$$

$$Z = -jX + \frac{jX(-jX + R_{load})}{R_{load}} \quad \textbf{(Equation 2.2)}$$

$$Z = \frac{(jX)^2}{R_{load}} = \frac{X^2}{R_{load}} \quad \textbf{(Equation 2.3)}$$

$Z = R_{source}$ for max power transfer

$$X = \sqrt{R_{load} \cdot Z} = \sqrt{R_{load} \cdot R_{source}} = \sqrt{180 \cdot 50} = 94.868 \quad \textbf{(Equation 2.4)}$$

Now since we have X as 94.868, let's calculate the capacitance and inductance values for this X. Here are the relevant equations for C and L values for our components **(Equations 3.1 to 3.4)**.

$$C = \frac{1}{\omega X} \textbf{(Equation 3.1)}$$

$$C = \frac{1}{2 \cdot \pi \cdot 10^7 \cdot 94.868} = 167.7pF \quad \textbf{(Equation 3.2)}$$

$$L = \frac{X}{\omega} \textbf{(Equation 3.3)}$$

$$L = \frac{94.868}{2 \cdot \pi \cdot 10^7} = 1.51\mu H \textbf{ (Equation 3.4)}$$

167.7pF and 1.51µH will be the used component values for the capacitors and the inductors in this specific lab.

1.2.2- π-Section Impedance Matching Circuit

Here you can see a schematic for a sample π-section circuit and the relevant equations **(Figure 1.2.3 & Equations 4.1 to 4.5)**:

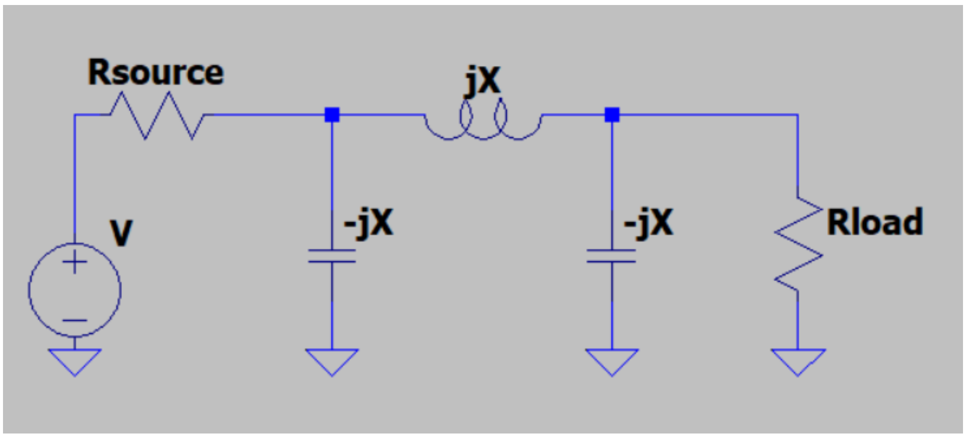

Figure 1.2.3: A Sample π-Section Circuit

Let the impedance seen by the source resistance be Z again.

$$Z = \frac{1}{\frac{1}{-jX} + \frac{1}{jX + \frac{1}{\frac{1}{-jX} + \frac{1}{R_{load}}}}} \textbf{ (Equation 4.1)}$$

$$Z = \frac{1}{\frac{1}{-jX}+\frac{1}{jX+\frac{-jX\cdot R_{load}}{R_{load}-jX}}} \quad \textbf{(Equation 4.2)}$$

$$Z = \frac{1}{\frac{1}{-jX}+\frac{R_{load}-jX}{X^2}} \quad \textbf{(Equation 4.3)}$$

$$Z = \frac{1}{\frac{R_{load}}{X^2}} = \frac{X^2}{R_{load}} \quad \textbf{(Equation 4.4)}$$

Z = R_{source} for max power transfer

$$X = \sqrt{R_{laod}\cdot Z} = \sqrt{R_{laod}\cdot R_{source}} = \sqrt{180\cdot 50} = 94.868 \quad \textbf{(Equation 4.5)}$$

Since the X and ω values are the same with the previous part, the capacitance and inductance values for the components will be the same. So, 167.7pF and 1.51µH will be the used component values for the capacitors and the inductors just as the T-section circuit.

The maximum power that the voltage source can provide can be calculated with the following equations **(Equations 5.1 & 5.2)**:

$$V_{max} = 10V \quad \textbf{(Equation 5.1)}$$

$$P_{source} = \frac{V_{max}^2}{2\cdot R_{eq}} = \frac{10^2}{2\cdot(50+50)} = 500mW \, \textbf{(Equation 5.2)}$$

The equivalent resistance value (R_{eq}) is chosen as 100Ω because the 180 Ω load resistance will be turned into 50 Ω because of the impedance matching circuit elements. In **Equation 5.2**, what we found is that 500mW is the amount of maximum instantaneous power our voltage source can provide at an instant. Since the source and load resistances are equal to each other, 250mW of average power should theoretically be on both resistors.

1.3- Simulation:

Here is the T-section impedance matching circuit I've built and the necessary simulation results **(Figures 1.3.1 to 1.3.7)**:

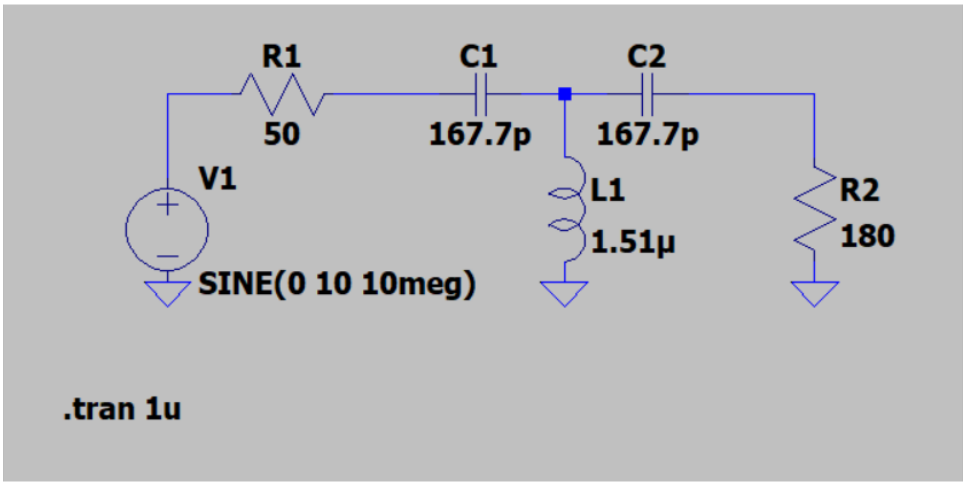

Figure 1.3.1: The Specific T-Section Circuit I Have Designed

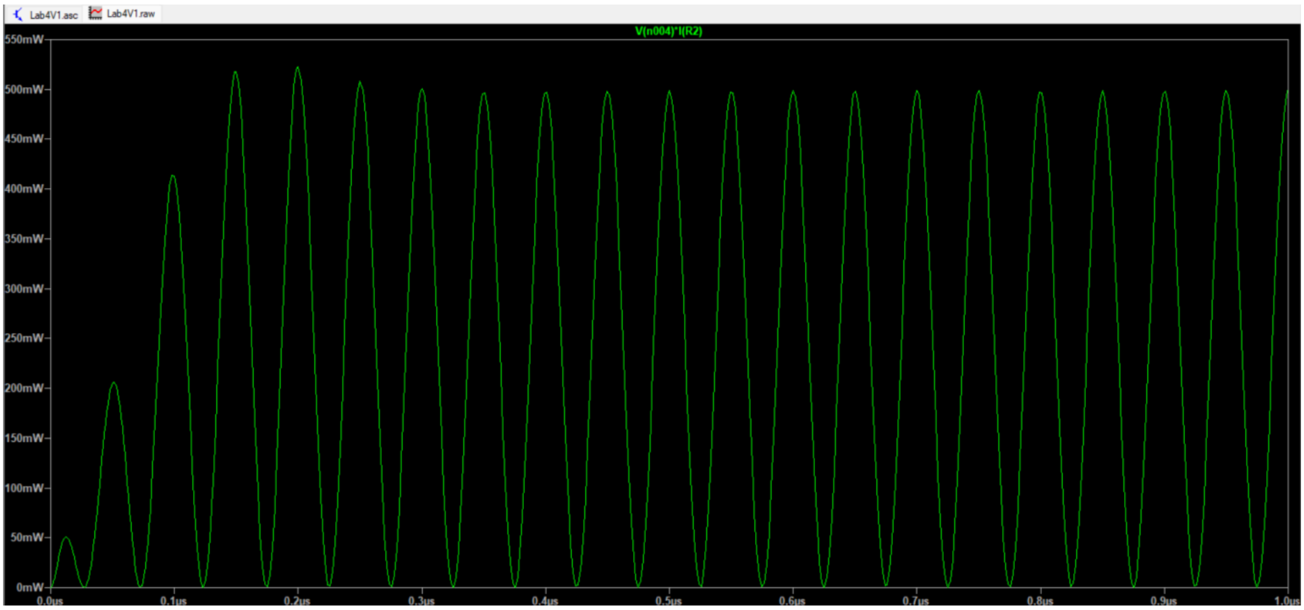

Figure 1.3.2: The Power on the Load Resistor of the T-Section Circuit

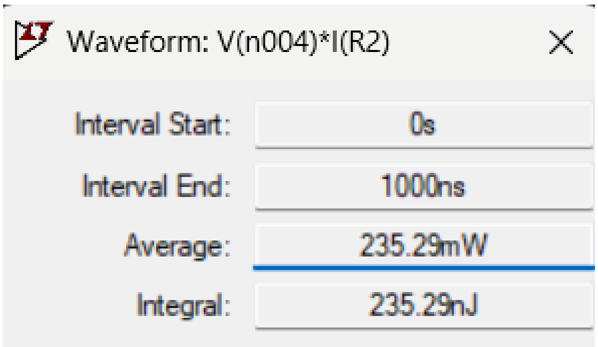

Figure 1.3.3: The Average Power on the Load Resistor of the T-Section Circuit –235.29mW

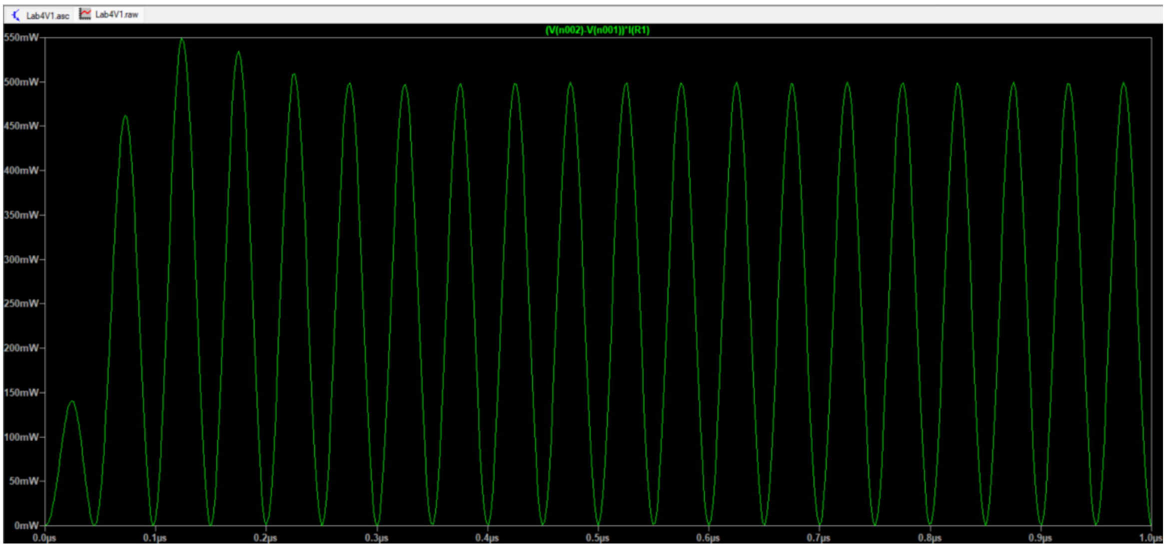

Figure 1.3.4: The Power on the Source Resistor of the T-Section Circuit

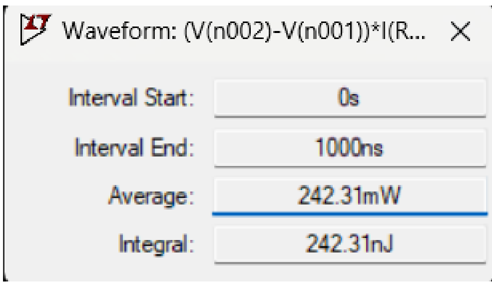

Figure 1.3.5: The Average Power on the Source Resistor of the T-Section Circuit –242.31mW

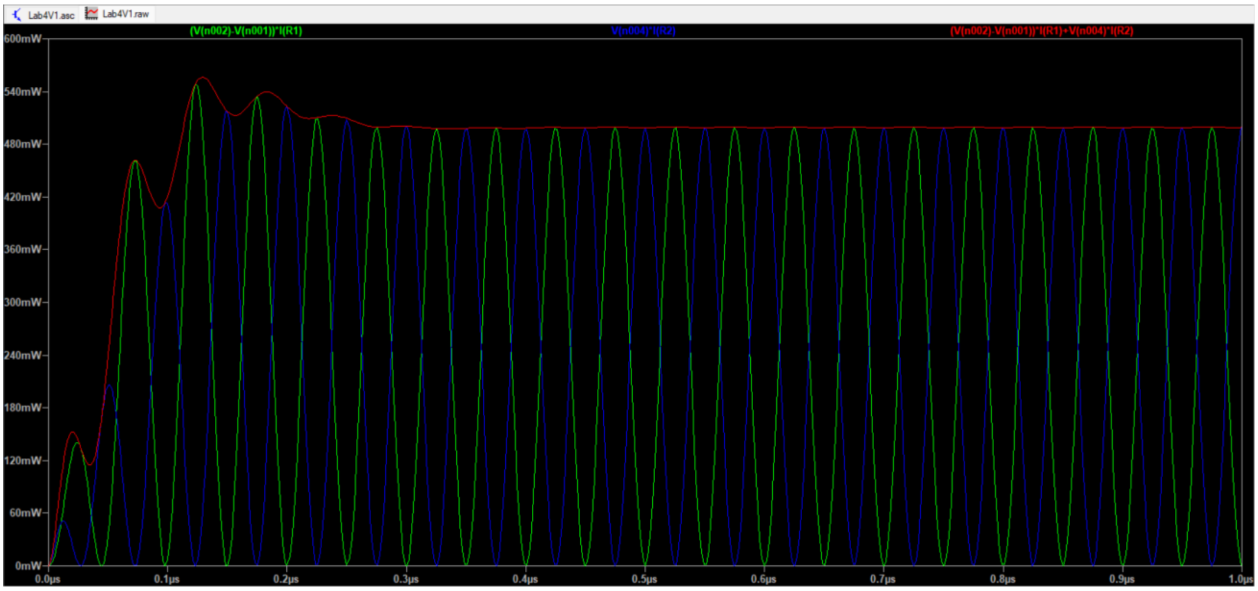

Figure 1.3.6: The Power on the Source (Green Line) and Load (Blue Line) Resistors and their Total (Red Line)

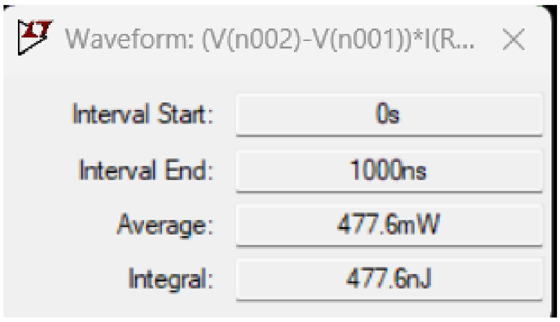

Figure 1.3.7: The Average Total Power on Both Resistors in the T-Section Circuit – 477.60mW

We found total power drawn from the voltage source to be 477.6mW. 235.29mW of this power went to the load resistor. This means that 49.26% of the total power drawn from the voltage source went to the load resistor **(Equation 6.1)**. Also, the theoretical value for the average power on the load resistor was 250mW. We have a 5.88% error margin for the power on the resistor **(Equation 6.2)**.

$$\frac{235.29}{477.6} = 49.26\% \qquad \textbf{(Equation 6.1)}$$

$$\frac{250-235.29}{250} = 5.88\%\ error \qquad \textbf{(Equation 6.2)}$$

Now, let's move on to the π-section impedance matching circuit I've built and the necessary simulation results **(Figures 1.3.8 to 1.3.14)**:

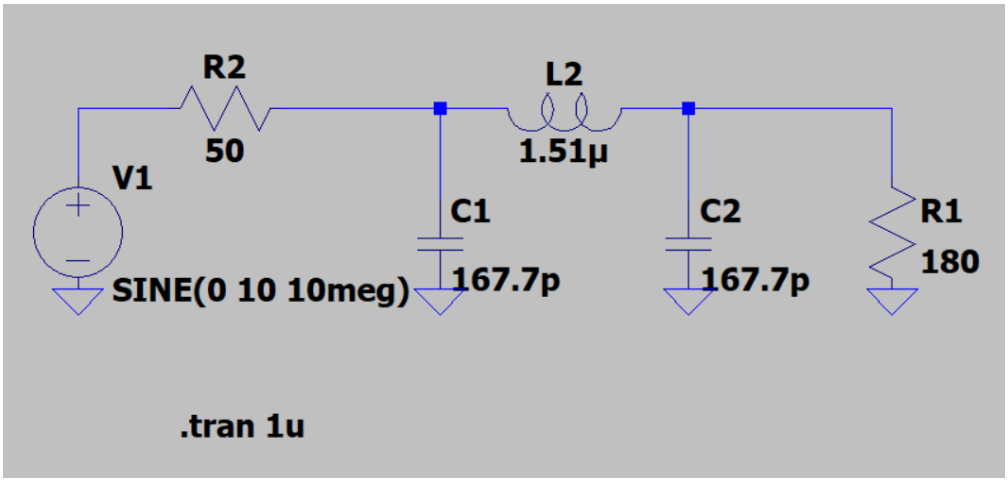

Figure 1.3.8: The Specific π-Section Circuit I Have Designed

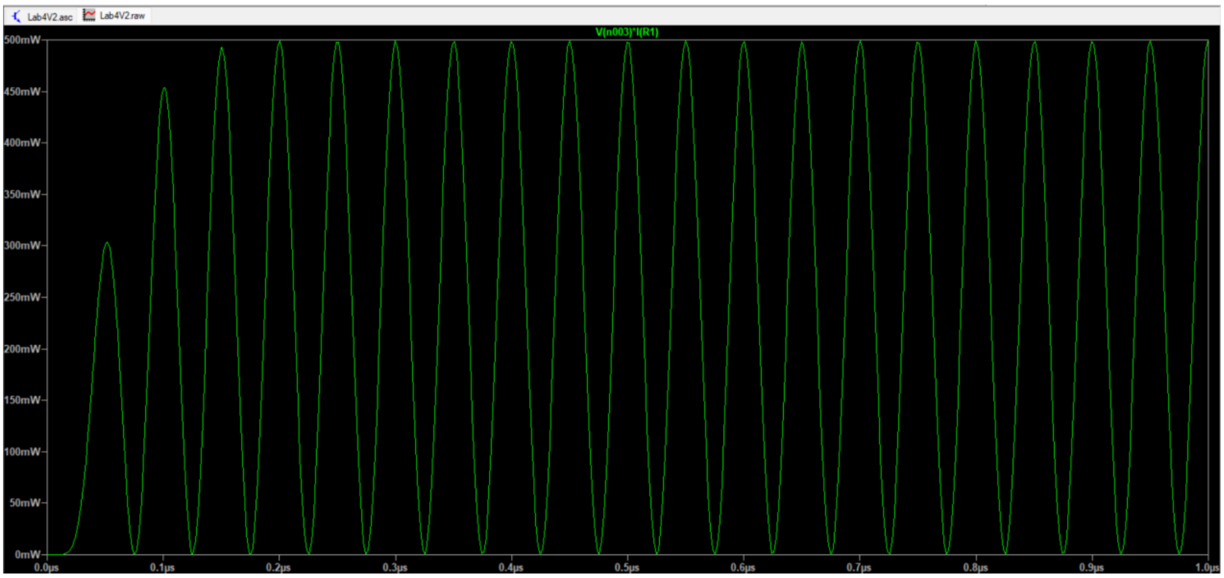

Figure 1.3.9: The Power on the Load Resistor of the π -Section Circuit

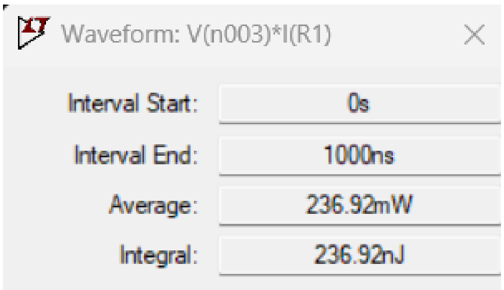

Figure 1.3.10: The Average Power on the Load Resistor of the π-Section Circuit –236.92mW

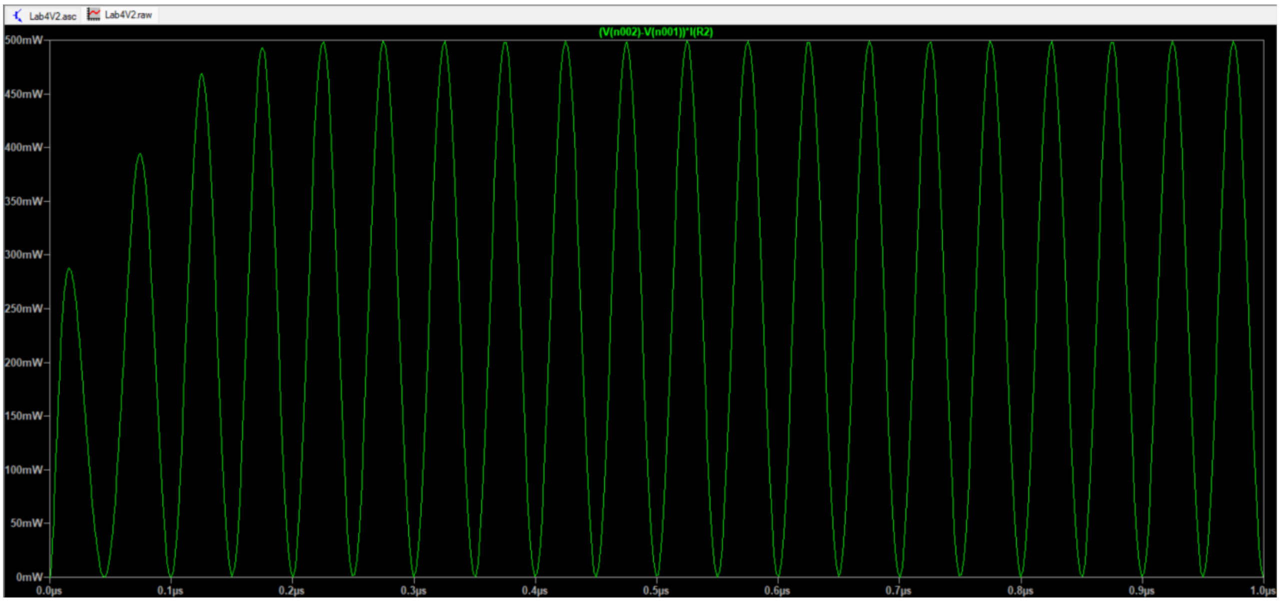

Figure 1.3.11: The Power on the Source Resistor of the π -Section Circuit

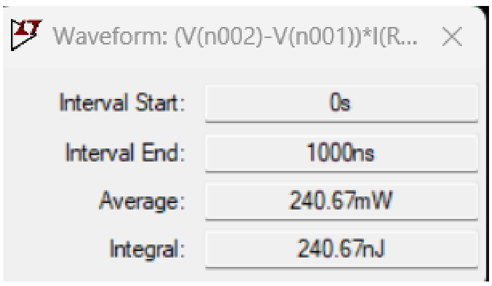

Figure 1.3.12: The Average Power on the Source Resistor of the π-Section Circuit –240.67mW

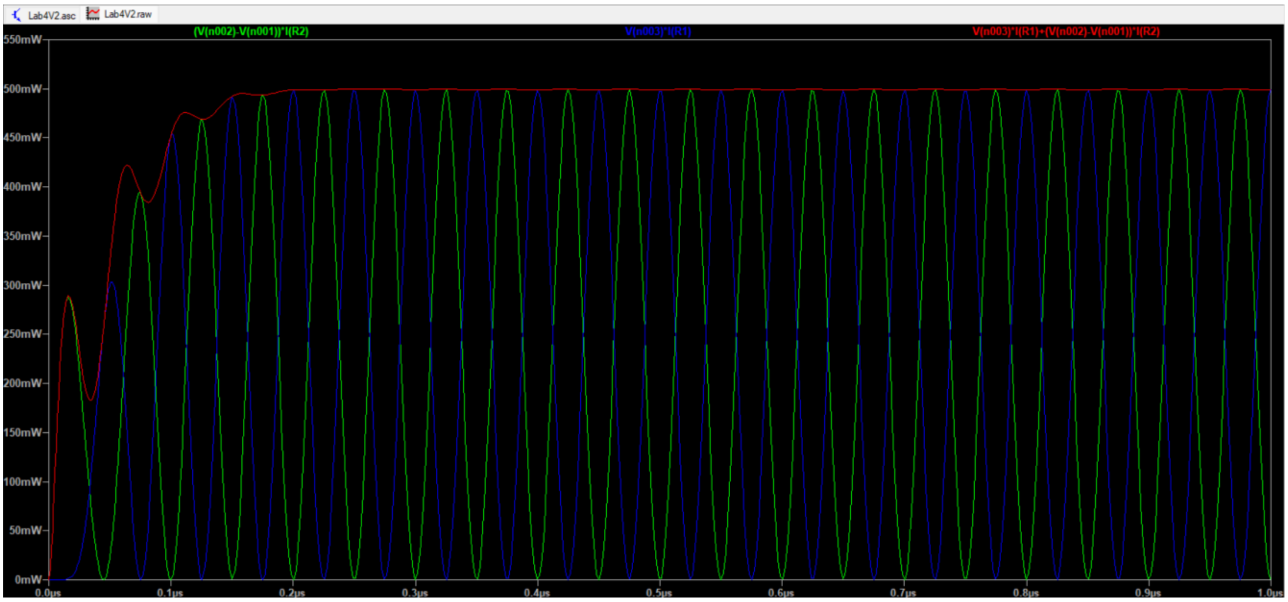

Figure 1.3.13: The Power on the Source (Green Line) and Load (Blue Line) Resistors and their Total (Red Line)

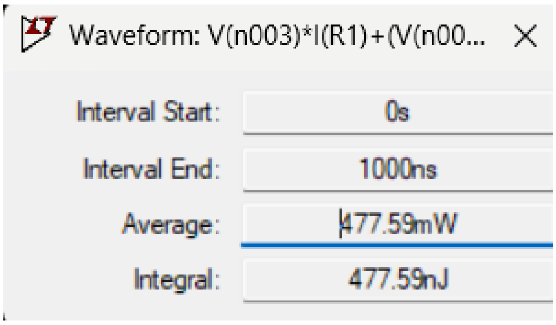

Figure 1.3.14: The Average Total Power on Both Resistors in the π-Section Circuit – 477.59mW

The We found total power drawn from the voltage source to be 477.59mW. 236.92mW of this power went to the load resistor. This means that 49.60% of the total power drawn from the voltage source went to the load resistor **(Equation 7.1)**. Also, the theoretical value for the average power on the load resistor was 250mW. We have a 5.23% error margin for the power on the resistor **(Equation 7.2)**.

$$\frac{236.92}{477.59} = 49.60\% \qquad \textbf{(Equation 7.1)}$$

$$\frac{250-236.92}{250} = 5.23\%\ error \qquad \textbf{(Equation 7.2)}$$

Here are the tables that summarize the results obtained from the simulation part **(Tables 1&2)**:

	Theoretical Total Power Drawn from the Voltage Source	Total Power Drawn from the Voltage Source	Error
T-Section	500mW	477.60mW	4.480%
Π-Section	500mW	477.59mW	4.482%

Table 1

	Theoretical Load Power	Load Power	Error
T-Section	250mW	235.29mW	5.88%
Π-Section	250mW	236.92mW	5.23%

Table 2

2- Hardware Implementation

2.1- Introduction:

I used ceramic capacitors and axial inductors present in the laboratory to implement my designs. I provided a 5V Pk-Pk sinusoidal signal with a frequency of 10MHz. Here are the two impedance matching circuits I have implemented on a breadboard **(Figures 2.1.1 & 2.1.2)**:

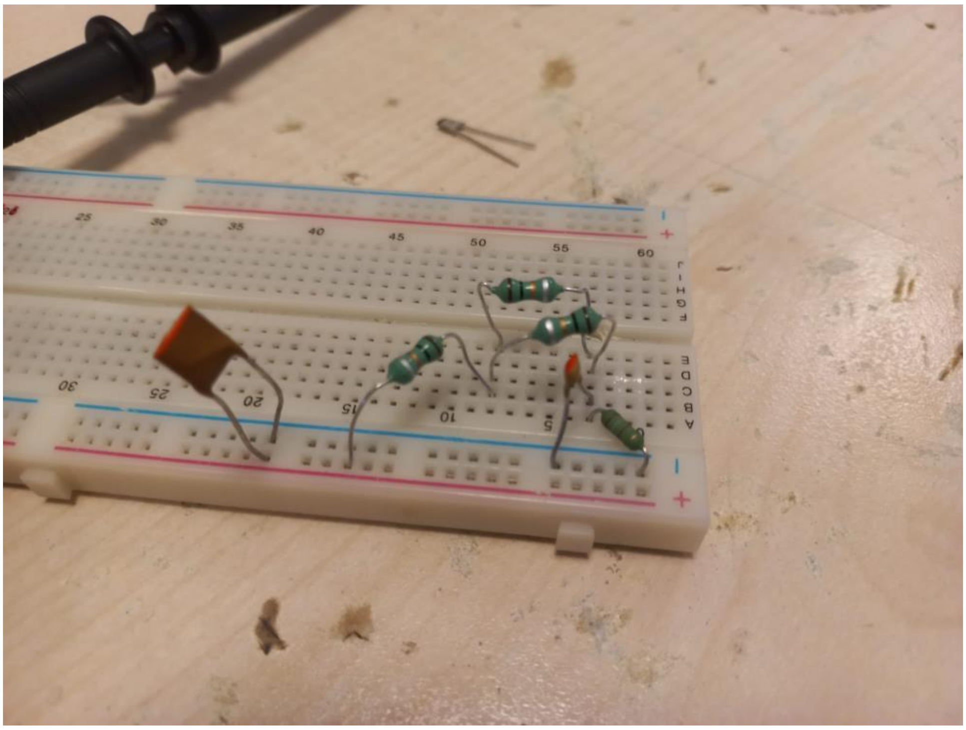

Figure 2.1.1: The π-Section Circuit

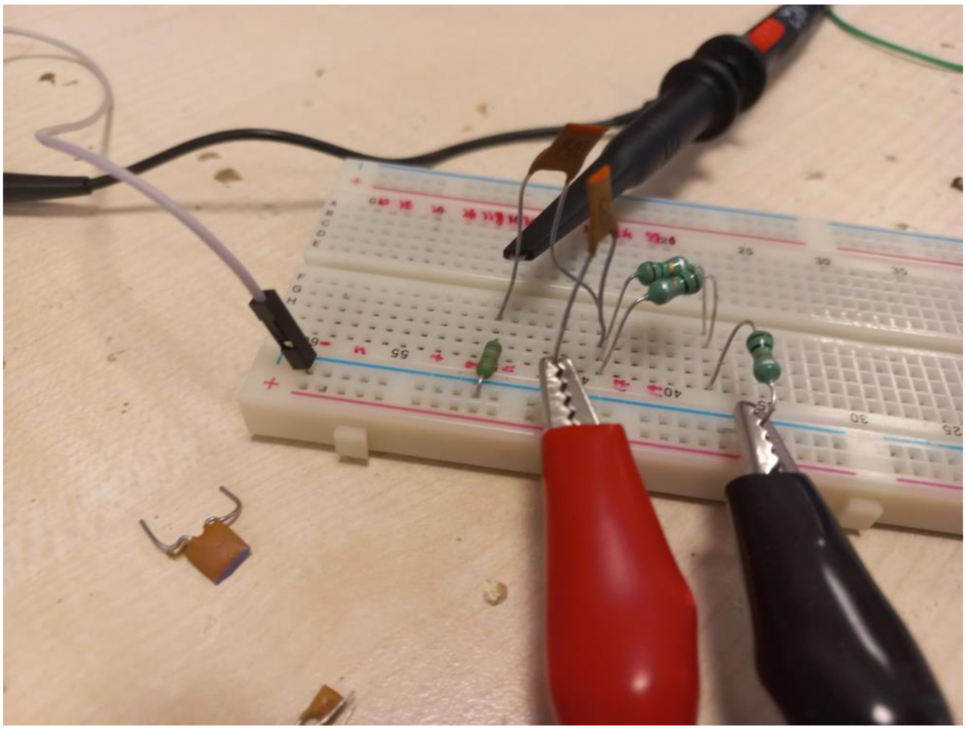

Figure 2.1.2: The T-Section Circuit

2.2- Results:

First, let's calculate the power dissipated on a 47Ω resistor. Here you can see the voltage across the resistor and the equations for the power dissipated on the resistor **(Figure 2.2.1 & Equation 8)**:

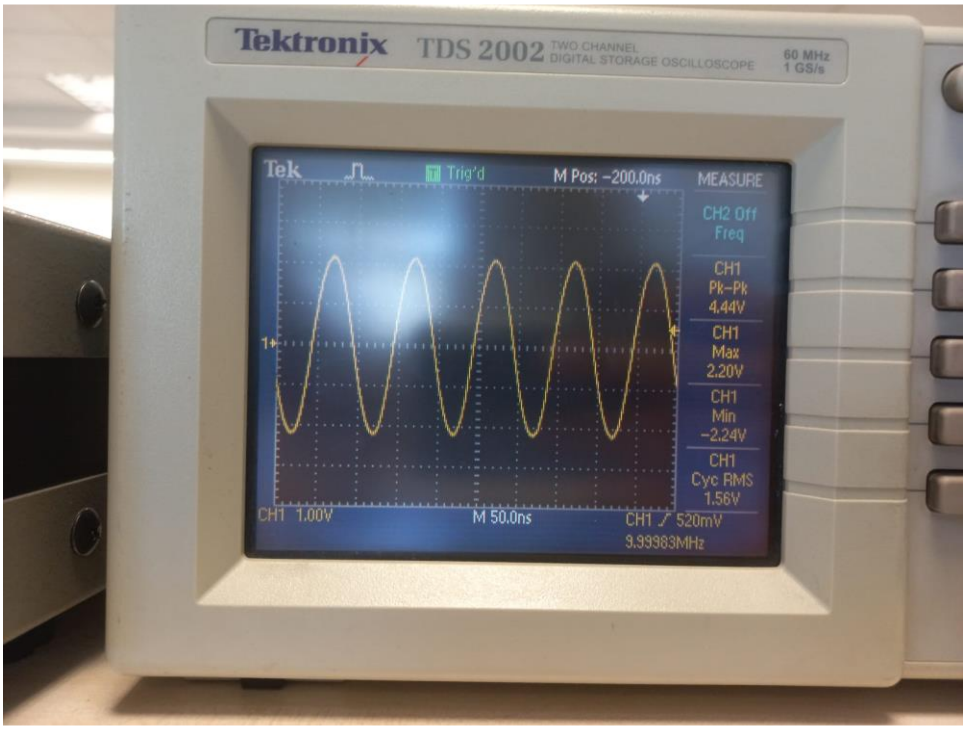

Figure 2.2.1: The Signal Generator when Connected to a 47Ω Resistor – 4.44V

$$P = \frac{V^2}{2R} = \frac{2.20 \cdot 2.20}{2 \cdot 47} = 51.48mW \qquad \textbf{(Equation 8)}$$

Here you can see the power output of the signal generator in dBm when connected to the impedance matching circuits and the equations for converting dBm to Watts **(Figure 2.2.2 & Equations 9.1&9.2)**:

Figure 2.2.2: The Signal Generator Power Output when Connected to the Impedance Matching Circuits

$$P_{Watts} = \frac{10^{\frac{P_{dBm}}{10}}}{1000}; P_{dBm} = 10 \cdot \log(P_{Watts}) + 30 \qquad \textbf{(Equation 9.1)}$$

$$P_{Watts} = \frac{10^{\frac{17.96}{10}}}{1000} = 62.51mW \quad \textbf{(Equation 9.2)}$$

This means that the signal generator is providing 62.51mW of power to outside when connected to impedance matching circuits.

Here you can see the voltage outputs on the load resistor of both impedance matching circuits and the relevant equations for the power on those resistors. **(Figures 2.2.3 & 2.2.4 and Equations 10.1 & 10.2)**:

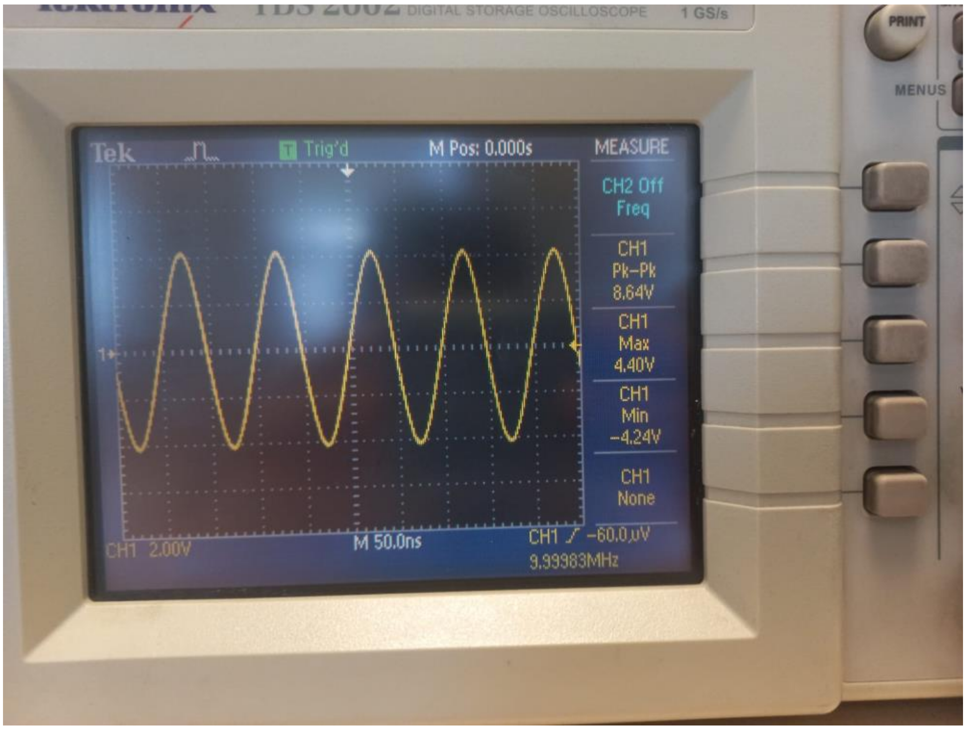

Figure 2.2.3: The Voltage on the Load Resistance of the T-Section Circuit; 4.40V

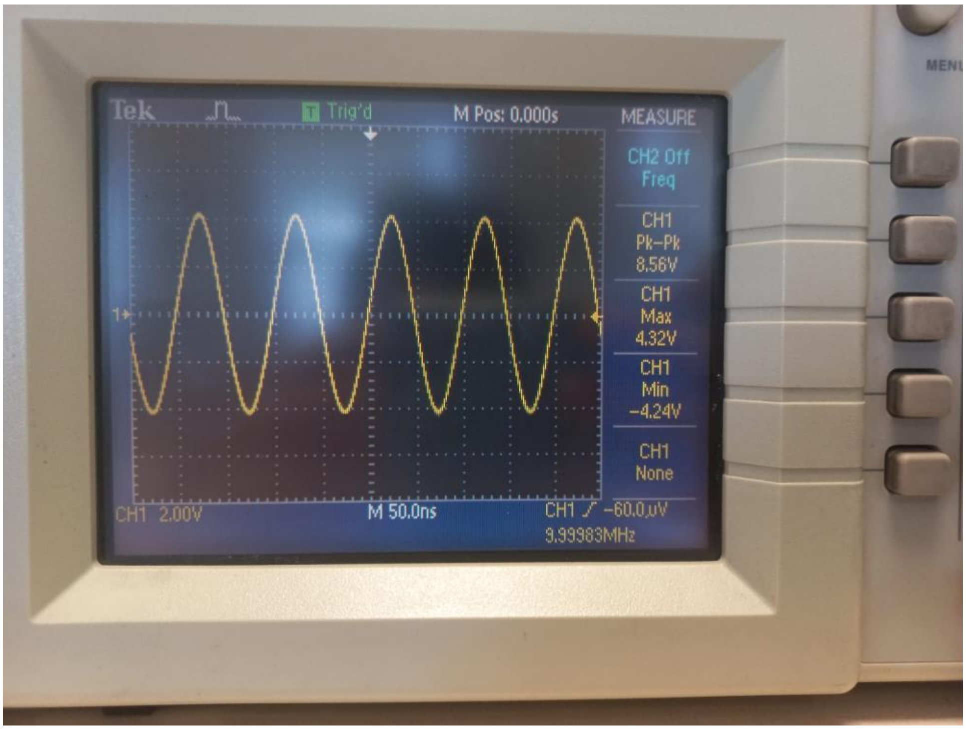

Figure 2.2.4: The Voltage on the Load Resistance of the π-Section Circuit; 4.32V

$$P = \frac{V^2}{2R} = \frac{4.40 \cdot 4.40}{2 \cdot 180} = 53.77mW \qquad \textbf{(Equation 10.1)}$$

$$P = \frac{V^2}{2R} = \frac{4.32 \cdot 4.32}{2 \cdot 180} = 51.84mW \qquad \textbf{(Equation 10.2)}$$

Here you can see the table of results for the hardware part **(Table 3)**:

	Power Drawn from the Signal Generator	Power on the Load	Error
T-Section	62.51mW	53.77mW	13.98%
Π-Section	62.51mW	51.84mW	17.06%

3- Conclusion

In this lab, we were assigned to design at least two passive linear circuits to transfer maximum power to a 180Ω load from a voltage source with a serial output impedance 50Ω at a frequency between 5 and 10Mhz. I've chosen the frequency to be 10MHz. There were also many methods I could use in order to achieve maximum power transfer such as a narrowband filter, a wideband filter, an L-section, a T-section, a π-section and so on These were all valid options to choose from. I finally decided to use a T-section and a π-section in my circuits.

In the software lab, I got 5.88% and 5.23% errors. But it is important to note that these errors were due to the simulation data starting from 0 seconds. In other words, the initial conditions for the inductor and the capacitor affected the power on the load. As you could see from the simulation data, the power on the load got more stabilized as the time went on in the simulation in both circuits. Therefore, even though they were under the 10% error limit, the error percentages are kind of misleading in the software part.

In the hardware lab, we came up against bigger errors: 13.98% and 17.06%. These errors are more meaningful since the oscilloscope measures the voltage continuously and therefore, initial conditions of the inductors and the capacitors are not affecting the results on the oscilloscope screen. The main reason for these error margins were probably the additional inductance and resistance caused from hardware components such as the breadboard. Also, the capacitance and inductance values of the selected components might differ from their standard values, and this might be a huge reason for the error margins. Nevertheless, just as the software part, in the hardware part my designs got under the 20% error limit. Therefore, the lab was a net success in my opinion.