Bilkent University

Electrical and Electronics Department

EE321-02 Lab 4 Report:

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Introduction:

This lab consisted of 9 parts. We were assigned with designing a practical FIR filter, testing its frequency response, and performing some filtering operations. The specifications of the filter can be seen in **Figure 1**:

* Bandpass filter

* Real valued $h[n]$.

* Length of the filter is $L = 11 + N_1$.

* Cutoff frequencies are $\min\{\frac{\pi}{M_1}, \frac{\pi}{M_2}\}$ and $\max\{\frac{\pi}{M_1}, \frac{\pi}{M_2}\}$ radians.

* A stopband and a passband, which are as flat as possible, are desirable.

* Causal; $h[n] = 0$ for $n < 0$. Also, $h[n] = 0$ for $n \geq L$.

Figure 1: The Filter Specifications

My Bilkent ID is 22201689. Therefore, N1=8, N2=6, M1=10 and M2=8. The filter length is 19. The passband is between $\left(\frac{\pi}{10}, \frac{\pi}{8}\right)$ and $\left(-\frac{\pi}{10}, -\frac{\pi}{8}\right)$. The reason for this is the following property of the z-transform (**(1)** and **(2)**). If x[k] is real, then **(3)** holds.

$$X(e^{jw}) = \sum_{k=-\infty}^{\infty} x[k] \cdot e^{-jwk} \qquad \textbf{(1)}$$

$$X^*(e^{jw}) = \sum_{k=-\infty}^{\infty} x^*[k] \cdot e^{-jwk} \qquad \textbf{(2)}$$

$$\text{If } x[k] = x^*[k]; \qquad X^*(e^{jw}) = X(e^{jw}) \quad \textbf{(3)}$$

This means that if x[k] is real, then $X(e^{jw})$ is conjugate symmetric. In other words, the zeros of the passband should be symmetric to the x-axis. Since the length of my filter is 19, there are 19 zeros of the system.

Q1:

I carefully distributed the zeros along the unit circle such that the passbands were not suppressed. Then I calculated h[n] digitally using Matlab. I used **(4)** to calculate the impulse response of the system where a_k's are the zeros of the system.

$$\frac{\prod_{k=0}^{18}(z-a_k)}{z^{19}} = H(z) = \sum_{k=-\infty}^{\infty} h[k] \cdot z^{-k} = \sum_{k=0}^{18} h[k] \cdot z^{-k} \textbf{(4)}$$

Here you can see the zeros of bandpass filter **(Figure 2)**:

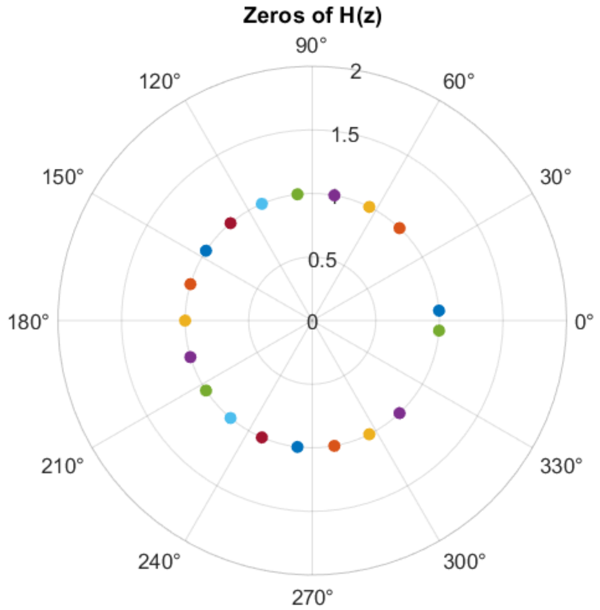

Figure 2: Zeros of the Filter

Here are the array of the impulse response and the plot of the impulse response **(Figures 3.1 & 3.2)**:

```
h =

  Columns 1 through 13

    1.0000    2.2857    3.3208    3.6765    3.1203    1.6747   -0.3867   -2.6088   -4.4734   -5.5341   -5.5341   -4.4734   -2.6088

  Columns 14 through 20

   -0.3867    1.6747    3.1203    3.6765    3.3208    2.2857    1.0000
```

Figure 3.1: Array of h[n]

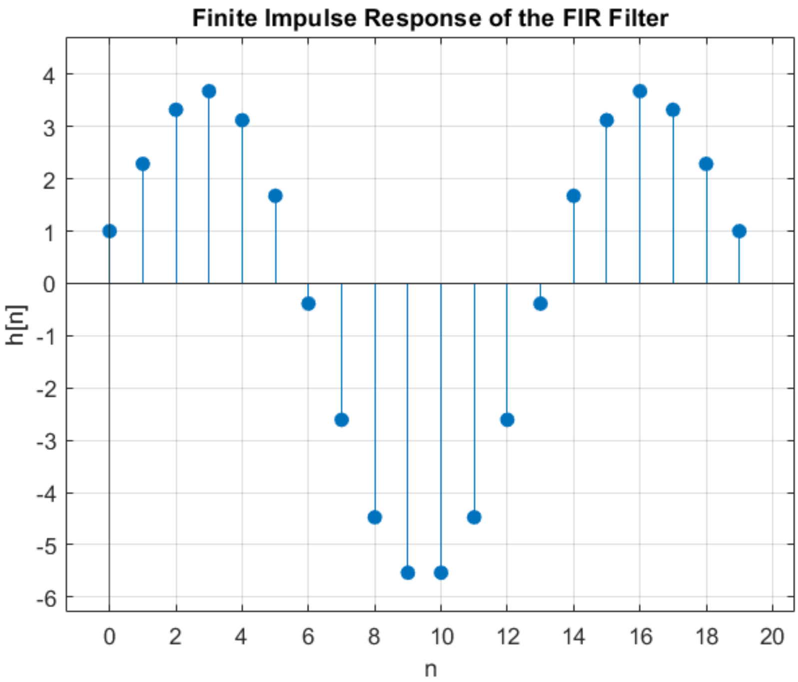

Figure 3.2: Plot of h[n]

As you can see, the h[n] is purely real as specified in the lab manual. Now let's look at the magnitude and phase plots of $H(e^{jw})$ **(Figures 4.1&4.2)**:

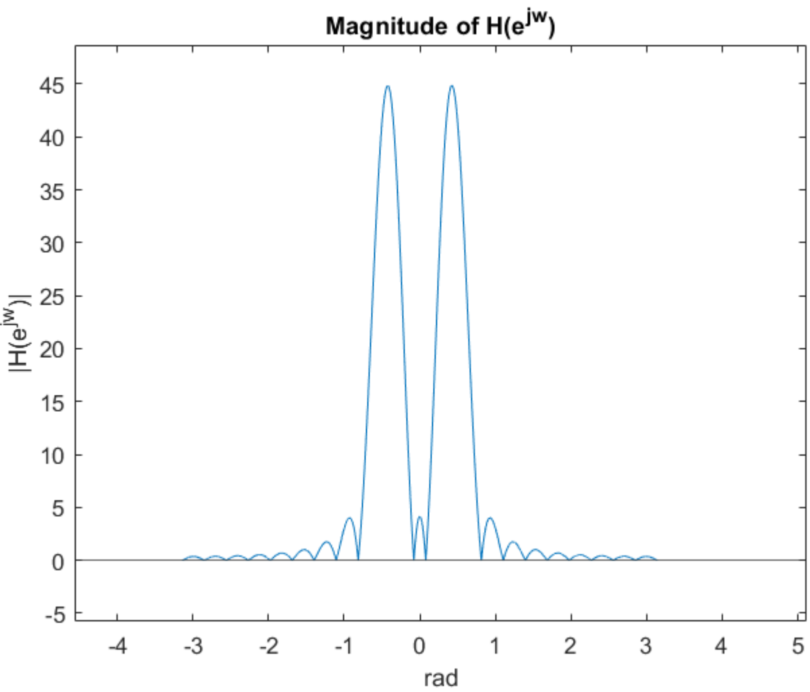

Figure 4.1: Magnitude Plot of H(e^{jw})

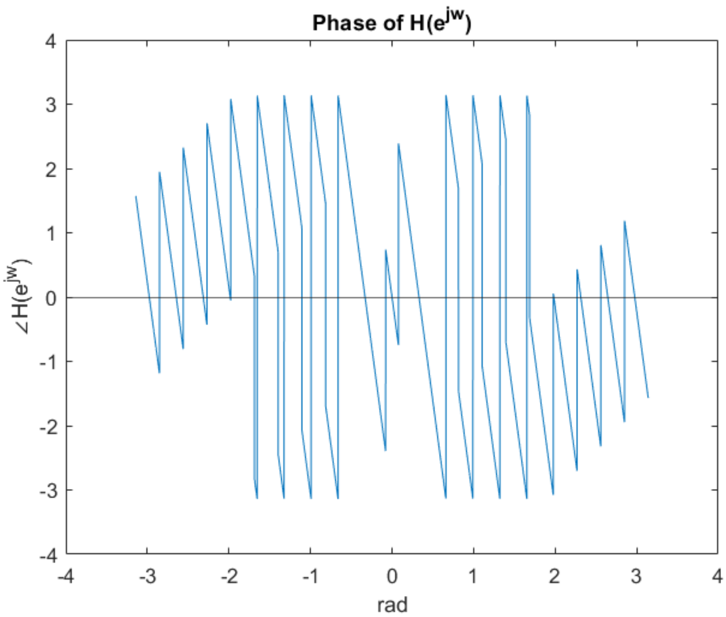

Figure 4.2: Phase Plot of H(e^{jw})

Q2:

Let,

$$x_{\propto}(t) = \cos(\propto \cdot t^2)\,;\ where\ t\ \in (-\infty, \infty)\ and\ \propto = 1$$

Then,

$$x_\propto(t) = \cos(t^2)\,;\ where\ t\ \in (-\infty, \infty)$$

Let’s check the periodicity of this function:

$$x_\propto(t) = x_\propto(t+T) = \cos(t^2) = \cos((t+T)^2)$$

$$\cos(t^2 + 2Tt + T^2) = \cos(t^2 + 2\pi k)\,;\ where\ k\ \in\ Z$$

$$T^2 + 2Tt\ = 2\pi k\,;\ where\ k\ \in\ Z$$

$$T = \frac{-2t\ \pm (4t^2 - 8\pi k)^{\frac{1}{2}}}{2}\ ;\ where\ k\ \in\ Z$$

This shows that T is a function of time and is not constant. There are some non-integer time values where there cannot be found an integer k to satisfy the equation. Therefore, we conclude that this signal is not periodic in its analog form.

The instantenous frequency of the signal is:

$$w_i(t)\ =\ \frac{d(t^2)}{dt} = 2t$$

Here is the sampled version of the signal with sampling period $T_s = \sqrt{\frac{\pi}{\propto \cdot 512}} = \sqrt{\frac{\pi}{512}} \cong$ $0.078\ s$ (x1[n]):

$$x_1[n] = \cos\left[\frac{n^2\pi}{512}\right]$$

Let’s check the periodicity of the sampled signal.

$$\cos\left[\frac{n^2\pi}{512}\right] = \cos\left[\frac{(n+N)^2\pi}{512}\right]$$

$$\frac{N^2 + 2Nn}{512} = 2k\,;\ where\ n,k\ \in\ Z$$

$$N\ +\ 2n\ =\ \frac{1024k}{N}\ ;\ where\ n,k\ \in\ Z$$

$$n\ =\ \frac{512k}{N} - \frac{N}{2}\,;\ where\ n,k\ \in\ Z$$

$$N_{min} = 512$$

$$n\ =\ k - 256\,;\ where\ n,k\ \in\ Z$$

There is always an integer k that satisfies this equation. Hence, we can state that the sampled signal is periodic with a fundamental period of N=512. Here you can see the plot of xf[n] **(Figure 5)**:

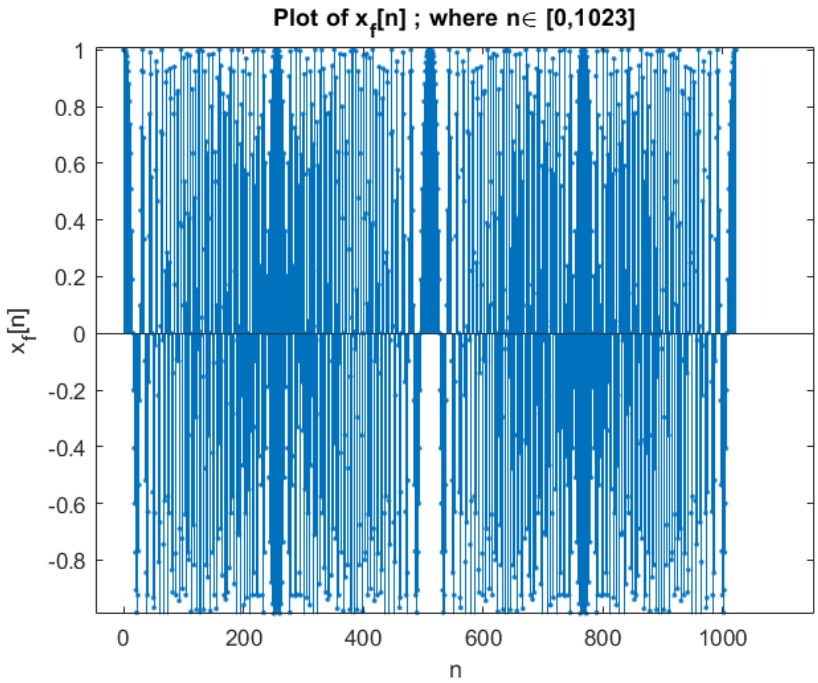

Figure 5: Plot of xf[n]

The analog signal and the sampled signal does not look the same at all. The analog signal's frequency varies with respect to time whereas the sampled signal is periodic. Sampling the analog signal brings aliasing with itself. While the sampled signal may capture certain aspects like the amplitude and general shape of the oscillations of the analog signal; it does not retain the exact behaviour or smooth chirp characteristics of the original analog signal.

Q3:

$x_g[n]$ is a discretized signal which:

$$x_g[n] = \cos(\propto \cdot (n \cdot T_s)^2) \ ; \ where\, n \in [0,8192] \, and \, T_s = 1000 \, rad^{-2}$$

Here you can see the plot of $x_g[n]$ **(Figure 6)**:

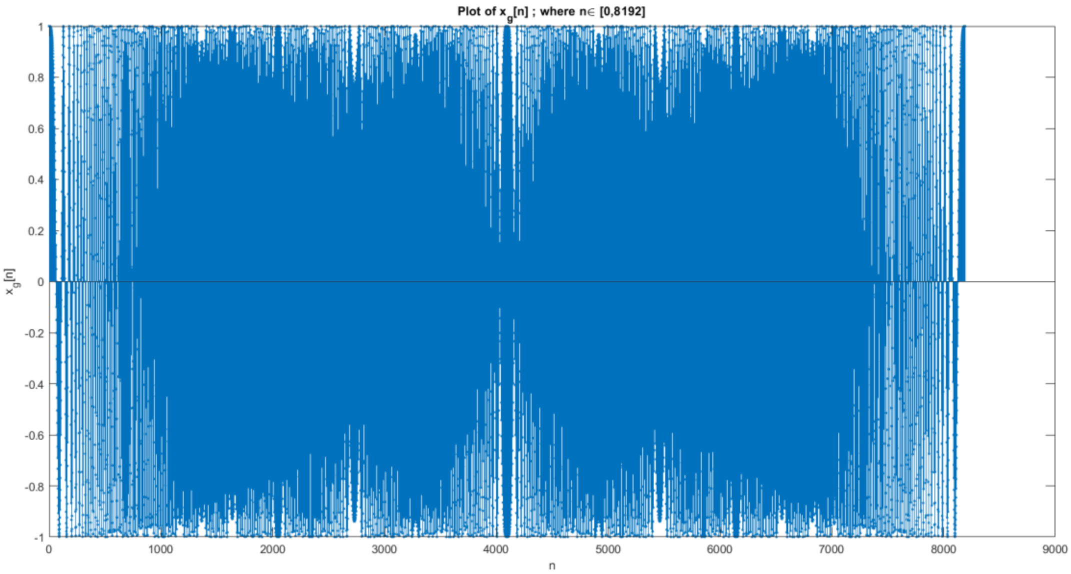

Figure 6: Plot of xg[n]

Q4:

Here is the output when the filter is applied to xf[n] **(Figure 7)**:

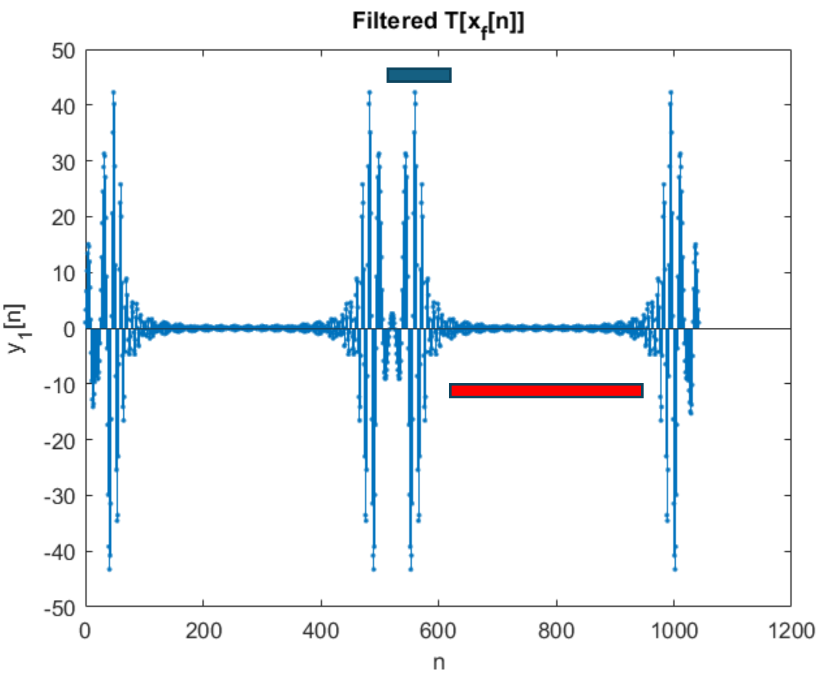

Figure 7: Plot of y1[n]

The blue rectangle roughly shows the transient part of the signal, the red line roughly shows the steady state part of the signal. Transient part of the system output is the finite duration where the signal approaches to a bounded value and steady state duration is where the signal has approached a stable value interval. There are time dependent-sudden changes in the transient part, and they are generally dominated by initial conditions.

The chirp signal $\cos(\propto \cdot t^2)$ has an instantenous frequency of $2 \cdot \propto \cdot t$. Therefore, instantaneous frequency of the chirp signal can be exactly the point it is examined for $\propto = \frac{1}{2}$. The output of the filter by using the chirp signal as input basically gives the behaviour of the filter for every time instance t. So technically, y1[n] shows us a frequency response of the filter.

The quality of this frequency response lies in the sampling rate. The sampling rate must be high to ensure that very less aliasing or truncation happens to the chirp signal. If sampling rate is not chosen carefully, the frequency response we obtain would be meaningless.

Also, chirp signal is not a stable signal. We should be careful when convolving an unstable signal with an impulse response of an LTI system. This might result in divergence and unwanted physical outcomes for the system.

Here is the output y2[n] when xg[n] is applied to the filter **(Figure 8)**:

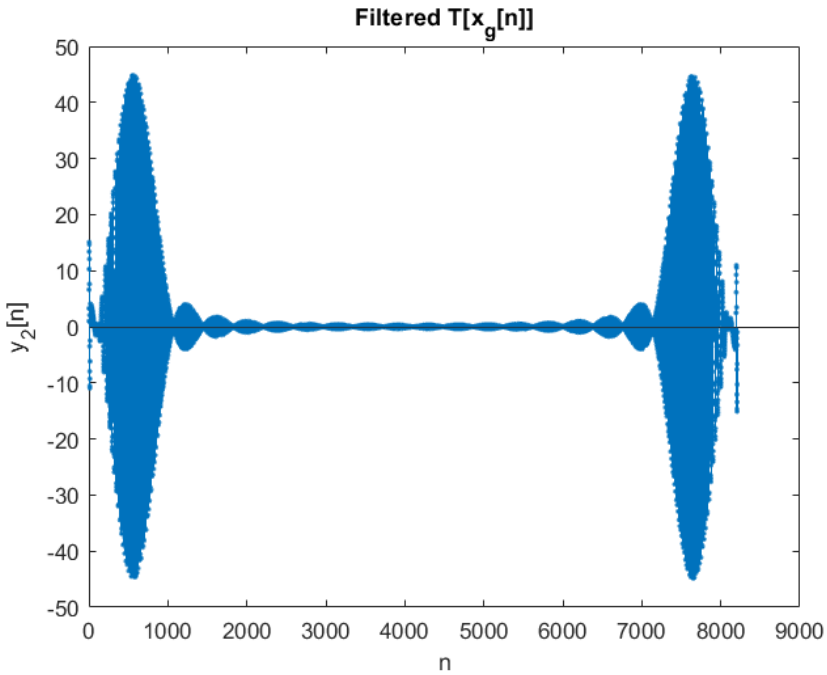

Figure 7: Plot of y2[n]

Q5:

The analog signal xa(t) is a two-sided non-periodic chirp signal, for which the instantaneous frequency increases with time, yielding a sound of a continuously increasing pitch. On the other hand, the sampled signal xr(t), which is obtained by periodic sampling of xa(t), is periodic, and thus the sound produced by xr(t) is distinctly different from xa(t). Specifically, whereas xa(t) produces a pitch that continuously rises because of its increasing instantaneous frequency, xr(t) produces a periodically rising and falling pitch. This is because the sampling in time imparts periodicity in frequency, which yields a time-domain representation for xr(t) to repeat itself. In addition, aliasing effects can reinforce this periodic behavior by folding higher frequencies back into the audible range. Listening to the audio actually confirms these differences, as xr(t) exhibits a cyclic tonal pattern, while xa(t) maintains a non-repeating, ever-increasing pitch.

Q6:

The signals yr(t) and y2[n] are related because yr(t) is obtained by interpolating the discrete signal y2[n] to reconstruct a continuous-time approximation. Interpolation allows yr(t) to appear continuous while retaining the periodic structure of y2[n]. Both signals are periodic because the sampling process enforces periodicity. The discrete signal y2[n] is formed by sampling the original signal within a finite interval, and this periodicity is reflected in the reconstruction process. Although yr(t) is continuous and y2[n] is discrete, their periodic natures create similar repeating patterns that make them equivalent when considering their repeated sound characteristics.

Q7:

The equivalent analog system that produces yr(t) from xa(t) is not exactly the low pass filter we designed initially. However, the cut-off frequencies of this equivalent analog system, which is a low-pass filter, are the same as our initial filter. These cutoff frequencies are $\frac{\pi}{10}$ and $\frac{\pi}{8}$as well as $\frac{-\pi}{8}$and $\frac{-\pi}{10}$. This is because of the real nature of the impulse responses of both systems. This realness forces the frequency responses of these signals to be conjugate symmetric. These cut-off frequencies can be seen in the following figure **(Figure 8)**:

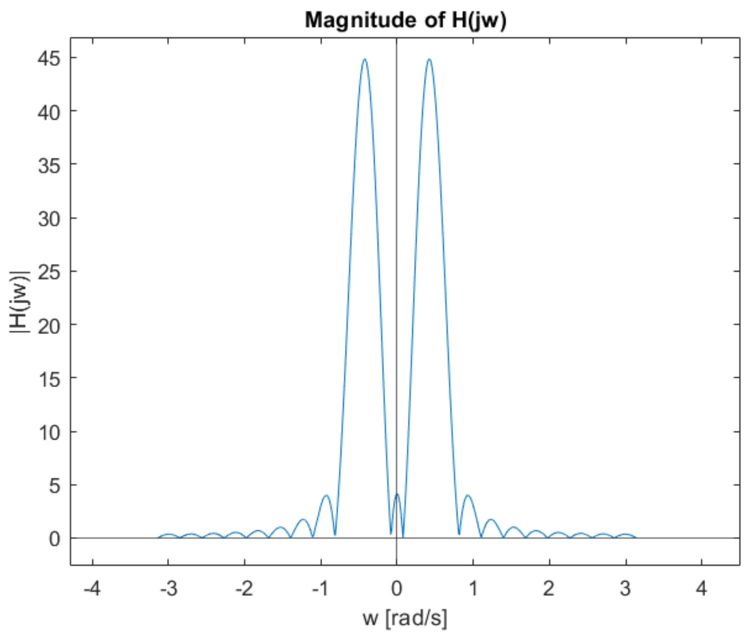

Figure 8: Magnitude Plot of H(jw)

Q8:

The filter we designed passes only some low frequency components of the incoming signal. Therefore, only a small range of frequency components gets passed through from the music. This causes the output to consist of fewer frequency components. The output is sharper than from the original music since many of the frequency components on the stopbands are filtered.

Q9:

My filtered voice was a lot sharper than my original recorded voice. This is because the sound signals at the frequencies on the stopbands are filtered.

Conclusion:

This lab consisted of 9 parts. We were assigned with designing a practical FIR filter, testing its frequency response, and performing some filtering operations. The lab was a total success. Every specification was met, and every question was answered.

I believe the lab was very helpful in understanding how FIR filters work. I learned a lot about practical filter design and the characteristics of filters.