# Bilkent University

# Electrical and Electronics Department

# EE321-02 Lab 2 Report:

17/10/2024

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## Introduction:

This lab consisted of 6 parts. We worked with a system whose impulse response was given to us in the lab manual. Then, we calculated the different outputs of the system for different input signals.

The function “**impuls.m**” takes the length as an input and gives the desired discrete impulse response as output as a vector. The function converts the continuous impulse response into a discrete one in itself. **(Appendix 1)**

The function “**fmcconvo.m**” works exactly the same as the in-built matlab function “conv”. Fmc stands for my name – Fatih Mehmet Çetin. The function takes two vectors as input and returns their discrete convolutive output based on a shifted multiplication algorithm. **(Appendix 2)**

The matlab file **“anal.mat”** is the file containing the analytical solutions of the outputs. **(Appendix 3)**

## Q1:

Here is the plot for the continuous equation which is used to calculate the discrete impulse response of the system and the relevant code **(Figures 1.1&1.2)**:

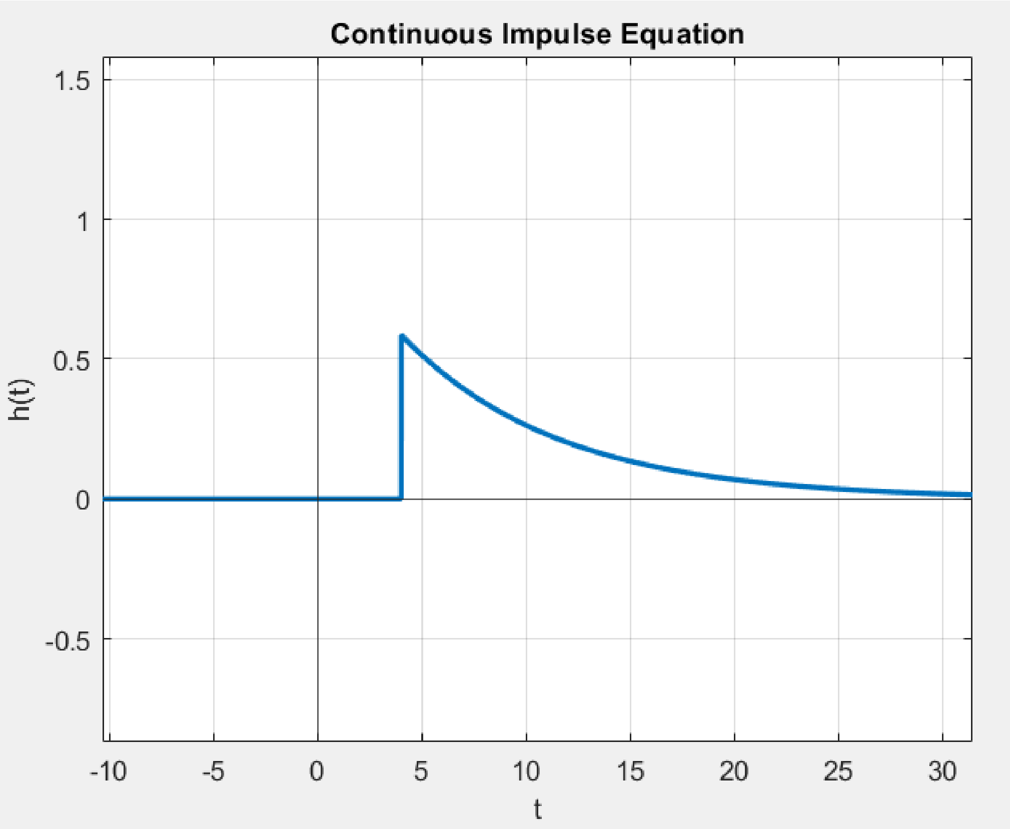


**Figure 1.1: The Continuous Equation for the Impulse Function**

```
>> h(n) = piecewise(n<4, 0, (7/8)^(n));
fplot(h,'LineWidth',2);
grid on;
xlabel('t');
ylabel('h(t)');
title('Continuous Impulse Equation');
xline(0);
yline(0);
```

**Figure 1.2: The code for the plot**

After these steps, here is the obtained plot of the discrete impulse response of the system **(Figure 1.3)**:

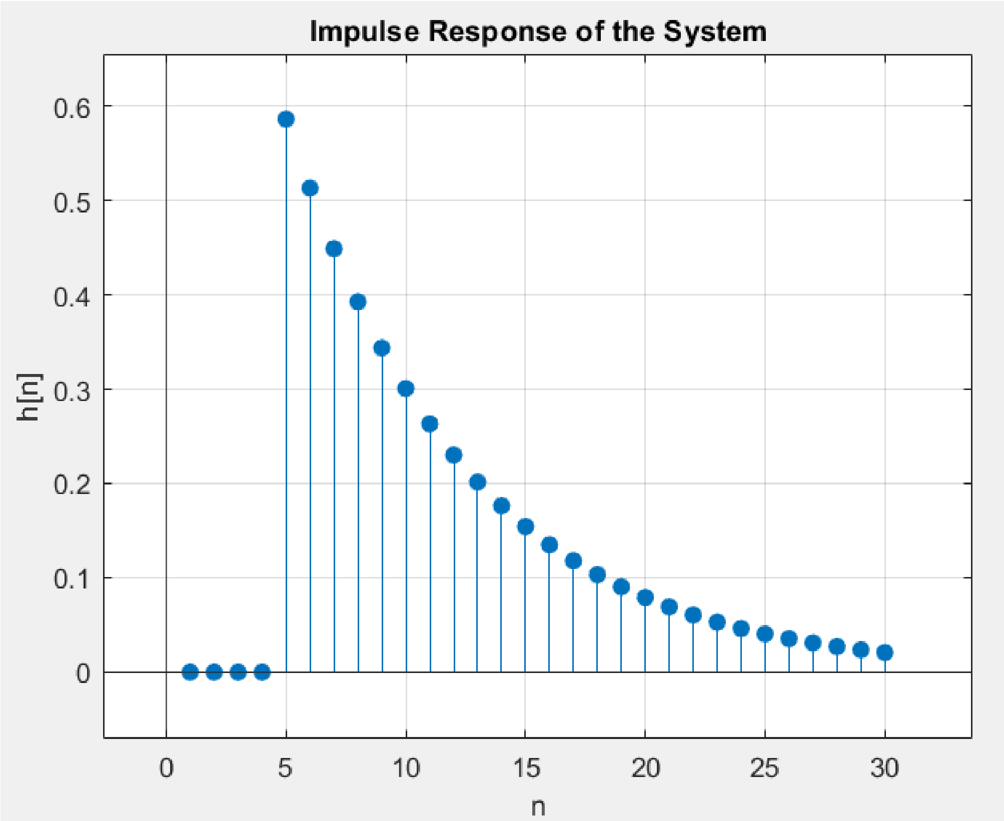


**Figure 1.3: The Impulse Response of the System**

The system is definitely causal, because all the values before t=0, is 0 in the impulse function. In other words, no element determines the value of another previous element in time domain.

The system is indeed stable. Because the system does not produce an unbounded output for a bounded input. In other words, the integral for the impulse response does not diverge.

**a)**

Here is the input signal for part a, and the system response **(Figures 1.a.1 & 1.a.2)**:

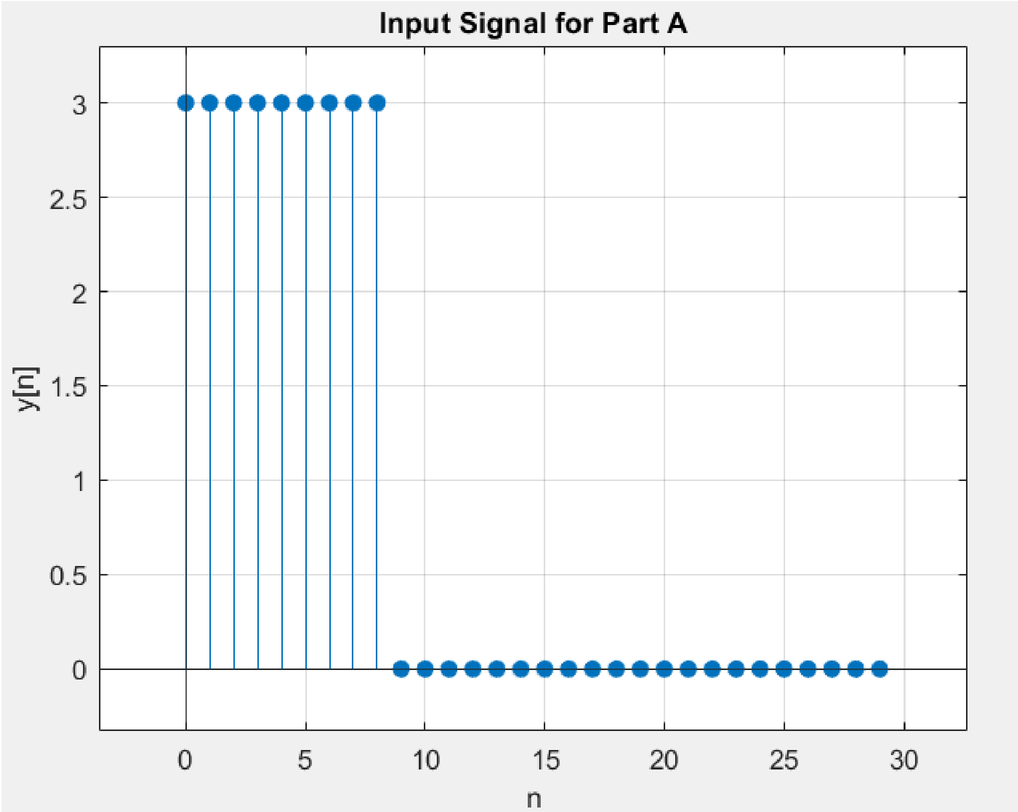


**Figure 1.a.1: The Input Signal for Part A**

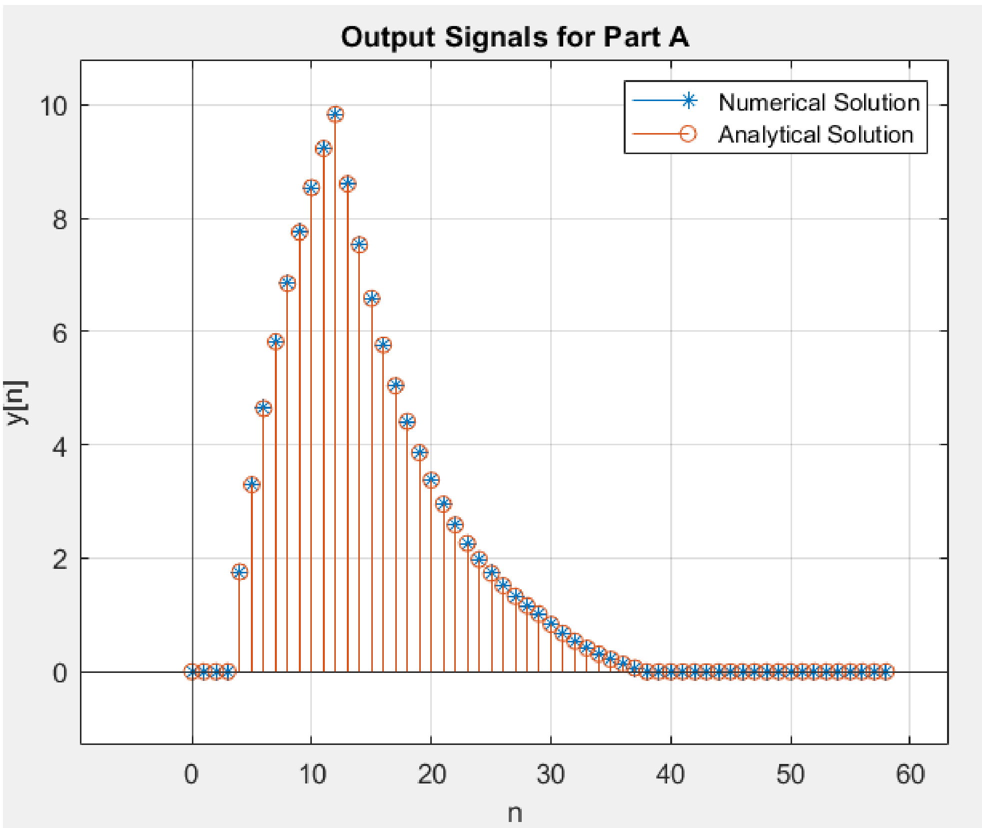


**Figure 1.a.2: The Output Signals for Part A**

**b)**

Here is the input signal for part b, and the system response **(Figures 1.b.1 & 1.b.2)**:

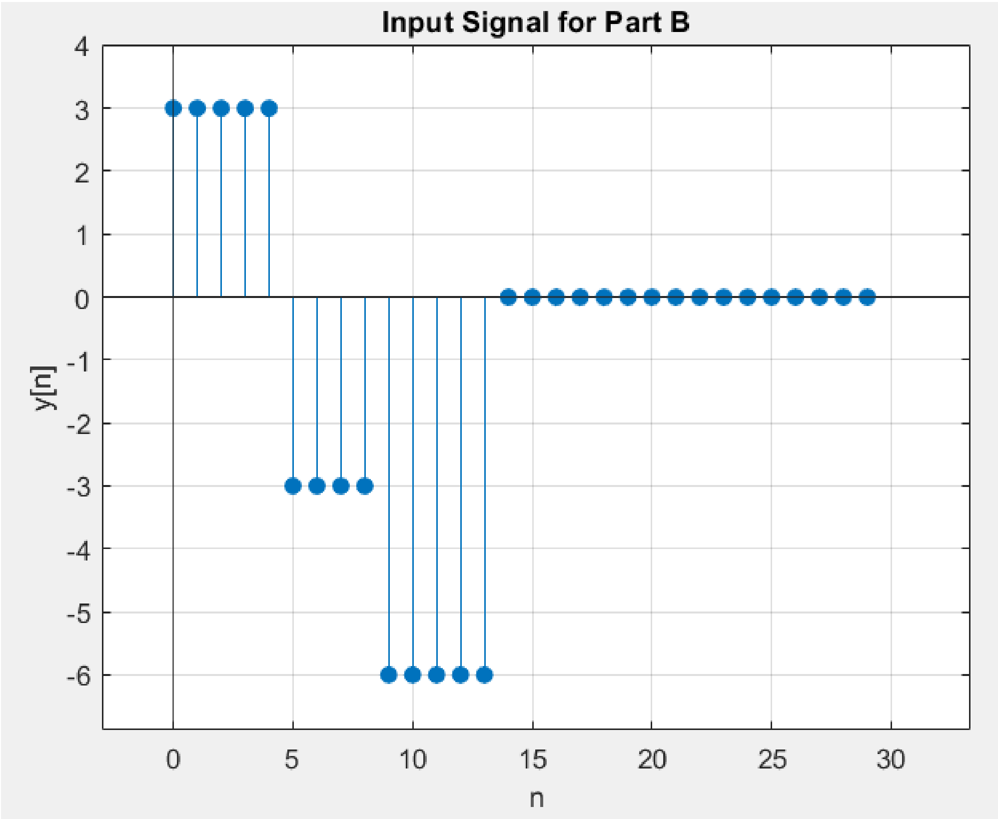


**Figure 1.b.1: The Input Signal for Part B**

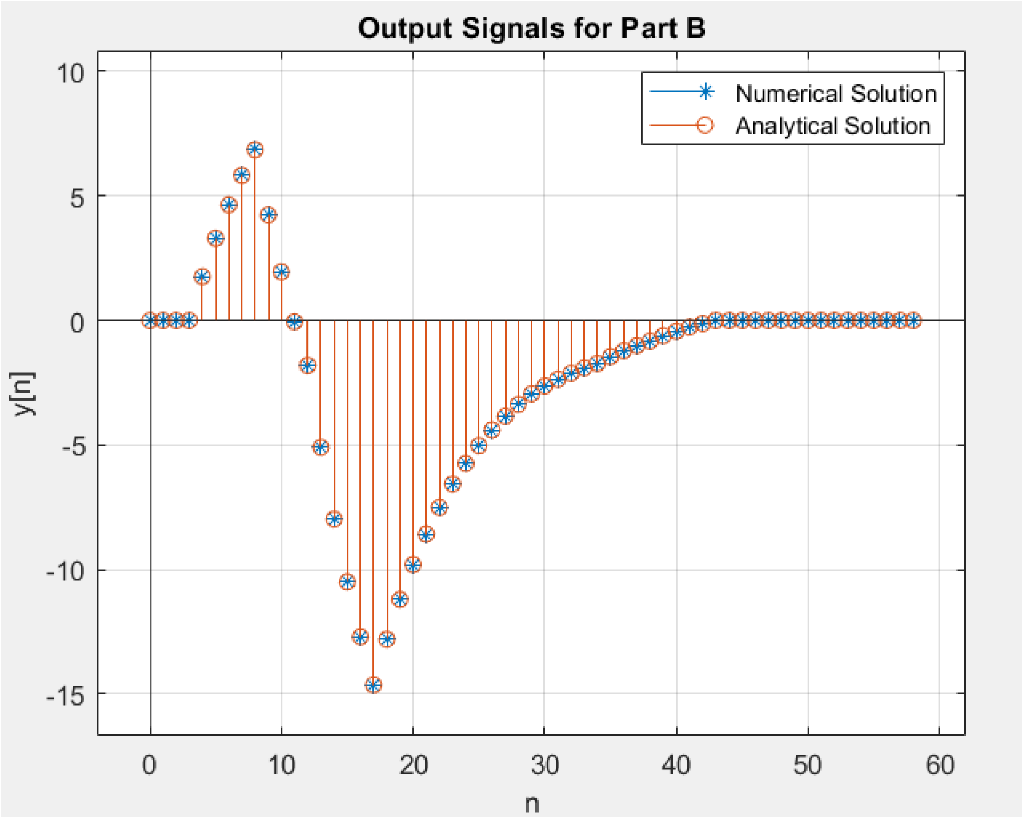


**Figure 1.b.2: The Output Signals for Part B**

**c)**

Here is the input signal for part c, and the system response **(Figures 1.c.1 & 1.c.2)**:

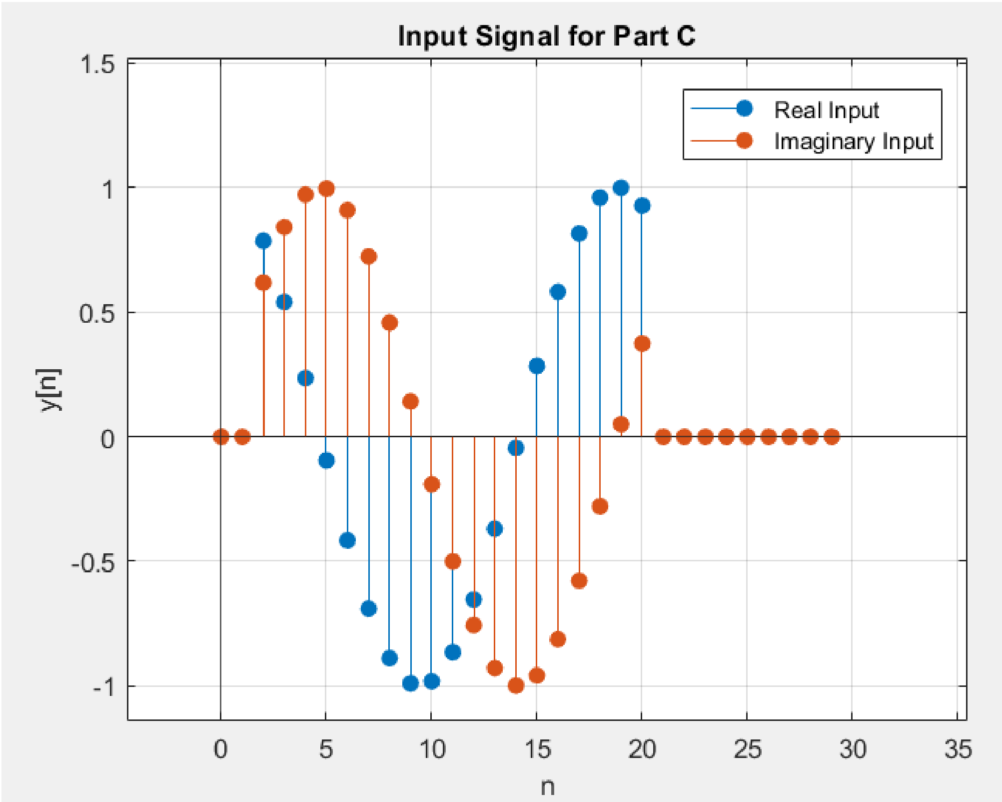


**Figure 1.c.1: The Input Signal for Part C**

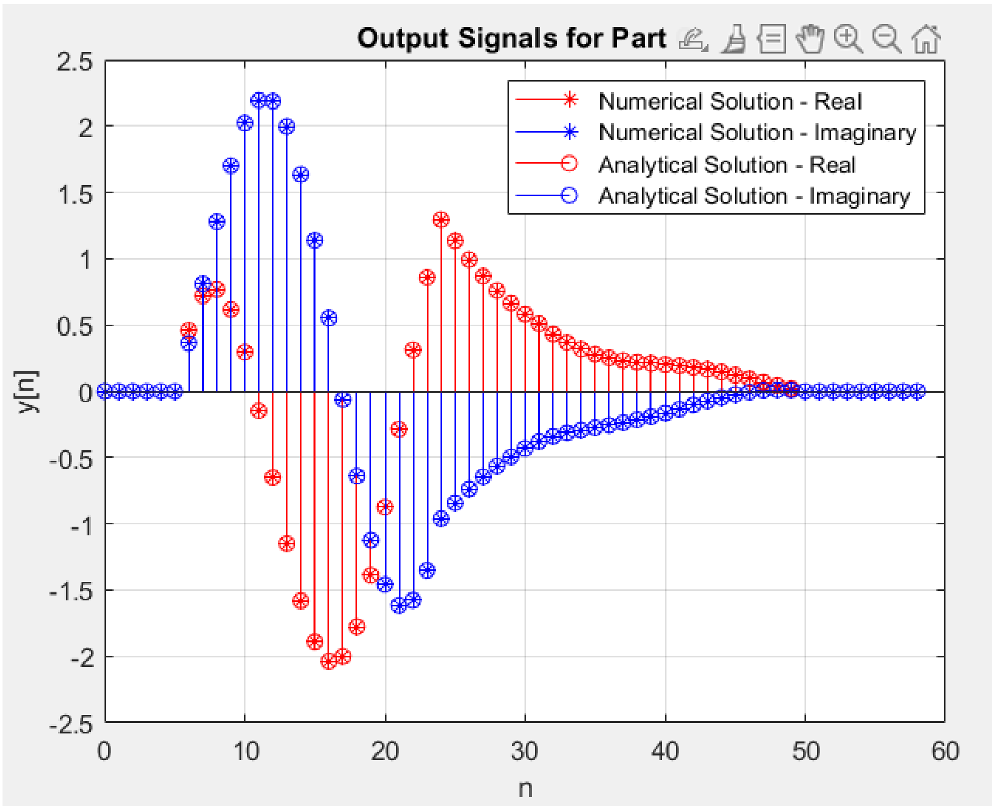


**Figure 1.c.2: The Output Signals for Part C**

One remark we can make here is that convolution operation is a linear operation for imaginary numbers which means the real elements get convolved with each other and imaginary elements are convolved with each other.

**d)**

Here is the input signal for part d, and the system response **(Figures 1.d.1 & 1.d.2)**:

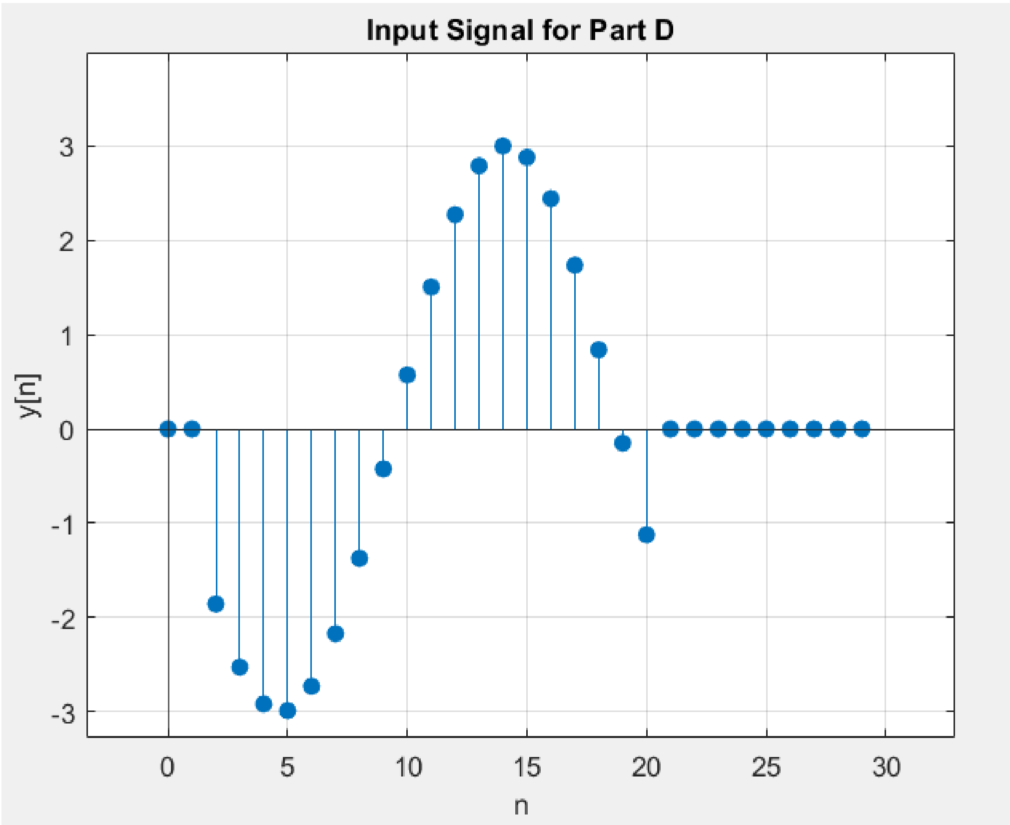


**Figure 1.d.1: The Input Signal for Part D**

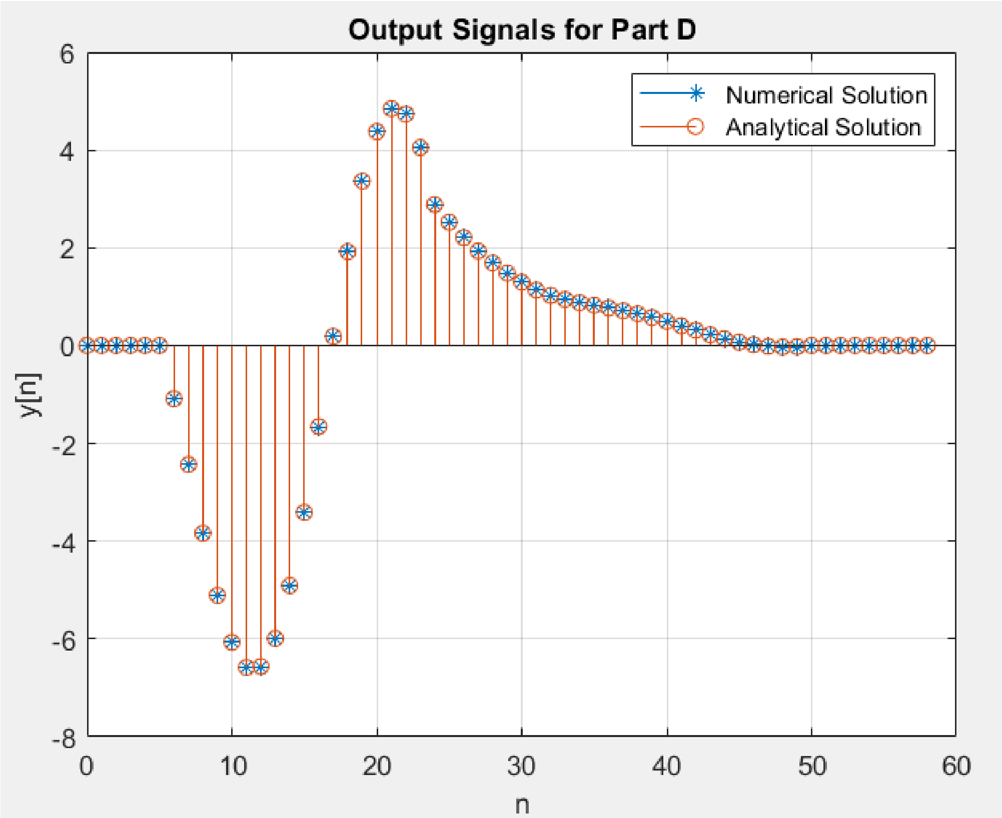


**Figure 1.d.2: The Output Signals for Part D**

Please note that the input signal in part D, is identical to the imaginary component of the input signal multiplied by -3 **(Figure 1.d.3)**. The output in the figure is negligible and is caused by MATLAB's rounding algorithms when calculating sine cosine etc. Here is also the figure that shows us the output relationships of these signals are also identical **(Figure 1.d.4)**. Finally, we can state that the convolution operation is independent for real and imaginary components of a complex valued signal.

```
>> sum(x_4-(-3*imag(x_3)),'All')

ans =

  -4.9960e-16
```

**Figure 1.d.3: The Matlab script that shows us the relationship between x_3 and x_4**

```
>> sum(out4-(-3*imag(out3)),'All')

ans =

  -2.4945e-15
```

**Figure 1.d.4: The Matlab script that shows us the relationship between out3 and out4**

**e)**

Here is the input signal for part e, and the system response **(Figures 1.e.1 & 1.e.2)**:

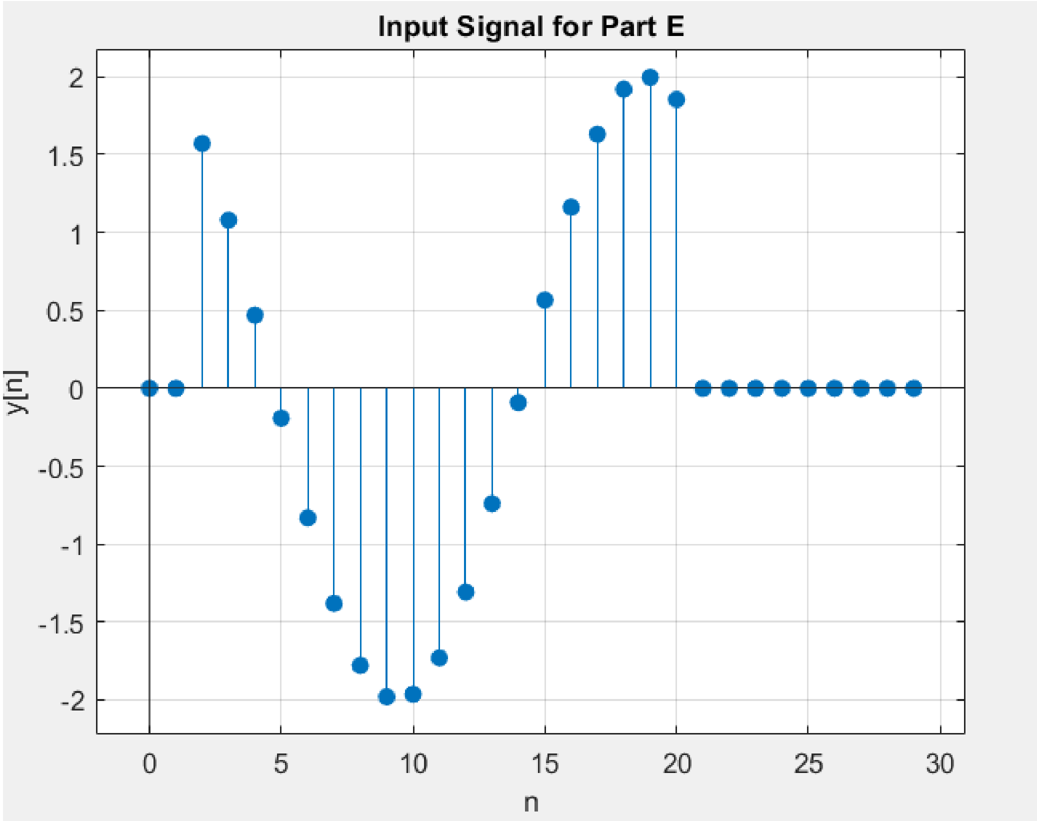


**Figure 1.e.1: The Input Signal for Part E**

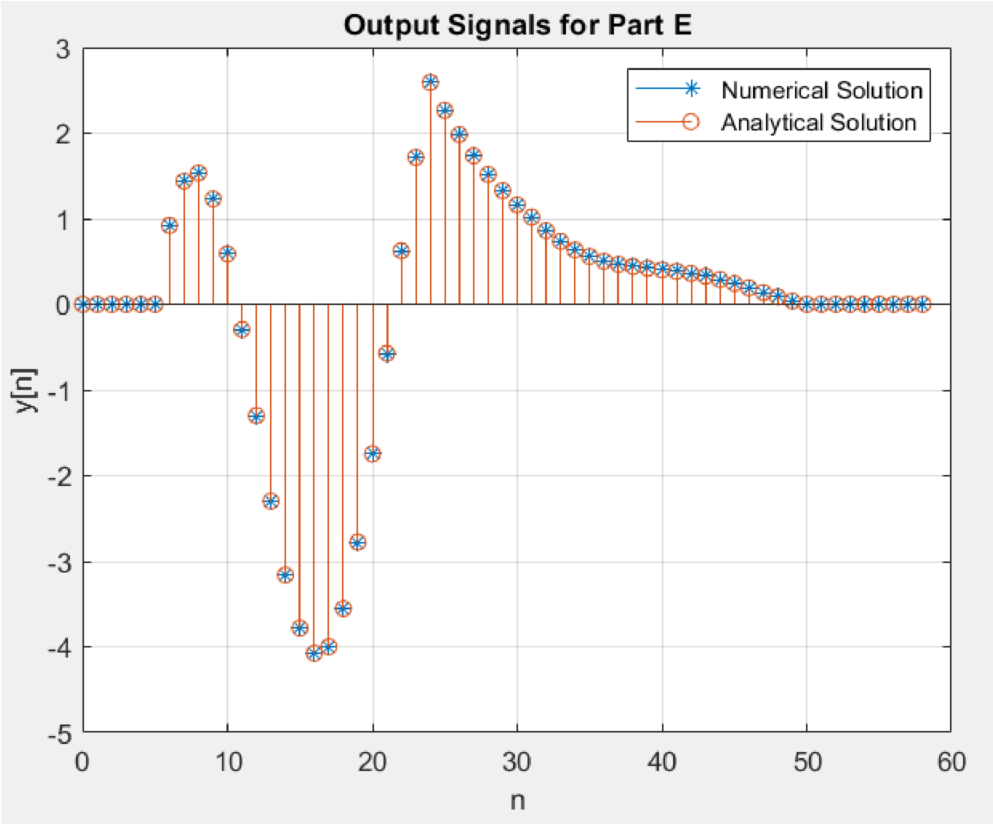


**Figure 1.e.2: The Output Signals for Part E**

Please note that the input signal in part E, is identical to the real component of the input signal multiplied by 2 **(Figure 1.e.3)**. The output in the figure is zero. Here is also the figure that shows us the output relationships of these signals are also identical **(Figure 1.e.4)**. Finally, we can state that the convolution operation is independent for real and imaginary components of a complex valued signal just as we did in part D.

```
>> sum(x_5-(2*real(x_3)),'All')

ans =

     0
```

**Figure 1.e.3: The Matlab script that shows us the relationship between x_3 and x_5**

```
>> sum(out5-(2*real(out3)),'All')

ans =

     0
```

**Figure 1.e.4: The Matlab script that shows us the relationship between out3 and out5**

**f)**

Here is the input signal for part f, and the system response **(Figures 1.f.1 & 1.f.2)**:

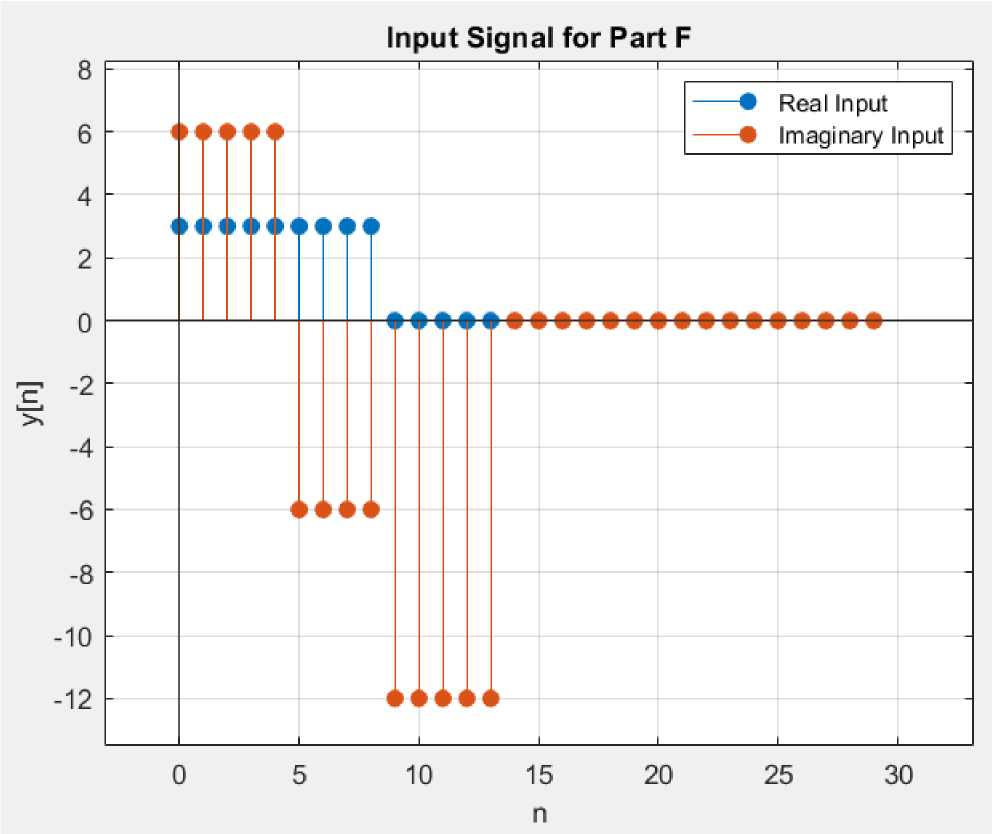


**Figure 1.f.1: The Input Signal for Part F**

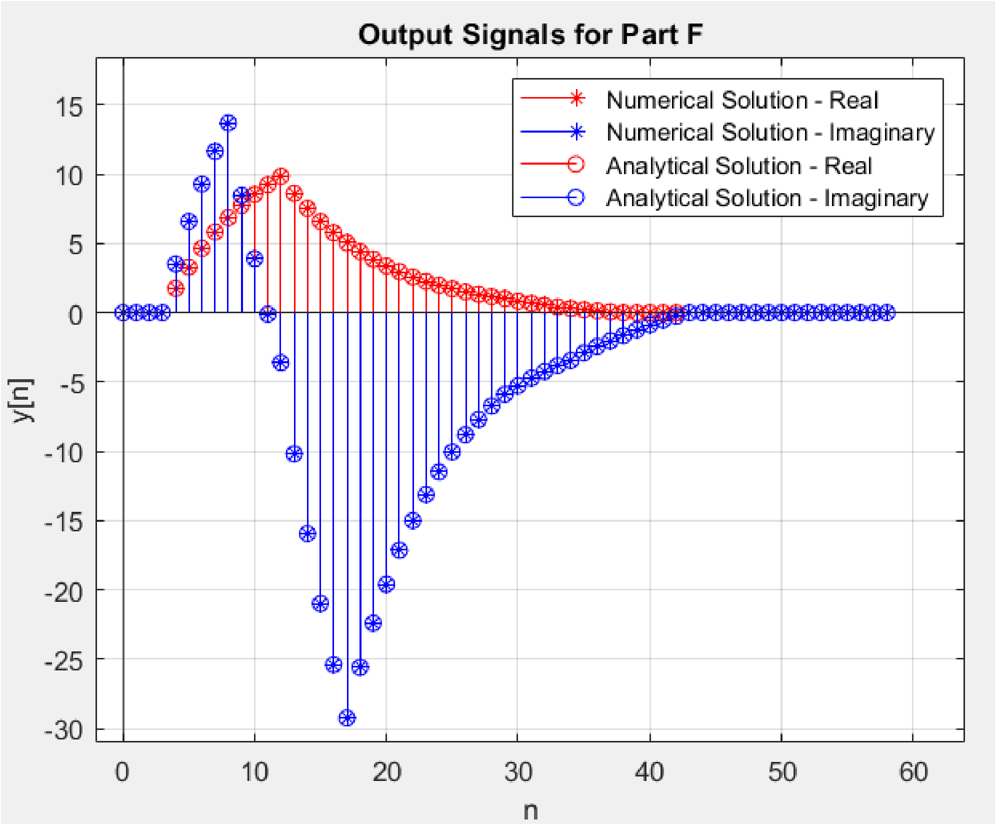


**Figure 1.f.2: The Output Signal for Part F**

The following figures show us that the complex convolution operation is linear considering its real and imaginary components **(Figures 1.f.3 & 1.f.4)**.

```
>> isequal(x_6, (x_1+2i*x_2))

ans =

  logical

   1
```

**Figure 1.f.3: The Matlab script that shows us the relationship between x_1, x_2 and x_6**

```
>> isequal(out6, (out1+2i*out2))

ans =

  logical

   1
```

**Figure 1.f.4: The Matlab script that shows us the relationship between out1, out2 and out6**

## Conclusion:

This lab consisted of 6 parts. We worked with a system whose impulse response was given to us in the lab manual. Then, we calculated the different outputs of the system for different input signals.

I believe this lab was very helpful for understanding the very essence of the convolution operation as well as LTI systems. I learned very useful information on some properties of the convolution operation. I also learned how to 3D-plot in MATLAB.

## Appendices:

1. https://github.com/fmcetin7/Bilkent-EEE-321/blob/main/lab%202/impuls.m
2. https://github.com/fmcetin7/Bilkent-EEE-321/blob/main/lab%202/fmcconvo.m
3. https://github.com/fmcetin7/Bilkent-EEE-321/blob/main/lab%202/anal.mat

# On-Paper Solutions: (Starting From Next Page)

EE-321 Lab 2 Analytical Solutions

a) $y_1[n] = x[n] * h[n] = \sum_{m=0}^{\infty} 3 \cdot \left(\frac{7}{8}\right)^{n-m} \cdot u[n-m-4]$

$$y_1[n] = \begin{cases} 0 & , \; n < 4 \\ 3 \cdot \sum_{m=4}^{n} 3 \cdot (7/8)^m & , \; 4 \leq n < 12 \\ 3 \cdot \sum_{m=n-8}^{n} \left(\frac{7}{8}\right)^m & , \; n \geq 12 \end{cases}$$

b) $y_2[n] = x_2[n] * h[n] = y_1[n] - 2 \cdot y_1[n-5]$

$$y_2[n] = \begin{cases} 0 & , \; n < 4 \\ 3 \cdot \sum_{m=4}^{n} (7/8)^m & , \; 4 \leq n < 9 \\ 3 \cdot \sum_{m=4}^{n} (7/8)^m - 6 \cdot \sum_{m=4}^{n-5} (7/8)^m & , \; 9 \leq n < 12 \\ 3 \cdot \sum_{m=n-8}^{n} \left(\frac{7}{8}\right)^m - 6 \cdot \sum_{m=4}^{n-5} \left(\frac{7}{8}\right)^m & , \; 12 \leq n < 17 \\ 3 \cdot \sum_{m=n-8}^{n} (7/8)^m - 6 \cdot \sum_{m=n-13}^{n-5} (7/8)^m & , \; n \geq 17 \end{cases}$$

c) $y_3[n] = x_3[n] * h[n] = \sum_{m=2}^{20} e^{jm/3} \cdot \left(\frac{7}{8}\right)^{n-m} \cdot u[n-m-4]$

$$y_3[n] = \begin{cases} 0 & , \quad n < 6 \\ \sum_{m=2}^{n-4} e^{jm/3} \cdot \left(\frac{7}{8}\right)^{n-m} & , \quad 6 \leq n < 24 \\ \sum_{m=2}^{20} e^{jm/3} \cdot \left(\frac{7}{8}\right)^{n-m} & , \quad n \geq 24 \end{cases}$$

d) $y_4[n] = x_4[n] * h[n] = -3 \cdot \mathrm{Im}\{y_3[n]\}$

$$y_4[n] = \begin{cases} 0 & , \quad n < 6 \\ -3 \cdot \sum_{m=2}^{n-4} \left(\frac{7}{8}\right)^{n-m} \cdot \sin\left(\frac{m}{3}\right) & , \quad 6 \leq n < 24 \\ -3 \cdot \sum_{m=2}^{20} \left(\frac{7}{8}\right)^{n-m} \cdot \sin\left(\frac{m}{3}\right) & , \quad n \geq 24 \end{cases}$$

e) $y_5[n] = x_5[n] * h[n] = 2 \cdot \mathrm{Re}\{y_3[n]\}$

$$y_5[n] = \begin{cases} 0 & , \quad n < 6 \\ 2 \cdot \sum_{m=2}^{n-4} \left(\frac{7}{8}\right)^{n-m} \cdot \cos\left(\frac{m}{3}\right) & , \quad 6 \leq n < 24 \\ 2 \cdot \sum_{m=2}^{20} \left(\frac{7}{8}\right)^{n-m} \cdot \cos\left(\frac{m}{3}\right) & , \quad n \geq 24 \end{cases}$$

$$y_6[n] = x_6[n] * h[n] = y_1[n] + 2j \cdot y_2[n]$$

$$y_6[n] = \begin{cases} 0 & , n < 4 \\ (1+2j)\cdot 3 \cdot \sum_{m=4}^{n} \left(\frac{7}{8}\right)^m & , 4 \leq n < 9 \\ (1+2j)\cdot 3 \cdot \sum_{m=4}^{n} \left(\frac{7}{8}\right)^m - 2j \cdot 6 \sum_{m=4}^{n-5} \left(\frac{7}{8}\right)^m & , 9 \leq n < 12 \\ (1+2j)\cdot 3 \cdot \sum_{m=n-8}^{n} \left(\frac{7}{8}\right)^m - 2j \cdot 6 \cdot \sum_{m=4}^{n-5} \left(\frac{7}{8}\right)^m & , 12 \leq n < 17 \\ (1+2j)\cdot 3 \cdot \sum_{m=n-8}^{n} \left(\frac{7}{8}\right)^m - 2j \cdot 6 \cdot \sum_{m=n-13}^{n-5} \left(\frac{7}{8}\right)^m & , n \geq 17 \end{cases}$$

→ Remark : $y_6[n] = y_1[n] + 2j \cdot \left(y_1[n] - 2 \cdot y_1[n-5]\right)$

Therefore : $\boxed{y_6[n] = (1+2j) \cdot y_1[n] - 4j \cdot y_1[n-5]}$