Bilkent University

Electrical and Electronics Department

EE321-02 Lab 6 Report:

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Introduction:

The lab consisted of 8 parts. We were assigned with designing a practical IIR filter, testing its frequency response, and performing some filtering operations.

The matlab file **"lab6.mat"** is the file where all my matlab code lies in for this lab. **(Appendix 1)**

My Bilkent ID is 22201689. Therefore, N1=6, N2=1, M1=8 and M2=3. The filter order is 11. The passband is between $\left(\frac{\pi}{8},\frac{\pi}{3}\right)and\left(-\frac{\pi}{8},-\frac{\pi}{3}\right)$. The reason for this is the following property of the z-transform (**(1)** and **(2)**). If x[k] is real, then **(3)** holds.

$$X\left(e^{jw}\right)=\sum_{k=-\infty}^{\infty} x[k]\cdot e^{-jwk} \qquad \textbf{(1)}$$

$$X^*\left(e^{jw}\right)=\sum_{k=-\infty}^{\infty} x^*[k]\cdot e^{-jwk} \qquad \textbf{(2)}$$

$$\text{If } x[k] = x^*[k]; \qquad X^*\left(e^{jw}\right)=X\left(e^{jw}\right) \qquad \textbf{(3)}$$

This means that if x[k] is real, then $X(e^{jw})$ is conjugate symmetric. In other words, the zeros of the passband should be symmetric to the x-axis. Since the order of my filter is 11, there are 11 poles and 11 zeros of the system.

Q1:

I carefully distributed the zeros and poles along and within the unit circle such that the passbands were not suppressed. Then I calculated h[n] digitally using Matlab. I used **(4)** to calculate the impulse response of the system where a_k's are the zeros of the system.

$$H(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{11} b_k \cdot z^{-k}}{\sum_{k=1}^{11} a_k \cdot z^{-k}} \qquad \textbf{(4)}$$

H(z) can be defined in terms of the polynomials which are A and B. Roots of B are the zeros of the system and roots of A are the poles of the system. Here you can see the pole-zero plot of the bandpass filter **(Figure 1)**:

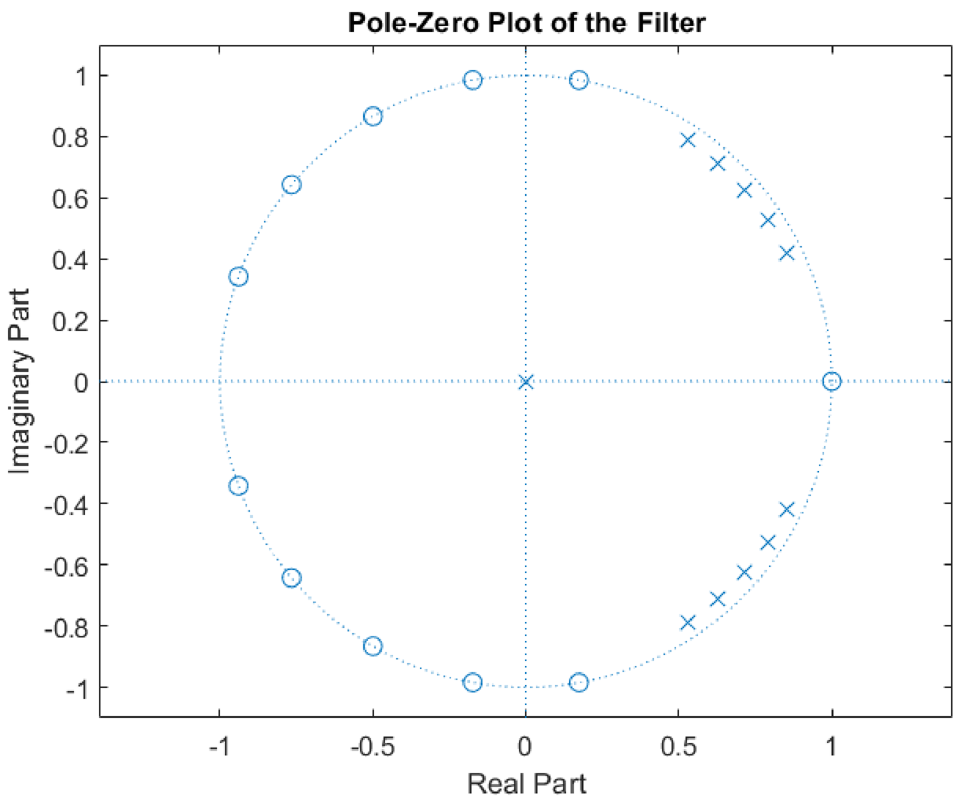

Figure 1: The Pole-Zero Plot of the Bandpass Filter

Here is a reasonable amount of the impulse response as an array and the plot of a reasonable part of the impulse response (**Figures 2.1 & 2.2)**:

```
h =

   1.0e+03 *

  Columns 1 through 13

    0.0010    0.0101    0.0523    0.1831    0.4810    0.9933    1.6370    2.1128    1.9376    0.6678   -1.7385   -4.6360   -6.7515

  Columns 14 through 26

   -6.7068   -3.8181    1.2850    6.6823    9.9590    9.4237    5.0714   -1.2962   -6.9169   -9.4302   -8.0082   -3.6512    1.4554

  Columns 27 through 39

    5.1019    6.0753    4.5395    1.6904   -1.0078   -2.5373   -2.6559   -1.7775   -0.5872    0.3461    0.7725    0.7461    0.4789

  Columns 40 through 52

    0.1830   -0.0192   -0.1040   -0.1063   -0.0737   -0.0395   -0.0168   -0.0056   -0.0014   -0.0002   -0.0001   -0.0004   -0.0013

  Columns 53 through 65

   -0.0035   -0.0072   -0.0119   -0.0154   -0.0141   -0.0049    0.0126    0.0337    0.0491    0.0488    0.0278   -0.0093   -0.0486

  Columns 66 through 78

   -0.0724   -0.0685   -0.0369    0.0094    0.0503    0.0685    0.0582    0.0265   -0.0106   -0.0371   -0.0442   -0.0330   -0.0123
```

Figure 2.1: Array of a portion of h[n]

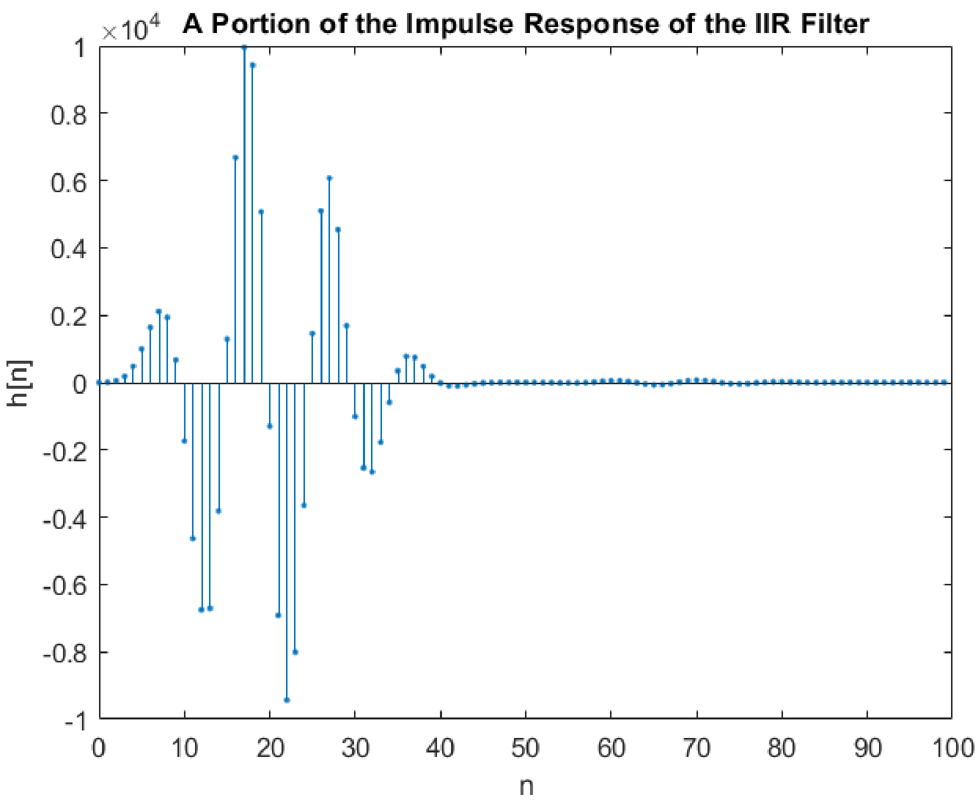

Figure 2.2: Plot of a portion of h[n]

As you can see, the h[n] is purely real as specified in the lab manual. Now let's look at the magnitude and phase plots of $H(e^{jw})$ **(Figures 3.1&3.2)**:

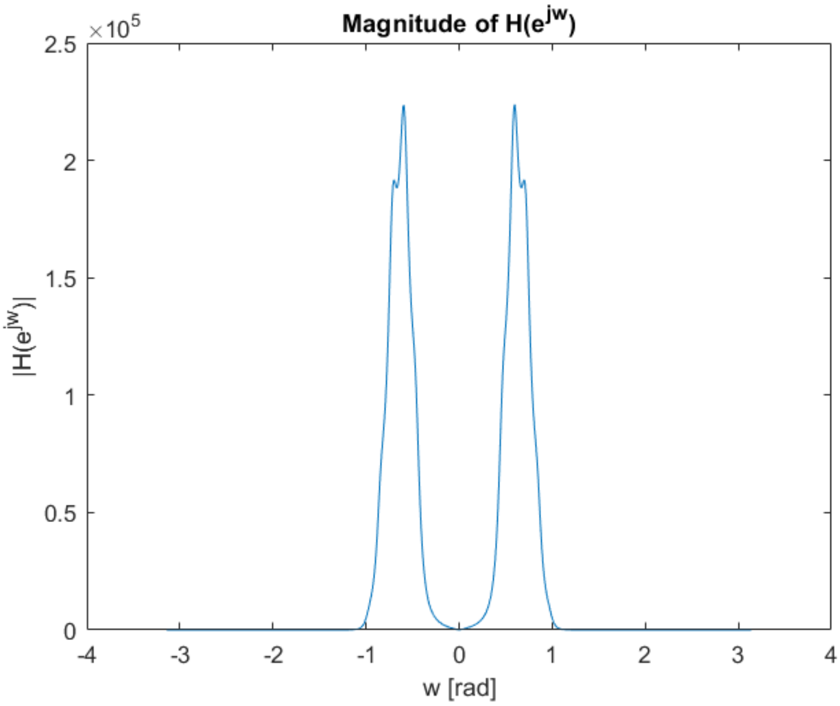

Figure 3.1: Magnitude Plot of $H(e^{jw})$

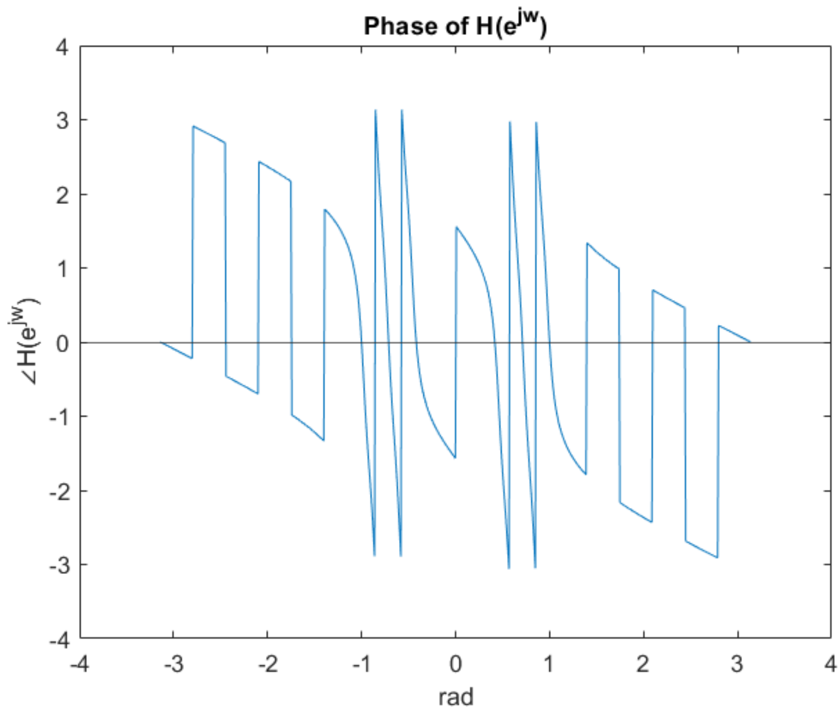

Figure 3.2: Phase Plot of $H(e^{jw})$

Q2:

In the last lab, we proved that although their analog versions are not periodic, discrete chirp signals are periodic. Here you can see the plots for x1[n] and x2[n] **(Figures 4.1&4.2)**:

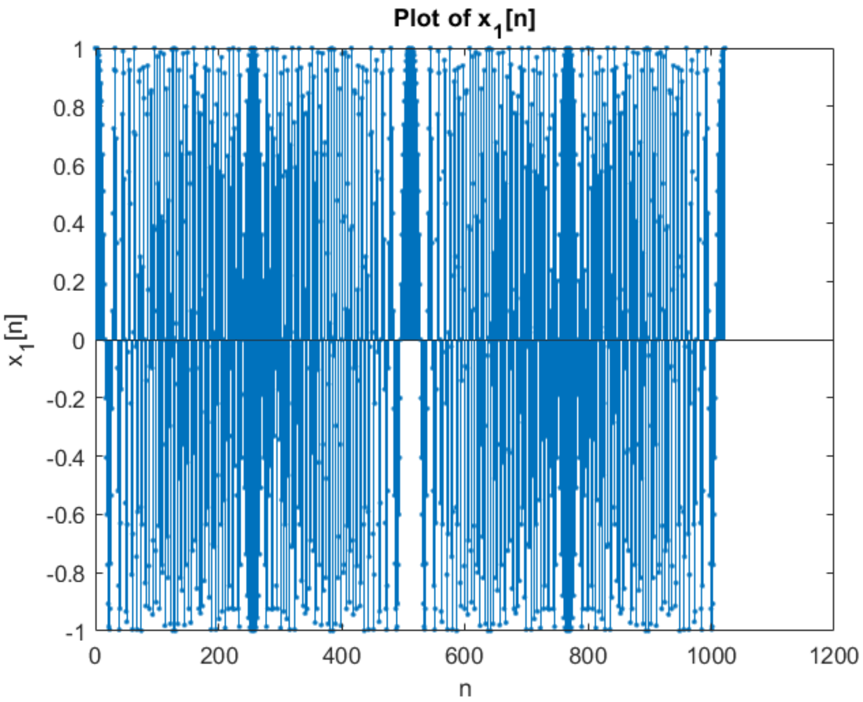

Figure 4.1: $x_1[n] \;=\; \cos\left(\frac{\pi n^2}{512}\right);\; for\; n \in [0,1023]$

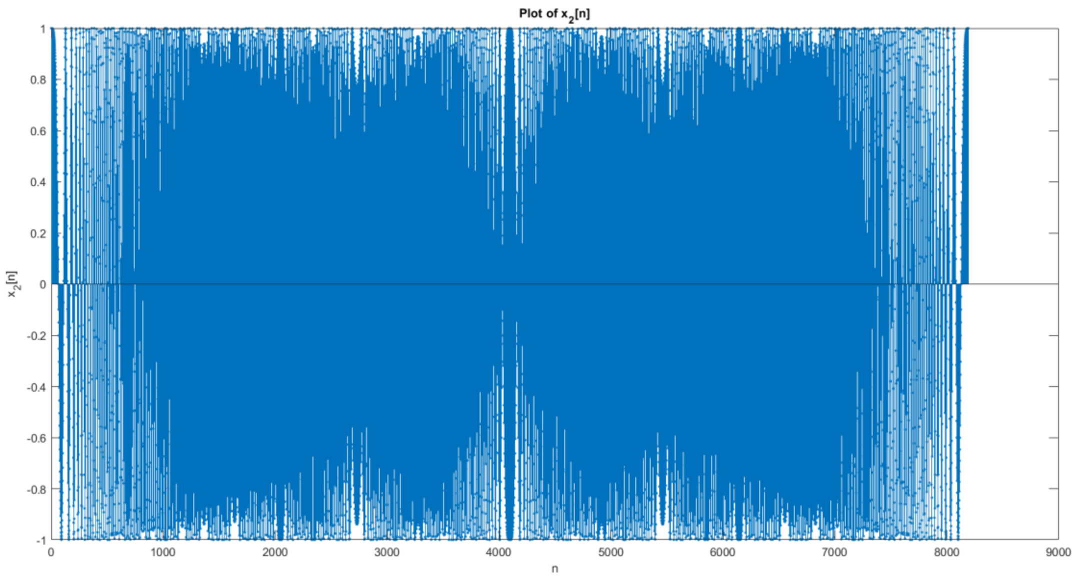

Figure 4.2: $x_1[n] \;=\; \cos\left(\frac{\pi n^2}{8192}\right);\; for\; n \in [0,8191]$

Q3:

$x_1[n]$ and $x_2[n]$ are passed through the filter via a recursion loop. Here is the difference equation that defines both y[n]'s **(5)**. The first term tells us about the contribution of the past inputs whereas the second term tells us about the contribution of past outputs.

$$y[n] \; = \; \sum_{i=0}^{10} B(k+1) \cdot x[n-k] \; + \; \sum_{j\,=\,0}^{11} a(k+1) \cdot y[n-k] \; \textbf{(5)}$$

Here are the plots of the outputs when the inputs are passed through the filter **(Figures 5.1&5.2)**:

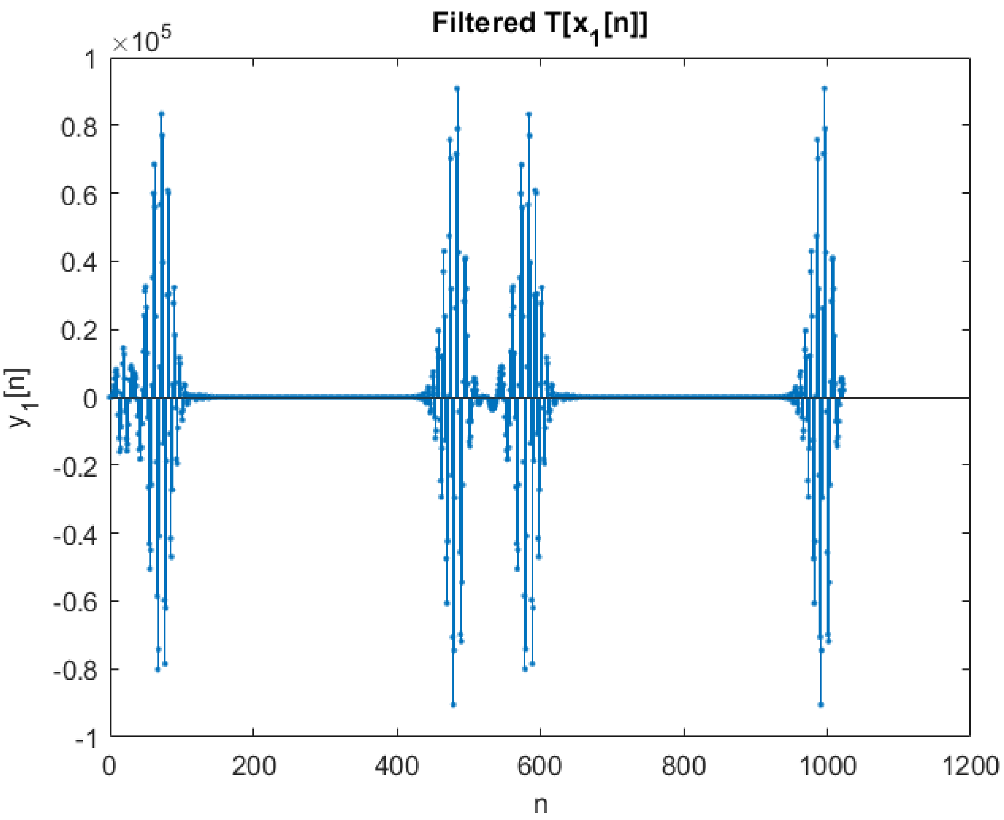

Figure 5.1: Plot of y1[n]

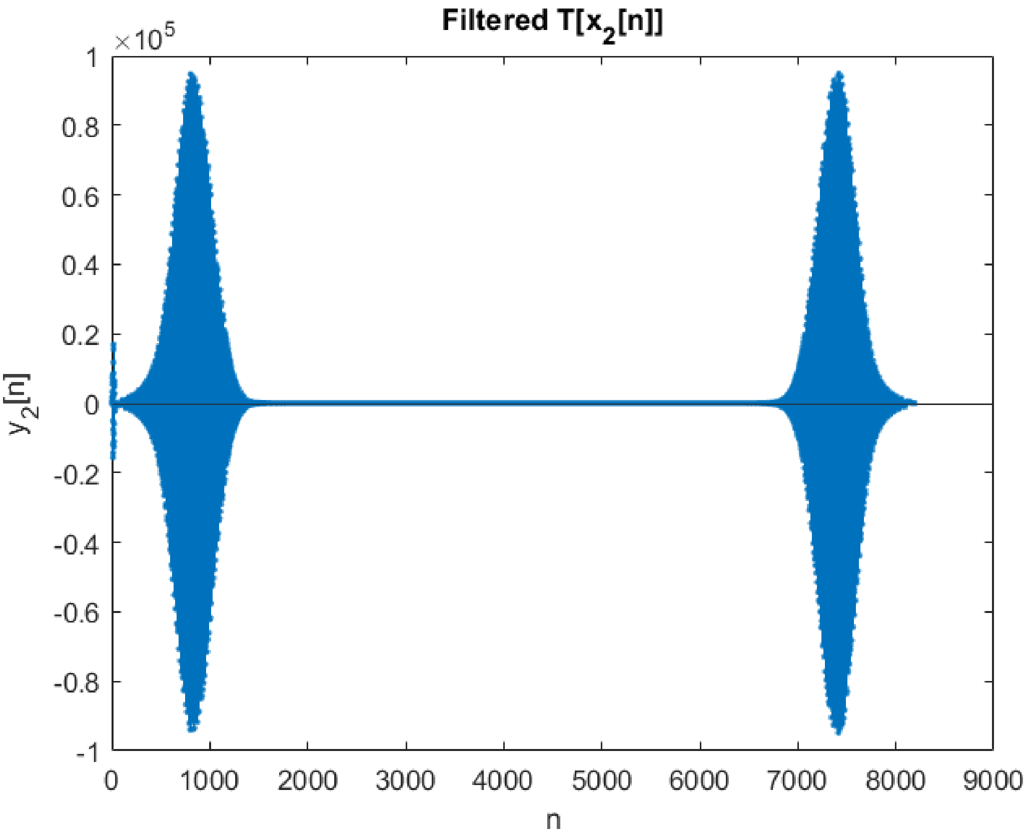

Figure 5.2: Plot of y2[n]

X1 is sampled with 2.5ms and x2 is sampled with 0.62 ms. Both outputs show the passband and stopbands very clearly. The IIR filter responses are more ideal than the FIR filter responses. This is because we have more zeros and poles to work with, and to design the filter with. This helps us obtain a filter closer to an ideal filter.

The chirp signal $\cos(\propto \cdot t2)$ has an instantenous frequency of $2 \cdot \propto \cdot t$. Therefore, instantaneous frequency of the chirp signal can be exactly the point it is examined for $\propto = 1/2$. The output of the filter by using the chirp signal as input basically gives the behaviour of the filter for every time instance t. So technically, y1[n] shows us a frequency response of the filter.

The quality of this frequency response lies in the sampling rate. The sampling rate must be high to ensure that very less aliasing or truncation happens to the chirp signal. If sampling rate is not chosen carefully, the frequency response we obtain would be meaningless.

Also, chirp signal is not a stable signal. We should be careful when convolving an unstable signal with an impulse response of an LTI system. This might result in divergence and unwanted physical outcomes for the system.

Q4:

The analog signal xa(t) is a two-sided non-periodic chirp signal, for which the instantaneous frequency increases with time, yielding a sound of a continuously increasing pitch. On the other hand, the sampled signal xr(t), which is obtained by periodic sampling of xa(t), is periodic, and thus the sound produced by xr(t) is distinctly different from xa(t). Specifically, whereas xa(t) produces a pitch that continuously rises because of its increasing instantaneous frequency, xr(t) produces a periodically rising and falling pitch. This is because the sampling in time imparts periodicity in frequency, which yields a time-domain representation for xr(t) to repeat itself. In addition, aliasing effects can reinforce this periodic behavior by folding higher frequencies back into the audible range. Listening to the audio actually confirms these differences, as xr(t) exhibits a cyclic tonal pattern, while xa(t) maintains a non-repeating, ever-increasing pitch.

Q5:

The signals yr(t) and y2[n] are related because yr(t) is obtained by interpolating the discrete signal y2[n] to reconstruct a continuous-time approximation. Interpolation allows yr(t) to appear continuous while retaining the periodic structure of y2[n]. Both signals are periodic because the sampling process enforces periodicity. The discrete signal y2[n] is formed by sampling the original signal within a finite interval, and this periodicity is reflected in the reconstruction process. Although yr(t) is continuous and y2[n] is discrete, their periodic natures create similar repeating patterns that make them equivalent when considering their repeated sound characteristics.

Q6:

The equivalent analog system that produces yr(t) from xa(t) is not exactly the low pass filter we designed initially. However, the cut-off frequencies of this equivalent analog system, which is a low-pass filter, are the same as our initial filter. These cutoff frequencies are $\left(\frac{\pi}{8},\frac{\pi}{3}\right)\ and\ \left(-\frac{\pi}{8},-\frac{\pi}{3}\right)$. This is because of the real nature of the impulse responses of both systems. This realness forces the frequency responses of these signals to be conjugate symmetric. These cut-off frequencies can be seen in the following figure **(Figure 6)**:

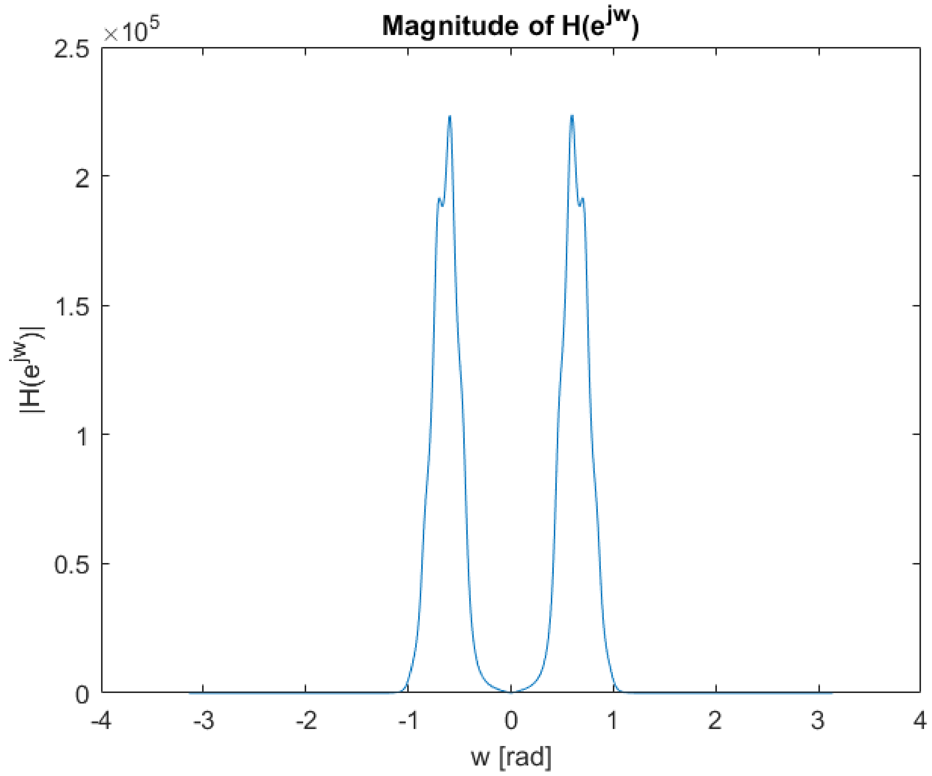

Figure 6: Magnitude Plot of H(jw)

Q7:

The filter we designed passes only some low frequency components of the incoming signal. Therefore, only a small range of frequency components gets passed through from the music. This causes the output to consist of fewer frequency components. The output is sharper than from the original music since many of the frequency components on the stopbands are filtered.

Q8:

My filtered voice was a lot sharper than my original recorded voice. This is because the sound signals at the frequencies on the stopbands are filtered. The filtered voice was even sharper than the FIR filtered voice of myself.

Conclusion:

This lab consisted of 8 parts. We were assigned with designing a practical FIR filter, testing its frequency response, and performing some filtering operations. The lab was a total success. Every specification was met, and every question was answered.

I believe the lab was very helpful in understanding how IIR filters work. I learned a lot about practical filter design and the characteristics of filters. I also learned about the differences IIR and FIR filters in out last 2 labs.

Appendix:

https://github.com/fmcetin7/Bilkent-EEE-321/blob/main/lab6/lab6.mat