EEE342 Lab 3 Report

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1. Introduction

The purpose of this lab is to understand how gain, phase and delay margins can be estimated by using mathematical models.

This lab consists of three parts. In the first part, we come up with a controller for the same DC motor we work with in the first two labs. Then, we mathematically calculate gain, phase and delay margins of the controlled DC motor system using Matlab. In the second and third parts, we apply some gain and some delay to the physical system and observe the stability of the system to see whether the margins we calculated in part 1 are actually correct.

2. Laboratory Content

2.1. Part 1

In lab 1, we came up with the following transfer function for the DC motor.

$$G_{motor}(s) = \frac{133}{s + 9.70}$$

Then, in lab 2 we introduced a pade approximation for the DC motor which added some delay to the first lab approximation. Here is the pade approximation of the DC motor.

$$G_p(s) = \frac{133}{s + 9.70} \cdot \frac{1 - 0.05s}{1 + 0.05s}$$

The lab report gives us an example controller as following:

$$G_c(s) = \frac{1}{s + \tau_{LPF}} \cdot \frac{K_c \cdot (s + 80)}{s}$$

After inputting the tau and Kc variables, we come up with the following controller:

$$G_c(s) = \frac{1}{s + 3 \cdot 9.70} \cdot \frac{\frac{2 \cdot 9.70}{133.33} \cdot (s + 80)}{s}$$

After some simplification the controller becomes:

$$G_c(s) = \frac{0.1455 \cdot s + 11.6403}{s^2 + 29.1s}$$

The final transfer function for the system with the controller becomes the following:

$$G(s) = G_c(s) \cdot G_p(s)$$
$$= \frac{-0.097 \cdot s^2 + 11.64 \cdot s + 0.0015}{0.05 \cdot s^4 + 1.194 \cdot s^3 + 40.21 \cdot s^2 + 282.27 \cdot s}$$

Now, we plot the magnitude and phase plots of the G(s) using the bode function in matlab. Here you can see the bode plots of G(s):

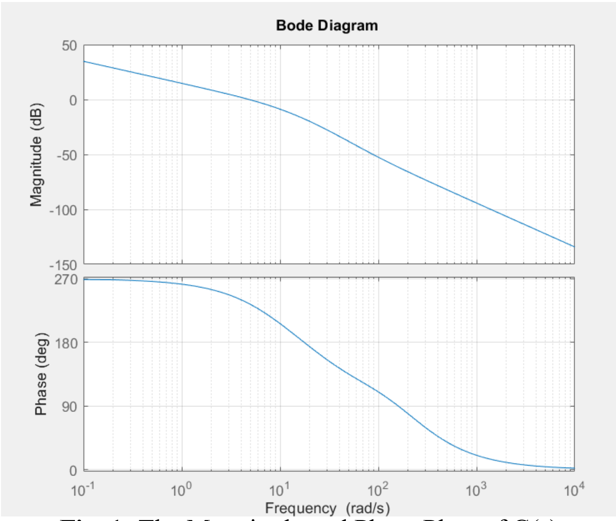

Fig. 1: The Magnitude and Phase Plots of G(s)

We can calculate gain, phase and delay margins from the graphs. For the gain margin, please observe the following graph:

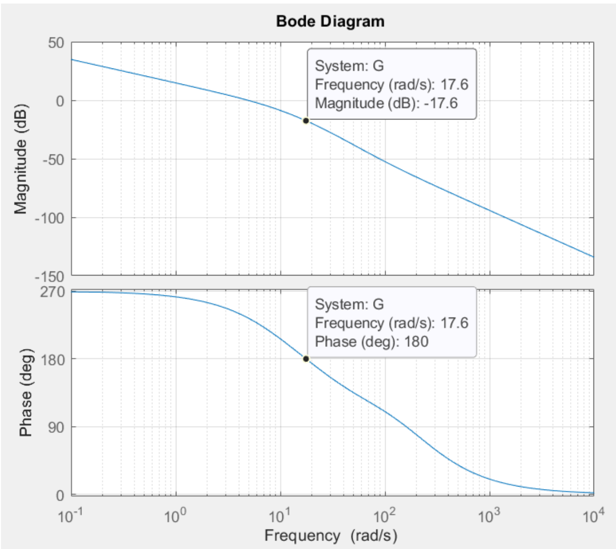

Fig. 2: Gain Margin Calculations of G(s)

Fig. 2 shows us that the frequency where phase shift is -180° is 17.6[rad/sec]. Looking at the magnitude plot, the gain at 17.6[rad/sec] is –17.6dB. Then we use the following formula to calculate the gain margin

$$17.6 = 20 \cdot \log(K) \Rightarrow GM = K = 10^{\left(\frac{17.6}{20}\right)}$$

This calculation gives us the gain margin K as **7.5832**.

For the phase margin, we observe the following graph:

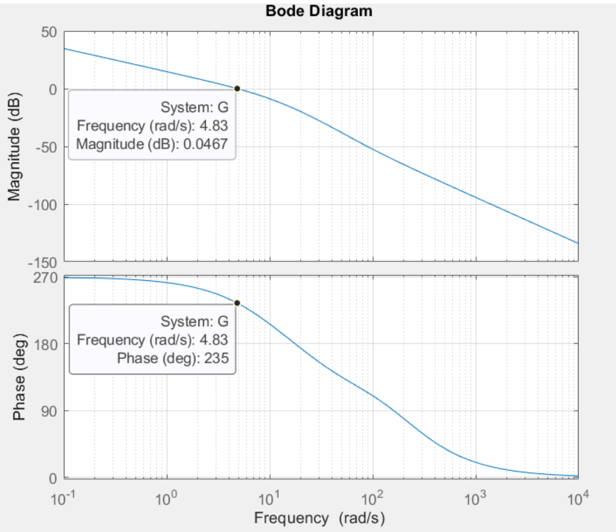

Fig. 3: Phase Margin Calculations of G(s)

Fig. 3 shows us that the frequency where magnitude is 0dB is 4.83[rad/sec]. Looking at the phase plot, the phase at 4.83[rad/sec] is 235°. Then we use the following formula to calculate the phase margin.

$$PM = 235° - 180° = 55°$$

This calculation gives us the phase margin as 55°.

Then, we calculate the delay margin as following:

$$DM = \frac{PM\,[rad]}{\omega_{PM}};\ \omega_{PM} = 4.83\left[\frac{rad}{sec}\right]$$

$$PM[rad] = \frac{PM[\deg\ ]\cdot\pi}{180} = \frac{55\cdot\pi}{180} = 0.9599$$

$$DM = \frac{0.9599}{4.83} = 0.1987$$

This calculation gives us the delay margin as 0.1987 sec.

Since we have all the margins at hand, we can verify our calculations using matlab's built-in "allmargin" command.

```
  GainMargin: 7.5832
 GMFrequency: 17.6163
 PhaseMargin: 54.6116
 PMFrequency: 4.8580
 DelayMargin: 0.1962
 DMFrequency: 4.8580
      Stable: 1
```

Fig. 4: Allmargin of G(s)

The difference between the margins that we calculated, and Matlab calculated are pretty consistent and within a 1% error limit.

2.2. Part 2

In the first part of the lab, we calculated the gain margin of the system to be K=7.58.

In this part we manually apply a gain near the calculated gain margin to observe whether the system becomes unstable after the calculated gain margin. The following three figures have three values of K, and they involve the highest value of K such that the system is stable, the value of K such that the system is marginally stable and finally the smallest value of K such that the system is unstable.

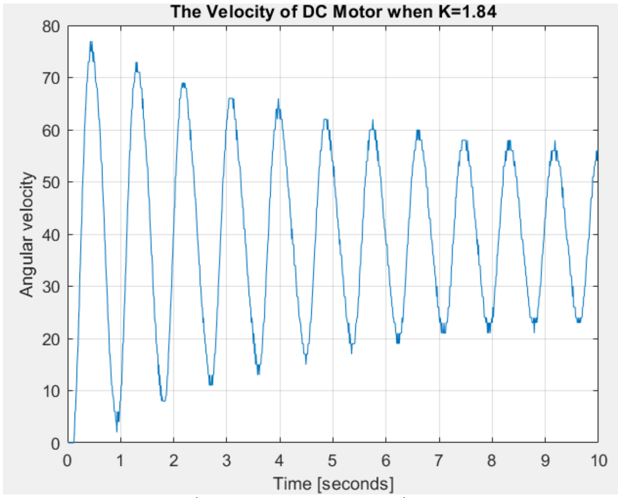

Fig. 5: The system output when K=1.84

In Fig. 5, we observe that the system is stabilizing the oscillations as time goes on. K=1.84 is the highest value of K such that the system is stable.

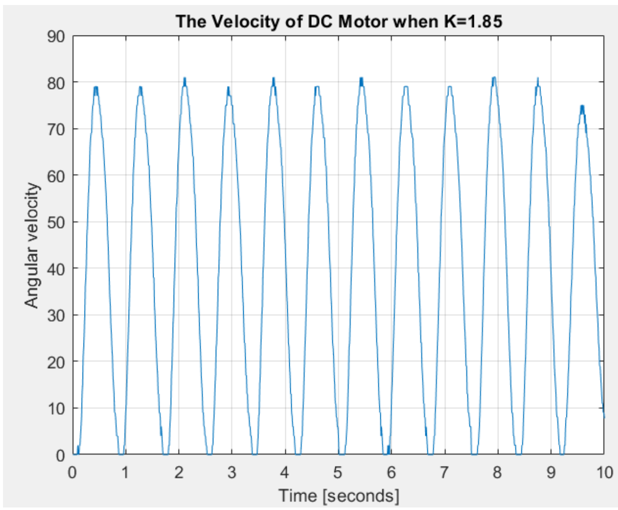

Fig. 6: The system output when K=1.85

In Fig. 6, we observe that the system is not stabilizing the oscillations as time goes on. K=1.85 is the value of K such that the system is marginally stable.

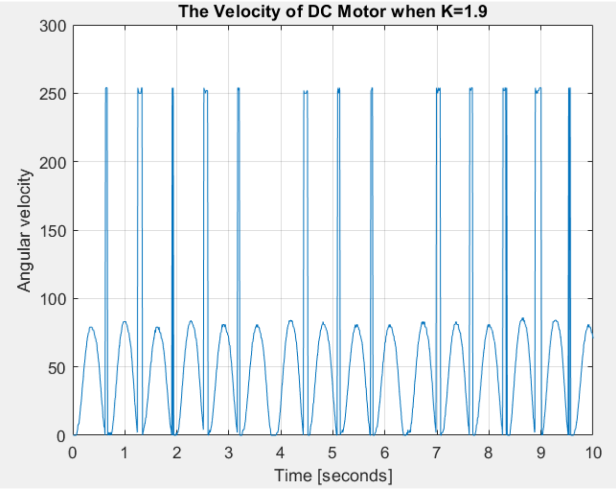

Fig. 7: The system output when K=1.9

In Fig. 7, we observe that the system is not stabilizing, and the oscillations grow bigger as time goes on. K=1.9 is the lowest value of K such that the system is unstable.

In our manual observations, we found the gain margin to be K = 1.85. There is a pretty radical difference in terms of the calculated GM and manually observed GM. The error in between can be written as following:

$$e_{GM} = \frac{7.58 - 1.85}{7.58} = \%75.59$$

This error can be caused by many things. We assume the model for the system is non-linear. However, the real system can be highly non-linear because of the saturation of amplifiers, friction in gears etc. We might have wrongly modeled some high-frequency approximations for the system.

2.3. Part 3

In the first part of the lab, we calculated the gain margin of the system to be DM=0.1962 seconds.

In this part we manually apply a delay near the calculated delay margin to observe whether the system becomes unstable after the calculated delay margin. The following three figures have three values of DM, and they involve the highest value of DM such that the system is stable, the value of DM such that the system is marginally stable and finally the smallest value of DM such that the system is unstable.

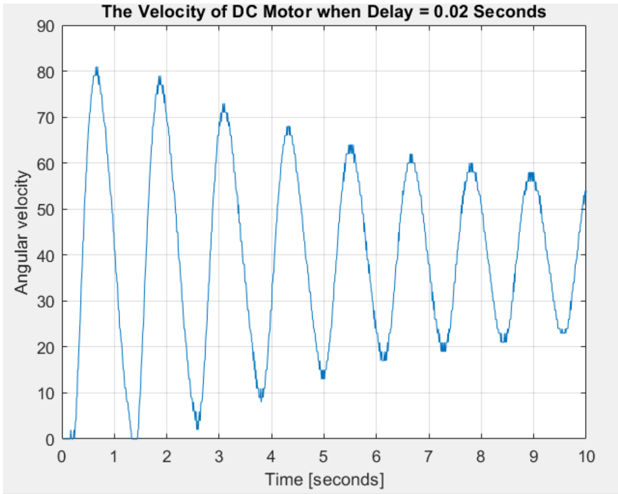

Fig. 8: The system output when delay is 0.02 seconds

In Fig. 8, we observe that the system is stabilizing the oscillations as time goes on. Delay = 0.02 seconds is the highest value of the delay such that the system is stable.

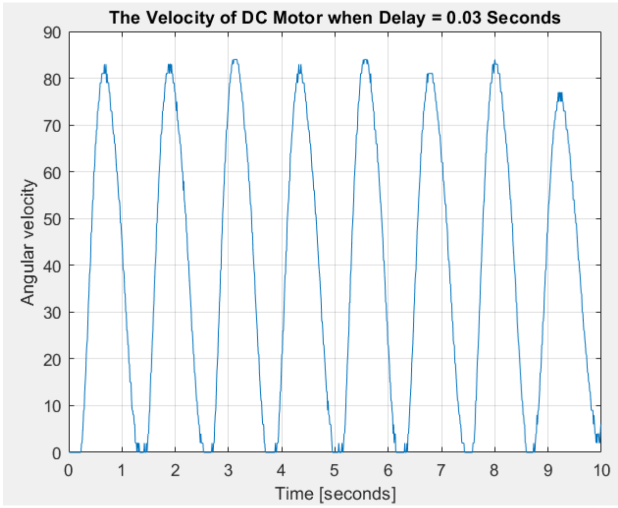

Fig. 9: The system output when delay is 0.024 seconds

In Fig. 9, we observe that the system is not stabilizing the oscillations as time goes on. Delay = 0.24 seconds is the value of the delay such that the system is marginally stable.

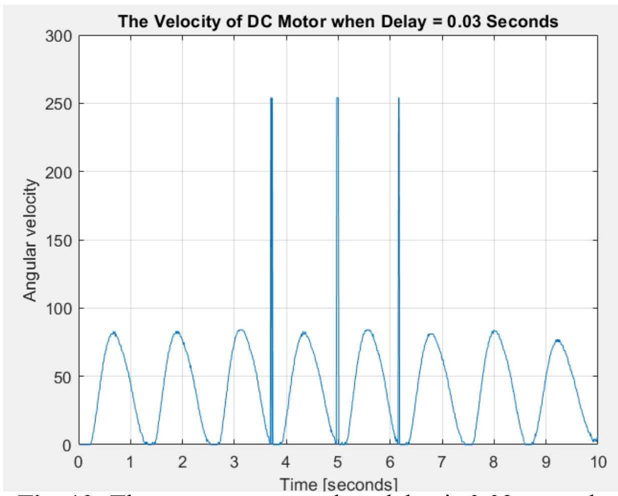

Fig. 10: The system output when delay is 0.03 seconds

In Fig. 10, we observe that the system is not stabilizing, and the oscillations grow bigger as time goes on. Delay = 0.024 seconds is the lowest value of the delay such that the system is unstable.

In our manual observations, we found the delay margin to be 0.024 seconds. There is a pretty radical difference in terms of the calculated DM and manually observed DM. The error in between can be written as following:

$$e_{DM} = \frac{0.19 - 0.024}{0.19} = \%87.36$$

This error can be caused by many things. The delay margin is particularly sensitive because it assumes a pure time delay applied to the plant or controller. In practice, a small delay can lead to a much larger phase lag at different frequencies. Therefore, a small delay at high frequencies can destabilize the system much earlier than the theoretical delay margin suggests.

The system may operate around varying conditions. Load, temperature, supply voltage... All affect how the system and the margins. Also, in our digital control system, additional delays may arise from sampling periods, computation time and communication delays. These may accumulate and make the effective delay greater than what we manually introduce or model.

3. Conclusion

The purpose of this lab was to understand how gain, phase and delay margins could be estimated by using mathematical models.

This lab consisted of three parts. In the first part, we came up with a controller for the same DC motor we worked with in the first two labs. Then, we mathematically calculated gain, phase and delay margins of the controlled DC motor system using Matlab. In the second and third parts, we applied some gain and some delay to the physical system and observed the stability of the system to see whether the margins we had calculated in part 1 were actually correct.

We observed radical differences between the calculated and manually observed gain and delay margins. There could be many reasons for these errors. The non-linearity in the real-life model, the poorly modeled high frequency effects in the system, the surrounding conditions –temperature, humidity, load, supply voltage, sampling periods and communication delays in digital control systems might all cause these radical error margins.

4. Appendices

Matlab Code:

```
%% PART 1:
Kg = 133.33/9.7;
tau_p = 1/9.7;
K_c = 2/Kg;
tau_lpf = 3/tau_p;
A = tf(1,[1 tau_lpf]);
B = tf([K_c K_c*80],[1 0]);
G_c= A*B;
G_p = tf(133.33,[1 9.7]);
G_delay = tf([-0.005 1],[0.005 1]);
G_pdelay = G_p * G_delay;
G = G_c*G_pdelay;
G2=G_c*G_p;
% bode(G);
% grid on;
% allmargin(G);
%%PART 2
[numgc, dengc] = tfdata(G_c);
t=1:1001;
t=t/100;
% vel = nan(1,1001);
% for i = 1:1001
% vel(i) = velocity.data(:,:,i);
% end
vel1=vel;
vel5=vel;
vel3=vel;
vel2=vel;
vel1_3=vel;
vel1_6=vel;
vel1_8=vel;
vel1_9=vel; %unstable K=1.9
vel1_82=vel;
vel1_83=vel;
vel1_84=vel;%stable K=1.84
vel1_85=vel; %marginally stable K=1.85
% plot(t,vel1_84);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
%  title('The  Velocity  of  DC  Motor  when
K=1.84');
% plot(t,vel1_85);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
%  title('The  Velocity  of  DC  Motor  when
K=1.85');
% plot(t,vel1_9);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
%  title('The  Velocity  of  DC  Motor  when
K=1.9');
%%PART 3
% vel = nan(1,1001);
% for i = 1:1001
% vel(i) = velocity.data(:,:,i);
```

```
% end
vel05=vel;
vel04=vel;
vel03=vel; %unstable delay = 0.03 sec
vel02=vel; %stable delay=0.02 sec
vel024=vel;  %marginally  stable  delay  =
0.024 sec
% plot(t,vel02);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
% title('The Velocity of DC Motor when De-
lay = 0.02 Seconds');
% plot(t,vel024);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
% title('The Velocity of DC Motor when De-
lay = 0.03 Seconds');
% plot(t,vel03);
% grid on;
% xlim([0 10]);
% xlabel('Time [seconds]');
% ylabel('Angular velocity');
% title('The Velocity of DC Motor when De-
lay = 0.03 Seconds');
```