EE342 Lab-2 Report

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1. Introduction

This lab consisted of 3 main parts.The aim was to input several sinusoidals to the DC motor system, each with a different frequency.

Basically, we observed the frequency response of the system and conducted frequency domain analysis on the system.

2. Laboratory Content

2.1. Part 1

In the first lab we found the transfer function of the DC motor to be **(1)**:

$$G(s) = \frac{133.33}{s+9.70} \quad \textbf{(1)}$$

The system has a pole at $s = -9.7$. The frequency response of the system is

$$H(jw) = \frac{133.33}{s+9.70} \qquad \textbf{(2)}$$

Here are the equations that govern the magnitude and the phase of the DC motor's transfer function **(3) & (4)**.

$$|H(jw)| = \frac{133.33}{\sqrt{\omega^2+9.7^2}} \qquad \textbf{(3)}$$

$$\angle H(jw) = \tan^{-1}\left(\frac{Im\{H(jw)\}}{Re\{H(jw)\}}\right) \qquad \textbf{(4)}$$

Here are the magnitude and the phase plots of the frequency response of the DC motor. Note that the x-axis which represents frequency are plotted with logarithmic scale.

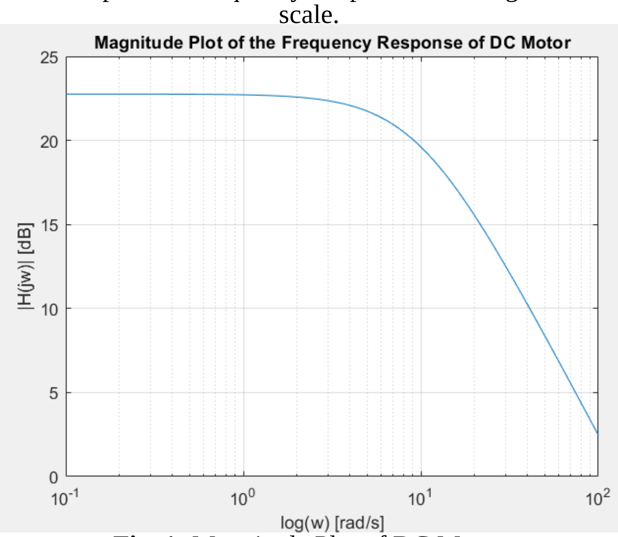

Fig. 1: Magnitude Plot of DC Motor

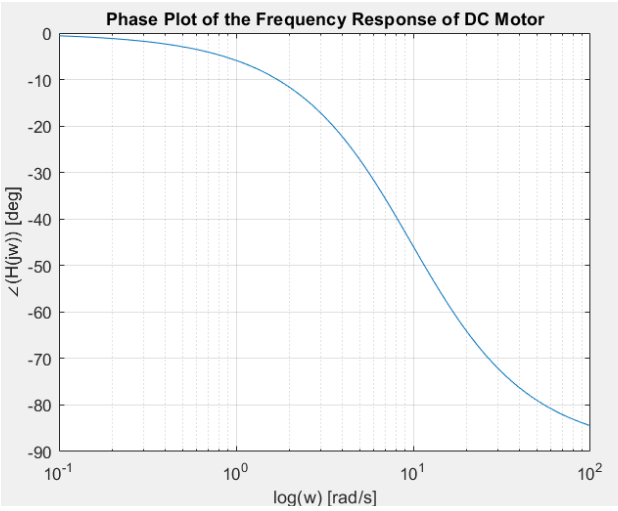

Fig. 2: Phase Plot of DC Motor

The DC motor system acts like a low-pass filter. The phase difference starts getting higher at higher frequencies. Now these theoretical assumptions will be tested in the lab environment in the following chapters of this lab work.

2.2. Part 2

In this part we will apply discrete sinusoidal signals with different frequencies to the DC motor as input. Then we will observe how the system responds to each input. Table 1 shows the applied frequencies of each input.

TABLE 1: FREQUENCY - DURATION TABLE

Angular Frequency [rad/s]	Simulation Duration (s)
0.1	70
0.3	70
1	25
3	25
10	10
30	10
100	10

The magnitude and phase responses have been found by the steps given to us in the lab manual. We used the FFT operation for each of the different input frequencies and used the indexes of the maximum values in the FFT of the signals to find the phase and the magnitude of the output signal.

Now we will input 7 different frequencies from Table 1 to the system one by one.

2.2.1. ω = 0.1 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 0.1 [rad/s].

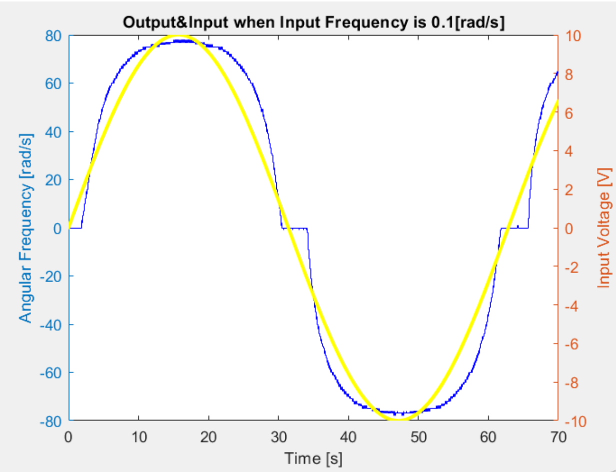

Fig. 3: Output & Input graphs when the input frequency is 0.1 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

As can be seen in the graph, the output signal is a proper low frequency signal which is amplified. There are some points where the output stood stable, the times when the input voltage is being reversed. This can be caused by two things. The friction within the mechanical system of the motor and the open/rise time for the MOSFET's and BJT's within the Arduino set.

Here are the magnitude and the phase of the response of the system to 0.1[rad/s] input frequency.

$$|H(j\,0.1)| = 18.7749$$
$$\angle H(j0.1) = -3.08°$$

2.2.2. ω = 0.3 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 0.3 [rad/s].

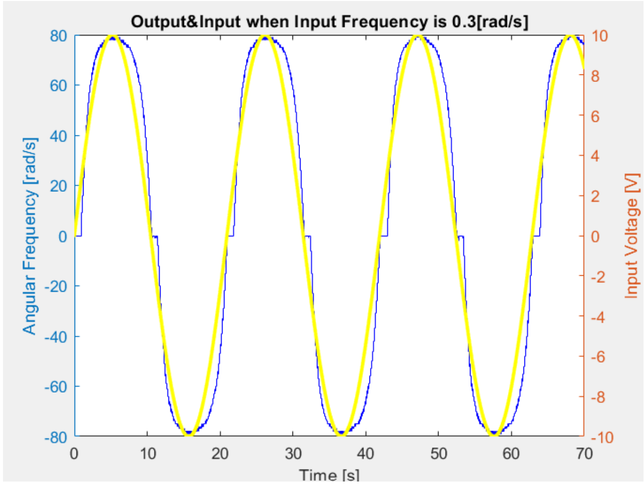

Fig. 4: Output & Input graphs when the input frequency is 0.3 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

Here are the magnitude and the phase of the response of the system to 0.3[rad/s] input frequency.

$$|H(j\,0.3)| = 18.852$$
$$\angle H(j0.3) = -7.30°$$

2.2.3. ω = 1 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 1 [rad/s].

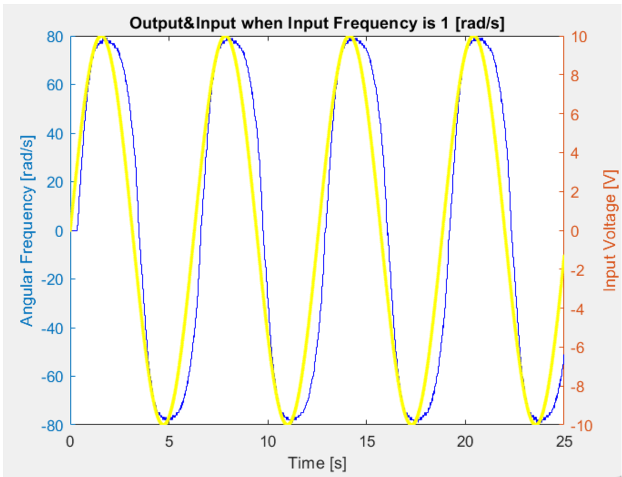

Fig. 5: Output & Input graphs when the input frequency is 1 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals. Also note that the simulation duration has gotten shorter at this step from 70 seconds to 25 seconds.
There is a slight phase shift that can be observed on the graph. This is caused by the increasing frequency and the system's delay while responding to input voltage.

Here are the magnitude and the phase of the response of the system to 1 [rad/s] input frequency.

$$|H(j\,1)| \;=\; 18.8571$$
$$\angle H(j\,1) \;=\; -\,18.49°$$

2.2.4. ω = 3 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 3 [rad/s].

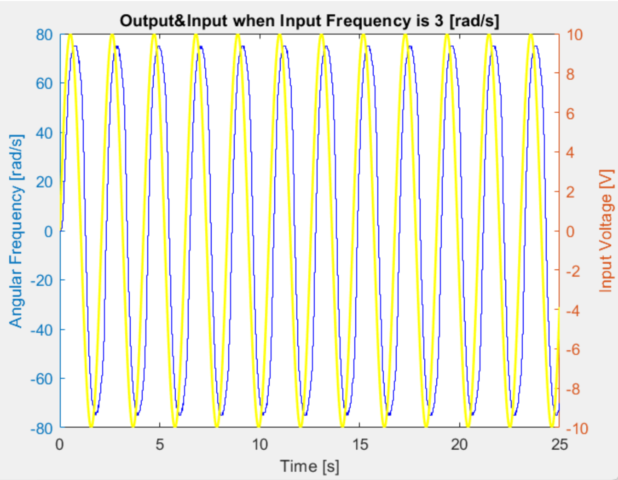

Fig. 6: Output & Input graphs when the input frequency is 3 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

Here are the magnitude and the phase of the response of the system to 3 [rad/s] input frequency.

$$|H(j\,3)| \;=\; 18.25$$
$$\angle H(j\,3) \;=\; -\,40.29°$$

2.2.5. ω = 10 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 10 [rad/s].

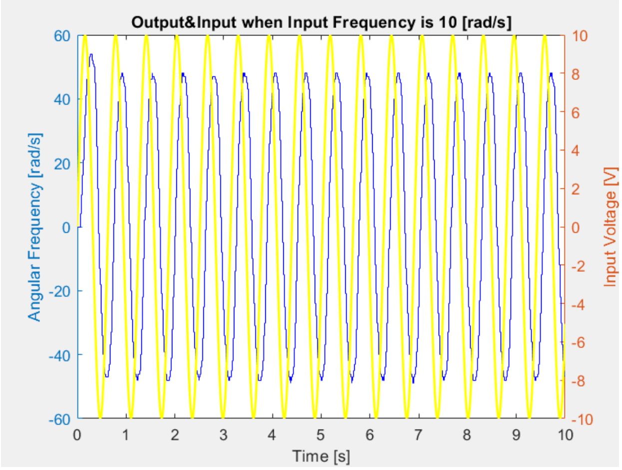

Fig. 7: Output & Input graphs when the input frequency is 10 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

There is a drop in the output of the system. DC motor rotates with a smaller angular frequency compared to lower frequency inputs' outputs. This phenomenon is expected since the DC motor system acts like an LPF.

Here are the magnitude and the phase of the response of the system to 10 [rad/s] input frequency.

$$|H(j\,10)| \;=\; 14.02$$
$$\angle H(j\,10) \;=\; -\,69.93°$$

2.2.6. ω = 30 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 30 [rad/s].

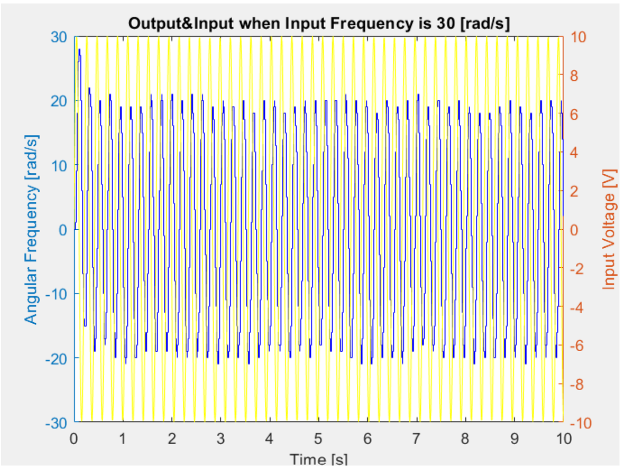

Fig. 8: Output & Input graphs when the input frequency is 30 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

Also note that the simulation duration has gotten shorter at this step from 25 seconds to 10 seconds.

Here are the magnitude and the phase of the response of the system to 30 [rad/s] input frequency.

$$|H(j\,30)| \;=\; 6.14$$

$$\angle H(j\,30) \;=\; -96.66$$

2.2.7. ω = 100 [rad/s]

Here you can see the input – output plot when the angular frequency is equal to 100 [rad/s].

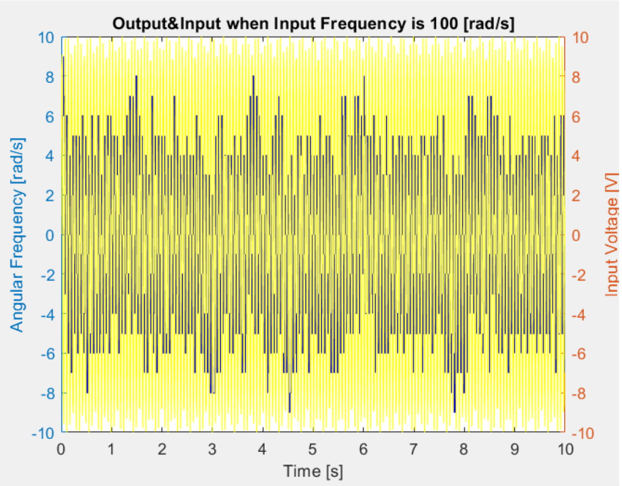

Fig. 9: Output & Input graphs when the input frequency is 100 [rad/s]

The blue signal on the graph is the output angular frequency signal. The yellow signal on the graph is the input voltage. Note that y-axes are different for two signals.

Here are the magnitude and the phase of the response of the system to 100 [rad/s] input frequency.

$$|H(j\,100)| \;=\; -5.32$$

$$\angle H(j\,100) \;=\; -147.05$$

At this point, we plotted the analytical datapoints on top of the theoretical magnitude and phase plots at Fig.1 and Fig.2. Here you can see the newly obtained graphs.

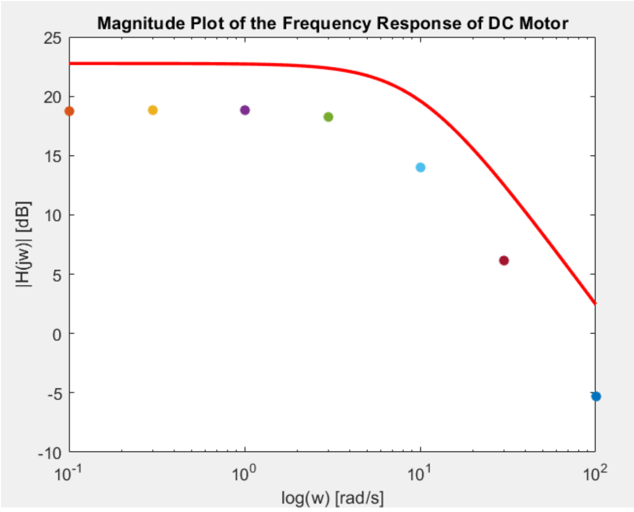

Fig. 10: Magnitude Plot of the DC Motor as well as the obtained datapoints

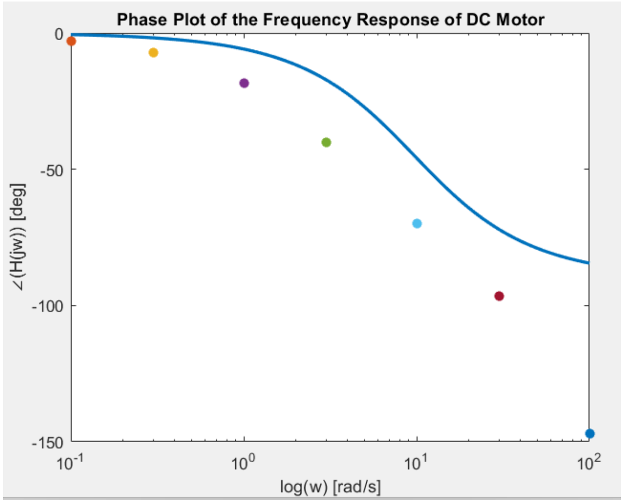

Fig. 11: Phase Plot of the DC Motor as well as the obtained datapoints

The magnitude of the frequency response aligns with the theoretical results, though it remains slightly below the expected amplification levels. This discrepancy may be attributed to physical properties or external disturbances affecting the motor system.

However, the phase response shows an unusual deviation at the highest measured frequency of 100 rad/s. Instead of following the expected trend, the phase response drops significantly, leading to a notable difference between the theoretical and experimental results.

This issue arises due to a 10ms processing delay in the system, which becomes more pronounced at higher frequencies, causing a substantial phase shift. This phenomenon will be examined further in the next section of the lab report.

2.3. Part 3

In order to approximate the 10ms system delay we are introducing a new transfer function for the system.

$$G_{delayed}(s) = G(s) \cdot \frac{1-0.005s}{1+0.005s} \quad \textbf{(5)}$$

The new approximation does not change the magnitude of the initial transfer function. But it drastically changes the phase especially at higher frequencies. Here are the revisions for Fig.10 and Fig.11; initial transfer function, newly obtained transfer function and the analytical datapoints are all plotted together.

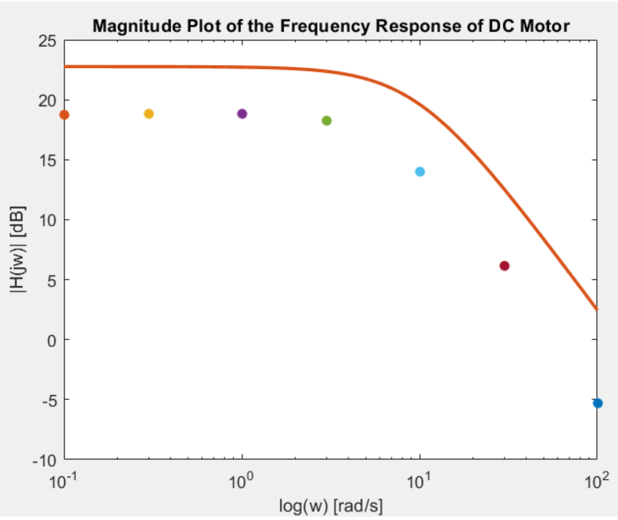

Fig. 12: Magnitude Plot of the DC Motor as well as the obtained datapoints and the new approximation for the delayed transfer function

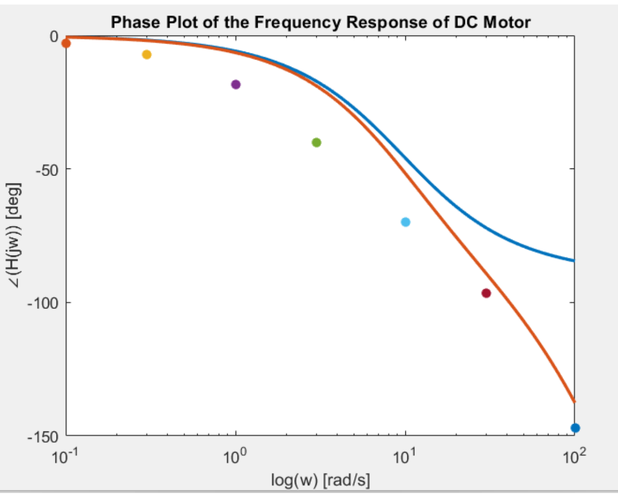

Fig. 13: Phase Plot of the DC Motor as well as the obtained datapoints and the new approximation for the delayed transfer function

After adjusting the theoretical data, graphs align with the experimental results, confirming that the difference between them was due to an incomplete understanding of the system.

3. Conclusion

This lab consisted of 3 main parts. The aim was to input several sinusoidals to the DC motor system, each with a different frequency. We observed the frequency response of the system and conducted frequency domain analysis on the system.

There were some points where the output did not change even though the input was changing, i.e. the times when input voltage was being reversed. This can be caused by two things. The friction within the mechanical system of the motor and the open/rise time for the MOSFET's and BJT's within the Arduino set.

There was a slight phase shift that can be observed on the graphs on Q2. These were caused by the increasing frequency and the system's delay to respond to the input voltage. After introducing a 10ms delay in the new trasnfer function, this problem was solved.

I think the lab was a total success. All the graphs were obtained and plotted completely. The frequency domain analysis on the DC motor was successfully done.

4. Appendices

```
w = logspace(-1,2,100);
A=133.3;
B=9.70;
G=nan(1,100);
for k = 1:100
s = 1i * w(k);
G(k) = A / (s+B);
end
% semilogx(w,20*log10(abs(G)));
% grid on
% xlabel('log(w) [rad/s]');
% ylabel('|H(jw)| [dB]');
% title('Magnitude Plot of the Frequency
Response of DC Motor');
% semilogx(w,angle(G)*180/pi)
% grid on
% xlabel('log(w) [rad/s]');
% ylabel('∠(H(jw)) [deg]');
% title('Phase Plot of the Frequency Re-
sponse of DC Motor');
angular_frequency =0.1;
duration     = 70;
t = 0:0.01:duration;
input01 =    10*sin(angular_frequency  *
t);
%vel01=out.velocity;
% yyaxis left;
% plot(vel01,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input01,'y.');
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 0.1[rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
% vel03=out.velocity;
angular_frequency =0.3;
duration     = 70;
```

```
t = 0:0.01:duration;
input03 =    10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel03,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input03,'y.');
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 0.3[rad/s]');
% ylabel('Input Voltage [V]');
% grid on;
% hold off;
% vel1=out.velocity;
angular_frequency =1;
duration     = 25;
t = 0:0.01:duration;
input1 =     10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel1,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input1,'y.');
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 1 [rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
% vel3=out.velocity;
angular_frequency =3;
duration     = 25;
t = 0:0.01:duration;
input3 =     10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel3,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input3,'y','LineWidth',1.5);
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 3 [rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
% vel10=out.velocity;
angular_frequency =10;
duration     = 10;
t = 0:0.01:duration;
input10 =    10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel10,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input10,'y','LineWidth',1.5);
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 10 [rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
% vel30=out.velocity;
angular_frequency =30;
duration     = 10;
t = 0:0.01:duration;
input30 =    10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel30,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input30,'y');
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 30 [rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
% vel100=out.velocity;
angular_frequency =100;
duration     = 10;
t = 0:0.01:duration;
input100 =   10*sin(angular_frequency  *
t);
% yyaxis left;
% plot(vel100,'b');
% ylabel('Angular Frequency [rad/s]');
% hold on;
% yyaxis right;
% plot(t,input100,'y');
% hold off;
% xlabel('Time [s]');
% title('Output&Input when Input Frequency
is 100 [rad/s]');
% ylabel('Input Voltage [V]');
% hold off;
[maxVel01, maxVelIndex01] =
max(abs(fft(vel01.data)));
[maxVel03, maxVelIndex03] =
max(abs(fft(vel03.data)));
[maxVel1, maxVelIndex1] =
max(abs(fft(vel1.data)));
[maxVel3, maxVelIndex3] =
max(abs(fft(vel3.data)));
[maxVel10, maxVelIndex10] =
max(abs(fft(vel10.data)));
[maxVel30, maxVelIndex30] =
max(abs(fft(vel30.data)));
```

```
[maxVel100, maxVelIndex100] =
max(abs(fft(vel100.data)));
[maxinp01, maxinpIndex01] =
max(abs(fft(input01)));
[maxinp03, maxinpIndex03] =
max(abs(fft(input03)));
[maxinp1, maxinpIndex1] = max(abs(fft(in-
put1)));
[maxinp3, maxinpIndex3] = max(abs(fft(in-
put3)));
[maxinp10, maxinpIndex10] =
max(abs(fft(input10)));
[maxinp30, maxinpIndex30] =
max(abs(fft(input30)));
[maxinp100, maxinpIndex100] =
max(abs(fft(input100)));
k01= maxVel01/maxinp01;
k03= maxVel03/maxinp03;
k1= maxVel1/maxinp1;
k3= maxVel3/maxinp3;
k10= maxVel10/maxinp10;
k30= maxVel30/maxinp30;
k100= maxVel100/maxinp100;
angvel01 = angle(fft(vel01.data));
angvel03 = angle(fft(vel03.data));
angvel1 = angle(fft(vel1.data));
angvel3 = angle(fft(vel3.data));
angvel10 = angle(fft(vel10.data));
angvel30 = angle(fft(vel30.data));
angvel100 = angle(fft(vel100.data));
anginp01 = angle(fft(input01));
anginp03 = angle(fft(input03));
anginp1 = angle(fft(input1));
anginp3 = angle(fft(input3));
anginp10 = angle(fft(input10));
anginp30 = angle(fft(input30));
anginp100 = angle(fft(input100));
ang01 = angvel01(maxVelIndex01) - an-
ginp01(maxinpIndex01);
ang03 = angvel03(maxVelIndex03) - an-
ginp03(maxinpIndex03);
ang1 = angvel1(maxVelIndex1) - an-
ginp1(maxinpIndex1);
ang3 = angvel3(maxVelIndex3) - an-
ginp3(maxinpIndex3);
ang10 = angvel10(maxVelIndex10) - an-
ginp10(maxinpIndex10);
ang30 = (angvel30(maxVelIndex30) - an-
ginp30(maxinpIndex30));
ang100 = (angvel100(maxVelIndex100) - an-
ginp100(maxinpIndex100));
semilogx(w,20*log10(abs(G)));
hold on;
plot(0.1,20*log10(k01),'*');
plot(0.3,20*log10(k03),'*');
plot(1,20*log10(k1),'*');
plot(3,20*log10(k3),'*');
plot(10,20*log10(k10),'*');
plot(30,20*log10(k30),'*');
plot(100,20*log10(k100),'*');
hold off;
xlabel('');
semilogx(w,angle(G)*180/pi);
hold on;
plot(0.1,ang01*180/pi,'*');
plot(0.3,ang03*180/pi,'*');
plot(1,ang1*180/pi,'*');
plot(3,ang3*180/pi,'*');
plot(10,ang10*180/pi,'*');
plot(30,ang30*180/pi-360,'*');
plot(100,ang100*180/pi-360,'*');
hold off;
A=133.3;
B=9.70;
w = logspace(-1,2,100);
G2=nan(1,100);
for k = 1:100
s = 1i * w(k);
G2(k) = A * (1-0.005*s) / (s+B) /
(1+0.005*s);
end
% semilogx(w,20*log10(abs(G)),'r','Lin-
eWidth',2);
% hold on;
% plot(0.1,20*log10(k01),'*','Lin-
eWidth',2);
% plot(0.3,20*log10(k03),'*','Lin-
eWidth',2);
% plot(1,20*log10(k1),'*','LineWidth',2);
% plot(3,20*log10(k3),'*','LineWidth',2);
% plot(10,20*log10(k10),'*','Lin-
eWidth',2);
% plot(30,20*log10(k30),'*','Lin-
eWidth',2);
% plot(100,20*log10(k100),'*','Lin-
eWidth',2);
% semilogx(w,20*log10(abs(G2)),'Lin-
eWidth',2);
% hold off;
% xlabel('log(w) [rad/s]');
% ylabel('|H(jw)| [dB]');
% title('Magnitude Plot of the Frequency
Response of DC Motor');
%
% semilogx(w,angle(G)*180/pi,'Lin-
eWidth',2);
% hold on;
% plot(0.1,ang01*180/pi,'*','Lin-
eWidth',2);
% plot(0.3,ang03*180/pi,'*','Lin-
eWidth',2);
% plot(1,ang1*180/pi,'*','LineWidth',2);
% plot(3,ang3*180/pi,'*','LineWidth',2);
% plot(10,ang10*180/pi,'*','LineWidth',2);
% plot(30,ang30*180/pi-360,'*','Lin-
eWidth',2);
% plot(100,ang100*180/pi-360,'*','Lin-
eWidth',2);
```

```
% semilogx(w,angle(G2)*180/pi,'Lin-
eWidth',2);
% hold off;
% xlabel('log(w) [rad/s]');
% ylabel('∠(H(jw)) [deg]');
% title('Phase Plot of the Frequency Re-
sponse of DC Motor');
```