Explain the use of the arcsine transformation. How and why does it work?

Cohen's h measures the distance between two proportions. It can be used to determine if the difference in proportions is small, medium, or large. It can also determine if the proportion difference is meaningful, and calculate study sample size.

Transforms $P_1 - P_2$ to $\phi_1 - \phi_2 = h$

$\phi = 2 \arcsin\sqrt{p}$

$h = \phi_1 - \phi_2$

It is able to stabalize the variance, as it becomes approximatly constant after the transformation.

Use MoM and/or MLE to find estimators

Exponential $\exp(\lambda)$

MLE: $L(\lambda; x_1, \ldots, x_n) = \prod_{i=1}^{n} f_X(x_i; \lambda)$

$= \prod_{i=1}^{n} \lambda \exp(-\lambda x_i)$

$= \lambda^n \exp(-\lambda \sum_{i=1}^{n} x_i)$

$\xrightarrow{\text{log}}$ likelihood $\ell(\lambda; x_1, \ldots, x_n) = \ln(L(\lambda; x_1, \ldots, x_n))$

$= \ln(\lambda^n \exp(-\lambda \sum_{i=1}^{n} x_i))$

$= \ln(\lambda^n) + \ln(\exp(-\lambda \sum_{i=1}^{n} x_i))$

$= n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i$

$\xrightarrow{\text{optimization}}$ $\hat{\lambda} = \arg\max \ell(\lambda; x_1, \ldots, x_n)$

$\frac{d}{d\lambda} \ell(\lambda; x_1, \ldots, x_n) = \frac{d}{d\lambda}(n \ln(\lambda) - \lambda \sum_{i=1}^{n} x_i)$

$= \frac{n}{\lambda} - \sum_{i=1}^{n} x_i = 0 \rightarrow$

$\lambda = \frac{n}{\sum_{i=1}^{n} x_i}$

$$f(x) = \begin{cases} (1-\theta)+2\theta x & \text{for } 0<x<1 \\ 0 & \text{else} \end{cases}$$

Using MoM for θ

$$E(x) = \int_0^1 x[(1-\theta)+2\theta x]\,dx = \frac{\theta+3}{6} \text{ (wolfram)}$$

$$\bar{X} = \frac{\theta+3}{6} \rightarrow \theta = 6\bar{X}-3$$

Using MLE for θ

$$L(\theta) = \prod_{i=1}^{n} [(1-\theta)+2\theta X]$$

$$\xrightarrow{\text{log likelihood}} \ln(L(\theta)) = \sum_{i=1}^{n} \ln(1-\theta+2\theta x)$$

$$\frac{d\ln(L(\theta))}{d\theta} = \sum_{i=1}^{n} \frac{-1+2x_i}{1-\theta+2\theta x_i}$$

$$= \sum_{i=1}^{n} \frac{-1+2x_i}{1-\theta(-1+2x_i)}$$

$$\text{maxamize} \rightarrow = \sum_{i=1}^{n} \frac{1}{\frac{1}{-1+2x_i}-\theta} = 0$$

no closed form solution