

Inference and hypothesis testing

EDS 222

Tamma Carleton

Fall 2021

Announcements/check-in

- Assignments to slow down to give space for final project (details on the next slide)
 - Assignment 04 will cover the next two weeks of material

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- Increasing divergence between course material and IMS textbook (including today's lecture)

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 - Assignment 04 will cover the next two weeks of material
- Increasing divergence between course material and IMS textbook (including today's lecture)
- Some additional nuances on interactions and multiple linear regression at end of week 05 slide deck, for your reference (e.g., multicollinearity)

Final project

Goal:

Apply **some of** the statistical concepts you have learned in this course to **answer an environmental data science question**.^{*}

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Two parts:

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2. 3-5 minute in-class presentation

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Apply **some of** the statistical concepts you have learned in this course to **answer an environmental data science question**.^{*}

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Please run your idea by me via email, Slack, or in person before committing so that I can help assess whether it is feasible and appropriate!

[*]: Your project *must* include concepts from the second half of the course.

Final project

Some example topics:

- Are political views on climate change associated with recent natural disaster exposure?

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- Are political views on climate change associated with recent natural disaster exposure?
- Detecting trends in air quality for disadvantaged groups across California
- Spatial patterns of deforestation during COVID-19

--

- Are there gendered health effects of wildfire smoke?

Today

Thinking about uncertainty

Sampling distributions

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Hypothesis testing: conceptual foundations

Null hypotheses, alternative hypotheses, rejecting the null

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Hypothesis testing: in practice

The Central Limit Theorem, standard errors, Z-scores, p-values

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Confidence

Confidence intervals

Thinking about uncertainty

Why does uncertainty matter?

All our sample statistics (e.g., sample means, regression parameters) are uncertain

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- We have a *randomly drawn sample* and are trying to learn about the population from our sample
- But our sample statistics would have been different had we randomly drawn a different set of observations!
- This is **natural variability** and it means that all our sample statistics are uncertain estimates of population parameters, even if they are unbiased (e.g., no convenience sampling, no systematic non-response, etc.)

Why does uncertainty matter?

Key question: Is our estimate indicating anything more than sampling variability or "noise"?

- This is the question **statistical inference** and **hypothesis testing** are trying to answer

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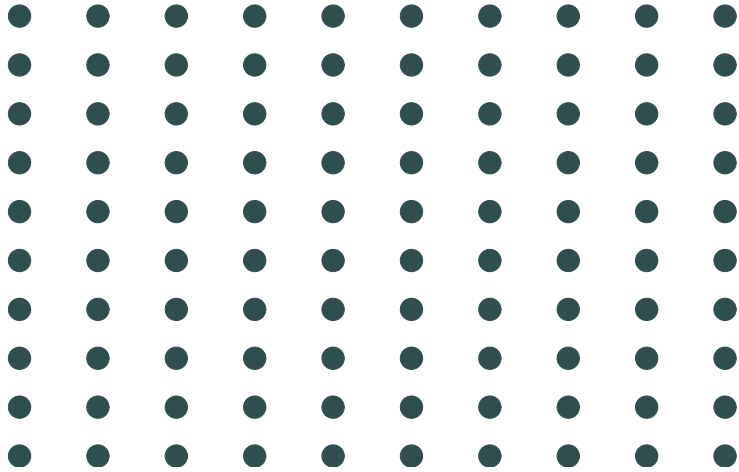
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- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few high-earning women in this group?

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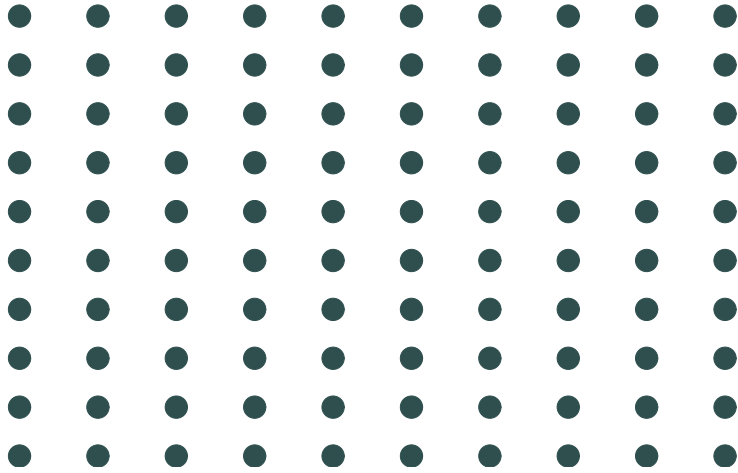
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- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few high-earning women in this group?
- If we collected another independent sample of 100, would the gap be the same?

Population vs. sample

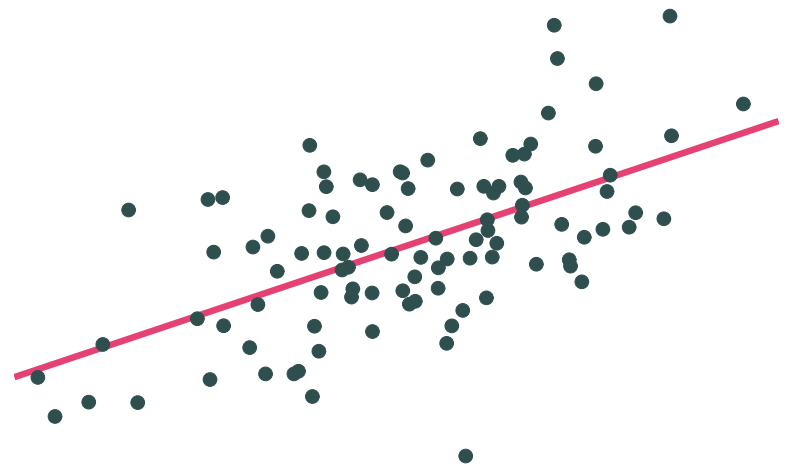


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Population vs. sample



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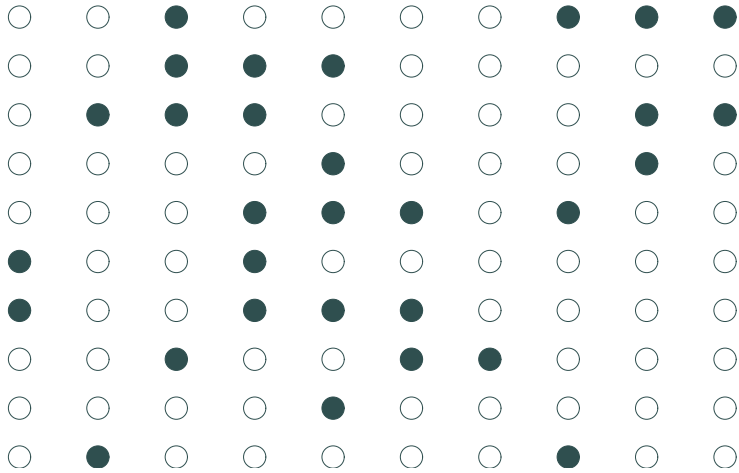


Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

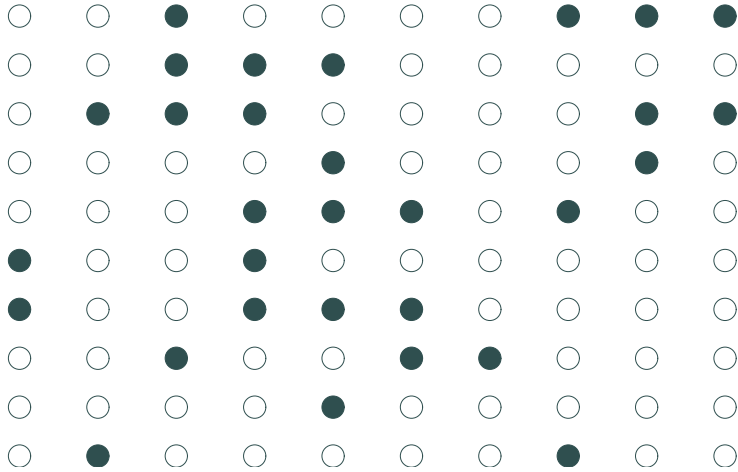
$$y_i = \beta_0 + \beta_1 x_i + u_i$$

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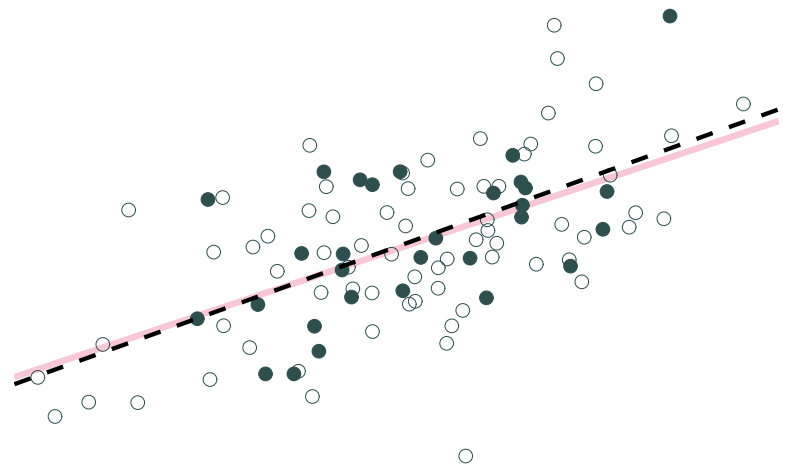


Sample 1: 30 random individuals

Population vs. sample



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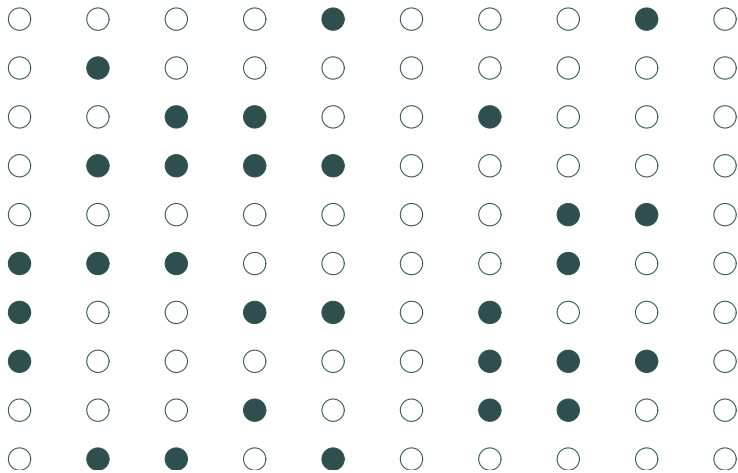
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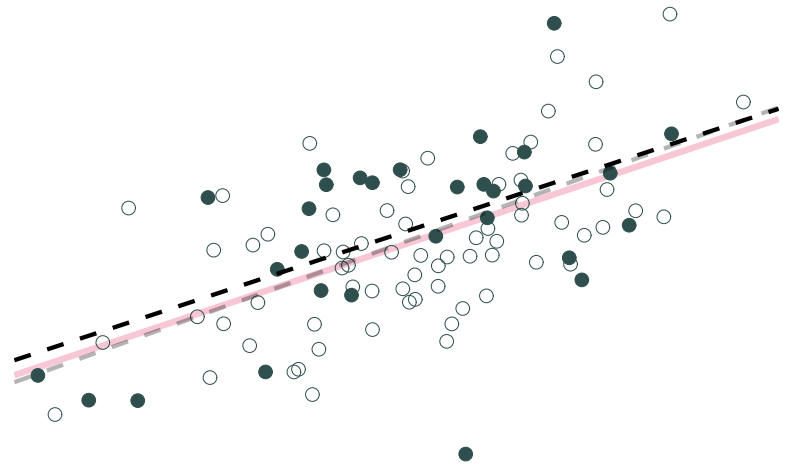
$$\hat{y}_i = 2.36 + 0.61x_i$$

Population vs. sample

count: false



Sample 2: 30 random individuals



Population relationship

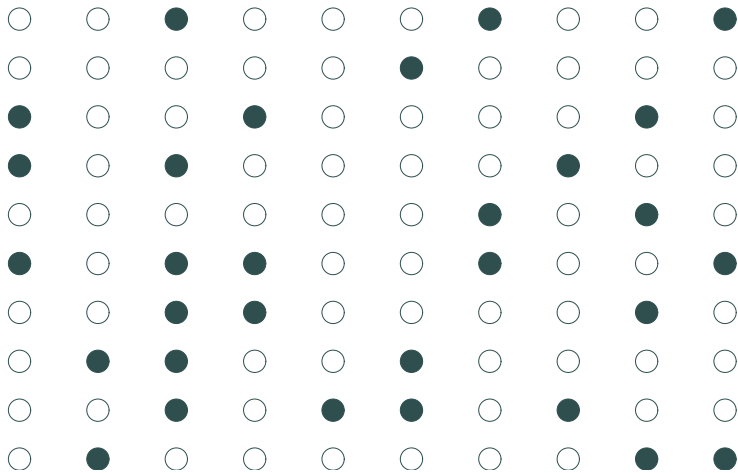
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Sample relationship

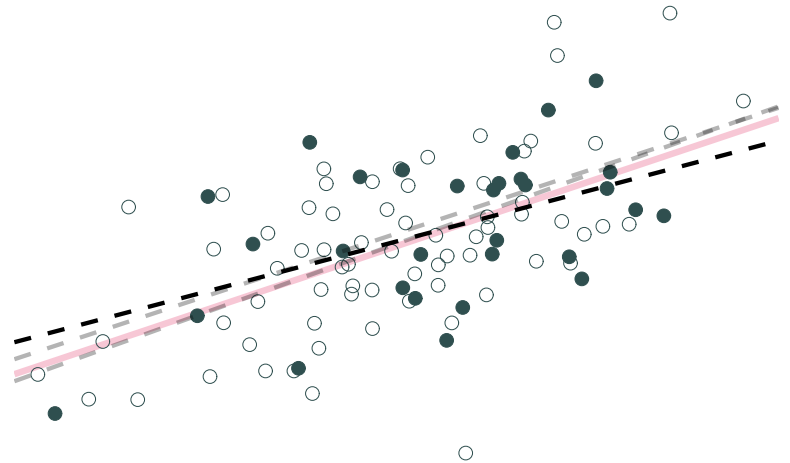
$$\hat{y}_i = 2.79 + 0.56x_i$$

Population vs. sample

count: false



Sample 3: 30 random individuals



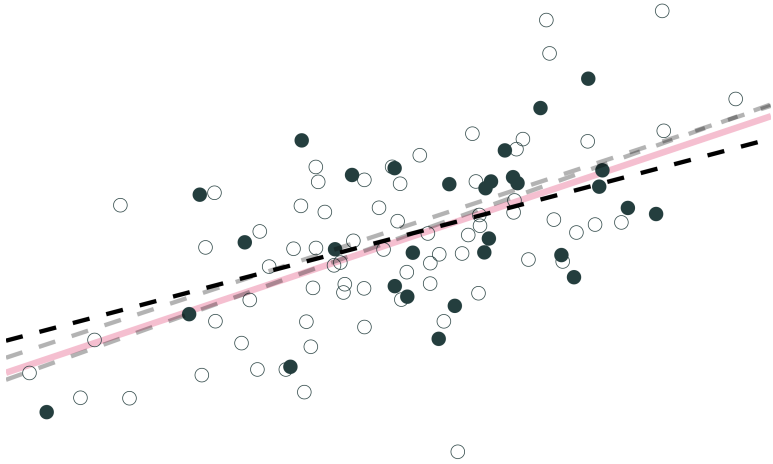
Population relationship

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Sample relationship

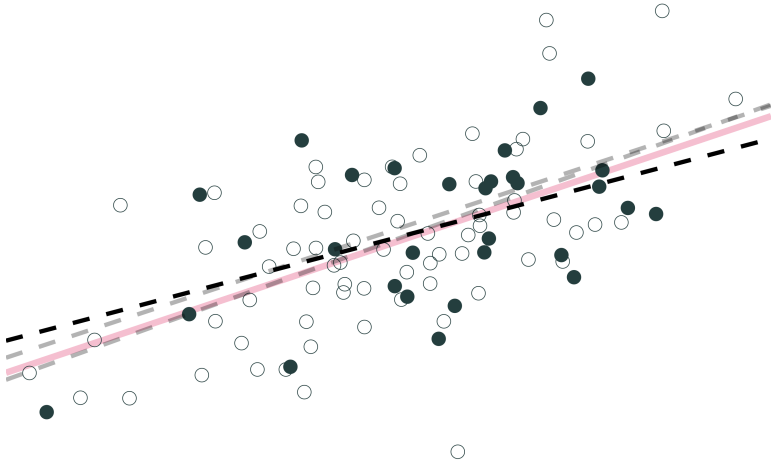
$$\hat{y}_i = 3.21 + 0.45x_i$$

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- On **average**, our regression lines match the population line very nicely.
- However, **individual lines** (samples) can really miss the mark.
- Differences between individual samples and the population lead to **uncertainty** for the statistician

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Keeping track of uncertainty allows us to test hypotheses about the population using just our sample

Hypothesis testing: conceptual foundations

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- H_A : **Alternative hypothesis** A hypothesis that the measured quantity is nonzero. In other words: whatever I recovered in my sample is due to true population differences or effects.

Hypothesis testing: Example

Example: Are runners getting slower or faster over time?

Random sample of 100 runners from the 2017 Cherry Blossom 10-mile race.

- Mean 2006 finish time for all runners: 93.29 minutes
- Mean 2017 finish time: 98.78 minutes
- Standard deviation 2017 finish time: 16.59 minutes
- H_0 : The average time was the same in the two years. That is, $\mu = 93.29$
- H_A : The average time was not the same in the two years. That is, $\mu \neq 93.29$

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Here, the difference in mean finish time between 2017 sample and the 2006 census of runners is:

- $98.78 - 93.29 = 5.49$

Hypothesis test asks if this point estimate is actually different from zero once we account for sampling variability

Hypothesis testing: Rejecting the null

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Rejecting the null involves both a **point estimate** and a measure of **uncertainty** or spread in your data

Hypothesis testing: in practice

Hypothesis testing in five steps

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5. Based on #4, either **reject** or **fail to reject** the null

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We already know all about these two steps.

- Null and alternative hypotheses will depend entirely on the statistical question of interest.
- Data collection and point estimates (e.g., means, regression parameters, variances, etc.) we have studied at length in this class

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Let's turn to some definitions.

Sampling distribution

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A **sampling distribution** is the distribution of all possible values of a sample statistic from samples of a given size from a given population.

- The sampling distribution describes how sample statistics (e.g., mean, regression parameters) vary from one sample (or study) to the next
- This is *not* the same as the **data distribution**!
 - Distribution of your data = distribution within one sample (e.g., gives you *one* sample mean)
 - Sampling distribution = distribution across samples (e.g., gives you *many* sample means)

Sampling distribution

For example, recall our regression above, where the population model is:

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For example, recall our regression above, where the population model is:

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- A regression using one sample gives us *one* set of coefficients, called the **point estimates**: $\hat{\beta}_0 = 2.36$ and $\hat{\beta}_1 = 0.61$
- If we could collect 1000 samples and run that regression 1,000 times, we would recover the **sampling distribution** for each coefficient

Sampling distribution

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But how do we obtain one of these?

You only have one sample of data! Where does the sampling distribution come from?

--

We derive the sampling distribution from applying the Central Limit Theorem

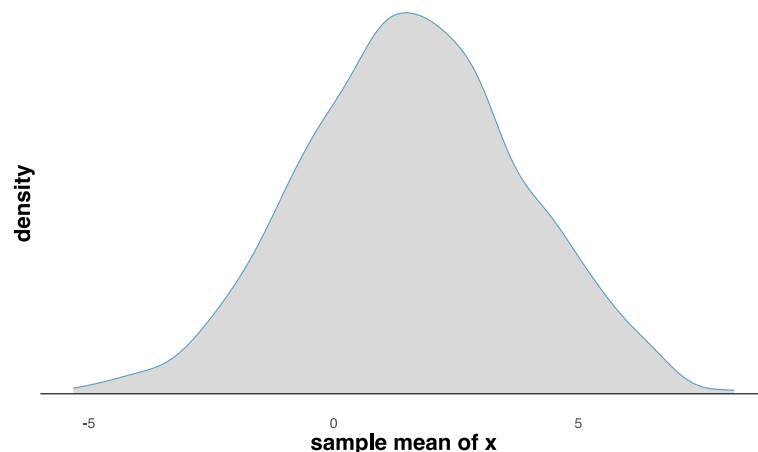
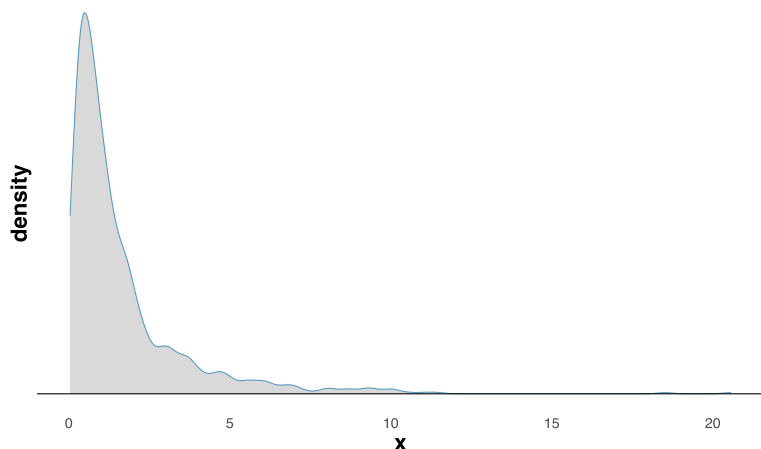
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This is true *even if* the underlying data are not normally distributed!



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Question: How big does our sample need to be?

Answer: Rule of thumb is $n \geq 30$

But this is not a hard and fast rule! Be cautious about hypothesis testing and inference with small sample sizes.

Standard errors

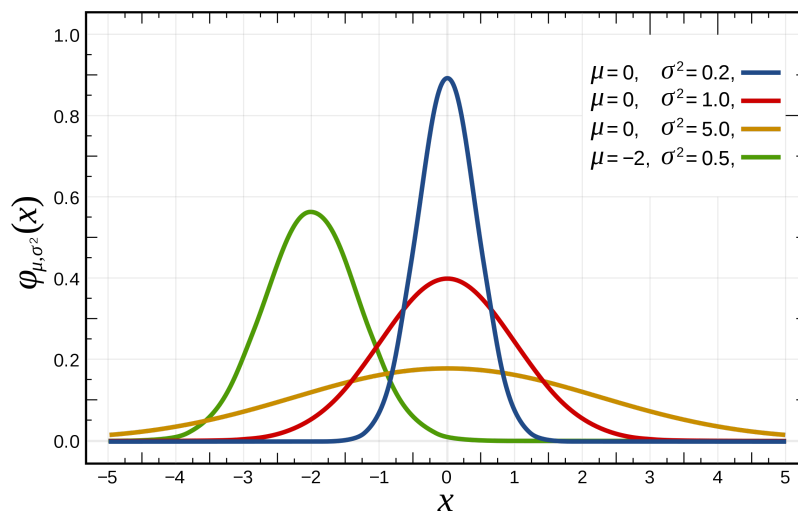
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but there are so many normal distributions!

We need to know the μ and σ of our sampling distribution in order to fully model the variability of our statistic.



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For example:

- H_0 : 10-mile finish times in 2017 have the same mean as 2006. $\mu = 93.29$
- H_0 : Male and female wages have a mean *difference* of zero. $\mu = 0$.
- H_0 : There is *no effect* of neonicotinoid use on colony collapse disorder.
 $\mu = \beta_1 = 0$.

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For example, if we are interested in a sample mean, our friend the **Central Limit Theorem** tells us that:

$$SE = \frac{s^2}{\sqrt{n}}$$

Q: What happens to the standard error as sample size increases?
Why?

Standard error for regression slope

The standard deviation of your sampling distribution is called the **standard error**

How you calculate the standard error depends on the research question.

For example, if we are interested in a regression slope, the CLT plus some algebra tell us that:

$$SE = \sqrt{\text{var}(\hat{\beta}_1)} = \sqrt{\frac{s^2}{\sum_i (x_i - \bar{x})^2}}$$

Q: What happens to the standard error as sample size increases?
Why?

SE for comparing two means

The standard deviation of your sampling distribution is called the **standard error**

How you calculate the standard error depends on the research question.

For example, if we are interested in the difference between two means, the CLT plus some algebra tell us that:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

In all these cases, the SE is the standard deviation of the sampling distribution!

Summary: Standard errors

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If we could collect many samples from the population, and we computed our statistic for each sample (e.g., mean, slope coefficient), we could construct the **sampling distribution**.

The **standard error** is our estimate of the the standard deviation of the sampling distribution. We can never actually collect hundreds of independent samples, so we use our single sample to approximate the true sampling distribution standard deviation, leveraging the **Central Limit Theorem**

Standard error measures how dispersed our sample statistic is around the population parameter of interest (highly dispersed = large SE = a lot of uncertainty about the population parameter from our one sample)

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Hypothesis testing in five steps

Step 4: quantify the probability that your sample statistic differs from the null by the observed amount, if the null were true

- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .

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Step 4: quantify the probability that your sample statistic differs from the null by the observed amount, if the null were true

- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .
- This should tell me something about **how unlikely it was** that I happened to draw my point estimate if the null were true, right?

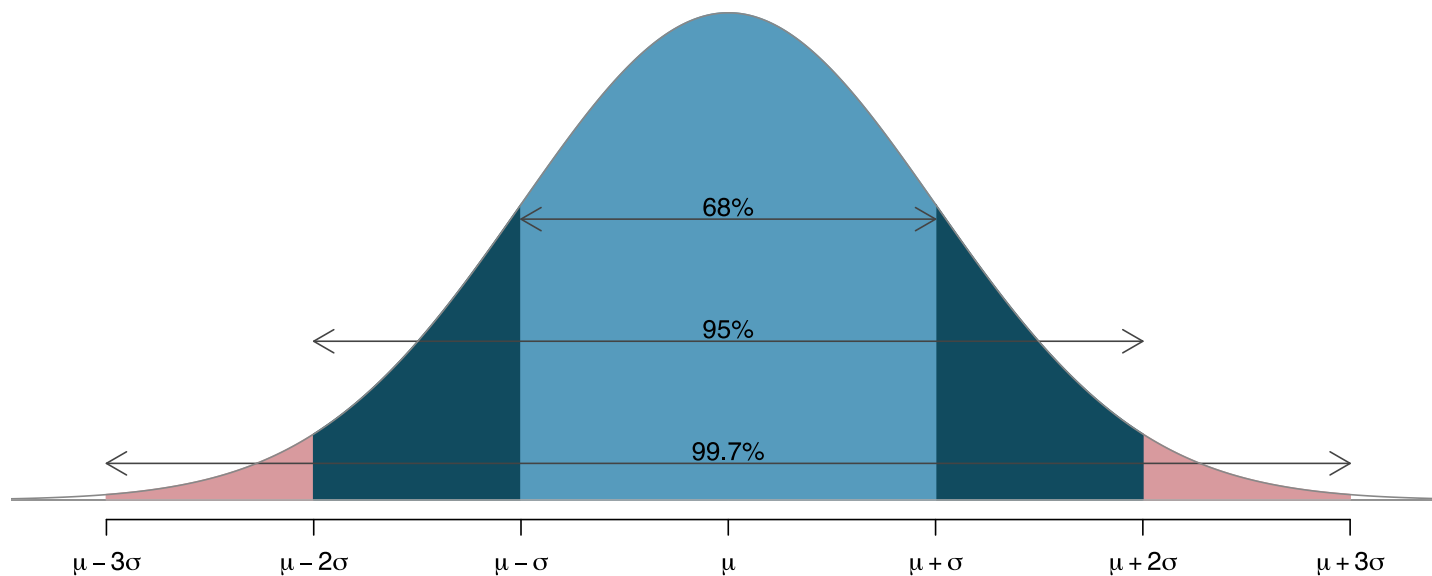
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- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE .
- This should tell me something about **how unlikely it was** that I happened to draw my point estimate if the null were true, right?
- Yep! But we need a couple more definitions to get all the way there.

The 68-95-99.7 rule

For a normal distribution:



Probabilities for falling within 1, 2, and 3 standard deviations of the mean in a normal distribution.

Z-score

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Z-score: How many standard deviations is a value from the mean?

$$z = \frac{x_i - \mu}{\sigma}$$

Z-score for hypothesis testing

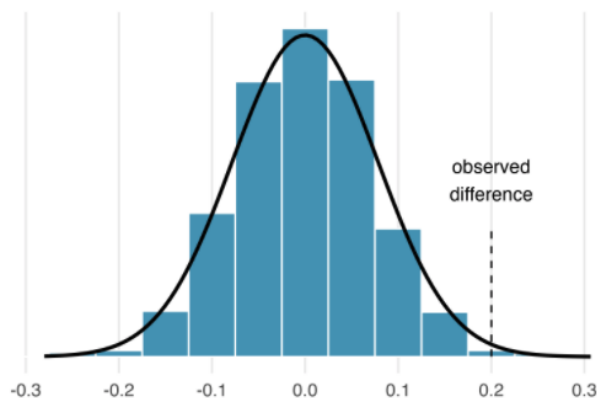
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Z-score for hypothesis testing

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Z-score for hypothesis testing: How many standard deviations is a point estimate from the null?

$$z = \frac{\text{point estimate} - \text{null value}}{SE}$$



Quantifying probabilities: p -value

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Quantifying probabilities: p -value

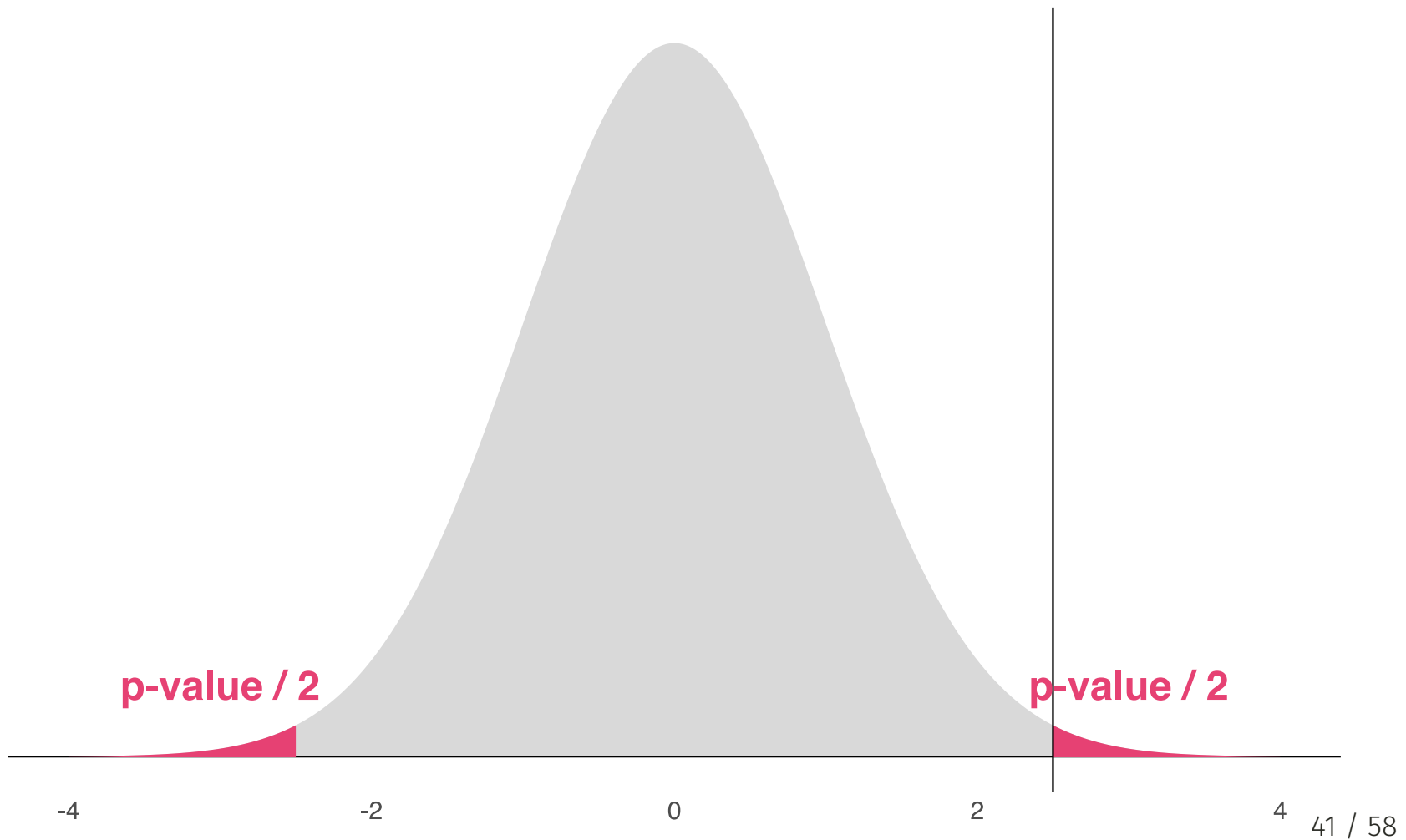
- The Z-score is also called the **test statistic**
- The Z-score/test statistic allows us to compute the **p -value**:

p -value is the probability of getting a point estimate *at least as extreme* as ours **if the null hypothesis were true**.

$$p - value = Pr(Z < -|z| \text{ or } Z > |z|) = 2 * Pr(Z > |z|)$$

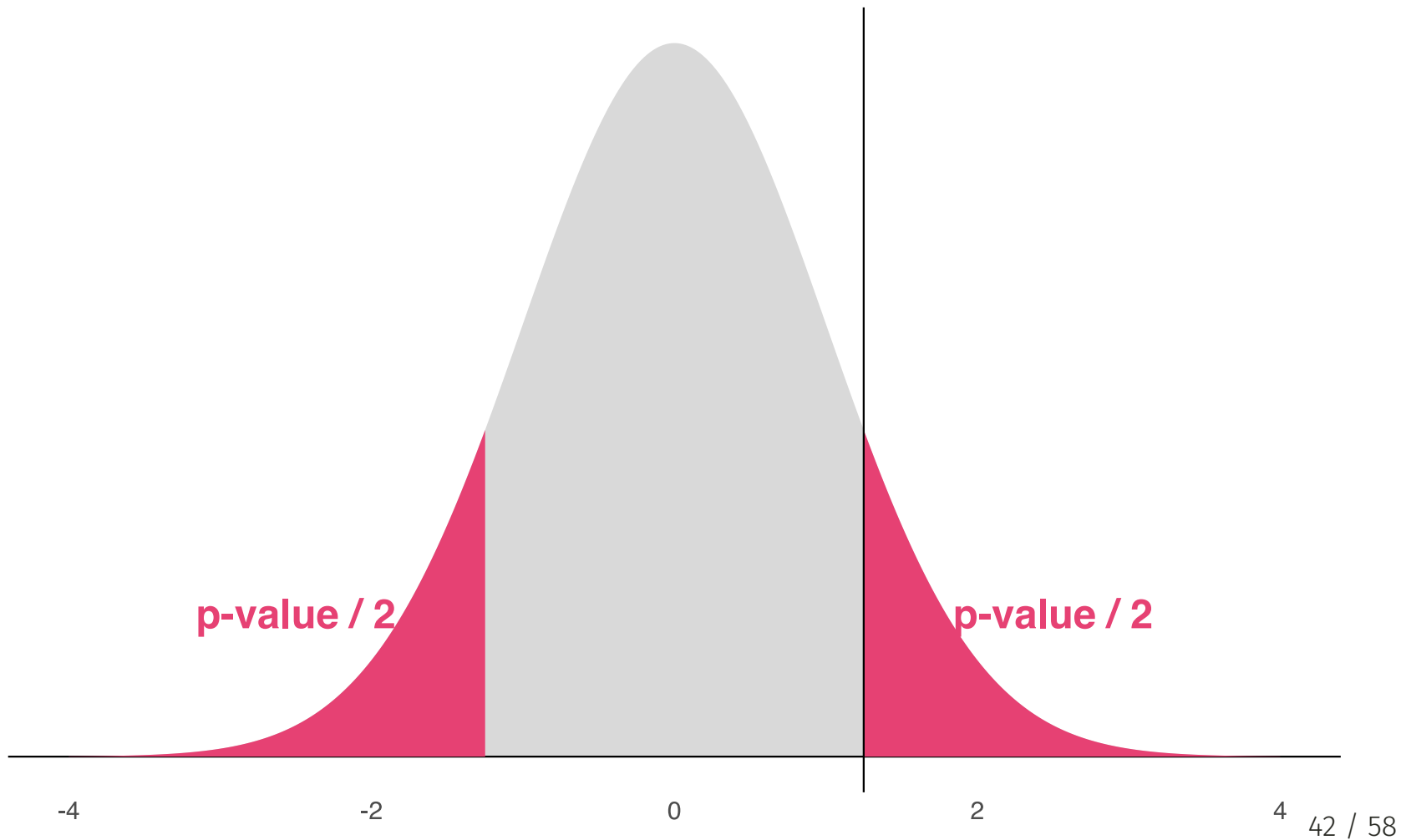
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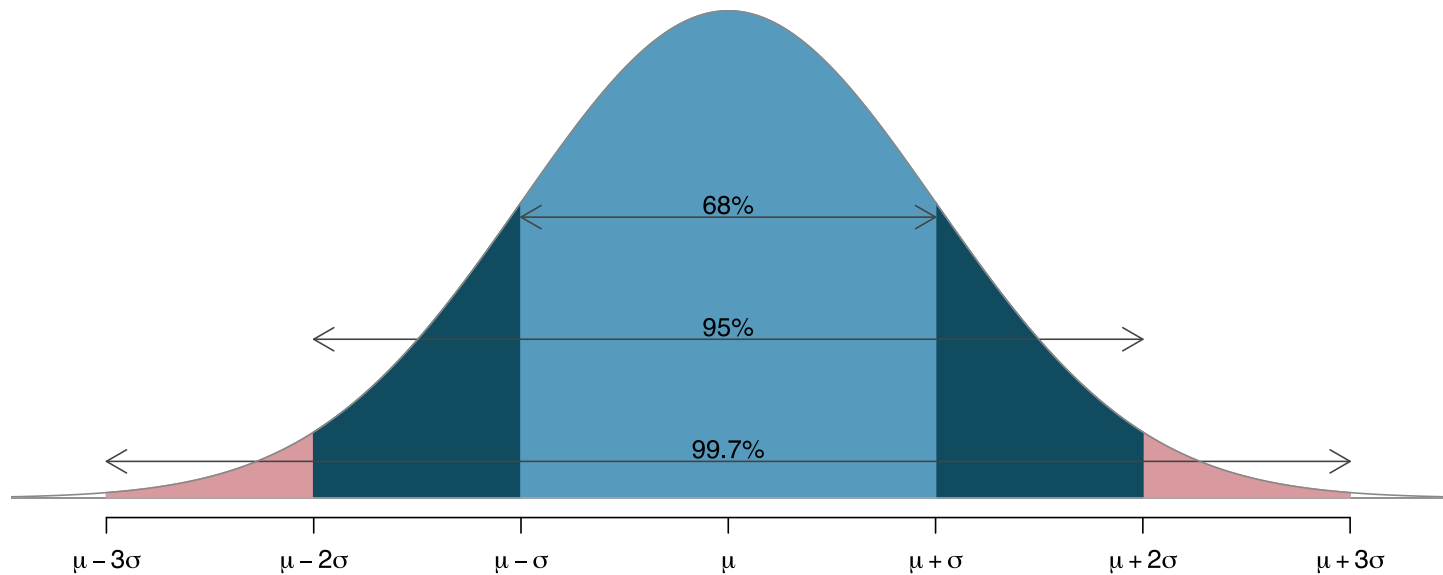
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How do I compute a p -value from a test statistic?

- **In math:** Integrate the sampling distribution's probability density function between $-\infty$ and $-|z|$; multiply by 2
- **In R:** `pnorm()`, `t.test()`, `summary(lm())`, ...

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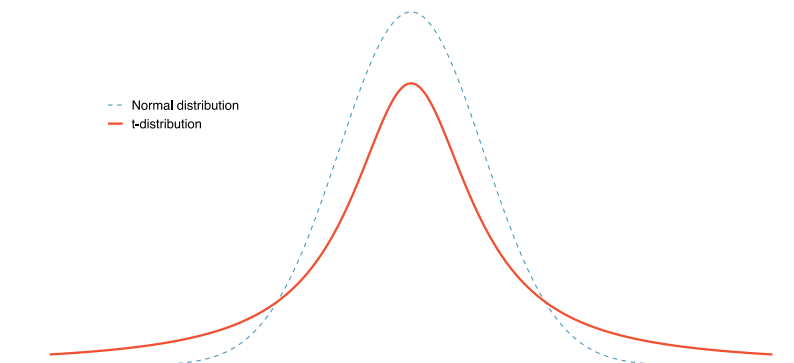
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1. Observations in our sample are **independent**
2. We have a **large enough sample** (at the very least $n \geq 30$)

When n is relatively small, we can still proceed, we just need to use a t -distribution (and T-score -- use `pt()` in `R`) instead of a normal distribution (and Z-score)



Hypothesis testing in five steps

The general framework for implementing a hypothesis test is:

1. **Define the null** and alternative hypotheses
2. Collect data and compute the **point estimate of the statistic**
3. Model the **variability of the statistic**
4. Given this variability, **quantify the probability that your sample statistic differs from the null** by the observed amount, if the null were true
5. Based on #4, either **reject** or **fail to reject** the null

Can we finally test something?

Step 5: Based on the p -value, either **reject** or **fail to reject** the null

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- Traditionally, we use a **significance level** of $\alpha = 0.05$
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- In general, reject the null if $p < \alpha$.
 - Other common α s: 0.01, 0.1

Statistical significance

We say a point estimate is "statistically significant" when:

$$p < \alpha$$

For example:

"[W]e find a **statistically-significant** effect whereby increases in surface UV intensity lowers subsequent COVID-19 growth rates...we estimate that a 1 kJm⁻²hr⁻¹ increase in local UV reduces local COVID-19 growth rates by .09 (±.04, $p = .01$) percentage points over the ensuing 17 days." --- Carleton *et al.*, 2021

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- Failing to reject the null tells us we do not have sufficient evidence to prove there is an effect or a difference

Constructing confidence intervals

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- Effect sizes are important! We care not just about whether a treatment effects an outcome, but by *how much*

A **confidence interval** is a range of plausible values where we may find the true population value.

- It tells us something about the magnitude of the parameter of interest, as well as our uncertainty around our estimate

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When the sampling distribution of a point estimate can be modeled as normal, the point estimate we observe will be within 1.96 standard errors of the true value of interest about 95% of the time (think back to the 68-95-99.7 rule).

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Thus, a 95% confidence interval for such a point estimate can be constructed:

$$\text{point estimate} \pm 1.96 * SE$$

We can be 95% confident this interval captures the true value.

Also can see this as: `2*pnorm(-1.96) = .05`

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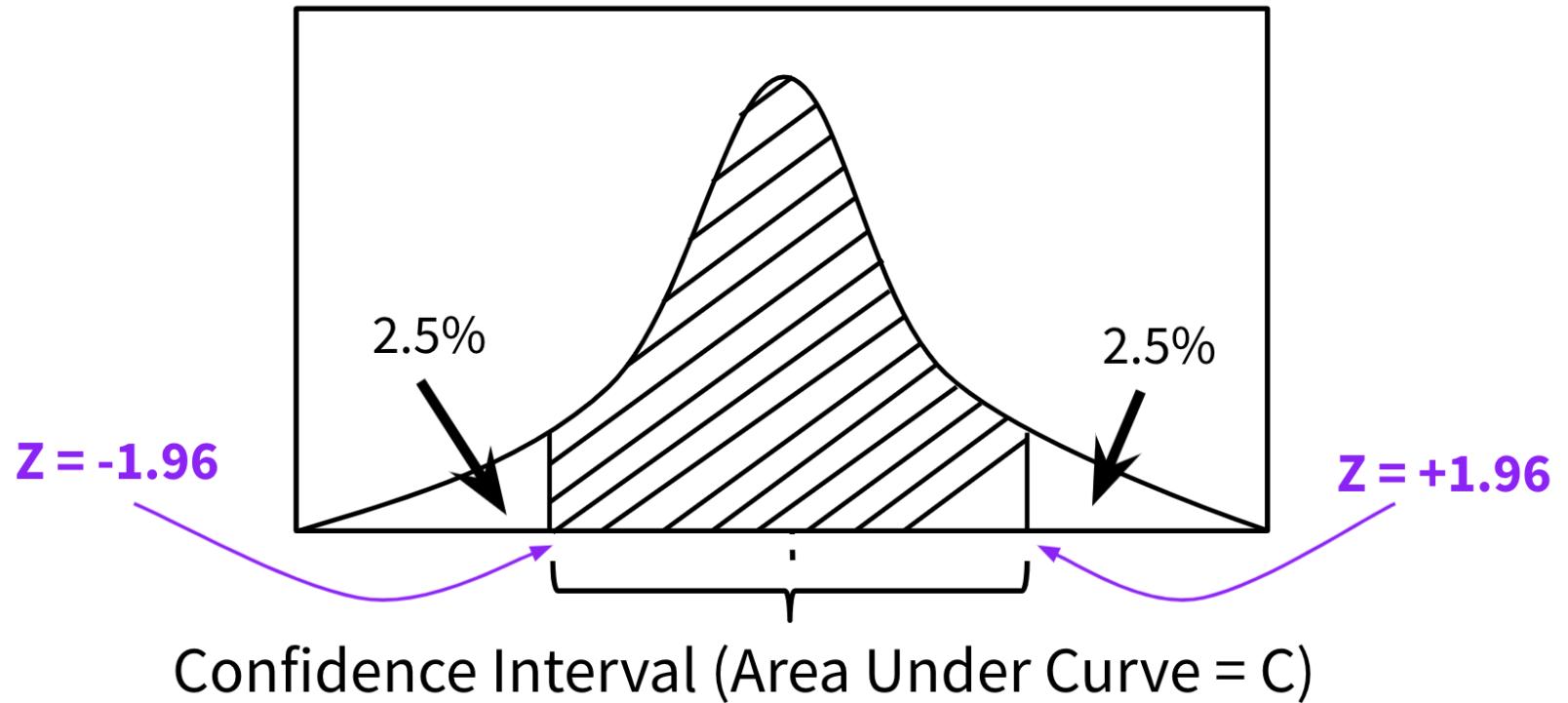
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Why do the z-scores get larger for higher confidence intervals?

A 95% confidence interval visual



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A 95% confidence interval tells you there is a **95% chance that your interval includes the true population parameter**.

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Why is this a big deal?

The population parameter *is not random*. So it either **is or is not** inside your CI.

Slides created via the R package **xaringan**.

Some slide components were borrowed from **Ed Rubin's** awesome course materials.