# ISA 444: Business Forecasting

## 12 - Seasonal Decomposition and Forecasting

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# Outline

# Quick Refresher on Chapter 04 so Far

**Main Learning Outcomes Discussed in Previous Chapter**

☑ **Explain when to use an additive vs. multiplicative model for a time series.**

☒ **Use classic decomposition methods to detrend and deseasonalize a time series.**

# Learning Objectives for Today's Class

## Main Learning Outcomes

- Use classic decomposition methods to detrend and deseasonalize a time series.
- Use Holt-Winters method to forecast a time series with a seasonal component.
- Evaluate the application of different smoothing methods applied to a time series, and determine the best performing method.

# Outline

# Background: Centered Moving Averages

Calculate the CMA(3), where you center the moving average in the middle of the moving window.

| Q | Bike Sales | MA3 |
|---|---|---|
| 1.00 | 10.00 | —- |
| 2.00 | 31.00 | —- |
| 3.00 | 43.00 | —- |
| 4.00 | 16.00 | —- |
| 1.00 | 11.00 | —- |
| 2.00 | 33.00 | —- |
| 3.00 | 45.00 | —- |
| 4.00 | 17.00 | —- |
| 1.00 | 14.00 | —- |
| 2.00 | 36.00 | —- |
| 3.00 | 50.00 | —- |
| 4.00 | 21.00 | —- |
| 1.00 | 19.00 | —- |
| 2.00 | 41.00 | —- |
| 3.00 | 55.00 | —- |
| 4.00 | 25.00 | —- |

# Decomposition Methods

- Decomposition methods are used to "decompose" a time series into its components.
- Decomposition methods are generally poor forecasting methods, but they work well for:
  - exploring and visualizing time series data
  - detrending and/or deseasonalizing data
- Decomposition methods may be applied to multiplicative or additive time series.

# Pure Decomposition Process for an Additive Time Series

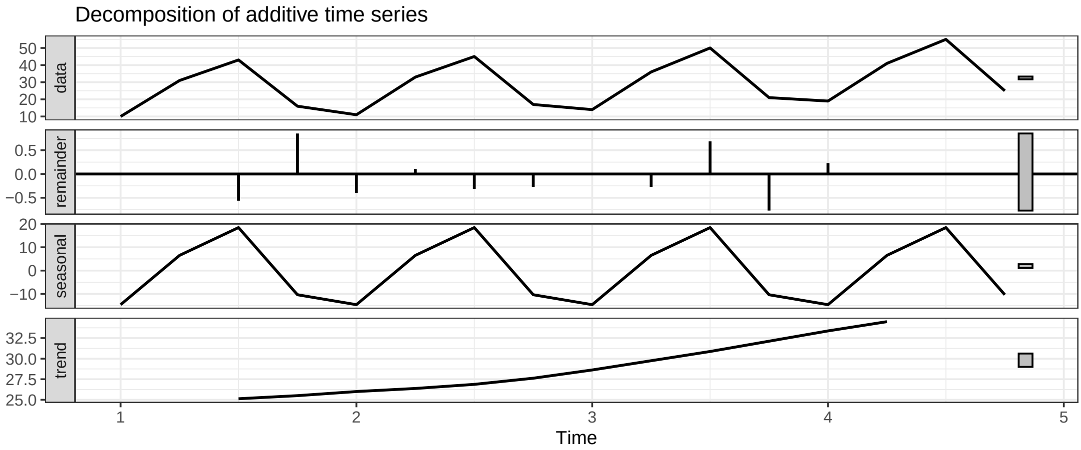
- **Estimate the trend** by calculating the centered moving average for a window of width $K$, denoted as CMA($K$). Note you will lose $(K-1)/2$ observations at the beginning and end of the series if $K$ is odd; suppose $K = 3$, so we lose one observation at the beginning and the end.
- **Detrend the series** by subtracting the CMA from the corresponding observations.
- **Estimate the initial seasonal factors** by calculating the average value of the detrended series for each quarter, month, day, etc. (depending on the season length).
- **Standardize the seasonal factors** by computing their averages and then setting the final seasonal factor for each season equal to the initial value minus the overall average.
- **Estimate the error term** by subtracting seasonal factor from the detrended series for each corresponding season.

# A Non-Graded Class Activity: Decomposing the 11-BikeSalesR.xlsx

Based on the procedure described above, please use Excel to perform the aforementioned five steps.

# A Live Demo of Using R as an alternative

In class, we will use R to decompose the series and obtain the following plot



# Notes on the `decompose()` in R

- The `decompose()` function in R uses a slightly different algorithm than your textbook presents.[1]
- The MA used to compute the trend estimate is a $2 \times m$ moving average. This means that for quarterly data, a $2 \times 4$ moving average is computed. First a MA(4) is computed, then a MA(2) of the MA(4) is computed. This is used to estimate the trend.
- The seasonal components are computed as usual and centered.

[1] Slide is from Dr. Allison Jones-Farmer's lecture notes, Miami University, Spring 2020.

# Pure Decomposition Process for a Multiplicative Model:

- **Estimate the trend** by calculating the centered moving average for a window of width $K$ (i.e., CMA(K)). For now, let us assume that $k = 3$.
- **Detrend the series** dividing the observations $2, .., (n-1)$ from the their corresponding CMA(3).
- **Estimate the initial seasonal factors** by calculating the average value of the detrended series for each quarter, month, day, etc. (depending on the season length).
- **Standardize the seasonal factor** by computing their averages and then setting the final seasonal factor for each season equal to the initial value divided by the overall average.
- **Estimate the error term** by dividing the detrended series by the seasonal factor for each corresponding season.

# Limitations to Decomposition

- Decomposition is widely used in practice but is not a good forecasting method.
- Decomposition methods are useful for visualizing your data and exploratory data analysis.
- Trend estimates are from moving averages and are not available for the first few and last few observations.
- Decomposition methods assume that the seasonal factors occur regularly from season to season over every period. This may not be true over the long run.
- Decomposition methods are not robust to unusual or spurious patterns that may occur in the data.

Because of these limitations, we need a better forecasting method for seasonal data![2]

[2] Slide is from Dr. Allison Jones-Farmer's lecture notes, Miami University, Spring 2020.

# Outline

# Definition and Basic Principles

If a time series has a linear trend with a local trend ( $\beta_1$ ,growth rate) and a local seasonal pattern ($SN_t$) that may be changing over time, we can use the Holt-Winters exponential smoothing method for forecasting to accommodate the seasonal pattern.

The Holt-Winters method accommodates time series data with a **local level**, a **local trend**, and a **local seasonal pattern**, all of which are slowly changing over time. There are both additive and multiplicative versions of the Holt-Winters method.

# Additive Holt-Winters Smoothing Method [1]

To compute the FORECAST, we will use three smoothing constants, $\alpha$ , to smooth the level, $\beta$ , the smoothing constant to smooth the trend, and $\gamma$ to smooth the seasonal pattern of length/frequency $m$ (e.g. day-of-the-week pattern, $m = 7$; quarterly pattern, $m = 4$; monthly pattern, $m = 12$).

The estimate of the **level** is:

$$l_t = \alpha(y_t - sn_{t-L}) + (1-\alpha)[l_{t-1} + b_{t-1}] \tag{1}$$

The estimate of the **trend** is:

$$b_t = \beta[l_t - l_{t-1}] + (1-\beta)b_{t-1} \tag{2}$$

# Additive Holt-Winters Smoothing Method [2]

The estimate of the **seasonal pattern** is:

$$sn_t = \gamma[y_t - l_t] + (1-\gamma)sn_{t-L} \tag{3}$$

To estimate the **point forecast** for time $t+h$ time periods ahead made in time $t$:

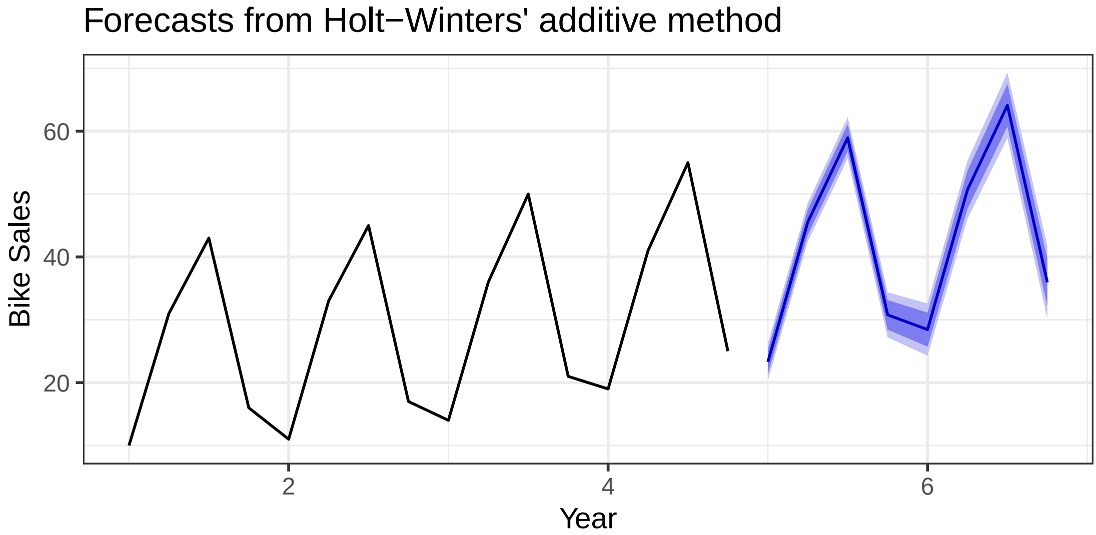
$$\hat{y}_{t+h}(t) = l_t + h \times b_t + sn_{t+h-L} \tag{4}$$

where $sn_{t+h-L}$ is the most recent estimate of the seasonal pattern for the season corresponding to the time period $t+h$.

# Comments on the Use of Software for Holt-Winters Method

- **Starting values:** We will need three sets of starting values; one for the Level, one for the Trend, and a set for $m$ Seasonal Components. There are no two statistical packages that compute starting values in the same way! *Therefore, be comfortable on the fact that there will be some slight differences in the error values when compared to your textbook.*
- As we have done throughout the semester, we will be using R. The function used is titled `hw()`, which gets loaded from the forecast package (which we load when we run the command `pacman::p_load(fpp2)`).
- Details on the method used to compute starting values in the `hw()` function can be found in the Rstudio documentation by typing `?forecast::hw()` at the command prompt.

# Live Demo: Holt Winters (Additive) on the BikeSales Data



# Multiplicative Holt-Winters Smoothing Method [1]

To compute the FORECAST, we will use three smoothing constants, $\alpha$ , to smooth the level, $\beta$ , the smoothing constant to smooth the trend, and $\gamma$ to smooth the seasonal pattern of length/frequency $m$ (e.g. day-of-the-week pattern, $m = 7$; quarterly pattern, $m = 4$; monthly pattern, $m = 12$).

The estimate of the **level** is:

$$l_t = \alpha(y_t/sn_{t-L}) + (1-\alpha)[l_{t-1} + b_{t-1}] \tag{5}$$

The estimate of the **trend** is:

$$b_t = \beta[l_t - l_{t-1}] + (1-\beta)b_{t-1} \tag{6}$$

# Multiplicative Holt-Winters Smoothing Method [2]

The estimate of the **seasonal pattern** is:

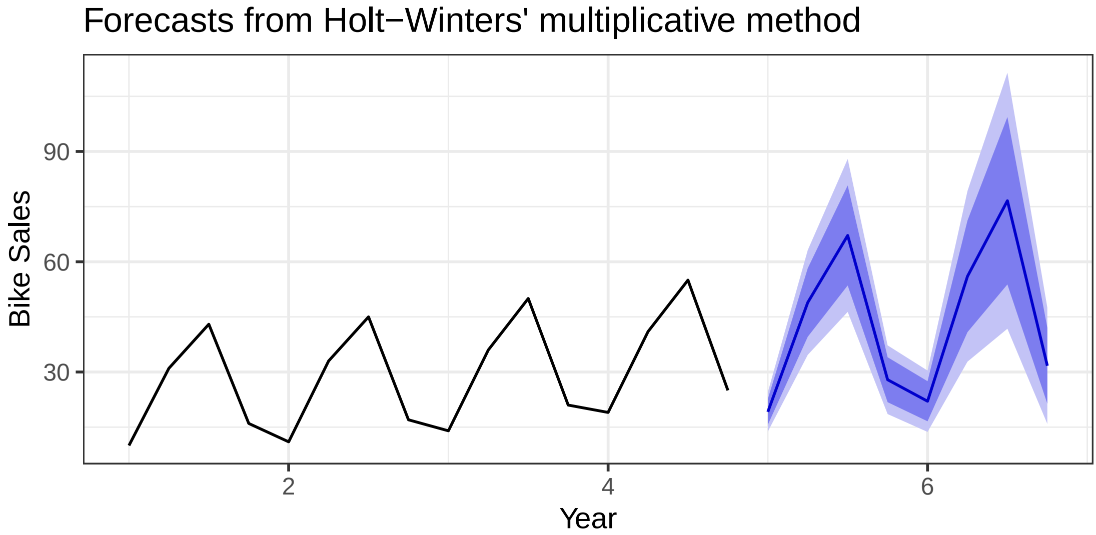
$$sn_t = \gamma[y_t/l_t] + (1-\gamma)sn_{t-L} \quad (7)$$

To estimate the **point forecast** for time $t+h$ time periods ahead made in time $t$:

$$\hat{y}_{t+h}(t) = (l_t + h \times b_t) \times sn_{t+h-L} \quad (8)$$

where $sn_{t+h-L}$ is the most recent estimate of the seasonal pattern for the season corresponding to the time period $t+h$.

# Live Demo: Holt Winters (Multiplicative) on BikeSales



# Live Demo: Accuracy Comparison

| | ME | RMSE | MAE | MPE | MAPE | MASE | ACF1 |
|---|---|---|---|---|---|---|---|
| Additive HW | 0.60 | 1.06 | 0.89 | 2.49 | 3.88 | 0.27 | 0.01 |
| Multiplicative HW | -0.07 | 1.94 | 1.67 | -1.75 | 7.86 | 0.50 | 0.18 |

# Outline

# Summary of Main Points

**Main Learning Outcomes**

- **Use classic decomposition methods to detrend and deseasonalize a time series.**
- **Use Holt-Winters method to forecast a time series with a seasonal component.**
- **Evaluate the application of different smoothing methods applied to a time series, and determine the best performing method.**

# Things to Do for Next Class

- Thoroughly read Chapter 4.1-4.4 and 4.6-4.7 of our textbook.
- Go through the slides, examples and make sure you have a good understanding of what we have covered.
- Complete the practice assignment (see details in next slide).

# Practice/Non-Graded Assignment [1]

Use the file '12-Job_openings.xlsx' for this exercise.

1. Convert the data to an appropriate time series and plot the series over time.
2. Use simple additive decomposition visualize the series (see the `type` argument in `decompose()`). Plot the decomposition. Print the seasonal factors. What are the units of the seasonal factors?
3. Repeat with multiplicative decomposition (see the `type` argument in `decompose()`). Plot the decomposition. Print the seasonal factors. What are the units of the seasonal factors?
4. Split the series into a training sample (2001-2014) and a validation sample (2015).

# Practice/Non-Graded Assignment [2]

5. Use Linear Exponential Smoothing (holt's method) to fit an optimal model to the training sample, forecasting for the next 12 periods. Plot the series, forecast, and retrospective fitted values.
6. Use Holt-Winters Smoothing to fit an optimal model to the training sample, forecasting for the next 12 periods. Plot the series, forecast, and retrospective fitted values.
7. Assuming you need to forecast with a horizon of 12 months, compare the validation sample accuracy of Holt's method to Holt Winters on the 2015 data only.

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