ISA 444: Business Forecasting

15 - Stationarity

Fadel M. Megahed

Associate Professor
Department of Information Systems and Analytics
Farmer School of Business
Miami University
Email: fmegahed@miamioh.edu
Office Hours: Click here to schedule an appointment

Fall 2020

Outline

Quick Refresher on What we Covered in Chapter 06 so Far [1]

Main Learning Outcomes Discussed in Chapter 06

- ☑ Define the population mean, and variance of a random variable.
- ☑ Define the population covariance, and correlation between two random variables.
- ☑ Define the population autocovariance and autocorrelation of a random variable.
- ☑ Use sample estimates of the population mean, variance, covariance, and correlation.
- ☑ Explain the properties of the large sample distribution of the sample ACF.
- ☑ Use the large sample distribution of the sample ACF to identify significant autocorrelation in a time series.

Quick Refresher on What we Covered in Chapter 06 so Far [2]

Main Learning Outcomes Discussed in Chapter 06

☑ Determine if a sample ACF plot "cuts off" or "dies down".

☑ Determine if a sample ACF plot "cuts off" or "dies down".

☑ Explain how sample partial autocorrelation is calculated.

☑ Define the term "weakly stationary" and recognize time series that do not fit this definition.

☑ Use the sample ACF plot to identify a nonstationary time series.

Recall: A Formal Definition for Stationarity

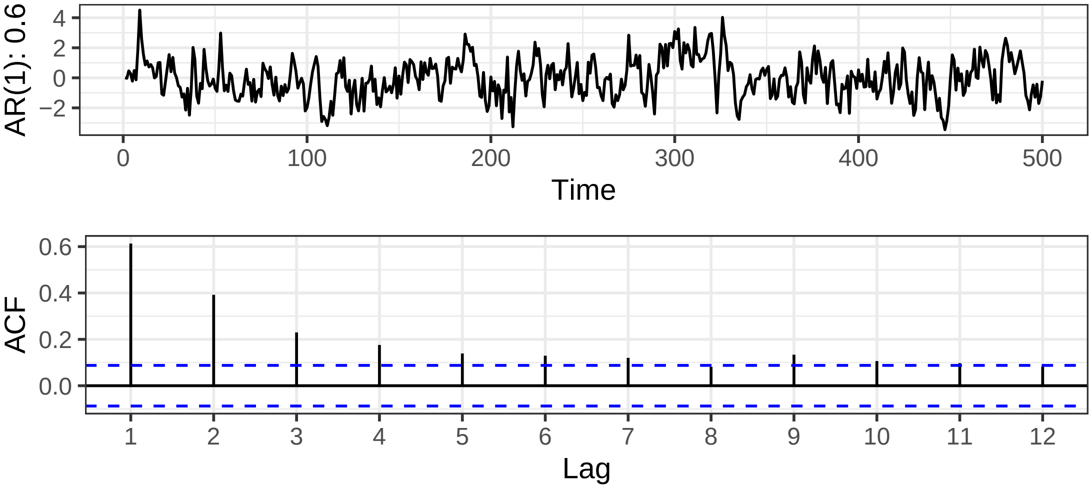
Weak Stationarity: A weakly stationary time series is a finite variance process such that:

1. The mean, μ_t, is constant and does not depend on the time t; and
2. The autocovariance function, $\gamma(s, t)$ depends on s and t only through their difference $|s - t|$.

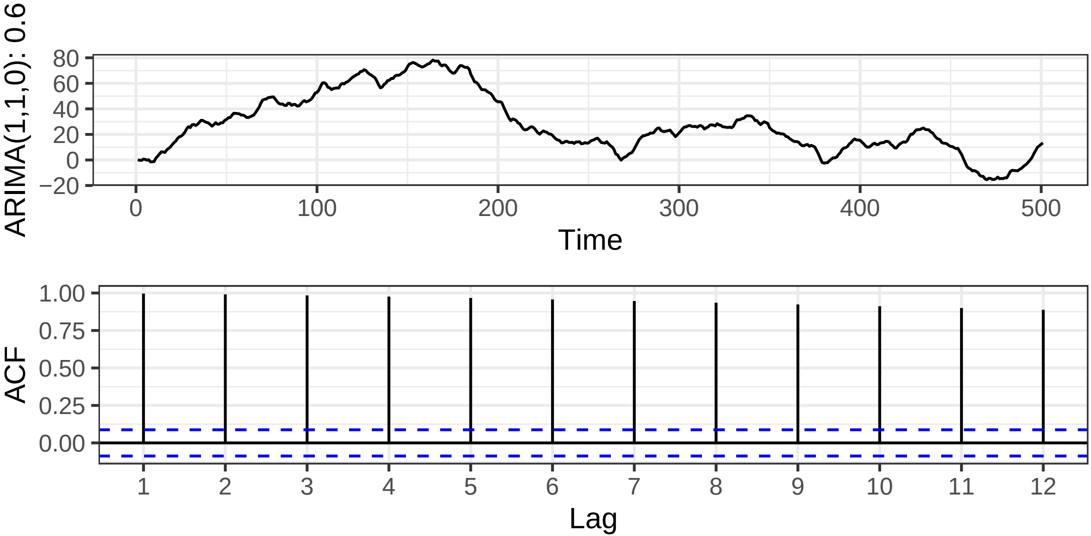
We will use the term “stationary” to refer to weak stationarity.

- The concept of weak stationarity forms the basis of much of the foundation for time series modeling.
- The fundamental properties (1 & 2) required for weak stationarity are satisfied by many of the models that are widely used.

Recall: A Visual Explanation of Stationarity



Recall: A Visual Explanation of Nonstationarity



Learning Objectives for Today's Class

Main Learning Outcomes

- **Apply transformations to a nonstationary time series to bring it into stationarity.**
- **Recognize and explain a random walk model (both with and without a drift).**
- **Recognize a random walk model from an ACF plot.**
- **Conduct formal tests for stationarity using the ADF and KPSS tests.**

Outline

What to do when we have a Nonstationary series?

In order to model a time series, it must usually be in a state of stationarity. If the time series is not stationary, you must transform it to achieve stationarity.[1]

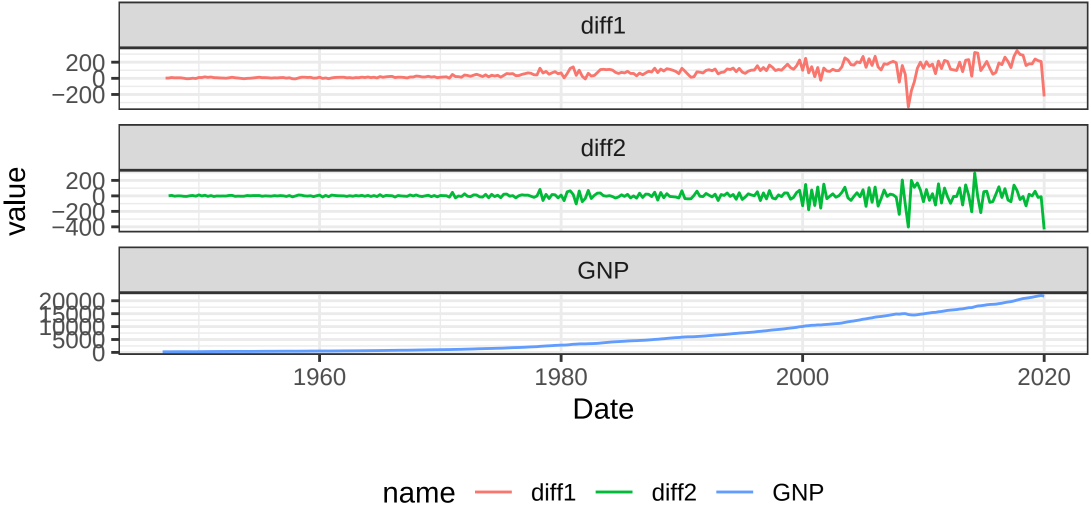
Successive **differencing** is typically used to achieve stationarity.

First Differences: $y_t' = y_t - y_{t-1}$.

Second Differences" $y_t'' = y_t' - y_{t-1}'$.

[1] Slides are based on Dr. Allison Jones-Farmer's Handouts for ISA 444, Spring 2020.

A Live Example: Examining the US GNP



So Why Does Differences Work?

Because many nonstationary time series have features of a random walk.

Random Walk Model [1]

Random Walk with Drift: A random walk is a model such that successive differences (first differences) are independent.

The classic random walk model:

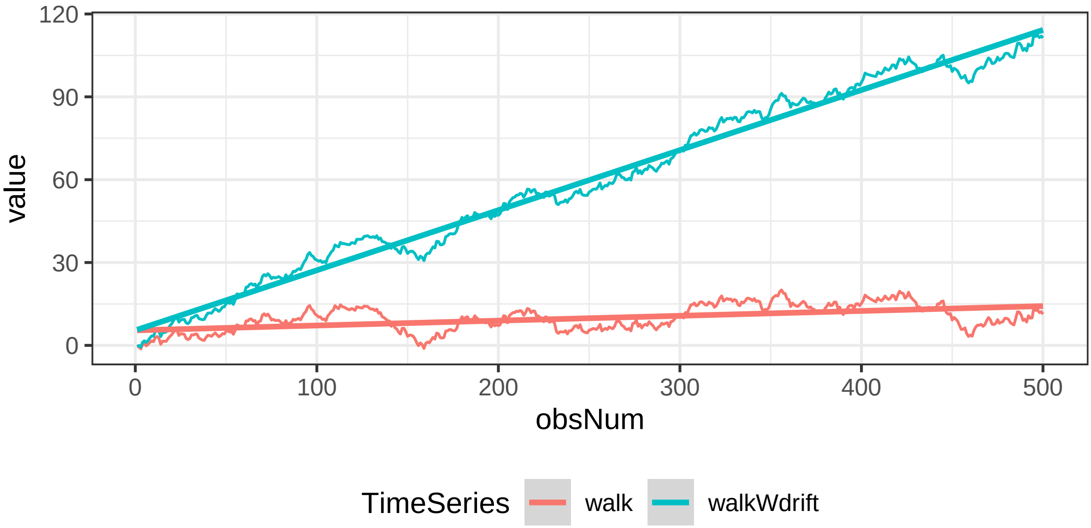
$$Y_t = Y_{t-1} + \epsilon_t$$

A random walk with a drift:

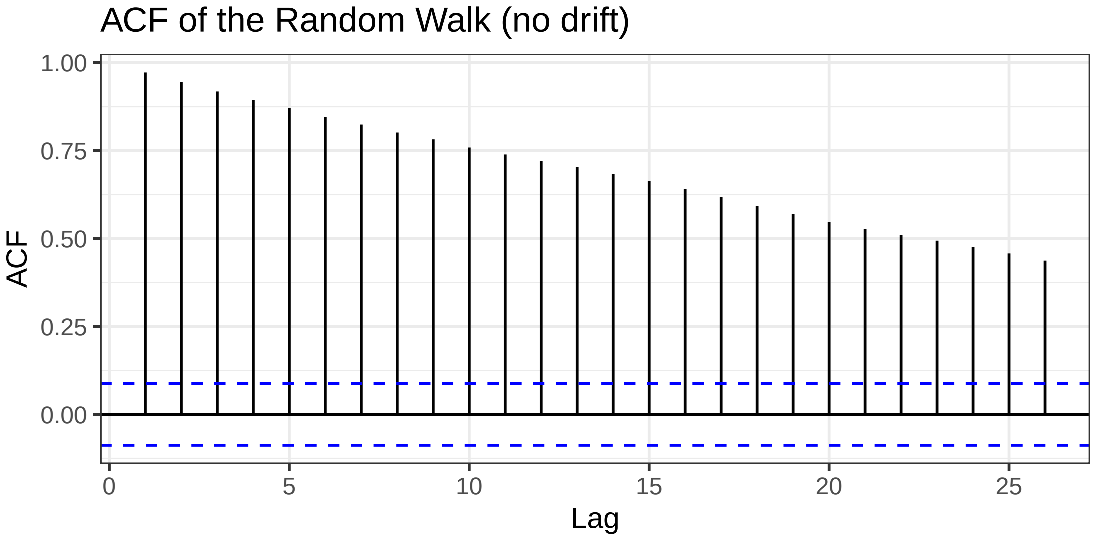
$$Y_t = \delta + Y_{t-1} + \epsilon_t$$

Notes: When $\delta = 0$, the value of the current observation is just the value of the prior observation plus random noise.

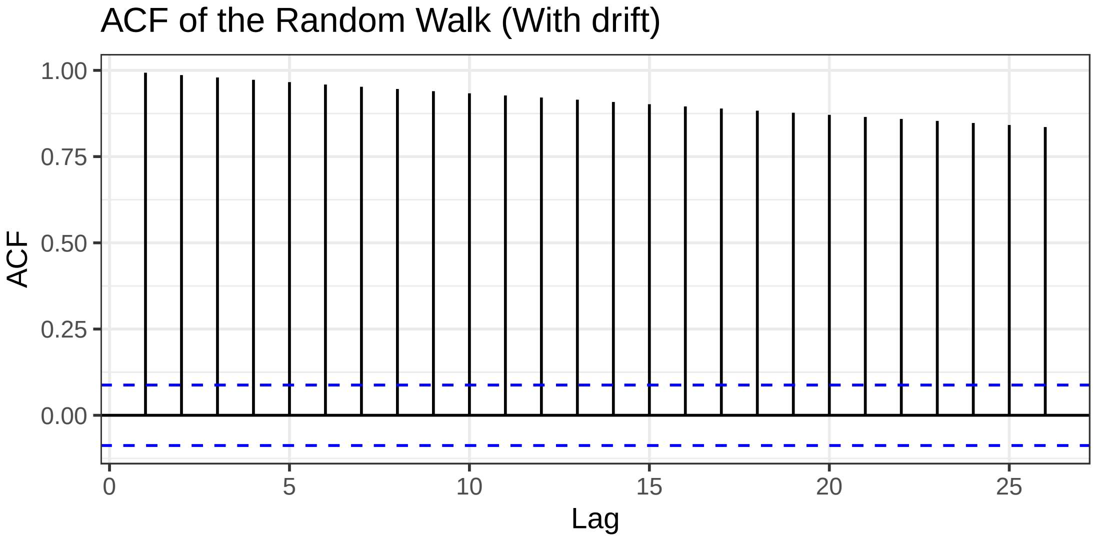
Random Walk Model [2]



Random Walk Model [3]



Random Walk Model [4]



Outline

Basic Idea

Unit Root Test: One way to objectively determine if differencing is required is to use a unit root test. A unit root is a feature of a stochastic process that indicates a time series is nonstationary.

Augmented Dickey Fuller (ADF) Test [1]

The **Augmented Dickey Fuller (ADF) Test** tests whether or not there is a unit root. The hypotheses are as follows:

Ho: The series is nonstationary

Ha: The series is stationary

Thus, if we have a *SMALL p-value*, we reject the null hypothesis and conclude **STATIONARITY**.

Augmented Dickey Fuller (ADF) Test [2]

In R, we will use the function `adf.test()` from the package tseries.

```
pacman::p_load(tseries)
adf.test(gnp$GNP)
```

```
##
##  Augmented Dickey-Fuller Test
##
## data:  gnp$GNP
## Dickey-Fuller = 0.10388, Lag order = 6, p-value = 0.99
## alternative hypothesis: stationary
```

So what do we conclude from the test above?

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test [1]

The **Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test** tests whether or not there is a unit root. The hypotheses are as follows:

Ho: The series is stationary

Ha: The series is nonstationary

Thus, if we have a *SMALL p-value*, we reject the null hypothesis and conclude **NONSTATIONARITY**.

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test [2]

In R, we will use the function `kpss.test()` from the package tseries.

```
pacman::p_load(tseries)
kpss.test(gnp$GNP)
```

```
##
##  KPSS Test for Level Stationarity
##
## data:  gnp$GNP
## KPSS Level = 4.5081, Truncation lag parameter = 5, p-value = 0.01
```

So what do we conclude from the test above?

Successive Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Tests

As a followup to the case when the `kpss.test()` is rejected (or alternatively when you do not reject the `adf.test()`), you can utilize the `ndiffs()` from the package forecast, which uses a series of the KPSS tests in a sequence to determine the appropriate number of first differences required for a nonseasonal time series.

`ndiffs()` returns the number of first differences needed to achieve stationarity according to the KPSS test.

```
pacman::p_load(fpp2) # fpps loads the forecast package as well
ndiffs(gnp$GNP)
```

```
## [1] 2
```

According to the `ndiffs() function`, **two successive differences are recommended to transform the GNP data into stationarity**.

Outline

Summary of Main Points

Main Learning Outcomes

- **Apply transformations to a nonstationary time series to bring it into stationarity.**
- **Recognize and explain a random walk model (both with and without a drift).**
- **Recognize a random walk model from an ACF plot.**
- **Conduct formal tests for stationarity using the ADF and KPSS tests.**

Things to Do to Prepare for the Exam

- Thoroughly read Chapters 1-4, 6.1, and 6.3-6.4 of our textbook.
- Go through the slides, examples and make sure you have a good understanding of what we have covered.
- Go over the interactive tutorial (will be posted by close of business 10-13-2020).

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