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On monitoring industrial processes under feedback control

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Abstract

The concurrent use of statistical process control and engineering process control involves monitoring manipulated and controlled variables. One multivariate control chart may handle the statistical monitoring of all variables, but observing the manipulated and controlled variables in separate control charts may improve understanding of how disturbances and the controller performance affect the process. In this article, we illustrate how step and ramp disturbances manifest themselves in a single-input–single-output system by studying their resulting signatures in the controlled and manipulated variables. The system is controlled by variations of the widely used proportional-integral-derivative(PID) control scheme. Implications for applying control charts for these scenarios are discussed.

KEYWORDS

control charts, disturbance signatures, engineering process control (EPC), proportional-integral-derivative (PID), statistical process control (SPC)

1 | INTRODUCTION

Statistical process control (SPC) and engineering process control (EPC) have developed more or less independently but with the same overarching goal of reducing process variability. SPC methods typically employ control charts to monitor that a product quality characteristic or important process variable remains close to a nominal value. If control charts signal a statistically significant change in the process mean and/or variability, the SPC methodology assumes that off-line process analysis will be able to identify sources of variation, so-called assignable causes. Given that the root cause can be identified, problem-solving methods can then be used to remove or reduce effects of the variation sources. Conversely, EPC attempts to make a process insensitive to external disturbances by continuously adjusting a process input (manipulated variable) to ensure that an output variable (controlled variable) remains on target (the controller set point).

The assignable causes in SPC usually arise from external disturbances. Such disturbances will increase probabilities for out-of-control signals in control charts. An out-of-control signal should trigger further investigation and corrective action, and given a successful remedial action, the reduced variability improves the process performance. Controllers in EPC continuously adjust the process to minimize deviations of a controlled variable from its set point due to various unexpected and/or unplanned external phenomena. The control action stems from the manipulation of a related and less sensitive variable thereby transferring the variability from the controlled variable to the manipulated variable.

Knowledge of the causal relationships between such manipulated and controlled variables and of the process dynamics is therefore important to determine the required EPC adjustments. Although the continuous adjustments of the manipulated variable may be able to keep the controlled variable at its set point, they may come at some increased cost that we would like to avoid.

The currently high and increasing level of automation leads to complex production environments where a combination of EPC and SPC may be needed to improve the processes, as for example, in semiconductor manufacturing.[1] Accordingly, there have been attempts to develop a unified tool for process improvement that concurrently uses EPC for process adjustments and SPC for process monitoring.[2,3] MacGregor[4] suggested that engineers could use control charts to monitor a process that was already under feedback control. For additional background and discussions, see, for example, Vander Wiel,[5] MacGregor[6], Tucker et al.,[7] Fultin et al.,[8] Del Castillo,[9] and Vining.[10]

To apply SPC naïvely in a process under feedback control without considering how the implemented feedback control should affect the choice of variables to monitor is risky. A control chart applied to a tightly controlled variable in an EPC scheme will often result in a small 'window of opportunity' or in a failure to detect out-of-control situations due to the continuous adjustments of the manipulated variable.[11] In the SPC literature, there are two basic recommended approaches to deal with the potential masking effect that EPC may have on process disturbances. The first approach suggests monitoring the difference between the controlled variable and set point value, that is, the control error.[12–14] Keats et al.[13] showed that a control chart that monitors the control error detects sources of variation for which the feedback controller does not compensate. The study included integral (I), proportional-integral (PI), and proportional-integral-derivative (PID) control schemes. Montgomery et al.[12,14] drew similar conclusions for feedforward control schemes. The second approach is to monitor the manipulated variable itself.[2,15,16] Tsung and Tsui[17] demonstrate that monitoring the manipulated variable may be more appropriate than monitoring the controlled variable for some processes and vice versa for others. Therefore, a combined approach that jointly monitors the control error and the manipulated variable (or the controlled and the manipulated variables) using a multivariate control chart is also possible.[18,19] A combined approach increases the chances that the control chart will issue an out-of-control signal. The out-of-control signal may be issued either when the controller fails to compensate for the disturbance completely or when the manipulated variable deviates from its normal (expected) operating condition.

The main aim of this article is to provide further insight and guidelines to an analyst who wants to apply SPC on a system under feedback control. We focus on outlining and illustrating what we in this article will call 'disturbance signatures' (e.g., mean shifts or trends) and how these will manifest themselves in the controlled and/or manipulated variables at steady state for step and ramp disturbances. We limit our study to a single-input–single-output (SISO) system controlled by variations of the widely used PID control scheme. Step and ramp disturbances represent two general classes of disturbances and control engineers often use such disturbances to quantify feedback control systems in practice. The controlled and manipulated variables are also monitored using control charts. We argue that monitoring both the manipulated and controlled variables in separate univariate charts instead of using a combined, multivariate chart may increase understanding of the out-of-control condition of the process and also makes it possible to evaluate controller performance. We use two simulated examples of SISO systems in Matlab/Simulink® to illustrate the disturbance signatures in the controlled and manipulated variables and the implications for process monitoring.

2 | PRELIMINARIES ON EPC

Feedback control schemes mitigate unwanted deviations of a process variable by adjusting a related manipulated variable, that is, a process input. These adjustments (the actuator signals) depend on the implemented control scheme and the output error fed back to the controller. The output error is the difference between the actual measured value of the process variable and its set point. Feedback control systems are also referred to as closed-loop systems. Figure 1 shows a general block diagram of a closed-loop system.[20] Conventionally, the variables are expressed in the frequency domain through their Laplace transformed quantities:

- $y(s)$ is the controlled variable to be kept at its set point value $y_{sp}(s)$,
- $u(s)$ is the actuator signal of the controller, that is, the manipulated variable,
- $y_m(s)$ is the measured output variable,
- $e(s)$ is the output error, that is, $e(s) = y_{sp}(s) - y_m(s)$, and
- $d(s)$ is the disturbance signal affecting the process.

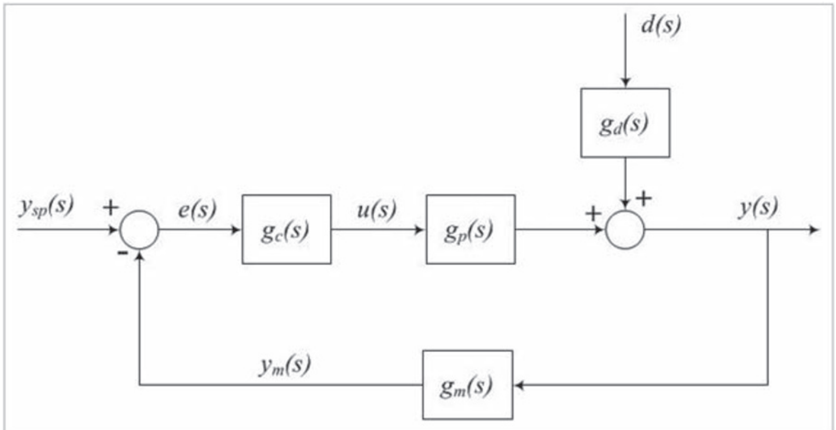

FIGURE 1 Block diagram of a closed-loop system subject to a disturbance

By definition, all Laplace transformed variables are also in deviation form, that is, each variable represents its deviation from a corresponding steady-state value.

The dynamics of the various elements in the feedback loop are defined through their transfer function models as

- $g_p(s)$ is the transfer function of the process plant,
- $g_c(s)$ is the transfer function of the controller,
- $g_m(s)$ is the transfer function of the measuring element (e.g., a sensor) and,
- $g_d(s)$ is the transfer function describing how the disturbance influences the output.

2.1 | Transfer function of the controller

The implemented EPC action defines the controller model (e.g., a transfer function model, $g_c(s)$). The outcome of the EPC action is evaluated considering several criteria such as closed-loop stability and performance. The speed of the response, the degree of overshoot and damping, as well as the ability of the control system to eliminate the steady-state error are often important aspects of controller performance evaluation.[20] Below, we briefly discuss the properties of the common PID controller. The PID controller is typically deployed in one of three control modes—P, PI, or PID—depending on the system requirements. For convenience and ease of exposition, we will assume that the transfer function of the measuring element, $g_m(s) = 1$. We will thus have $y_m(s) = y(s)$.

2.1.1 | P controller

The P mode is the simplest form of a feedback controller. The relationship between the manipulated variable and control error is expressed proportionally as

$$g_c(s) = \frac{u(s)}{e(s)} = k_c \tag{1}$$

where the constant k_c denotes the proportional gain. The P controller has the main advantage of having to tune only one parameter (k_c). However, the P controller can produce a steady-state error. That is, a non-zero difference, $e(s) \neq 0$, between the set point and measured output will remain indefinitely as long as the disturbance persists.

2.1.2 | PI controller

The PI controller combines the proportional and integral control actions according to the transfer function:

$$g_c(s) = \frac{u(s)}{e(s)} = k_c\left(1 + \frac{1}{\tau_I s}\right) \tag{2}$$

where τ_I is the integral time. In the PI control mode, the integral part of the control action works to eliminate the steady-state error. However, tuning of τ_I is critical, as a too large value can lead to long settling times and a too small value can produce an oscillatory response of the controlled variable.

2.1.3 | PID controller

A PID controller combines the proportional, integral, and derivative control actions. The transfer function of this combined control action is given by

$$g_c(s) = \frac{u(s)}{e(s)} = k_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{3}$$

where τ_D represents the derivative time. A PID controller has the added advantage of balancing the aggressive integral action by providing an anticipatory element through the derivative action.[20]

2.2 | Transfer function model of the process plant

One of the core components of a controlled system is the model of the process plant , representing the dynamic behavior of the controlled variable of interest in response to a specific input (manipulated) variable. The transfer function of the process plant is usually expressed in the following general model

$$g_p(s) = \frac{k_p\,(b_0 s + 1)(b_1 s + 1)\ldots(b_m s + 1)}{(a_0 s + 1)(a_1 s + 1)\ldots(a_n s + 1)} \tag{4}$$

where k_p is the process gain, and the as and bs can be viewed as time constants associated with the underlying physical phenomena. The constants m and n represent the order of the numerator and denominator polynomials, and their difference denotes the relative order of the model. For causal systems, $m \leq n$.

2.3 | Closed-loop systems subject to a disturbance

Closed-loop systems are designed to satisfy particular control objectives such as stability and performance while addressing two specific problems: set-point tracking and disturbance rejection. In this article, our focus is on the disturbance rejection problem, and we thus assume that the set point is held constant. We will study the effect of external disturbances on systems controlled by variations of PID control modes and show that in some cases, even though the controller attempts to mitigate unwanted variations by adjusting the manipulated variable, the controlled variable may have a resulting steady-state error depending on the disturbance type. Therefore, steady-state analysis of both the controlled and manipulated variables in systems where EPC is implemented is relevant for understanding how SPC will work if applied to that system. Specifically, the steady-state analysis is important to understand how SPC can help detect the out-of-control signals that may result from disturbance signatures in the controlled and manipulated variables. In what follows, we focus on analyzing the steady-state behavior of the controlled and manipulated variables in closed-loop systems subject to a disturbance. We examine the effects of using P, PI, and PID controllers on the behavior of the controlled and manipulated variables for step and ramp disturbances. As most textbooks on process control provide the mathematical background, we give here only essential concepts for purposes of completeness. Appendix A provides a more detailed derivation.

2.4 | Steady-state response of controlled and manipulated variables

The following assumptions are made:

- the set point $y_{sp}(s)$ is constant over time, that is, $y_{sp}(s) = 0$,
- the set point $y_{sp}(s)$ and the disturbance $d(s)$ are handled independently, and
- the closed-loop system is stable.

As shown in Appendix A, using the final value theorem, the steady-state error e_{ss} and the steady-state values (the long-term value after the transient dynamics have settled) of the controlled and manipulated variables, y_{ss} and u_{ss}, are determined to be

$$e_{\mathrm{ss}} = \lim_{s \to 0} se(s) = \lim_{s \to 0} s\left[-\frac{g_d(s)}{1+g_p(s)g_c(s)}d(s)\right] \tag{5}$$

$$y_{\mathrm{ss}} = \lim_{s \to 0} \mathrm{sy}(s) = \lim_{s \to 0} s\left[\frac{g_d(s)}{1+g_p(s)g_c(s)}d(s)\right] = -e_{\mathrm{ss}} \tag{6}$$

$$u_{\mathrm{ss}} = \lim_{s \to 0} su(s) = \lim_{s \to 0} s\left[-\frac{g_d(s)g_c(s)}{1+g_p(s)g_c(s)}d(s)\right] \tag{7}$$

The steady-state values of the controlled and manipulated variables of a closed-loop system can be determined using Equations 5–7 if the implemented controller $g_c(s)$, the process plant model $g_p(s)$, the disturbance model $g_d(s)$, and the disturbance signal $d(s)$ are known. Again, for convenience, and without loss of generalization, we assume that $g_d(s) = 1$.

2.4.1 | Scenario I: Step disturbance

In this first scenario, suppose that a step disturbance of magnitude $\bar{d}$ affects the system in Figure 1, that is, $d(s) = \bar{d}/s$. The steady-state values of the controlled and manipulated variables can now be determined for a given control mode—P, PI, or PID—using the model of the process plantin Equation 4 and Equations 5–7. Table 1 presents the steady-state error e_{ss}, and the steady-state values of the controlled and manipulated variables, y_{ss} and u_{ss}, when a P, PI, or PID control mode is in place and a step disturbance of magnitude $\bar{d}$ affects the system.

Table 1 shows that a P control mode produces a steady-state error e_{ss} proportional to $\bar{d}$ (magnitude of the disturbance) and inversely proportional to k_c (proportional gain). Even though the controller adjusts the manipulated variable continuously, ($u_{ss} \neq 0$), the controller cannot keep the controlled variable at the set point.

On the contrary, both the PI and PID control modes produce a steady-state error $e_{ss} = 0$, which means that both control modes are able to remove the step disturbance effect from the controlled variable by adjusting the manipulated variable ($u_{ss} \neq 0$). Note that for ease of discussion, we intentionally avoided adding a random component to the system. In the presence of a small amount of random noise, e_{ss}, y_{ss}, and u_{ss} also show the presence of noise, slightly fluctuating around their steady-state values. However, when the amount of random noise is not negligible, the control action

TABLE 1 Steady-state error (e_{ss}) and steady-state values of the controlled variable (y_{ss}) and manipulated variable (u_{ss}) of a closed-loop system subject to a step disturbance when a P, PI, or PID control mode is in place.

Control mode	Steady-state error	Controlled variable	Manipulated variable
	e_{ss}	y_{ss}	u_{ss}
P	$-\frac{\bar{d}}{1+k_c k_p}$	$\frac{\bar{d}}{1+k_c k_p}$	$-\frac{k_c \bar{d}}{1+k_c k_p}$
PI, PID	0	$0\ (= y_{sp})$	$-\frac{\bar{d}}{k_p}$

design should also consider noise attenuation. Hereafter, we assume that the random noise affecting the system is small and that the controller can cope with it.

2.4.2 | Scenario II: Ramp disturbance

In this second scenario, suppose that a ramp disturbance of a slope $\hat{d}$ affects the system, that is, $d(s) = \hat{d}/s^2$. Similar to the step disturbance scenario, the steady-state values e_{ss}, y_{ss}, and u_{ss} (given in Table 2) can be determined for the P, PI, and PID control actions when a ramp disturbance affects the system. Note that when a P action is in place, the steady-state values e_{ss}, y_{ss}, and u_{ss} do not converge to a finite value. It is thus evident that a P control mode is not suitable if ramp disturbances are expected. When a PI or a PID control action is in place, the steady-state value of the controlled variable y_{ss} converges to a constant finite value, although different from the set point value y_{sp}, proportional to $\hat{d}$ (slope of the ramp) and τ_I (the integral time constant) and inversely proportional to k_c (the proportional gain of the controller). In summary, a PID control scheme cannot remove the steady-state error for a ramp disturbance.

3 | SPC AND EPC USED CONCURRENTLY

As mentioned earlier, the aim of EPC is to reduce variation by keeping the variable of interest around a set point through continuous adjustments of another variable. SPC also aims at reducing variation, but in this case, the aim is the detection and subsequently the removal of the disturbance from the system that causes more than an expected amount of variation. In that regard, EPC can be seen as relieving the symptom (i.e., excessive variation) without necessarily identifying and removing the cause of the problem. Its prevalence has primarily been due to its ease of application at a low cost. However, continuous monitoring of the process via SPC can be more effective than EPC alone when the disturbance is persistent, for example, when the resulting disturbance signature in the manipulated variable is a mean shift and adjustments are relatively costly.

SPC is by nature a real-time scheme. That is, process surveillance is performed as an on-going process. The idea is to focus on a variable of interest or a statistic directly related to the state of the process and based on its current value declare the process in-control or out-of-control. Hence, SPC can ultimately be seen as a decision-making scheme and as is the case with any decision made, two potential errors can occur: labeling a process out-of-control when in fact it is in-control (false alarm or false positive) and vice versa (missed detection or false negative). The probability of these errors happening can be calculated if the distribution of the statistic being monitored is known. In most cases, even if the distribution is identified, its parameter(s) needs to be estimated. For that, an off-line study (also called Phase I study) is performed where the data are collected following the data collection scheme set for the real-time monitoring and parameters are estimated accordingly. These estimates can be used to calculate the probabilities of a false alarm or a missed detection for a given change in the distribution parameters. The uncertainties associated with the parameter estimation also affect the calculation of these probabilities, but this is beyond the scope of this article.

In SPC, the primary decision-making tool is a control chart on which the statistic of interest is plotted along with a threshold(s) (also called the control limits) within which the statistic is expected to be for an in-control process. The threshold is obtained through the probability distribution of the statistic. Akin to Type I and Type II errors in hypothesis testing, the decision errors (false alarm and missed detection) compete leading to a choice of threshold where a balance between the probabilities of these two errors is established. Similarly, the commonly used performance measure for a control chart is its average run length (ARL), the expected amount of observations collected before an out-of-

TABLE 2 Steady-state error (e_{ss}) and steady-state values of the controlled variable (y_{ss}) and manipulated variable (u_{ss}) of a closed-loop system subject to a ramp disturbance when a P, PI, or PID is in place. Note that the signs of ∞ need to be changed to the opposite signs if $\hat{d} < 0$

Control mode	Steady-state error	Controlled variable	Manipulated variable
	e_{ss}	y_{ss}	u_{ss}
P	$-\infty$	$+\infty$	$-\infty$
PI, PID	$-\frac{\tau_I}{k_c k_p}\hat{d}$	$\frac{\tau_I}{k_c k_p}\hat{d}$	$-\infty$

control signal is seen. For an in-control process (and assuming that the observations are independent), the ARL (called ARL_0) is the inverse of the false alarm rate, and for an out-of-control process, it is the inverse of the probability of detection (ARL_1). For further details on control charts, we refer the reader to Montgomery[21].

3.1 | Control charts for individual observations

The Shewhart chart and the time-weighted control charts, such as the cumulative sum (CUSUM) control chart[22] or the exponentially weighted moving average (EWMA) control chart,[23] are commonly applied univariate control charts for individual observations. In this article, we elaborate on the CUSUM chart in somewhat more detail below as it performs well in detecting small shifts in the mean of a process, which we will encounter in the upcoming examples.

As the name indicates, the statistic for the CUSUM chart is obtained through the accumulated deviation from the expectation. Since in this work we mainly focus on the shift in the process mean, we apply CUSUM on the deviations of the observations from their expectation. To avoid scaling issues, we standardize the variable by taking its average out and dividing it by the sample standard deviation both obtained from the Phase I study. For the standardized CUSUM chart, the following two statistics are recorded for upward and downward shifts in the mean, respectively:

$$\begin{aligned} C_i^+ &= max\left[0, \frac{x_i - \mu_0}{\sigma} - k + C_{i-1}^+\right] \\ C_i^- &= min\left[0, \frac{x_i - \mu_0}{\sigma} + k + C_{i-1}^-\right] \end{aligned} \tag{9}$$

where μ_0 is the target value and σ is the standard deviation of the process variable x estimated, respectively, using the average and the sample standard deviation, and k is the reference or slack variable. The slack variable is used to avoid excessive false alarms and often chosen to be halfway between the target mean (μ_0) and out-of-control mean (μ_1) that is of interest for fast detection. The shift is often given in standard deviation units $\mu_1 = \mu_0 + \delta\sigma$. For the standardized variable, we then have $k = \delta/2$. Furthermore, the starting values are $C_0^+ = C_0^- = 0$.

The process is deemed out-of-control if either C_i^+ or C_i^- exceeds a critical value h. In many practical applications, $h = 5$ is often recommended as it provides a good balance of the in-control run length (ARL_0) and a short out-of-control run length (ARL_1) for shifts of 1σ in the process mean. The reader is referred to Montgomery[21] for further details about the CUSUM chart.

3.2 | Disturbance signatures in the controlled and manipulated variables

Depending on the applied control mode, (P, PI, or PID), the steady-state error in the controlled variable due to a step disturbance can be eliminated partially or completely through continuous adjustments of the manipulated variable (see Table 1). Moreover, Table 2 shows that variations of the PID control scheme, at best, only reduces the steady-state error in the controlled variable that a ramp disturbance induces. Consequently, there are circumstances where the disturbance signature appears in both the controlled and manipulated variables. If control charts are applied to both the controlled and manipulated variables, then both may issue an out-of-control signal, albeit for different reasons. The choice of control mode will also influence how the two charts will signal for different types of disturbances. Moreover, the disturbance types and magnitudes and the control chart and its parameters influence the charts' detection abilities. General recommendations of which control charts and control chart parameters settings are most appropriate for each variable in a process under feedback control are hard to give. However, if we know which control action is in place (P, PI, or PID), we would know a priori in which variables a step or a ramp disturbance will manifest itself and what kind of signature to expect (mean shift or trend). Such knowledge can guide the choice of a control chart and eventually the disturbance identification during the monitoring phase (Phase II).

Table 3 indicates on which variables (manipulated and/or controlled) the signature of a step or ramp disturbance can be found depending on the control mode used (P, PI, or PID) and if this signature is of the type 'mean shift' or 'trend'.

As shown in Table 3, the signature of a step disturbance can be found as a mean shift solely in the manipulated variable when the PI or PID control modes are used, whereas the signature of a step or ramp disturbance will be visible as

TABLE 3 Signatures of step and ramp disturbances on the controlled and manipulated variables depending on the control mode (P, PI, or PID).

	Step disturbance		Ramp disturbance	
Control mode	Controlled variable	Manipulated variable	Controlled variable	Manipulated variable
P	Mean shift	Mean shift	Trend	Trend
PI, PID	No signature	Mean shift	Mean shift	Trend

mean shifts or trends in both the controlled and manipulated variables in the remaining cases. In the former case, a properly chosen and parameterized control chart applied to the manipulated variable should issue an out-of-control signal if the controller is working properly. In latter cases, control charts on both the controlled and manipulated variables can be expected to issue out-of-control signals. Consequently, the typical approaches of monitoring either only the manipulated variable or both the manipulated and controlled variables in one multivariate control chart are expected to result in an out-of-control signal for all cases in Table 3. However, it should be underscored that the multivariate chart in itself would be less informative as monitoring the controlled and manipulated variables separately allows for more insight, for example, regarding how well the controller is performing, or if the controller is malfunctioning as well as to offering clues of what type of disturbance may be affecting the system.

4 | EXAMPLE 1—HEAT-EXCHANGER WITH A P CONTROLLER

In this example, we will study how the disturbance signatures manifest themselves in the controlled and manipulated variables of a simulated SISO system controlled by a proportional (P) controller to exemplify some of the theoretical results presented in Tables 1–3.

4.1 | Heat-exchanger and controller transfer functions

The model of the process plant, $g_p(s)$, obtained empirically by Romagnoli and Palazoglu,[20] represents a heat exchanger where the input–output relationship between the exit temperature (°C) of the process stream and the steam flow rate (ml/s) is expressed as

$$g_p(s) = \frac{\text{Temperature}}{\text{Steam Flow Rate}} = \frac{2.58e^{-14.61s}}{33.73s+1} \tag{10}$$

Note that using the properties of logarithms, it can be shown that Equation 10 satisfies Equation 5.

In this example, we have assumed that the heat exchanger is operating in closed-loop with a P controller:

$$g_c(s) = \frac{u(s)}{e(s)} = k_c = 0.8315 \tag{11}$$

where the proportional gain, k_c, was tuned using the Ziegler–Nichols technique. The reader is referred to Romagnoli and Palazoglu[20] for additional information about the Ziegler–Nichols technique for tuning PID controllers. The system was simulated in Matlab/Simulink® adding a random, normally distributed noise with zero mean and constant variance, $\sigma^2 = 0.002$.

4.2 | Scenario I: Step disturbance

A dataset including 840 samples of the controlled and manipulated variables was produced through simulation. A step-change disturbance of magnitude $\bar{d} = 0.25$ was introduced at the 440th observation and onwards.

4.2.1 | Disturbance signatures in the controlled and manipulated variables

The left half of Table 4 provides the mean, standard deviation, and steady-state values of the temperature (controlled variable) and steam flow rate (manipulated variable) when no disturbance is present, $\bar{d} = 0$. The mean and the standard deviation were calculated by removing the start-up phase of the process that was deemed to be completed after 40 samples, creating a Phase I data set consisting of 400 observations. All theoretical steady-state values, y_{ss} and u_{ss}, were calculated using the formulas in Table 1.

As shown in the left half of Table 4, the theoretical steady-state values y_{ss} and u_{ss} are similar to the respective mean values of the controlled and manipulated variables. Differences from the theoretical steady-state values are due to the added random noise. As the mean temperature is close to zero, it is clear that the P controller is able to keep the controlled variable at its set point when the process is running without any disturbance ($\bar{d} = 0$).

The right half of Table 4 provides the theoretical steady-state values y_{ss} and u_{ss} when the process is affected by a step disturbance of magnitude $\bar{d} = 0.25$ as well as the estimated means and standard deviations of the controlled and manipulated variables. The P controller is no longer able to maintain the temperature at the desired set point value. The mean of the controlled variable when the disturbance is active is 0.090, close to the theoretical steady-state value. As outlined in Table 3, when a P controller is used a step disturbance results in a mean shift on both the controlled and manipulated variables.

Figure 2a,b provide the time series plots of the 800 observations of the temperature (controlled variable) and steam flow rate (manipulated variable). Both variables show a transient and then a clear, sustained shift after the introduction of the step disturbance. For this scenario we do not provide control charts as any univariate control chart would be able to detect these apparent mean shifts in the variables quickly.

4.3 | Scenario II: Ramp disturbance

A new dataset with 840 observations of the controlled and manipulated variables from the heat-exchanger example was generated. This time, a ramp disturbance with a slope $\hat{d} = 0.01$ was introduced at the 440th observation and onwards.

TABLE 4 Mean, standard deviation, and steady-state values of the controlled and manipulated variables when $\bar{d} = 0$ and $\bar{d} = 0.25$ (step disturbance).

	Phase I: $\bar{d} = 0$				Phase II: Step disturbance $\bar{d} = 0.25$			
Variable	**Mean**	**SD**	**Steady-state value**		**Mean**	**SD**	**Steady-state value**	
Temperature (°C):Controlled	0.001	0.047	y_{ss}	0	0.090	0.045	y_{ss}	0.080
Steam flow rate (ml/s):Manipulated	−1.066	38.87	u_{ss}	0	−74.937	37.74	u_{ss}	−66.109

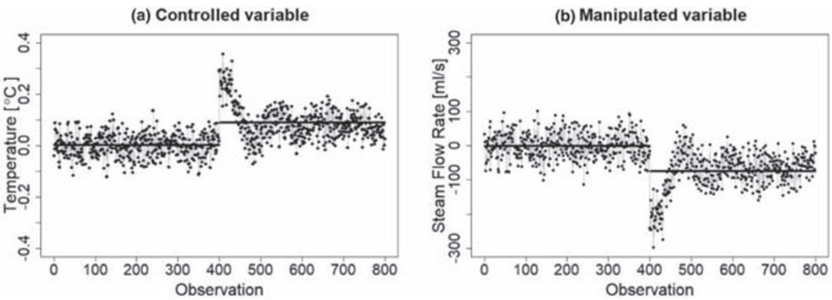

FIGURE 2 Time series plots of the controlled (a) and manipulated (b) variables. The step disturbance of magnitude $\bar{d} = 0.25$ occurs at the 400th observation. The horizontal lines in the time series plots indicate the mean values of the controlled and manipulated variables in Phase I and Phase II

4.3.1 | Disturbance signatures in the controlled and manipulated variables

Table 5 shows the mean, standard deviation, and the theoretical steady-state values of the controlled and manipulated variable without the disturbance ($\hat{d} = 0$) and after the disturbance is introduced ($\hat{d} = 0.01$). Again, the mean and the standard deviation when there is no active disturbance were calculated by removing the first 40 observations of the start-up phase. The theoretical steady-state values y_{ss} and u_{ss} are zero when $\hat{d} = 0$ (see Table 1) and drawn from Table 2 when $\hat{d} = 0.01$.

As expected, the simulation results summarized in Table 5 confirm those presented in Table 3. The P controller cannot keep the temperature at its set point when the ramp disturbance is introduced, and the disturbance signature is visible on the mean values of both the controlled and manipulated variables in Phase II. Note that the theoretical steady-state values of the controlled and manipulated variables approach infinity as the ramp disturbance approaches infinity. That is, the values of the controlled (manipulated) variable keep increasing (decreasing) as long as the ramp disturbance continues to increase (decrease). Figure 3a,b provide a graphical representation of this behavior where the temperature continues to increase and move away from its set point while the steam flow rate keeps decreasing to counteract the disturbance. Similar to the previous scenario, these trends in both variables are apparent, so we do not provide control charts. Any univariate control chart would be able to signal an out-of-control situation in these variables quickly.

4.4 | Remarks on the heat-exchanger example

The heat-exchanger example illustrates that for the P control mode, the adjustments of the manipulated variable at best reduces the effect of a step disturbance on the controlled variable while a ramp disturbance will affect the controlled variable with a continuously increasing difference between the controlled variable and its set point over time. Consequently, the signatures of a step or ramp disturbance are present on both the controlled and manipulated variables for the P control mode. When introducing control charts to monitor a process controlled by the P control mode, it may suffice just to monitor the controlled variable in a univariate chart. Since the disturbance signal in the controlled variable may be a small mean shift, a robust choice may be to use a time-weighted control chart, such as a CUSUM chart to increase detection capability. Regarding diagnosing disturbances, note that the disturbance signatures on the controlled and manipulated variables keep their step and ramp characteristics in both cases. As we have illustrated, a step disturbance induces mean level shifts on both the controlled and manipulated variables for the P control mode whereas a ramp disturbance induces an increasing/decreasing trend in both variables. The analyst can thus use these known patterns to classify which type of disturbance is affecting the process, given the knowledge that the P control mode is in use.

TABLE 5 Mean, standard deviation, and steady-state values of the controlled and manipulated variables when $\hat{d} = 0$ and $\hat{d} = 0.01$ (ramp disturbance).

	Phase I: $\hat{d} = 0$				Phase II: Ramp disturbance $\hat{d} = 0.01$			
Variable	Mean	SD	Steady-state value		Mean	SD	Steady-state value	
Temperature (°C): Controlled	0.0004	0.045	y_{ss}	0	0.4244	0.044	y_{ss}	$+\infty$
Steam flow rate (ml/s): Manipulated	−0.3322	37.695	u_{ss}	0	−352.873	37.196	u_{ss}	$-\infty$

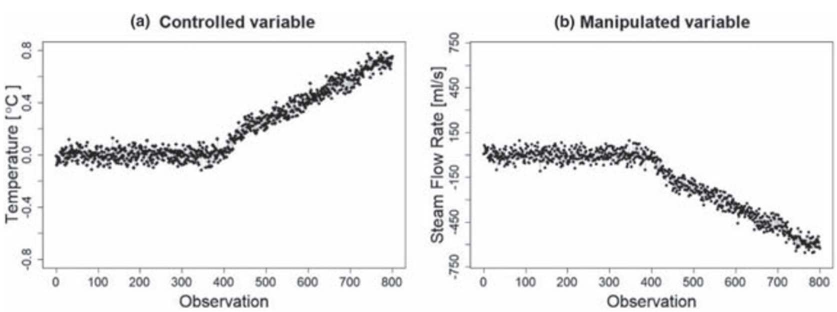

FIGURE 3 Time series plots of the controlled (a) and manipulated (b) variables. A ramp disturbance of a slope $\hat{d} = 0.01$ occurs at 400th observation

5 | EXAMPLE 2—STEEL ROLLING MILL WITH A PI CONTROLLER

In this second example, we further illustrate the theoretical results presented in Tables 1–3 but now with a different example using the PI control mode.

5.1 | Steel rolling mill and controller transfer functions

Figure 4 depicts a steel rolling mill where steel bars pass through a pair of rolls to reduce their thickness. Each time a new bar engages the rolls, the load change produces a torque on the rolls that reduces their speed. This unwanted speed reduction can be avoided by designing a feedback control scheme that keeps the roll speed (the controlled variable) at the desired set point.[24] The steel rolling mills are usually equipped with a DC motor and a speed controller to maintain the speed of the rolls. That way, the DC motor can convert the electromotive force (the manipulated variable) into rotational energy according to the error fed back to the controller. The resulting feedback control scheme has a block diagram like the one shown in Figure 1, where the torque on the rolls due to the load change can be interpreted as a (known) constant disturbance.

The described system was implemented in Matlab/Simulink®, introducing a normally distributed random noise with zero mean and constant variance $\sigma^2 = 6 \times 10^{-5}$ chosen subjectively to provide a realistic simulation. The variance level was picked by trial and error. The employed DC motor has the following transfer function

$$g_p(s) = \frac{\text{Speed}}{\text{Voltage}} = \frac{2.857}{(\tau s + 1)} = \frac{2.857}{(0.086s + 1)} \tag{12}$$

A Matlab model of the DC motor by Elshamy[25] is available in the Matlab central file repository. For additional information about the transfer function and typical constants of the DC motor, see Dorf and Bishop (pp.70–73).[24]

A PI controller with the following transfer function

$$g_c(\mathrm{s}) = \mathrm{k_c}\left(1 + \frac{1}{\tau_I s}\right) = 0.175\left(1 + \frac{1}{0.086s}\right) \tag{13}$$

was implemented to keep the roll speed at the set point value. The controller parameters were tuned using the internal model control (IMC) rule by setting $\lambda = \tau/2$, which indicates a twice as fast closed-loop response time compared to the open loop. The reader is referred to Romagnoli and Palazoglu[20] for additional information about the IMC rules for tuning PID controllers.

5.2 | Scenario I: Step disturbance

Observations of the roll speed (controlled variable) and the voltage (manipulated variable) were collected in sequence during a continuous simulation of the process. Again, 840 observations were generated. The first 440 observations were collected under normal operating conditions, that is, $\bar{d} = 0$. The first 40 observations were excluded to remove the start-

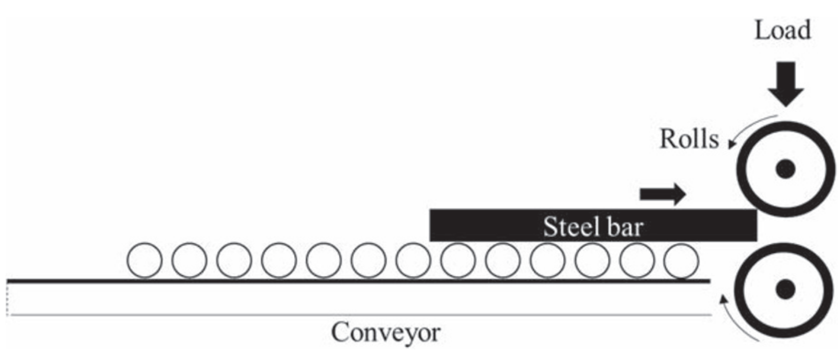

FIGURE 4 Steel rolling mill.
Figure inspired by Dorf and Bishop (2011)

up phase thus creating a Phase I data set of 400 observations. The last 400 observations constitute the Phase II dataset. A step disturbance in the torque of magnitude $\bar{d} = -0.0025$ was introduced at 440th observation and onwards.

5.2.1 | Disturbance signatures in the controlled and manipulated variables

Table 6 shows the mean, standard deviation, and theoretical steady-state values of the controlled and manipulated variables in Phase I and Phase II. The theoretical steady-state values y_{ss} and u_{ss} were calculated based on formulas in Table 1.

From Table 6, we see that mean values of the controlled and manipulated variables in Phase I are the same or similar to the theoretical steady-state values y_{ss} and u_{ss} indicating stable process operation during which the PI controller is able to keep the controlled variable at the set point. The mean value of the manipulated variable is close to the theoretical value, u_{ss}, and the small difference is due to the random noise. Table 6 also provides the theoretical steady-state values y_{ss} and u_{ss} in Phase II. Note that in a real process, these values are not known in advance since the disturbance types and magnitudes are unknown beforehand. However, in the specific situation that the analyst can assume that the disturbance is a step change, formulas in Table 1 may actually be used to estimate the magnitude of the disturbance $\bar{d}$ using the mean of the controlled (or manipulated) variable in Phase II as an estimate of the steady-state value y_{ss} (or u_{ss}).

Table 3 as well as the mean values of Phase II in Table 6 suggest that for the PI control mode, the signature of a step-change disturbance should only be visible as a mean shift in the manipulated variable. This means that the PI controller is able to keep the disturbance from affecting the controlled variable, and the disturbance signature is only identifiable in the manipulated variable.

5.2.2 | Monitoring the controlled and manipulated variables in control charts

Figure 5a,b show the time-series plots of the controlled and manipulated variables in both Phases I and II. A visual inspection of the time-series plots shows that the controlled variable does not exhibit a clear shift when the step disturbance is introduced. However, the time series plot of the manipulated variable seems to exhibit a slightly higher mean value after the step disturbance is introduced.

The Phase I and Phase II datasets with 400 observations each of the controlled and manipulated variables were used to create the control charts. Two CUSUM charts were applied to monitor the controlled and manipulated variables in Phase II, see Figure 5c,d. Note that in this example (as well as in the previous one), the observations in Phase I (no disturbance) are independent and normally distributed. Without active disturbances, the random variability of the controlled and manipulated variables comes from the added random, normally distributed noise. Throughout this example, we used the common choices of $k = 0.5$ and $h = 5$ for the CUSUM charts.[21] Other choices are possible, but here, we are mainly interested in illustrating how commonly used univariate control charts can be applied to the controlled and manipulated variables and not in optimizing the sensitivity of the charts. Under the assumption of time-independent observations, the selected CUSUM charts' parameters would result in an in-control ARL (ARL_0) of 465 observations.[21] We also tested other control charts with comparable detection abilities, such as a EWMA chart, but the results were similar. We excluded illustrations in other control charts to save space as a different choice of control chart does not change the conclusions.

From Figure 5c, we see that the CUSUM chart for the controlled variable does not issue any out-of-control signal. However, the CUSUM chart for the manipulated variable issues an out-of-control signal as the CUSUM passes the

TABLE 6 Mean, standard deviation, and steady-state values of the rolls' speed and voltage in Phase I and Phase II (step disturbance of magnitude $\bar{d} = -0.0025$).

	Phase I: $\bar{d} = 0$				Phase II: Step disturbance $\bar{d} = -0.0025$			
Variable	Mean	SD	Steady-state value		Mean	SD	Steady-state value	
Speed (rad/s): Controlled	0.000	1.278	y_{ss}	0	0.000	1.294	y_{ss}	0
Voltage (mV): Manipulated	0.042	0.007	u_{ss}	0	0.974	0.007	u_{ss}	0.875

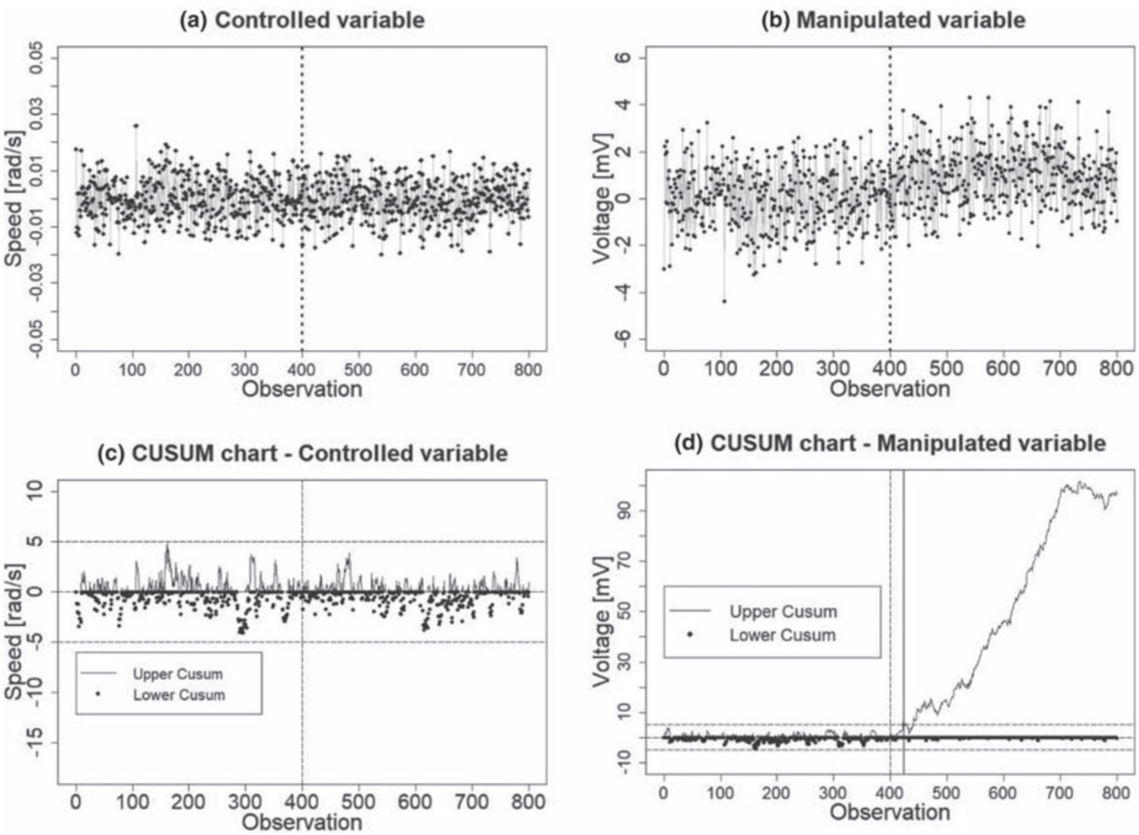

FIGURE 5 Time series plots of the speed (a) and voltage (b) variables. A step-change disturbance of magnitude $\bar{d} = -0.0025$ occurs at the 400th observation. The vertical dotted lines divide observations in Phase I and Phase II. (c) CUSUM chart for the controlled variable. (d) CUSUM chart for the manipulated variable. The vertical solid line indicates the out-of-control signal at the 424th observation

control limit at the 424th observation in Figure 5d. From the analysis of the control charts, it is possible to conclude that the controlled variable is in control operating close to or at its desired set point and that the PI controller prevents the disturbance from affecting the controlled variable. The signature of the disturbance is instead displaced to the manipulated variable. The analyst may at this point undertake a root cause search for the disturbance if the sustained control action is causing unwanted costs or other negative consequences.

5.3 | Scenario II: Ramp disturbance

Another dataset of 840 observations was once again produced for both the controlled and manipulated variables following the same criteria of the previous step disturbance scenario. Again, the first 40 observations were removed, and the Phase I dataset includes 400 observations. This time, a ramp disturbance with a slope $\hat{d} = -0.025$ was introduced at 440th observation and onwards.

5.3.1 | Disturbance signatures in the controlled and manipulated variables

Table 7 shows the mean, standard deviation, and theoretical steady-state values of the controlled and manipulated variables in Phase I ($\hat{d} = 0$) and in Phase II ($\hat{d} = -0.025$). The theoretical steady-state values, y_{ss} and u_{ss}, are zero in Phase I whereas their values in Phase II were obtained using the formulas in Table 2.

TABLE 7 Mean, standard deviation, and steady-state values of the rolls' speed and voltage in Phase I and Phase II (ramp disturbance with a slope $\hat{d} = -0.025$).

	Phase I: $\hat{d} = 0$				Phase II: Ramp disturbance $\hat{d} = -0.025$			
Variable	**Mean**	**SD**	**Steady-state value**		**Mean**	**SD**	**Steady-state value**	
Speed (rad/s): Controlled	0.000	0.0077	y_{ss}	0	−0.004	0.0073	y_{ss}	−0.004
Voltage (mV): Manipulated	−0.059	1.348	u_{ss}	0	8.096	1.270	u_{ss}	$+\infty$

5.3.2 | Monitoring the controlled and manipulated variables in control charts

Figure 6a,b show the time-series plots of the controlled and manipulated variables. A visual inspection of the time-series plots shows that after the disturbance introduction (400th observation) there is an obvious increasing trend in the manipulated variable and a slight decrease in the mean of the controlled variable. In this example, the PI controller does a fair job of keeping the roll speed close to its set point by rapidly increasing the voltage. Based on theoretical results summarized in Tables 2–3, the signature of a ramp disturbance should remain on both the controlled and manipulated variables, but a signature in the controlled variable is perhaps not evident from a visual inspection of Figure 6a. Therefore, we move on to analyze the control charts for the controlled and manipulated variables. Again, we used a CUSUM chart for both the controlled and the manipulated variables using the same control chart parameters as in the previous step disturbance scenario, see Figure 6c,d.

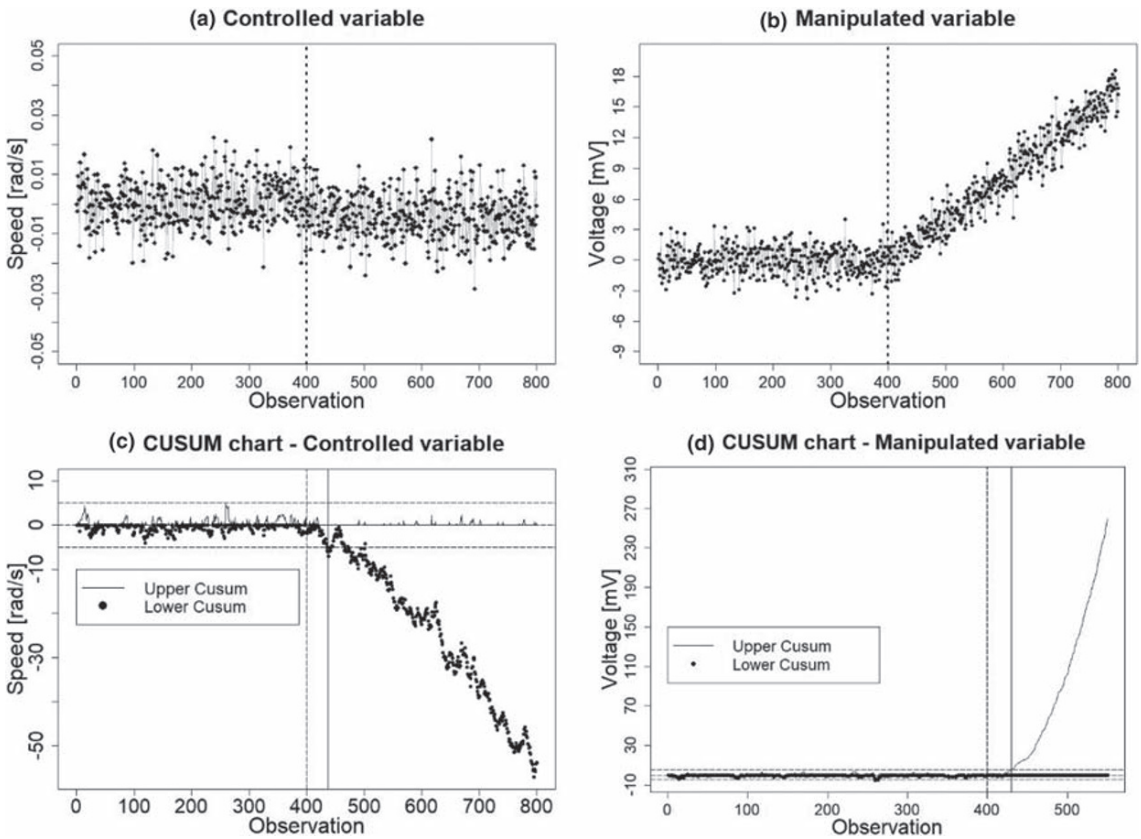

FIGURE 6 Time series plots of the speed (a) and voltage (b) variables. A ramp disturbance with a slope $\hat{d} = -0.025$ is introduced at the 400th observation. The vertical dotted lines divide Phase I and Phase II data. (c) and (d) are CUSUM charts for the controlled and manipulated variables, respectively. The vertical solid lines indicate the out-of-control signal at the 430th observation for the controlled variable and at 437th observation for the manipulated variable

The CUSUM chart for the controlled variable issues an out-of-control signal indicating a decrease in the mean of the controlled variable, see Figure 6 (c). Not surprisingly, the CUSUM chart in Figure 6 (d) for the manipulated variable also issues an out-of-control signal with an ever-increasing voltage. In a real-life scenario, however, the voltage would only be allowed to increase to a certain limit before the process would be shut down.

5.4 | Remarks on the steel rolling mill example

The steel rolling mill example shows how for a given implemented control scheme and process plant, monitoring the controlled and manipulated variables separately may be crucial for understanding and interpreting out-of-control situations in a closed-loop system. Monitoring both the controlled and the manipulated variables in separate charts allows the analyst to evaluate the performance of the controller but also to develop an increased understanding of which type of disturbance is active in the system (step or ramp). For the PI (or PID) control mode, a step disturbance results in a mean shift signature in the manipulated variable only. However, a ramp disturbance induces a mean level shift in the controlled variable and an increasing or decreasing trend in the manipulated variable. In which of the variables a disturbance signal can be detected depends on the control mode used, the type of disturbance affecting the system, and the choice of control chart, as we have illustrated in the examples above and in Tables 1–3. By illustrating the behavior for a P control mode in Example 1 and a PI control mode in Example 2 for the step and ramp disturbances, we intentionally exemplify all interesting disturbance signatures covered in Table 3: ‘no signature’, ‘mean shift’, and ‘trend’.

A generalization on the proper choice of control chart for a general EPC application is not self-evident. The Shewhart chart for individual observations would be fast to signal when a large shift occurs, such as a dramatic step change. However, at times, the remaining signals in the controlled and/or manipulated variables may be much smaller and time-weighted control charts, such as a CUSUM or EWMA chart, would be recommended for fast and effective shift detection.

6 | CONCLUSIONS AND DISCUSSION

This article explores and discusses the concurrent use of EPC and SPC, and more specifically, the implications of monitoring variables from a system under feedback control through control charts. From an SPC perspective, the control action increases complexity and influences the behavior of the process variables when a disturbance affects the process. The analyst may even fail to detect a disturbance affecting the system when monitoring only the controlled variable in an EPC scheme. This mistake may occur since the controlled variable in a feedback controller usually is ‘the’ important process output that the naïve analyst may think warrants monitoring through SPC.

In this article, we provide formulas for calculating the theoretical steady-state values of the controlled and manipulated variables in a SISO system for the P, PI, and PID control modes, given that the system is affected by step or ramp disturbances. In the two simulated examples, we illustrate how step and ramp disturbance signatures manifest themselves in the controlled and manipulated variables for the above-mentioned control modes. The control mode used, the disturbance type, and the choice of control chart determine whether the disturbance signature can be detected in the controlled and/or manipulated variables. For step disturbances, PI and PID controllers can maintain the controlled variable on target by adjusting the manipulated variable thereby displacing the disturbance signature to the manipulated variable. For P controllers and for ramp disturbances combined with all tested control modes, the disturbance affects both the controlled and the manipulated variables.

Consequently, irrespective of the control mode applied, properly chosen and parameterized control charts monitoring the controlled and/or the manipulated variables should be able to signal out-of-control situations for step and ramp disturbances affecting the system. Our recommendation is to monitor both the controlled and manipulated variables when applying SPC on a process under feedback control. A combined study of the disturbance signatures in both control charts can also give important information on how well the controller is performing in terms of disturbance elimination as well as clues of what type of disturbance is affecting the system (step or ramp). Indeed, for the P, PI, and PID control modes, univariate control charts monitoring only the manipulated variable may also issue out-of-control signals for step and ramp disturbances. Another alternative is to use a bivariate chart based on both the controlled and manipulated variables. However, these choices would potentially be at the expense of gaining a deeper process insight. For example, if the control chart for the controlled variable is in control and the control chart for the manipulated variable is out-of-control, one can infer that the controller is performing satisfactorily in keeping the controlled variable at its set point by transferring the disturbance (variability) from the controlled variable to the manipulated variable. The disturbance signature in the manipulated variable may, of course, be a lingering problem if the sustained adjustment incurs increased costs or other negative consequences. The goal of SPC in this case is to eliminate the assignable cause to cut potential unwanted costs of corrective adjustments by the controller. Moreover, if the control charts for both manipulated and controlled variables are out-of-control, we may conclude either that the controller is not working or that the controller is working but unable to counteract the disturbance completely. In this case, understanding and eliminating the assignable cause is even more important to reduce unwanted costs of waste, resource consumption, production of off-spec products, safety risk, or to consider potential changes on the controller design if, for example, the assignable cause occurs frequently.

This article focuses on SISO systems. Future research will explore multivariate systems and the potential of dividing controlled and manipulated variables in separate multivariate control charts for increased process insight.

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REFERENCES

1. Janakiram M, Bert Keats J. Combining SPC and EPC in a hybrid industry. *J Qual Technol*. 1998;30(3):189-200.
2. Box GEP, Kramer T. Statistical process monitoring and feedback adjustment: a discussion. *Dent Tech*. 1992;34(3):251-267. https://doi.org/10.2307/1270028, http://www.jstor.org/stable/1270028
3. Box GEP, Luceño A. *Statistical control by monitoring and feedback adjustment*. New York: Wiley; 1997.
4. MacGregor JF. On-line statistical process control. *Chem Eng Prog*. 1988;84(10):21-31.
5. Wiel V, Scott A, Tucker WT, Faltin FW, Doganaksoy N. Algorithmic statistical process control: concepts and an application. *Dent Tech*. 1992;34(3):286-297.
6. MacGregor JF. Statistical process monitoring and feedback adjustment: discussion. *Dent Tech*. 1992;34(3):273-275. https://doi.org/10.2307/1270030, http://www.jstor.org/stable/1270030
7. Tucker WT, Faltin FW, Vander SA, Wiel. Algorithmic statistical process control: an elaboration. *Dent Tech*. 1993;35(4):363-375.
8. Faltin FW, Hahn GJ, Tucker WT, Vander Wiel SA. Algorithmic Statistical Process Control: Some Practical Observations. *Int Stat Rev/-Revue Internationale De Statistique*. 1993;67-80.
9. Del Castillo E. *Statistical Process Adjustment for Quality Control*. 369 Wiley-Interscience; 2002.
10. Vining G. Technical advice: statistical process control and automatic/engineering process control. *Qual Eng*. 2010;22(3):222-224.
11. Vander Wiel SA. Monitoring processes that wander using integrated moving average models. *Dent Tech*. 1996;38(2):139-151.
12. Montgomery DC, Bert Keats J, Runger GC, Messina WS. Integrating statistical process control and engineering process control. *J Qual Technol*. 1994;26(2):79-87.
13. Keats JB, Montgomery DC, Runger GC, Messina WI. Feedback control and statistical process monitoring. *Int J Reliab Qual Saf Eng*. 1996;03(03):231-241. https://doi.org/10.1142/S0218539396000168
14. Montgomery DC, Keats JB, Yatskievitch M, Messina WS. Integrating statistical process monitoring with feedforward control. *Qual Reliab Eng Int*. 2000;16(6):515-525. doi: AID-QRE359>3.0.CO;2-I. https://doi.org/10.1002/1099-1638(200011/12)16:63.0.CO;2-I
15. Capilla C, Ferrer A, Romero R, Hualda A. Integration of statistical and engineering process control in a continuous polymerization process. *Dent Tech*. 1999;41(1):14-28. https://www.scopus.com/inward/record.uri?eid=2-s2.0-0033080104&partnerID=40&md5=af38a26559849b6fbf58e22401749298
16. Capaci F, Vanhatalo E, Kulahci M, Bergquist B. The revised Tennessee Eastman process simulator as Testbed for SPC and DoE methods. *Qual Eng*. 2019;31(2):212-229. https://doi.org/10.1080/08982112.2018.1461905
17. Tsung F, Tsui K-L. A mean-shift pattern study on integration of SPC and APC for process monitoring. *IIE Trans*. 2003;35(3):231-242. https://doi.org/10.1080/07408170304365
18. Tsung F, Shi J, Wu CFJ. Joint monitoring of PID-controlled processes. *J Qual Technol*. 1999;31(3):275-285.
19. Tsung F. Improving automatic-controlled process quality using adaptive principal component monitoring. *Qual Reliab Eng Int*. 1999;15(2):135-142.
20. Romagnoli JA, Palazoglu A. *Introduction to Process Control*. 2nd ed. New York: CRC press, Taylor & Francis Group; 2012.
21. Montgomery D. *Statistical Process Control: A Modern Introduction*. 7th ed. Hoboken, New Jersey: Wiley; 2012.
22. Page ES. Continuous inspection schemes. *Biometrika*. 1954;41(1/2):100-115.
23. Roberts SW. Control chart tests based on geometric moving averages. *Dent Tech*. 1959;1(3):239-250.
24. Dorf RC, Bishop RH. *Modern Control Systems*. 12th ed. New Jersey: Pearson; 2011.
25. Elshamy W. DC Motor Model (Simulink) - File Exchange - MATLAB Central, accessed Jul 12, 2018, https://se.mathworks.com/matlabcentral/fileexchange/11587

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APPENDIX A.

As shown in the block diagram of Figure 1, the effect of $y_{sp}(s)$ and $d(s)$ on the controlled variable can be expressed through the representation of the set point tracking and disturbance rejection problems. This yields the additive output response as

$$y(s) = \frac{g_p(s)g_c(s)}{1+g_p(s)g_c(s)}\, y_{\mathrm{sp}}(s) + \frac{g_d(s)}{1+g_p(s)g_c(s)}\, d(s) \tag{A.1}$$

$$y(s) = g_{sp}(s)y_{sp}(s) + g_d(s)d(s)$$

where $g_{sp}(s)$ and $g_d(s)$ are the closed-loop transfer functions for the set point response and the disturbance response, respectively. Since the set point is assumed to be constant ($y_{sp}(s) = 0$), we are left with the expression,

$$y(s) = \frac{g_d(s)}{1 + g_p(s)g_c(s)} \, d(s) \tag{A.2}$$

The steady-state value of the controlled variable y_{ss} can then be determined using the final value theorem:

$$y_{ss} = \lim_{t \to +\infty} y(t) = \lim_{s \to 0} s\,[y(s)] = \lim_{s \to 0} s\,[g_d(s)d(s)] \tag{A.3}$$

or, incorporating Equation A.2 in Equation A.3:

$$y_{ss} = \lim_{s \to 0} s \left[\frac{g_d(s)}{1 + g_p(s)g_c(s)} \, d(s)\right] \tag{A.4}$$

where the steady-state value of the controlled variable y_{ss} becomes a function of the controller, $g_c(s)$, the process model $g_p(s)$, and the disturbance model, $g_d(s)$ as well as the disturbance signal. If y_{ss} is non-zero, it represents the steady-state error. Similarly, the steady-state value of the manipulated variable is calculated as,

$$u_{ss} = \lim_{t \to +\infty} u(t) = \lim_{s \to 0} s[u(s)] = \lim_{s \to 0} s[e(s)g_c(s)]$$

Since the error term is expressed as,

$$e(s) = y_{sp}(s) - y(s)$$

This yields,

$$u_{ss} = \lim_{s \to 0} s \left[\frac{-g_d(s)}{1 + g_p(s)g_c(s)} \, d(s)\right] g_c(s) = \lim_{s \to 0} s \left[\frac{-g_d(s)g_c(s)}{1 + g_p(s)g_c(s)} \, d(s)\right] \tag{A.6}$$