Received: 8 October 2023 | Revised: 11 November 2023 | Accepted: 14 November 2023

DOI: 10.1002/qre.3475

SPECIAL ISSUE ARTICLE

WILEY

Reliability analysis of a loading dependent system with cascading failures considering overloads

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Funding information
UTFORSK, Grant/Award Number: UTF-2017-four-year/10058; China Scholarship Council, Grant/Award Number: 202006150011; IKTPLUSS, Grant/Award Number: 309628

Abstract
In many production facilities, multiple components have to work together to share the overall workload on the entire system, leading to loading dependence and higher vulnerability to cascading failures. Additionally, overloading of one component can expedite the failures of others, exemplifying another form of loading dependence. In this study, we develop a system reliability analysis model for loading dependent systems considering overloads based on the Multi-state CASCADE model. By incorporating a time variable, the tailor-made model is able to characterize the duration of each generation in the cascading process, along with the cumulative time of the whole cascading process until the system collapse. A combination of analytical and simulation techniques is then employed to investigate how various potential influencing factors of loading dependence and cascading processes influence the system reliability. The results demonstrate that the effectiveness of proposed method in estimating the system reliability of the loading dependent system considering overloads. Such findings can improve the decision-makings of reliability prediction, system design, and maintenance optimization, especially in scenarios involving the loading dependent with cascading failures.

KEYWORDS
cascading failures, cascading time, multi-state CASCADE model, overloading, system reliability

1 | INTRODUCTION

Modern production systems become increasingly complicated with more interconnected devices and components. Interactions and dependences between various components can increase the likelihood of failures. When one component fails, the failure might propagate and cause failures of other components. This phenomenon is known as a cascading failure (CAF). CAFs are the major threat to electric power transmission systems,[1,2] transportation systems,[3] healthcare infrastructure systems,[4] safety instrumented systems,[5] chemical industry clusters,[6,7] and other complex network systems.[8–10] For example, the 2003 blackout in the Northeastern United States was initially triggered by the tripping of multiple power transmission lines, and finally led to considerable traffic congestion and communication breakdowns due to dependencies between various systems.[11] Another typical event triggered by CAFs is the domino accident that took place in Mexico in 1984.[12] This event led to a chain reaction, resulting in 12 to 20 subsequent accidents, and this catastrophic sequence of events claimed the lives of 650 people. Concerning the damages brought by CAFs, it is essential to consider the mechanism and consequence of potential CAFs in system design, system reliability analysis and maintenance optimization.

CAFs can be attributed to the structural or functional dependence among various components within a complex system. One of the prime examples of systems exhibiting such dependencies is a loading dependent system, in which all components collectively share the overall workload of the entire system.[13] CAFs have been found common in loading dependent systems.[14–16] Keizer et al.[14] explored a parallel and redundant system that experiences failure dependence due to load sharing and economic dependence. The study varies the extent of load sharing and the degradation process to uncover crucial insights into the optimal maintenance strategy. Brown et al.[15] proposed an innovative spatial model aimed at assessing the reliability of load-sharing systems, accounting for spatial dependence and proximity effects, which is suitable for systems, whether they can provide distance-related information or not. Sharifi et al.[16] presented a novel matrix-based approach for the multi-state and load-sharing components to calculate the system inspection cost. These contributions have served as inspiration for researchers to focus on the loading dependence mechanism and CAFs.

For a loading dependent system, degradation of components and the systems due to overloading "nodes" are very common, which nevertheless is not well studied in current research. Evaluation of the overloading state of components needs to be generalized. In a loading dependent system, "overloading" typically refers to a scenario where a component experiences the operational workloads surpassing its intended or specified capacity. The overloading state of a component may be caused by a sudden outside disturbance or by additional workloads allocated from other components during the cascading process, or by capacity decrement due to component degradation. These overloading components are frequently overlooked in comparison to outright failures, because they still continue to be functioning or at least partly functioning. However, despite not presenting as severe a risk as complete failures, these overloads can pose hazards and require costly maintenance or replacement of the associated components if not addressed promptly. Besides, overloading components may allocate loads to the others, and reduce the performance of more components, accelerate their deterioration, or even result in substantial failures. The following two examples can illustrate such situations. In a power system, if a transformer is overloading, it can lead to overheating or potential damage. Other transformers in the system are therefore required to share more workloads, possibly approaching or exceeding their expected capacity. In terms of a traffic network, if there is a traffic jam on a major road, causing an "overloading" major road, vehicles are then forced to pass through other roads, which triggers an increase in traffic flow on other roads and an intensification of congestion. These examples also demonstrate that the study of CAFs for loading dependent systems, considering overloading components, remains worthy to be investigated.

To address the above issue concerning overloading components, an extended multi-state CASCADE Model[13] has been developed based on some studies of classical CASCADE models.[17–19] Such a model[13] involves discussions on three types of stop scenarios of the cascading process. It reflects the cascading process mechanism more practically and provides a reference for cascading probability analysis of loading dependent systems subjected to CAFs affected by overloading components. However, some special scenarios need to be further explored, including the cascading scenarios where the cascading process stops and the system fails. Such a study is crucial for analyzing system reliability. Further investigations of the previous model on system reliability analysis are thus stimulated.

Reliability describes the ability of a system to sustain its regular operation in a specific period without failures. System reliability analysis can offer important information to guide design, operation, and maintenance strategies. There has been an uprising interest in the research of reliability analysis for loading dependent systems in recent years.[2,20–24] Some researches consider the internal degradation of the components in loading dependent systems. For example, Duan et al.[2] developed a novel cascading failure model to uncover the influence of route-choosing behavior on traffic network reliability with consideration of overload failures. Zhao et al.[20] explored a framework for modeling and analyzing the reliability of load-sharing systems consisting of identical components. Some other works include both the internal degradation and external shocks simultaneously. For example, Guo et al.[21] proposed an analytical model to compute the reliability with local load-sharing effect and shock processes for consecutive k-out-of-n: F systems. Nezakati et al.[24] investigated the conditional distribution considering the soft and hard failures, and developed a reliability model for the load sharing k-out-of-n system. Despite the varying approaches, the contributions outlined above collectively emphasize the importance of reliability analysis for loading dependent systems and their relevance to a wide range of complex systems. This has motivated us to enhance the multi-state CASCADE model to incorporate a system reliability perspective.

In the analysis of system reliability, it is essential to consider the failure scenarios and the duration for such a scenario to occur. For a loading dependent system with CAFs, the time for the system to fail can be naturally assumed to be closely related with the duration of the cascading process. The duration for a cascading process to proceed could be referred to as cascading time. The previously proposed model solely accounted for varying evolving scenarios of the cascading process, without considering the time for each generation or the time for the overall duration, let alone emphasizing the time at which system failure occurs during the cascading process. In the new model, we consider that there is a period for each generation in the cascading process. The cascading time of each generation, the duration of the whole cascading process, and the probability that the loading dependent system fails are also calculated. By integrating the cascading time and failure probability, the system reliability is expected to be estimated. Some discussions about reliability analysis of a loading dependent system considering overloads are given with case studies.

The rest of the paper is organized as follows. In Section 2, detailed descriptions of the theoretical basis and our previous works are presented. The method to consider cascading period and the system reliability function is discussed in Section 3. Section 4 illustrates the reliability analysis results by the case study, and conclusions are presented in Section 5.

2 | MULTI-STATE CASCADE MODEL WITH CASCADING TIME

This section provides the theoretical basis of this study by illustrating the multi-state CASCADE model briefly. The mechanism of multi-state CASCADE model considering overloading components in loading dependent systems and the cascading scenarios of cascading process are performed in our previous contribution.[13] In this model, the components in a loading dependent system have three states or performance levels: Normally Working, Overloading and Failed, which can be determined by the ratio of load to capacity, denoted as load/capacity ratio for abbreviation. The capacity of components decreases when the cascading failures propagate, due to the naturally degradation of components. The load on components depends on the initial workload, the sudden outside disturbance, and additional loads from overloading and failed components. The initial workload refers to the load that a component bears during its normal operation before encountering a sudden disturbance. The sudden outside disturbance can be a suddenly environmental change, such as temperature and pressure, etc., or manifest as unexpected damage, such as pollution or strikes. Additional loads arise due to overloads or the failures of other components with loading dependence. In a loading dependent system, when some components fail, they become incapable of handling the expected workloads, and additional loads are assigned to the components that are still functioning. Considering the actual situation, overloading components cannot bear all the expected workloads well, and also in turn allocate additional loads to the components that are still functioning. Therefore, this article considers the intermediate state between the Normally Working and Failed, defining as the Overloading state.

The introduction of a new state can bring challenges in modeling since it is difficult to achieve a classification that perfectly aligns with real-world situations. In addition, the cost required for detailed differentiation of component states when the model is applied in practice is also substantial. According to existing research,[13] despite the fact that overloading components exert certain influence on the cascading process, their impact on the probability distribution of the number of failed components and system reliability is less pronounced when compared to failed components. Therefore, although the value of the additional loads depends on the actual state of the component, it is of little significance to determine the additional loads based on the specific actual states of the overloading component. This study simplifies the additional loads into two types: those from failed components and those from overloading components.

This model acts as the foundation for our subsequent reliability analysis. According to the steps of the multi-state CASCADE model, a new algorithm that accounts for cascading time is structured as the following steps:

Step 0. All components are normally working initially.

Step 1. An initial outside disturbance d to all components triggers the initial event.

Step 2. Check states for each component i. If the load/capacity ratio of component is less than r^*, then it is working well. When the load/capacity ratio exceeds 1, the component fails. Otherwise, the component is overloading. Suppose that there are n_{fj} failed components and noj overloading components in the jth generation.

Step 3. The capacity of functioning components decreases due to natural degradation. The additional loads due to each failure and each overloading component in this generation on every functioning component in next generation are respectively.

Step 4. l_f and l_o. Additional loads $l_j = n_{fj}\, l_f + n_{oj} l_o$ are allocated and added to every functioning component. In this step, the new state of each component could be obtained according to the ratio of new workload and new capacity.

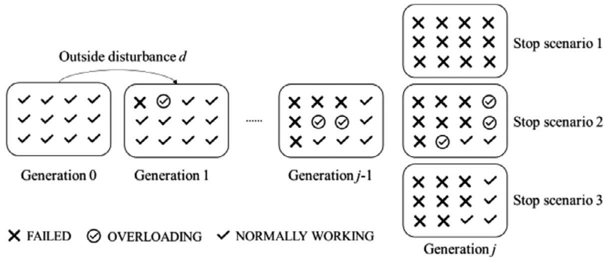

FIGURE 1 Cascading process of multi-state CASCADE model.

Step 5. Record the cascading time for every generation. Assume that the interval time of each generation in the cascading process is Y_j.

Step 6. If $n_{fj} = 0$, there is no more new failures in the jth generation, and the cascading process stops. Then the total cascading time from the start until the cascading process stops can be calculated by accumulation. Otherwise, the cascading process proceeds, then go to the next generation and iterate from step 2.

In this multi-state CASCADE model, some assumptions are made as below:

1. The system contains a finite number of components, denoted as n.
2. Every component within the system is identical, exchangeable, and nonrepairable.
3. The initial capacity of the components $c_0 = 1$, and their random loads l_i are uniformly distributed in $[0, 1]$.
4. As the cascading process proceeds, the capacity of each functioning component naturally decreases. The reduction in capacity for each generation is denoted as c_d.
5. The cascading process starts when there is a failure at time $T_0 = 0$.

This cascading process is shown in Figure 1. According to the above algorithm, the cascading process stops in the jth generation if and only if when there are no subsequent failed components in the generation j+1. If the remaining components tend to fail after a period, we consider it as a new cascading process with a new generation 0. When the cascading process ends, this does not imply that all components fail. However, when all the components fail in the jth generation of the cascading process, the system fails, and the cascading process stops in the jth generation since there are no new failures in the generation j+1.

There are plenty of cascading scenarios for the cascading process to proceed. When the cascading process stops, there are three types of scenarios, denoted as stop scenarios:

·Stop scenario 1: All components fail (cascading process stops, the system fails);

·Stop scenario 2: The load/capacity ratio of the functioning component is less than the failure threshold, and there exist some overloading components (cascading process stops, the system does not fail);

·Stop scenario 3: The load/capacity ratio of the functioning component is less than r^*, and all components work normally (cascading process stops, the system does not fail).

Based on the multi-state CASCADE model proposed in Ref.,[13] the probability that the number of failed components and overloading components in every generation follows $(s_0,\ s_1, \ldots s_j)$ until the jth generation is given by Equation (1).

$$P\left[S_j = s_j, \cdots, S_0 = s_0\right] = \frac{n!}{n_{f0}!n_{o0}!n_{w0}!}\alpha_0^{n_{f0}}\beta_0^{n_{o0}}\gamma_0^{n_{w0}}\frac{(n-u_0)!}{n_{f1}!n_{o1}!n_{w1}!}\alpha_1^{n_{f1}}\beta_1^{n_{o1}}\gamma_1^{n_{w1}}\cdots\frac{\left(n-u_{(j-1)}\right)!}{n_{fj}!n_{oj}!n_{wj}!}\alpha_j^{n_{fj}}\beta_j^{n_{oj}}\gamma_j^{n_{wj}} \tag{1}$$

In Equation (1), n_{fj}, n_{oj}, n_{wj} are respectively the numbers of failed components, overloading components and normally working components in the jth generation. u_j is the total number of the failed components until the jth generation. v_j is the number of overloading components in the jth generation. α, β, and γ are the indices used to abbreviate the probability of components in different states. More illustrations about the indices and this equation could be referred to Ref.[13]

Suppose that cascading process stops and all components fail in the jth generation, it could be obtained that $n_{f(j+1)} = 0$, then $0 = n_{f(j+1)} = n_{f(j+2)} = \ldots$. Besides, since $u_j = n$ in this stop scenario, there are no more subsequent failures in following generations. In this case, we have

$$P\left[S_{j+1} = s_{j+1} \mid S_j = s_j, \dots, S_0 = s_0\right] = 1 \tag{2}$$

for $n_{f(j+1)} = 0$.

By multiplying Equation (1) with Equation (2), we could derive Equation (3) to verify the distribution associated with the stop scenario 1.

$$P\left[S_{j+1} = s_{j+1}, \cdots, S_0 = s_0\right] = \frac{n!}{n_{f0}!n_{o0}!n_{w0}!}\alpha_0^{n_{f0}}\beta_0^{n_{o0}}\gamma_0^{n_{w0}}\frac{(n-u_0)!}{n_{f1}!n_{o1}!n_{w1}!}\alpha_1^{n_{f1}}\beta_1^{n_{o1}}\gamma_1^{n_{w1}}\cdots\frac{\left(n-u_{(j-1)}\right)!}{n_{fj}!n_{oj}!n_{wj}!}\alpha_j^{n_{fj}}\beta_j^{n_{oj}}\gamma_j^{n_{wj}} \tag{3}$$

The probability distribution of cascading scenarios, represented by $(s_0,\ s_1, \dots s_{j+1})$, is provided by the multi-state CASCADE model. Using this model, the probability distribution of overall scenarios where the cascading process stops in the jth generation could be identified. Furthermore, with the inclusion of cascading time, the system reliability analysis could then be conducted.

3 | SYSTEM RELIABILITY ANALYSIS

This section provides the reliability analysis of a loading dependent system based on the multi-state CASCADE model. As distinguished before, the criterion to determine if the cascading process stops in the jth generation is whether there are new failed components in the generation j+1. The criterion to determine if the system fails is whether all the components fail. The situations in which the cascading process ends following stop scenario 1 are examined to assess system reliability.

Since the cascading process can evolve in various ways, there are several scenarios where the cascading process stops and the system fails in the jth generation. It is necessary to determine the probability of each scenario resulting in system failure in the jth generation. The overall system failure probability is accomplished by summing the probabilities of all the scenarios resulting in system failure in the jth generation. Equation (3) represents one cascading scenario of the cascading process, which follows $(s_0,\ s_1, \dots s_j)$. By summing all the cascading scenarios where the cascading process ends and the system fails in the jth generation, we can obtain Equation (4). The cascading scenarios encompass situations where there are varying number of failed and overloading components for each generation. The equation represents the probability distribution of S_j, as shown the probability that the cascading process ends and the system fails at the jth generation, no matter how the cascading process proceeds before the jth generation. The probability distribution of S_j is crucial to evaluate the system reliability when the cascading process proceeds to the jth generation.

$$\begin{aligned} P\left[S_j = s_j\right] = &\sum_{n_{f0}=1}^{n-j}\sum_{n_{o0}=0}^{n-n_{f0}-j}\sum_{n_{f1}=1}^{n-u_0-(j-1)}\sum_{n_{o1}=0}^{n-u_1-(j-1)}\cdots\sum_{n_{f(j-1)}=1}^{n-u_{(j-2)}-1}\sum_{n_{o(j-1)}=0}^{n-u_{(j-1)}-1} \\ &\times\frac{n!}{n_{f0}!n_{o0}!n_{w0}!}\alpha_0^{n_{f0}}\beta_0^{n_{o0}}\gamma_0^{n_{w0}}\frac{(n-u_0)!}{n_{f1}!n_{o1}!n_{w1}!}\alpha_1^{n_{f1}}\beta_1^{n_{o1}}\gamma_1^{n_{w1}}\cdot\cdot\frac{\left(n-u_{(j-1)}\right)!}{n_{fj}!n_{oj}!n_{wj}!}\alpha_j^{n_{fj}}\beta_j^{n_{oj}}\gamma_j^{n_{wj}} \end{aligned} \tag{4}$$

The probability that the cascading process stops in the Jth generation is

$$P\,(j = J) = P\left(u_J = n, n_{fJ} \neq 0\right) + P\left(u_J < n, n_{fJ} \geq 1, n_{f(J+1)} = 0\right) \tag{5}$$

In Equation (5), $P(u_J = n, n_{fJ} \neq 0)$ implies the probability of the stop scenario 1. $u_J = n$ implies the event that the total number of failed components until the Jth generation is n, which means that all the components fail until the generation J. $n_{fJ} \neq 0$ implies the event that the number of failed components in the Jth generation is not 0, which means that there are still new failures in the Jth generation. In addition, $u_J < n, n_{fJ} \geq 1$, and $n_{f(J+1)} = 0$ separately implies the event that the total number of failed components until the Jth generation is less than n, the event that there are at least one failed component in the Jth generation, and the event that there are no new failed components in the generation J+1. These restrictions exhibit a scenario that the cascading process stops, but the system does not fail in the Jth generation, whose probability could be denoted by $P(u_J < n, n_{fJ} \geq 1,\ \ n_{f(J+1)} = 0)$.

Therefore, the probability that the system fails and cascading process stops in the Jth generation is

$$P\left(j=J,\ u_J=n\right)=P\left(u_J=n,n_{fJ}\neq 0\right) \tag{6}$$

Since the cascading time is further considered in the model, and the cascading time is closely related to the generation j of the cascading process, the index J where the cascading process stops is the key factor to analyze the system reliability. The cascading events occur at time $\{T_0, T_1, \ldots T_j\}$, and T_j is duration of cascading process from the start to the jth generation $T_j = T_{j-1} + Y_j$. Assume that the cascading time Y_j for every generation follows an exponential distribution[19] with probability density function

$$f_{\mathrm{Y}}(t)=\mu \mathrm{e}^{-\mu t} \tag{7}$$

for $\mu > 0$, where μ is the rate parameter, which could be changed to control the cascading time distribution.

Then the cumulative probability distribution function that all components fail in the Jth generation at time t could be denoted as

$$F_{\mathrm{Y}}{}^{(J+1)}(t)=1-\mathrm{e}^{-\mu t} \tag{8}$$

The system reliability can be represented as the probability that the system is still working until time t, and could then be evaluated using the following equation

$$R(t)=1-\sum_{j=0}^{n-1}P(U_j=n,\ T_j<t)=1-\sum_{J=0}^{n-1}F_Y{}^{(J+1)}(t)\cdot P\left(J=j,u_J=n\right) \tag{9}$$

where $P(U_j = n, T_j < t)$ represents the probability that the system fails before time t. Through the integration of the Equations (4), (8), and (9), the system reliability over time could be obtained.

This model could be more general to extend the assumption of system failure. For instance, it can encompass scenarios where the system fails if a specific number of components fail, as in the case of a k-out-of-n system where the system fails when k components out of n fail. The only difference shown by the model for a k-out-of-n system is the stop scenario 1 where the cascading process stops if the total number of failed components is no less than k.

In this case, the probability that the system fails and cascading process stops in the Jth generation is

$$P\left(j=J,u_J\geq k\right)=P\left(u_J\geq k,n_{fJ}\geq 1,\ \ n_{f(J+1)}=0\right) \tag{10}$$

The system reliability for a k-out-of-n system can be represented as Equation (11). Through the integration of the Equations (4), (8), and (11), the reliability for the k-out-of-n system over time could be obtained.

$$R(t)=1-\sum_{j=0}^{k-1}P(U_j\geq k,T_j<t)=1-\sum_{J=0}^{k-1}F_Y{}^{(J+1)}(t)\cdot P\left(u_J\geq k,n_{fJ}\geq 1,\ \ n_{f(J+1)}=0\right) \tag{11}$$

The above outputs improved the multi-state CASCADE model by inducing cascading time and offers failure probability estimation of the loading system considering different cascading scenarios. The improved model can be used to assess the system condition, optimize system design and maintenance activities to increase reliability. The following section will provide some numerical examples for further illustration.

4 NUMERICAL EXAMPLES

To provide guidance on system design and operation, the effects of variation of some parameters on system reliability are examined in this section. Numerical examples are studied with coding in MATLAB. These case studies are mostly sensitivity analysis, meaning that when one parameter is analyzed, the other parameters remain constant.

Table 1 shows the parameter benchmark. In this study, load redistribution is the main driving force of the cascading process. The parameters c_0 and c_{d} imply the properties of the components themselves and the natural degradation, which

TABLE 1 Parameter benchmark of the examples.

Parameter	d	c_0	c_d	n	l_f	l_o	r^*	μ
Value	0.3	1	0.05	100	0.05	0.01	0.8	0.2

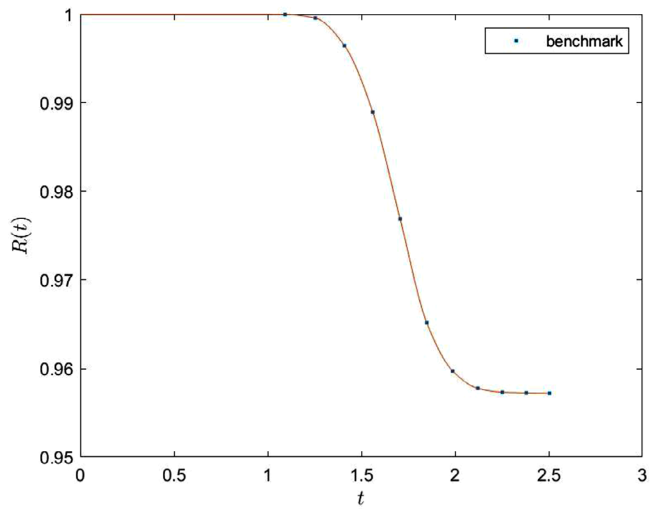

FIGURE 2 System reliability over time as a benchmark.

are relatively independent of the load redistribution mechanism. Therefore, the case studies do not delve into the impact of c_0 and c_d on the system reliability. Values of the other parameters are drawn from our previous work.[13] For example, the total number of the components is fixed as 100, and the overloading threshold is set as 0.8 in the benchmark. However, since the focus of this study is on studying the stop scenario 1 and system reliability, the values that increase the likelihood of stop scenario 1 occurring are favored. Therefore, the values of initial disturbance and the loading increments in this numerical example are set much larger than that in previous example.[13]

Based on the benchmark, a reliability curve is drawn to show the main properties of the reliability over time, as shown in Figure 2.

In overall, the system reliability experiences a minor decline at the start, a sharp drop in the middle phase, and another slight decrease towards the end of the curve. Towards the end, there seems to be a trend for the curve to remain constant. Such a curve can be explained as follows: At the onset of the cascading process, the probability of system failure is determined by adding the initial disturbance to the initial workload of components. As a result, there is a very low likelihood of all components failing simultaneously in the beginning, meaning that the system reliability is close to 1. As the cascading process proceeds, more generations of the cascading process imply more additional workloads on the functioning (overloading and normally working) components, and such load redistribution causes more components to fail, leading to a rapid decline in the system reliability. The total number of generations J remains stable for a given initial disturbance d in the scenario where the cascading process stops and the system fails. Consequently, as the cascading process slows down, and the reliability curve approaches its tail, the system reliability gradually reaches a stable value, which could be abbreviated as the minimum stable system reliability in our study.

According to Figure 2, it is also found that the curve stops at a specific time, instead of extending further. This is because that the curve only demonstrates the system reliability within a single cascading process that terminates at a specific time due to various cascading scenarios. As mentioned before, we only consider one cascading process until it stops. If the remaining components tend to fail after a period, we mark it as a new cascading process, which is not included in this model. This clarifies why the curve comes to a halt at a specific time, rather than advancing continuously. The duration of cascading process stopping in stop scenario 1 could also be employed to help to evaluate system reliability, because the system is generally expected to maintain normal operation for longer period.

System reliability, as defined, can be assessed using two primary metrics: the probability of system failure and the operating time before failures. These aspects can be described in terms of the duration cascading process stopping in stop scenario 1 and the minimum stable system reliability. This prompts us to emphasize these two aspects when performing sensitivity analysis.

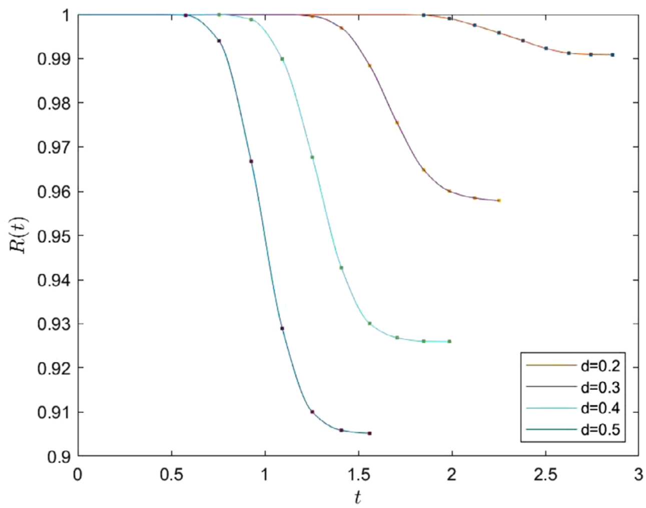

FIGURE 3 System reliability with different initial disturbance.

4.1 | Effect of initial disturbance

This subsection illustrates the change of system reliability with different initial disturbances. According to the previous work, stop scenario 1 was reported to occur only when there is a sufficient initial disturbance of at least 0.2. Besides, the analysis of system reliability is performed when the system fails under stop scenario 1, which can occur with different initial disturbances valued at $d = 0.2, 0.3, 0.4, 0.5$.

Figure 3 presents the variations in system reliability with various initial disturbances. Through a detailed comparison of these four curves, some findings could be obtained. The system reliability is lower when the initial disturbance value is larger at the same timepoint. Besides, the minimum stable system reliability is also lower when the initial disturbance value is larger. The explanation is provided as follows: As the initial disturbance increases, the workloads of more components tend to surpass the failure threshold, resulting in lower system reliability. Apart from the system reliability, it is also found that the cascading process stops and the system fails in shorter time when the initial disturbance value is larger.

The results emphasize the importance of controlling the initial disturbance to improve the system reliability and provide some managerial implications. In practice, it is costly to strive for an extremely high system reliability approaching a value of 1. However, it is equally unwise to ignore the impact of disturbances. Thus, we can determine an acceptable range for external disturbance when the system can tolerate a certain level of reliability. An example from the solar panel system could be taken to demonstrates that how the proposed model serves as an effective tool in system reliability prediction and maintenance optimization. The solar panel system is a loading dependent system with CAFs, where the performance of the panel is affected by a variety of external disturbances, including light intensity, temperature, and contaminants. In terms of the external disturbance contaminants, the pursuit of maintaining 100% power output can result in high cleaning and maintenance costs. On the contrary, by employing the proposed model, the system reliability under varying initial disturbances can be estimated. The estimation results, when combined with system design specifications and standards, allows for the determination of an acceptable range of system reliability. Subsequently, maintenance strategies for regular solar panel cleaning can be customized to minimize costs while ensuring that the system can tolerate a certain degree of surface contamination without causing a significant decline in system reliability.

4.2 | Effect of total number of components

The proposed model is now used to examine the cascading process and system reliability changes in various systems with different total number of components. Figure 4 displays the changes in system reliability observed when altering the total numbers of components $n = 50, 100, 150, 200$.

Figure 4 illustrates a similar trend of all the curves, which is consistent with the trend of the curves in the last subsection. When comparing the four curves, it is observed that as n increases, the system reliability curves shift toward the right. Specifically, the curves begin to decline at a later time and reach the halt point at a later time as well. Additionally, for larger

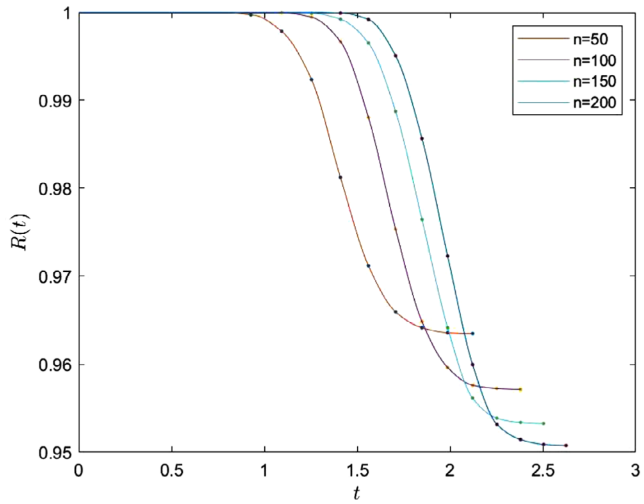

FIGURE 4 System reliability with different total number of components.

values of n, the system reliability displays a lower value once it stabilizes and reaches the endpoint. In brief, an increased number of components in a system lead to lower minimum system reliability and an extended cascading process. This implies that when there are more components within a system, the likelihood of system failure increases, but the time it takes for the system to eventually fail is longer. The aforementioned findings can be explained by that when there are more components in a system, it takes more time for all the components to fail and also takes more time for a cascading process to end in stop scenario 1. In a system with more components, a longer cascading process implies more possibilities for the process to proceed, resulting in an increased likelihood of system failure and lower system reliability.

Such findings also can remind engineers that during the design phase of a system with CAFs, reducing the number of components to a suitable range may improve the minimum stable system reliability and assist in sustaining the system operation within an acceptable timeframe. In term of the operation phase, some suggestions can be provided: when time is limited, priority is given to maintaining systems with fewer components, while when the primary aim is to improve system reliability, it is advisable to allocate more maintenance resources to systems with more components.

4.3 | Effect of loading increments

In this subsection, we utilize varying values of l_f and l_o for different configurations to compare the impacts of two types of loading increments. The loading increments are the additional loads from the failed or overloading components to the remaining functioning components, representing the dependence among components. Based on the assumption that the initial workload of the component lies in $[0, 1]$, the values of l_f and l_o also lie in $[0, 1]$, and the value of l_o must not surpass l_f to align the configurations with reality. In this example, we firstly set $l_o = 0.01$, and set different loading increments l_f as: 0.03, 0.05, 0.07, and 0.09. Then, we set different loading increments l_o as 0.01, 0.02, 0.03, and 0.04 for $l_f = 0.05$.

The changes of system reliability under two types of loading increments are respectively depicted in Figures 5 and 6. It is noteworthy that the curves closely overlap during the previous part of the cascading process, and discernible differences only as the process nears its tails. The results show that the loading increments have a stronger influence on the system reliability as the cascading process proceeds. The reason for this finding can be attributed to the following. At every generation, the two kinds of load increments are added to the functioning components, resulting in increased cumulative loads as the cascading process proceeds over time. Furthermore, the dissimilarity in cumulative loads induced by distinct load increments becomes more apparent, amplifying the variance in their effect on the system reliability. Besides, As shown in Figures 5 and 6, higher values of loading increments result in reduced system reliability. Higher loading increments lead to increased additional loads on functioning components, and lead to a higher likelihood of their failures. This ultimately leads to lower system reliability. This outcome implies that the loading increment has a certain degree of impact on the system reliability, but the extent of this influence is observable only when it is considerably large.

Another finding is that the impact on the system reliability due to failed components and that due to overloading components are similar to a certain extent. According to the proposed model, both kinds of loading increments are assigned to

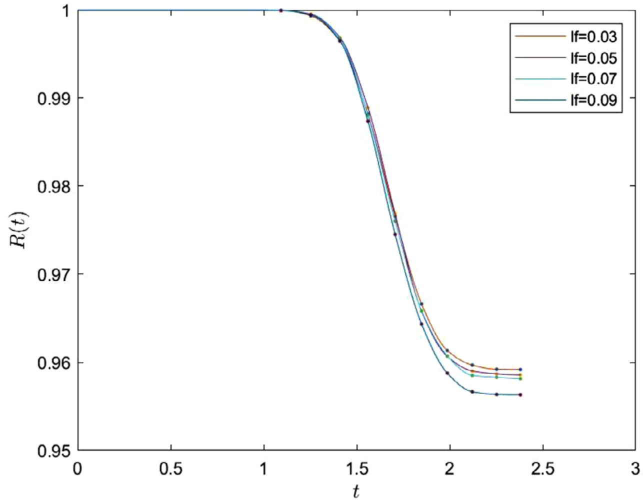

FIGURE 5 System reliability with different loading increment from failed components.

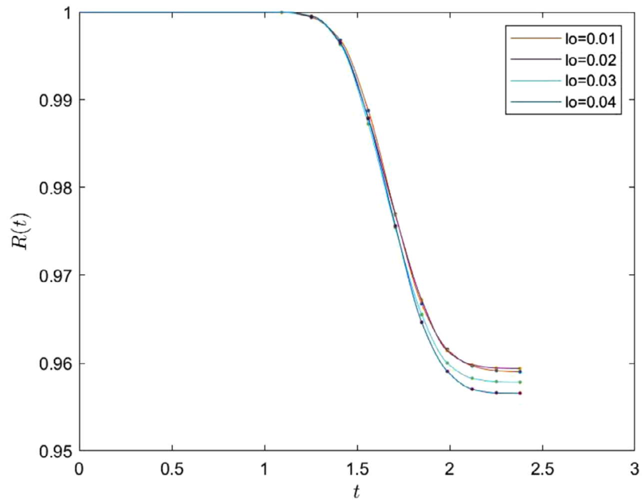

FIGURE 6 System reliability with different loading increment from overloading components.

the remaining functioning components in the same way. This distribution way ensures that the influence of these loading increments on the component should be comparable.

Based on such findings, firstly, management need to attempt to avoid subsequent failures by developing a more reasonable workload allocation strategy. Taking a pipeline system as an example, when a pipeline ruptures or becomes blocked, other pipelines will bear more flow, in other words, additional workload. This extra workload can be adjusted through valve regulation and reasonable diversion measures. Besides, given the roughly equal impact of both loading increments changes on system reliability, the strategy with higher cost-effectiveness can be chosen by comparing the costs associated with controlling the two kinds of loading increments.

4.4 | Effect of overloading threshold

By setting that the overloading threshold r^* varies from 0.5 to 0.9, the changes of system reliability are observed as shown in Figure 7.

According to Figure 7, we can observe that as the overloading threshold increases or decreases, the system reliability curve changes, but not in a systematic manner. In other words, we cannot make a definitive conclusion that the overload threshold has a significant impact on the system reliability. This finding aligns with our prior research result, where we

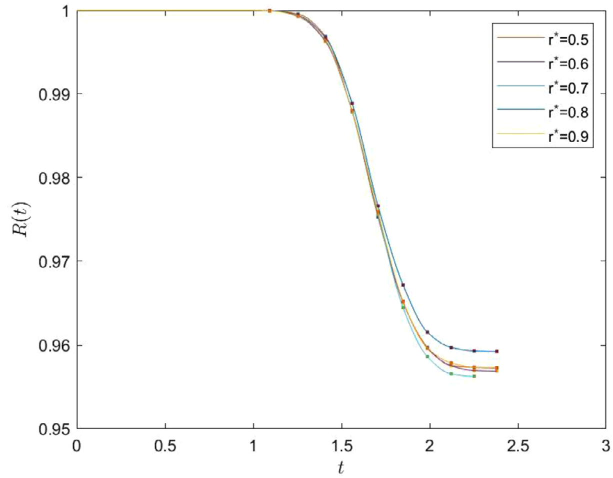

FIGURE 7 System reliability with different overloading threshold.

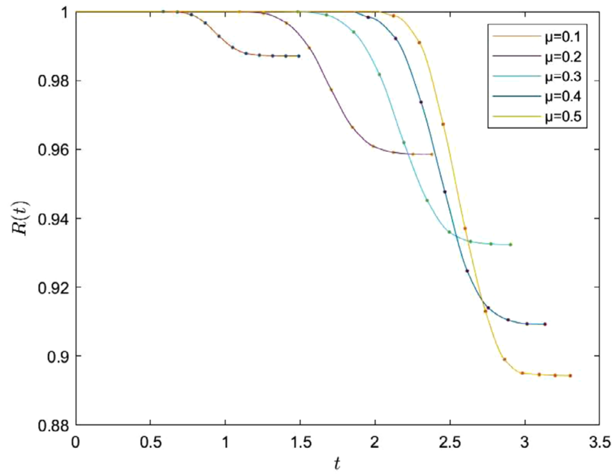

FIGURE 8 System reliability with different cascading time.

discovered that altering the overloading threshold mainly influences the probability distribution range of the number of overloading components, but has minimal impact on the probability distribution range of failed components, which is closely linked to system reliability.

4.5 | Effect of cascading time

The developmental features of the cascading process may vary for distinct systems, environments, and failure modes. In various cascading scenarios, alternative probability distributions may be considered. This paper assumes that the exponential probability distribution governs the cascading time taken for the development of each generation in the system cascading process. To investigate the effect of cascading time, an example is given by changing the rate parameter μ of the probability density function. Set that the value of rate parameter μ varies from 0.1 to 0.5, and the changes of system reliability are observed as shown in Figure 8.

Figure 8 displays how system reliability changes with different cascading time. As the value of μ increases, it can be observed that the system reliability curves shift towards the right. This implies that in situations where μ is high, the evolving of the cascading process takes more time. Besides, the minimum stable system reliability decreases when μ is high. In mathematical terms, μ represents the scale parameter of the exponential distribution. As μ increases, the value of $f_Y(t)$ near the origin also increases, signifying that the time needed for each generation at the start of the cascading process increases. Consequently, with an increase in the μ value, the initial cascading process requires more time to proceed,

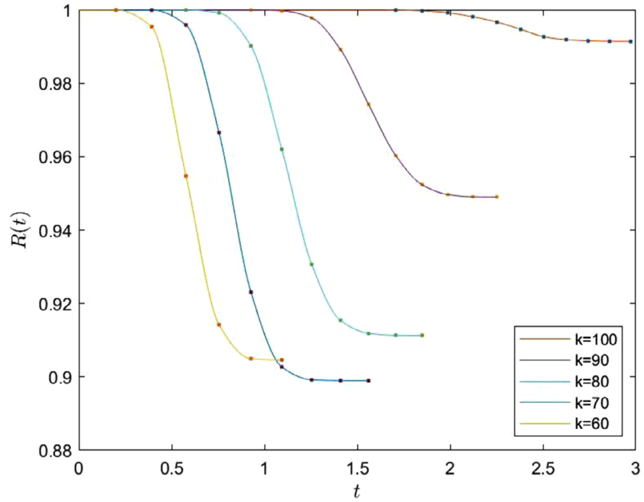

FIGURE 9 System reliability with different parameter k in a k-out-of-n system.

causing the corresponding system reliability curve to display its decline at a later period. This, in turn, contributes to an extended total duration for the development of the whole cascading process. In addition, as μ increases, the value of $f_Y(t)$ far away from the origin decreases, implying that as the cascading process proceeds, the time needed for each generation, and the time for causing the failed components, decreases, resulting in a faster rate for system degradation. This also accounts for the phenomenon that a higher value of μ leads to a more significant slope in the declining section of the system reliability curve.

This finding offers insights for system design and management. Regarding system design, simplifying the internal components and avoiding a tightly-packed layout can be effective in reducing cascading time. In terms of management, systems with higher rate parameters and fast-developing cascading processes should undergo more frequent inspections and maintenance activities. Moreover, systems with different rate parameters have distinct reliability curves, leading to different results in risk assessments. These dissimilar outcomes are crucial references for managers when making decisions.

4.6 | Effect of parameter k in a k-out-of-n system

The above subsections discuss the reliability analysis for the system where system failure occurs only when all components fail. In this part, the value of k is changed from 60 to 100 to display the reliability variations of the k-out-of-n system consisting of 100 components in total.

From Figure 9, when the value of k decreases, the system reliability curves shift toward the left. Initially, the minimum stable system reliability experiences a gradual decline, but it increases when $k = 60$. Following is the explanation for this phenomenon. When k decreases, it signifies that a smaller number of failed components can lead to system failure. Firstly, the system will therefore fail in a shorter period, reducing the duration of the cascading process and causing the reliability curves to shift leftward. Secondly, the system becomes more prone to failures, meaning the probability of system failure increases, thereby decreasing the minimum stable system reliability. However, when k decreases to a certain extent, the system fails very quickly, increasing the value of system reliability. According to the definition of system reliability, it depends on the combined influence of the probability of system failure and the operating time before failures. Therefore, in this case, even though the probability of system failure increases, its impact on system reliability is not as significant as the effect of the duration for the system to fail, leading to an increase in the minimum system reliability value instead.

5 | CONCLUSION AND FUTURE WORKS

In this paper, we have proposed a new reliability index for evaluating the system reliability of a loading dependent system considering overloading state based on the multi-state CASCADE model. Cascading time is well considered in such a method. The numerical example is conducted to examine the system reliability model and demonstrate the impacts of different factors on the cascading process and the system reliability: Alterations in the initial disturbance, the total number of components, the cascading time distribution, and the parameter k in a k-out-of-n system all significantly influence both the system reliability and the duration of the cascading process. On the other hand, the variation of the loading increments only exhibits an influence on the system reliability when the cascading process approaches its end due to influence accumulation. Notably, neither the system reliability nor the duration of the cascading process remains unaffected by the overloading thresholds of the components. These findings can help maintenance crews and managers make more informed decisions in terms of system design and operational management when considering cascade time and reliability.

Some relevant topics are worth examining in future studies. Firstly, given that the model we propose are mainly used for the multi-component systems with simple structures, further investigations for complex systems such as series-parallel system or parallel-series system in engineering applications are encouraged. In addition, from a comprehensive point of view, the system may still operate after the cascading process end. Therefore, the system reliability analysis could be examined after the first cascading process ends. Thirdly, since the environmental factors are dynamic and may cause multiple outside disturbances, this model can be extended to allow a series of outside disturbances which happen at different time points before the system fails. The extended model will consider the probabilities of more cascading scenarios. These cascading scenarios are categorized based on different numbers of outside disturbances, and are subsequently subdivided into two types, the scenario where the cascading process ends before the arrival of the last disturbance, as well as the scenario where the cascading process ends after the last disturbance has arrived. All scenarios will be examined and the probabilities of cascading scenarios where the system fails are accumulated to provide a final calculation of system reliability.

NOMENCLATURE

c_0	initial capacity of the component
c_d	capacity decrement of the functioning components during each generation
c_j	capacity of the component in the jth generation
d	the value of the initial disturbance
j	generation of the cascading process, $j = 0,1, 2, \ldots.$
J	the generation that the cascading process stops
k	the number of components out of the total number of components that need to be functioning for the entire system to function.
l_f	the loading increment from a failed component
l_i	the initial workload on component i
l_j	the loading increments from all the failed and overloading component in the jth generation
l_o	the loading increment from an overloading component
n	total number of the components in a system
n_fj	number of the failed components in the jth generation
n_oj	number of the overloading components in the jth generation
n_wj	number of the working components in the jth generation
$r*$	overloading threshold of the component
$R(t)$	the system reliability with time t
s_j	the case that there are n_fj failed components, n_oj overloading components, and n_wj normally working components in the jth generation
t	cascading time, and $t = 0$ when the cascading process starts
T_j	the time interval from the initiation of the cascading process to the jth generation
u	total number of the failed components
v	total number of the overloading components
Y_j	the cascading time for generation j
α, β, γ	the indices used to abbreviate the probability of components in three different states
μ	rate parameter of the exponential distribution characterizing the cascading time

ACKNOWLEDGMENTS

Y. Zhao thanks China Scholarship Council (No. 202006150011). The authors are also grateful to IKTPLUSS program (No. 309628) of Norway, and the UTFORSK program (UTF-2017-four-year/10058) managed by DIKU, Norway.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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How to cite this article: Zhao Y, Sun T, Liu Y. Reliability analysis of a loading dependent system with cascading failures considering overloads. *Qual Reliab Engng Int*. 2023;1-15. https://doi.org/10.1002/qre.3475

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