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Semiparametric MEWMA for Phase II profile monitoring

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Abstract

A control chart is one of the statistical process techniques that is used to monitor different processes. Some processes are characterized by functions or profiles, and a profile is a functional relationship between the dependent and independent variable(s) used to monitor the quality of the process. Several research studies were conducted on linear profiling where only fixed effects are considered. However, in this research, we focus on random effects as they represent the differences between profiles and thus are more proper for interpretation. Two approaches are proposed in this study for Phase II profile monitoring; the first approach is the nonparametric via residuals and the second is the semiparametric approach, where this technique combines the parametric estimates with a portion of the nonparametric estimates to the residuals. Usually, parametric estimations lead to biased estimates when the model is misspecified, whereas nonparametric estimates may give high variances, and thus semiparametric estimates are preferred. New nonparametric and semiparametric multivariate exponential weighted moving average (MEWMA) control charts are introduced and their performances compared to the parametric approach for different samples and shift sizes, and the correlation between and within profiles was considered. The average run length (ARL) and average time to signal (ATS) criteria are used for choosing the best approach. Simulation studies and real datasets were utilized for comparing the performance of the proposed MEWMA charts.

KEYWORDS

ARL, ATS, linear mixed models, MEWMA, misspecification, model robust regression 2, profile monitoring

1 | INTRODUCTION

Profile monitoring is when the quality characteristic of the process is represented by a relationship between the response variable and the independent variables, where this relationship is known as profile, and their profiles are being monitored by control charts. Several research studies have been conducted on profile monitoring, for example, Mahmoud and Woodall[1] used profile monitoring in linear calibration, where different concentrations were collected by analytical chemistry procedures. The curves were fitted by linear models to guarantee that the measurement equipment is calibrated. Nonlinear profiles were considered by Williams et al,[2] where particleboard density was measured at equally spaced locations, and the result was higher density at the edges compared to the density in the middle. Zou et al[3] used wavelet transforms (which are more complicated compared to linear or nonlinear models) to approximate the profile of the forging cycle of a stamping process. The wavelet coefficients were monitored to determine whether the forging cycles differ from each other or not. Profiles can be represented as simple, multiple, mixed, linear, or nonlinear, depending on the shape of our profile of interest. So far, most of the researchers considered simple linear profiles where only fixed effects were monitored, for example, Kang and Albin,[4] Kim et al,[5] Saghaei et al,[6] Noorossana et al[7] and Mahmood et al.[8] In fact, monitoring fixed effects do not show the variations between profiles, it only considers the similarities between profiles assuming all profiles are similar, which is an unrealistic assumption. Thus, in this research, we are monitoring linear mixed models (LMM), where these models consider fixed and random effects, as well as they are very flexible models where the correlation within and between profiles are taken into account (more information about LMM is found in Verbeke and Molenberghs[9]). Monitoring LMM was conducted in many studies; for example, Jensen et al,[10] Abdel-Salam,[11] Qiu et al,[12] Narvand et al,[13] and Siddiqui and Abdel-Salam.[14]

Most state-of-the-art in the profile monitoring assume that LMMs are correctly fitted with no misspecification (parametric approaches), but in fact in real-life situations this assumption is not realistic and may lead to poor results. Thus researchers chose nonparametric (NP) and semiparametric (SP) approaches as alternative approaches for avoiding this assumption, as was conducted by Abdel-Salam,[11] Abdel-Salam et al,[15] and Siddiqui and Abdel-Salam,[14] where they showed that the NP and SP approaches performed better than the parametric approach when there is model misspecification for Phase I profile monitoring. Hence, NP and SP (Model Robust Regression 2 [MRR2] introduced by Mays et al[16]) approaches will be introduced in this research for monitoring LMM in Phase II in this study. The multivariate exponential weighted moving average (MEWMA) control charts will be modified based on the three regression approaches, which are: parametric, NP, and SP. We considered uncorrelated and correlated profiles for different profile sizes and different sample sizes to check whether they have effects on monitoring profiles or not by comparing the performances of the three MEWMA for detecting model misspecification and slope shifts using two evaluation measures, which are: average run length (ARL) and average time to signal (ATS), where the ARL is the average samples required until the first out-of-control signals and ATS represents the number of time-periods that occur until the first out-of-control signals. The chart with the smallest values of ARL and ATS means it is the best chart and has high sensitivity in detecting shifts. These two measures were calculated through Monte Carlo simulations. Also, this study is motivated by a real data application, which is about the vertical density of practical boards, where the density is measured by a profilometer that uses a laser device to take a series of measurements across the thickness of the board. A profilometer takes multiple measurements on a sample, usually a 2×2-inch piece. Where, the relationship between the response variable, board density, and the explanatory variable, depth, is being monitored (Abdel-Salam[11]).

The rest of this paper is organized as follows: LMMs and the three approaches used for estimating the profiles (parametric, NP, and SP) are given in Section 2, the proposed NP and SP MEWMA control charts are illustrated in Section 3, and the Monte Carlo simulation and the real data application are given in Sections 4 and 5, respectively. Finally, we conclude in Section 6.

2 | LINEAR MIXED MODELS

LMMs are popular and flexible models as they contain both fixed and random effects, and it takes into account the autocorrelation within and between profiles. We can differentiate between these two effects (or coefficients) as follows:

1. Fixed effects: When the effects are identical constants for all profiles.
2. Random effects: Account for different effects from profile to profile.

LMMs are like general linear models where the response variable is obtained by adding fixed effects, random effects, and an error term together. The form of LMM is given by

$$y_{ij} = \beta_0 + \beta_1\ x_{1ij} + \beta_2 x_{2ij} \dots \beta_n x_{nij} + b_{i1} z_{1ij} + b_{i2} z_{2ij} \dots b_{in} z_{nij} + \varepsilon_{ij}, \tag{1}$$

where y_{ij} represents the response variable (profile of interest) for a particular case (ith profile and jth sample). The x_{1ij}, $x_{2ij} \dots\ x_{nij}$ are the fixed-effect predictors, and the fixed-effect coefficients are $\beta_1, \beta_2 \dots \beta_n$, while $z_{1ij}, z_{2ij} \dots\ z_{nij}$ are the random-effect predictors and $b_{i1}, b_{i2} \dots b_{in}$ refer to the random-effect coefficients. ε_{ij} represents the error term of sample j in profile i, and it is assumed to have a multivariate normal distribution.

In profile monitoring, the LMM of the ith profile is represented in matrix notation as follows:

$$y_i = X_i\ \beta + Z_i b_i + \epsilon_i\ ,\quad i = 1, 2, \ldots, m, \tag{2}$$

where $y_i(n_i \times 1)$ is the ith profile of interest vector, $X_i(n_i \times p)$ is the matrix of fixed regressors, while $Z_i(n_i \times q)$ represents the matrix of the random regressors. $\beta(p \times 1)$ is the fixed-effects vector and b_i is the random-effects vector of the ith profile where $b_i \sim \mathrm{MN}(0, D)$, where $D(q \times q)$ is a positive definite matrix. Moreover, the $\epsilon_i(n_i \times 1)$ is the error terms vector where $\epsilon_i \sim \mathrm{MN}(0, R_i)$, where R_i is often assumed to be autoregressive (AR) or compound symmetry (CS) if the errors are correlated, while $R_i = \sigma^2\ I$ if errors are uncorrelated. When we are assuming that the errors follow this multivariate normal distribution, then consequently $y_i \sim \mathrm{MN}(X_i\beta, V_i)$, where $V_i = Z_i\ D_i Z_i' + R_i$ is the estimated variance-covariance matrix. More details about LMM are found in Schabenberger and Pierce.[17]

The estimations of β and b_i are $\hat{\beta}$ and $\hat{b}_i$, respectively. In 1950s, Charles Henderson provided the best linear unbiased estimate of fixed effects and best linear unbiased predictions of random effects, BLUE and BLUP, respectively. In our study, we are using three approaches for fitting the model to obtain BLUE and BLUP. These approaches are: parametric approach (generalized least square method), NP approach for the residuals using the penalized spline method and SP approach based on the MRR2. Then, we calculated the multivariate exponential weighted average (MEWMA) control chart based on the parametric, NP and SP approaches.

2.1 | Parametric estimation approach (P)

The generalized least square method is one of the parametric methods that is used to estimate the fixed and random effects. As mentioned previously, $\hat{\beta}$ and $\hat{b}_i$ are the fixed and random-effects estimates, respectively, the estimated profile average is $\hat{y}_{i,P}^{\mathrm{PA}} = X_i\ \hat{\beta}_p$ and the estimated specific profile prediction is $\hat{y}_{i,P}^{\mathrm{SP}} = X_i\ \hat{\beta}_p + Z_i \hat{b}_{i,p}$, for more details see.[23,24] If the variance-covariance matrices V_i, D, and R_i are known, it can be shown that BLUE is given as follows:

$$\hat{\beta}_p = \left(\sum_{i=1}^{m} X'_i V_i^{-1} X_i \right)^{-1} \left(\sum_{i=1}^{m} X'_i V_i^{-1} y_i \right) \tag{3}$$

and the BLUP is given as follows:

$$\hat{b}_{i,p} = DZ_i' V_i^{-1} \left(y_i - X_i \hat{\beta}_p \right). \tag{4}$$

Since we are only interested in the BLUPs for constructing our MEWMA control charts, thus, the mean and variance are given as follows:

$$\bar{\hat{b}}_P = \frac{\sum_{i=1}^{m} \hat{b}_{i,P}}{m}, \tag{5}$$

$$\hat{\Sigma}_P = \frac{\sum_{i=1}^{m-1} \left(\hat{b}_{i+1,P} - \hat{b}_{i,P} \right) \left(\hat{b}_{i,+1,P} - \hat{b}_{i,P} \right)'}{2\,(m-1)}. \tag{6}$$

Furthermore, in this study we will be using MRR2 approach where it is a combination between the parametric and NP fits. MRR2 is explained in Section 2.3, but first we need to calculate the residuals from the parametric estimation and fitted nonparametrically in order to obtain MRR2 estimation, as it will be explained later.

2.2 | Nonparametric residuals estimation approach (NPR)

The parametric methods usually relying on several assumptions, such as the model is correctly specified, errors are normally distributed, and a linear relationship. However, practically, these assumptions are not realistic, as the user is frequently unsure about the actual model, the underlying distribution of the data may not always be known or may not be assumed correctly, which may result in biased estimations. Thus, they may be misleading as they are based on unrealistic assumptions. The NP approach is an alternative technique used due to its flexibility, as it eliminates the assumptions of the parametric approach. There are different methods available in the NP approach used to fit the model between the dependent variable and the independent variables. In this study, we will focus on the penalized spline (P-spline) method for fitting the residuals obtained from the parametric approach, as it is a well-known method due to its flexibility. A spline is a continuous function that is used to generate a smooth curve to pass through a set of points; usually, the n-degree polynomials are generated. Thus p-degree spline functions are required to join these polynomials, and the polynomials are tied together by knots to produce a smooth curve joining them. The number of knots and their locations must be chosen accurately in order to avoid over- or underfitting the data, but as no optimal solution exists to the number of knots or their locations; penalization is used to put weights on the splines to avoid overfitting and at the same time to allow the accurate fit to the data.

The general LMM formula for this approach is given as

$$yij = f\left(x_{ij}\right) +,\ \xi_i\left(x_{ij}\right) + \varepsilon_{ij},\quad i = 1,2,\dots,m \text{ and } j = 1,2,\dots,n, \tag{7}$$

where y_{ij} is our response variable, $f(x_{ij})$ is the overall profile average and $\xi_i(x_{ij})$ is a smoother function where it represents the random difference between ith specific prediction and the profile average. In our study, we are interested in fitting the residuals (r). Thus we replaced $f(x_{ij})$ and $\xi_i(x_{ij})$ by $f(r_{ij})$ and $\xi_i(r_{ij})$, where $r = y - \hat{y}^p$ and it represents the residuals from the estimated parametric LMM. $f(r_{ij})$ and $\xi_i(r_{ij})$ are defined, respectively, as

$$f\left(r_{ij}\right) \approx \beta_0 + \sum_{l=1}^{p} \beta_i r_{ij}^{l} + \sum_{k=1}^{k_1} u_k \left| r_{ij} - K_k \right|^p,\quad i = 1,2,\dots,m \text{ and } j = 1,2,\dots,n_i, \tag{8}$$

$$\xi_i\left(r_{ij}\right) \approx b_{i0} + \sum_{l=1}^{p} b_{ij} r_{ij}^{l} + \sum_{k=1}^{k_2} t_{ipk} \left| r_{ij} - K_k \right|^p,\quad i = 1,2,\dots,m \text{ and } j = 1,2,\dots,n_i. \tag{9}$$

For $f(r_{ij})$, p is the order of polynomial basis, k_1 is the number of knots used and $\mathbf{K_1}, \dots, \mathbf{K_{k1}}$ are the locations of the knots. The $\sum_{l=1}^{p} \beta_i r_{ij}^{l}$ represents the parametric component, while $\sum_{k=1}^{k_1} u_k |r_{ij} - r_k|^p$ refers to the B-spline component. Moreover, $\xi_i(r_{ij})$ represents the B-spline basis for a specific prediction curve, and here p is the order of polynomial basis with the intercept and the parameters of random effects. $b_{i0} + \sum_{l=1}^{p} b_{ij} r_{ij}^{l}$ is the parametric random-effects component, while $\sum_{k=1}^{k_2} t_{ipk} |r_{ij} - K_k|^p$ refers to the B-spline component, where k_2 is the number of knots used. The estimated profile average is $\hat{y}_{i,\mathrm{NPR}}^{\mathrm{PA}} = X_i\,\hat{\beta} + Z_i\hat{u}$ and the estimated specific-profile prediction is $\hat{y}_{i,\mathrm{NPR}}^{\mathrm{SP}} = X_i\,\hat{\beta} + Z_i\hat{u} + X_i\hat{b}_i + E_i\hat{t}_i$. For obtaining the estimated random-effects $\hat{\gamma}_{i,NPR}$, we are going to combine the penalized spline estimates $\hat{b}_i$ and $\hat{t}_i$, where they are estimated random effects for the spline component vector and the knot location for the ith profile vector, respectively, such that they follow multivariate normal distributions, that is $(b_{i0},\ b_{i1}, \dots b_{ip})' \sim \mathrm{MN}(0, \Sigma_b)$ and $t_{ipk2} \sim \mathrm{MN}(0, \sigma_t^2)$. Thus, the estimated random-effects vector using residuals P-spline regression along with its mean and variance-covariance matrices are given as, respectively,

$$\hat{\gamma}_{i,\mathrm{NPR}} = \left[\hat{b}_i\ \hat{t}_i\right]', \tag{10}$$

$$\bar{\gamma}_{\mathrm{NPR}} = \frac{\sum_{i=1}^{m} \hat{\gamma}_{i,\mathrm{NPR}}}{m}, \tag{11}$$

$$\hat{\Sigma}_{\mathrm{NPR}} = \frac{\sum_{i=1}^{m-1} \left(\hat{\gamma}_{i+1,\mathrm{NPR}} - \hat{\gamma}_{i,\mathrm{NPR}}\right)\left(\hat{\gamma}_{i,+1,\mathrm{NPR}} - \hat{\gamma}_{i,\mathrm{NPR}}\right)'}{2\,(m-1)}. \tag{12}$$

More details about this approach are found given by Abdel-Salam[11] and Siddiqui and Abdel-Salam.[14]

2.3 | Semiparametric estimation approach (SPR)

In this section, we are using MRR2 approach that was proposed by Mays et al,[16] and extended to Phase II profile monitoring. MRR2 combines the advantages of the parametric and the NP techniques and reduces their disadvantages. This technique combines the random effects obtained using the parametric technique (Equation 4) with a portion of the random effects obtained using the NPR technique (Equation 10), where this portion is the mixing parameter: $\lambda \in [0,1]$. The estimated random-effects vector by MRR2, along with its mean and variance-covariance matrices are obtained as well, respectively,

$$\hat{\psi}_{i,\mathrm{SPR}} = \begin{bmatrix} \hat{b}_{i,\mathrm{p}} \\ \hat{\lambda}\hat{\gamma}_{i,\mathrm{NPR}} \end{bmatrix}', \tag{13}$$

$$\bar{\psi}_{\mathrm{SPR}} = \frac{\sum_{i=1}^{m} \hat{\psi}_{i,\mathrm{SPR}}}{m}, \tag{14}$$

$$\hat{\Sigma}_{\mathrm{SPR}} = \frac{\sum_{i=1}^{m-1} \left(\hat{\psi}_{i+1,\mathrm{SPR}} - \hat{\psi}_{i,\mathrm{SPR}}\right)\left(\hat{\psi}_{i+1,\mathrm{SPR}} - \hat{\psi}_{i,\mathrm{SPR}}\right)'}{2(m-1)}, \tag{15}$$

where $\hat{b}_{i,\mathrm{p}}$ is the estimated parametric random-effects vector and $\hat{\gamma}_{i,\mathrm{NPR}}$ is the estimated NP random-effects vector. $\bar{\psi}_{\mathrm{SPR}}$ and $\hat{\Sigma}_{\mathrm{SPR}}$ represent the average vector and the successive-differences variance-covariance matrix for the estimated SP random effects, respectively.

Furthermore, the mixing parameter λ is unknown and can be estimated based on the data to obtain $\hat{\lambda}$. Our estimated $\hat{\lambda}$ is a modified version of Waterman et al[18] for λ based on population average (PA) and cluster specific (CS), and these estimators are given by, respectively,

$$\hat{\lambda}_{\mathrm{PA,SPR}} = \frac{(\hat{r})'\left(y - \hat{y}^{\mathrm{PA}}_{\mathrm{Par}}\right)}{(\hat{r})'(\hat{r})}, \tag{16}$$

$$\hat{\lambda}_{\mathrm{SP,SPR}} = \frac{(\hat{r})'\left(y - \hat{y}^{\mathrm{SP}}_{\mathrm{Par}}\right)}{(\hat{r})'(\hat{r})}, \tag{17}$$

where $\hat{r}$ represents the NP fitted residuals vector, y represents the true values vector, $\hat{y}^{\mathrm{PA}}_{\mathrm{Par}}$ represents the fitted PA vector using parametric technique, and $\hat{y}^{\mathrm{CS}}_{\mathrm{Par}}$ represents the fitted CS vector by the parametric technique. The SP estimated PA is $\hat{y}^{\mathrm{PA}}_{i,\mathrm{SPR}} = \hat{y}^{\mathrm{PA}}_{i,\mathrm{Par}} + \hat{\lambda}_{\mathrm{PA,SPR}}\hat{r}$ and the estimated CS is $\hat{y}^{\mathrm{CS}}_{i,\mathrm{SPR}} = \hat{y}^{\mathrm{CS}}_{i,\mathrm{Par}} + \hat{\lambda}_{\mathrm{CS,SPR}}\hat{r}$. This technique is called mixed model robust residuals profile monitoring (MMRRPM) (details are given by Siddiqui and Abdel-Salam,[14] Abdel-Salam[28] and Wan and Birch[19]).

3 | PROPOSED MEWMA CONTROL CHARTS

In 1992, Lowry et al developed a multivariate EWMA (MEWMA) control chart (Montgomery[25]), where MEWMAs are vectors given by

$$\mathrm{MEWMA}_i = WX_i + (I - W)\,\mathrm{MEWMA}_{i-1}, \tag{18}$$

where $\text{MEWMA}_0 = 0$, X_i is a vector of observations and $W = \text{diag}(w_1, w_2, \ldots, w_p)$, $0 < w_i \le 1$, $i = 1, 2, \ldots, p$, where w is a parameter that regulates the magnitude of smoothing; usually they are chosen to be small for quicker detection of small shifts (Runger and Prabhu,[20] Noorossana[26] and Niaki[27]). The MEWMA signals when any of the T_i^2 exceeds the desired control limits.

$$T_i^2 = (\text{MEWMA}_i)' \Sigma_{\text{MEWMA}_i}^{-1} (\text{MEWMA}_i), \tag{19}$$

where $\Sigma_{\text{MEWMA}_i}^{-1}$ is the inverse of the covariance matrix of MEWMA_i; if $w_1 = w_2 = \ldots = w_p$, then $\Sigma_{\text{MEWMA}_i} = \frac{w}{2-w}\left[1-(1-w)^{2i}\right]\Sigma_x$, as $i \to \infty$: $\Sigma_{\text{MEWMA}_i} = \frac{w}{2-w}\Sigma_x$. If $w_1 \ne w_2 \ldots \ne w_p$, then $\Sigma_{\text{MEWMA}_i}(k, L) = w_k\, w_L \frac{[1-(1-w_k)^i(1-w_L)^i]}{(w_k+w_L-w_k w_L)}\sigma_{kL}$. More details regarding how to design the MEWMA chart are found in Runger and Prabhu.[20]

The MEWMA formulas are modified to get the proposed MEWMA for the parametric, NPR, and SPR MMRRPM techniques, as shown below.

3.1 | Parametric estimated random effects (PMEWMA)

$$\text{MEWMA}_{i,\text{P}} = W\hat{b}_{i,P} + (I - W)\,\text{MEWMA}_{i-1,\text{P}}, \tag{20}$$

$$T_{i,\text{P}}^2 = (\text{MEWMA}_{i,\text{P}})' \Sigma_{\text{MEWMA}_{i,\text{P}}}^{-1} \left(\text{MEWMA}_{i,\text{P}}\right), \tag{21}$$

where $\text{MEWMA}_{0,\text{P}} = 0$, $\hat{b}_{i,\text{P}}$ is the estimated parametric random-effects vector for the ith profile.

3.2 | Nonparametric estimated residuals random effects (NPRMEWMA)

$$\text{MEWMA}_{i,\text{NPR}} = W\hat{\gamma}_{i,\text{NPR}} + (I - W)\,\text{MEWMA}_{i-1,\text{NPR}}, \tag{22}$$

$$T_{i,\text{NPR}}^2 = (\text{MEWMA}_{i,\text{NPR}})' \Sigma_{\text{MEWMA}_{i,\text{NPR}}}^{-1} \left(\text{MEWMA}_{i,\text{NPR}}\right), \tag{23}$$

where $\text{MEWMA}_{0,\text{NPR}} = 0, \hat{\gamma}_{i,\text{NPR}}$ is the estimated NPRs random effects vector for the ith profile.

3.3 | Semiparametric estimated random-effects MMRRPM (SPRMEWMA)

$$\text{MEWMA}_{i,\text{SPR}} = W\hat{\psi}_{i,\text{SPR}} + (I - W)\,\text{MEWMA}_{i-1,\text{SPR}}, \tag{24}$$

$$T_{i,\text{SPR}}^2 = (\text{MEWMA}_{i,\text{SPR}})' \Sigma_{\text{MEWMA}_{i,\text{SPR}}}^{-1} (\text{MEWMA}_{i.\text{SPR}}), \tag{25}$$

where $\text{MEWMA}_{0,\text{SP}} = 0, \hat{\psi}_{i,\text{MMRPM}}$ is the estimated SP MRR1 random-effects vector for the ith profile.

TABLE 1 Mixing parameter averages, SIMSEs, and (standard errors) for $m = 300$ at different n and γ for uncorrelated ($\rho = 0$) profile datasets

m	n	γ	$\bar{\lambda}_{\text{MMRRPM}}$	P	NPR	MMRRPM
300	**10**	**0.00**	0.000	**2.45** (0.44)	3.95 (0.59)	2.45 (0.44)
		0.25	0.214	5.40 (0.45)	6.52 (0.60)	5.33 (0.45)
		0.50	0.460	14.36 (0.51)	14.26 (0.65)	13.70 (0.48)
		0.75	0.774	29.67 (0.73)	28.49 (0.74)	27.68 (0.59)
		1.00	0.973	51.70 (0.85)	48.31 (0.85)	48.64 (0.73)
300	**20**	**0.00**	0.000	**0.83** (0.24)	2.16 (0.40)	0.83 (0.24)
		0.25	0.496	3.83 (0.26)	4.98 (0.43)	3.80 (0.26)
		0.50	0.592	12.85 (0.31)	13.49 (0.53)	12.64 (0.34)
		0.75	0.818	28.01 (0.43)	28.22 (0.67)	27.58 (0.48)
		1.00	0.955	49.45 (0.61)	45.58 (0.81)	45.33 (0.66)

TABLE 2 Mixing parameter averages, SIMSEs, and (standard errors) for $m = 300$ at different n and γ for moderately correlated ($\rho = 0.5$) profile datasets

m	n	γ	$\bar{\lambda}_{\text{MMRRPM}}$	P	NPR	MMRRPM
300	**10**	**0.00**	0.000	**4.82** (0.95)	7.94 (1.27)	**4.82** (0.95)
		0.25	0.317	7.64 (0.93)	10.51 (1.26)	**7.63** (0.93)
		0.50	0.537	16.25 (0.76)	18.24 (1.28)	**16.06** (0.90)
		0.75	0.730	30.99 (0.88)	31.16 (1.32)	**29.84** (0.93)
		1.00	0.903	52.27 (1.15)	49.25 (1.38)	**48.74** (1.05)
300	**20**	**0.00**	0.000	**2.18** (0.63)	5.24 (1.00)	**2.18** (0.63)
		0.25	0.429	5.13 (0.64)	8.05 (1.03)	**5.12** (0.64)
		0.50	0.615	14.03 (0.65)	16.54 (1.12)	**13.96** (0.67)
		0.75	0.853	28.99 (0.71)	30.71 (1.25)	**28.62** (0.76)
		1.00	0.923	50.13 (0.84)	50.58 (1.41)	**48.99** (0.93)

4 | MONTE CARLO SIMULATION

In this section, we are showing the simulation steps carried out for comparing the performances of the parametric, NPRs, and SP MEWMA charts. All the simulation steps were performed using SAS program, where GLIMMIX procedures were used for estimating the parametric random effects and the fitted NPRs using the P-spline technique based on a quadratic model adapted from the literature given in Equation (26) (Abdel-Salam,[11] Siddiqui and Abdel-Salam,[14] and Waterman[21]). Then, after estimating the parametric and the NP random effects, the estimated mixing parameter $\hat{\lambda}$ for the SP MMRRPM is estimated to be used in combining parametric and a portion of the NP random effects for obtaining the estimated MMRRPM random effects.

$$y_{ij} = (5 + b_{i1})\, x_{ij} + (2 + b_{i2})\,(x_{ij} - 5.5)^2 + \gamma \left[10 \sin\left(\frac{\pi(x_{ij}-1)}{2.25}\right) + b_{i3}\right] + \in_{ij},$$
$$i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n_i, \tag{26}$$

where j and i represent the observations and profiles, respectively, the selected profile sizes to be $m = 300$ and 600, and the sample sizes to be $n = 10$ and 20. The y_{ij} is the profile of interest and x_{ij} represents the independent variables where they were chosen to take 1-10 integer values. The γ in the model represents the misspecification amount inserted to the model, and we chose it to be between low and high ($\gamma = 0.0, 0.25, 0.50, 0.75, 1.00$) consistent with Abdel-Salam[11] and Siddiqui and Abdel-Salam.[14] b_{i1}, b_{i2} and $b_{i3} \sim N(0, 0.5)$ represent the random effects and ϵ_{ij} are the random errors such that $\epsilon_{ij} \sim (0, R_i)$, where R_i is the variance-covariance matrix and it is equal to $R_i = \sigma^2 I$ for uncorrelated data $\rho = 0.0$, where the error variances are assumed to be $\sigma^2 = 16$, while for correlated data, R_i takes the form of autoregressive model AR(1), where the correlation levels were chosen to be low, moderate, and high with values $\rho = 0.2, 0.5$, and 0.8, respectively. The

TABLE 3 Out-of-control ARL for uncorrelated ($\rho = 0.0$) profile datasets for different model misspecification levels (γ) and shift sizes (δ) ($m = 300$, $n = 10$, $l = 40$)

γ	δ	T^2_{P}	T^2_{NPR}	T^2_{SPR}
0.00	**0.00**	200.00	200.00	**200.00**
	0.10	150.48	158.60	**125.40**
	0.20	113.42	120.38	**90.90**
	0.30	72.94	89.84	**64.29**
	0.50	37.20	48.25	**31.25**
0.25	**0.00**	199.38	175.30	**149.07**
	0.10	147.49	142.64	**119.62**
	0.20	112.67	103.64	**89.95**
	0.30	70.98	58.98	**45.28**
	0.50	38.71	31.53	**24.72**
0.50	**0.00**	196.49	147.89	**135.63**
	0.10	141.09	115.73	**104.59**
	0.20	112.16	89.52	**68.46**
	0.30	78.35	52.75	**35.31**
	0.50	40.37	23.98	**19.49**
0.75	**0.00**	196.63	134.99	**123.47**
	0.10	158.73	100.79	**90.03**
	0.20	123.28	70.31	**52.31**
	0.30	78.09	37.34	**24.73**
	0.50	42.69	19.56	**11.01**
1.00	**0.00**	198.69	126.10	**104.42**
	0.10	142.61	90.15	**78.33**
	0.20	110.20	60.06	**37.00**
	0.30	70.60	28.11	**11.79**
	0.50	35.59	15.18	**6.71**
UCL		*32.77*	*46.60*	*32.77*

sin in the model is the deviation from an assumed quadratic model. Furthermore, we computed the simulated integrated mean square error (SIMSE) along with their standard errors. The SIMSE is a goodness of fit measure, and it is given in Equation (27):

$$\mathrm{SIMSE} = \frac{1}{m} \sum_{i=1}^{m} (y_i - \hat{y}_i)' (y_i - \hat{y}_i), \tag{27}$$

where it computes the average of the squared differences between the actual and the estimated profiles. The smaller the SIMSE value, the better the estimation is; because it means that there is no big difference between the true and estimated values. The SIMSE and standard error results are shown in Tables 1 and 2 for uncorrelated $\rho = 0.0$ and moderately correlated $\rho = 0.5$ data, respectively; only these two cases are shown for $m = 300$ and $n = 10, 20$ because the results are almost the same for $m = 600$ and also due to the space limitation. All the other results are available and can be requested from the authors.

From Tables 1 and 2, the smallest SIMSEs are given in bold, we can notice that for $\gamma = 0.0$ both parametric and MMRRPM techniques gave the same SIMSE value, this is because the mixing parameter is estimated to be zero ($\hat{\lambda} = 0$). Furthermore, as the misspecification level increases, one can see that the SIMSE values of SP get closer to the NP values, and the estimated mixing parameter getting larger and closer to one, and thus a bigger portion of the NP estimation will be counted. Also, for both $\rho = 0.0$ and $\rho = 0.5$, as sample size n gets larger, the SIMSE values get smaller for all the three approaches, as well as the standard error values. We have noticed that as the correlation increases, the SIMSE values also increase for different m and n. Moreover, in all the cases, MMRRPM approaches had the least SIMSEs and this means

TABLE 4 Out-of-control ARL for uncorrelated ($\rho = 0.0$) profile datasets for different model misspecification levels (γ) and shift sizes (δ) ($m = 300$, $n = 20$, $l = 40$)

γ	δ	T^2_{P}	T^2_{NPR}	T^2_{SPR}
0.00	**0.00**	**200.00**	**200.00**	**200.00**
	0.10	150.85	153.53	**139.60**
	0.20	117.48	126.98	**106.97**
	0.30	75.14	83.53	**66.29**
	0.50	43.8	49.10	**34.85**
0.25	**0.00**	199.89	178.37	**153.49**
	0.10	152.06	140.75	**114.21**
	0.20	112.63	105.36	**84.21**
	0.30	76.89	64.54	**42.66**
	0.50	40.13	32.59	**21.78**
0.50	**0.00**	199.4	163.54	**142.53**
	0.10	154.70	117.98	**106.53**
	0.20	115.69	87.92	**69.21**
	0.30	79.16	60.44	**38.67**
	0.50	44.77	31.09	**16.68**
0.75	**0.00**	199.46	146.84	**133.49**
	0.10	150.13	105.49	**100.44**
	0.20	110.52	75.25	**70.78**
	0.30	77.06	47.04	**36.45**
	0.50	46.07	25.34	**21.72**
1.00	**0.00**	196.79	135.01	**110.09**
	0.10	149.47	93.48	**79.53**
	0.20	118.17	67.51	**45.17**
	0.30	80.59	35.98	**19.59**
	0.50	47.37	20.84	**11.63**
UCL		*33.09*	*46.02*	*33.09*

its estimation is the closest to the actual values with the smallest errors. Therefore, MMRRPM gives more precise results compared to the other two approaches.

Now, the MEWMA charts in Equations (21), (23), and (25) for the parametric, NP, and MMRRPM, respectively, are constructed by substituting the random effects obtained for $\gamma = 0.0$ with no shift $\delta = 0.0$; to obtain the desired upper control limits (UCL) by setting the in-control ARL, $\mathrm{ARL}_0 = 200$. For simplicity, the smoothing parameters within each MEWMA control chart were selected to be equal ($w_1 = w_2 = \ldots = w_p$), and in our numerical study, they were chosen to be 0.2 as was chosen by Kim et al[5] and Narvand et al.[13] The control limits for the parametric and MMRRPM approaches are the same because, as mentioned previously, the mixing parameter is 0 when $\gamma = 0.0$. Then, after obtaining the UCLs for all charts, different amounts of shifts ($\delta = 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.50$) are introduced in the slope parameter to the last L profiles and the out-of-control ARL_1, and ATS values are computed for comparing the sensitivities of these three control charts among all misspecification levels for the uncorrelated and correlated data with a different combination of profiles and sample sizes. Again, due to space limitation, the uncorrelated and moderately correlated ARL results will be shown only for specific shift sizes ($\delta = 0.10, 0.20, 0.30, 0.50$) of MEWMA charts for $m = 300$ with $n = 10, 20$; these results are given in Tables 3-6. We can see that MMRRPM performance is better than both the parametric and the NP performances when there is no model misspecification, as its ARL_1 values are the least, except when $m = 300$ and $n = 10$ for $\rho = 0.5$, where it had exactly the same performance as the parametric MEWMA chart. Furthermore, when the misspecification level increases, the sensitivity of the NP MEWMA chart increases compared to the parametric MEWMA, as it is able to detect all amounts of shift quickly. Thus the NP chart is superior compared to the parametric chart when there is model misspecification, but in fact, it is not better than MMRRPM chart as its ARL_1 values are larger. We have found that neither the correlation level nor the profile size and the sample size have any effects on the results. We can conclude that the SP

TABLE 5 Out-of-control ARL for moderate autocorrelated ($\rho = 0.5$) profile datasets for different model misspecification levels (γ) and shift sizes (δ) ($m = 300$, $n = 10$, $l = 40$)

γ	δ	T^2_{P}	T^2_{NPR}	T^2_{SPR}
0.00	**0.00**	**200.00**	**200.00**	**200.00**
	0.10	150.85	154.78	**150.85**
	0.20	117.23	120.24	**117.23**
	0.30	90.65	95.96	**90.65**
	0.50	53.27	57.05	**53.27**
0.25	**0.00**	198.66	168.85	**148.14**
	0.10	147.98	130.89	**111.13**
	0.20	112.52	97.57	**94.20**
	0.30	85.41	70.11	**68.24**
	0.50	51.14	39.09	**33.23**
0.50	**0.00**	195.74	142.29	**133.47**
	0.10	142.70	99.74	**100.94**
	0.20	109.32	76.92	**75.52**
	0.30	78.65	61.16	**51.10**
	0.50	49.75	33.91	**26.00**
0.75	**0.00**	198.27	130.62	**111.26**
	0.10	144.33	94.22	**82.02**
	0.20	105.49	58.84	**56.42**
	0.30	76.91	37.10	**32.52**
	0.50	49.66	16.61	**14.63**
1.00	**0.00**	196.64	104.62	**90.11**
	0.10	143.46	75.84	**69.46**
	0.20	104.53	45.54	**41.24**
	0.30	77.59	20.76	**14.68**
	0.50	50.46	10.63	**6.54**
UCL		*33.02*	*47.51*	*33.02*

(MEWMA) control chart is the best in detecting model misspecification among all shifts. Thus, it is recommended to be used for monitoring profiles in Phase II.

5 | REAL DATA APPLICATION

The vertical density profile (VDP) data were used by Walker and Wright[22] as shown in Figure 1, where the density of 24 wood boards was measured as it is important for maintaining good quality. Each VDP contains 314 measurements taken 0.002 inches apart.

The response variable (Y) is the board density and the explanatory variable (X) is the depth measured at an interval of 0.002 inches between every two consecutive measures. The profile between the density and the depth was fitted using the LMM shown in Equation (28), which is used for obtaining control limits by using the same simulation scenario mentioned previously. Also, the normality assumption was checked for all the approaches and the plotting histograms for the residuals obtained from each method. From Figure 2, one can see that the parametric histogram is highly skewed to the left, this is an indication of a lack-of-fit using this technique, on the other hand, the histograms of the NP and SP approaches showed almost normal distributions.

$$y_{ij} = 53.70 + b_{0j} + \left(-67.55 + b_{1j}\right) x_{ij} + \left(109.15 + b_{2j}\right) {x_{ij}}^2 + \varepsilon_{ij}, \tag{28}$$

where $b_{0j} \sim N(0,\ 1.62)$, $\ b_{1j} \sim N(0,\ 46.80)$, and $b_{2j} \sim N(0,\ 121.19)$.

TABLE 6 Out-of-control ARL for moderate autocorrelated ($\rho = 0.5$) profile datasets for different model misspecification levels (γ) and shift sizes (δ) ($m = 300$, $n = 20$, $l = 40$)

γ	δ	T^2_{P}	T^2_{NPR}	T^2_{SPR}
0.00	**0.00**	200.00	200.00	**200.00**
	0.10	143.57	144.79	**143.57**
	0.20	110.51	109.79	**110.51**
	0.30	75.58	80.67	**75.58**
	0.50	46.38	50.06	**44.38**
0.25	**0.00**	197.10	171.20	**163.26**
	0.10	140.58	139.77	**124.77**
	0.20	104.01	94.56	**86.11**
	0.30	72.87	54.97	**52.51**
	0.50	44.43	35.49	**29.81**
0.50	**0.00**	197.09	159.68	**146.83**
	0.10	142.67	113.57	**110.73**
	0.20	100.81	72.88	**72.09**
	0.30	73.79	42.34	**40.25**
	0.50	44.50	24.60	**21.85**
0.75	**0.00**	198.71	145.30	**130.95**
	0.10	140.74	106.77	**102.21**
	0.20	100.40	69.43	**65.06**
	0.30	69.21	40.76	**39.67**
	0.50	40.54	21.71	**21.07**
1.00	**0.00**	197.19	137.09	**114.31**
	0.10	140.87	96.25	**78.55**
	0.20	100.58	64.71	**47.41**
	0.30	70.16	23.36	**17.82**
	0.50	41.67	12.61	**6.73**
UCL		*32.74*	*46.59*	*32.74*

Moreover, we fitted the profiles using the three approaches. In Figure 3, we can see that the parametric approach provides a smooth fit for the data. While in fact, the data have some fluctuations, and therefore, this approach is not capable of modeling this profile data. The MMRRPM technique presents the fluctuations and considers peaks and dips, thus it fits the data better. Furthermore, the mean square error (MSE) for each technique was computed: the parametric approach gives MSE 0.181, the MSE for NPRs is 0.0305, and the SP (MMRRPM) MSE equals 0.0305. The parametric fit has the highest MSE compared to other techniques, while both the NPRs and SP (MMRRPM) techniques have the least MSE.

As was done in simulation, the UCLs for the proposed MEWMA charts in Equations (21), (23), and (25) for the parametric, NPRs, and MMRRPM, respectively, were found to achieve the desired $\mathrm{ARL}_0 = 200$. The performances of these charts are examined by considering the first 16 profiles as in-control, and the last eight profiles were used to show an out-of-control condition for a slope shift coefficient of 0.5. The sensitivities of the proposed NP and SP MEWMA charts are represented in Figure 4. The in-control profiles are represented in blue, while the out-of-control profiles are represented in red. It can be seen that the PMEWMA chart required five profiles to detect an out-of-control process, while the NPRMEWMA required almost three profiles. This leads to better performance of the NP chart compared to the parametric chart. Furthermore, the SPRMEWMA chart needed two profiles only to detect an out-of-control process, and there exists one false alarm (profile 11). We can conclude that the sensitivity of the SP technique is more, and its shift detection is faster compared to the parametric and NP techniques for the real dataset.

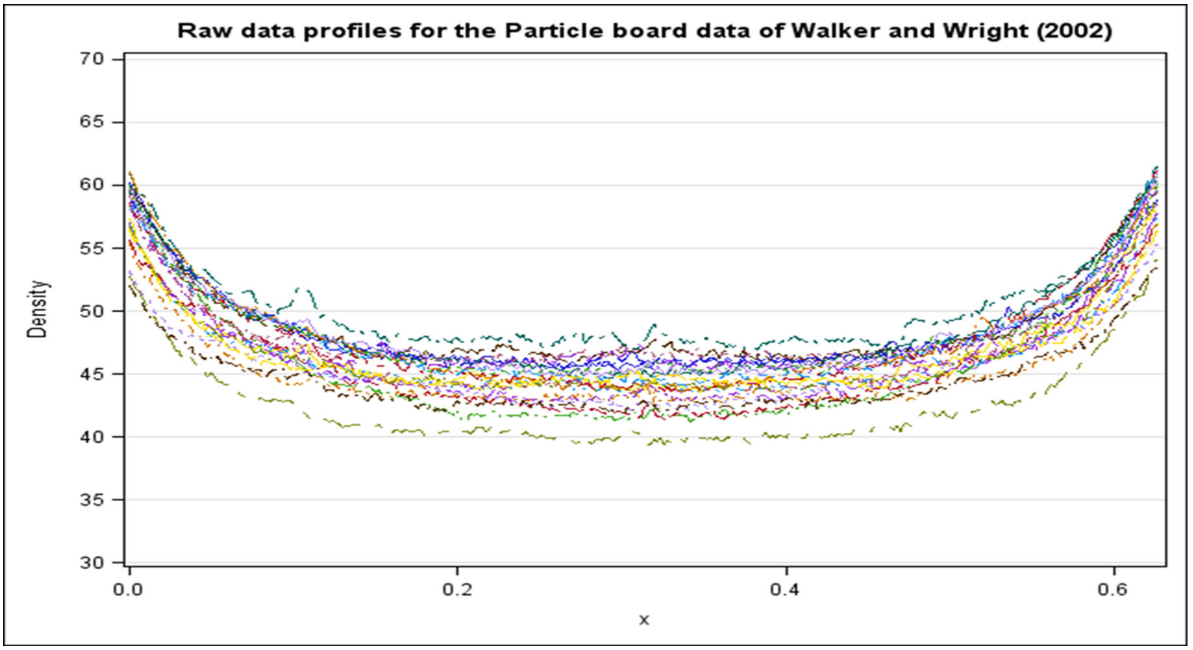

FIGURE 1 Raw dataset for vertical density boards

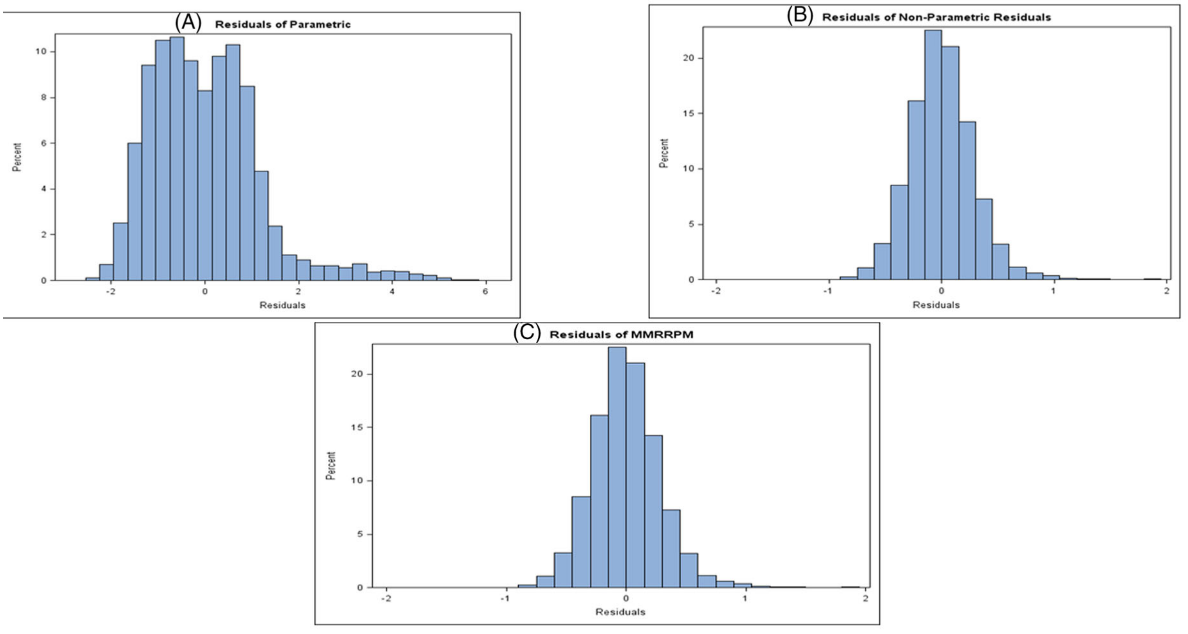

FIGURE 2 Vertical density profile residuals. (A) Parametric fit. (B) Nonparametric fit on residuals. (C) Semiparametric fit MMRRPM

6 | CONCLUSION

In this study, we have proposed the NP and SP approaches for monitoring LMMs for correlated and uncorrelated profiles in Phase II analysis. Our proposed NP and SP MEWMA control charts were constructed based on the estimated random effects for monitoring profiles. The performances of the NP and SP MEWMA charts were compared to the parametric approach using ARL criterion, where model misspecification was taken into consideration. Also, we considered different amounts of shifts inserted in the slope, to check the charts abilities in detecting out-of-control profiles. Comprehensive Monte Carlo simulation studies were done and a real data application was conducted. We found that the proposed SP MEWMA control chart had the best performance compared to the other two approaches. Moreover, the NP MEWMA

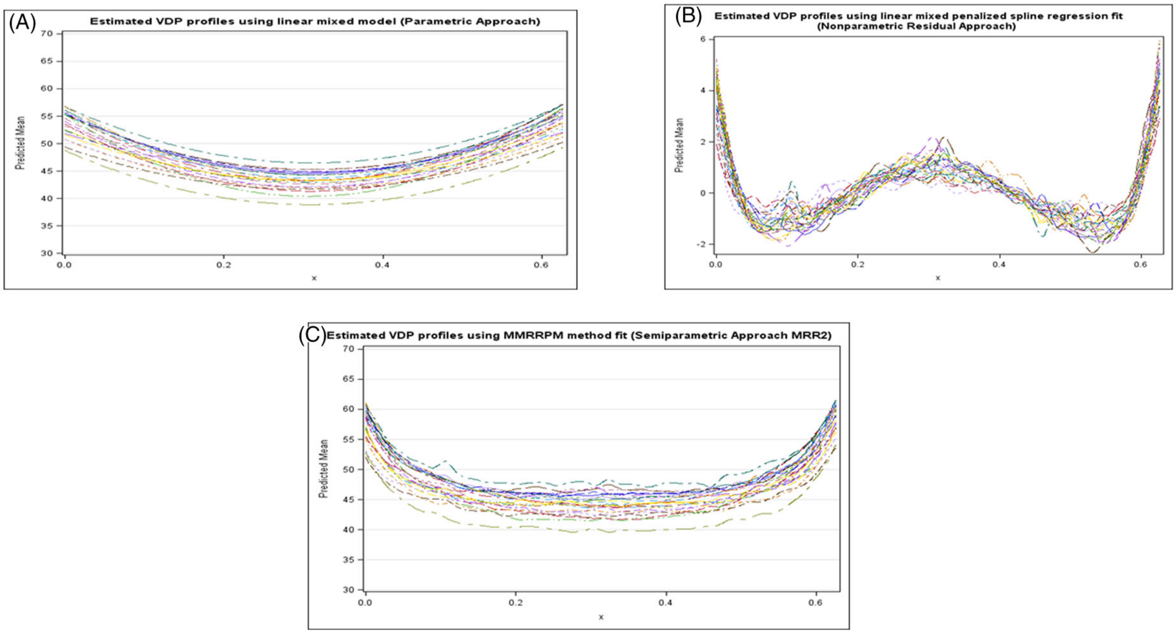

FIGURE 3 Vertical density profiles. (A) Parametric fit. (B) Nonparametric fit on residuals. (C) Semiparametric fit MMRRPM

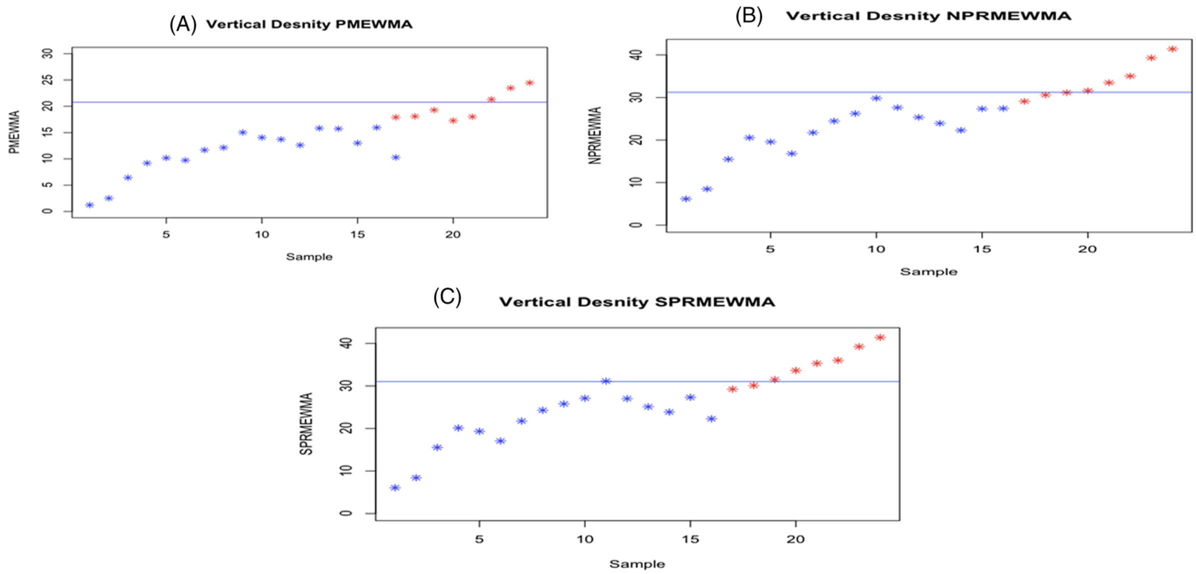

FIGURE 4 Vertical density MEWMA. (A) Parametric MEWMA. (B) Nonparametric residuals MEWMA. (C) Semiparametric MEWMA

control chart had a better performance than the parametric one, when there was model misspecification. Also, we computed SIMSE for the three charts, and again the SP approach had the least SIMSE results and thus it was the best. We recommend using the SP technique for faster detection of an out-of-control profile and for future recommendations; we suggest that the shift may be inserted in the intercept or variance-covariance matrix instead of introducing it into the slope. Moreover, the MEWMA charts may be based on the fitted values instead of the random effects only and their performances may be compared.

DATA AVAILABILITY STATEMENT

Data source: Data are available upon request from the authors.

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