ISA 444: Business Forecasting

10: Nonseasonal Smoothing and Forecasting

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Quick Refresher from Last Class

- ✅ Recognize time series that are appropriate for simple exponential smoothing (SES).
- ✅ Use SES for smoothing and forecasting.

Recap: Assignment 08

Go to www.menti.com/alwo42su6bo3

I read P.74-75 before attempting HV

Recap: Assignment 08

mended practice, we used an out-of-sample evaluation. In other words, we fit the first part of the series to the data and then examined the performance of the forecasting method by seeing how well it worked on later observations. In this example, we used a hold-out sample of more than 50 percent of the observations because our primary focus was on evaluating forecasting methods; a larger hold-out sample provides a better basis for comparisons. We compared five methods:

1. MA(3): A moving average of three terms
2. MA(8): A moving average of eight terms
3. SES(0.2): Simple exponential smoothing with $\alpha = 0.2$
4. SES(0.5): Simple exponential smoothing with $\alpha = 0.5$
5. SES(opt): Simple exponential smoothing with $\alpha = 0.728$

For each of the SES sets of forecasts, we used the first observation as the starting value, as in the previous example. We then generated the one-step-ahead forecasts for 36 weeks (weeks 27–62), starting at forecast origin $t = 26$. Only method 5 requires any parameter estimation. The summary results appear in Table 3.5. The best performance on each criterion is shown in bold. In this case, the best-fitting optimal SES scheme performs best on all counts. However, it may well happen that the different criteria lead to different conclusions. ■

Table 3.5 Summary Error Measures for One-Step-Ahead Forecasts of WFJ Sales Data (*Hold-out sample, weeks 27–62*)

Method	MAE	RMSE	MAPE

Recap: Assignment 08

We will use my tablet to explain the book's strategy for evaluating the performance of the five different forecasting procedures. **In my opinion, this strategy makes a lot of sense.**

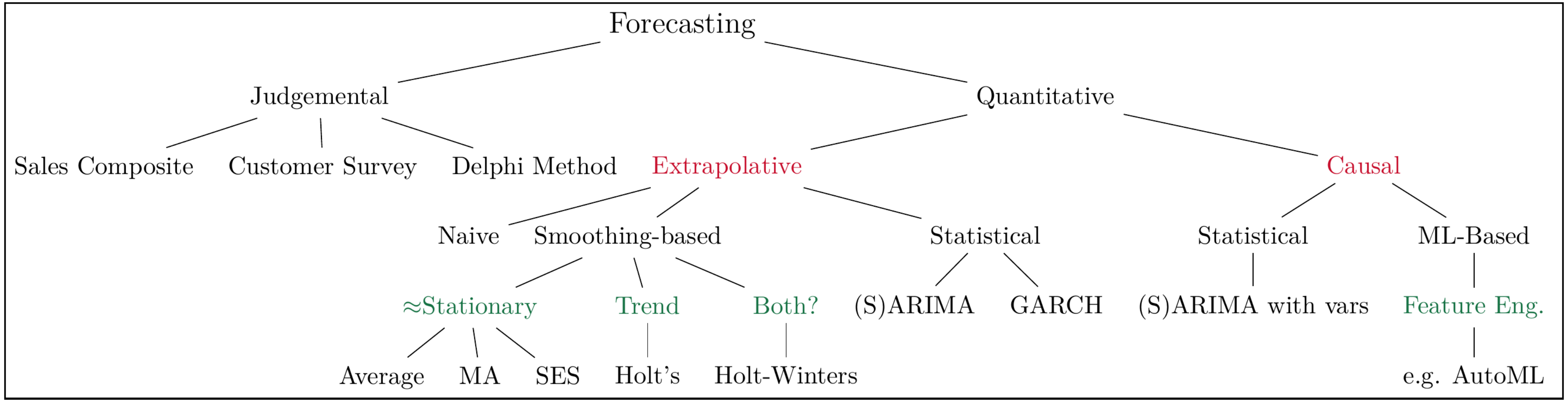
Learning Objectives for Today's Class

- Recognize time series that are appropriate for linear exponential smoothing (LES).
- Use SES for smoothing and forecasting.

Linear Exponential Smoothing

(LES, Holt's Method or Double Exponential Smoothing)

Overview of Univariate Forecasting Methods



A 10,000 foot view of forecasting techniques

Notes: My (incomplete) classification of **univariate** forecasting techniques, i.e., they exclude popular approaches used in multivariate time series forecasting.

Definition and Basic Principles

Linear Exponential Smoothing (LES) is a method used for one-step-ahead forecasting of a time series when there **is a local trend, but no** seasonal pattern.

A "global" trend occurs when a trend is increasing or decreasing at a nearly constant rate as in a simple linear regression model:

$$y_t = \beta_0 + \beta_1 t + \epsilon_t$$

A "local" trend occurs when a linear trend is increasing or decreasing at a nonconstant rate. LES, also referred to as Holt's Method or double exponential smoothing, is appropriate when the level (β_0) of the series is slowly changing as with SES, and the trend is also changing over time.

To compute the **forecast** we will use two smoothing constants, α, to smooth the level, and β, the smoothing constant to smooth the trend.

Definition and Basic Principles

The estimate of the **level** is:

$$l_t = \alpha y_t + (1 - \alpha)[l_{t-1} + b_{t-1}]$$

The estimate of the **trend** is:

$$b_t = \beta[l_t - l_{t-1}] + (1 - \beta)b_{t-1}$$

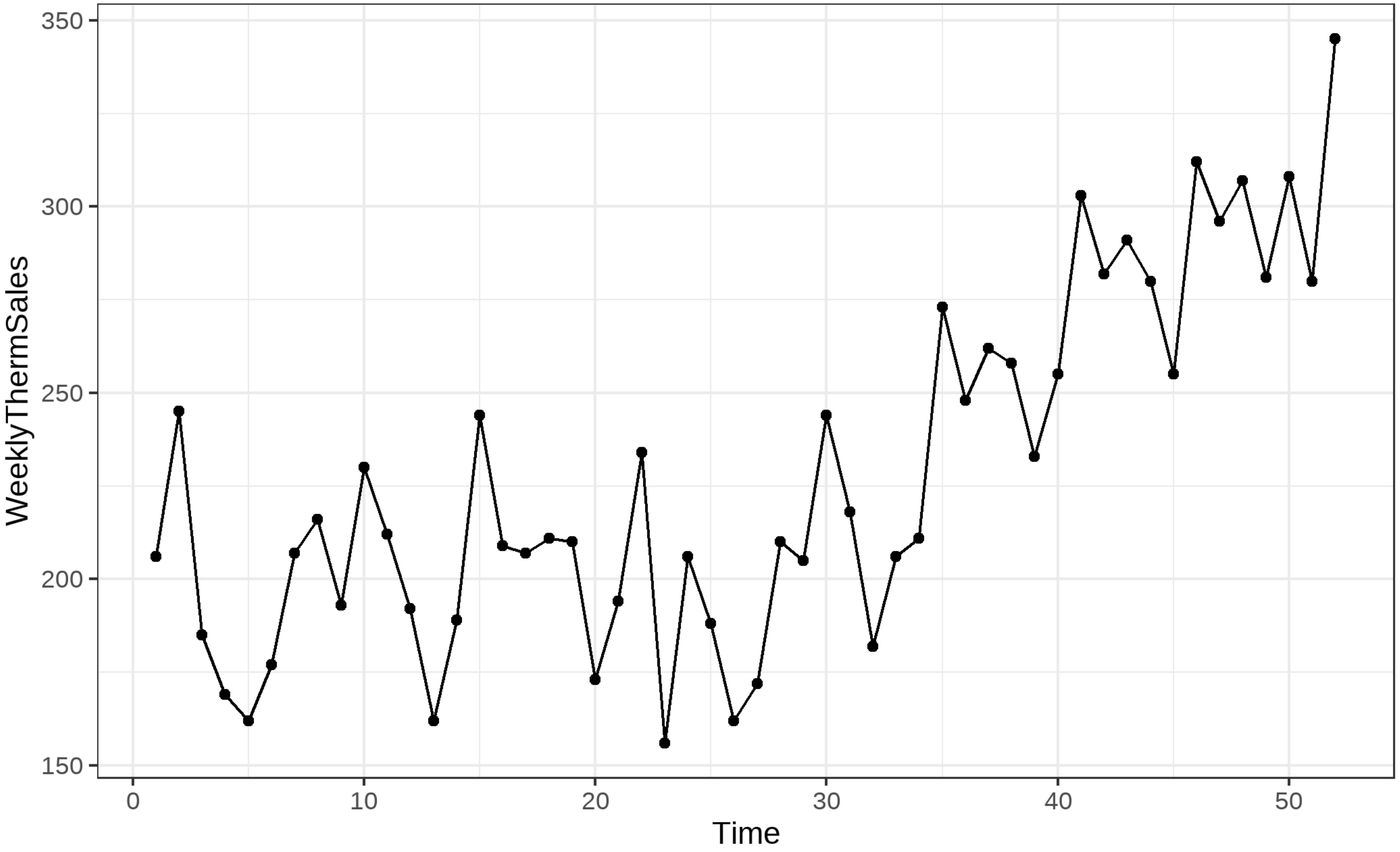
To estimate the **point forecast** for time $t + h$ time periods ahead made in time t:

$$\hat{y}_{t+h}(t) = l_t + (h \times b_t)$$

What needs to be Determined/Optimized for?

- **Starting value** for the level, L_0 and the starting value of the trend, B_0:
  - When fitting "by hand" you can use a training sample and fit a simple linear trend regression, $\hat{y}_t = b_0 + b_1 t$, to obtain initial estimates of L_0 and B_0.
  - $L_0 = b_0$, the intercept from a simple regression equation.
  - $B_0 = b_1$, the slope from a simple regression equation.
- The value of the **smoothing constants** for the: level, α, and trend, β:
  - $0 < \alpha < 1$, and $0 < \beta < 1$;
  - The values for α and β may be chosen to be the same or different.
  - Often the choices of the smoothing constants are arbitrary.
  - α and β can also be chosen by minimizing the mean squared one-step ahead forecast error (MSE) or equivalently, the square root of the mean squared one-step ahead forecast error (RMSE).

Out-of-Class: Weekly Therm Sales Example



Out-of-Class: Weekly Therm Sales Example

Let us use the first 26 points in the dataset to estimate both L_0 and B_0.

```
time = therm_sales$Time[1:26]
weekly_sales = therm_sales$WeeklyThermSales[1:26]

reg_model = lm(weekly_sales ~ time)

round(summary(reg_model)$coefficients, digits = 3)
```

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  202.625     10.320  19.634    0.000
## time          -0.368      0.668  -0.551    0.587
```

Out-of-Class: Weekly Therm Sales Example

Based on the information in the previous slide, please fill the table below. For the purpose of our example, please use $\alpha = 0.2$ and $\beta = 0.1$. Please create and fill this table in the Excel file.

```
## # A tibble: 9 × 5
##    Time WeeklyThermSales Level Trend `1-step ahead Forecast`
##   <dbl>            <dbl> <chr> <chr> <chr>
## 1     0               NA ...   ...   ...
## 2     1              206 ...   ...   ...
## 3     2              245 ...   ...   ...
## 4     3              185 ...   ...   ...
## 5     4              169 ...   ...   ...
## 6     5              162 ...   ...   ...
## 7     6              177 ...   ...   ...
## 8     7              207 ...   ...   ...
## 9     8              216 ...   ...   ...
```

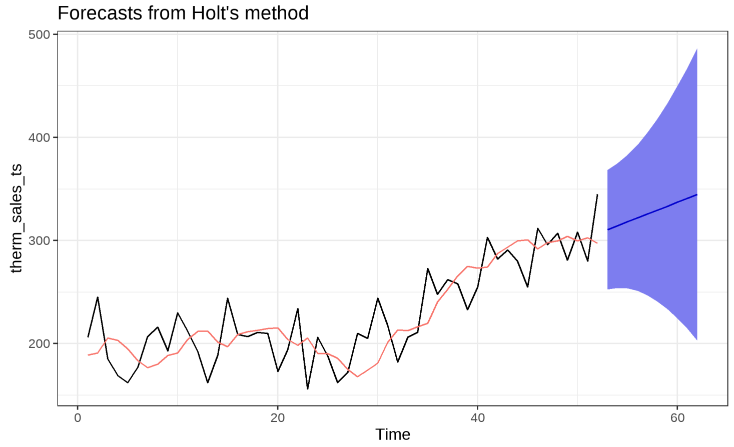
Weekly Therm Sales: Using R

```r
therm_sales =
  readxl::read_excel("../../data/Weekly_Therm_Sales.xlsx")

therm_sales_ts =
  ts(therm_sales$WeeklyThermSales, start = 1, frequency = 1)

# fitting holt model is similar to ses (two smoothing constants thoug
les_model = forecast::holt(
  therm_sales_ts, alpha = 0.2, beta = 0.1, level = 95, h=10
  )

# visualize the forecasts and fitted values
forecast::autoplot(les_model) + # produces original data + forecast
  forecast::autolayer(les_model$fitted) + # fitted values
  ggplot2::theme_bw() +
  ggplot2::theme(legend.position = 'none')
```



Weekly Therm Sales: Using R

```
therm_sales$les_f = les_model$fitted

head(therm_sales)
```

```
## # A tibble: 6 × 3
##    Time WeeklyThermSales les_f
##   <dbl>            <dbl> <dbl>
## 1     1              206  189.
## 2     2              245  191.
## 3     3              185  206.
## 4     4              169  203.
## 5     5              162  195.
## 6     6              177  183.
```

```
forecast::accuracy(les_model)
```

```
##                   ME     RMSE      MAE
## Training set 1.35195 28.36988 22.5766
```

Demo: Optimal α and β for WFJ Sales

Let us examine the data for the WFJ Sales Example (i.e., Example 3.2 in our textbook). Per the textbook example, we will use the first the 26 observations as the estimation sample. **Note that we will now apply LES instead of the SES approach we examined last class.**

Exam 01 Prep and Advice

Open Discussion

Recap

Summary of Main Points

By now, you should be able to do the following:

- Recognize time series that are appropriate for simple exponential smoothing (SES).
- Use SES for smoothing and forecasting.

Things to Do to Prepare for Our Next Class

- **Recommended:** Thoroughly read Chapter 3.1-3.4 of our reference book.
- **Required:** Complete assignment09.