

# Dem 260: Mathematical Demography with Applications

## Reading list and class calendar (v1.0)

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### Contents

1	Introduction to Demographic Heterogeneity (Jan 23)	3
2	Multiplicative fixed frailty (Jan 30)	4
3	Consequences of Gamma frailty (Feb 6)	5
4	Student Presentations on Applications (Feb 13)	6
5	Guest lecture on plateaus (Feb 20)	8
6	Fertility mixtures (Feb 27)	9
7	Branching Processes and Extinction (March 5)	10
8	Branching Processes and Survival (March 12)	11
9	Fisher-Wright: Random Parents (March 19)	11
10	No Class (April 2)	13
11	Incorporating Population Dynamics (April 9)	13
12	Coalescence (April 16)	14
13	PAA (April 23)	15
14	Student Presentations on Applications (April 30)	15

# Notes

- The first version of this calendar includes more detail at the beginning of the semester (Weeks 1 - 6) than later on. We will discuss whether we want to do advanced topics in weeks 11 and 12 – or cover another topic such as entropy or population dynamics of fertility decline.
- This version is missing links to data sets.
- Readings that have no link will be made available through scans.
- Problem sets are not yet included

# 1 Introduction to Demographic Heterogeneity (Jan 23)

## Topics

- Defining demographic heterogeneity
- Dynamics of population growth with two sub-groups
- Keyfitz's result  $\bar{r}'(t) = \sigma_r^2(t)$ .
- Ken's model of Poisson heterogeneity

## Resources

- James W. Vaupel and Anatoli I. Yashin (1985) "Heterogeneity and Its Ruses: Some Surprising Effects of Selection on Population Dynamics" *American Statistician* 39(3):176-185. <https://www.jstor.org/stable/2683925> [Very accessible intro and overview. Almost no math, lots of pictures.]
- Kenneth W. Wachter, *Essential Demographic Methods*, Section 8.8: Frailty Models. [An overview of Gamma frailty in just 4 pages. (Note: Ken told me there is a mistake in some of the math – can you find it?)]
- Nathan Keyfitz (1977 [or later edition]), *Applied Mathematical Demography*, [Keyfitz derivation that growth rate change equals the variance of growth rates.]

## Practice

- Taylor series approximation of two sub-group population growth
- Determinants of speed of convergence in Poisson-Exponential model
- Application of Poisson-Exponential model to Africa?

## 2 Multiplicative fixed frailty (Jan 30)

### Topics

- Varieties of heterogeneity
- Consequences of multiplicative fixed frailty including an extension of Keyfitz's result
- The gamma-frailty case

### Resources

- James W. Vaupel and Trifon I. Missov (2014) [V&M ] “Unobserved population Heterogeneity: A review of Formal Relationships” *Demographic Research* 31(22):659-688  
<https://www.demographic-research.org/Volumes/Vol31/22/>  
[A self-contained ‘primer’ with results and their derivations that cover most everything we’re doing in weeks 1-3]
- Germán Rodríguez (2001[2005]) “Unobserved Heterogeneity” [hand-out] <https://data.princeton.edu/pop509/UnobservedHeterogeneity.pdf>  
[Detailed hand-out for Princeton class. Many of the same results as V&M , but with alternative derivations and notation. Also includes inversion formula and extensions beyond Gamma frailty such as “Inverse Gaussian Frailty”]

### Practice

- Derivation of ...
- Gamma mathematics and exploration
- Simulation of Uniform-Gompertz and Gamma-Gompertz
- CenSoc income analysis of 1905 cohort?

### 3 Consequences of Gamma frailty (Feb 6)

#### Topics

- Plateaus
- Cross-overs
- Mortality Improvement

#### Resources

- Youtube: ... gamma video
- V&M Sections 5,6 and 7.
- Rodriguez, section 3 “The Inversion Formula”

#### Practice

- Inversion app
- Simulation [TBD]
- CenSoc analysis of Black-White crossover for 1905 cohort?

## 4 Student Presentations on Applications (Feb 13)

### Resources

- Crossovers

- Ansley J. Coale and Ellen E. Kisker (1986) “Mortality Crossovers: Reality or Bad Data?” *Population Studies* 40(3):389-401. <https://www.jstor.org/stable/2174582>.

[Non-technical overview of data issues that could create the appearance of mortality cross-overs.]

- Kenneth G. Manton and Eric Stollard (1981) “Methods for Evaluating the Heterogeneity of Aging Processes in Human Populations Using Vital Statistics Data: Explaining the Black/White Mortality Crossover by a Model of Mortality Selection” *Human Biology* 53(1):47-67.

[You can skim most of the math, which we have already covered, and focus on pages 55-66, especially the justification for the parameter choice of the Gamma and the empirical results. Is their method equivalent to using the inversion formula?]

- Rodriguez’s shiny app on Black-White Crossover using Inversion Formula <https://data.princeton.edu/pop509/heterogeneityApp>

[You might want to look at link to code at the bottom of the App to understand details. Note: the app uses period mortality as a proxy for cohort data.]

- Plateaus

- David R. Steinsaltz and Kenneth W. Wachter (2006) “Understanding Mortality Rate Deceleration and Heterogeneity,” *Mathematical Population Studies* 12:19-37. [Will also be covered by Ken’s guest lecture.]

- Barbi et al. (2018) “The plateau of human mortality: Demography of Longevity pioneers,” *Science* 360(6396):1459-1461. <https://science.sciencemag.org/content/360/6396/1459>

[Recent paper claiming to be “the best evidence to date for the existence of extreme-age mortality plateaus in humans”]

- Rising Inequality

- Hilary Waldron (2007) “Trends in Mortality Differentials and Life Expectancy for Male Social Security-Covered Workers, by Socioeconomic Status” *Social Security Bulletin* 67(3):1-28. [https://papers.ssrn.com/sol3/Delivery.cfm/SSRN\\_ID1153977\\_code787227.pdf?abstractid=1153977&mirid=1](https://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID1153977_code787227.pdf?abstractid=1153977&mirid=1)

[Age-cohort model suggesting growing inequality by income, but includes many caveats about heterogeneity. Our challenge is to apply models of heterogeneity to this issue, particularly the result on mortality improvement over time.]

**Practice** Each team will present

- Summary of reading
- 1 mathematical derivation or simulation
- Data analysis (HMD for Plateaus, CenSoc for crossovers, Simulation for inequality trends)

## 5 Guest lecture on plateaus (Feb 20)

Ken Wachter has agreed to give a guest lecture. Everyone should read the two papers on mortality plateaus from the previous week.

**Practice** TBD



## 6 Fertility mixtures (Feb 27)

### Topics

- Tempo effects and demographic heterogeneity
- Estimating Mixture Models with the EM algorithm
- Application to the United States

### Resources

- Rachel Sullivan (2005) “The age pattern of first-birth rates among US women: the bimodal 1990s,” *Demography*, 42(2), 259-273. <https://www.jstor.org/stable/4147346>  
[An early paper (by a Berkeley Demog student!) focusing on first birth hazards.]
- Marion Burkimsher (2017) “Evolution of the shape of the fertility curve: Why might some countries develop a bimodal curve?” *Demographic Research*, 37(11):295-324. <https://www.demographic-research.org/volumes/vol37/11/default.htm>  
[A descriptive paper, which you can mostly skip. But see especially section 7, where she argues that her findings contradict Sullivan for the United States.]
- Hastie and Tibersashi (2017) “The EM Algorithm” section 8.5 of *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Pages 272-276 of ES-LII\_print12.pdf available at <https://web.stanford.edu/~hastie/ElemStatLearn/>  
[A textbook example of expectation-maximization algorithm applied to mixture of two normals].
- Victor Lavrenko. Youtube video :”EM algorithm: how it works” [https://www.youtube.com/watch?v=REypj2sy\\_5U](https://www.youtube.com/watch?v=REypj2sy_5U)  
[Fitting mixture models with EM algorithm – with great yellow and blue graphics. Try this first before reading Hastie and Tibersashi]

### Practice

- Estimation of Mixture Model to other countries (e.g., Australia, Netherlands, Spain, using HFD)
- First birth analysis: does heterogeneous hazards give the same answer as heterogeneous density?

## 7 Branching Processes and Extinction (March 5)

### Topics

- What is a branching process
- Generating functions for reproduction
- Extinction probabilities

### Resources

- Charles M. Grinstead and J. Laurie Snell (2006) *Introduction to Probability* <https://math.dartmouth.edu/~prob/prob/prob.pdf>

[An intermediate/advanced undergraduate textbook with good section on Branching Processes, with references to Harris and Keyfitz below.]

- Theodore E. Harris (1964) *The Theory of Branching Processes* RAND Report version available on-line at <https://www.rand.org/content/dam/rand/pubs/reports/2009/R381.pdf>

[Classic reference, but readable if you take your time. We will only read chapter 1 and will only consider the very easiest material.]

- Surname Extinction: When will we all be "Smiths"? <https://www.youtube.com/watch?v=5p-Jdjo7sSQ>

[Popular science video providing good motivation, but without authority.]

- Is Your Surname about to Go Extinct? <https://blogs.ancestry.com/cm/is-your-rare-surname->

[A brief blog entry, mentioning of "endangered names."]

- Nathan Keyfitz (1968[1977]) "The Branching Process as a Population Model," Chapter 18 in *Introduction to the Mathematics of Populations with Revisions*, pages 399-412. (I will make a scan available).

[Another introduction to branching process with the details of the empirical example cited by Grinstead and Snell]

### Practice

- Analytical analysis of model with only 0, 1, and 2 offspring.
- Simulation
- 1880 to 1940 Complete Census Count analysis (of African-American last names)(part 1)

## 8 Branching Processes and Survival (March 12)

### Topics

- Geometric parity: a tractable parity distribution
- Distribution of surviving family sizes
- Time to extinction

### Resources

- See previous week
- Analytical analysis of geometric model
- More simulation
- 1880 to 1940 Complete Census Count analysis (of African-American last names)(part 2)

## 9 Fisher-Wright: Random Parents (March 19)

### Topics

- Assumptions
- Extinction probabilities
- Distribution of surviving lines
- Diffusion Approximation
- Assessment of Hahn and Bentley's application to naming patterns

### Resources

- Blog "Introduction to the Wright-Fisher Model" [https://stephens999.github.io/fiveMinuteStats/wright\\_fisher\\_model.html#pre-requisites](https://stephens999.github.io/fiveMinuteStats/wright_fisher_model.html#pre-requisites)
- Hahn, M. W. and Bentley, R. A. (2003). Drift as a mechanism for cultural change: An example from baby names. Proceedings of the Royal Society of London B, 270, S120S123. <https://hahnlab.sitehost.iu.edu/Publications/HahnBentley2003.pdf>

[Evolutionary anthropologists arguing that the neutral explanation of the Fisher-Wright model is consistent with the distribution of 1st names. What other quantitative or qualitative features of 1st name fashion could be used to try to reject the neutral model?]

[Note to self: McElreath cites this paper in Statistical Rethinking but I'm not sure of the context]

- Joe Felsenstein (2017) *Theoretical Evolutionary Genetics: A Draft Text* <http://evolution.gs.washington.edu/gs562/2017/pgbook.pdf>

[Very complete “lecture notes” for graduate genetics course. Lots of good commentary, does not assume a lot of math background, but lots of content and can be difficult to read a piece by itself.]

- Simulation
- Are US 1st names really consistent with Fisher-Wright? (Analysis of Social Security babynames data)

**Spring Break (March 26)**

**10 No Class (April 2)**

**11 Incorporating Population Dynamics (April 9)**

**Topics**

- Mutation
- Migration
- Population Growth

**Resources**

- Joe Felsenstein (2017) *Theoretical Evolutionary Genetics: A Draft Text* <http://evolution.gs.washington.edu/gs562/2017/pgbook.pdf>

## 12 Coalescence (April 16)

### Topics

- Definition of Most Recent Common Ancestor (“MRCA”)
- Coalescence under Fisher-Wright
- Applications

### Resources

- Felsenstein Chapter 10, section 4 (X.4) [Nice figure for Fisher Wright]
- Applications [to be added]

### **13 PAA (April 23)**

### **14 Student Presentations on Applications (April 30)**

Empirical presentations on Branching Processes and Fisher-Wright based on simulation, mathematical analysis, and/or data analysis of first or last names in the United States.