Problem Set 2

Joshua R. Goldstein
UC Berkeley
Dem260 "Mathematical Demography"
Spring 2020

January 30, 2020

Everyone should do the "un-starred" problems. The "starred" are optional. They aren't necessarily difficult, but if you want to do less, feel free to not do these. The "double star" problems might be more advanced or time-consuming. Great if anyone wants to take a stab, but totally optional.

P.S. Feel free to team up for any of the problems, but especially the "starred" ones.

1. True/False: The variance of the population distribution of deaths will always be larger than that of the baseline. Explain your answer briefly.

2. Use simulation in R to produce plots of the uniform, gamma, and U-shaped beta distribution. Describe in a sentence, each, how the population hazard behaves at older ages. (See "frailty_simulator.R" for sample code).

3. Does the behavior of the uniform at older ages look like a population with two (proportional) sub-groups? What do you think is driving this? (This is an open-ended question. You should feel free to use mathematics, intuition, or any other approach to answer.)

∗ 4. Does the behavior of the beta at older ages look like the gamma at older ages? What do you think is driving this? (Also open ended)

5. At what age do population hazards start to diverge from the baseline in the the three models? Is it fair to say that half the cohort has to have died before unobserved heterogeneity plays a role?

6. Extend the simulation code to include life expectancy at age x. (This requires some programming.)

∗ 7. Extend the simulation code to include the average frailty of the surviving at age x, $\bar{z}(x)$. (Note: this requires some more difficulty programming, and I would recommend keeping your N fairly small.)

∗ 8. Extend the simulation code to histograms of frailty of survivors at different ages. Does the uniform stay uniform? How about the other distributions?

9. Use the method of completing the gamma to get the mean of the gamma distribution. (Hint: I believe there are youtube examples of this).

10. Derive V&M equation 13, extending Keyfitz's result. Did your derivation require you to assume proportional hazards; if so, where?

∗11. Derive V&M equation 20, extending Keyfitz's result to proportional changes in the population hazard. Did your derivation require you to assume proportional hazards; if so, where?

∗12. Describe a strategy for simulating cross-overs in the aggregate hazards of two groups, which have baseline hazards that don't cross. If you want, write code and produce a plot.