Chapter 2
Amplified Changes: An Analysis of Four Dynamic Fertility Models

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2.1 Introduction

In this chapter, we provide overviews of four models for the dynamics of fertility change. These models are applicable to populations with low and moderate birth rates, populations that have already completed the demographic transition and are now subject to the ups-and-downs of period fertility.

Implicit in each model is its own story of the behavior driving fertility. Each model offers its own lessons about how relatively small changes in one aspect of fertility (e.g., timing or targets) can produce large changes in another aspect such as total fertility.

Demographers use these models to try to understand why fertility change is occurring and to make informed predictions about the future level of births. Ideally, such models would allow a better understanding of the determinants of fertility change. But, as we will see, the stylized assumptions behind each model are better suited for providing insights into the variety of forces that influence fertility rather than the creation of measures that are free of "distortions" or "bias."

In this chapter, we present the models in the order that they were developed. We begin with Norman Ryder's (1964) classic formulation of the translation between period and cohort measures of fertility. Ryder himself was strongly committed to giving priority to the cohort perspective on fertility (Ryder 1965, cf. Ní Bhrolcháin

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R. Schoen (ed.), *Dynamic Demographic Analysis*,
The Springer Series on Demographic Methods and Population Analysis 39,
DOI 10.1007/978-3-319-26603-9_2

1992). The formal nature of his model is bidirectional, allowing translation from periods to cohorts or vice versa.

The second model we present is Ronald Lee's moving-target formulation. This model is distinctly more behavioral, translating between targeted cohort fertility goals and the flow of births seen in period fertility. Lee's model can be seen as translation model, but it is not translating between total cohort fertility and total period fertility, but rather between the "desired fertility" or "target" of a cohort and observed period fertility.

The modern work on fertility dynamics was pioneered by Bongaarts and Feeney (1998), whose approach has been extended by many, including Kohler and Philipov (2001), and Bongaarts and Sobotka (2012). Here the implicit behavioral model is not about changing targets of completed family size but about the timing of planned children. Bongaarts and Feeney build on Ryder's general insight about fertility timing to show that in the particular case where births are shifted from one period to another, the period fertility rate will change even if there is no change in completed fertility.

Finally, we present our own work (Goldstein and Cassidy 2014), which can be seen as the re-application of Bongaarts and Feeney's thinking to the timing of cohort fertility. Instead of relatively high-frequency variation in period birth timing, we focus on gradual changes of the timing of cohort fertility. The result is that period fertility is influenced by cohort timing in a manner similar to what Ryder first envisioned, but in the more modern context of shifting births.

All of these models have in common a simplified view of how fertility change occurs. A remarkable lesson told by all of these formulations is that small changes in timing or targets can produce large fluctuations in period fertility. This is what we see as the take-home lesson of the last half-century of fertility models. It is not that one model is right and the others are wrong, but rather that all of the models tell us period fertility is remarkably sensitive to changes in other underlying aspects of the fertility process.

The sensitivity of period fertility is seen by some (e.g. Sobotka and Lutz 2010) as reason to abandon the total fertility rate as a meaningful demographic measure. However, population processes like the size of birth cohorts and the shape of the population age pyramid are driven by period fertility. So even though period fertility may be fickle and fleeting, it is important enough to merit the attention that demographers have given it.

2.2 Fertility Models

When describing the dynamics of fertility over time, demographers distinguish between cohort and period measures. Cohort measures apply to the life-times of people born at the same time, whereas period measures are cross-sectional, and apply across the ages of people alive at the same moment in time. Cohort measures are more "real" in the sense that they summarize the lifetime experience of a group of individuals. But period measures are also important, in that the rate of birth at a given moment in time determines the unrolling of demographic history that is seen in a population's age structure. The period total fertility rate is the most commonly used measure of fertility.

Beginning with Ryder, demographers have also distinguished between demographic change brought about by changes in "tempo" (the timing of births) and changes in "quantum" (the level or intensity of births). Quantum can be operationalized as a change in the level of fertility across all ages. In contrast, a tempo change can be thought of as an adjustment in the timing of childbirth that shifts the mean age at birth up or down without raising or lowering the fertility schedule. One can distinguish between a *period* quantum change, which is an increase or decrease between periods, and a *cohort* quantum change, where the adjustment varies by cohort. Similarly, we can separate *period* tempo changes from *cohort* tempo changes, where the former refers to a rigid shift in the period fertility schedule, while the later assumes the shift operates on the cohort fertility schedule.

Tempo and quantum are idealized concepts, and the dichotomy between the two cannot be maintained when switching between cohort and period perspectives. For example, it is possible that a sequence of changes in cohort quantum, when viewed from the period perspective, could be indistinguishable from a period tempo change. Likewise, a sequence of period quantum changes could appear from the cohort perspective to be a cohort tempo change. The aim of some models, for example the period-shift model of Bongaarts and Feeney or our own cohort-shift model, is to make tempo and quantum changes separable. In these cases, the goal is to obtain a "pure" measure of quantum that is not influenced by timing changes. While actual changes in fertility may be a blend of tempo, quantum, and other transformations, the distinction between tempo and quantum remains a valuable interpretative tool for demographers. By classifying changes into these two basic types, dynamic demographic models distinguish decisions about when to have children from decisions about how many children to have.

Each of the models we present will have some of its own notation, however some common notation can be used across all the models. Denote the fertility rate at age a and time t by $f(a,t)$. Births are the product of the fertility rate and person-years of exposure, so that a woman exposed for 1 year to a fertility rate of 0.2 would be expected to average 0.2 births. We use $c = t - a$ to denote the birth year of a cohort, so that the fertility rate at age a of the cohort born at time c is given by $g(a,c) = f(a, c + a)$. The (period) total fertility rate in year t is defined as the sum of age-specific rates over the whole age range,

$$TFR(t) := \int_{\alpha}^{\beta} f(a,t)\,da \tag{2.1}$$

where α and β are the lower and upper biological limits of childbearing. The completed total fertility rate for the cohort born in year c is given by

$$CTFR(c) := \int_{\alpha}^{\beta} f(a, c+a)\, da = \int_{\alpha}^{\beta} g(a, c)\, da. \tag{2.2}$$

We let $f_0(a)$ denote a normalized, standard baseline fertility schedule that sums to one, giving the baseline probability density of births by age. The models discussed in Sects. 2.5 and 2.6 assume the existence of period quantum effect, i.e. a level change in period fertility as a consequence of events that are independent of timing. We use $q(t)$ to denote such an effect.

2.3 Ryder's Approximation

Norman Ryder's classic result (1964) pioneered the formal analysis of the relationship between period and cohort total fertility in the context of changing age-specific rates.

Without changes in fertility timing, it is easy to imagine that cohort total fertility is a moving average of period total fertility. If each year the schedule of fertility has the same shape, with an unchanging share $f_0(a)$ having children at age a every year, and period total fertility equal to $q(t)$, then the surface of fertility over age and time will have the form

$$f(a, t) = q(t) f_0(a) \tag{2.3}$$

and cohort total fertility of the cohort born in year c will be

$$CTFR(c) = \int_{\alpha}^{\beta} f(a, c+a)\, da = \int_{\alpha}^{\beta} q(c+a) f_0(a)\, da. \tag{2.4}$$

Thus, cohort total fertility is a moving average of period total fertility with weights equal to the share of childbearing occurring at each age.

An analogous relationship could be written expressing period total fertility as a moving average of cohort totals, with the assumption of constant cohort age-distribution.[1] In either case, however, there should be no consistent divergence of cohort and period fertility – the long run averages of cohort and period fertility should be roughly equal, particularly if the appropriate correspondence between dates and ages is made.

The innovation of Ryder was to introduce the possibility of a changing age schedule. Any surface of age specific rates can be written as a Taylor series

[1] If we posit a world with fluctuating cohort quantum $q(c)$ and fertility rates given by $f(a, t) = q(t-a) f_0(a)$, then period total fertility rate is a moving average of cohort total fertility. In most countries we see greater variability in period measures than in cohort measures, suggesting that the model with fluctuating period quantum is a better approximation of reality.

$$f(a,t) = f(a,0) + f'(a)t + \dots, \tag{2.5}$$

where the $f'(a)$ term is the rate of change with respect to time in fertility at age a. Taking only these first two terms, some algebraic manipulation[2] shows that for the cohort born in year t,

$$CTFR(t) \approx TFR(t + \mu_c)/(1 - \mu_c'), \tag{2.6}$$

where μ_c is the cohort mean age of childbearing of the cohort born in year t and μ_c' is its time-derivative. Note that in this correspondence, the dating of periods and cohorts are offset. In (2.6), the fertility of the cohort born in year t is compared with period fertility in year $t + \mu_c$, several decades later. An analogous expression can be written in terms of the period age of childbearing.[3] The same translations can also be written for parity-specific fertility.

The importance of Ryder's result is that it allows us to understand how prolonged changes in fertility timing can produce a situation in which the period and cohort total fertility rates are consistently different from each other. During the Baby Boom, the mean age of birth was falling at an annual pace of roughly a tenth of a year, raising observed period fertility by about 10 % over the completed fertility of corresponding cohorts. More recently, the postponement of fertility has been at a similar pace of about a tenth per year, making period fertility some 10 % smaller than cohort fertility.

Ryder's formulation is completely general, in the sense that any continuous and differential time series in age-specific rates can be expressed as a Taylor series. His specific result relating the total fertility of periods and cohorts arises in the simple case of linear change but is completely general as to the age pattern of this linear change, allowing for changes in the level ("quantum") of completed cohort fertility as well as in period fertility. One can view this result either as an exact expression of linear change or as a first order (linear) approximation of more complicated changes. The tempo-models introduced more recently by Bongaarts and Feeney, Kohler and Philipov, and Goldstein and Cassidy (among others) add more structure as to the specific nature of age-specific change. However, as we will

[2]Set μ_c to be $\dfrac{\int af(a,t+a)\,da}{CTFR(t)}$ and use the first to two terms of (2.5) to show that $\mu_c' = \dfrac{\int(a-\mu_c)f'(a)\,da}{CTFR(t)}$. Note that $CTFR(t) = \int f(a,0)\,da + t\int f'(a)\,da + \int af'(a)\,da$. From Eq. (2.1), we get $TFR(t+\mu_c) = \int f(a,0)\,da + t\int f'(a)\,da + \int \mu_c f'(a)\,da = CTFR(t) - \int af'(a)\,da + \int \mu_c f'(a)\,da = CTFR(t)(1-\mu_c')$. It follows that $CTFR(t) = \dfrac{TFR(t+\mu_c)}{1-\mu_c'}$.

[3]Letting $\mu_p'(t)$ be the derivative of the period mean age of childbearing at time t, we obtain

$$CTFR(t - \mu_p) \approx TFR(t) \times (1 + \mu_p'(t)).$$

With this translation, one can estimate cohort fertility from easily obtainable period measures.

see, they produce similar, even if not identical, estimates of the relationship between period and cohort rates (or tempo-adjusted and observed period rates) to that found in Ryder's classic expression. The formal interpretation of this similarity is that Ryder's first order approximation can be applied to any model of fertility change. A more substantive interpretation is that the importance of changes in timing revealed by Ryder are of such importance that they manifest themselves in a wide family of models for fertility change.

2.4 Moving Targets

We now turn to Ronald Lee's more behavioral formulation of fertility dynamics, modeling the process by which desired family size translates into period quantum. His (1980) "moving-target" model has women updating their family size targets from one period to the next and shows the consequences of changing intentions on the flow of births as measured by fertility rates. Lee shows how this process can be modeled as a differential equation and uses his model, among other things, to show how relatively large changes in period total fertility can result from small changes in fertility goals.

Lee's moving-target model allows desired completed family size, $D(t)$, to change from year to year in response to broad socioeconomic forces, which are assumed to influence all ages equally. This framework is in contrast to what could be called a "fixed-target" model, in which women's target family size is determined early in life and does not change. The moving-target model relies on the simplifying assumption that all cohorts alive in year t share the same desired completed family size. Based on their predictions about the future socioeconomic climate and other factors, couples make decisions about an ideal family size, and changes in these decisions lead to revisions in the target. Since predictions of the future are constructed out of current experiences, this model allows women to revise their target each year. Thus D is function of the year t. Survey data indicates that for the period beginning in 1946 through the mid 1960s, the value of D for women in the U.S. rose, but then declined rapidly in the 1970s.

Lee's model for fertility can be thought of as analogous to the industrial process of inventory and manufacture, in which we can imagine large swings in production based on the gap between actual and desired inventories. When the demand for children outstrips the current supply, women can respond by increasing their fertility. There are, however, some obvious biological limitations to viewing fertility changes as a sequence of stock adjustments. In particular, if supply were to exceed demand, there is no way to reduce inventory.[4] Consequently, the analogy works

[4] Changes in additional desired fertility create an important asymmetry for the moving-target model. When desired family size is rising, women can respond with adjustments to their fertility intentions that take into account the new ideal family size. In fact, as desired family size rises,

best if we imagine that the inventory is always less than the demand, and it is the changing size of this gap that leads to variation in production.

In the moving-target model, a cohort's fertility in a given year is a function of the gap between each cohort's average achieved fertility and the value of D for that year. This gap, called additional desired fertility, varies both by year and by cohort. It is defined as the difference between the current desired family size and a cohort's average accumulated fertility to date. For example, if 2.6 children represented the average desired family size in the year 1980, and if women from the cohort of 1955 had an average of 1.9 children by 1980, then for this cohort the additional desired fertility is 0.7 children. The assumption of a single target for women of all ages means that different cohorts would experience the same goal of 2.6 children, but because each cohort can have a different accumulated fertility, additional desired fertility varies by age within a given year.

In order to make the model identifiable in a simple way, Lee's formulation makes the additional assumption that, for women aged 25 and over, the annual birth rate is a fixed fraction of the additional desired fertility. Based on a comparison between survey evidence of additional desired fertility and birth rates in the subsequent year, Lee approximates this fixed fraction, denoted λ, as 0.18. Extensions of the model could account for variation in λ with age.

We now turn to a formal development of the moving-target model. Let $C(a, c)$ denote cumulated fertility of the cohort born in year c at age a, i.e. $C(a, c) := \int_0^a g(x, c)dx$. (Inversely, one can obtain age-specific fertility rates from the cumulative cohort fertility by differentiating, with the partial derivative $\frac{\partial}{\partial a}C(a, c)$ equal to $g(a, c)$). We use $A(a, c)$ to denote the additional desired fertility for women in cohort c, i.e.,

$$A(a, c) = D(a + c) - C(a, c). \tag{2.7}$$

The fixed fraction assumption for the rate at which desired fertility is realized means that, for $a \geq 25$, we have $g(a, c) = \lambda A(a, c)$. Multiplying Eq. (2.7) by λ yields

$$g(a, c) = \lambda D(a + c) - \lambda C(a, c). \tag{2.8}$$

For a fixed cohort c, we can differentiate this equation with respect to age a to produce

$$\frac{\partial}{\partial a}g(a, c) = \lambda D'(a + c) - \lambda g(a, c). \tag{2.9}$$

women who had perhaps thought that their family was complete, might change their minds and plan for additional children. In contrast, if desired family size is declining, some women who have completed childbearing may find themselves with children that were wanted when conceived, but are now part of a larger family size than the current ideal. Lee calls this phenomenon "the irreversibility of fertility," and formulates an alternate version of the moving-target model to address this issue.

This first order linear differential equation can be solved for $g(a,c)$ to obtain

$$g(a,c) = \lambda e^{-\lambda a}\left[\int_0^a e^{\lambda u} D'(u+c)du + D(c)\right]. \tag{2.10}$$

This formula presents fertility rates as an outcome of previous change in desired family size. For our purposes, the importance of this result is not that it allows us to calculate fertility rates from known targets. Rather, this model allows us to understand, in a formal way, the complex interplay of changing desires, past behaviors, and period measures. It presents observed fertility rates as the outcome of an evolving history of target family sizes. The model implies that women can change their fertility intentions each year and that their fertility choices are determined by the gap between the current ideal size and the fertility already achieved as a consequence of their pursuit of previous targets. This means that observed fertility rates are not simply a lagged version of desired family size, with period total fertility replicating the desired family size of a few years earlier. Instead, the model presents period total fertility as a function of the rate of change in the desired family size. We explore implications of this relationship next.

Implications of the moving-target model The moving-target model has several interesting consequences, provided that desired fertility D changes smoothly over time. These consequences are illustrated in detail in Lee (1980), where the implications of both a linear and a sinusoidal shape for D are investigated. We discuss just a few of these consequences here, including one that is, at least superficially, counterintuitive.

First, the underlying assumptions in the moving-target model have implications for the analysis of fertility change over time. If we accept the central tenet that desired family size is a function of period rather than of cohort, then a cohort's completed fertility should not be interpreted as a measure of that cohort's fixed intentions throughout its reproductive years. Rather, the completed fertility of a cohort is the outcome of changing desires, and may tell us nothing about the aspirations of the cohort when it began its reproductive years.

Second, when desired family size D is rising, TFR is greater than D, and when desired family size is falling, TFR is less than D. Although this phenomena can be demonstrated by substituting various specific functions of desired family size into Eq. (2.10), it can also be understood as a consequence of the fact that additional desired fertility is the difference between current desires and previously accumulated fertility. When desired family size is rising, cumulative fertility C will lag behind current desires because C is the consequence of behavior in previous years when desired family size was lower. It follows that women at all ages, in order to catch up with the latest desired target, will need to increase their fertility by more than would have been necessary had the latest value of D been in place in previous years. Similarly, if family size is falling, then cumulative fertility will represent births that occurred in years when the goal was higher, and women will need to reduce births to avoid overshooting the new target.

Third, when desired family size fluctuates, *TFR* will fluctuate with greater amplitude, so that in effect changes in *TFR* are exaggerated versions of changes in *D*. This fact follows from the previous result, since *TFR* will be larger than desired family size when desired family size is increasing but become smaller than desired family size when desired family size is decreasing.

Fourth, when desired family size fluctuates, turning points in *TFR* may precede those in desired family size by several years. Intuitively, we might have thought of *TFR* as responding to changes in the desired family size, and so we might expect that turning points in *TFR* would follow turning points in *D*. However, *TFR* is a sum of fertility rates that are determined not by the value of *D*, but rather by the difference between *D* and the *C*. From Eq. (2.7), we see that fertility rates, and consequently *TFR*, will be highest when the difference between desired family size and cumulative fertility is greatest. If desired family size has been increasing but begins to slow down, this gap will shrink, and *TFR* will begin to drop. By the time that desired family size peaks, *TFR* will already be dropping.

2.5 Period Shifts

In this section and Sect. 2.6 we present two models in which births are shifted, either postponed or advanced. One approach is period-based and the other is cohort-based. Much of the material in these sections is drawn directly from Goldstein and Cassidy (2014), where a more detailed comparison of these models can be found.

The period-shift model of fertility change appears in the appendix to Bongaarts and Feeney (1998). An alternate derivation is provided in Rodriguez (2006). Bongaarts and Feeney were interested in eliminating distortion in *TFR* caused by changes in the timing of births. Their influential paper introduced a method for correcting these distortions, and inaugurated a debate that continues to this day about how to adjust period measures of fertility and whether such adjustments provide a meaningful picture of fertility behavior. Some objections to the Bongaarts-Feeney adjustment technique have been raised (Kim and Schoen 2000), and modifications to the adjustment procedure have been proposed (Kohler and Philipov 2001; Bongaarts and Feeney 2006). In this chapter, our focus is on the assumptions underlying this adjustment procedure and the consequences of these assumptions, not on the arguments for or against tempo adjustment.

The period-shift model aims to get at a "true" period quantum $q(t)$ that is exogenous to the model. Bongaarts and Feeney's formulation assumes that the timing of fertility is a function only of the year t, so that women of all ages in a given year will postpone or advance the timing of childbirth by the same amount. This shift in timing acts on a base-line fertility schedule f_0, and thus the shape of the fertility schedule is assumed to be constant, although both its position and level are allowed to change. Period-based postponement is represented by $R(t)$, which denotes the total number of years by which women in period t have shifted their fertility, i.e., if $R(2010) = 3$ then women in the year 2010 have postponed childbirth by 3 years relative to the baseline schedule. Period fertility rates are determined both by the timing changes, via R, and by changes in the period quantum $q(t)$. R' denotes the derivative $\frac{dR}{dt}$.

The period-shift model of fertility Although originally presented as an adjustment procedure, it is possible to formalize Bongaarts and Feeney's approach in terms of a model of fertility change over time, with the age-period surface of fertility described as

$$f(a,t) = f_0(a - R(t))(1 - R'(t))q(t). \tag{2.11}$$

Notice that the period-shift model includes the term $(1 - R'(t))$ that lowers or raises the fertility level depending on whether women have delayed or advanced fertility. This term appears because timing changes that occur within cohorts spread or consolidate period births.

A derivation of the model, due to Rodriguez (2006), is as follows. Let $F_0(a) = \int_0^a f_0(x)dx$ be the cumulative fertility for the baseline schedule. We assume $F_0(\beta) = 1$, where β is sufficiently large so as to be well beyond the last age of fertility, leaving room for shifts. Under the period-shift model, the shifted cumulative fertility schedule for women born in year c and currently of age a is $F_0(a - R(c + a))$. Differentiating cumulative fertility with respect to age a gives us $f_0(a - R(c+a))(1 - R'(c + a))$. To obtain the observed fertility rate $f(a,t)$, we include period quantum effects $q(t)$ and rewrite cohort in terms of period and age.

If $f(a,t)$ is given by Eq. (2.11), then the total fertility rate will simplify to a product of independent quantum and tempo terms.

$$TFR(t) = \int_\alpha^\beta f(a,t)da = q(t)(1 - R'(t)). \tag{2.12}$$

If we wish to determine the value of q that would have been observed in period t had there been no postponement, we need to estimate R'. This turns out to be relatively easy, since Eq. (2.11) implies that $R' = \mu_p'(t)$, where $\mu_p(t)$ denotes mean age at birth in period t. This derivative can be approximated by comparing the mean age at birth in year t with that in adjacent periods. Thus we have the shift-adjusted total fertility rate

$$TFR^*(t) := TFR(t)/(1 - \mu_p'(t)). \tag{2.13}$$

If the hypotheses of the period-shift model hold, then $TFR^*(t)$ will be equal to $q(t)$, and this adjustment technique will reveal the true period quantum. In practice, the period mean age at birth is sometimes influenced by changes in parity composition, since first births tend to occur to women at younger ages while higher parity births occur to older women. For example, if women in year t choose to have smaller family sizes, then they will forgo higher order births, and this will lower the mean age at birth even though there may be no change in the timing of births of any fixed order. For this reason, Bongaarts and Feeney recommend that (2.13) be used to calculate TFR_i^* separately for each birth order i, with adjusted total fertility of all orders being the sum of the adjusted parity-specific total fertility rates.

Bongaarts and Feeney's tempo adjustment Eq. (2.13) is strikingly reminiscent of Ryder's period-cohort translation relationship given by Eq. (2.6). Indeed, when Ryder's relationship is re-expressed in terms of changes in the period mean age μ_p, one obtains

$$CTFR(t - \mu_p) \approx TFR(t) \times (1 + \mu_p'(t)).$$

These expressions are nearly the same, but with a minor and a major difference. The minor difference is that one expression is *divided* by $1 - \mu_p'$, whereas the other is *multiplied* by $1 + \mu_p'$, but for small values of μ_p' this numerical difference is very small. The major difference is that the period-shifts approach is giving us a formula for tempo-adjusted period fertility, whereas the Ryder approach gives us a formula for cohort fertility. The lesson we draw from this similarity is not that tempo-adjusted period fertility is a measure of cohort fertility. Indeed, Bongaarts and Feeney repeatedly caution against this interpretation. Rather, we take the approach that the first-order approximation of age-specific fertility change is a good representation of a wider class of models, including tempo-shifts. The other commonality, as we have emphasized, is that both approaches show the influence of timing changes, in which small changes in the pace of timing change are consistent with large changes in the summary measure of interest.

The story implicit in the period-shift model has several appealing features. Fluctuations in fertility rates are represented as the result of two forces, both period driven. The first is quantum, which is understood to be the outcome of period specific factors outside of this model. The second force is the timing of births, which can be postponed or advanced in response to changing socioeconomic conditions. Since both of the forces are functions of period only, this model does not require one to know the complete fertility history for any of the cohorts active in year t. Rather, one can examine fertility rates in adjacent years to measure the mean age at birth, and from this demographers can uncover the value of the quantum q and then measure the size of the timing influence as the difference between TFR and q.

The hypothesis that changes in timing are a purely period-driven phenomenon means that a recent history of postponement has no long-term impact on the total fertility rate. For example, if we assume that q is constant, then the end of a period of shifting means that $R'(t)$ will be zero after 1 year, and so all fluctuations in the total fertility rate will cease. In this way, the period-shift model asserts that timing decisions made in the past do not impose on the future. In contrast, the moving target model in Sect. 2.4 and the cohort-shift model in Sect. 2.6 each imply that decisions made in the past can continue to influence fertility rates for years.

Any useful formal model will require some simplifying assumptions. The strongest, and therefore most questionable, assumption in the period-shift model is the idea that women of all ages will shift their timing by the same amount in a given year. However, it is plausible that women would respond to a period shock in different ways, with, for example, women at younger ages choosing to postpone more in response to an economic slowdown, while women at older ages would postpone less or not at all. A further challenge to this assumption is that it allows for the complete independence of behavior at different ages across a given cohort. So, for example, postponement at younger ages has no direct implication for the fertility of the same cohort as it ages. For example, a decision to postpone childbirth until after completing college or after establishing a career has implications for years beyond the time when the decision was made. It is with these criticisms in mind that we turn to the cohort-shift model.

2.6 Cohort Shifts

The cohort-shift model (Goldstein and Cassidy 2014) decomposes observed fertility into an interaction of cohort-based decisions about fertility timing and period-based decisions about the intensity of fertility. Fluctuations in period fertility are thus understood as the result of both cohort tempo, i.e. changes in period fertility that results from the timing of births in cohorts, and period quantum, i.e. level changes in period fertility as a consequence of events that are independent of age and cohort. Like the period-shift model, the cohort-shift model treats quantum q as an exogenous function of period t. However, timing changes are now seen as a cohort, rather than a period, phenomenon.

Cohort-based postponement is introduced through a shift function $S(c)$, the cohort analog of the period shift R. The shift $S(c)$ indicates the amount by which women from cohort c have shifted their fertility relative to the baseline schedule $f_0(a)$.[5] For example, if "33 is the new 30" for the cohort of 1960, then $S(1960) = 3$.

The cohort-shift model of fertility The model of the fertility age-period surface consistent with the cohort-shift model is

$$f(a,t) = f_0(a - S(t-a))q(t). \tag{2.14}$$

This model is derived as follows. The shifted cumulative fertility schedule for the cohort-shift model is $F_0(a - S(c))$. Differentiating cumulative fertility with respect to age a gives us $f_0(a - S(c))$, the fertility rate for cohort c at age a in the absence of any period quantum effects. To obtain the observed fertility rate $f(a,t)$, we include period quantum effects and rewrite in terms of period and age. Note that unlike the case in the period-shift model, there is no need for an R' term or its cohort equivalent in the cohort-shift description of the fertility surface. This is because, unlike the case with period shifts, cohort shifts do not spread or consolidate births within cohorts.

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[5] Simultaneous period and cohort postponement can be modeled as a sum of R and S. This combined model is discussed in Goldstein and Cassidy (2014).

The cohort-shift model implies its own tempo-adjusted measure of period fertility, denoted $TFR^{\dagger}(t)$. This is a period measure, and should not be used to approximate levels of cohort fertility. Rather, $TFR^{\dagger}(t)$ represents the total fertility rate that would have been observed if the cohorts active during year t had neither postponed nor accelerated their fertility schedules.

Under the assumptions of the cohort-shift model (2.14), $q(t)$ can be recovered from observed rates by defining the shift-adjusted period total fertility rate as

$$TFR^{\dagger}(t) := \int_0^{\omega} f(a,t)(1 + S'(t-a))da, \tag{2.15}$$

where $S'(t-a)$ is the derivative of $S(c)$ evaluated at $t-a$, the incremental shift relevant for the cohort aged a at time t. To show that $TFR^{\dagger}(t) = q(t)$, use (2.14) to replace $f(a,t)$ with $f_0(a - S(t-a))q(t)$. Substituting $w = a - S(t-a)$ and $dw = (1 + S'(t-a))da$ gives the desired result. Thus, the shift-adjusted $TFR^{\dagger}(t)$ recaptures the period TFR that would have been observed in the absence of cohort shifts.

An interpretation of the effect of cohort shifts is that they compress (or dilate) the period cross-section of the age-period fertility surface. The adjustment factor $(1 + S'(t-a))$ allows the passage of time in the period to return to its original pace, as if there had been no changes in timing.

The tempo-adjusted measure $TFR^{\dagger}$ bears some resemblance to the Average Completed Fertility (ACF) measure developed by Butz and Ward (1979) and Ryder (1980), and subsequently analyzed in Schoen (2004). ACF, a weighted average of completed cohort fertility rates, is equal to the TFR divided by a quantity called the timing index. The timing index $TI(t)$ is the sum of the ratios of fertility rates at each age in period t to the completed fertility rate for that particular cohort, i.e. $TI(t) = \int_{\alpha}^{\beta} f(a,t)/CTFR(t-a)\, da$. In the special case of constant period quantum q, the hypotheses of the cohort-shift model imply that $ACF(t)$ and $TFR^{\dagger}(t)$ will both be equal to q. However these two measures are not in general equivalent.

A drawback of the cohort approach is that estimation of the adjustment factor $1 + S'$ is not as easy as is estimation of the corresponding factor R' for the period-shift model. Here we mention just one of two estimation methods developed in Goldstein and Cassidy (2014). If completed cohort fertility data is available, S' can be calculated as the change in the cohort mean age at birth. If the available cohort fertility data is truncated, it is still possible to estimate S', as detailed in the appendix to this chapter.

The cohort-shift model shares with the period-shift model a strong assumption about the age-distribution of fertility. Both use an unchanging baseline fertility age-schedule $f_0(a)$, although in the case of the cohort-shift model, the realized cohort age-schedule is also modifiable via period quantum, $q(t)$. Neither the period-shift assumptions nor the cohort-shift assumptions hold perfectly. Goldstein and Cassidy (2014) use goodness-of-fit comparisons to argue that at least in the case of the Netherlands, the cohort shift description appears to fit the observed fertility surface better. On the other hand, Bongaarts and Sobotka (2012) argue for the superiority of the period-shift model.

The cohort-shift model represents the timing of births as the outcome of shifts that describe how much each cohort may have advanced or delayed its fertility schedule. Different generations are allowed to have different plans for the timing of childbearing, and these plans play out over the course of their lives. These underlying schedules of intended fertility then encounter period driven events or shocks that may ultimately reduce or increase the cohort's total fertility. This model, a mixture of cohort and period influences, captures both the lifetime implications of cohort fertility intentions and the immediate responses to unanticipated period events. The interplay of cohort plans and period events then produces the variety in the observed fertility surface.

The cohort-shift model allows postponement choices made in the past to contribute to a 'fertility momentum' that plays out in terms of evolving fertility rates over the life of cohorts. Fertility momentum characterizes how fertility rates might change even after fertility quantum and shifts are fixed at current levels. We can define fertility momentum as the ratio of a future stationary total fertility rate to the total fertility rate in year t_0. In the numerator we have the total fertility rate that would eventually occur if quantum and shifts (period or cohort) are fixed at the most recent levels. Cohort shifts are fixed by setting $S(c) = S(t_0 - 15)$ for all $c \geq t_0 - 15$. In either case, the numerator is evaluated when the timing of fertility is constant. Thus in a period shift world we have fertility momentum

$$\frac{q(t_0)}{TFR(t_0)} = \frac{TFR^*(t_0)}{TFR(t_0)},$$

while the cohort-shift model yields momentum

$$\frac{q(t_0)}{TFR(t_0)} = \frac{TFR^\dagger(t_0)}{TFR(t_0)}.$$

The two shift models of postponement differ significantly in the lag time until fertility rates have stabilized. If shifts in the timing of births are a function purely of period, then fixing current quantum and shifts implies a stabilization of fertility rates within 1 year. In contrast, if shifts are represented as a cohort driven process, then fertility rates will continue to evolve for 30 or more years before stabilizing. Just as with population momentum, this evolution under fixed conditions is a result of the age structure within the population. The cohort-shift model represents postponement as a process played out over a lifetime, and this implies a continuing transformation in future fertility rates until all active cohorts follow the same fertility schedule.

The cohort formulation of fertility postponement has inherent implications for future fertility rates, and so can be used to forecast *TFR* under the assumption of no changes in quantum. For all currently active cohorts, we look at the timing of births to deduce the intended fertility schedule for these cohorts. We then assume that the newer cohorts who have not yet begun to give birth will follow the timing of the most recent cohorts. By projecting fertility schedules for each cohort individually, we can predict *TFR* in subsequent years. If the currently active cohorts have experienced different schedules, then we will see fertility rates evolve for several years even if quantum and shifts are now fixed.

2.7 An Illustrative Application to the United States

In this section, we apply the moving-target model, the period-shift model, and the cohort-shift model to the last several decades of period fertility change in the United States. We base our calculations on data from the Human Fertility Database (HFD, www.humanfertility.org), an excellent source of accurate and detailed fertility information for a number of low and moderate fertility countries.

Before presenting the results, we first provide a few more details about fitting the models to observed fertility rates. As discussed in Lee (1977), the moving-target model allows us to use period fertility rates to track changes in desired family size over time. One can calculate implied period level of desired fertility (D) by solving Eq. (2.8) to obtain

$$D(t) = \frac{g(a,t-a)}{\lambda} + C(a,t-a). \tag{2.16}$$

This provides a distinct estimate of desired fertility for each cohort in a given year. We reduce these estimates to a single period value by averaging across ages 25–35.

For the Bongaarts-Feeney tempo adjustment, we use their recommended approach of calculating tempo-adjusted total fertility for each parity separately and then let the estimate of tempo-adjusted quantum TFR^* be the sum of these parity-specific quantities. For the cohort shift estimator, we do use all-parity fertility rates in combination with the truncated mean estimator derived in Goldstein and Cassidy (2014) and described in the appendix to this chapter. We do not show any estimates from the Ryder model. However, if one considered the Ryder-like relationship $TFR = CTFR \times (1 - \mu_p')$, then the Bongaarts-Feeney results could also be interpreted as first-order estimates of the Cohort *TFR*.

Figure 2.1 shows the period total fertility rate observed in the United States from 1970 to 2010 along with the period measures from the models under consideration. We see in panel (a) that period fertility fell rapidly at the onset of the 1970s, a continuation of the baby-bust that began in the mid-1960s. This rapid fertility decline occurred at a time of new contraceptive technologies (the pill), liberalization of abortion law (Roe v. Wade), rapid change in female education and career ambitions, and economic change (the Oil Shock), a perfect storm of forces that understandably produced changes in birth rates. However, the continuation of low fertility throughout the 1970s and 1980s is less understandable in terms of forces influencing birth rates, particularly since the upswing in fertility at the end of the 1980s did not coincide with much in the way of remarkable social or economic

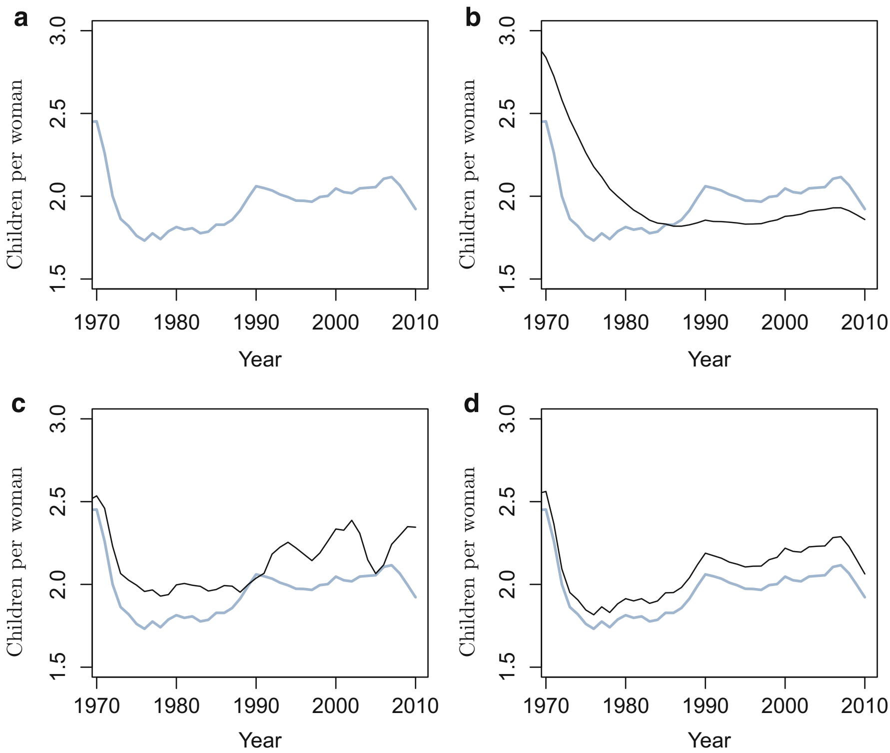

Fig. 2.1 *TFR*, desired family size, *TFR** and *TFR*† for the United States from 1970 to 2010. (**a**) Observed TFR. (**b**) Period target fertility D. (**c**) Period tempo-adjusted TFR*. (**d**) Cohort tempo-adjusted TFR†

changes. Since 1990 the slight dip of the Bush recession and the slight increase during the Clinton years and into the bubble economy of the second Bush's second term are all consistent with broad economic trends, as is the sudden decline of fertility since the onset of the Great Recession.

Panel (b) shows the estimated values of *D*, period desired fertility, obtained by applying Lee's moving-target model. The historical unfolding of desired fertility is considerably simpler than the *TFR*, with a steady decline from 1970 to 1980, and nearly three decades of stability thereafter, until the onset of the recession. According to Lee's model, the level of period fertility will be related to the rate of change in *D*. Thus, the rise in period fertility in around 1990 corresponds to an end to the decline in *D*. Likewise, the low level of births during the 1970s and 1980s is attributable to steady decline in desired fertility. We note also that the bottoming out of period fertility occurs before the minimum of *D*, a feature of Lee's model that we discussed above.

The level of desired fertility since about 1980 is quite low, well below the observed *TFR*. This results because we estimate *D* using Lee's value, from the

1960s, for the fraction λ of unachieved, desired fertility that occurs in each year. More plausible *levels* of D could be obtained by using an updated estimate, and, perhaps, by including age-specific variation in λ. In the context of fertility postponement, it seems likely to us that λ might well vary over time, in general taking on smaller values when fertility shifts to older ages.

Panel (c) shows the estimated values of TFR^*, period-shift-adjusted period total fertility, obtained from the Bongaarts-Feeney formula. These estimates show a markedly higher period fertility rate during the baby-bust years of the 1970s, on the order of 2.0 children per woman. The story the period-shift model tells is that the below-replacement period fertility of the 1970s and 1980s was to a large extent the result of fertility postponement, which, once taken into account, suggests that period fertility remained at near replacement levels. The period since 1990 suggest that there was a substantial increase in the quantum of fertility in the 1990s, up to as high as 2.3 children per woman, before fluctuating somewhat eratically since 2000. These "seemingly random fluctuations" (Bongaarts and Feeney 2006) are a feature of the TFR^* adjustment method, addressable by smoothing or by alternative calculation procedures (Bongaarts and Sobotka 2012) which have the effect of smoothing.

Panel (d) shows the estimated values of $TFR^\dagger$, cohort-shift-adjusted period total fertility, obtained from the Goldstein-Cassidy formula. The cohort adjustment, because it combines tempo effects from multiple cohorts in a single period, is considerably smoother than the period adjustment. We see during the Baby Bust years that its estimated value lies between the observed *TFR* and the period-tempo adjusted value. The cohort-shift adjusted measure tracks the observed period variation but at an offset equal to the average cohort shift effect. For the case of the United States this suggests that there is still a detectable tempo effect depressing fertility rates. Increases of 0.1 children would be achieved simply from a slowdown in postponement. A disadvantage of the cohort formulation is that by construction it cannot attribute much of the decline in period fertility during the recession to a sudden increase in postponement. In contrast to the Bongaarts-Feeney approach, which is perhaps oversensitive to period changes in timing, the Goldstein-Cassidy approach will tend to dampen period-to-period changes.

Taken together, we see that each of these models tells a somewhat different story about the last half century of fertility change in the United States. While the tempo-shift models attribute the low fertility of the 1970s and 1980s to changes in fertility timing, the moving-target model shows that low levels of period fertility can also result from changing fertility goals even in the absence of timing changes. For more recent decades, the period postponement approach suggest that without postponement we would have seen quite a high demand for children in the 1990s, whereas the cohort approach suggests a shrinking effect of postponement after 1990 such that observed and tempo-adjusted period fertility come close to convergence.

In terms of the recession, both the cohort model and the moving-target approach suggest a marked decline in underlying period quantum, whereas the period-shifts approach offers at least the possibility that much of the decline was "just" the postponement of births.

2.8 Discussion

Period fertility can vary substantially even when there are relatively small changes in actual completed family sizes. The Baby Boom and Bust were both larger in period terms than cohort changes would imply. "Lowest low" fertility (Kohler et al. 2002) and its apparent end in some countries (Goldstein et al. 2009), both represent larger swings in period fertility than changes in cohort fertility (Myrskylä et al. 2013) would imply.

The models of fertility dynamics that we have presented here offer different explanations for variation in period fertility. They all share in common the feature that the flow of period fertility is sensitive to the rate of change (the derivative) of other quantities like fertility timing or fertility targets. Table 2.1 summarizes the four models, showing for each model the mathematical formulation of period fertility rates and a related property discussed in this chapter.

Each model has something new to teach us. The generality of Ryder's formulation is useful in showing us the fundamental dependency of period fertility on changes in fertility timing. The formal nature of period-cohort translation lacks any behavioral basis, but for this reason Ryder's logic is applicable to a wide range of different reasons for fertility change. Lee's model of moving targets, which we have included here despite its general absence in the modern tempo-shifts literature, is more behavioral in its conception. The particular version that Lee develops, and the application we make of it, is based on fairly stylized assumptions, which could be relaxed in future applications. Nonetheless, even in its stylized form, the moving-target model gives us a valuable perspective on how the timing of period fertility changes will relate to changes in fertility desires, with amplification, lags, and other features.

The models of tempo-shifts in births give us another way of thinking about the relationship between individual behavior and aggregate fertility. The sum of individual decisions to time births differently, postponing or advancing otherwise planned births, changes the stream of births arriving in a time period, producing changes in period fertility simply as a result of timing changes. Both the period and cohort formulations of these shift-models include the simplifying assumption that all ages shift together. These can be relaxed (e.g. Kohler and Philipov 2001) but there is a tradeoff between model flexibility and estimation. More flexible models can be more sensitive to small violations of assumptions, producing erratic estimates

Table 2.1 Models of dynamic fertility change

Model	$f(a,t)$ equation	Property
Ryder	$f(a,0) + f'(a)t + \cdots$	$CTFR(t) \approx TFR(t+\mu_c)/(1-\mu_c')$
Moving-target	$\lambda e^{-\lambda a}\left[\int_0^a e^{\lambda u} D'(u+t-a)du + D(t-a)\right]$	$D(t) = f(a,t)/\lambda + C(a,t-a)$
Period-shift	$q(t)f_0(a-R(t))(1-R'(t))$	$TFR^*(t) = TFR(t)/(1-R')$
Cohort-shift	$q(t)f_0(a-S(t-a))$	$TFR^\dagger(t) = \int f(a,t)(1+S')\,da$

of tempo-adjusted fertility. This is the fertility modelers version of the statistician's bias-variance tradeoff.

There are also attempts to improve estimation of shift models by looking at different measures of fertility rates, including parity-specific hazards and new kinds of rates taking different sub-populations as the exposure to risk (Bongaarts and Sobotka 2012). These approaches may result in better estimators, but they can also run into difficulties in interpretation. Developments are still occurring, and so we should not rule out the possibility that there are better approaches. But we are skeptical that there is a "magic bullet" that will allow us to get substantially more precise or accurate estimates of the effect of tempo changes. The general principle found by Ryder that period fertility would be proportional to changes in mean ages will hold in all of these models and beyond this there is, we believe, only limited room to obtain a more detailed description.

On the other hand we do believe, as of this writing in 2015, that there are still many fruitful directions to explore in the dynamic modeling of fertility. The model introduced by Alkema (2011) of fertility transitions, currently being used as the basis for the United Nations probabilistic population forecasts, could benefit from more behavioral foundations. The possibility of changing timing needs to be incorporated into Lee's moving-target model in order to have it apply to the modern context. Improved methods of cohort forecasting, building on Myrskylä et al. (2013) and Schmertmann et al. (2014), should make it possible to include the analysis of more recent cohorts. The possibilities for gaining further insight into individual aspects of fertility change have not been exhausted. The recent literature on fertility intentions has focused on prospective plans (Morgan et al. 2011). One avenue worth consideration is to ask people about their plans retrospectively and about how the unfolding of their lives changed their behavior, in terms of timing, in terms of desired total fertility, and in other dimensions. Finally, the recent- and in some countries ongoing- recession offers a chance to see the inadequacies in current formulations and to create new theories to help us make sense of changing fertility.

Appendix: Tempo Adjustment for the Cohort-Shift Model

Tempo adjustment for the cohort-shift model of fertility requires estimation of the adjustment factor $1+S'$. The first step in this estimation is to control for the influence of changes in the period quantum q on the cohort mean age at birth. Our 'quantum-normalized fertility rates' $\tilde{f}(a,t)$ are calculated as follows. In each period t there is an age (or ages) with the peak fertility rate, and we denote this rate by $m(t)$. We can calculate $m(t)$ from observed data as $\text{Maximum}_a\{f(a,t)\}$. We then define the quantum-normalized rate as

$$\tilde{f}(a,t) := \frac{f(a,t)}{m(t)}.$$

Let m_0 denote the peak value of the baseline schedule f_0. Under the assumptions of purely cohort shifts, $m(t) = m_0 q(t)$ and the quantum-normalized rates are

$$\tilde{f}(a,t) = \frac{1}{m_0} f_0(a - S(t-a)). \tag{2.17}$$

If the cohort born in year c has completed its years of fertility, we can use the change in the cohort quantum-normalized mean age at birth to estimate $S'(c)$. For cohorts that have not yet completed the fertile years, we can estimate $S'(c)$ using the change in mean age at birth truncated to the latest year of data.

Our formula depends on several quantities. Let $\mu(c)$ be the quantum-normalized mean age at birth for cohort c as of latest available age, i.e.

$$\mu(c) = \frac{\int_l^h x\tilde{f}(x, c+x)\, dx}{\int_l^h \tilde{f}(x, c+x)\, dx} \tag{2.18}$$

where l is the lowest age of available fertility data for cohort c, and h is the highest age of available fertility data for cohort c. Let $v(a,c)$ be the proportion of cohort c's truncated fertility that occurs at age a calculated from the quantum-normalized schedules, i.e.

$$v(a,c) = \frac{\tilde{f}(a, c+a)}{\int_l^h \tilde{f}(x, c+x)\, dx}, \tag{2.19}$$

Let $\mu'(c)$ be the derivative of $\mu(c)$. In practice this can be estimated using

$$\frac{\mu(c+1) - \mu(c-1)}{2}$$

provided the same l and h are used for cohorts $c+1$ and $c-1$.

We then use this estimate of $S'(c)$:

$$S'(c) = \frac{\mu'(c)}{1 + v(h,c)(\mu(c) - h) + v(l,c)(l - \mu(c))}. \tag{2.20}$$

A detailed derivation of this formula can be found in Goldstein and Cassidy (2014).

References

Alkema, L. (2011). Probabilistic projections of the total fertility rate for all countries. *Demography, 48*(3), 815–839.

Bongaarts, J., & Feeney, G. (1998). On the quantum and tempo of fertility. *Population and Development Review, 24*, 271–291.

Bongaarts, J., & Feeney, G. (2006). The quantum and tempo of life-cycle events. In *Vienna yearbook of population research*, vol. 4 (pp. 115–151). Austrian Academy of Sciences Press.

Bongaarts, J., & Sobotka, T. (2012). A demographic explanation for the recent rise in European fertility. *Population Development Review, 38*, 83–120.

Butz, W., & Ward, M. (1979). The emergence of countercyclical U.S. fertility. *American Economic Review, 69*, 318–328.

Goldstein, J., & Cassidy, T. (2014). A cohort model of fertility postponement. *Demography, 51*(5), 1797–1819.

Goldstein, J., Sobotka, T., & Jasilioniene, A. (2009). The end of "Lowest-low" fertility? *Population and Development Review, 35*(4), 663–699.

Kim, Y., & Schoen, R. (2000). On the quantum and tempo of fertility: Limits to the Bongaarts-Feeney adjustment. *Population and Development Review, 26*, 554–559.

Kohler, H., & Philipov, D. (2001). Variance effects in the Bongaarts-Feeney formula. *Demography, 38*(1), 1–16.

Kohler, H., Billari, F., & Ortega, J. (2002). The emergence of lowest-low fertility in Europe during the 1990s. *Population and Development Review, 28*(4), 641–680.

Lee, R. (1977). Target fertility, contraception, and aggregate rates: Toward a formal synthesis. *Demography, 14*(4), 455–479.

Lee, R. (1980). Aiming at a moving target: Period fertility and changing reproductive goals. *Population Studies, 34*, 205–226.

Morgan, P., Sobatka, T., & Testa, M. (Eds.). (2011). *Vienna yearbook of population research* (Special issue on reproductive decision-making). Vienna: Austrian Academy of Sciences.

Myrskylä, M., Goldstein, J., & Cheng, Y. (2013). New cohort fertility forecasts for the developed world: Rises, falls, and reversals. *Population and Development Review, 39*(1), 31–56.

Ní Bhrolcháin, M. (1992). Period paramount? A critique of the cohort approach to fertility. *Population and Development Review, 18*(4), 599–629.

Rodriguez, G. (2006). Demographic translation and tempo effects. *Demographic Research, 14*(article 6), 85–110.

Ryder, N. (1964). The process of demographic translation. *Demography, 1*(1), 74–82.

Ryder, N. (1965). The cohort as a concept in the study of social change. *American Sociological Review, 30*(6), 843–861.

Ryder, N. (1980). Components of temporal variations in American fertility. In R. Hiorns (Ed.), *Demographic patterns in developed societies* (pp. 15–54). London: Taylor and Francis.

Schmertmann, C., Zagheni, E., Goldstein, J., & Myrskylä, M. (2014). Bayesian forecasting of cohort fertility. *Journal of the American Statistical Association, 109*(506), 500–513.

Schoen, R. (2004). Timing effects and the interpretation of period fertility. *Demography, 41*(4), 801–819.

Sobotka, T., & Lutz, W. (2010). Misleading policy messages from the period TFR: Should we stop using it? *Comparative Population Studies, 35*(3), 637–664.