Problem Set for Week 8

Joshua R. Goldstein
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(This week, everyone should do all of the problems.)

For the following problems, assume $p_0 = .3$, $p_1 = .3$, and $p_2 = .4$. and $h(z) = p_0 + p_1 z + p_2 z^2$. P

1. Multiply out $h(z)^3$ algebraically and explain how the coefficient on z^4 consists of all of the possible ways for 3 father's to produce a total of 4 sons.

2. Multiply out $h_2(z) = h(h(z))$ algebraically and explain how the coefficient on z^2 consists of all of the possible ways for a woman to have 2 grand-daughters.

3. Write an R-program to reproduce the 20 entries of the table at https://en.wikipedia.org/wiki/Branching_process.

4. Use the quadratic formula to solve for d, the probability of ultimate extinction:

$$d = p_0 + p_1 d + p_2 d^2$$

   What do you get for d given our p_k values above? Does it correspond to the same value one gets by using iteration, as in the Wikipedia table?

5. Simulate a critical branching process such that $m = 1$ by reversing the p_1 and p_2 values we're using. Check that $m = 1$. You can use the "branch()" code in the slides.

   (a) See if you can do a big number of trials, 1000? For many generations, 30, 50, 100?

   (b) What fraction of lines become extinct?

(c) What is the distribution of surviving lines? (Hint: Choose a time that is is not so distant that few lines survive)

(d) What is the mean and variance of of Z_{10}? What about Z_{20} and Z_{30}? What will happen as $n \to \infty$?