Mortality Crossovers

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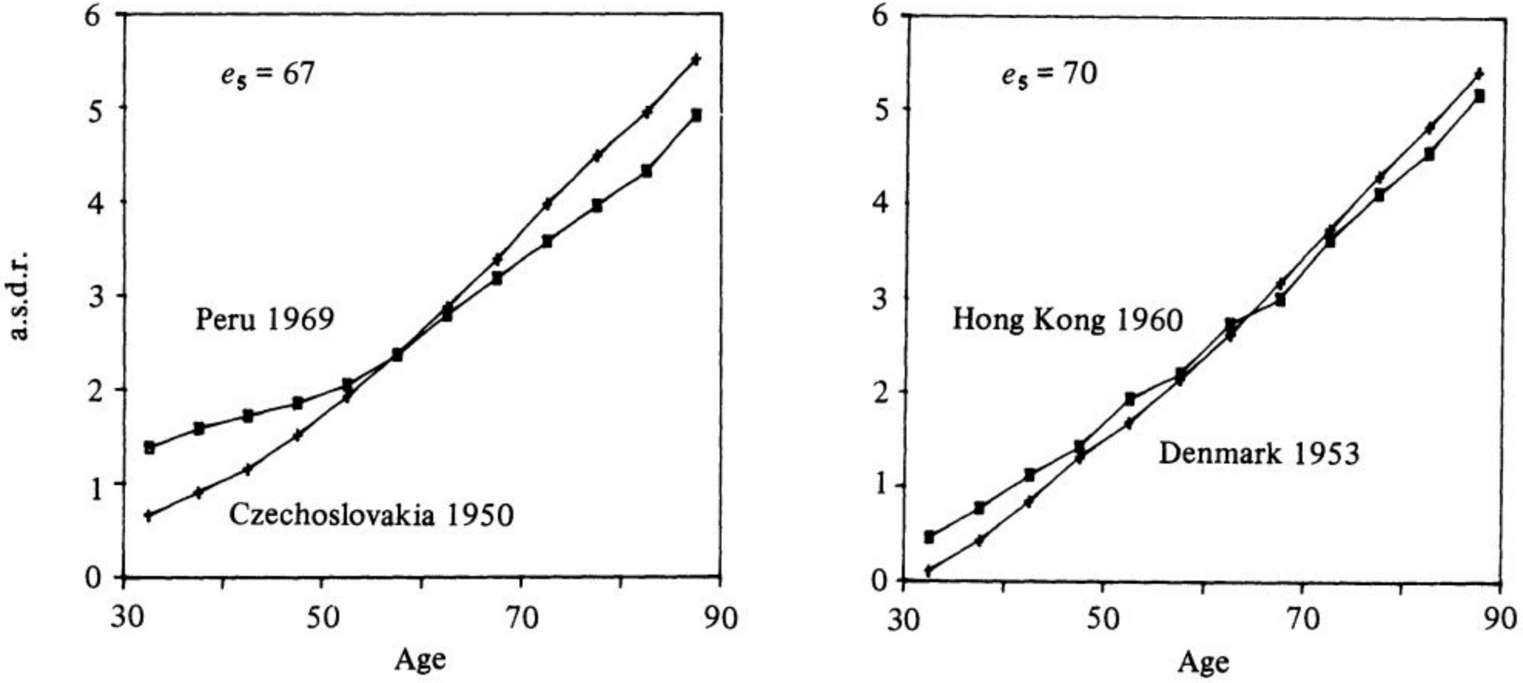
Roadmap

- Paper Presentations
  - Coale and Kisker (1986): Mortality Crossovers: Reality or Bad Data?
  - Manton and Stallard (1981): Methods for Evaluating the Heterogeneity of Aging Processes in Human Populations Using Vital Statistics Data
- Empirical examples of crossovers with CenSoc data
- Investigate quality of age of death reporting in CenSoc

Mortality Crossovers: Reality or Bad Data?

Ansley J. Coale and Ellen Eliason Kisker
(1986)

Mortality crossovers



Mortality Crossovers

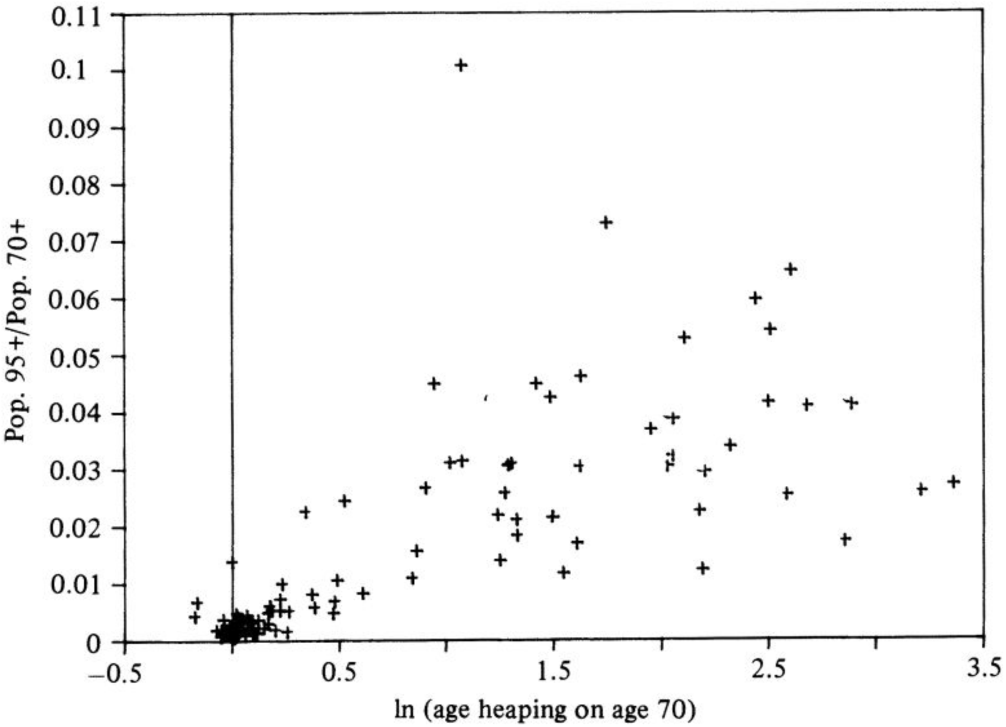
- Selection / heterogeneity:
  - Elimination of the frailer members of the population at younger ages leaves only the very robust with lower mortality rates
- Level playing fields at older ages
  - Social Security, Medicare, etc.
- **Bad data**
  - Misreporting age of death can lead to biased estimates of mortality rates at older ages

Age Heaping

- General pattern of age misstatement, most often rounding up to nearest 5 or 10
- Begins with a modest **upward** transfer at age 60 or 70, increases rapidly with age

Code	Label	egypt 1996	guatem 1981
050	50	111,574	3,779
051	51	20,364	1,106
052	52	30,051	1,700
053	53	20,216	1,333
054	54	19,165	1,280
055	55	75,201	2,128
056	56	20,560	1,237
057	57	15,889	914
058	58	21,429	1,334
059	59	14,276	786
060	60	77,266	2,897
061	61	13,359	610
062	62	19,200	917
063	63	16,648	795
064	64	13,075	629
065	65	57,534	1,356
066	66	11,013	617
067	67	11,332	519
068	68	7,861	667
069	69	4,810	403
070	70	42,807	1,256
071	71	4,684	260
072	72	6,346	444
073	73	4,573	327
074	74	3,284	314

Implications of Age Heaping



$$\text{age heaping on age 70} = \frac{\text{Pop}_{70}}{(\text{Pop}_{69} + \text{Pop}_{71})/2}$$

Takeaways

- Age overstatement at advanced ages is common and downwardly biases estimates of mortality rates
- Age heaping is associated with age overstatement
- Low quality mortality data can artificially create a mortality crossover

Methods for Evaluating the Heterogeneity of Aging Processes in Human Populations Using Vital Statistics Data: Explaining the Black/White Mortality Crossover by a Model of Mortality Selection

Kenneth G. Manton and Eric Stallard

(1981)

Summary

- A model to compute the ratio of Black and White individual age specific mortality risks (within sex) to determine if the adjustments of heterogeneity and mortality selection is sufficient to remove the crossover.
- Data from the U.S. Black and White populations for the period 1935 to 1975.
- Mortality crossover (Blacks having relatively lower mortality rates) at age 75.
- Could be explained under the proposed model.
- Data quality? Variety of evidence supporting the existence of a crossover.
- Consequently, careful consideration should be made of the population mechanisms by which the crossover might occur.

A model of selection

- Life tables are separately calculated for the Black and White populations in the U.S. over the period 1935 to 1975 based upon the assumptions:
    - Each population is heterogeneous.
    - The initial distribution of individuals in each population is identical (within sex) with respect to variables relevant to longevity.
    - Individual's environmental conditions are fixed at birth.
- Operationally, they modified standard life table calculations (Chiang, 1968) to reflect the dependence of mortality rates at advanced ages upon the selection of earlier mortality levels on a heterogeneous population.

A little bit of math: Assumptions

- The following partial differential equation governs the change of the distribution as cohort age:

$$\frac{\partial f_x(z)}{\partial x} = f_x(z)(\overline{\mu}_x - \mu_x(z)) \tag{1}$$

- Each person retains the value of z (longevity characteristics) given at birth.
- Functional forms

$$\mu_x(z) = z\mu_x(1) = z\mu_x \tag{2}$$

- Thus z may be taken to be a measure of relative (to the standard individual) frailty or "susceptibility to death." Alternately, 1/z may be considered as a measure of vitality or "robustness."

A little bit of math: Variance and frailty relation

We also have the following definitions:

$$\overline{\mu}_x = \bar{z}\mu_x \tag{3}$$

$$\bar{z}_x = \int_0^\infty z f_x(z) dz \tag{4}$$

Therefore, we can say that $\frac{\partial \bar{z}_x}{\partial x} = -\mu_x \sigma_x^2(z)$

$$\frac{\partial f_x(z)}{\partial x} = f_x(z)(\bar{z}\mu_x - z\mu_x) \tag{5}$$

$$\frac{\partial \bar{z}_x}{\partial x} = \frac{\partial \int z f_x(z) dz}{\partial x} \tag{6}$$

$$= \int z \frac{\partial f_x(z)}{\partial x} dz \tag{7}$$

$$= \int z f_x(z)(\bar{z}\mu_x - z\mu_x) dz \tag{8}$$

A little bit of math

$$= \mu_x \left(\int z f_x(z) \bar{z} dz - \int z^2 f_x(z) dz \right) \quad (9)$$

$$= -\mu_x \left(\int z^2 f_x(z) dz - \bar{z} \int z f_x(z) dz \right) \quad (10)$$

$$= -\mu_x \left(\int z^2 f_x(z) dz - \bar{z}^2 \right) \quad (11)$$

$$\frac{\partial \bar{z}_x}{\partial x} = -\mu_x \sigma_x^2(z) \quad (12)$$

This means the "frailer" population members (with high z's) are being selected earlier than their more "robust" contemporaries (with low z's)

A little bit of math: Gamma distribution

The proportionality assumption has implications for $f_x(z)$

- Mortality cannot be negative, then z must be positive.
- Average endowment, $\bar{z}_0$, for longevity, it follows that $\overline{\mu}_0 = \mu_0$. Hence, $\bar{z}_0 = 1$
- It would be desireble that the paramteres of f_x be unchacheged for any x.

$$f_x(z) = z^{k-1}\lambda_x^k exp(-z\lambda_x)/\Gamma(K) \tag{13}$$

With mean $\bar{z}_x = k/\lambda_x$ and variance:

$$\sigma_x^2(z) = \bar{z}_x^2/k = k/\lambda^2 \tag{14}$$

ASPD and the relative risk

From the variance of the gamma, the average mortality and the definition of $\bar{s}_x$, we have:

$$\sigma_x^2(z) = \bar{z}_x^2/k \tag{15}$$

$$\bar{\mu}_x = \bar{z}_x \mu_x \tag{16}$$

$$\bar{s}_x = exp\left(-\int_0^x \bar{\mu}_t dt\right) \tag{17}$$

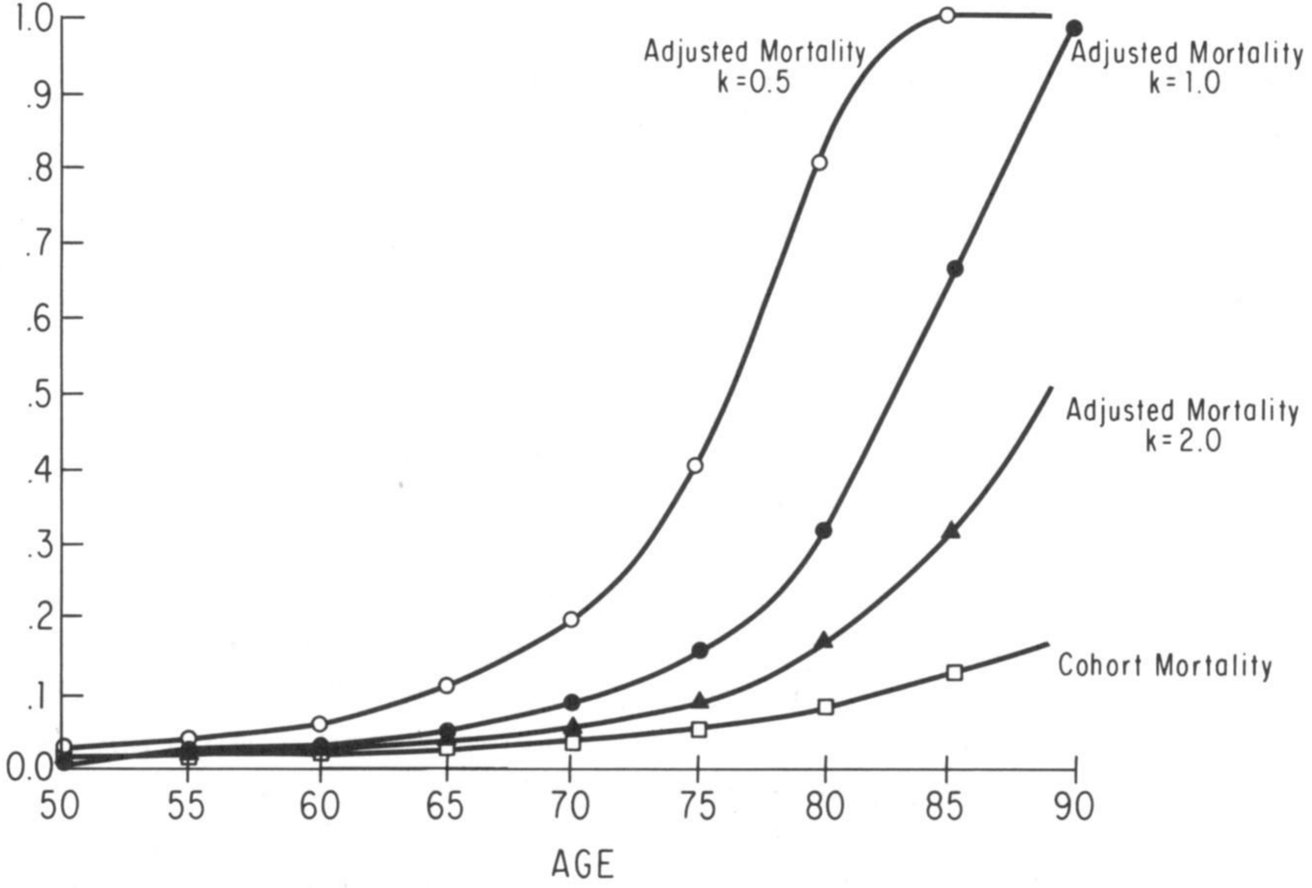
$$nq_x(z) = 1 - exp\left(\frac{kz}{\bar{s}_x^{1/k}} - \frac{kz}{\bar{s}_{x+n}^{1/k}}\right) \tag{18}$$

$$\bar{r}_x = \frac{\mu_{x1}}{\mu_{x2}}\left(\frac{\bar{s}_{x1}}{\bar{s}_{x2}}\right)^{1/k} \tag{19}$$

Parameter k

- Select values of k focus upon the biological rather than statistical
    a. Biological dimensions underlying longevity are normally distributed at birth.
    b. Any deviation from an "optimal" biological point will be associated with decreased survival.
    c. Conditionally on age, mortality will be a quadratic function
    d. Each individual's endowment for longevity (z) is fixed at birth.
- The value of κ is the result of n, number of dimensions relevant to longevity.
- The relation of n to the gamma shape parameter is simply n=2k.
- Lower n, the greater is the heterogeneity (higher variance of gamma)
- The values of n used are 1 and 2, suggesting that longevity is unidimensional (k=0.5) and bidimensional (k=1), respectively.

Cohort and Individual Age Specific Mortality Probabilities for the 1875 White Female Birth Cohort



Age-Specific Mortality Risk Ratios—Black Males vs. White Males—for the Years 1935, 1955, and 1975

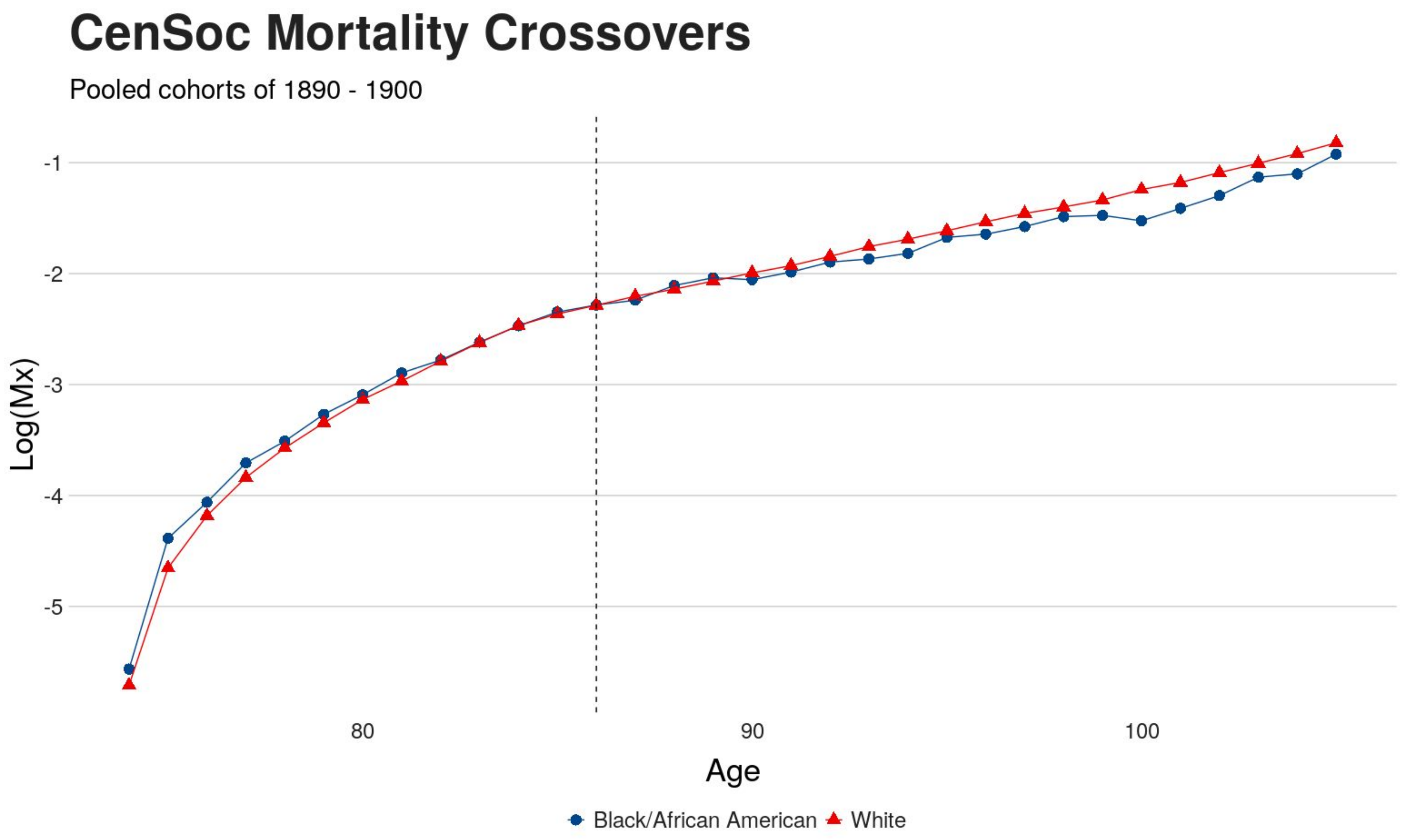
	Observed Ratios			Bidimensional Model (k = 1.0)			Unidimensional Model (k = 0.5)		
Age	1935	1955	1975	1935	1955	1975	1935	1955	1975
0	1.26	1.62	2.04	1.26	1.63	2.05	1.27	1.65	2.07
5	1.18	1.60	1.44	1.19	1.63	1.47	1.21	1.66	1.50
10	1.31	1.27	1.47	1.32	1.30	1.50	1.33	1.34	1.54
15	1.91	1.40	1.30	1.91	1.45	1.33	1.91	1.49	1.36
20	2.48	1.52	1.44	2.65	1.56	1.48	2.83	1.60	1.52
25	2.66	1.88	2.51	3.12	1.92	2.60	3.66	1.96	2.69
30	2.65	2.19	3.01	3.31	2.26	3.16	4.15	2.34	3.33
35	2.66	2.27	2.93	3.53	2.38	3.13	4.67	2.50	3.35
40	2.43	2.04	2.84	3.34	2.33	3.06	4.59	2.67	3.30
45	2.29	1.78	2.14	3.25	2.28	2.34	4.62	2.92	2.56
50	2.05	1.61	2.00	3.02	2.25	2.26	4.45	3.15	2.55
55	1.67	1.56	1.66	2.57	2.38	1.95	3.96	3.65	2.28
60	1.29	1.23	1.57	2.05	2.03	2.05	3.25	3.34	2.68
65	1.25	1.28	1.31	1.98	2.17	1.93	3.12	3.67	2.85
70	1.14	1.16	1.24	1.78	2.05	2.04	2.79	3.64	3.35
75	1.00	.97	1.27	1.60	1.72	2.43	2.55	3.04	4.63
80	.92	.84	.97	1.52	1.38	1.85	2.52	2.27	3.56
84	.81	.75	.64	1.39	1.03	1.09	2.36	1.41	1.84

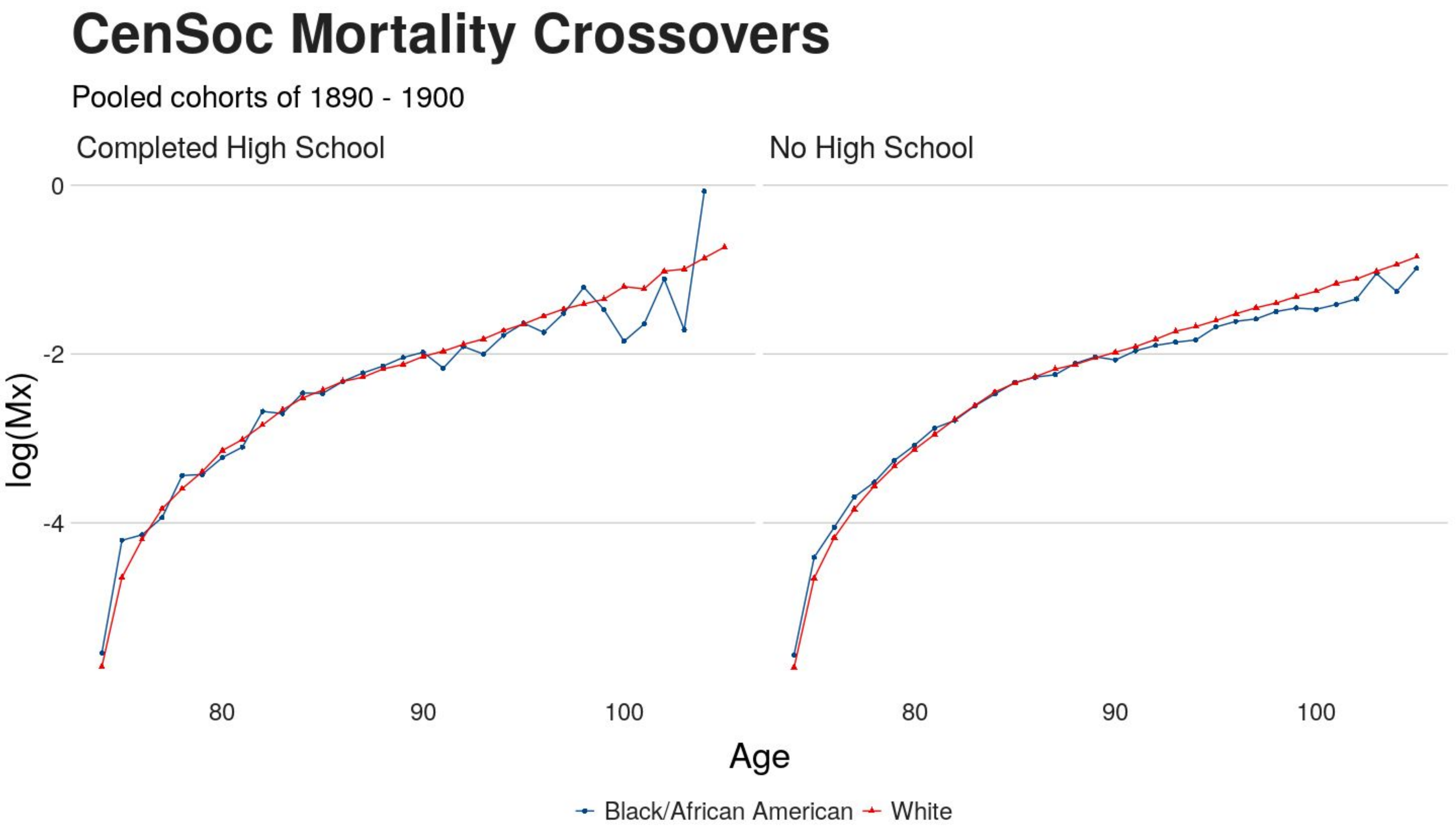
Age-Specific Mortality Risk Ratios—Black Females vs. White Females—for the Years 1935, 1955, and 1975

	Observed Ratios			Bidimensional Model (k = 1.0)			Unidimensional Model (k = 0.5)		
Age	1935	1955	1975	1935	1955	1975	1935	1955	1975
0	1.31	1.72	2.24	1.31	1.74	2.26	1.32	1.75	2.27
5	1.29	1.76	1.26	1.30	1.79	1.28	1.31	1.82	1.31
10	1.46	1.52	1.27	1.47	1.55	1.30	1.49	1.59	1.32
15	2.79	1.89	1.19	2.81	1.94	1.22	2.84	2.00	1.24
20	3.20	2.41	1.59	3.44	2.47	1.63	3.70	2.54	1.66
25	3.19	2.78	2.27	3.74	2.86	2.33	4.39	2.94	2.38
30	3.00	3.07	2.33	3.76	3.22	2.41	4.72	3.38	2.49
35	3.08	3.17	2.80	4.13	3.39	2.94	5.53	3.64	3.08
40	2.93	2.95	2.50	4.11	3.44	2.65	5.78	4.01	2.80
45	2.73	2.64	2.14	3.96	3.43	2.31	5.75	4.46	2.49
50	2.41	2.45	2.14	3.58	3.48	2.40	5.32	4.96	2.69
55	1.91	2.21	1.97	2.90	3.48	2.30	4.40	5.48	2.69
60	1.43	1.75	1.88	2.19	2.97	2.47	3.36	5.02	3.23
65	1.30	1.70	1.56	1.98	3.00	2.36	3.02	5.27	3.58
70	1.10	1.27	1.67	1.66	2.35	2.94	2.50	4.35	5.19
75	.84	.93	1.74	1.22	1.66	3.68	1.78	2.96	7.80
80	.68	.73	1.01	.88	1.14	2.06	1.14	1.78	4.22
84	.55	.60	.69	.60	.75	1.24	.65	.94	2.23

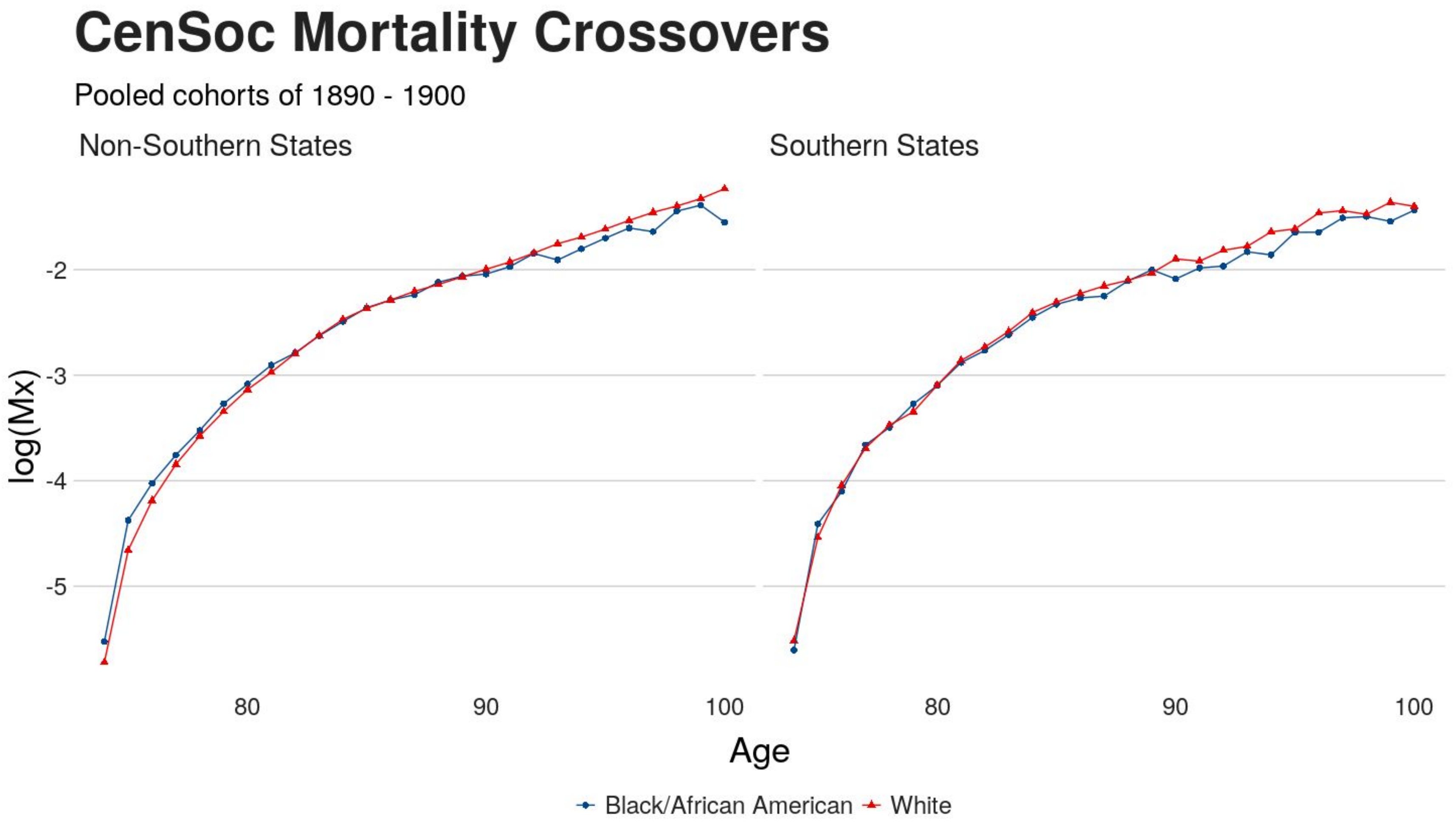
Takeaways

- It seems that the crossover at advanced ages for males is an artifact of the early differential mortality selection.
- An explanation for this differentials at older ages is the relatively more rapid reduction in individual white male mortality
- For black females we can see that they were worse off than males between 25-45 (the childbearing years) at 1935.
- Different Zs will result in a divergence between the increase with age of the cohort mortality rates and the age increase in the probabilities of death for individuals within the cohort.
- This because the earlier selection of the less "robust" population members, implies that individuals age "faster" than their cohorts

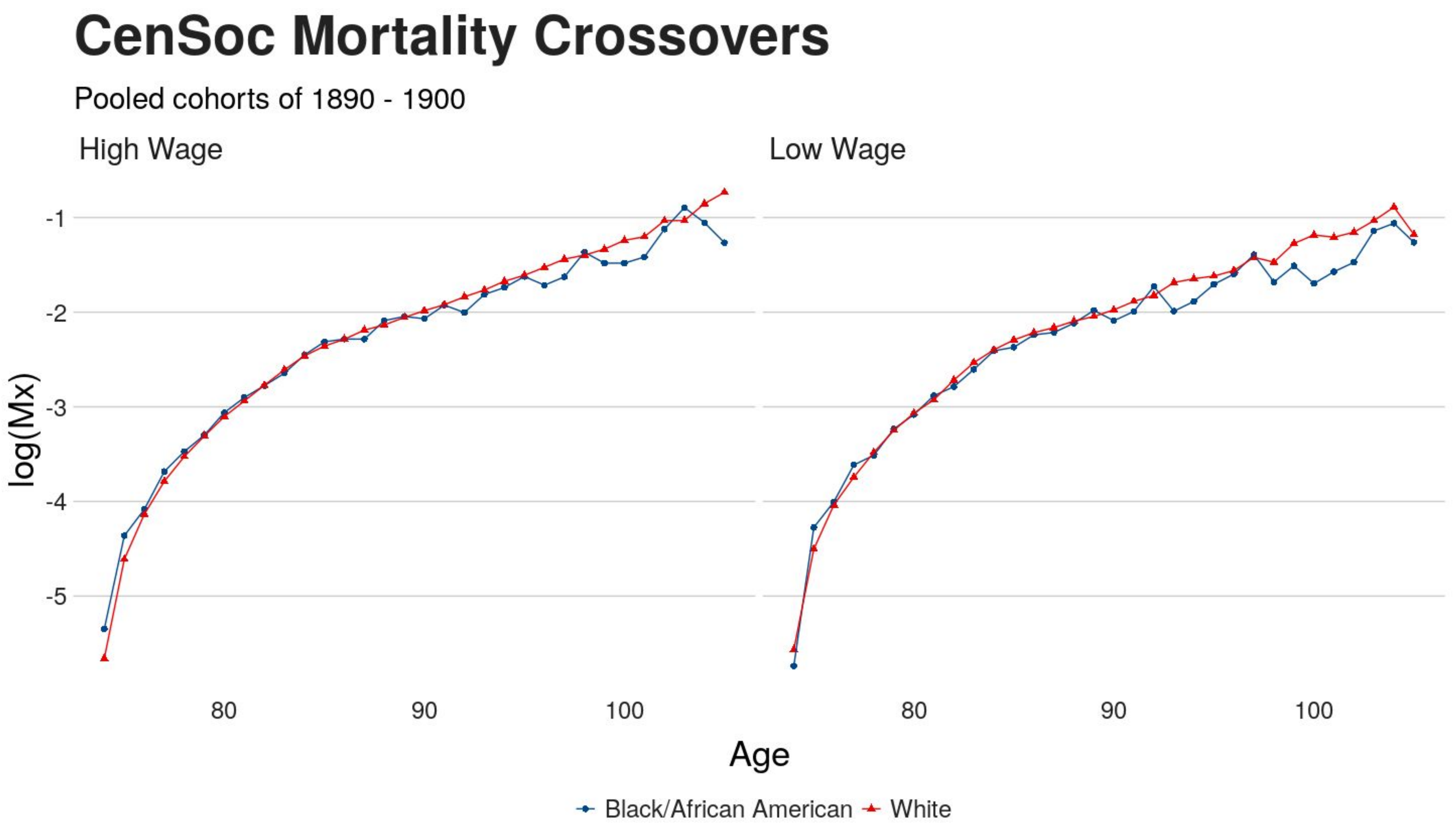

CenSoc Mortality Crossovers
Pooled cohorts of 1890 - 1900
Log(Mx)
-1
-2
-3
-4
-5
80
90
100
Age
Black/African American
White



CenSoc Mortality Crossovers
Pooled cohorts of 1890 - 1900
Completed High School
No High School
0
-2
-4
log(Mx)
80
90
100
Age
Black/African American
White



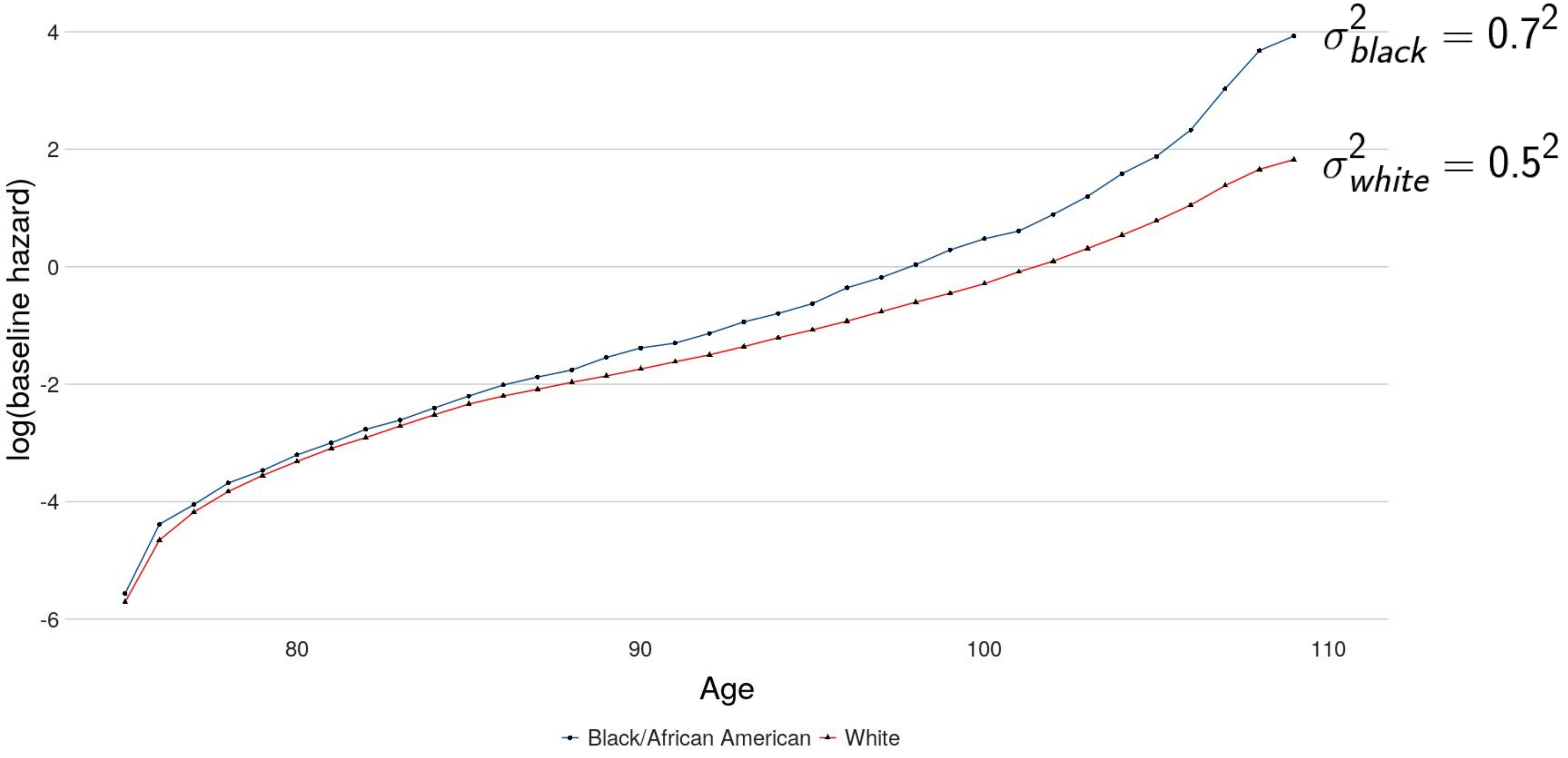
CenSoc Mortality Crossovers
Pooled cohorts of 1890 - 1900
Non-Southern States
Southern States
log(Mx)
-2
-3
-4
-5
80
90
100
80
90
100
Age
Black/African American
White


CenSoc Mortality Crossovers
Pooled cohorts of 1890 - 1900
High Wage
Low Wage
log(Mx)
-1
-2
-3
-4
-5
80
90
100
Age
Black/African American
White

Eliminating the Crossover

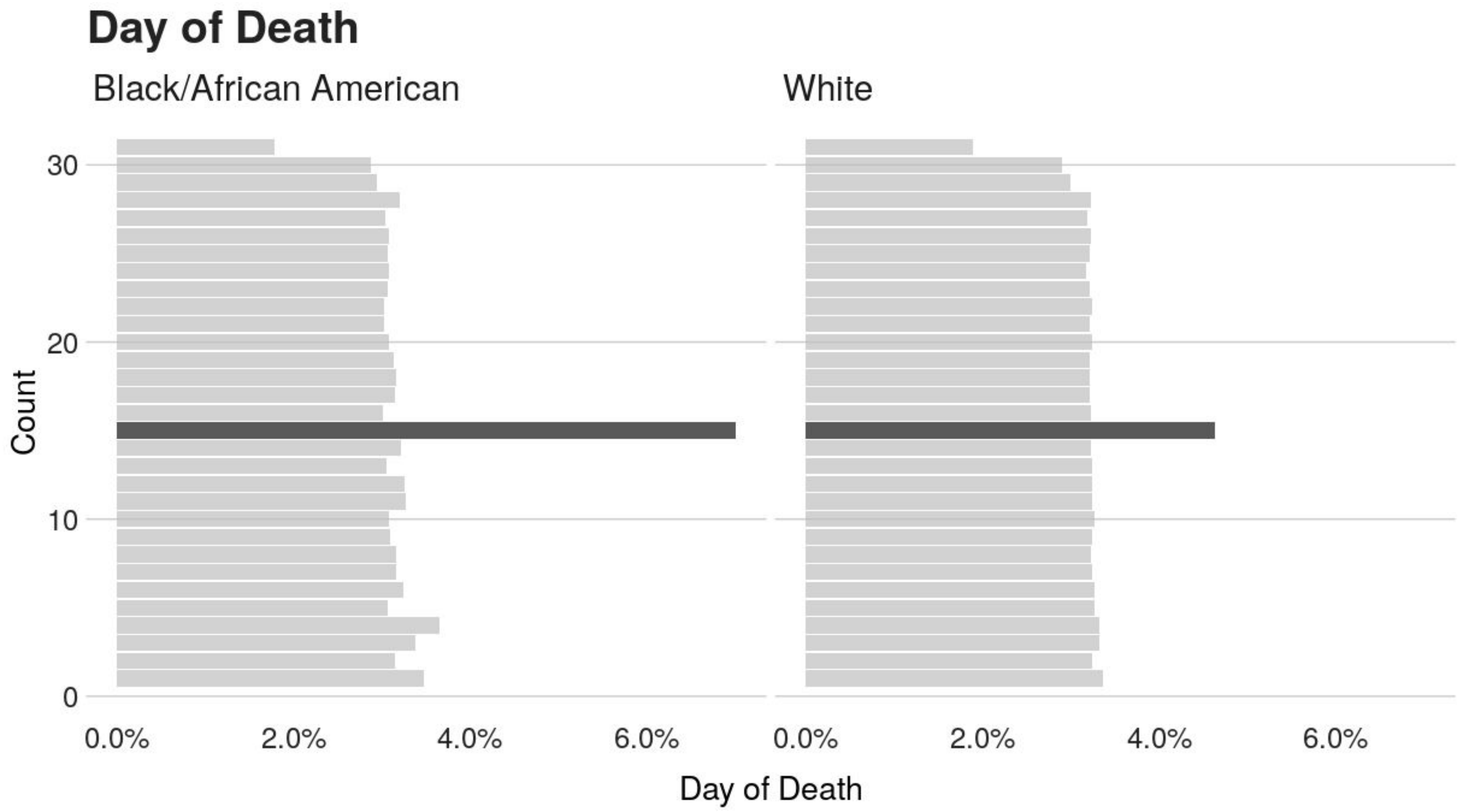
Pooled cohorts of 1890 - 1900

$$\mu_0(x) = \bar{\mu}(x) e^{\sigma^2 \bar{H}(x)}$$

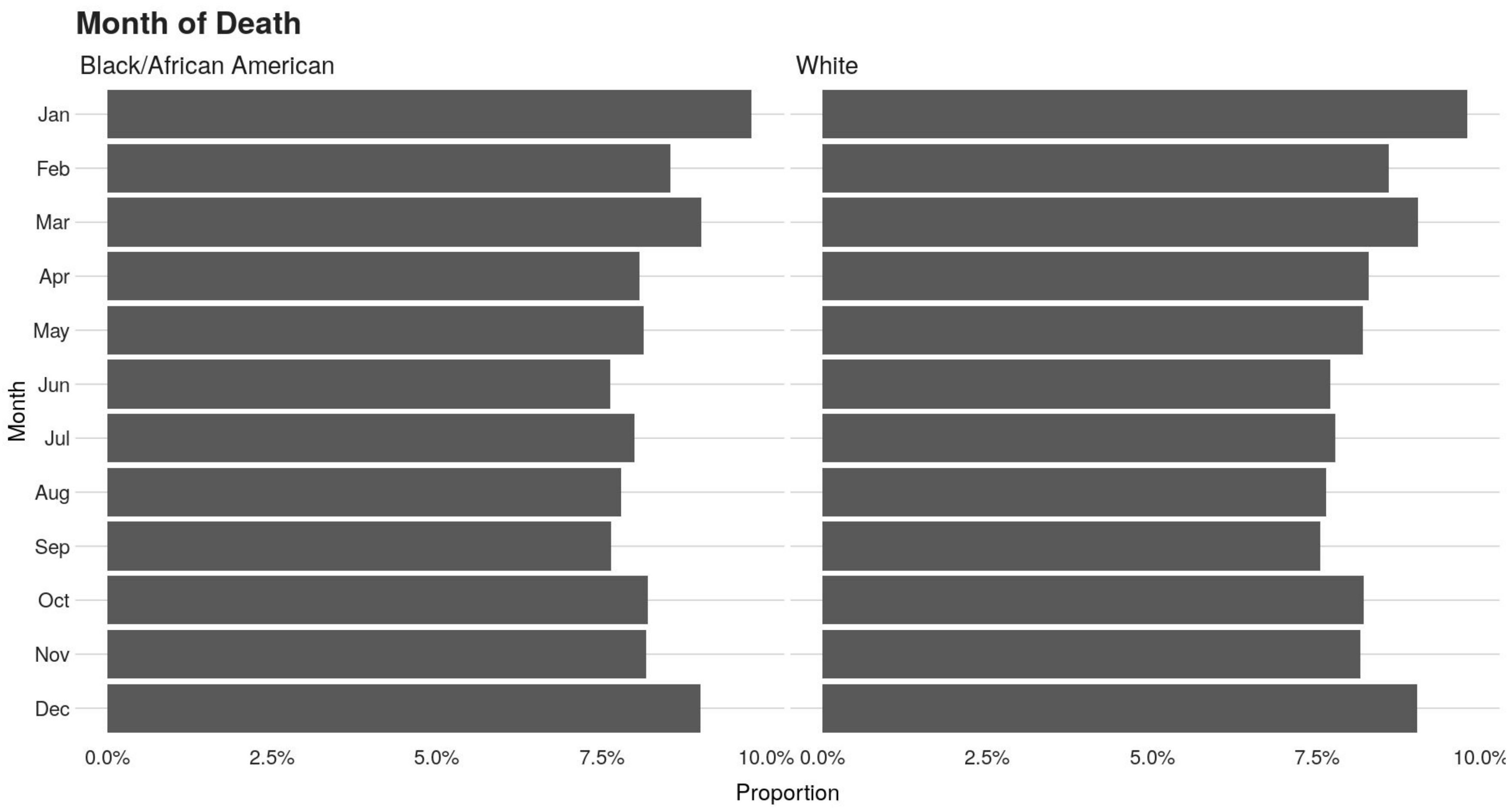


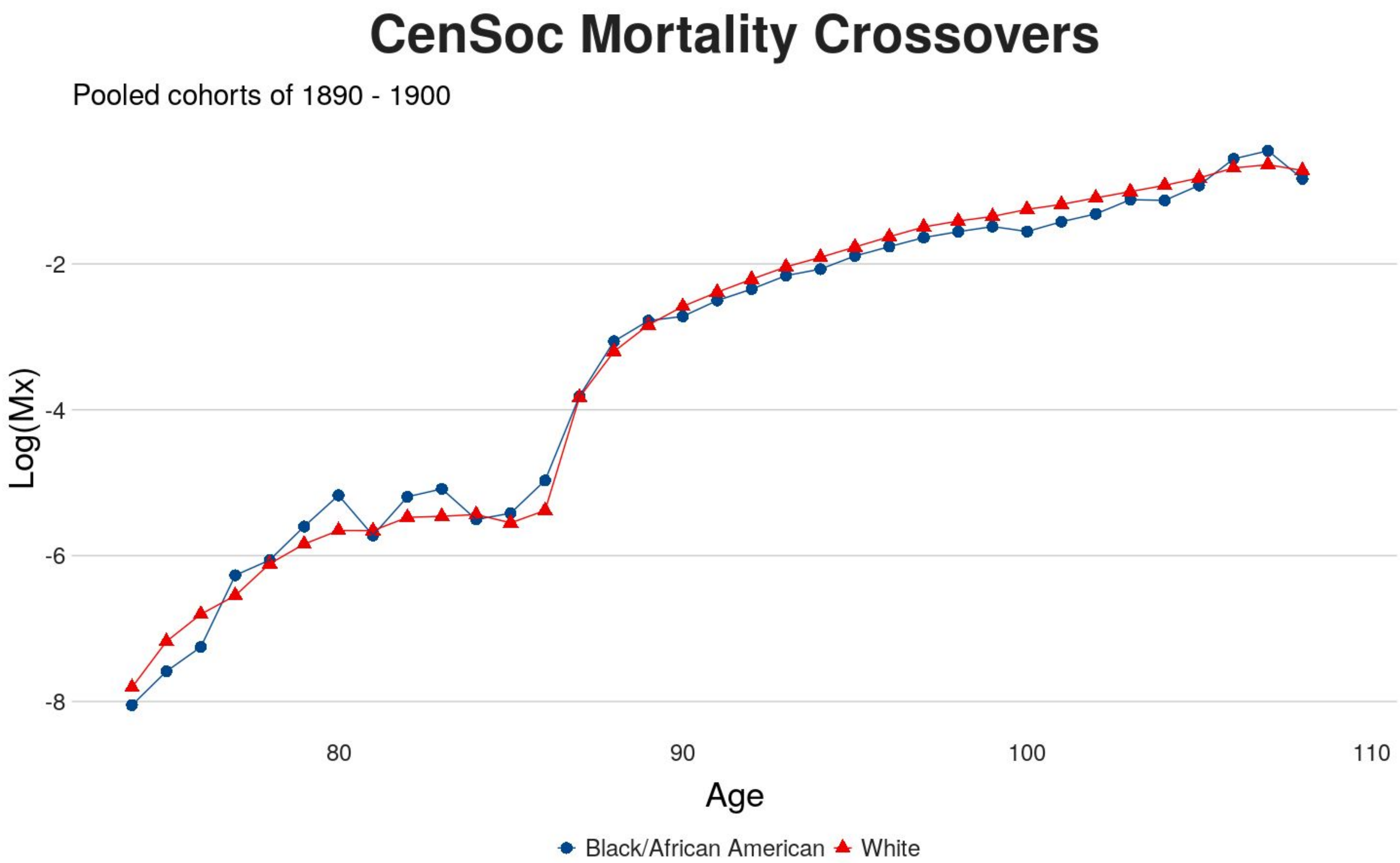
Death data in CenSoc file less reliable for Blacks?

- 57% of black people in pooled 1890-1900 cohort are missing death days (day of the month)
- 51% of white people missing death days
- No missing death months
- Missing day of birth far less common (about 0.005% of records)
- Are missing dates indicative of poor data?



Day of Death
Black/African American
White
Count
30
20
10
0
0.0%
2.0%
4.0%
6.0%
0.0%
2.0%
4.0%
6.0%
Day of Death


Month of Death
Black/African American
White
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
Month
0.0%
2.5%
5.0%
7.5%
10.0%
Proportion

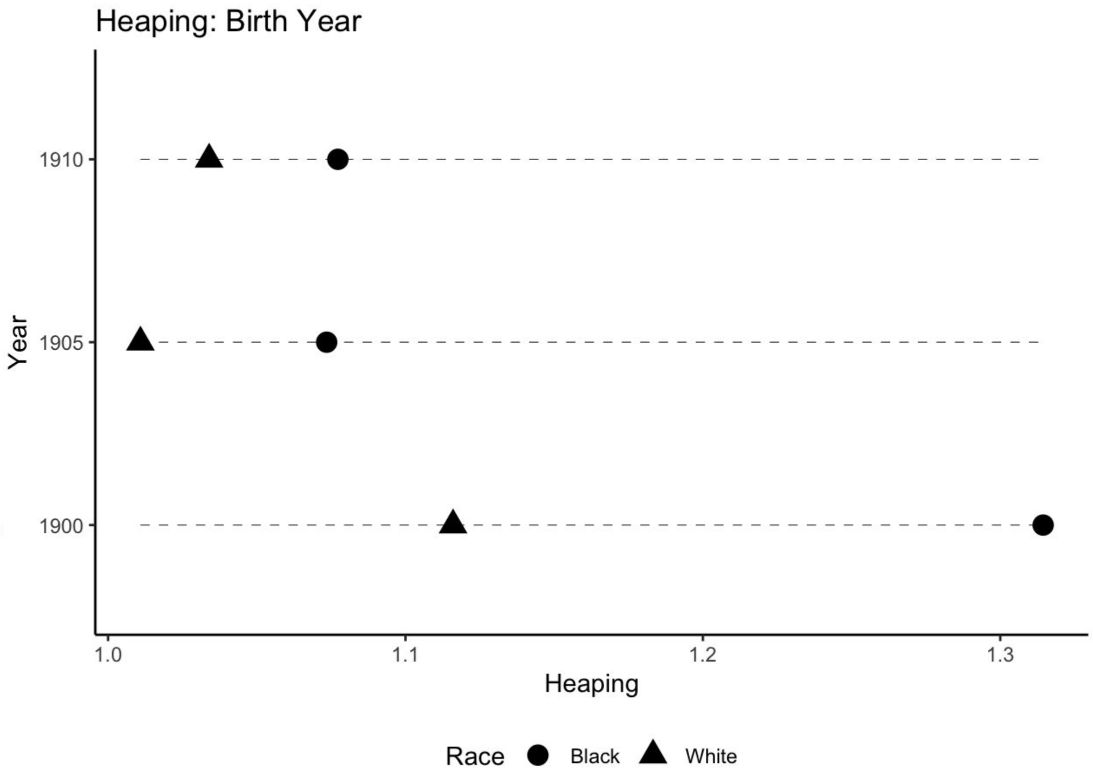


CenSoc Mortality Crossovers
Pooled cohorts of 1890 - 1900
Log(Mx)
-2
-4
-6
-8
80
90
100
110
Age
Black/African American
White

Heaping for reported Birth Year

- No Heaping on Death Year
- No Heaping on Age of Death
- Heaping on Birth Year

$$\text{Heaping (1900)} = \frac{\text{B}_{1900}}{(\text{B}_{1899} + \text{B}_{1901})/2}$$



Questions

For review:

What assumptions do we make when modeling multiplicative fixed frailty? Do they make sense?

For discussion:

Why is the black-white crossover of substantive interest? Any alternate theories for why we observe it?