Problem Set 3

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Everyone should do the "un-starred" problems. The "starred" are optional. They aren't necessarily difficult, but if you want to do less, feel free to not do these. The "double star" problems might be more advanced or time-consuming. Great if anyone wants to take a stab, but totally optional.

P.S. Feel free to team up for any of the problems, but especially the "starred" ones.

1. Under gamma frailty, we obtained an explicit expression for average frailty by age for any baseline hazard schedule.

$$\bar{z} = \frac{1}{1 + \sigma^2 H_0(x)}$$

   Assume baseline mortality is Gompertz (say with $a = 10^-4$ and $b = 1/12$). Try a couple of different values of σ^2 (but make sure one of these values is $1/7$ for comparability with the next problem). Describe what happens to average frailty at older ages. Does it decrease exponentially? If so, is there an age at which the rate of decrease equals (or at least comes very close to) the exponential rate of increase in baseline hazards b? Does this age depend on σ^2?

2. Obtain from the Human Mortality Database a schedule of single-year-of-age, cohort mortality rates for females born in 1880 in Italy. Use the "inversion formula" for the gamma distribution to obtain the baseline hazards implied by $\sigma^2 = 1/7$. Plot the observed and implied baseline schedule. Plot the average frailty by age. Do your results resemble or differ from the Gompertz case above?

3. Derive V&M 's result (5E)

$$\bar{R}(x) \equiv \frac{\bar{\mu}_2(x)}{\bar{\mu}_1(x)} = \frac{R + R\sigma_1^2 H_1(x)}{1 + R\sigma_2^2 H_1(x)}$$

4. Simulate this cross over with two proportional Gompertz schedules, with different frailty variances. Can you get a cross-over? If so, does it occur when cumulative hazard satisfy the condition (in small font) at the end of 5E?

∗ 5. Use simulation to say what the determinants of the age of crossover are in terms of the respective frailty variances, R, and the baseline Gompertz schedule.

∗∗ 6. ? Use mathematics to say what the determinants of the age of crossover are in terms of the respective frailty variances, R, and a baseline Gompertz schedule.to to say what the determinants of the age

∗ 7. Get two Italian cohorts 20 years apart and calculate the rate of mortality improvement by age $\rho(x)$ that you observe and that which you would have observed had there been no frailty. For frailty, assume gamma-distributed with $\sigma^2 = 1/5$.

8. Extend the CenSoc demonstration of changing characteristics with age in at least one of the following ways

   (a) Use years of education instead of wage income.

   (b) Use both years of education and wage income.

   (c) Analyze Blacks and Whites separately using wage income? Is the variance of "observed heterogeneity" ($\hat{z}_{obs}$) larger for one group. Discuss briefly.

9. If you are in the cross-over group,

   (a) See if there is a Black-White mortality crossover (you might want to try several different cohorts or groupings of cohorts).

   (b) Does the cross-over go away if you add controls (e.g. wage income or education)?

(c) Is there a level of frailty that makes the cross-over disappear up to age 100?

10. If you are in the plateaus group,

    (a) Use HMD's cohort schedules to see if you can detect plateaus at older ages. Do they appear more for men or women? Do they appear more for recent cohorts than older cohorts?

    (b) Fit the Gamma-Gompertz above age 60 to one or more of the plateau'd (or plateau-ing) schedules you find. What parameter estimates do you obtain? Graph the observed and fitted schedules and discuss briefly? What do you think of the parameter values?

    ∗(c) Is there any evidence of plateau in the CenSoc cohort of 1900? Discuss.

11. If you are in inequality/rate-of-progress group,

    (a) Using the explicit formula for the gamma-Gompertz population hazard, plot a graph of the population log-hazards of two groups with equal variance gamma frailty, one of which has hazards that are 25 percent more than the other at age 65. Choose parameters so you get a plausible value of $\bar{S}(x)$ (e.g., $e(65) = 15$ years). Halve the baseline mortality of both groups. What happens to the gap between the two groups at age 65? If it changes, is there another age at which the original 25 percent gap is found?

    (b) Waldron mentions frequently that heterogeneity might be an alternative explanation of her results. What kind of explanation is she thinking of and how would it work? Give a quotation from her that comes closely to explaining her thinking.

    (c) Use simulation to see if you can generate some of the patterns in Table 1.

    (d) Discuss the structure of the model used to generate Chart 3. Does the fact that later cohorts are observed only in the early years make unobserved heterogeneity a concern? Are there assumptions of linearity or other model assumptions that one should take into account when interpreting Chart 3?

Here is a guide for the presentation next week:

- Aim for about 20 minutes of material. We'll have 3 presentations, with discussion (and we'll probably have some overflow lecture from this week).
- Briefly summarize (e.g 5 to 10 min) the papers you read and what you learned from them.
- Present some empirical investigation or simulation that helped you understand or find evidence supporting or refuting the basic point the papers were making.
- Pose a question for the group for discussion.