Fertility Heterogeneity: Tempo Distortions and Distorted Tempo Dem260 Math Demog Spring 2020 Lecture 6

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Agenda for today

1. A tempo simulation
2. Bongaarts and Feeney's formula
3. An application to the United States
* Cookie Break
4. Two Americas?
5. EM algorithm for unmixing mixtures
6. An application to two Americas.

A common theme for 1st half of semester

What we see is superficial. Heterogeneous models reveal what's "really" going on. (Or do they?)

- Until today, population hazards mislead
- Today, homogeneous fertility misleads

2nd half of the semester will reverse perspectives

- We see differences we see in genotypes, in lineages, in names.
- These could be due to "real" differences (heterogeneity).
- But they could also be due to luck. Everyone is the same but stochastic outcomes differ.
- Our models of individual-level randomness will have predicted dynamics, which are themselves interesting but can also be used as a "null" to compare to observations.

Fertility postponement, a very simple example

Baseline

- A population has a history of 1 birth per day
- When women turn age 25, they have a daughter.
- This gives us a constant stream of births, 365 per year.

Postponement

- Starting on Jan. 1, 020, everyone postponements childbearing an additional month, until they are aged 25 1/12.
- How many births will there be in 2020?
- How many births in 2021?

Continuous postponement, a shiny simulation

`https://shiny.demog.berkeley.edu/josh/tempo/`

$R(t)$ Cumulative postponment

$r(t)$ Incremental postponement $r(t) = R'(t)$

What is a formula for recovering original birth stream?

$$\hat{B}_{orig} = B_{obs} \times (1 + R'(t))$$

or

$$\hat{B}_{orig} = B_{obs} \times 1/\left[1 - R'(t)\right]?$$

Note: this idea of "recovering original" is one way to think about tempo adjustment.

A bigger microsimulation

- Each period will have births across a range of ages
- We'll randomly generate the original planned birthdays
- Then we'll shift by a continuous function $R(t)$.

Bongaarts and Feeney's model

$$f(a, t) = f_0(a - R(t))(1 - R'(t))q(t)$$

Bongaarts and Feeney's model

$$f(a,t) = f_0(a - R(t))(1 - R'(t))q(t)$$

$f(a,t)$ birth rate of women aged a in period t

f_0 A constant baseline schedule (can be norm'd to sum to 1).

$q(t)$ A period intensity parameter: "quantum"

$R(t)$ Cumulative shift.

An example

$$f(a,t) = f_0(a - R(t))(1 - R'(t))q(t)$$

- $R_{2019} = 3$
- $R'_{2019} = .1$
- $q(2019) = 1$

Give an expression for $f(28, 2019)$.

A derivation (due to Rodriguez)

Assume no quantum effects.
Take a cohort with cumulative fertility

$$F_0(a) = \int_0^a f(x)\,dx$$

Now put in shifts so that observed fertility is from an age $R(t)$ years earlier. ("28" is the new "25"!)

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$$F(a,t) = F_0(a - R(t)) = F_0(a - R(c + a))$$

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Differentiate with respect to age (which for a cohort is also time t), using chain rule

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$$f(a,t) = f_0(a - R(t))\left[1 - R'(t)\right]$$

Bingo!

Quantum comes at the end

Let's re-notate our constant quantum result

$$f_0(a, t|R(t)) = f_0(a - R(t))\left[1 - R'(t)\right]$$

Then we can incorporate period quantum on the shifted surface:

$$f(a, t) = f_0(a, t|R(t))q(t) = f_0(a - R(t))\left[1 - R'(t)\right] q(t)$$

Note: If we vary quantum before shifts, then $q(t)$ will bleed into neighboring years. (a small effect, but makes model messier).

Tempo-adjusted TFR: counter-factual, TFR in absence of timing changes

$$TFR(t) = \int_0^\infty f(a, t)\, da$$

Substituting our shifted birth rates with quantum

$$TFR(t) = \int_0^\infty f_0(a - R(t)) \left[1 - R'(t)\right] q(t) \, \boldsymbol{da}$$

gives?

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$$TFR(t) = TFR_0 \left[1 - R'(t)\right] q(t)$$

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gives?

$$TFR(t) = TFR_0 \left[1 - R'(t)\right] q(t)$$

WLG, define $TFR_0 = 1$, then

$$q(t) = \frac{TFR(t)}{1 - R'(t)} \equiv TFR^*(t)$$

Voila, the BF formula

How do period schedules change?

For homework

$$f(a,t) = f_0(a - R(t))\left[1 - R'(t)\right]$$

What does

$$\frac{\partial}{\partial t}\log f(a,t) = ?$$

Let's sketch

A diagnostic

for homework

“Uniform” shifts

- BF model assumes all ages shift by $R(t)$.
- BF model assumes all ages rise or fall by same quantum $q(t)$
- Violating these assumptions means change in mean age will not just reflect “tempo”.
- Example: What happens if people have fewer higher order births?

BF recommendation for achieving uniformity

Separate estimates for each birth order, and then combine:

$$TFR^*(t) = \sum_i TFR_i^*(t) = \sum_i \frac{TFR_i(t)}{1 - r_i(t)}$$

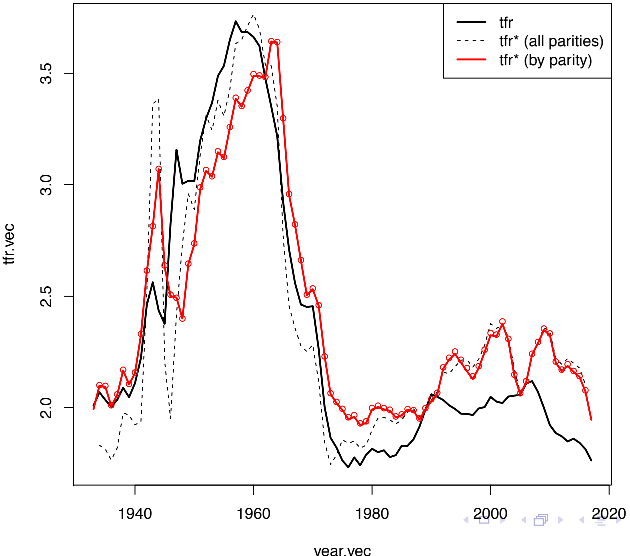
This will protect against order-specific quantum effects.

An Application to the United States

- We'll use HFD data
- We'll do tempo adjustment for all births
- We'll redo by birth order

`usa_tempo.R`

Figure: BF tempo adjustment of US total fertility



Conclusions

- Baby boom smaller if we account for "pre-ponement".
- Fertility lull in 1970s and 80s disappears if we account for "postponement"
- Birth order disaggregation improves estimates of shifts from changes in mean age
- What happened with the recession?

Cookie Break

Animation

Let's look at births (all orders).

`fat_movie.pdf`

Mixing

Let's look at 1st births, again as if their are two latent groups: A and B. (These could be "early moms" / "late moms", non-college / college, pre-marital / marital, lower-class / upper class, ...)

`fat_mix_movie.pdf`

Youtube

In R

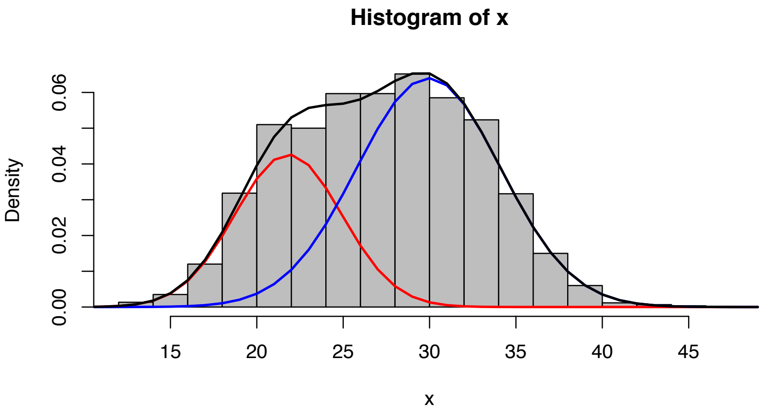
```
library(mixtools)

## mixtools package, version 1.1.0, Released
2017-03-10
## This package is based upon work supported by the
National Science Foundation under Grant No.
SES-0518772.
```

In R

```
## simulate 2 normals
N <- 1000
x1 <- rnorm(N, mean = 22, sd = 3) ##
x2 <- rnorm(2*N, mean = 30, sd = 4)
## combine them
x <- c(x1,x2)
## use EM to infer mixture
out <- normalmixEM(x,
                   lambda = c(.5, .5),
                   mu = c(15, 35),
                   sigma = c(5,5))

## number of iterations= 330

print(out$mu)

## [1] 21.82306 29.94661

print(out$sigma)

## [1] 3.094665 4.171042

print(out$lambda)

## [1] 0.3308744 0.6691256
```

Seems to work great.

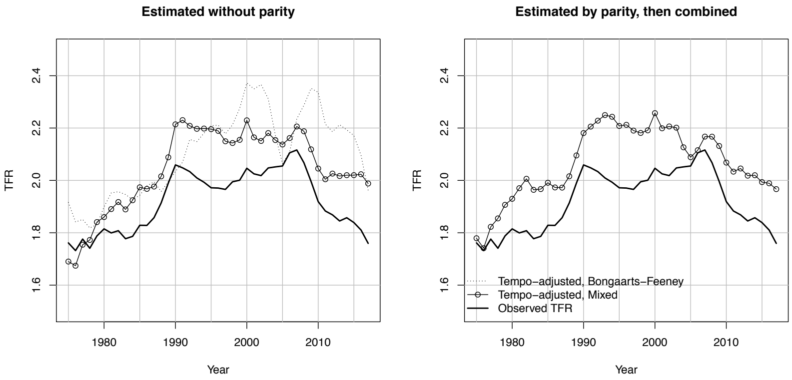
Visualization



An algorithm for tempo adjustment of mixtures

1. Fit normal mixture to each year.
2. Refit using constant variance (average). This assures shape invariance of each component, fulfilling BF assumption.
3. Estimate BF separately for A and B, and combine.

Figure: Results (preliminary)



Identifiability?

Main points

- Postponement dilutes period births, lowers TFR
- Tempo-adjustment tries to “put births back in”
- Changes in mean work fine if “shape” doesn’t change
- Shape can change through heterogeneity
- With strong assumptions, we can identify heterogeneity
- Declining quantum for young and postponement for old appears to be the story

Caveats

- Who are these latent groups? Do you start out in one and end up in the other? Do you stay in one your whole life?
- How do we project forward?
- Can we use other indicators (e.g., social class, education, marriage) to get same results?

Next time, Branching Processes

Either Dartmouth textbook reading, or Harris classic.