Problem Set 1

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Everyone should do the "un-starred" problems. The "starred" are optional. They aren't necessarily difficult, but if you want to do less, feel free to not do these. The "double star" problems might be more advanced or time-consuming. Great if anyone wants to take a stab, but totally optional.

1. True or False? "If every part grows exponentially at its own rate, then the whole will also grow exponentially." Explain your answer briefly.

2. List what you think are two of the best arguments in favor of doing a disaggregated projection? What are two of the best arguments in favor of doing an aggregated projection?

3. For Keyfitz's example of the United States and Mexico: ($r_U = .75/100$; $r_M = 3.5/100$; $K_U(1970) = 200$; $K_M(1970) = 50$).

   (a) Use the first few years of the projection to veryify that rate of change in the aggregate growth rate equals the variance of the growth rate. Does it matter what time points and period you consider?

   (b) The growth rate changes over the course of the 50 years, but there is a constant growth rate that will produce the exact same population after 50 years.[1] A reasonable choice of the constant growth rate to apply is the value of the changing growth rate at year 25 (half-way through the period). We can estimate this using a Taylor series approximation:

$$\bar{r}(25) \approx \bar{r}(0) + 25\bar{r}'(0) + (25^2)/2\bar{r}''(0)$$

[1] If you don't believe this, feel free to ask me why or see Wikipedia "Mean Value Theorem" under "First mean value theorem for integrals"

i. We know that $\bar{r}'(0) = \sigma_r^2(0)$. Show that $\bar{r}''(0)$ equals $\bar{r}_3 - \ldots$

ii. Calculate the combined US-Mexico population after 50 years according to the following five (5) methods, plot the total population after 50 years according to these 5 methods on a graph

   A. the dissaggregated ("true") forecast, with each country growing at its own rate

   B. the aggregated forecast, pretending it's one country, growing at $\bar{r}(0)$ for 50 years

   C. the aggregated forecast, growing at the "true" value of $\bar{r}(25)$ for the whole period. (Use the value of $\bar{r}(25)$ that you calculate from the disaggregated forecast).)

   D. the aggregated forecast, growing at the first-order Taylor series estimate of $\widehat{\bar{r}}(25)$ for the whole period.

   E. the aggregated forecast, growing at the second-order Taylor series estimate of $\widehat{\bar{r}}(25)$ for the whole period.

   What conclusions do you draw?

4. For Ken's Poisson-Exponential model,

   (a) What is the closed-form expression for $\bar{r}(t)$?

   *(b) What is the variance of the growth rate $\sigma_r^2(t)$?

   **(c) (Advanced?) Show that under the model the growth rates are Poisson distributed at all times t. Write an expression for the Poisson parameter $\lambda(t)$.

   *(d) Let's call the difference between $\bar{r}(t)$ and $\bar{r}(0)$ the "distortion index." Write down an expression for the distortion index. What variables and parameters in the model does it depend on? Are there any variables or parameters that it doesn't depend on?

** 5. Project idea (Totally optional! At the very most, I would just do the first part for this week. But If someone might want to build on this to do their research project later in the term) : Use Ken's Poisson-Exponential model to do a projection of some region of the world (e.g., Sub-Saharan Africa) or the world as a whole, fitting the Poisson to the observed distribution of growth rates. (If you want to do this, please ask me for the UN projection file.) Here are some questions to explore:

(a) What does the observed distribution of growth rates look like?

(b) Fit the Poisson approach – perhaps matching the mean and variance and maximum growth rate. Does this approach provide a good description to the distribution of growth rates you observe?

(c) Compare the projection of Ken's model to the disaggregated and the aggregated projection you would obtain by assuming growth rates stayed constant.

(d) Describe your ideas for how to expand this project to changing growth rates – what are some challenges?