Macro, Money, and Finance: a continuous-time approach

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This course is a graduate-level field course and covers an entire semester (∼15-16 weeks). [PRELIMINARY AND INCOMPLETE]

Grading:

- Problem sets (60%)
- Presentation and referee report (15%)
- Research proposal (25%)

Overview:

1. **A Brief History of Macroeconomics and Finance** A helicopter view of how the fields of macroeconomics and finance have evolved and their convergence into continuous-time macro models with financial frictions. Why continuous time modeling?

2. **Stochastic Calculus Basics**

   Measure and probability theory: basic concepts, Standard Brownian Motion, Martingales, Stochastic integral, Ito processes, Ito diffusion, Stochastic Differentiation: Ito formula, Generator of an Ito diffusion, Stochastic optimization: Dynamic Programming, Martingale Representation Theorem, Girsanov Theorem, Kolmogorov Forward Equation, Backward Stochastic Differential Equations (BSDE), Forward-Backward Stochastic Differential Equation (FBSDE), Stochastic Optimization: Stochastic Maximum Principle.

3. **Optimization: Consumption and Portfolio Choice**

   We solve a fairly general single-agent consumption-portfolio choice using three approaches: [i] Dynamic Programming (HJB equation), [ii] Stochastic Maximum Principle (the Hamiltonian), [iii] the Martingale Method.

4. **A Simple Real Macro Model with Heterogeneous Agents**

   We illustrate how to solve a general equilibrium within this literature using a simple two sector model (Basak Cuoco 1998). This simple model allows for closed form (global) solutions mainly because of (i) the impossibility of less productive agents to manage the productive input (capital) and (ii) logarithmic preferences. Unfortunately, the model lacks some desired properties to study financial crises (e.g., endogenous risk).

- S. Basak and D. Cuoco. An equilibrium model with restricted stock market participation. *Review of Financial Studies*, 11(2):309–341, 1998

5. **Endogenous Risk Dynamics in Real Macro Model with Heterogeneous Agents**

   We solve a version of the model in Brunnermeier Sannikov (2016), which allows for capital miss-allocation and non-myopic preferences. The model delivers a normal regime around the (stochastic steady state) and a crisis regime when the key sector in the economy (the financial sector) is under-capitalized. The model features endogenous (time-varying) risk and price of risk. Also, it allows to study fire-sales, liquidity spirals, and fat tails. It delivers a volatility paradox and rationalizes how a financial innovation could cause instability. Additional toolboxes: price-taking social planner and change of numeraire.

   - M. K. Brunnermeier and Y. Sannikov. Macro, money, and finance: A continuous-time approach. In *Handbook of Macroeconomics*, volume 2, pages 1497–1545. 2016a
   - M. K. Brunnermeier and Y. Sannikov. A macroeconomic model with a financial sector. *American Economic Review*, 104(2):379–421, 2014. Link.

6. **Numerical methods for continuous-time models (finite differences)**: Introduction: a general class of equations, examples: valuation equation and HJB, Forward and Backward Equations: HJB and KFE, Finite Difference Schemes: Key Principles, Finite Difference Operator and sign of Matrix M, Explicit Scheme, Implicit Scheme, Stationary Value Function in a Single Step, KFE using Matrix M, General Class of HJB in One Dimension, Solving HJB, Non-monotone Schemes (what can go wrong?), Valuation Equation in m Dimensions, Convex Positive Semidefinite Cone, Some Geometry in Two Dimensions, Algorithm for the 2nd-order Term in Two Dimensions, Assembling M and Solving the Valuation Equation, Solving HJB Equation in m Dimensions.

7. **Multiplicity and Stochastic Stability**

   In a simplified version of the model with endogenous risk dynamics, we show that the model allows for equilibria influenced by sunspots or "rational sentiment" (i.e., equilibria that are not Markov in the fundamental states of the economy). We show that rational sentiment helps resolve two puzzling features plaguing models emphasizing balance sheets: [i] financial crises emerge suddenly, featuring large volatility spikes and asset-price declines; [ii] asset price booms, with below-average risk premia, predict busts and financial crises. We introduce basic concepts and a useful lemma of stochastic stability theory.

   - F. Mendo and P. Khorrami. Rational sentiments and financial frictions. *Working Paper*, February 2023
   - F. Mendo and P. Khorrami. Fear and volatility at the zero lower bound. *Working Paper*, 2022

- R. Khasminskii. *Stochastic stability of differential equations*, volume 66. Springer Science & Business Media, 2011

8. **Epstein Zin preferences and Jumps**

   We generalize the model with endogenous risk dynamics to allow for recursive EZ preferences. Basic stochastic calculus with jumps. We extend the single agent consumption-portfolio problem to include the case where the asset returns are sensitive to jumps. We use jumps in three different contexts: [i] Multiplicity. We extend the information set to include a Poisson shock and show that the model can have equilibria in which equilibrium outcome respond to those shocks. This applies to both, equilibria recursive in fundamentals and those that are not. [ii] Real Poisson shock. We extend the model to add a capital destruction shock that arrives according to a Poisson shock. [iii] Model with financial panics.

   - W. Li. Public liquidity and financial crises. Working Paper, 2021
   - F. Mendo. Risky low-volatility environments and the stability paradox. *Working Paper*, 2020

9. **One sector Monetary Model and Idiosyncratic Risk**

   - M. K. Brunnermeier, S. Merkel, and Y. Sannikov. Safe assets: A dynamic retrading perspective. Working Paper, 2022a
   - M. K. Brunnermeier, S. Merkel, and Y. Sannikov. The fiscal theory of the price level with a bubble. Working Paper, 2022b

10. **The I Theory of Money with Heterogenous Agents**

    - M. K. Brunnermeier and Y. Sannikov. The i theory of money. *Working Paper*, August 2016b. doi: 10.3386/w22533

11. **Applications**

    - D. Silva and D. Duarte. Machine learning for continuous-time finance. Working Paper, 2023
    - M. Maggiori. Financial intermediation, international risk sharing, and reserve currencies. *American Economic Review*, 107(1):3038–3071, 2017
    - S. Di Tella and R. Hall. Risk premium shocks can create inefficient recessions. *Review of Economic Studies*, 0:1–35, 2021. doi: 10.1093/restud/rdab049
    - S. Di Tella. Risk premia and the real effects of money. *American Economic Review*, 110(7):1995–2040, 2020. doi: 10.1257/aer.20180203
    - D. Vayanos and J.-L. Vila. A preferred-habitat model of the term structure of interest rates. *Econometrica*, 89(1):77–112, 2021. doi: 10.3982/ECTA17440
    - P. Khorrami. The risk of risk-sharing: Diversification and boom-bust cycles. *Working Paper*, January 2020

- A. d'Avernas, Q. Vandeweyer, and M. Darracq Paries. Central banking with shadow banks. *Working Paper*, February 2023
- Z. He and A. Krishnamurthy. Intermediary asset pricing. *American Economic Review*, 103(2):732–770, 2013
- A. Krishnamurthy and W. Li. Dissecting mechanisms of financial crises: Intermediation and sentiment. *Working Paper*, 2021
- S. Di Tella. Uncertainty shocks and balance sheet recessions. *Journal of Political Economy*, 125(6):2038–2081, 2017
- Z. He and A. Krishnamurthy. A model of capital and crises. *Review of Economic Studies*, 79(2):735–777, 2012
- G. Gopalakrishna. A macro-finance model with realistic crisis dynamics. Working Paper, 2022b
- S. Di Tella. Why are banks exposed to monetary policy? *American Economic Review*, 110(7):1995–2040, 2021. doi: 10.1257/aer.20180203

12. **More on Numerical Methods**

- G. Gopalakrishna. Aliens and continuous time economies. Working Paper, 2022a
- G. Gopalakrishna, S. J. Lee, and T. Papamichalis. Heterogeneous beliefs, risk amplification, and asset returns. Working Paper, 2023
- J. Fernández-Villaverde, S. Hurtado, and G. Nuño. Financial frictions and the wealth distribution. *Econometrica*, 91(3):869–901, 2023
- A. d'Avernas, D. Petersen, and Q. Vandeweyer. A solution method for continuous-time models. Working Paper, 2022

Bibliography

S. Basak and D. Cuoco. An equilibrium model with restricted stock market participation. *Review of Financial Studies*, 11(2):309–341, 1998.

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