Heterogeneity and Market Incompleteness in Macroeconomics *

PUC Rio, MA/PhD 2023

Course Information

Instructor: Fernando Mendo (fmendolopez@gmail.com)

Time: Tuesdays, 4pm-7pm. **Location:** Room F306

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Office Hours: By appointment

Course Description

This course corresponds to the second half of Macroeconomics II, the second course of the first-year graduate-level macroeconomics sequence. It covers 7-8 weeks of classes (3-4 hours per week). The course introduces students to macroeconomic questions in terms of distributions rather than just aggregates. For this, we develop macroeconomic theories in which the relevant state variable is a distribution, i.e., heterogeneous agent models. Given that analytic characterizations of these models is limited, the course highlights the numerical methods required to solve these heterogeneous agent models. The course has a hands-on approach and by the end of the course, the student should be able to use this type of models in its own research. The workhorse model for the course is the income fluctuation problem, which we later embed into a general equilibrium model. Heterogeneity will be relevant in this framework due to markets incompleteness. This workhorse model emphasizes heterogeneity in the household sector but the methods developed can be applied to other dimensions of heterogeneity (e.g., firms or financial institutions).

Preliminary and subject to changes. Incomplete set of references.

Evaluation

Problem sets will account for 40% of the overall grade (of Macroeconomics II). The remaining 60% will correspond to problem sets of the first half of the course (40%) and a final assignment (20%).

Course Outline

Useful background readings for the course: Heathcote *et al.* (2009), Guvenen (2012), Quadrini and Rios-Rull (2014).

1. **A Brief History of Macroeconomics.** A helicopter view of how the field of macroeconomics has evolved.

2. **Income Fluctuation Problem.** We study the consumption-saving problem of a single-agent who faces a stochastic income stream and can trade only a risk-free bond subject to a borrowing constraint. We characterize the long-term behavior of his consumption and asset holdings, which depends on stochastic properties of income and the agent's patience relative to the market interest rate. We review the Permanent Income Hypothesis which emerges in a especial case of this income fluctuation problem. We introduce the notion of precautionary savings and relate it to the convexity of marginal utility (prudence), We show that precautionary savings can arise even without prudence as long as borrowing constraints may bind in some state of the world.

   References:

   Book / Lecture notes: Ljungqvist and Sargent (2018, ch. "Self-Insurance"), Lecture notes by Fabrizio Perri (lectures 5 and 6), available here.

   Theory papers: Leland (1968), Sandmo (1970), Chamberlain and Wilson (2000), Kimball (1990), Schechtman (1976), Schechtman and Escudero (1977), Sibley (1975), Yaari (1976), Bewley (1977). Empirical (implications of full-insurance): Cochrane (1991), Schulhofer-Wohl (2011), Mace (1991), Hall (1978), Blundell and Preston (1998).

   Numerical Methods. We present a set of simple numerical techniques to solve for the

consumption and saving policy functions in the recursive formulation of the income fluctuation problem. We consider the following methods: Value Function Iteration, Euler Equation Iteration, Endogenous Grid Method.

References: Tauchen (1986), Suen and Kopecki (2010), Lkhagvasuren (2012), Lkhagvasuren and Gospodinov (2014), Carroll (2006), Judd (1998), Miranda and Fackler (2002), Heer and Maubner (2005).

3. **The Neoclassical Growth Model with Incomplete Markets.** We begin with a refresher of the representative agent (RA) neoclassical growth model and highlight the infinitely-elastic long-term capital supply. Then, we analyze the equilibrium of an economy populated by a continuum of agents who face an income fluctuation problem. We consider three alternatives for the asset supply: bonds in zero net supply (Huggett model), bonds or money in positive net supply (Bewley model), and capital in positive net supply (Aiyagari model). The latter includes labor and constitutes the Neoclassical Growth Model with incomplete markets. We analytically characterize (as much as possible) the stationary equilibrium and present numerical procedures to solve for it. We also study numerical procedures to solve for transition dynamics.

   References: Ljungqvist and Sargent (2018, ch. "Incomplete Markets Models"), Imrohoroglu (1989), Huggett (1993), Aiyagari (1994), Heer and Maubner (2005), Uhlig (1996), Laitner (1992). Constrained efficiency in the Aiyagari model: Hong *et al.* (2012).

4. **Continuous Time Methods.** We begin by stating the potential advantages of continuous time for macroeconomic modeling. We then introduce the required methods. (1) Deterministic optimization: Hamiltonian and Hamilton-Jacobi-Bellman equation. (2) Deterministic numerical solution: the finite difference method. (3) Diffusion and Poisson processes. (4) Ito's Lemma. (5) Stochastic optimization: HJB equation. (6) Kolmogorov Forward Equations. (7) Stochastic numerical solution: the finite difference method revisited.

   References: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 1: Background and Overview, Hamiltonians and Phase Diagrams, Lecture 3: Hamilton-Jacobi-Bellman Equations, Lecture 4: Diffusion Processes,

Stochastic HJB Equations and Kolmogorov Forward Equations). Acemoglu (2009, Ch. 7, "Review of the Theory of Optimal Control"),

Numerical Methods: Kushner and Dupuis (1992), Candler (1999). **Codes**: heterogeneous agent models in continuous time using finite difference methods available at Ben Moll's webpage (click here).

5. **A Workhorse Model of Income and Wealth Distribution in Macroeconomics**

   We study the continuous time version of a Bewley-Huggett economy with a two-state process for income. We show additional analytical results regarding the income fluctuation problem faced by individuals. In particular, we characterize the consumption and saving behavior near the borrowing constraint. We provide a uniqueness result for the stationary equilibrium under the condition that the IES is equal or greater than one. We then discuss the numerical approach (finite differences) to solve for the stationary equilibrium and transitional dynamics.

   References: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lectures 7 and 8: The Workhorse Model of Income and Wealth Distribution in Macroeconomics). Achdou *et al.* (2022), including Online Appendix available here and codes available here.

6. **HANK — Heterogeneous Agent New Keynesian Models.**

   We combine two workhorses of modern macroeconomics: New Keynesian models (Gali, Gertler, Woodford) and Bewley models (Aiyagari, Bewley, Huggett). We discuss the Kaplan-Moll-Violante incarnation, but there are many others. The building blocks in this framework are: (1) uninsurable idiosyncratic income risk, (2) nominal price rigidities,and (3) assets with different degrees of liquidity. It constitutes a framework for quantitative analysis of aggregate shocks and macroeconomic policy. We focus on the transmission mechanism for conventional monetary policy.

   References: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 9: HANK — Heterogeneous Agent New Keynesian Models). Kaplan *et al.* (2018).

Macroeconomic implications of heterogeneity: Cavalcanti *et al.* (2021), Broer *et al.* (2020), Auclert (2019), Luetticke (2021), Farhi and Werning (2019), Auclert *et al.* (2020), Nuño and Thomas (2020), McKay and Wieland (2021), Laibson *et al.* (2021), Beraja *et al.* (2019), Lagakos *et al.* (2021).

7. **Firm Heterogeneity and Stopping Time Problems.** We discuss (the continuous time version of) a workhorse model of firm dynamics: Hopenhayn (1992). We study stopping time problems in continuous time. We discuss the standard solution approach: "smooth pasting", but we focus on a more powerful approach: HJB "Variational Inequality."

   References: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 10: Firm Heterogeneity, Distribution and Dynamics; Stopping Time Problems).

   Theories of Firm Size Distribution: Hopenhayn (1992), Luttmer (2007), Bertola and Caballero (1994), Caballero and Engel (1999), Khan and Thomas (2008), Winberry (2016), Zwick and Mahon (2017).

   Methods: Stopping time problems. Stokey (2008), Dixit (1993), Dixit and Pindyck (1994).

Bibliography

ACEMOGLU, D. (2009). *Introduction to modern economic growth*. Princeton, NJ [u.a.]: Princeton Univ. Press.

ACHDOU, Y., HAN, J., LASRY, J.-M., LIONS, P.-L. and MOLL, B. (2022). Income and wealth distribution in macroeconomics: A continuous-time approach. *The Review of Economic Studies*, **89**, 45–86.

AIYAGARI, R. (1994). Uninsured idiosyncratic risk and aggregate saving. *Quarterly Journal of Economics*.

AUCLERT, A. (2019). Monetary policy and the redistribution channel. *American Economic Review*.

—, ROGNLIE, M. and STRAUB, L. (2020). Micro jumps, macro humps: Monetary policy and business cycles in an estimated hank model.

BERAJA, M., FUSTER, A., HUST, E. and VAVRA, J. (2019). Regional heterogeneity and the refinancing channel of monetary policy. *The Quarterly Journal of Economics*.

BERTOLA, G. and CABALLERO, R. (1994). Irreversibility and aggregate investment.

BEWLEY, T. (1977). The permanent income hypothesis: A theoretical formulation. *Journal of Economic Theory*.

BLUNDELL, R. and PRESTON, I. (1998). Consumption inequality and income uncertainty. *Quarterly Journal of Economics*.

BROER, T., HANSEN, N.-J. H., KRUSELL, P. and ÖBERG, E. (2020). The new keynesian transmission mechanism: A heterogeneous-agent perspective. *Review of Economic Studies*.

CABALLERO, R. J. and ENGEL, E. M. R. A. (1999). Explaining investment dynamics in u.s. manufacturing: A generalized (s,s) approach. *Econometrica*.

CANDLER, W. (1999). Finite-difference methods for dynamic programming problems. In R. Marimon and A. Scott (eds.), *Computational Methods for the Study of Dynamic Economies*, Cambridge, England: Cambridge University Press.

CARROLL, C. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*.

CAVALCANTI, T., KABOSKI, J., MARTINS, B. and SANTOS, C. (2021). *Dispersion in Financing Costs and Development*. Tech. rep., code is available at http://benjaminmoll.com/ckmscode/.

CHAMBERLAIN, G. and WILSON, C. (2000). Optimal intertemporal consumption under uncertainty. *Review of Economic Dynamics*.

COCHRANE, J. (1991). A simple test of consumption insurance. *Journal of Political Economy*.

DIXIT, A. K. (1993). *The Art of Smooth Pasting*, *Fundamentals of Pure and Applied Economics*, vol. 55. Routledge.

— and PINDYCK, R. S. (1994). *Investment Under Uncertainty*. Princeton University Press.

FARHI, E. and WERNING, I. (2019). Monetary policy, bounded rationality, and incomplete markets. *American Economic Review*.

GUVENEN, F. (2012). Macroeconomics with heterogeneity: A practical guide. *Richmond Fed QR*.

HALL, R. (1978). Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence. *Journal of Political Economy*.

HEATHCOTE, J., STORESLETTEN, K. and VIOLANTE, G. (2009). Quantitative macroeconomics with heterogeneous households. *Annual Review of Economics*.

HEER, B. and MAUBNER, A. (2005). *DGE Modelling, Computational Methods and Applications*. Springer.

HONG, J., DAVILA, J., KRUSELL, P. and RIOS-RULL, J.-V. (2012). Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks. *Econometrica*.

HOPENHAYN, H. A. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*.

HUGGETT, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*.

IMROHOROGLU, A. (1989). The costs of business cycles with indivisibilities and liquidity constraints. *Journal of Political Economy*.

JUDD, K. (1998). *Numerical Methods in Economics*. MIT Press.

KAPLAN, G., MOLL, B. and VIOLANTE, G. L. (2018). Monetary Policy According to HANK. *American Economic Review*, **108** (3), 697–743.

KHAN, A. and THOMAS, J. K. (2008). Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics. *Econometrica*.

KIMBALL, M. (1990). Precautionary saving in the small and in the large. *Econometrica*.

KUSHNER, H. J. and DUPUIS, P. (1992). *Numerical Methods for Stochastic Control Problems in Continuous Time*. Springer.

LAGAKOS, D., MOBARAK, A. M. and WAUGH, M. E. (2021). The welfare effects of encouraging rural-urban migration. *Econometrica*, code is available at https://github.com/lnsongxf/welfaremigration.

LAIBSON, D., MAXTED, P. and MOLL, B. (2021). Present bias amplifies the household balance-sheet channels of macroeconomic policy.

LAITNER, J. (1992). Random earnings differences, lifetime liquidity constraints, and altruistic intergenerational transfers. *Journal of Economic Theory*.

LELAND, H. (1968). Savings and uncertainty: The precautionary demand for saving. *Quarterly Journal of Economics*.

LJUNGQVIST, L. and SARGENT, T. J. (2018). *Recursive Macroeconomic Theory, fourth edition*. MIT Press, illustrated edn.

LKHAGVASUREN, D. (2012). *Key Moments in the Rowenhorst Method*. Mimeo.

— and GOSPODINOV, N. (2014). A moment-matching method for approximating var processes by finite-state markov chains. *Journal of Applied Econometrics*.

LUETTICKE, R. (2021). Transmission of monetary policy with heterogeneity in household portfolios. *AEJ: Macroeconomics*.

LUTTMER, E. F. P. (2007). Selection, growth, and the size distribution of firms. *Quarterly Journal of Economics*.

MACE, B. (1991). Full insurance in the presence of aggregate uncertainty. *Journal of Political Economy*.

MCKAY, A. and WIELAND, J. (2021). Lumpy durable consumption demand and the limited ammunition of monetary policy. *Econometrica*.

MIRANDA, M. and FACKLER, P. (2002). *Applied Computational Economics and Finance*. MIT Press.

NUÑO, G. and THOMAS, C. (2020). *Optimal Monetary Policy with Heterogeneous Agents*. Tech. rep., working paper. Click https://example.com/working-paper-2020.

QUADRINI, V. and RIOS-RULL, J.-V. (2014). Inequality in macroeconomics. In *Handbook of Income Distribution*, *14*.

SANDMO, A. (1970). The effect of uncertainty on saving decisions. *Review of Economic Studies*.

SCHECHTMAN, J. (1976). An income fluctuation problem. *Journal of Economic Theory*.

— and ESCUDERO, V. (1977). Some results on an income fluctuation problem. *Journal of Economic Theory*.

SCHULHOFER-WOHL, S. (2011). Heterogeneity and tests of risk sharing. *Journal of Political Economy*.

SIBLEY, D. (1975). Permanent and transitory income effects in a model of optimal consumption with wage income uncertainty. *Journal of Economic Theory*.

STOKEY, N. L. (2008). Exercising an option. In *The Economics of Inaction: Stochastic Control Models with Fixed Costs*, *6*.

SUEN, R. and KOPECKI, K. (2010). Finite state markov-chain approximations to highly persistent processes. *Review of Economic Dynamics*.

TAUCHEN, G. (1986). Finite state markov chain approximations to univariate and vector autoregressions. *Economic Letters*.

UHLIG, H. (1996). A law of large numbers for large economies. *Economic Theory*.

WINBERRY, T. (2016). Lumpy investment, business cycles, and stimulus policy.

YAARI, M. E. (1976). A law of large numbers in the theory of consumer's choice under uncertainty. *Journal of Economic Theory*.

ZWICK, E. and MAHON, J. E. (2017). Tax policy and heterogeneous investment behavior. *American Economic Review*.

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