Fear, Real Indeterminacy and Policy Responses

Paymon Khorrami (Duke Fuqua)
Fernando Mendo (PUC Rio)

18 June 2023

Motivation

- New Keynesian model is a leading framework in macroeconomics
  ⇒ inflation, aggregate demand stimulus, monetary policy, among others
- This model is plagued by well-known equilibrium multiplicities
  ⇒ equilibrium selection key for positive and normative predictions
- Monetary and fiscal policy central to equilibrium selection but no consensus on a single approach.

  ⇒ Active monetary policy: interest rate rule that responds sufficiently aggressively to output and inflation (Taylor principle)

  ⇒ Active fiscal policy: tax/spending policy that does not accommodate changes in public debt services

New light on this controversy

- Study textbook New Keynesian model but no linearization around its steady state to study local dynamics.
  ⇒ study its true nonlinear, stochastic form and global dynamics.
- New insights about multiplicity and policies that can eliminate/trim it.

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*

  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit

  $\Rightarrow$ beliefs about inflation dynamics not key

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*
  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit
  $\Rightarrow$ beliefs about inflation dynamics not key
- Monetary policy rule that only responds to inflation and output is unable to trim these new equilibria, *regardless of aggressiveness*.

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*
  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit
  $\Rightarrow$ beliefs about inflation dynamics not key
- Monetary policy rule that only responds to inflation and output is unable to trim these new equilibria, *regardless of aggressiveness*.
- Monetary rule targeting volatility (risk premia) can restore determinacy ...

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*

  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit

  $\Rightarrow$ beliefs about inflation dynamics not key
- Monetary policy rule that only responds to inflation and output is unable to trim these new equilibria, *regardless of aggressiveness*.
- Monetary rule targeting volatility (risk premia) can restore determinacy ... but *unfeasible with a lower bound* to interest rates

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*
  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit
  $\Rightarrow$ beliefs about inflation dynamics not key
- Monetary policy rule that only responds to inflation and output is unable to trim these new equilibria, *regardless of aggressiveness*.
- Monetary rule targeting volatility (risk premia) can restore determinacy ... but *unfeasible with a lower bound* to interest rates
- Active fiscal policy can kill self-fulfilling volatility
  $\Rightarrow$ FTPL approach contributes to equilibrium selection beyond disciplining price level dynamics

This paper

- Revisit the canonical New Keynesian model and study the *global dynamics* of its fully *stochastic nonlinear* version
- Unveil a new class of equilibria characterized by *self-fulfilled beliefs about volatility*
  $\Rightarrow$ real (not nominal) phenomenon, survive in the price rigid limit
  $\Rightarrow$ beliefs about inflation dynamics not key
- Monetary policy rule that only responds to inflation and output is unable to trim these new equilibria, *regardless of aggressiveness*.
- Monetary rule targeting volatility (risk premia) can restore determinacy ... but *unfeasible with a lower bound* to interest rates
- Active fiscal policy can kill self-fulfilling volatility
  $\Rightarrow$ FTPL approach contributes to equilibrium selection beyond disciplining price level dynamics
- Favors equilibrium selection criteria linked to fiscal rather than monetary policy

Related literature

New Keynesian indeterminacies:
Sargent and Wallace [1975], Benhabib et al. [2001], Benigno and Fornaro [2018], among many others.

Lee and Dordal i Carreras [2023] also studies a nonlinear NK model with volatility driving the multiplicity, and argue that "active" Taylor rules do not restore determinacy.
$\Rightarrow$ Only entertain equilibria local to steady state
$\Rightarrow$ Do not explore a lower bound on interest rates or active fiscal policy.

Fiscal theory of the price level (FTPL):
Leeper [1991], Sims [1994], Woodford [1994], Woodford [1995], Brunnermeier et al. [2020], Bassetto and Cui [2018], Cochrane [2023].

Overview

- Canonical New Keynesian model (without fundamental shocks)
- Deterministic analysis
  - Conventional indeterminacy analysis
  - Aggressive Taylor rule restore uniqueness
- Stochastic analysis
  - A new class of equilibria
  - Aggressive Taylor rule cannot restore uniqueness
  - Risk targeting: can restore uniqueness (if no lower bound on i_t)
- Active fiscal (FTPL)
  - Simple example: primary surplus proportional to output
  - Beyond simple fiscal rule

Model

Canonical New Keynesian model

- Uncertainty. For simplicity, only an extrinsic Brownian motion Z_t.
- Household's preferences

$$\mathbb{E}\Big[\int_0^\infty e^{-\rho t}\Big(\log(C_t) - \frac{L_t^{1+\varphi}}{1+\varphi}\Big)dt\Big]$$

- Technology: $Y_t = L_t$.
    - Continuum of intermediate-goods firms with linear labor technology.
    - Elasticity of substitution across intermediate goods is constant.
- Price setting
    - Monopolistic competition among intermediate good firms.
    - Sticky prices a la Rotemberg (quadratic adjustment costs).
    - Profits rebated lump-sum to household.
    - Competitive final-goods sector.

Canonical New Keynesian model

- Inflation and output gap: $\pi_t := \dot{P}_t / P_t$ and $x_t := \log(Y_t/Y^*)$ where Y^* is flexible-price output and P_t the price level.

$$dx_t = \mu_{x,t} dt + \sigma_{x,t} dZ_t$$
$$d\pi_t = \mu_{\pi,t} dt + \sigma_{\pi,t} dZ_t$$

  for some $\mu_x, \sigma_x, \mu_\pi, \sigma_\pi$ to be determined in equilibrium.

- Monetary policy: Taylor rule

$$\iota_t = \bar{\iota} + \Phi(x_t, \pi_t) \tag{MP}$$

  for some target rate $\bar{\iota}$ and some response function that satisfies $\Phi(0,0) = 0$

- Complete financial markets.
  - Real stochastic discount factor M_t induced by real interest rate $r_t = \iota_t - \pi_t$ and price of risk h_t associated to risk Z_t.
  - Risk-free bond in zero net supply.

Equilibrium

An *equilibrium* is processes $(C_t, Y_t, L_t, B_t, M_t, W_t, P_t, \iota_t, r_t, \pi_t)_{t\geq 0}$, such that

(i) Taking (M_t, W_t, P_t) as given, consumers choose $(C_t, L_t)_{t\geq 0}$ to maximize objective subject to their lifetime budget and No-Ponzi constraints

$$\Pi_0 + \mathbb{E}\Big[\int_0^\infty M_t \frac{W_t L_t}{P_t} dt\Big] \geq \mathbb{E}\Big[\int_0^\infty M_t C_t dt\Big]$$
$$\lim_{T\to\infty} M_T \frac{B_T}{P_T} \geq 0,$$

where Π represents the real present-value of producer profits and B represents the bond-holdings of the consumer.

(ii) Firms set prices optimally.

(iii) Markets clear, namely $C_t = Y_t = L_t$ and $B_t = 0$.

(iv) The central bank follows the interest rate rule.

Consumption-savings decision (Euler equation)

$$\mu_{x,t} = \iota_t - \pi_t - \rho + \frac{1}{2}\sigma_{x,t}^2 \tag{IS}$$

- The volatility term is assocated to:
  - precautionary savings due to prudence $u''' > 0$
  - the non-linear transformation: $x_t = \log(C_t/C^*)$
- If $u''' = 0$ and we study C_t dynamics directly $\Rightarrow$ no volatility term
- Literature usually disregards $\sigma_{x,t}^2$ by linearizing the IS equation

$$\mu_{x,t} = \iota_t - \pi_t - \rho \tag{linear IS}$$

Price setting behavior (Phillips curve)

$$\mu_{\pi,t} = \rho\pi_t - \kappa\left(\frac{e^{(1+\varphi)x_t} - 1}{1+\varphi}\right) \tag{PC}$$

- κ associated to degree of price flexibility ($\kappa \to 0$ fully sticky prices).
- Fully flexible prices ($\kappa \to \infty$) implies $x_t = 0 \ \forall t \geq 0$.

- Linearized version

$$\mu_{\pi,t} = \rho\pi_t - \kappa x_t \tag{linear PC}$$

Equilibrium characterization

Lemma

Suppose processes $(x_t, \pi_t, \iota_t)_{t \geq 0}$ *satisfy (IS) + (PC) + (MP).*

- *If* $|x_t| < \infty$ *almost-surely for all finite horizon* $\Rightarrow$ *an equilibrium*
- *If* $|x_t| = \infty$ *for some finite* t *with positive probability* $\Rightarrow$ *not an equilibrium*

- $x_t = \infty$ for finite t not compatible with household maximization.

Equilibrium characterization

Lemma

Suppose processes $(x_t, \pi_t, \iota_t)_{t\geq 0}$ satisfy (IS) + (PC) + (MP).

- *If $|x_t| < \infty$ almost-surely for all finite horizon $\Rightarrow$ an equilibrium*
- *If $|x_t| = \infty$ for some finite t with positive probability $\Rightarrow$ not an equilibrium*

- $x_t = \infty$ for finite t not compatible with household maximization.
- Fully flexible prices ($\kappa \to \infty$) benchmark
  - Unique outcomes for real variables. $x_t = 0$ and $r_t = \rho \; \forall t \geq 0$.
  - Inflation adjusts to ensure the proper real rate holds $\pi_t = i_t - \rho$.

Equilibrium characterization

Lemma

Suppose processes $(x_t, \pi_t, \iota_t)_{t\geq 0}$ satisfy (IS) + (PC) + (MP).

- *If $|x_t| < \infty$ almost-surely for all finite horizon $\Rightarrow$ an equilibrium*
- *If $|x_t| = \infty$ for some finite t with positive probability $\Rightarrow$ not an equilibrium*

- $x_t = \infty$ for finite t not compatible with household maximization.
- Fully flexible prices ($\kappa \to \infty$) benchmark
  - Unique outcomes for real variables. $x_t = 0$ and $r_t = \rho\ \forall t \geq 0$.
  - Inflation adjusts to ensure the proper real rate holds $\pi_t = i_t - \rho$.
- Sticky prices and equilibrium selection with monetary policy
  - Recall $\iota_t = \bar{\iota} + \Phi(x_t, \pi_t)$ with $\Phi(0,0) = 0$.
  - With target $\bar{\iota} = \rho$ and any function Φ, $x_t = \pi_t = 0$ is an equilibrium.
  - Can we find a function $\Phi(x_t, \pi_t)$ such that this is the only equilibria?

Deterministic Equilibria ($\sigma_x \equiv 0$)

Consider only equilibria without volatility $\sigma_x \equiv 0$.

Review: conventional indeterminacy in NK model

- Consider the linear version of the Phillips curve and specialize to a linear MP rule

$$\iota_t = \rho + \phi_x x_t + \phi_\pi \pi_t. \qquad \text{(linear MP)}$$

- The system becomes

$$\begin{bmatrix} \dot{x}_t \\ \dot{\pi}_t \end{bmatrix} = \mathcal{A} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix}, \quad \text{where} \quad \mathcal{A} := \begin{bmatrix} \phi_x & \phi_\pi - 1 \\ -\kappa & \rho \end{bmatrix}.$$

- The eigenvalues of $\mathcal{A}$ are both strictly positive, and the system unstable, if $\phi_x > 0$ and $\phi_\pi > 1$. The continuous-time version of conditions in Blanchard and Kahn [1980].

Deterministic Equilibria ($\sigma_x \equiv 0$)

- However, even when the Taylor rule is destabilizing, the outcome is still a valid equilibrium, as stressed by Cochrane [2011].
  - Nothing about the model rules out asymptotic explosions.
  - No matter how large the central bank picks ϕ_x and ϕ_π, the explosion will always only be at the infinite horizon.
  - Usually, explosive solutions are ruled out by an ad-hoc assumption. We do not do this. [But this is not paper about explosive solutions]

Proposition (Linear deterministic analysis)

Under (linear PC) and (linear MP), any (x_0, π_0) is consistent with a deterministic equilibrium.
If $\phi_x > 0$ and $\phi_\pi > 1$, then all deterministic equilibria explode asymptotically, except for the one with $(x_0, \pi_0) = (0, 0)$.

- We extend the analysis for the case of the non-linear Phillips curve. Results are the same.

Deterministic Equilibria ($\sigma_x \equiv 0$)

Trimming equilibria: a very active Taylor rule

- A linear Taylor rule does not ensure a unique equilibrium.
- Uniqueness requires an additional policy that pledges to "blow up the world" if proposed equilibrium is not followed (Cochrane [2011]).
- Such nuclear option can be a very aggressive Taylor rule, e.g.,

$$i_t = \rho + \frac{\phi_x}{2}(e^{x_t} - e^{-x_t}) + \pi$$

  which log-linearized form coincides with the linear Taylor rule studied.
- In this case,

$$x_t = \log\left(\frac{1 - Ke^{\phi_x t}}{1 + Ke^{\phi_x t}}\right)$$

  where $K = \frac{1-e^{x_0}}{1+e^{x_0}}$. This process diverges in *finite time* for any $x_0 \neq 0$: it explodes at time $T = -\phi_x^{-1}\log(|K|)$.

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

- A new class of equilibria emerge
  $\Rightarrow$ includes non-explosive, stationary equilibria

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

- A new class of equilibria emerge
  $\Rightarrow$ includes non-explosive, stationary equilibria
- Aggressive Taylor rule restore uniqueness?

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

- A new class of equilibria emerge
  $\Rightarrow$ includes non-explosive, stationary equilibria
- Aggressive Taylor rule restore uniqueness? No!

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

- A new class of equilibria emerge
  $\Rightarrow$ includes non-explosive, stationary equilibria
- Aggressive Taylor rule restore uniqueness? No!
- An extended Taylor rule that includes risk targeting can

Stochastic analysis

Deterministic analysis

- Linear Taylor rule $\Rightarrow$ deterministic multiplicity
- Aggressive Taylor rule (or no explosion clause) $\Rightarrow$ eliminates equilibrium multiplicity

What happens when we allow for $\sigma_x \neq 0$?

- A new class of equilibria emerge
  $\Rightarrow$ includes non-explosive, stationary equilibria
- Aggressive Taylor rule restore uniqueness? No!
- An extended Taylor rule that includes risk targeting can
  ... but not feasible with a lower bound on the interest rate.

A New Class of Sunspot Equilibria

- Assume prices are permanently rigid $\kappa \to 0$.
  - Focus on real indeterminacy rather than inflation indeterminacy.
  - Simple analytic analysis

- Consider the aggressive policy rule that restore uniqueness in the deterministic analysis

- An equilibrium is a process for x_t that satisfies (optimal consumption saving decision)

$$\mu_{x,t} = \phi_x(e^{x_t} - e^{-x_t}) + \frac{1}{2}\sigma_{x,t}^2$$

  and remains finite for all $t \geq 0$.

- Key: agents can have arbitrary beliefs about risk σ_x^2 and those influence optimal behavior (precautionary savings)
  - Beliefs about risk can bring "stability" to the system, e.g., large volatility when x_t large and negative

A New Class of Sunspot Equilibria

Simple example with transitory volatility

- Recall $y_t = e^{x_t} y^*$, so $x_t \to -\infty \iff y_t \to 0$
- Let agents believe / coordinate the following process for volatility

$$\sigma_x^2 = \begin{cases} \left(\frac{\nu}{y}\right)^2 + \phi_x \frac{1-y^2}{y}, & \text{if } y < 1; \\ 0, & \text{if } y \geq 1. \end{cases}$$

- Then

$$dy_t = \begin{cases} \frac{\nu^2}{y_t} dt + \sqrt{\nu^2 + \phi_x y_t (1 - y_t^2)} dZ_t, & \text{if } y_t < 1; \\ \phi_x (y_t^2 - 1) dt & \text{if } y_t \geq 1. \end{cases}$$

$\Rightarrow y_t > 0$ always (behaves as a Bessel(3) process as $y_t \to 0$)

- Family of equilibria indexed by $y_0 \in (0, 1)$ and $\nu > 0$
  - y_t bounces around the region $(0, 1)$ and eventually reaches $y_t = 1$ where is remains forever $\Rightarrow$ **transitory** sunspot volatility

A New Class of Sunspot Equilibria

Simple example with permanent volatility

- Slight modification to agents beliefs / coordination

$$\sigma_x^2 = \begin{cases} \left(\frac{\nu}{y}\right)^2 + \phi_x \frac{1-y^2}{y}, & \text{if } y < 1-\delta; \\ 0, & \text{if } y \geq 1-\delta. \end{cases}$$

- Then

$$dy_t = \begin{cases} \frac{\nu^2}{y_t} dt + \sqrt{\nu^2 + \phi_x y_t (1 - y_t^2)} dZ_t, & \text{if } y_t < 1-\delta \\ \phi_x (y_t^2 - 1) dt & \text{if } y_t \geq 1-\delta. \end{cases}$$

  $\Rightarrow y_t > 0$ always still

- Family of equilibria indexed by $y_0 \in (0,1)$, $\nu > 0$, $\delta \in (0,1)$
  - $y_0 \in (0, 1-\delta]$ bounces around this volatile region forever (visits $y_t = 1-\delta$ frequently)
  - $y_0 \in (1-\delta, 1)$ exits this deterministic region in finite time
    $\Rightarrow$ **permanent** sunspot volatility

A New Class of Sunspot Equilibria

Simple example with permanent volatility

- Stationary, non-explosive equilibria

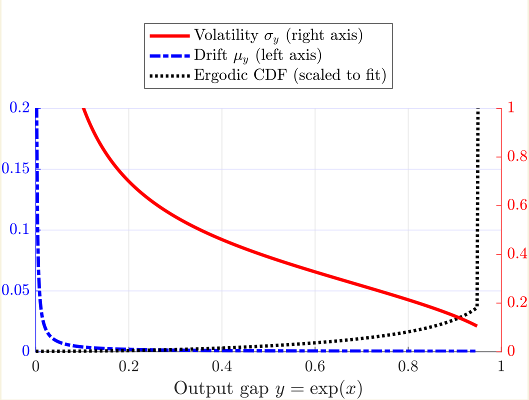

Figure: The resulting stationary CDF features a mass point at $y = 1 - \delta$. Parameters: $\rho = 0.02$, $\nu = 0.02$, $\delta = 0.05$, $\phi_x = 0.1$.

A New Class of Sunspot Equilibria

General case for the rigid price limit

- Equilibrium dynamics

$$dx_t = (\iota(x_t) - \rho + \sigma_{x,t}^2)dt + \sigma_{x,t}dZ_t$$

- Independently of interest rate rule $\iota(x_t)$, beliefs about volatility $\sigma_{x,t}$ can always stabilize the system for $x_t < 0$
  $\Rightarrow$ prevent $x_t \to -\infty$ in finite time.

Proposition (Multiplicity for any Taylor rule)

Suppose prices are rigid ($\kappa \to 0$).
For any Taylor rule $\iota(x)$ that is increasing in x, there exist a continuum of sunspot equilibria indexed by x_0 and the volatility function $\sigma_x(x)$

A New Class of Sunspot Equilibria: key features

Key feature: impact of volatility over consumption-saving decision

- Channel: precautionary savings
- Linearized version of the IS = no precautionary savings

$$dx_t = (\iota(x_t) - \pi_t - \rho)dt + \sigma_{x,t} dZ_t$$

  - analysis similar to the deterministic case (just adding some noise)
  - a sufficiently aggressive Taylor rule $\iota(x)$ restores equilibrium uniqueness!

$\Rightarrow$ it is not volatility per se but its role in consumption-saving decision

Key feature: demand determined output

- Phillips curve

$$\mu_{\pi,t} = \rho \pi_t - \kappa \Big(\frac{e^{(1+\varphi)x_t} - 1}{1+\varphi} \Big)$$

- Flexible price benchmark ($\kappa \to \infty$) implies $x_t = 0$ always.
- No role for beliefs about volatility

Risk targeting

- Extended monetary policy rule

$$\iota_t = \rho + \Phi(x_t, \pi_t) - \alpha(x)\sigma_{x,t}^2.$$

- Output dynamics

$$dy_t = y_t\Big[\Phi(x_t, \pi_t) - \pi_t + (1 - \alpha(x_t))\sigma_{x,t}^2\Big]dt + y_t\sigma_{x,t}dZ_t,$$

- $\alpha(x) = 1$ eliminates the role of volatility in consumption-saving decision $\Rightarrow$ aggressive Taylor rule Φ can restore uniqueness
- Consider $\alpha(x) := \alpha_- \mathbf{1}_{\{x<0\}} + \alpha_+ \mathbf{1}_{\{x>0\}}$.
  If $\alpha_- > 1 > \alpha_+$, volatility helps destabilize the system
  $\Rightarrow$ aggressive Taylor rule Φ can restore uniqueness

Proposition (Uniqueness with risk targeting)

Suppose prices are rigid ($\kappa \to 0$). With sufficiently strong risk premium targeting and sufficiently aggressive responsiveness to the output gap, the modified Taylor rules ensures that the unique equilibrium is $x_t = 0$.

Feasibility of aggressive Taylor rules: credible threats?

Are the extreme rules proposed to trim equilibria credible?

- “blow up the world” nuclear threats are generically not credible.
- the central bank commits to blow up the economy (in finite time) if the desired equilibrium path is not follow
- discussed by Cochrane [2011]

Feasibility of aggressive Taylor rules: Lower bound on ι_t

- Aggressive monetary rules that restore equilibrium uniqueness imply that $\iota_t \to -\infty$ as $x_t \to -\infty$
- Consider the price-rigid case and impose a lower bound

$$dx_t = \left[\underline{\iota} - \rho + \frac{1}{2}\sigma_{x,t}^2\right]dt + \sigma_{x,t}dZ_t, \quad \text{when} \quad x_t < 0$$

- For deterministic equilibria ($\sigma_x = 0$)
  - clearly x_t cannot diverge to $-\infty$ in finite time.
  - for $\underline{\iota} < \rho$, all equilibria with $x_0 < 0$ are explosive $\lim_{t\to\infty} x_t = -\infty$
  - exogenous assumption ruling out explosive equilibria $\Rightarrow$ uniqueness
- For stochastic equilibria ($\sigma_x \neq 0$)
  - Volatility helps to stabilize the system (increases the drift) but opens the door for a sequence of negative shocks.
  - Constant volatility $\Rightarrow x_t$ cannot diverge in finite time.
  - There are volatility process that induce non-explosive dynamics.
  - Exogenous assumption ruling out explosive equilibria $\not\Rightarrow$ uniqueness

Fiscal policy

- Previously, we abstract from fiscal policy.
- Let $S_t := \tau_t - \xi_t$ be the real primary deficit of the government and B_t the value of nominal government debt

$$\dot{B}_t = \iota_t B_t - P_t S_t$$

- Present value formula for government debt

$$\frac{B_t}{P_t} = \mathbb{E}_t\Big[\int_t^\infty \frac{M_u}{M_t} S_u du\Big], \qquad \text{(GD)}$$

  since TVC must hold for the representative agent.
- For now, let taxes τ_t and transfers ξ_t be lump-sum.
  $\Rightarrow$ equilibrium: same equations as before (IS, PC, MP) plus (GD)

- Passive fiscal: gov't chooses $\{S_t\}_{t\geq 0}$ taking (GD) as a constraint.
  $\Rightarrow$ all previous results hold (gov't debt has no role in eq. selection)

Fiscal policy: active fiscal

Active fiscal (FTPL): gov't chooses $\{S_t\}_{t\geq 0}$ independently of (GD)
$\Rightarrow$ Price level P_t must adjust to ensure gov't debt valuation holds.

Active fiscal equilibrium selection: a first example

- Consider $S_t = \bar{s}Y_t$ for some $\bar{s} > 0$
- Gov't debt valuation

$$\frac{B_t}{P_t} = \bar{s}\mathbb{E}_t \int_t^\infty e^{-\rho(u-t)} \frac{Y_t}{Y_u} Y_u du = \rho^{-1} \bar{s} e^{x_t} Y^*.$$

- Since B_t/P_t evolves locally deterministically, x_t also evolves locally deterministically, i.e., $\sigma_{x,t} = 0$.
    - $\sigma_\pi \neq 0$ possible but without real effects
    - holds for any monetary policy rule and even in the rigid-price limit!

Fiscal policy: active fiscal

Active fiscal equilibrium selection: a first example

- How active fiscal policy trims equilibria?
  - Disciplines beliefs about $\sigma_{x,t}$: must be consistent w/ debt valuation.
  - Aggregate demand adjusts such that (GD) holds.

- Role of monetary policy
  - Equilibrium will be deterministic but also depends on monetary policy.
  - Active monetary (e.g., linear Taylor rule with $\phi_x > 0$ and $\phi_\pi > 0$): equilibrium explodes asymptotically.
  - Passive monetary: equilibrium stable and non-explosive.

Beyond the simple fiscal rule.

Does active fiscal prevents sunspot vol. only with $S_t = \bar{s} Y_t$? No!
Result extends to ..

- Surplus-to-GDP ratio time-varying but exogenous
- Surplus-to-GDP ratio that depends on output gap (fully rigid prices)
- Long-term debt

Fiscal policy: active fiscal

Surplus to GDP time-varying but exogenous

- Let $S_t = s_t Y_t$ where

$$ds_t = \lambda(\bar{s} - s_t)dt + \sigma_{s,t} dZ^s_t,$$

  where Z^s is independent of the sunspot shock Z, and $\sigma_{s,t}$ is an arbitrary potentially stochastic volatility. Then,

$$\frac{B_t}{P_t} = \rho^{-1} e^{x_t} Y^* \Big[\frac{\rho}{\rho + \lambda} s_t + \frac{\lambda}{\rho + \lambda} \Big]$$

- Since B_t/P_t has no loading on the sunspot shock Z_t, and neither does s_t, we have that x_t is independent of Z_t.

Fiscal policy: active fiscal

Surplus to GDP that depends on output gap (fully rigid prices)

- Let $S_t = \zeta(x_t)Y_t$ with $\zeta(\cdot) > 0$. Then,

$$\frac{B_t}{P_t} = \rho^{-1} e^{x_t} Y^* \mathbb{E}_t \Big[\int_t^\infty \rho e^{-\rho(u-t)} \zeta(x_u) du \Big]$$

- With fully rigid prices, the expectation is a function of only x_t

$$\frac{B_t}{P_t} = \rho^{-1} e^{x_t} Y^* f(x_t)$$

- Again, we have that x_t is independent of Z_t (except for knife-edge cases).

Final thoughts

- We uncover a new type of fear-driven equilibria in NK models (i.e., self-fulfilled volatility)
- Benefit to working in continuous time and analyzing "globally"
- Monetary policy struggle as a equilibrium selection mechanism
- Using various examples, we have shown that active fiscal policy can kills real sunspot volatility
- Can unconventional monetary policy help? Maybe helps coordinating beliefs.