Central Bank Balance Sheet Policies: A Comparative Statics Approach*

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Abstract

This paper presents an equilibrium asset-demand system amenable for comparative-statics analysis of changes in the size and composition of central bank balance sheets. The analysis produces a set of sufficient statistics—key financial ratios and elasticities of asset flows with respect to interest rates and the price level. In frictionless financial markets without nominal rigidities, changes in the size or composition of the balance sheet size has no real effects. However, when financial markets are segmented and nominal rigidities exist, the estimated elasticities identify the strength of different channels by which changes in the size have effects. In turn, the composition of the central bank balance sheet is irrelevant unless (a) intermediaries are heterogeneously exposed to liquidity risk or (b) assets differ in risk or collateral properties. We estimate and calibrate these elasticities for the European Union and decompose the transmission of quantitative easing policies.

Key Words: Monetary Policy, Open-Market Operations

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1 Introduction

Ben Bernanke, the former US Fed chairman, famously jested that "The problem with QE is it works in practice, but it doesn't work in theory."[1] Since the time of that quote, macroeconomics has identified multiple channels under which central-bank balance-sheet policies impact the real economy. These mechanism operate by either altering the supply of collateral, effecting liquidity and credit risk exposures, relaxing financial constraints, or by signaling future fiscal consequences. At this stage, the challenge lies not in formulating theory, but in understanding the relevance of these different channels in terms of the real effects of the size and composition of open-market operations (OMO).

Humor aside, Bernanke's quote motivates the need to develop different frameworks to understand the relevance of the different transmission mechanisms of OMO. Indeed, real-world macroeconomic policy is about the use of range of different fiscal and monetary policy instruments. Among all, the area of greatest innovation in macroeconomic policy over the last decades, no doubt, has been the use of central bank balance sheet policies. For example, see the Figure 1 below which tracks the scale and composition of the balance sheets of major banks.[2] Yet, balance sheet policies have been carried out instinctively, without a well established framework that captures the relevance of different channels. We understand that the slope of the Phillips curve and the elasticity of intertemporal substitution determine the strength of the interest-rate channel, but we don't have a counterpart for OMO. This paper presents a new framework to produce such counterparts. We present an equilibrium model with a rich asset-demand system and heterogenous financial intermediaries. The framework is amenable for the comparative-statics analysis. The comparative-statics analysis delivers sufficient statistics that dictate the size and composition effects of balance sheet policies. These sufficient statistics are a set of key asset-market elasticities and bank financial ratios which, in turn, inform us about the strength of the different transmission channels of OMO.

The Framework. Our approach differs from the typical dynamic models used to analyze monetary policy. These models typically focus on a single friction and emphasize the equilibrium dynamics. The direct effects of policies are often obfuscated through feedback

[1] The quote is take from the transcript from a conversation at the Brookings institution in 2016 found here.

[2] In response to the 2008 financial crisis, the Federal Reserve under Bernanke lowered interest rates to near zero while continuously purchasing private assets in an attempt to preserve financial stability and promote lending. At the time, few models existed to analyze these unprecedented balance sheet policies. Most macro models assumed central banks control rates or money rather than banking system conditions. This theoretical gap was concerning because banks reacted by amassing reserves without expanding lending, contrary to policy aims. Many predicted high inflation a that time, though such fears where largely ignored.

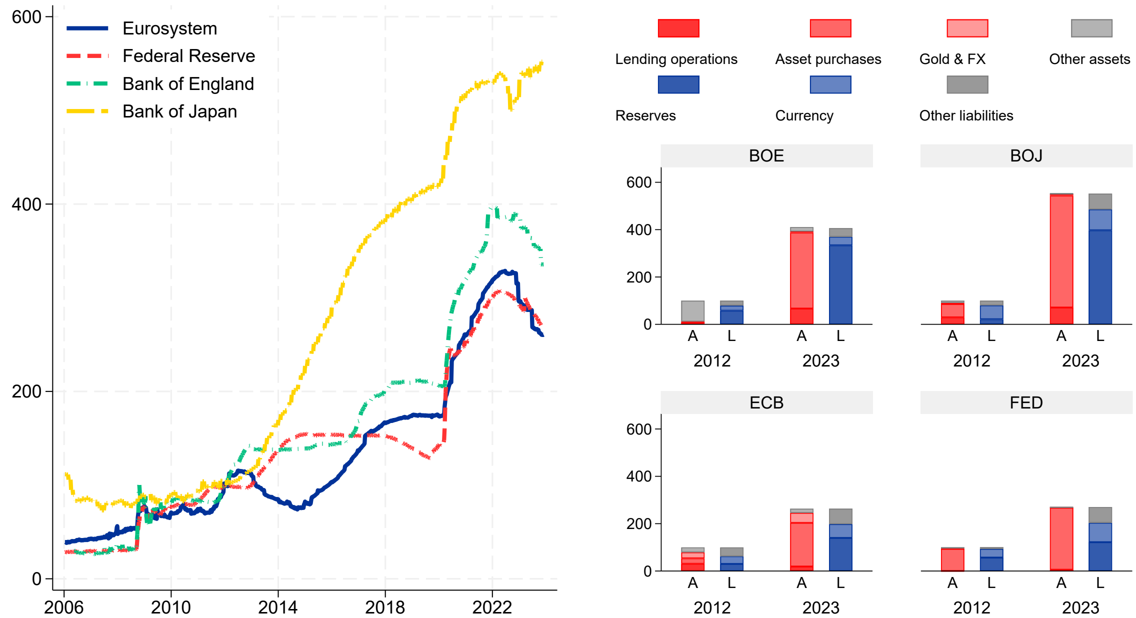

Figure 1: Total Asset Holdings of Major Central Banks

effects on expectations about future inflation, asset returns, or fiscal implications. Instead, our approach is to take the opposite route. We develop a static, microfounded framework that incorporates multiple frictions and handles heterogeneity across agents and asset properties. While the model admits multiple periods, there are no expectational feedback effects. This feature allows us to isolate the static effects of OMO and produce comparative statics.

The starting point of our framework is an asset demand system of heterogeneous loan demand and deposit supply schedules—in the spirit of xxx. We enrich that setting by incorporating an explicit set of heterogenous banks that intermediate the funds from the funding supply schedules to the loan demand schedules. Loan demands and deposit supplies are heterogeneous in their cross-bank-interest-rate elasticities. Moreover, loans can differ by their elasticities with respect to the price level. These properties capture loan and deposit market segmentation and exposures to inflation.

Banks face several real-world frictions. They are exposed to liquidity risk—the risk of inability to settle deposit outflows with their holdings central bank liabilities, reserves. We capture this through a simplified version of the withdrawal risk mechanism in Bianchi and Bigio (2020). As in Bianchi and Bigio (2020), a precautionary demand for reserves depends on liquidity frictions in the interbank market. Banks also face undiversifiable loan credit risk which is priced based on an exogenous discount factor. In addition, banks cannot freely raise new equity and face constraints on their leverage ratios, a simplified version of the funding frictions Gertler and Karadi (2011). Asset-quality heterogeneity is captures by differences in the pledgeability of loans as interbank market collateral or intrinsic credit risk. The banks' risk and liquidity management, induce loan supply, deposit demand, equity issuance, and reserve demand schedules that complement the asset demand system.

The central bank influences the equilibrium in the asset markets using its policy tools. It sets interest rates on discount window lending. By adjusting its policy rates, the central bank alters the costs of reserve shortfalls. We assume real interest rates are fixed over time, and that the central bank takes actions in futures days to keep inflation expectations anchored. The central bank also conducts open market operations (OMO), i.e. their purchases of loans for reserves. OMO shifts the real quantity of reserves in the banking system, thereby influencing the equilibrium in the rest of the asset demand system. With anchored inflation expectations, we can isolate the immediate impacts of OMO squarely. In general, OMO can influence lending and economic activity through multiple channels. But under certain specifications, OMO may have no real effects. We use our model to delineate the precise conditions under which OMO alters the supply and allocation of credit.

An equilibrium price level, a menu of interest rate rates, loans, deposits, and dividends solve the clearing conditions of the asset demand system. We perturb the equilibrium by introducing perturbations to the central bank's balance sheet. This allows us to generate comparative statics showing how the endogenous equilibrium variables respond to small OMO. The sensitivity of these responses depend on a number of sufficient statistics representing financial market frictions and nominal rigidities embedded in our model. The framework determines the sufficient statistics that govern the strength of real effects and nominal effects from asset purchases.

We analyze two potential effects of OMO: scale effects and composition effects. Scale effects refer to how the size of asset purchases impacts lending and economic outcomes, regardless of the composition of assets purchased. Composition effects regard how purchases of different assets have different effects, holding the size of purchases fixed.

Scale effects. We first examine scale effects abstracting away from heterogeneity which lets us state clear neutrality results. First, in a liquidity trap with banks satiated with reserves, OMO have no effects if they don't change the risk absorption by the central bank. This occurs because loans and reserves are perfect substitutes and, in fact, there are no price effects either. This demonstrates the importance of a downward-sloping demand curve for reserves. Second, with freely flexible bank equity, OMO also does not matter since banks can costlessly substitute equity for deposits. This highlights the need for an imperfectly elastic bank equity supply curve reflecting financing frictions.

However, we show reserve demand elasticity and equity financing rigidities alone are insufficient for the non-neutrality of OMO. Some form of price or wage stickiness is also needed, i.e. real loan demand or the bank equity must depend on the aggregate price level: Whereas real reserve balances ease liquidity and risk management for banks, translating this into real outcomes requires nominal rigidities to shift the asset-demand system. Otherwise, OMO operations only change the nominal size of the central bank's balance sheet, but no the real value of their liabilities. This neutrality holds even if the central bank attempts to buy risky assets.

With both financial market imperfections and nominal rigidities, the scale of OMO can affect lending and output. OMO carry real effects by tilting the liquidity and risk-management trade off of banks, with feedback effects dictated by the price elasticities. The degree of these real effects depends on the sufficient statistics summarized, the collection of key elasticities of embedded in the demand system and banks financial ratios.

Composition effects. There are may be no composition effects if there are scale effects. This happens when the effects of OMO are the same regardless of the composition of asset purchases. Naturally, to allow for composition effects, we must consider heterogeneity. Composition effects emerge from certain different forms of heterogeneity.

We first examine heterogeneity across agents: across banks and the sectors that demand loans and supply deposits. With risk and collateral properties across assets, but differences across banks' risk tolerances, balance sheet capacities, and liquidity risk exposures, we find a novel neutrality result: No composition effects arise except when banks differ in liquidity their risk exposures. Intuitively, which banks receive the liquidity influx from asset purchases affects interbank market stress levels based on bank risk. Thus, the relevant margin for composition effects is bank liquidity risk exposure. This is because only with that form agent heterogeneity does the asset composition of OMO affect the tightness the reserve demand schedules of banks. This result is important because it tells policy makers that OMO cannot stimulate particularly depressed sectors of the economy simply buy buying their bonds. We must consider the liquidity risk exposure and portfolios of different banks.

We first examine heterogeneity across banks and loans or deposits elasticities. With identical asset risk and collateral attributes, but differences across banks in risk tolerance, balance sheet capacity, and liquidity risk exposure, we find a novel neutrality result: Composition effects only emerge when banks have heterogeneous liquidity risk exposures. Intuitively, which specific banks receive the liquidity influx from an asset purchase impacts interbank market stress based on bank liquidity risk profiles. Thus, the key margin enabling composition effects is thus cross-bank differences in liquidity risk because this ultimately affects the reserve demand of different banks. An important implication is that the central bank cannot necessarily stimulate particular depressed sectors by purchasing loans to those sectors. Instead, we must consider how liquidity will distribute across banks and the sectoral portfolio exposures of banks respond to the distribution of liquidity.

We also examine the potential for composition effects stemming from heterogeneous asset attributes. Specifically, we analyze how differences in asset risk profiles and collateral values interact with central bank purchases. OMO focused on riskier assets can have larger economic impacts as long as the central bank absorbs the underlying credit risk. This result highlights how the effects of OMO differ between permanent outright asset purchases and short-term repurchase agreements, for example. The sensitivity of the composition effects resulting from risk absorption depends on bank attitudes toward risk and regulatory constraints around risk-taking. Additionally, we show that buying assets with lower collateral values has a disproportionate influence by affecting liquidity premiums in interbank markets.

A key takeaway from the compositional effects suggest that OMO should focus on the properties of the asset classes that are targeted and less so on the heterogeneity across sectors.

Quantitative Evaluation. We finally provide an example of a quantitative application of the methodology. First, we estimate a time-varying reserve demand function in Europe to empirically assess scale effects from purchasing riskless, but illiquid assets. We use a combination of indirect inference and cross-sectional bank data to obtain our sufficient statistics. With the sufficient statistics, we construct counterfactual time series regarding European Central Bank (ECB) OMO, taking as given the changes in the reserve demand elasticities changed. Finally, we study compositional effects by considering ECB purchases of either riskier loan portfolios or safe assets concentrated among highly liquidity exposed banks. This quantitative exercise generates model predictions about how much ECB credit easing policies can stimulate lending or translate to prices.

Connection with the Literature

Comparative Statics for Macroeconomic Policy Analysis. Outside of monetary policy, comparative statics have a tradition in macroeconomic policy evaluation. In international trade, Costinot and Rodriguez-Clare (2014) or Caliendo and Parro (2015) use build static models and estimate trade elasticities to the equilibrium effects of tariff reductions. Chetty, Guren, Manoli, and Weber (2013) use microestimates of extensive margin labor-supply elasticities to quantify the labor supply impacts of tax reforms. Baqaee and Farhi (2020) produce comparative statics about firm level distortions, which they combine firm-level data with input-output tables, to compute counterfactual effects on output. xxx use the same methodology to evaluate the role of oil sanctions. Comparative statics are not often used to evaluate monetary policy because the dynamic nature of these models complicates their computation. Here we isolate the dynamic feedback through expectations and allow for rich cross-sectional heterogeneity. The lack of dynamics is a drawback of our framework, as it is in trade, public finance, or input output models. Yet, there is no reason to think that comparative statics are not as useful to analyze monetary policy as they are in other areas.

Channels to Evaluate OMO. This paper is related to a large theoretical literature suggesting different channels through which central bank balance sheet policies can be effective. The most common channels include the following: Absorption of term risk (e.g. Chen, Curdia and Ferrero (2012), Vayanos and Vila (2021), Ray et al. (2019), Carlstrom, Fuerst and Paustian (2017)) or credit risk by the central bank (e.g. Silva (2020), Costain, Nuno and Thomas (2022)), relaxation of intermediary leverage constraints (e.g. Curdia and Woodford (2011) Gertler and Karadi (2011)) or increase in the supply of liquid reserves (e.g. Arce, Nuno, Thaler and Thomas (2020), Drechsler, Savov and Schnabl (2018), Bianchi and Bigio (2020)). This paper summarizes this literature by providing a highly tractable framework in which all these channels coexist.[3].

Empirical Analysis of OMO. [xxx]

Organization. Section 2 lays out the baseline economy with a representative bank, an a single credit demand and deposit supply condition. C introduces heterogeneity across banks and the asset-demand system. Section 4 studies a version where assets differ in their risk exposures and collateralization rates. Section 5 presents estimations of key elasticities, and uses the model to produce counterfactuals. Section 6 concludes.

[3]Another channel is that asset purchases may signal that future policy rates will be lower (e.g. Bhattarai, Eggertsson and Gafarov (2019)). We abstract from this channel for tractability.

2 Benchmark Economy

We study a two-period ($t = 0, 1$) economy comprising a representative household, firms, a continuum of banks, and a government. The model can be extended to an infinite-horizon economy where time-zero effects are identical to the ones here. For this reason, we focus on a two-period model. The economy is populated by a household that supplies funds, while firms demand loans, and banks serve as intermediaries, facilitating the flow of funds between saver households and borrower firms.

Notation. Let R^x denote the gross real rate, r^x denote its net rate, and i^x denote its nominal counterpart. For instance, $R^x = \frac{1+i^x}{1+\pi}$, and $r^x = R^x - 1$, where π represents the rate of inflation. Capital letters indicate nominal variables, while lowercase letters represent real variables. Time subscripts for assets and their returns are omitted when referring to transactions initiated at $t = 0$ and yielding payoffs at $t = 1$.

Information. The only feedback from future to current variables is expected inflation, which we keep fixed. We concentrate on time-zero policies, assuming that the central bank conducts policy measures to ensure reaching their inflation target from period 0 to period 1. Thus, the model abstracts away from any channel related to forward guidance.

2.1 Asset-Demand System: Deposit supply and loans demand

The representative household supplies real balances of deposit d, while firms demand real loans ℓ. The corresponding supply and demand schedules are given by $d\left(R^d\right)$ and $\ell\left(\bar{R}^\ell, P\right)$, where R^d is the real deposit rate, $\bar{R}^\ell$ is the expected return on loans, and P is the price level at period 0.

Let $\epsilon^d > 0$ denote the elasticity of deposit supply with respect to the the real deposit rate. Additionally, let $-\epsilon^\ell_R < 0$ represent the elasticity of loan demand concerning the expected return on loans, and $\epsilon^\ell_P(\lambda) \geq 0$ represent its elasticity concerning the price level P. The latter is influenced by wage stickiness, as a higher price level implies a decline in real wages. This elasticity is dependent on the degree of wage flexibility, denoted by $\lambda \in [0, 1]$. Moreover, $\lambda = 1$ signifies flexible wages, and $\epsilon^\ell_P(\lambda = 1) = 0$. Microfoundations for these functions are presented in Appendix A.

2.2 Banks

We turn our attention to the banks problem of banks. A continuum of banks indexed by $i \in [0,1]$ intermediate between the supply of deposits and the credit demand. Initially, banks are identical differing only on their initial portfolios. In a later section, we will consider the role of bank heterogeneity in their preferences and risk-management. Each bank is endowed with an initial portfolio of nominal assets and liabilities that mature at the beginning of period 0. Consequently, banks face exposure to potential surprises in the price level. Specifically, $e^{b,i}(P)$ represents the bank's equity at $t = 0$—prior to transfers/taxes, a function of the price level P. The function $e^{b,i}(P)$, thus, represents real equity and its exposure to changes in the price level. To simplify notation, we omit the individual bank index i in this subsection.

Given $e^{b}(P)$, the bank chooses a new portfolio of real loans, ℓ^{b}, real reserve holdings, m^{b}, and real deposits, d^{b}, and an amount of net dividends div_0. Its budget constraint at period 0 is

$$div_0 + m^b + \ell^b \quad = \quad e^b\left(P\right) + d^b + \frac{T_0^b}{P}. \tag{1}$$

T_0^b denotes transfers from the government to banks at period 0, which we introduce to clarify the mechanisms.

After the bank chooses its balance sheet components, it is affected by liquidity shocks: depositors transfer funds from one bank to another. Banks settle these payments in reserves. The reserve surplus at the central bank after the deposit transfers, s, are given by:

$$s = \begin{cases} m^b - \delta d^b & \text{with probability } 0.5 \\ m^b & \text{with probability } 0.5. \end{cases}$$

In particular, we assume that a bank faces a net outflow of share δ of its deposits with probability 1/2. If a bank receives an inflow, its balance remains at m^b until the position is cleared. Deposits are preserved within banks so banks that receive an inflow of deposits will eventually receive an increase in reserves. However, s refers to the funds available to the bank to lend in the interbank market.

Banks must maintain positive reserve balances at the end of the period. Hence, if this surplus is negative, banks need to borrow additional reserves. At the same time other banks will have a positive surplus which they may lend. This is the role of the interbank market. We model this market as an OTC market with matching frictions, following Bianchi and Bigio (2022). Without delving into the details, in this market banks that find trading partners trade at the (average) interbank rate R^{ib}, while banks that fail to find a trading partner need to either lend to the central bank in form of excess reserves at rate R^m or borrow from the central bank at the discount window at rate R^{dw}. The match finding probability depends on market tightness, defined as

$$\theta = -\frac{\overbrace{m-\delta d}^{\text{deficit}}}{\underbrace{m}_{\text{surplus}}}, \tag{2}$$

where $m \equiv M/P$ is aggregate real reserves, and d is aggregate real deposits in the banking system. Importantly, θ is an aggregate variable that banks take as given. Appendix XX discusses the details of the interbank market.

Given θ, the probability that a bank in surplus finds a match is $\psi^+(\theta)$ whereas the probability that a bank in deficit finds a match is $\psi^-(\theta)$. Depending on whether the bank needs to borrow or lend, the expected cost or return of this refinancing operation is

$$\chi(s;\theta) = \begin{cases} \chi^- \cdot s & \text{if } s \leq 0 \\ \chi^+ \cdot s & \text{if } s > 0 \end{cases} \tag{3}$$

where $\chi^+ \equiv \psi^+(\theta)\left(R^{ib}(\theta) - R^m\right)$ and $\chi^- \equiv \psi^-(\theta)\left(R^{ib}(\theta) - R^m\right) + (1-\psi^-(\theta))\left(R^{dw} - R^m\right)$. The marginal cost (benefit) of holding short (long) positions in reserves at the interbank market depends on aggregate liquidity conditions, as indicated by market tightness. The borrowing from the central bank is $w(s) = -(1-\psi^-(\theta))\, s\mathbb{I}_{[s\leq 0]}$.

In period 1, the assets and liabilities of banks mature, and the bank disburses its net worth, represented by

$$div_1(\omega, z) = \underbrace{R^\ell(z)\ell^b + R^m m^b - R^d d^b}_{\text{Expected Portfolio Returns}} + \underbrace{\chi(s(\omega)|\theta)}_{\text{Settlement Return}} + \frac{T_1^b(z)}{P(1+\pi)} \tag{4}$$

where ω indexes idiosyncratic uncertainty associated with the liquidity shock and z indexes aggregate uncertainty associated with the return on loans. These risk types are assumed to be independent. T_1^b represents the net nominal transfers from the government to the bank in period 1, π denotes inflation between period 0 and period 1, and $P(1+\pi)$ is the price level in period 1.

We assume that the asset called "loan" delivers a stochastic payoff at period 1, $x(z)$, following $x(z) \sim N(1, \sigma^2)$, where the mean has been normalized to one without loss of generality. Consequently, the return of a loan traded at priceq^ℓ at period 0 satisfies $R^\ell \sim N\left(\bar{R}^\ell, \left(\bar{R}^\ell\right)^2 \sigma^2\right)$, where $\bar{R}^\ell \equiv 1/q^\ell$ is the expected return on loans. The stochastic nature of the return of the asset called 'loan' implies that it represents a blend of funding sources for firms, encompassing those with state-dependent payments. This can be motivated by the uncertainty regarding the aggregate productivity of firms and a pledgeability constraint (see Appendix *A*). Furthermore, banks are subject to a leverage constraint.

While we abstract from a microfoundation, readers can conceptualize the constraint as either a straightforward regulatory constraint or as emerging from market discipline, as discussed in Bernanke, Gertler and Gilchrist (1999) and more recently in Gertler and Karadi (2011). We introduce the leverage constraint in a reduced form as a loss term in the bank's objective function: $\beta\Lambda\left(1 - \frac{m^b+\ell^b-d^b}{\kappa m^b+\ell^b}\right)(\kappa m^b + \ell^b)$, where $\Lambda' \geq 0$, $\Lambda'' \geq 0$, and $\Lambda(0) = 0$. Here $m^b + \ell^b - d^b$ is banks equity and $\kappa m^b + \ell^b$ are risk weighted assets, If $\Lambda(k)$ takes the form $\mathbb{I}_{\{k>\xi\}}\infty$, then this cost function is equivalent to an upper bound ξ on risk weighted leverage. Reserves have a lower weight κ, in line with the literature. For simplicity, we set κ to 0.

Given the described setup, banks solve the following problem

Problem 1 (Banks' problem)**.** *Banks solve the following static maximization:*

$$\max_{\{\ell^b, m^b, d^b, div_0\}} u(div_0) - \beta\Lambda\left(\frac{d^b - m^b}{\ell^b}\right)\ell^b + \beta\mathbb{CE}(div_1(\omega, z))$$

subject to (1) and (4).

In this objective, $\mathbb{CE}(x) = \phi^{-1}(\mathbb{E}(\phi(x)))$ is a certainty equivalent operator, for some risk-aversion specification captured by function $\phi(\cdot)$. We work with $\phi(x) = \gamma^{-1}(1 - \exp(-\gamma x))$, which delivers constant absolute risk aversion (CARA). In turn, we assume that $u(x) = \frac{x^{1-1/\psi}-1}{1-1/\psi}$. Under this payoff function, $1/\psi$ governs the elasticity of dividends (or retained earnings funding) to the profitability of the bank's investment opportunities. Recall that under this objective function:

$$\frac{1}{\psi} = -\frac{u''(div_0)\, div_0}{u'(div_0)}.$$

This parameter is one of the critical elasticities that govern the effect of asset purchases, and it connects with Modigliani-Miller theory, as we show below.

2.3 Government

The environment allows for a balance-sheet policy independent of the interest-rate policy.

Balance-Sheet Policy. We consider how OMO that exchange nominal loans for reserves affect the equilibrium. As emphasized in the literature, the impact of monetary policy interventions is contingent on assumptions about the fiscal side. Throughout our discussion, we use the term "government" to denote the entity executing all policy actions. However, given the focus on actions undertaken by the monetary authority, we occasionally refer to the government as the central bank, taking some literary license.

The government's budget constraint at period 0 is given by:

$$\frac{L^g}{P} + \frac{\int T_0^{b,i} di + T_0^h}{P} = \frac{M}{P} + e^g(P). \tag{5}$$

Here, $e^g(P)$ represents the government's equity. Similar to banks, the reference to P signifies the exposure of the central bank's balance sheet to changes in the price level. Real loans held by the central bank are denoted as $\ell^g = \frac{L^g}{P}$, the ratio of nominal loans to the price level. $T_0^{b,i}$ represents the net nominal transfers to bank i, assumed to be uniform across banks, i.e., $T_0^{b,i} = T_0^b \ \forall i \in [0,1]$. T_0^h represents the net nominal transfers to agents other than banks, and M denotes aggregate reserves in the central bank's balance sheet. At period 0, the government selects $\{L^g, M, T_0^h, T_0^b\}$ in accordance with budget constraint (5). For example, it can endogenously set three of these instruments and allow the remaining one to adjust.

The central bank's profits from the discount window are $R^{dw} w^g$, where $w^g = \int w(s(\omega_i))di$ represents total loans from the discount window. The profits from its portfolio are $R^\ell \ell^g - R^m \frac{M}{P}$. Therefore, the government's budget constraint for period 1 is:

$$\frac{T_1^b(z) + T_1^h(z)}{P(1+\pi)} = R^\ell(z)\ell^g - R^m \frac{M}{P} + (R^{dw} - R^m)w^g. \tag{6}$$

where we already assumed uniform transfers to all banks. The latter expression indicates that the central bank ultimately rebates any profits to agents in the economy and illustrates how the central bank ensures its target inflation rate π by backing up the value of reserves through taxes. Effectively, the central bank chooses an inflation rate π and ensures the return on reserves R^m aligns with it. At period 1, the government decides how to distribute profits (or losses) between banks and other agents. We assume that the fraction of profits going to banks is $1 - \varphi \in [0,1]$, i.e., $T_1^b / T_1^h = (1-\varphi)/\varphi$.

Interest-rate policy. The central bank controls the nominal interest paid on reserves (IOER) i^m and the nominal discount window rate i^{dw}. While prices are flexible today, inflation between periods 0 and 1 is fixed. Consequently, the central bank effectively manages the real rates $R^m = \frac{1+i^m}{1+\pi}$ and $R^{dw} = \frac{1+i^{dw}}{1+\pi}$.

2.4 Equilibrium

Market clearing. Market clearing for reserves, loans and deposits requires

$$\int m^{b,i} di = \frac{M}{P} \tag{7}$$

$$\ell^g + \int \ell^{b,i} di = \ell(\bar{R}^\ell, P) \tag{8}$$

$$\int d^{b,i} di = d(R^d). \tag{9}$$

The micro-foundations in the appendix show that if these markets clear, all other markets (i.e., the labor and goods markets) clear as well.

Equilibrium. The equilibrium is summarized as follows:

Definition 1 (Equilibrium)**.** *An equilibrium is a set of allocations for banks:*

$$\{\ell^{b,i}, m^{b,i}, d^{b,i}, div_0^i, div_1^i(z)\}_{i\in[0,1]},$$

and the private sector $\{\ell, d\}$*; government's policy choices*

$$\{L^g, M, T_0^h, T_0^b, i^m, i^{dw}, \pi, T_1^b(z), \varphi\};$$

and prices $\{R^d, \bar{R}^\ell, P\}$ *such that: (i) given prices, banks optimize; (ii) all markets clear: reserves, loans, and deposits; and (iii) the government satisfies its budget constraint each period.*

2.5 Equilibrium characterization

Next, we characterize the system of equilibrium conditions, following the spirit of classic comparative-statics analysis.

Bank's optimal decisions. As shown in Appendix B, bank i's first-order conditions can be approximated as follows:[4]

$$[\ell^b] : \bar{R}^\ell = \beta^{-1}u'\left(div_0^i\right) + \underbrace{\gamma\sigma^2\left(\bar{R}^\ell\right)^2\left(\ell^{b,i} + (1-\varphi)\frac{L^g}{P}\right)}_{\text{risk premium } \mathcal{RP}} - \underbrace{\left(\Lambda'\left(k^i\right)k^i - \Lambda\left(k^i\right)\right)}_{\text{balance sheet service } \mathcal{B}^a(k^i)} \quad (10)$$

$$[m^b] : R^m = \beta^{-1}u'\left(div_0^i\right) - \underbrace{\frac{1}{2}\Big[\chi^+(\theta) + \chi^-(\theta)\Big]}_{\text{liquidity service } \mathcal{L}^m} - \underbrace{\Lambda'\left(k^i\right)}_{\mathcal{B}^d(k^i)} \quad (11)$$

$$[d^b] : R^d = \beta^{-1}u'\left(div_0^i\right) - \underbrace{\frac{\delta}{2}\chi^-(\theta)}_{\text{liquidity risk } \mathcal{L}^d} - \underbrace{\Lambda'\left(k^i\right)}_{\text{balance sheet cost } \mathcal{B}^d(k^i)} \quad (12)$$

where $k^i \equiv \frac{d^{b,i}-m^{b,i}}{\ell^{b,i}}$ represents the deposits in excess of reserves-to-assets ratio of the bank, a measure of leverage.

The left-hand side of the expressions represents the equilibrium expected return on the asset (or liability), while the right-hand side corresponds to the required return for the bank to hold the asset (or finance via the liability). We denote $R^{f,i} \equiv \beta^{-1}u'\left(div_0^i\right)$ as it corresponds to the real rate the bank would require to invest in a risk-free asset absent balance-sheet constraints and liquidity considerations.

For loans, banks face two considerations. Firstly, a higher expected return is demanded to compensate for aggregate risk, as denoted by the term labeled risk premium ($\mathcal{RP} > 0$). Secondly, the required return diminishes as the expansion of assets—holding liabilities constant—leads to a reduction in the cost associated with the leverage constraint (i.e., the term labeled as balance sheet service satisfies $\mathcal{B}^a > 0$). For reserves, banks require a return below the risk-free rate. This is due to the liquidity service they provide during the settlement stage and the reduction in the cost associated with the leverage constraint since more reserves decreases the relevant measure of leverage. For deposits, the logic concerning the premiums is reversed, as the right-hand side corresponds to the return banks are willing to pay to fund themselves with this liability. This return diminishes due to the liquidity risk associated with deposits—withdrawals being proportional to deposits—and the fact that larger liabilities tighten the leverage constraint ($\mathcal{B}^d > 0$).

Let us further discuss these premia. The risk premium for loans depends on: (i) the risk

[4] As detailed in Appendix B, we simplify the banks' objective function by approximating the term associated with liquidity risk via a first-order Taylor expansion around $\chi^+ = \chi^- = 0$. Hence, higher order terms related to the liquidity premium drop out and the bank behaves risk neutral with respect to liquidity risk. The disregarded terms are proportional to $(\chi^+)^2$, $(\chi^-)^2$or $\chi^+\chi^-$.

aversion of banks, γ; (ii) the volatility of their return, $\sigma^2(\bar{R}^\ell)^2$; and (iii) the sensitivity of the bank's net worth to aggregate risk. This sensitivity, in turn, is influenced by the size of their loan portfolio, $\ell^{b,i}$, and the net transfer received in period 1, $T_1^b/(P(1+\pi))$, given that both are exposed to aggregate risk. Note that transfers to banks constitute a constant fraction $(1-\varphi)$ of the government's profits, which are stochastic due to its portfolio of loans L^g/P.

The liquidity service premium for reserves, denoted as $\mathcal{L}^m(\theta)$, comprises two components. This is because additional reserves increase the bank's benefit if the bank becomes a supplier in the interbank market (this marginal benefit is $\chi^+(\theta)$), and it decreases the liquidity cost if the bank needs additional reserves (this marginal cost is $\chi^-(\theta)$). Both the marginal benefit and the marginal cost depend on matching probabilities and the interbank rate, which, in turn, rely on market tightness θ. The interbank rate is also contingent on the corridor $[i^m, i^{dw}]$ chosen by the central bank.

Similarly, additional deposits increase the reserves deficit when the bank faces a negative liquidity shock. However, deposits play no role when the bank is a supplier of reserves in the interbank market. This stems from the assumption that the inflow of reserves generated by a positive liquidity shock is not accessible until the subsequent period. Consequently, the liquidity premium for deposits is denoted as $\mathcal{L}^d(\theta) = \frac{\delta}{2}\chi^-(\theta)$.

The elasticities of liquidity service and risk premia with respect to market tightness, denoted as

$$\epsilon_\theta^{\mathcal{L}^d} \equiv \frac{\partial \mathcal{L}^d}{\partial \theta}\left(\frac{\theta+1}{\mathcal{L}^d}\right), \quad \text{and} \quad \epsilon_\theta^{\mathcal{L}^m} \equiv \frac{\partial \mathcal{L}^m}{\partial \theta}\left(\frac{\theta+1}{\mathcal{L}^m}\right),$$

respectively, will play a pivotal role in our analysis of open market operations below. For convenience, we use elasticities with respect to a monotonic transformation of market tightness, i.e., $\theta + 1$.

In a similar fashion, define the elasticities with respect to bank's leverage of the premia associated to the (soft) leverage constraint as

$$\epsilon_k^{\mathcal{B}^a} \equiv \frac{\partial \mathcal{B}^a}{\partial k}\frac{k}{\mathcal{B}^a} > 0, \qquad \epsilon_k^{\mathcal{B}^d} = \frac{\partial \mathcal{B}^d}{\partial k}\frac{k}{\mathcal{B}^d} > 0$$

Recall that $\mathcal{B}^a(k) \equiv \Lambda'(k)\,k - \Lambda(k) > 0$, and $\mathcal{B}^d(k) \equiv \Lambda'(x) > 0$. Hence, the sign of the elasticities follow from the convexity of the leverage cost $\Lambda(k)$.

Aggregation. Bank's first-order conditions imply that all banks choose the same loans, dividends and deposits-to-assets ratio, i.e., $\ell^{b,i} = \ell^b$, $div_0^i = div_0$ and $d^{b.i}/(\ell^{b,i} + m^{b,i}) = k$ for all $i \in [0,1]$. Naturally, these constants correspond also to the aggregate outcomes of the banking sector, i.e., $\ell^b = \int \ell^{b,i} di$, $d^b = \int d^{b,i} di$, and $d^b/(\ell^b + m^b) = k$ where $m^b \equiv \int m^{b,i} di$ and $d^b \equiv \int d^{b,i} di$. The decisions for deposits and reserves depend on the bank's equity $e^{b,i}(P)$, so they are heterogeneous across banks.

Equilibrium System. Substituting the private sector schedules, namely, loan demand and deposit supply, along with the government budget constraint for the second period (6), into the system of optimality conditions (10) - (12), the resulting three equations, along with the government budget constraint for the first period (5), collectively define equilibrium outcomes for aggregate loans ℓ, aggregate deposits d, the price level P, and the endogenous policy choice among $\{M, T_0^h, T_0^b, L^g\}$. It is important to note that policy choices must ensure that (5) holds for any price level P. The following lemma formalizes this characterization.

Lemma 1 (Equilibrium characterization)**.** *Given policy decisions for $\{i^m, i^{dw}, \pi, \varphi\}$ and for all but one of $\{M, T_0^h, T_0^b, L^g\}$, the following system of equations determines equilibrium outcomes for aggregate loans ℓ, deposits d, price level P, and the endogenous policy outcome*

$$\bar{R}^\ell(\ell, P) = \beta^{-1} u'(div_0) + \gamma\sigma^2 \left(\bar{R}^\ell(\ell, P)\right)^2 \left(\ell - \varphi \frac{L^g}{P}\right) - \mathcal{B}^a \left(\frac{d - \frac{M}{P}}{\ell - \frac{L^g}{P}}\right) \tag{13}$$

$$R^m = \beta^{-1} u'(div_0) - \mathcal{L}^m(\theta(d, P|M)) - \mathcal{B}^d \left(\frac{d - \frac{M}{P}}{\ell - \frac{L^g}{P}}\right) \tag{14}$$

$$R^d(d) = \beta^{-1} u'(div_0) - \mathcal{L}^d(\theta(d, P|M)) - \mathcal{B}^d \left(\frac{d - \frac{M}{P}}{\ell - \frac{L^g}{P}}\right) \tag{15}$$

where aggregate dividends are:

$$div_0(P, d, \ell) \equiv d - \ell + e(P) - \frac{T_0^h}{P}, \tag{16}$$

and central bank purchases are:

$$\frac{L^g}{P} = \frac{M}{P} + e^g(P) - \frac{T_0^b + T_0^h}{P} \tag{17}$$

and where $e(P) \equiv e^b(P) + e^g(P)$ is the consolidated position of banks and the government, $\bar{R}^\ell(\ell, P)$ is implicitly defined by firms' loan demand, $R^d(d)$ is implicitly defined by the *representative household's deposit supply, and market tightness $\theta(d, P|M)$ is defined by equation (2).*

This lemma is very important, as it is the basis to perform comparative statics by perturbing the central bank's balance sheet. First, it shows that we can obtain an aggregate version of the equilibrium decisions of individual banks (10-12) and aggregating bank decisions into a single budget constraint. This is enough to characterize all equilibrium rates and is possible because bank's are only heterogeneous in their endowments. Second, a perturbation with respect to L^g can be used to study a policy counterfactual, which we do next and later use to approximate policy effects to small policy changes, as for example, done in the trade or taxation literatures.

2.6 Open-Market Operations as Perturbations

To investigate the comparative statics of OMO, we start from an equilibrium allocation and adjust policy choices to represent a marginal exchange of reserves for loans. Consider the differential form of budget constraint (5) in nominal terms

$$\mathtt{d}L^g + \mathtt{d}T_0^h + \mathtt{d}T_0^b = \mathtt{d}M + \mathtt{d}[e^g(P)P] \tag{18}$$

The policy perturbation must specify which policy variables among $\{M, T_0^h, T_0^b, L^g\}$ respond to the endogenous revaluation effects of the central bank's portfolio triggered by the change in the price level, i.e., the last term on the right-hand side of the latter equation. This revaluation effect corresponds to the change in the equity position of the central bank in response to an inflation surprise $\mathtt{d}P$. If all legacy assets and liabilities are nominal, there is no revaluation effect. For simplicity, we work under the assumption that the government has no legacy position, i.e., $e^g = 0$. Aligned with the real-world implementation of an *open market operation*, we assume that the purchase of loans is completely financed issuing reserves, and not through changes in transfers or taxes. Therefore, we analyze a policy that satisfies

$$\mathtt{d}L^g = \mathtt{d}M.$$

In particular, we focus on the responses of aggregate loans, $\mathtt{d}\ell$, aggregate deposits, $\mathtt{d}d$, and the price level $\mathtt{d}P$. For simplicity, we also assume away taxes or transfers in the first period, i.e., $T_0^h = T_0^b = 0$.

Remark: Comparative-Statics with respect to other policies. While not pursue in this project, the framework is convenient to study other policies. For example, we could consider an *helicopter drop* to banks ($\mathtt{d}T_0^b = \mathtt{d}M$) or households ($\mathtt{d}T_0^h = \mathtt{d}M$). We could also move away from the assumption that government's equity is not sensitive to inflation surprises but in that case we would need to specify which policy variable absorbs the revaluation effect. For example,. one alternative is that those gains or losses are transfer to banks, i.e., $\mathtt{d}T_0^b = \mathtt{d}[e^g(P)P]$, another one is that the central bank use the gains to purchase loans or finance the losses through loan sales, i.e., $\mathtt{d}L^g = \mathtt{d}M + \mathtt{d}[e^g(P)P]$. Finally, we could also alter the assumption that inflation is constant, which assumes policy changes at $t = 1$.

Remark: Inflation vs. Changes in the Price Level. It is crucial to distinguish between surprise changes in the price level and inflation. Notice that upon a policy perturbation, the response of the price level, $\frac{dP}{P}$, is the surprise change in the price level at $t = 0$, i.e. a counterfactual, and not inflation between the two periods in the model. Inflation between periods is defined as $(1 + \pi) = \frac{P'}{P}$, where P' is the price level in the second period. Therefore, $\frac{\mathtt{d}\pi}{1+\pi} = \frac{\mathtt{d}P'}{P'} - \frac{\mathtt{d}P}{P}$. Of course, if we fix the price level at some previous period, $t = -1$, the counterfactual would produce a change in inflation.

We assume $\mathtt{d}\pi = 0$, implying that the price level in the second period responds correspondingly to changes in the price level in period 0, i.e., $\mathtt{d}\frac{P'}{P'} = \mathtt{d}\frac{P}{P}$. A second remark is that by abstracting from the effect of inflation expectations, we suppress forward guidance: the model doesn't feature forward guidance, i.e., $\mathtt{d}\pi = 0$. Notice there is no link from future variables to today's equilibrium system.

To guarantee no response of inflation, the central bank must have the ability to commit to future inflation and must have the fiscal capacity to do so (i.e., must commit to adjusting transfers in the second period).

2.7 Comparative-Statics about OMO

In this section, we present our comparative static analysis, initiating with four benchmark neutrality results that highlight the required ingredients for Open Market Operations (OMO) to exert an impact on the total credit extended to the real sector and, consequently, on aggregate production. The key ingredients for the non-neutrality of OMO include: (i) financial frictions, (ii) nominal rigidities, and (iii) funding (equity or deposits) or investment (loans) schedules characterized by non-zero finite elasticities.

Let us begin by providing a precise definition of the neutrality of an OMO within our framework.

Definition 2 (Neutrality of OMO)**.** *An OMO is* ***neutral*** *when it does not affect the real aggregate value of credit extended to firms or of deposits held by households*

Financial frictions. The first benchmark we consider is an economy without financial frictions. In this case, OMOs are neutral. Specifically, OMOs remain neutral if they do not influence the way financial frictions impact banks' decisions. Let us delve into how OMOs affect banks' decisions under the financial frictions in our framework.

Firstly, let's examine the most conspicuous financial friction—the leverage constraint, contingent on risk-weighted leverage, i.e., the ratio of risk-weighted assets over equity. OMOs involve swapping loans for reserves on banks' balance sheets, reducing risk-weighted assets since reserves are less risky than loans. This distinctively influences the portfolio decisions of private banks by diminishing the marginal cost of expanding their balance sheets.

Secondly, consider the search friction in the interbank market for reserves. OMOs augment the supply of reserves in the interbank market, decreasing market tightness and subsequently reducing costs associated with potential deposit withdrawals. This channel, however, is absent when the interbank market is saturated with reserves, as is the case in a liquidity trap. In this scenario, liquidity premiums are zero and remain insensitive to changes in the supply of reserves.

Thirdly, examine market segmentation, where households cannot directly provide funding for firms. Consequently, all aggregate risk is borne by banks. An OMO shifts that risk from banks' balance sheets to the central bank's. The entity ultimately bearing the risk depends on the transfer/tax policy of the government. If a more favorable realization of the return on loans held by the central bank translates, at least partially, into more transfers (or less taxes) to households, then the OMO expands the risk-bearing capacity available for loans. In the model, this occurs whenever the share of transfers to households in the second period is positive, i.e., $\varphi > 0$. In such cases, we say that OMOs absorb risk.

The following proposition formalizes the necessity of financial frictions for the non-neutrality of OMOs:

Proposition 1 (Neutrality of OMO without financial frictions)**.** *OMOs are neutral if financial frictions are absent. i.e., when the model features*

1. *No balance-sheet constraints:* $\mathcal{B}^d = \mathcal{B}^a = 0$.

2. *Satiation in the interbank market:* $\mathcal{L}^m = \mathcal{L}^d = 0$.

3. *No risk absorption of central bank's loan purchases:* $\varphi = 0$.

Moreover, price level does not respond to OMO.

Nominal rigidities. The second benchmark we consider pertains to an economy devoid of nominal rigidities. In this case, OMOs are also neutral. Within our framework, we account for two distinct nominal rigidities: sticky wages and nominal financial contracts. The stickiness of wages is encapsulated by the elasticity of loan demand concerning the price level. When wages exhibit full flexibility, this elasticity is zero. On the other hand, nominal contracts constitute a nominal rigidity because their real payoffs are responsive to inflation surprises, thereby influencing wealth distribution. In our model, inflation surprises are only possible in the initial period, and the key redistribution of wealth is the one between agents capable of funding firms (banks and the central bank) and those unable to do so (households). Consequently, the nominal rigidity is characterized by the redistribution induced by inflation surprises among the relevant groups, specifically the sensitivity of the consolidated equity of banks and the central bank (in the first period) to the price level.

In the absence of nominal rigidities, an OMO fails to impact real outcomes as equilibrium is restored through a proportional increase in the price level. This renders the policy ineffective in altering the real reserves held by banks or the actual funding provided by the central bank to the private sector. It is important to note that we consider the case where $M = L^g$ for this section. The following proposition formalizes this observation, illustrating the indispensable role of nominal rigidities for the non-neutrality of OMOs:

Proposition 2 (Neutrality of OMO without nominal rigidities)**.** *OMOs are neutral if nominal rigidities are absent. i.e., when the model features*

1. *Fully flexible wages, i.e.,* $\lambda = 1$, *and therefore a loan demand insensitive to price level, i.e.,* $\epsilon^{\ell}_P = 0$.

2. *A consolidated equity position of private banks and the central bank not exposed to inflation surprises, i.e.,* $\partial e/\partial P = 0$.

Moreover, price level adjusts proportionally to the change in reserves, i.e., $\mathrm{d}P/P = \mathrm{d}M/M$.

Perfectly elastic funding and investment schedules. The third benchmark we consider involves an economy characterized by perfectly elastic funding sources (both internal and external) and a perfectly elastic investment schedule for banks. In this scenario, once again, OMOs exhibit neutrality. The assumption of deep pockets for banks, signifying a perfectly elastic equity funding, corresponds to the limit where the inter-temporal elasticity of substitution parameter ψ diverges towards infinity, resulting in $u'(div_0) = 1$. A perfectly elastic deposit funding corresponds to the limit where the elasticity of deposits concerning the interest rate ϵ^d diverges towards infinity, while a perfectly elastic investment schedule corresponds to the elasticity of loans with respect to the (expected) interest rate ϵ^ℓ_R diverging towards infinity. In these limit cases, banks can finance as much as desired at a constant rate through equity or deposits and invest in loans as much as desired at a constant expected return.

In this environment, OMOs have no impact on the real credit extended to firms, the real value of deposits, or the aggregate quantity of real reserves. These quantities are determined by the differences in the exogenous rates for equity funding, deposit funding, and the expected return rate in loans. In equilibrium, these exogenous differences must align with the associated risk, liquidity, and balance-sheet premiums. This alignment is only possible if aggregate loans, deposits, and real reserves assume particular values. OMOs generate a proportional change in price level that renders all real variables unaffected. The following proposition establishes the necessity of finite elasticities for the non-neutrality of OMOs.

Proposition 3 (Neutrality of OMO with perfectly elastic funding and investment schedules)**.** *OMOs are neutral if funding and investment schedules for banks are perfectly elastic. i.e., when the model features*

1. *Perfectly elastic internal funding (equity):* $\psi \to \infty$
2. *Perfectly elastic external funding (deposits):* $\epsilon^d \to \infty$
3. *Perfectly elastic investment schedule (loans):* $\epsilon^\ell_R \to \infty$

Moreover, price level adjusts proportionally to the change in reserves, i.e., $\mathtt{d}P/P = \mathtt{d}M/M$.

Perfectly inelastic funding and investment schedules. The last benchmark we consider is an economy where funding (equity and deposits) and investment (loans) schedules for banks are perfectly inelastic. Again, OMOs are neutral. It is obvious that deposits do not respond to OMO when housholds' supply is inelastic with respect to the deposit rate, the only variable affecting such supply. In the case of loans, firms' demand is inelastic with respect to the (expected) payment to banks but it still sensitive to changes in the price level (via changes in real wage). Hence, OMO can have an effect over aggregate credit if they generate inflation surprises. However, with perfectly inelastic equity funding, dividends cannot respond to the OMO and inflation surprises would imply a response of dividends.

Proposition 4 (Neutrality of OMO with perfectly inelastic funding and investment schedules)**.** *OMOs are neutral if funding and investment schedules for banks are perfectly inelastic. i.e., when the model features*

1. *Perfectly elastic internal funding (equity):* $\psi \to 0$
2. *Perfectly elastic external funding (deposits):* $\epsilon^d \to 0$
3. *Perfectly elastic investment schedule (loans):* $\epsilon^\ell_R \to 0$

Moreover, price level does not respond to OMO.

Comparison across benchmarks. While each benchmark leads to the neutrality of OMOs concerning total credit to firms and deposits held by households, they diverge in their predictions regarding the impact on the price level (inflation surprises) and real reserves. In the absence of financial frictions, the price level remains unresponsive to OMOs, as it is determined by the conditions that equate the interest rate on deposits, reserves, and the risk-adjusted return on loans. Consequently, real reserves increase after OMOs but this holds no real consequence since it assumed that the interbank market is saturated with reserves. With perfectly inelastic funding and investment schedules, price level is also unresponsive to OMOs. In this case, the change in real reserves do affect liquidity and balance-sheet premiums but optimal decisions are ensured by responses of the deposit rate, loan (expected) rate, and the risk-free rate. The first two have no effect over quantities and the latter has a vanishing effect over dividends as equity funding becomes perfectly inelastic.

In contrast, in benchmark economies with no nominal rigidities and perfectly elastic schedules, a positive inflation surprise proportional to the expansion of reserves occurs. In this scenario, real reserves and the real credit provided by the central bank to firms remain unaffected by the operation. The surprise inflation also lacks a direct impact due to the absence of nominal rigidities or the fixation of relevant returns owing to perfectly elastic schedules.

General Characterization of Comparative statics. Let us return to the baseline model. Lemma (1) provides an equilibrium system. We exploit the differential form of this system to produce comparative statics. We are looking for the marginal response of equilibrium outcomes $\mathtt{d}Y/Y = [\mathtt{d}\ell/\ell, , \mathtt{d}d/d, \mathtt{d}P/P]$ as functions of the exogenous marginal change $\mathtt{d}M/M$. Recall that we assume $e^g = T_0^h = T_0^b = 0$, which renders $L^g = M$. Consequently, $\mathtt{d}L^g/L^g = \mathtt{d}M/M$.

It is convenient to compute the differential of the consolidated budget:

$$\frac{\mathtt{d}div_0(P,d,\ell)}{div_0(P,d,\ell)} = \omega_e \frac{\mathtt{d}e(P)}{e(P)} + \omega_d \frac{\mathtt{d}d}{d} - \omega_\ell \frac{\mathtt{d}\ell}{\ell},$$

where

$$\omega_e \equiv \frac{1}{div_0/e}, \quad \omega_d \equiv \frac{d/e}{div_0/e}, \quad \omega_\ell \equiv \frac{\ell/e}{div_0/e}.$$

Observe that these ratios are functions of the dividend rate and the leverage of the entire financial system (including the central bank). This representation allows us to solve out for dividends.

It is also convenient to define private banks' leverage as

$$k \equiv 1 - \frac{e}{\ell^b} = \frac{d-m}{\ell - \ell^g}$$

Utilizing these definitions and the elasticities of liquidity premia to market tightness and balance sheet premiums to the bank's leverage, we derive a characterization for the response of external funding, credit, and the price level to an OMO. This characterization is presented in the following proposition.

Proposition 5. *Consider an open-market operation, then the responses of deposits, price level, and loans are:*

$$
\begin{bmatrix} \frac{\mathtt{d}d}{d} \\ \\ \frac{\mathtt{d}P}{P} \\ \\ \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \mathcal{A}_d & \mathcal{A}_p & \mathcal{A}_\ell \end{bmatrix}^{-1} \begin{bmatrix} \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} \frac{m}{\ell^b} \left(k^{-1} - 1\right) \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d} \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + \varphi\gamma \mathbb{V}(R^\ell) m + \beta^{d,a} \frac{m}{\ell^b} \left(k^{-1} - 1\right) \end{bmatrix} \frac{\mathtt{d}M}{M}
$$

where

$$
\mathcal{A}_d \equiv \begin{bmatrix} \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + R^f \psi^{-1} \omega_d + \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} \frac{d}{k\ell^b} \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d} - \left(R^d / \epsilon_{R^d}^d\right) \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + \beta^{d,a} \frac{d}{k\ell^b} \end{bmatrix}, \quad \mathcal{A}_\ell \equiv \begin{bmatrix} -R^f \psi^{-1} \omega_\ell - \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} \left(\frac{\ell}{\ell^b}\right) \\ \\ 0 \\ \\ \mu / \epsilon_{\bar{R}^\ell}^\ell + \gamma \mathbb{V}(R^\ell)\ell - \beta^{d,a} \left(\frac{\ell}{\ell^b}\right) \end{bmatrix},
$$

$$
\mathcal{A}_p \equiv \begin{bmatrix} \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + R^f \psi^{-1} \left(\epsilon_P^e \omega_e\right) + \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} \frac{m}{k\ell^b} \left\{\frac{\ell}{\ell^b} - \frac{d}{\ell^b}\right\} \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d} \\ \\ \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mu \left(\epsilon_P^\ell / \epsilon_{\bar{R}^\ell}^\ell\right) + \varphi\gamma \mathbb{V}(R^\ell)\ell^g + \beta^{d,a} \frac{m}{k\ell^b} \left\{\frac{\ell}{\ell^b} - \frac{d}{\ell^b}\right\} \end{bmatrix},
$$

and

$$
\begin{aligned}
\mu &\equiv \bar{R}^\ell - 2\gamma \mathbb{V}(R^\ell) \left(\ell - \varphi \ell^g\right) \\
\beta^{d,a} &\equiv \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} - \mathcal{B}^a \epsilon_k^{\mathcal{B}^a} = \Lambda''(k)(1-k) > 0
\end{aligned}
$$

This system encompasses the effects of open-market operations on intermediation. While the system is complicated to analyze, the following polar cases teach us about the general effects.

2.8 Polar Cases

In this subsection, we examine deviations from the benchmarks discussed in order to understand the role of each of the financial frictions, nominal rigidities, and funding/investment elasticities.

2.8.1 Funding and Investment Elasticities

We start by studying the effect of an OMO when only one of the three key funding/investment elasticities (equity, deposits, and loans) is finite and different from zero. We do this under two distinct scenarios for the other elasticities: perfectly elastic funding/investment schedules, and perfectly inelastic funding/investment schedules. This allows us to isolate the role of each elasticity and the relevant interactions with nominal rigidities and financial frictions.

Finite equity elasticity in a perfectly elastic economy: $\epsilon^d \to \infty, \epsilon^\ell_R \to \infty$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \zeta_\psi & 1-\zeta_\psi & (\alpha_\ell/\alpha_d)\zeta_\psi \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\begin{aligned} \zeta_\psi &\equiv \frac{(\epsilon^e_P \omega_e)\,\alpha_d}{(\epsilon^e_P\omega_e - \omega_d)\,\alpha_d + \omega_\ell\alpha_\ell + \psi\ell\gamma\mathbb{V}(R^\ell)\mathcal{B}^d\epsilon^{\mathcal{B}^d}_k(\ell - \varphi\ell^g)} \\ \alpha_d &\equiv R^f\ell\left(\beta^{d.a} - \ell^b\gamma\mathbb{V}(R^\ell)\right) \\ \alpha_\ell &\equiv R^f\ell\left(\beta^{d.a} - (\varphi\ell^g/\ell)\,\ell^b\gamma\mathbb{V}(R^\ell)\right) \end{aligned}$$

Note that when $\beta^{d.a} > \ell^b\gamma\mathbb{V}(R^\ell)$, we have that $\alpha_\ell > \alpha_d > 0$.

The relevant nominal rigidity is the sensitivity of consolidated equity to inflation surprises. Absent this nominal rigidity, i.e., $\epsilon^e_P\omega_e = 0$, the OMO becomes neutral, i.e., $\zeta = 0$. This follows from the fact that wage stickiness becomes irrelevant as loan demand becomes perfectly elastic with respect to the (expected) interest rate. Any necessary adjustment in the demand of loans happens through an almost negligible in the (expected) interest rate and not through changes in the real wage (due to inflation surprises).

The relevant financial frictions are the risk absorption channel and balance-sheet constraints. Notably, liquidity premiums resulting from search frictions in the interbank market play no role. This is due to the perfect elasticity of deposit funding, which makes market tightness entirely governed by interest rates—specifically, by the gap between the deposit rate and the reserves rate. In mathematical terms, $R^d - R^m = \mathcal{L}^m(\theta) - \mathcal{L}^d(\theta)$. Consequently, market tightness and, therefore, liquidity premiums remain unaffected by OMOs. The fact that market tightness cannot change after OMOs delivers that the elasticity of deposits plus the elasticity of price level add up to one, i.e., $\mathtt{d}\theta = 0$ implies $\frac{\mathtt{d}d}{d} - \frac{\mathtt{d}M}{M} + \frac{\mathtt{d}P}{P} = 0$.

Absent two relevant financial frictions, i.e., $\beta^{d.a} = \epsilon_k^{\mathcal{B}^d} = \varphi = 0$, OMOs become neutral in terms on aggregate credit to firms, i.e., $\alpha_\ell = 0$, but there is still an effect over real deposits and price level since $\zeta = \epsilon_P^e \omega_e / \left(\epsilon_P^e \omega_e - \omega_d\right)$.

Finite equity elasticity in a perfectly inelastic economy: $\epsilon^d \to 0, \epsilon_R^\ell \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} 0 & \xi_\psi & \epsilon_P^\ell \xi_\psi \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\xi_\psi \equiv \frac{-\alpha_0}{\psi \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} \ell (d-m) \epsilon_P^\ell - (d-m)\ell^b R^f \left(\epsilon_P^e \omega_e - \omega_\ell \epsilon_P^\ell\right) - \alpha_0}$$
$$\alpha_0 \equiv \psi \left[(\ell - d)\, m \mathcal{B}^d \epsilon_k^{\mathcal{B}^d} + (d-m)\ell^b \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} \right]$$

The relevant nominal rigidity is sticky wages. Evidently, absent this nominal rigidity, OMOs are neutral and since aggregate loans and deposits are pinned down by firms' loan demand and households deposit supply. The relevant financial frictions are the ones associated to the liquidity and balances-sheet premium on reserves. The rate on deposits and the rate on loans adjust to ensure banks' optimal conditions without influencing the aggregate quantity of loans or deposits. The price level and aggregate loans are pinned down by the demand of firms $\ell(P)$ and the condition for banks to hold reserves. Hence, without the premiums on reserves, i.e., $\mathcal{B}^d \epsilon_k^{\mathcal{B}^d} = \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} = 0$, OMOs are neutral.

Finite deposit elasticity in a perfectly elastic economy: $\psi \to \infty, \epsilon_R^\ell \to \infty$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \frac{\mathtt{d}M}{M},$$

i.e., OMOs are neutral, and price level responds proportionally to the expansion of reserves. It is sufficient for banks to have perfectly investment opportunity (loans) and equity funding for OMOs to be neutral.

Finite deposit elasticity in a perfectly inelastic economy: $\psi \to 0, \epsilon^{\ell}_{R} \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} -(\alpha_1/\alpha_2)\xi_d & \xi_d & \epsilon^{\ell}_{P}\xi_d \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\xi_d \equiv \frac{\alpha_2}{-\alpha_1 + \alpha_2 + R^d \left(\epsilon^e_P \omega_e - \omega_\ell \epsilon^{\ell}_P\right)}$$
$$\alpha_1 \equiv \left(\epsilon^e_P \omega_e - \omega_\ell \epsilon^{\ell}_P\right) \epsilon^d \left(\mathcal{L}^m \epsilon^{\mathcal{L}^m}_{\theta} - \mathcal{L}^d \epsilon^{\mathcal{L}^d}_{\theta}\right)$$
$$\alpha_2 \equiv \omega_d \epsilon^d \left(\mathcal{L}^m \epsilon^{\mathcal{L}^m}_{\theta} - \mathcal{L}^d \epsilon^{\mathcal{L}^d}_{\theta}\right)$$

The relevant nominal rigidity for the effect over aggregate loans is sticky wages. With flexible wages total aggregate loans would be fixed, so OMOs cannot influence them. In the case of the effect of OMOs over deposits, both nominal rigidities have a role. The key financial friction is the search friction in the interbank market, in particular, the gap between the liquidity premium (benefit) for reserves and the liquidity premium (cost) of deposits. If these two premiums are the same, e.g., when there is satiation and both are zero, then OMOs become neutral. In those cases, the deposit rate must coincide with the interest rate on reserves to prevent arbitrage, and this pins down the quantity of deposits (through households' supply). Since loans' demand is a function only of the price level (demand inelastic with respect to interest rate), dividends becomes a function of the price level alone. Since equity funding is inelastic, dividends must be fixed, hence price level and loan quantity cannot respond to OMOs.

Finite loan elasticity in a perfectly elastic economy: $\psi \to \infty, \epsilon^{d} \to \infty$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \zeta_\ell & 1-\zeta_\ell & \zeta_\ell \end{bmatrix} \frac{\mathtt{d}M}{M},$$

where

$$\zeta_\ell \equiv \frac{\mu \epsilon^{\ell}_P}{\mu \epsilon^{\ell}_P + \mu + \gamma \mathbb{V}(R^\ell)\left(\ell - \varphi \ell^g\right) \epsilon^{\ell}_R}$$

The key nominal rigidity is the presence of sticky prices. Since $u'(div_0) = 1$, dividends, and therefore consolidated equity, have no role on determining equilibrium outcomes. With fully flexible wages, OMOs would be neutral (the no nominal rigidity benchmark). While financial frictions seem irrelevant in this limiting case, their presence– in particular, the presence of liquidity premiums –allows the equilibrium to be well-defined in this limit. Otherwise, the policy choice for the interest rate on reserves must obey $R^m = R^d$. In this limit case, market tightness –hence, liquidity premiums– is determined by the gap $R^d - R^m$. Therefore, the ratio of deposits and real reserves (our measure of market tightness) cannot respond to OMOs. Loans must move proportionally to deposits and real reserves to keep balance-sheet premiums constant as they are pinned down by R^m and R^d alone.

Finite loan elasticity in a perfectly inelastic economy: $\psi \to 0, \epsilon^d \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} 0 & \xi_\ell & \left(\epsilon^e_P \omega_e / \omega_\ell\right) \xi_\ell \end{bmatrix} \frac{\mathtt{d}M}{M},$$

where

$$\xi_\ell \equiv \frac{\alpha_3}{\alpha_3 - (d-m)\left[\epsilon^\ell_P \omega_\ell (\ell - \ell^g)\mu + \epsilon^e_P \omega_e \left(\epsilon^\ell_R \ell \left(\beta^{d,a} - (\ell - m)\gamma \mathbb{V}(R^\ell)\right) - (\ell - m)\mu\right)\right]}$$
$$\alpha_3 \equiv \omega_\ell \epsilon^\ell_R \left[(\ell - d)\, m \beta^{d,a} + (d-m)\,(\ell - \ell^g)\left(\varphi \ell^g \gamma \mathbb{V}(R^\ell) + \mathcal{L}^m \epsilon^{\mathcal{L}^m}_\theta\right)\right]$$

The relevant nominal rigidity is the sensitivity of the consolidated budget to inflation surprises, i.e., financial contracts written in nominal terms. Absent this nominal rigidity OMOs would be neutral. Given the perfectly inelastic equity funding, dividends must be constant. Since deposits are fixed, constant dividends imply that loans financed through deposits, i.e., $\ell - e(P)$, must be constant. Hence, loans can only respond to OMOs if consolidated equity respond to inflation surprises. In this limiting case, the three financial frictions have a role, i.e., OMOs are not neutral as long as one of the three is present.

2.8.2 Financial frictions

We discuss the effect of OMOs when only one of the financial frictions is present. This allows us to understand the interaction of the financial friction with each of the nominal rigidities and the funding/investment elasticities.

Search friction in interbank market: $\mathcal{B}^a, \mathcal{B}^d \to 0, \varphi \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{R^d/\epsilon^d}{\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}}\right)^{-1} (1-\varsigma) & \varsigma & (\alpha_5/\alpha_4)\varsigma \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\begin{aligned} \varsigma &\equiv \frac{\alpha_4}{\alpha_4 + \alpha_5} \\ \alpha_4 &\equiv \omega_d(\mu + \ell\gamma\mathbb{V}(R^\ell)\epsilon_R^\ell) R^f \epsilon^d (\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}) + R^d \left(\psi\mu + \epsilon_R^\ell \left(\ell\gamma\mathbb{V}(R^\ell)\psi + \omega_\ell R^f\right)\right) \mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} \\ \alpha_5 &\equiv (\epsilon_P^e \omega_e)\, \epsilon_R^\ell R^d R^f \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m}\right) + \epsilon_P^\ell \mu \left[\epsilon^d \omega_d R^f \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right) + \psi R^d \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m}\right)\right] \end{aligned}$$

Let us focus on the effect of OMOs over aggregate credit to firms (see the expression for α_5). In this case, both nominal rigidities interact with the search friction in the interbank market, i.e., OMOs are not neutral even if one of the nominal rigidities is absent (i.e., when $\epsilon_P^e \omega_e = 0$ or $\epsilon_P^\ell = 0$). The sensitivity of consolidated equity to inflation surprises operates through the elasticity of loans to the interest rate (hence, the term $(\epsilon_P^e \omega_e)\, \epsilon_R^\ell$ in α_5). If loans are inelastic with respect to their (expected) interest rate, we need that they respond to inflation surprises, i.e., wage stickiness, for OMOs to influence credit. Otherwise, loan quantity is fixed. Wage stickiness operates through both funding elasticities (deposits and equity), i.e., only when both of them are zero, wage stickiness has no bearing on the impact of OMOs over total credit.

Risk absorption: $\mathcal{B}^a, \mathcal{B}^d \to 0, \mathcal{L}^m, \mathcal{L}^d \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} 0 & \varsigma_\varphi & (\epsilon_P^e \omega_e/\omega_\ell)\varsigma_\varphi \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\varsigma_\varphi \equiv \frac{\omega_\ell \epsilon_R^\ell \varphi\gamma\mathbb{V}(R^\ell)\ell^g}{\mu\left(\epsilon_P^e \omega_e - \omega_\ell \epsilon_P^\ell\right) + \epsilon_R^\ell \gamma\mathbb{V}(R^\ell)\left(\ell\epsilon_P^e \omega_e + \ell^g \varphi\omega_\ell\right)}$$

When the only financial friction is that households are segmented from the loans' market and central bank purchases allow them to indirectly participate, the key nominal rigidity are the nominal legacy positions, specifically the sensitivity of consolidated equity to inflation surprises. Absent this nominal rigidity, OMOs become neutral. In this limit case, dividends and deposits are determined by the return on reserves alone. Hence, if consolidated equity is not responsive to inflation surprises, loans cannot respond to OMOs.

Meanwhile, the key funding/investment elasticity is the elasticity of loans with respect to the interest rate. If this elasticity is zero, OMOs would be neutral. When loans respond to their interest rate, dividends can be kept constant by different combinations of responses for the loans' rate and the price level. If loans become unresponsive to the interest rate, the only way to keep dividends constant is for the price level and loans to remain constant.

Balance sheet constraint: $\varphi \to 0, \mathcal{L}^m, \mathcal{L}^d \to 0$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} 0 & \varsigma_\beta & (\alpha_7/\alpha_6)\varsigma_\beta \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\varsigma_\beta \equiv \frac{\alpha_6}{\alpha_6 + (d-m)\left(\ell\mu\psi\mathcal{B}^d\epsilon_k^{\mathcal{B}^d}\epsilon_P^\ell - R^f(\ell-\ell^g)\,\mu\left(\epsilon_P^e\omega_e - \omega_\ell\epsilon_P^\ell\right) + R^f\ell\left(\beta^{d,a} - (\ell-\ell^g)\,\gamma\mathbb{V}(R^\ell)\right)\right)}$$
$$\alpha_6 \equiv -(\ell-d)\,m\left(\psi\mathcal{B}^d\epsilon_k^{\mathcal{B}^d}\left(\mu + \ell\epsilon_R^\ell\gamma\mathbb{V}(R^\ell)\right) + \beta^{d,a}\omega_\ell\epsilon_R^\ell R^f\right)$$
$$\alpha_7 \equiv -(\ell-d)\,m\left(\epsilon_P^\ell\psi\mu\mathcal{B}^d\epsilon_k^{\mathcal{B}^d} + \epsilon_P^e\omega_e\epsilon_R^\ell\beta^{d,a}R^f\right)$$

In this scenario, both nominal rigidities are relevant, i.e., OMOs have an effect over aggregate credit as long as one of them is present. Sticky prices are relevant as long as equity funding is elastic, while nominal contracts have a role as long as firms' loan demand is elastic. In this limiting case, deposits and reserves must have the same return, so aggregate deposits are pinned down by the return on reserves. With perfectly inelastic equity funding, dividends must be constant, which imply that loans cannot respond to OMOs if consolidated equity does not respond to inflation surprises (aggregate loans are ultimately financed through deposits and consolidated equity). With a perfectly inelastic demand for loans, OMOs are neutral absent sticky wages because aggregate loans would be fixed by firms' demand.

Summary table. Table 1 summarizes the results regarding the neutrality of OMOs under finite and non-zero funding and investment elasticities. The "N" stands for neutrality or no effect, while "Y" indicates that there is a real effect of OMOs in terms of aggregate credit or deposits. The presence of one nominal rigidity and one financial friction is a necessary condition for the non-neutrality of OMOs. It is also a sufficient condition except for one combination (sticky wages and risk absorption). Nevertheless, exception disappears if we were to study the response of aggregate output (in the extended model discussed in Appendix XX) instead of the response of aggregate credit.

Table 1: Non-neutrality of OMOs (finite non-zero funding/investment elasticities)

		Nominal Rigidity		
		None	Nominal assets	Sticky wages
Financial Friction	None	**N**	**N**	**N**
	Liquidity friction	**N**	**Y**	**Y**
	Market segmentation (risk absorption)	**N**	**Y**	**N**
	Balance sheet constraint	**N**	**Y**	**Y**

2.8.3 Nominal rigidities

We have already discussed how nominal rigidities interact with financial friction and funding/investment elasticities in the preceding subsections, by contrasting the situation with the nominal rigidity and the limit where it vanishes. Next, we discuss two different limiting cases: the limits where the nominal rigidity is extreme.

Perfectly elastic consolidated equity (to price level): $\epsilon_P^e \omega_e \to \infty$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_8}{1+\alpha_8} & 0 & \varkappa_e \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\begin{aligned}
\varkappa_e &\equiv \frac{\epsilon_R^\ell \left[\epsilon^d \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right) \left(m\ell^b \varphi\gamma \mathbb{V}(R^\ell) - \ell\beta^{d,a}\right) - \alpha_9\right]}{\left(\epsilon_R^\ell \ell \left(\beta^{d,a} - \ell^b \gamma \mathbb{V}(R^\ell)\right) - \ell^b \mu\right) \left(R^d - \epsilon^d \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right)\right)} \\
\alpha_8 &\equiv -\left(\epsilon^d / R^d\right) \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right) \\
\alpha_9 &\equiv \beta^{d,a} R^d \frac{(\ell - d)\, m}{(d-m)} + R^d (\ell - m) \left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} + \varphi \ell^g \gamma \mathbb{V}(R^\ell)\right)
\end{aligned}$$

In this limiting case, the price level is fixed.

Comments (analyze and summarize main forces/insights):

- Sticky wages are irrelevant because the price level is fixed.
- All three financial frictions are relevant.

- OMO affect real reserves and therefore market tightness and liquidity premiums. It also affects the effective leverage of banks.

Perfectly elastic loans (to price level): $\epsilon^{\ell}_{P} \to \infty$. The effect of an OMO is

$$\begin{bmatrix} \frac{\mathtt{d}d}{d} & \frac{\mathtt{d}P}{P} & \frac{\mathtt{d}\ell}{\ell} \end{bmatrix} = \begin{bmatrix} \frac{\alpha_8}{1+\alpha_8} & 0 & \varkappa_\ell \end{bmatrix} \frac{\mathtt{d}M}{M}$$

where

$$\varkappa_\ell \equiv \frac{\psi\left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m}\right)\ell^b R^d - \epsilon^d\left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right)\omega_d R^f \ell^b - \alpha_9}{\left(\ell\psi\mathcal{B}^d \epsilon_k^{\mathcal{B}^d} + \ell^b \omega_\ell R^f\right)\left(R^d - \epsilon^d\left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right)\right)}$$

$$\alpha_9 \equiv \psi\mathcal{B}^d \epsilon_k^{\mathcal{B}^d}\left(\ell\epsilon^d\left(\mathcal{L}^m \epsilon_\theta^{\mathcal{L}^m} - \mathcal{L}^d \epsilon_\theta^{\mathcal{L}^d}\right) - \left(\frac{\ell - d}{d - m}\right) m R^d\right)$$

In this case, the price level is also fixed.

Comments (analyze and summarize main forces/insights):

- Liquidity friction and balance sheet constraint matter (only the deposit/ reserve premium).
- The other nominal rigidity is irrelevant, obvious given that prices are fixed.
- Need funding elasticities (either equity or deposits), investment elasticity irrelevant.
- The effect over deposits is the same as above, understand it better.

3 Geographical Segmentation

We now extend the model to allow for heterogeneity in the asset-demand system, a notional that we label geographical heterogeneity. With respect to the previous section, we allow for firms to have heterogeneous preferences over banks, which can capture geographical considerations. Similarly, households have heterogeneous preferences over deposits offered by different banks. Meanwhile, banks differ in their exposure to liquidity risk and access to external funding. We abstract from aggregate risk (so, no risk absorption channel) and the leverage constraint. Hence, the only financial friction will be the liquidity friction.

Geographical Asset-Demand System. We consider a set $\mathcal{J} = \{1, ..., J\}$ with typical element j of firms that demand loans from all banks and a set $\mathcal{H} = \{1, ..., H\}$ of households with typical element h that supply deposits. We also consider that there is a set $\mathcal{I} = [0, 1]$ of banks with typical element denoted by i. Firm's j loan demand from bank i depends on the interest rate this bank offers to the firm but also on the interest rates offered by other banks to firm j, i.e., $\ell^{j,i}\left(\left\{R^{\ell,j,i}\right\}_{i\in\mathcal{I}}, P\right)$.Deposit supply of household h to bank i depends on the interest rate the bank offers this household but also on the interest rate other banks offer her $d^{h,i}\left(\left\{R^{d,h,i}\right\}_{i\in\mathcal{I}}\right)$.

Banks. Whereas the literature has emphasized market power, here we assume that there's a competitive fringe of banks, all of whom take deposit and loan rates as given.

Problem 2. *The problem of bank i is given by:*

$$\max_{\left\{\ell^{b,j,i}\right\}_{j\in\mathcal{J}}, m^{b,i}, d^{b,i}, div_0^i, div_1^i} u_i(div_0^i) + \beta\mathbb{C}\mathbb{E}(div_1^i(\omega))$$

subject to

$$div_0^i = e^{b,i}(P) + \sum_{h\in\mathcal{H}} d^{b,h,i} + \frac{T_0^b}{P} - m^{b,i} - \sum_{j\in\mathcal{J}} \ell^{b,j,i}$$

$$div_1^i(\omega) = \sum_{j\in\mathcal{J}} R^{\ell,j,i}\ell^{b,j,i} + R^m m^{b,i} - \sum_{h\in\mathcal{H}} R^{d,h,i} d^{b,h,i} + \chi(s(\omega)|\theta) + \frac{T_1^b}{P(1+\pi)}$$

where

$$s = \begin{cases} m^i - \delta d^i & \textit{with probability} \quad p^i \\ m^i & \textit{with probability } (1 - p^i). \end{cases}$$

Interbank market. With heterogeneity among banks' exposure to deposit withdrawal shocks, we obtain a modified interbank market tightness, i.e.,

$$\theta\left(P, \left\{M^{b,i}, \sum_{h\in\mathcal{H}} d^{b,h,i}\right\}_{i\in\mathcal{I}}\right) = -\frac{\int_{i\in\mathcal{I}} p^i \left[M^{b,i}/P - \delta \sum_{h\in\mathcal{H}} d^{b,h,i}\right] di}{\int_{i\in\mathcal{I}} (1 - p^i)\left(M^{b,i}/P\right) di}. \tag{19}$$

When there is symmetry among these exposure, i.e., $p^i = 1/2$ for all banks, the latter expression reduces to the expression in the baseline model (2), which only depends on aggregate reserves and aggregate deposits.

Government. The central bank can buy loans from banks. Since firms have different preferences over loans from different banks, it can potentially matter not only the type of loan the central bank purchases but from whom it purchases them. Hence, the central bank's asset purchase policy involves deciding a portfolio over each firm-bank loan type. The central bank's budget constraint at period 0 is

$$\frac{\sum_{j\in\mathcal{J}}\left(\int_{i\in\mathcal{I}} L^{g,j,i}di\right)}{P} + \frac{T_0^b + T_0^h}{P} = \frac{M}{P} + e^g(P), \tag{20}$$

The government selects $\left\{\{L^{g,j,i}\}_{i,j\in\mathcal{I}\times\mathcal{J}}, M, T_0^h, T_0^b\right\}$ in accordance with budget constraint (20). For example, it can endogenously set three of these instruments and allow the remaining one to adjust. We define the total size of government loan position as

$$\mathcal{Q} \equiv \sum_{j\in\mathcal{J}}\left(\int_{i\in\mathcal{I}} L^{g,j,i}di\right)$$

and denote loan portfolio shares $\vartheta \in \mathbb{R}^J \times \mathbb{R}^{\mathcal{I}}$ where $\vartheta^{j,i} \equiv L^{g,i,j}/\mathcal{Q}$. As in the baseline model, the central bank controls the nominal interest paid on reserves (IOER) i^m and the nominal discount window rate i^{dw}. Moreover, while prices are flexible at time zero, inflation between periods 0 and 1 is fixed.

The budget constraint for period 1 is

$$\frac{T_1^b + T_1^h}{P(1+\pi)} = \sum_{j\in\mathcal{J}}\left(\int_{i\in\mathcal{I}} R^{\ell,j,i}\ell^{g,j,i}di\right) - R^m\frac{M}{P} + (R^{dw} - R^m)w^g.$$

We assume that T_1^b adjusts to ensure this condition holds.

Market clearing. Market clearing for reserves requires:

$$\int_{i\in\mathcal{I}} m^{b,i}di = \frac{M}{P}.$$

Market clearing for loans to firm j is given by:

$$\ell^{b,j,i} + \ell^{g,j,i} = \ell^{j,i} \qquad \forall j,i \in \mathcal{J}\times\mathcal{I}.$$

Finally, market clearing for deposits of household h is given by:

$$d^{b,h,i} = d^{h,i} \qquad \forall h,i \in \mathcal{H}\times\mathcal{I}.$$

Equilibrium. The equilibrium is summarized as follows:

Definition 3 (Equilibrium)**.** *An equilibrium is a set of allocations for banks:*

$$\left\{\left\{\ell^{b,j,i}\right\}_{j\in\mathcal{J}}, m^{b,i}, d^{b,i}, div_0^i, div_1^i\right\}_{i\in[0,1]},$$

and the private sector $\{\ell^{j,i}\}_{j,i\in\mathcal{J}\times\mathcal{I}}, \left\{d^{h,i}\right\}_{h,i\in\mathcal{H}\times\mathcal{I}}$*, government's policy choices*

$$\left\{\left\{L^{g,j,i}\right\}_{i,j\in\mathcal{I}\times\mathcal{J}}, M, T_0^h, T_0^b, i^m, i^{dw}, \pi, T_1^b, T_1^h\right\};$$

and prices $\left\{\left\{R^{d,h,i}\right\}_{h,i\in\mathcal{H}\times\mathcal{I}}, \left\{R^{\ell,j,i}\right\}_{j,i\in\mathcal{J}\times\mathcal{I}}, P\right\}$ *such that: (i) given prices, banks optimize; (ii) all markets clear: reserves, loans, and deposits; and (iii) the government satisfies its budget constraint each period.*

Bank's optimal decisions. The first-order conditions for bank i are given by:

$$\ell^{b,j,i}: \qquad R^{\ell,j,i} = \beta^{-1}u_i'\left(div_0^i\right) \qquad \forall j\in\mathcal{J} \tag{21}$$

$$m^{b,i}: \qquad R^m = \beta^{-1}u_i'\left(div_0^i\right) - \underbrace{\left[p^i\chi^+ + \left(1-p^i\right)\chi^-\right]}_{\text{liquidity service } \mathcal{L}^{m,i}} \tag{22}$$

$$d^{b,h,i}: \qquad R^{d,h,i} = \beta^{-1}u_i'\left(div_0^i\right) - \underbrace{\left(1-p^i\right)\delta\chi^-}_{\text{liquidity risk}\mathcal{L}^{d,i}} \qquad \forall h\in\mathcal{H} \tag{23}$$

The first condition implies that in equilibrium the bank offers all firms the same loan rate, i.e., $R^{\ell,i,j} = R^{\ell,i,j'}\ \forall j,j'\in\mathcal{J}\times\mathcal{J}$. Let $R^{\ell,i}$ denote this interest rate. The second and third condition imply that the bank offers the same deposit rate to all households, i.e., $R^{d,i,h} = R^{d,i,h'}\ \forall h,h'\in\mathcal{H}\times\mathcal{H}$. Let $R^{d,i}$ denote this interest rate.

Equilibrium System.

Lemma 2 (Equilibrium characterization)**.** *Given policy decisions for* $\{i^m, i^{dw}, \pi, T_1^b\}$ *and for all but one of* $\left\{M, T_0^h, T_0^b, \{L^{g,j,i}\}_{i,j\in\mathcal{I}\times\mathcal{J}}\right\}$*, the following system of equations determines equilibrium outcomes for aggregate loans* $\{\ell^{j,i}\}_{j,i\in\mathcal{J}\times\mathcal{I}}$*, reserves* $\{M^i\}_{i\in\mathcal{I}}$*, deposits* $\left\{d^{h,i}\right\}_{h,i\in\mathcal{H}\times\mathcal{I}}$*, interest rates* $\left\{R^{\ell,i}, R^{d,i}\right\}_{i\in\mathcal{I}}$*, price level* P*, and the endogenous policy outcome*

$$\begin{aligned} R^{\ell,i} &= \beta^{-1}u_i'\left(div_0^i\right) && \forall i\in\mathcal{I} \\ R^m &= \beta^{-1}u_i'\left(div_0^i\right) - \mathcal{L}^{i,m}\left(\theta\left(P,\left\{M^i,d^i\right\}_{i\in\mathcal{I}}\right)\right) && \forall i\in\mathcal{I} \\ R^{d,i} &= \beta^{-1}u_i'\left(div_0^i\right) - \mathcal{L}^{i,d}\left(\theta\left(P,\left\{M^i,d^i\right\}_{i\in\mathcal{I}}\right)\right) && \forall i\in\mathcal{I} \end{aligned}$$

where dividends are:

$$div_0^i = e^{b,i}(P) + d^i + \frac{T_0^b}{P} - \frac{M^i}{P} - \sum_{j\in\mathcal{J}} \left(\ell^{j,i} - \frac{L^{g,j,i}}{P} \right) \qquad \forall i \in \mathcal{I},$$

central bank purchases are:

$$\frac{\sum_{j\in\mathcal{J}} \left(\int_{i\in\mathcal{I}} L^{g,j,i} di \right)}{P} + \frac{T_0^b + T_0^h}{P} = \frac{M}{P} + e^g(P),$$

where $M = \int_{i\in\mathcal{I}} M^i di$*; and where deposits at bank* i *satisfy households' deposit supply schedules, i.e.,* $d^i \equiv \sum_{h\in\mathcal{H}} d^{h,i} \left(\left\{ R^{d,i} \right\}_{i\in\mathcal{I}} \right)$*, total loans to firm* j *satisfy its loan demand schedule* $\ell^{j,i} \left(\left\{ R^{\ell,,i} \right\}_{i\in\mathcal{I}}, P \right)$*, and market tightness* $\theta \left(P, \{m^i, d^i\}_{i\in\mathcal{I}} \right)$ *is defined by equation (19).*

Open market operations (OMO). We consider OMO in which the central bank buys loans (according to portfolio $\hat{\vartheta} \in \mathbb{R}^J \times \mathbb{R}^{\mathcal{I}}$) in exchange for reserves, i.e.,

$$dM = \left[\sum_{j\in\mathcal{J}} \left(\int_{i\in\mathcal{I}} \hat{\vartheta}^{j,i} di \right) \right] (d\mathcal{Q})$$

The starting point is an equilibrium situation in which the government portfolio of loans has weights ϑ. As in the baseline model, for simplicity, we assume $e^g = T_0^b = T_0^h = 0$ for now on.

Comparative Statics. We have the following neutrality result.

Proposition 6 (Irrelevance of Asset Composition)**.** *Let* $p^i = 1/2 \quad \forall i \in \mathcal{I}$*, so that banks are equally exposed to liquidity risk. Then, the composition of an OMO does not matter, only the overall size of the operation.*

When the exposure to liquidity risk is homogeneous among banks, market tightness does not depend on the distribution of reserves or deposits over banks. In this case, an OMO that only reshuffles reserves among banks and switches ownership of loans between private banks and the central bank, i.e., an OMO that satisfies $\sum_{j\in\mathcal{J}} \left(\int_{i\in\mathcal{I}} \hat{\vartheta}^{j,i} di \right) = 0$, is neutral. This result implies that for the aggregate effects of OMOs, it is not relevant the type of assets purchased or from whom they are purchased.

We also have the converse result

Proposition 7 (Relevance of Asset Composition). *Let $p^i \neq p^u$ for some $i, u \in \mathcal{I} \times \mathcal{I}$, then the composition of the OMO can matter for its aggregate effects.*

In this case, even when the OMO is self-financed, i.e., when $\sum_{j \in \mathcal{J}} \left(\int_{i \in \mathcal{I}} \hat{\vartheta}^{j,i} di \right) = 0$, it can have aggregate effects if it reshuffles reserves among banks with different exposure to liquidity risk. The reason is that such operation alters the market tightness in the interbank market affecting liquidity premiums.

4 Asset Heterogeneity

In this section, we extend the baseline model to consider heterogeneity among assets. In particular, we examine the situations where assets are heterogeneous in terms of their risk profiles and when they have different liquidity properties. For simplicity, we abstract from the leverage constraint on banks.

4.1 Risk Heterogeneity

Consider a bank that can invest in J types of loans. Each loans promises a payoff of $x^j(z^j)$ at $t = 1$ and $x \sim N(1_{Jx1}, \Sigma^2_{JxJ})$ where Σ^2 is an arbitrary (positive semi-definite) co-variance matrix. Loan demands depend on the expected return of the loan $\bar{R}^{\ell,j}$ and the price level (due to sticky wages), i.e., $\ell^j \left(\bar{R}^{\ell,j}, P \right)$. Recall that the expected return is the inverse of the price of the bond, i.e., $\bar{R}^{\ell,j} \equiv 1/q^j$. We follow the same formulation for the attitude towards risk as in the baseline model.

Banks.

Problem 3. *The problem of a bank is given by:*

$$\max_{\{\ell^b, m^b, d^b, div_0, div_1\}} u(div_0) + \beta \mathbb{CE}(div_1(\omega))$$

subject to

$$div_0 = e^b(P) + d^b + \frac{T_0^b}{P} - m^b - \ell^b$$

$$div_1(\omega, z) = \sum_j R^{\ell,j}(z^j)\ell^{b,j} + R^m m^b - R^d d^b + \chi(s(\omega)|\theta) + \frac{T_1^b(z)}{P(1+\pi)}$$

where

$$R^{\ell}(z) \sim N(1, \mathcal{D}(\bar{R}^{\ell})\Sigma^2\mathcal{D}(\bar{R}^{\ell}))$$

and $\mathcal{D}(R^{\ell})_{JxJ}$ *is the diagonal matrix with the expected returns in the main diagonal. Denote* $\mathbb{V}\left(R^{\ell}(z)\right) \equiv \mathcal{D}(\bar{R}^{\ell})\Sigma^2\mathcal{D}(\bar{R}^{\ell})$.

Government. The central bank's budget constraints for periods 0 is

$$\frac{\sum_j L^{g,j}}{P} + \frac{T_0^b + T_0^h}{P} = \frac{M}{P} + e^g(P) \tag{24}$$

The government selects $\left\{\{L^{g,j}\}_{j\in\mathcal{J}}, M, T_0^h, T_0^b\right\}$ in accordance with budget constraint (24). For example, it can endogenously set three of these instruments and allow the remaining one to adjust. We define the total size of government loan position as

$$\mathcal{Q} \equiv \sum_{j\in\mathcal{J}} L^{g,j}$$

and denote loan portfolio shares $\vartheta \in \mathbb{R}^J$ where $\vartheta^{j,i} \equiv L^{g,j}/\mathcal{Q}$. As in the baseline model, the central bank controls the nominal interest paid on reserves (IOER) i^m and the nominal discount window rate i^{dw}. Moreover, while prices are flexible at time zero, inflation between periods 0 and 1 is fixed. The government's budget constraints for periods 1

$$\frac{T_1^b(z) + T_1^h(z)}{P(1+\pi)} = \sum_j R^{\ell,j}(z^j)\ell^{g,j} - R^m\frac{M}{P} + (R^{dw} - R^m)w^g$$

As in the baseline model, we assume that $T_1^b(z)/T_1^h(z) = (1-\varphi)/\varphi$. Importantly, transfers in the second period depend on the returns on the central bank's portfolio of loans.

Market clearing. Market clearing for loans to firm j is given by:

$$\int_{i\in\mathcal{I}} \ell^{b,j,i}di + \ell^{g,j} = \ell^j \qquad \forall j \in \mathcal{J}.$$

Market clearing for reserves and deposits are analogous as the ones in the baseline model.

Equilibrium. The equilibrium is summarized as follows:

Definition 4 (Equilibrium)**.** *An equilibrium is a set of allocations for banks:*

$$\left\{\left\{\ell^{b,j,i}\right\}_{j\in\mathcal{J}}, m^{b,i}, d^{b,i}, div_0^i, div_1^i(z)\right\}_{i\in[0,1]},$$

and the private sector $\left\{\left\{\ell^j\right\}_{j\in\mathcal{J}}, d\right\}$*; government's policy choices*

$$\left\{\left\{\ell^{g,j}\right\}_{j\in\mathcal{J}}, M, T_0^h, T_0^b, i^m, i^{dw}, \pi, T_1^b(z), \varphi\right\};$$

and prices $\left\{R^d, \left\{\bar{R}^{\ell,j}\right\}_{j\in\mathcal{J}}, P\right\}$ *such that: (i) given prices, banks optimize; (ii) all markets clear: reserves, loans, and deposits; and (iii) the government satisfies its budget constraint each period.*

Bank's optimal decisions. In this framework, we can write

$$\mathbb{CE}(div_1(\omega,z)) = \sum_j \bar{R}^{\ell,j}\ell^{b,j} - \frac{\gamma}{2}\left[\mathbb{V}\left(\left(\ell^b\right)^T R^\ell(z) + \frac{T_1^b(z)}{P(1+\pi)}\right)\right] + \frac{\bar{T}_1^b}{P(1+\pi)}...$$
$$... + R^m m^b - R^d d^b + 0.5\chi^-(\delta d - m) - 0.5\chi^+ m + h\left(\chi^-, \chi^+\right)$$

where $\frac{h\left(\chi^-,\chi^+\right)}{\sqrt{(\chi^-)^2+(\chi^+)^2}} \to 0$ as $(\chi^-, \chi^+) \to 0$. Then, the optimal portfolio decision with respect to loans is characterized by

$$\begin{aligned}\bar{R}^\ell &= \beta^{-1}u'(div_0)1_{Jx1} + \frac{\gamma}{2}\nabla_{\ell^b}\left[\mathbb{V}\left(\left(\ell^b\right)^T R^\ell(z) + \frac{T_1^b(z)}{P(1+\pi)}\right)\right] \\ &= \beta^{-1}u'(div_0)1_{Jx1} + \gamma\mathbb{V}\left(R^\ell(z)\right)\left(\ell^b + (1-\varphi)\ell^g\right)\end{aligned}$$

where the second line follows from replacing transfers to banks $T_1^b(z)$ using the government's budget constraint.

Equilibrium characterization.

Lemma 3 (Equilibrium characterization)**.** *Given policy decisions for* $\{i^m, i^{dw}, \pi, \varphi\}$ *and for all but one of* $\{M, T_0^h, T_0^b, L^g\}$*, the following system of equations determines equilibrium* *outcomes for loans* $\{\ell^j\}_{j\in\mathcal{J}}$*, deposits* d*, price level* P*, and the endogenous policy outcome*

$$\begin{aligned}
\bar{R}^{\ell}(\ell,P) &= \beta^{-1}u'(div_0)1_{Jx1}+\gamma\mathcal{D}(\bar{R}^{\ell})\Sigma^2\mathcal{D}(\bar{R}^{\ell})\left(\ell-\varphi\frac{L^g}{P}\right)\\
R^m &= \beta^{-1}u'(div_0)-\mathcal{L}^m(\theta(d,P|M))\\
R^d(d) &= \beta^{-1}u'(div_0)-\mathcal{L}^d(\theta(d,P|M))\\
\frac{(L^g)^T 1_{Jx1}}{P} &= \frac{M}{P}+e^g(P)-\frac{T_0^b+T_0^h}{P}
\end{aligned}$$

where aggregate dividends are:

$$div_0(P,d,\ell)\equiv d-\ell^T 1_{Jx1}+e(P)-\frac{T_0^h}{P}$$

and where $e(P)\equiv e^b(P)+e^g(P)$ *is the consolidated position of banks and the government,* $\bar{R}^{\ell,j}(\ell^j,P)$*, i.e., the typical element of vector* $\bar{R}^{\ell}(\ell,P)$*, is implicitly defined by firms' loan demand,* $R^d(d)$ *is implicitly defined by the representative household's deposit supply, and market tightness* $\theta(d,P|M)$ *is defined by equation (2) as in the baseline model.*

Open market operations (OMO). We consider OMO in which the central bank buys loans (according to portfolio $\hat{\vartheta}\in\mathbb{R}^J$) in exchange for reserves, i.e.,

$$dM=\left(\hat{\vartheta}\cdot 1_{Jx1}\right)(d\mathcal{Q})$$

The starting point is an equilibrium situation in which the government portfolio of loans has weights ϑ. As in the baseline model, for simplicity, we assume $e^g=T_0^b=T_0^h=0$ for now on.

Comparative Statics. We have the following results connected to the neutrality of OMOs in benchmark cases

Proposition 8 (Neutrality of OMO)**.** *OMOs are neutral in the following cases:*

1. *Financial frictions are absent:* $\mathcal{L}^m=\mathcal{L}^d=\varphi=0$.
2. *Nominal frictions are absent and weights of the loan portfolio purchase in the OMO coincides with the weights of the current loan portfolio of the central bank, i.e.,* $\hat{\vartheta}=\vartheta$.

Comment: Include a proposition that highlights that in contrast to the case with heterogeneous preferences over loans and deposits from different banks, here the composition of the intervention matters.

4.2 Collateral Uses

Consider a bank that can invest in J type of loans. In this case, each loan is characterized by a fraction η^j that can be used as collateral. For simplicity, we abstract from the risk absorption channel (e.g., loans are not risky) and the leverage constraint. Loan demands depend on the return of the loan $R^{\ell,j}$ and the price level (due to sticky wages)$\ell^j(R^{\ell,j}, P)$. The key assumption is that a bank short of reserves can sell the loan after it suffers a withdrawal. For example, we can think of this as the market for repos. In this case, after the bank chooses its balance sheet components and liquidity shocks determine the positions, a bank short of reserves can sell the fraction ψ^j of its loans of type j. Banks in surplus buy the sold assets. In this market:

$$\sum_{j\in\mathcal{J}} x^j = \sum_{j\in\mathcal{J}} \psi^j \ell^j,$$

x^j are purchases of loans of type j by banks with surplus.

Results. [TBA]

5 Quantifying the Effects

This section presents estimates of the reserve-demand elasticity and predictions about the effects of QE in Europe, which we can be used as indirect targets to calibrate the model.

5.1 Reserve-Demand Elasticities

In this section, we present empirical information on the key elasticities uncovered in the model.

A primary statistic of interest is the elasticity of short-term interest rates with respect to the supply of central bank reserves. Going back to the stochastic model for banks' reserve management by Poole (1968), the demand curve for central bank reserves is typically modelled as a non-linear relationship between the spread of overnight interest rates over the central bank's policy rate and the supply of central bank reserves. For computational convenience, the relationship can be approximated with the following parametric functional form:

$$\text{Short-term rate}_t - \text{policy rate}_t = \alpha + \beta \frac{\ln(\text{Excess reserves})_t}{\text{Total assets}_t} + u_t \tag{25}$$

where the short-term rate refers to the EONIA until [October 2019] and the EURSTR thereafter and the policy rate refers to the interest rate on the main refinancing operations (MRO). In order to account for the changing width of the corridor of the ECB's interest rate on its lending operations (the MRO rate) and the remuneration of excess reserves (the deposit facility rate, DFR), the spread is normalised to a corridor width of 50 basis points. Excess reserves are defined as reserve holdings in the current account and deposit facility above minimum reserve requirements, expressed as a fraction of banks' total assets and adjusted for the possibility of averaging minimum reserve holdings over a reserve maintenance period.[5]

Estimating equation (5.1) for the euro area points to a β of around -8.9, suggesting that a one percentage point-increase in excess reserves reduces the spread of the overnight rate to the ECB's policy rate by around 0.1% (see column (1) in Table 5.1). By definition, the relationship is non-linear. Figure 2 plots the data and fitted relationship (red line), showing that the overnight interest rate is highly sensitive to changes in the supply of reserves when liquidity is scarce, but reaches a satiation point at around 3.5-4% of banks' total assets after which the slope of the demand curve becomes flat. This is considerable lower than the satiation point found in the US, which recent estimates put at around 8-10% of bank assets (Afonso, Giannone, Spada and Williams, 2023), but in line with previous estimates for the euro area (Altavilla, Rostagno and Schumacher, 2023). The lower holdings is likely

[5] In the euro area, banks need to comply with minimum reserve requirements only on average during the time between the Governing Council's policy meetings, the so-called "reserve maintenance periods". This allows front- or backloading of reserve holdings in order to comply with the average requirement. In order to account for this, excess reserves are computed as total reserve holdings (in current account and deposit facility) in excess of the *remaining* reserve holdings needed to fulfil the requirement on average over the maintenance period.

Table 2: Reserve demand estimation

	(1) Baseline	(2) LS-VJ	(3) Deposit-adjusted EL
log(Excess liquidity/Total assets) (B)	-8.917***	-9.258***	
	(0.0474)	(0.0561)	
log(Deposits/Total assets) (C)		69.21***	
		(2.306)	
Deposit-adjusted excess liquidity			-9.258***
			(0.0532)
Constant (A)	-80.36***	-52.70***	-52.70***
	(0.206)	(0.807)	(0.0979)
Observations	8189	8189	8189
R^2	0.837	0.850	0.850

Standard errors in parentheses. $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Figure 2: Reserve demand as a function of excess liquidity

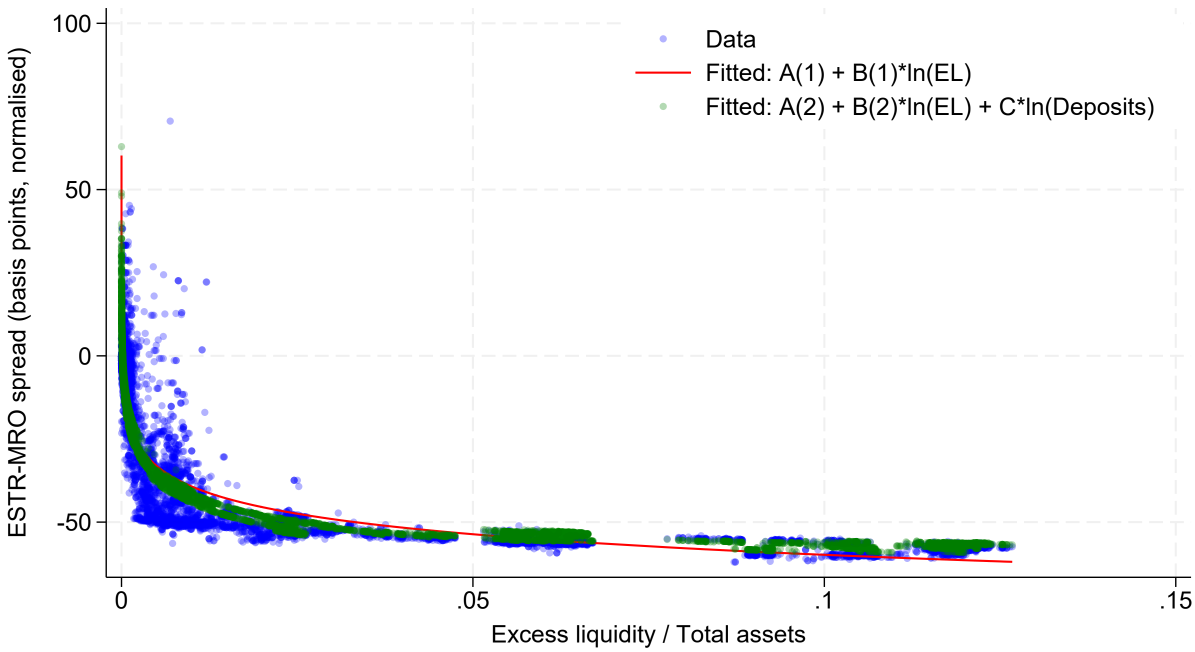

Chart notes tba

a reflection of the larger size of the euro area banking system relative to size of the economy in the two jurisdictions.

For the US, recent contributions have stressed the importance of adjusting the reserve supply for banks' deposit funding to capture the effective liquidity conditions in the short-term money market (Lopez-Salido and Vissing-Jorgensen, 2023). For robustness purposes, we therefore also estimate:

$$\text{Short-term rate}_t - \text{policy rate}_t = \alpha + \beta \frac{\ln(\text{Excess reserves})_t}{\text{Total assets}_t} + \gamma \frac{\ln(\text{Deposits})_t}{\text{Total assets}_t} + u_t \tag{26}$$

with total deposits as a fraction of banks' total assets added to the relationship. Column (2) in Table 5.1 reports the results, and the green dots in Figure 2 show the fitted relationship based on this specification. For the euro area, the results are largely unchanged. While the coefficient on deposits is significant, the estimated β remains largely unchanged and

Figure 3: Reserve demand as a function of deposit-adjusted excess liquidity

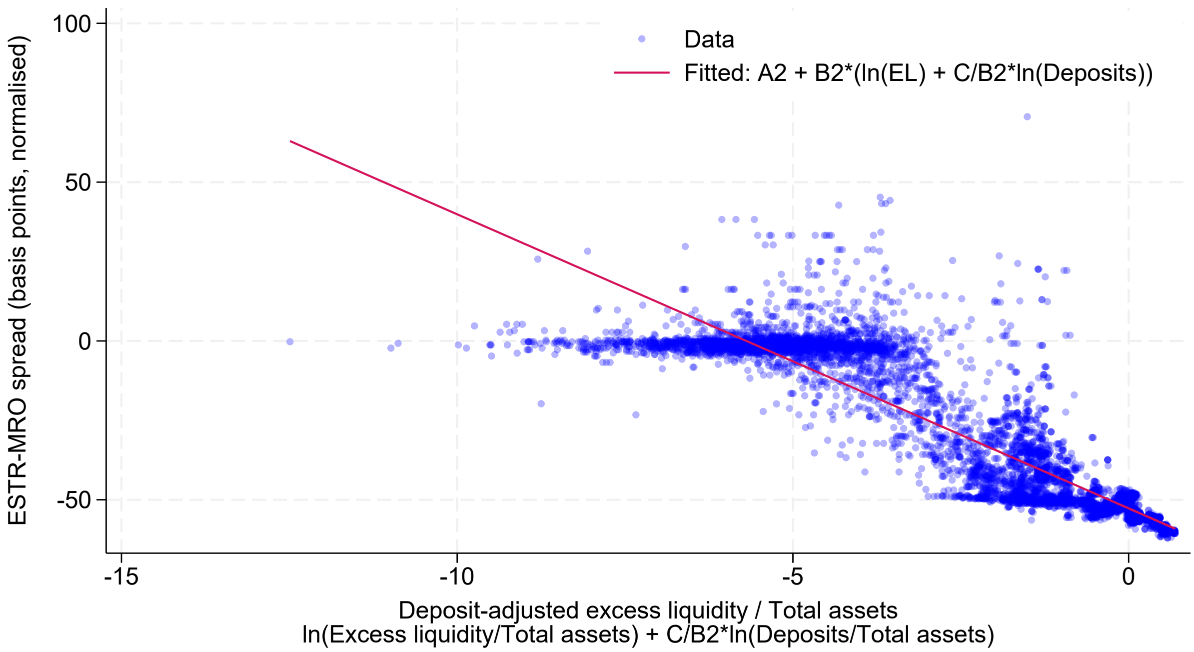

Chart notes tba.

the share of explained variation increases only marginally. Figure 3 visualises the fitted relationship as a function of the deposit-adjusted excess liquidity holdings, defined as the sum of the log of excess liquidity and the log of excess liquidity adjusted for the elasticity of reserve demand with respect to deposits.

5.2 Aggregate Response Estimation

We bring the predicted comparative statics to the data in two steps. First, we establish benchmark results, showing that (unexpected) QE shocks have expansionary effects on output and inflation. Second, in line with the the predictions of the previous section, we condition these estimates on the prevalence of risk premia in the market, showing that the effects are stronger if risk premia are higher.

The first step consists in estimating a basic impulse-response function of macroeconomic aggregates (output and inflation) to monetary policy (QE) shocks (Ramey, 2016), using the local projection method developed by Jorda (2005):

$$y_{t+h} - y_{t-1} = \alpha_h + \sum_{i=0}^{l} [\beta_{h,i} \Delta E_{t-i}[QE] + \gamma_{\mathbf{h,i}}{}' \mathcal{R}_t] + \delta_{\mathbf{h}}{}' X_t + u_{t+h} \tag{27}$$

where y_t refers to the two variables of interest, which we measure as the log of real GDP and the year-on-year change in the Harmonised Index of Consumer Prices (HICP).

The QE shock refers to the change in the expected peak QE holdings of the ECB. Expectations are derived from surveys of market analysts held in the run-up to the ECB Governing Council's monetary policy meetings.[6] Specifically, we derive the median path of survey respondents' expected QE holdings, isolate the maximum value, and compute the first difference in these expectations. Adopting a forward-looking measure of *expected* QE holdings reflects that the transmission of QE programmes works not only contemporaneously via realised purchases – e.g. through the injection of reserves, as stressed in the previous section – but also in anticipation of *future* purchases, e.g. through the portfolio rebalancing channel. Our approach in this regard is similar to Kim, Laubach and Wei (2020), who use survey-based measures of expected LSAP purchases by the Fed.

$\mathcal{R}$ is a 3-by-1 vector of risk premia. For liquidity risk, we include in this vector the spread between the euro area overnight interest rate to the deposit facility rate (DFR); to capture credit risk, we include the average 10-year sovereign spread over the corresponding risk-free OIS rate; and in order to measure duration risk, we include the 10-year term premium. X_t is a vector of control variables which includes 3 lags of y; an indicator variable capturing the height of the pandemic in the sample period, to account for the outsized volatility in GDP and inflation during that period; the lagged DFR, in order to capture the proximity of policy rates to the effective lower bound on interest rates; and the lag of a (log) commodity price index to dampen the price puzzle in the impulse response functions. The sample period ranges from September 2014, when the ECB announced the initial components of its first large-scale QE programme (the "Asset Purchase Programme", APP), to December 2022. The data frequency is monthly.[7]

Figures (4) and (5) report the results. The left panel in each chart shows the response of inflation and output to a one percentage point increase in the expected peak QE holdings. The results suggest expansionary effects. Inflation rises by around 0.2 percentage points about one year after the monetary policy shock, before receding back to zero after around

[6]For the period from 2014 to [2019], we use the market analyst surveys conducted by Bloomberg. For the period from [2019] onwards, we use the data from the ECB's Survey of Monetary Analysts (SMA).

[7]Variables observed at lower than frequency (GDP) are interpolated to monthly frequencies.

Figure 4: Response of inflation to QE shock

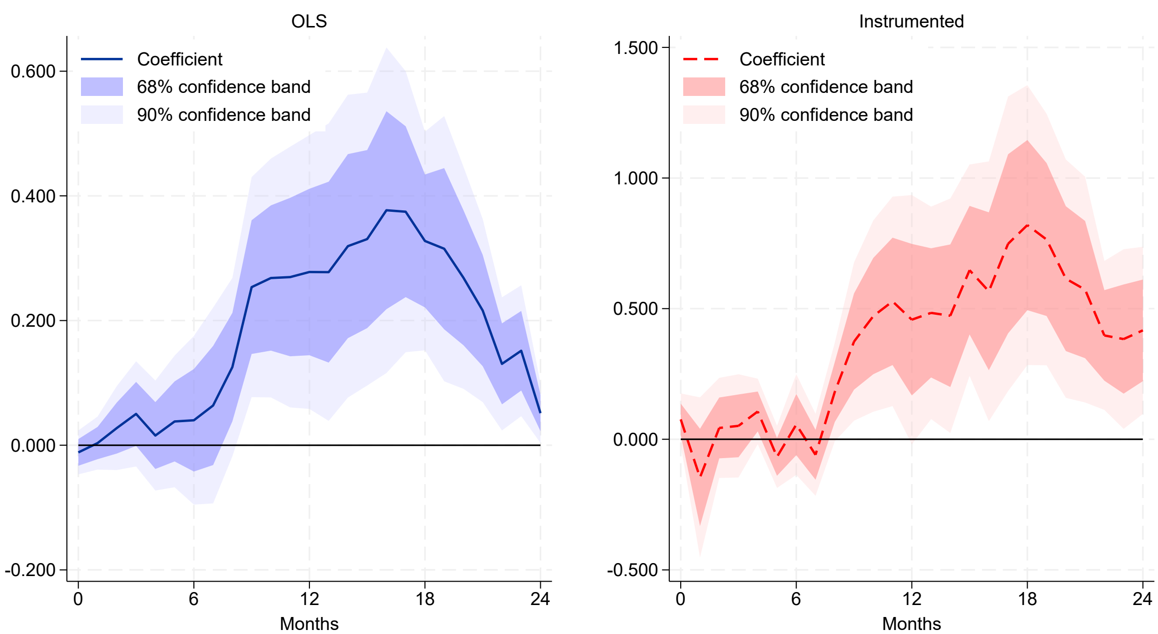

Chart shows the estimated response to a QE-shock impulse equal to one percent of GDP. Specifically, the chart plots $\beta_{h,0}$ in eq. (5.2). The response variable is year-on-year change in the HICP.

Figure 5: Response of output to QE shock

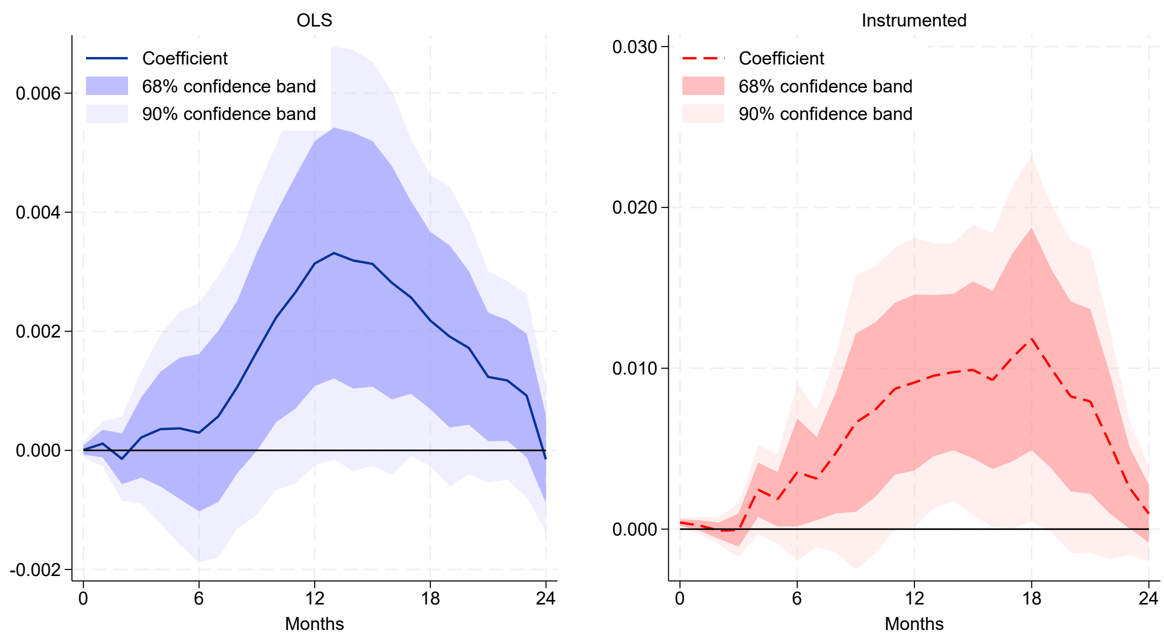

Chart shows the estimated response to a QE-shock impulse equal to one percent of GDP. Specifically, the chart plots $\beta_{h,0}$ in eq. (5.2). The response variable is log(real GDP).

two years. Similarly, the impact of a one percentage point increase in the expected QE holdings on GDP is ca. 0.2% after one year, but drops back marginally faster after alread around 18 months. The magnitude and profile of these estimates are squarely in line with the findings in the preceding literature, e.g. **?**, who find effects of 0.3% on output and inflation in the UK and 0.6% in the US, respectively, or Rostagno, Altavilla, Carboni, Lemke, Motto and Guilhem (2021), who estimate an effect on euro area GDP of around 0.1% and summarise the findings in the literature as pointing to a plausible range of 0.02% to 0.2%.

A plausible concern about the validity of these results is that shifts in expected QE holdings are endogenous responses to changes in the macroeconomic outlook, rather than monetary policy-induced shocks.[8] In order to address this concern, we instrument the changes in expected QE holdings with the intraday change in long-term risk-free interest rates

[8] During the first phase of large-scale net asset purchases from January 2015 to December 2018, the ECB explicitly linked the duration of its QE programme to a "sustained adjustment in the path of inflation." If market participants internalised this as the ECB's reaction function, they may have adjusted their expectations in response to the inflation outlook.

in a narrow window around ECB policy announcements, similar in methodology to Kim et al. (2020) but using the database of euro area monetary policy shocks created by Altavilla, Brugnolini, Gurkaynak, Motto and Ragusa (2019). Specifically, we instrument $E_t[QE] = \theta \Delta OIS_t^{10y} + \nu_t$. The implicit exclusion restriction assumes that changes of long-term risk-free yields in a time window from 15 minutes before the ECB's monetary policy press release to 15 minutes after the end of the corresponding press conferences affect macroeconomic outcomes mainly through the shift in QE expectations. While long-term interest rates also contain an expectations component incorporating expectations about the future path of interest rates, innovations in these tenors have been largely attributed to QE announcements both in the euro area and the US (Gurkaynak, Sack and Swanson, 2005; Altavilla et al., 2019). In order to further strengthen the identification of monetary policy shocks, we restrict the instrumental variable to observations in which the market reaction of the OIS rate goes into the opposite direction than a broad euro area stock market index (STOXX50). Following Jarocinski and Karadi (2020), this sign restriction on the intraday market reaction isolates events in which the monetary policy shock at least outweighs the potential information shock contained in the central bank announcement.

The right panels in Figures (4) and (5) report the results of the instrumental variables approach. The shape of the estimated impulse-response functions is remarkably similar to the simple OLS case. However, the magnitude of the estimated effect is considerably larger both for output and inflation, and surrounded by large uncertainty. This reflects the fact that the instrument is relatively [weak], as suggested by standard statistical diagnostics [**To be added.**]. Given the comparable trajectory of the OLS and IV-based estimations, we employ the OLS-based estimate as the benchmark specification in our following analyses. [In any case, the results are broadly robust to employing the IV-based estimates. [**To be checked.**]]

The second step of our analysis considers the interaction between risk premia and the effectiveness of QE policies stressed in our model. In order to confirm whether the comparative statics are consistent with dynamics in the data, we condition the effect of the monetary policy (QE) shocks as described above on the level of the risk premia:

$$y_{t+h} - y_{t-1} = \alpha_h + \sum_{i=0}^{l} \left[\beta_{h,i} \Delta E_{t-i}[QE] + \phi'_{\mathbf{h},\mathbf{i}} \left(E_{t-i}[QE] \times \mathcal{R}_{t-i} \right) + {\gamma_{\mathbf{h},\mathbf{i}}}' \mathcal{R}_{t-i} \right] + {\delta_{\mathbf{h}}}' X_t + u_{t+h} \tag{28}$$

Figure 6: Conditional response of inflation to QE shock

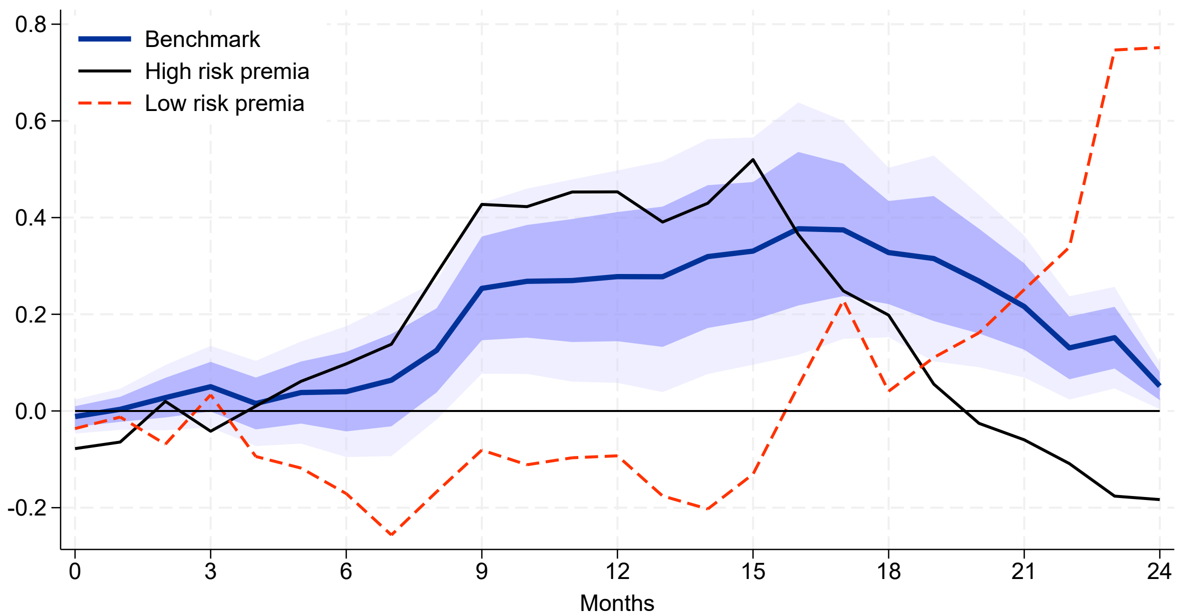

Chart shows the estimated response to a QE-shock impulse equal to one percent of GDP. Specifically, the chart plots $\beta_{h,0} + \phi'_{h,0}\mathcal{R}$ in eq. (5.2). The response variable is the year-on-year change in the HICP.

Figure 7: Conditional response of output to QE shock

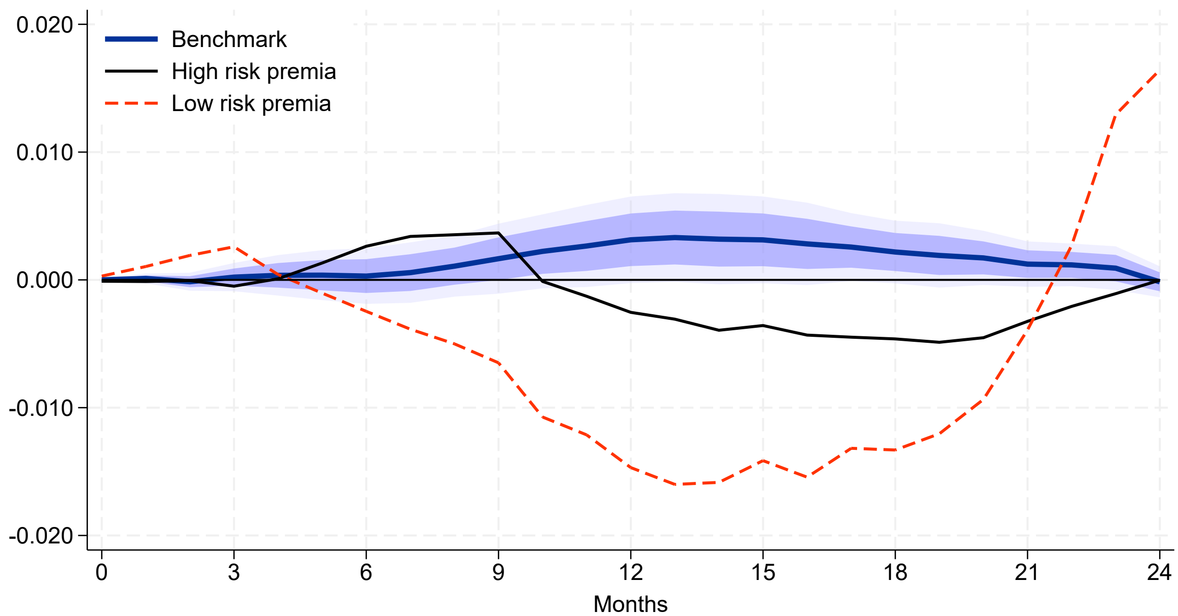

Chart shows the estimated response to a QE-shock impulse equal to one percent of GDP. Specifically, the chart plots $\beta_{h,0} + \phi'_{h,0}\mathcal{R}$ in eq. (5.2). The response variable is log(real GDP).

Effectively, the interaction term implies a continuous version of the state-contingent local projections approach popularised by **?**. The response at horizon h to a QE shock observed at time t is given by $\beta_{h,0} + \phi'_{h,0}\mathcal{R}$, which is contingent on the values of the risk premia contained in $\mathcal{R}$. In order to illustrate the state-contingency, we illustrate the impulse-response function at the 90th and at the 10th percentile of the in-sample distribution of the three risk premia.

The estimated conditionalities are consistent with the predicted comparative statics. The results are shown in Figures (6) and (7), together with the unconditional specification for reference. With respect to inflation, in circumstances in which risk premia are high – i.e., if the EONIA-DFR spread, the 10-year term premium and the 10-year average sovereign spread are at the 90th percentile of their distribution – the response of inflation to a one percentage-point higher expected QE/GDP ratio is at its peak about 50% higher compared to the unconditional specification. At the opposite extreme, with risk premia at exceptionally low levels equal to the 10th percentile of their distribution, the response is muted, oscillating around zero except for a counter-intuitive uptick at the far end of the response horizon.

Qualitatively, a similar result emerges for the impact on GDP. The response in the high risk premia-state is notably stronger than in the low risk premia-state. However, the shape of the impulse response function becomes counter-intuitive, suggesting a contractionary response to a QE impulse. **[To be revised]**

6 Conclusion

[TBA]

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Online Appendix

Not for Publication

A Microfoundation for Deposit Supply and Loan Demand

We examine infinite-horizon problems for the representative household and firms, which can be straightforwardly simplified into a two-period setup.

A.1 Household

A representative household maximizes its discounted lifetime utility by choosing consumption of two types of goods, c_t^d and c_t^h, flexible labor supply ($\hat{h}_t$), and investments in deposits (D_{t+1}) and firm shares (Σ_{t+1})

$$V_t^h\left(D_t, \Sigma_t\right) = \max_{c_t^d, c_t^h, \hat{h}_t, D_{t+1}, \Sigma_{t+1}} U\left(c_t^d\right) + c_t^h - \hat{\pi}\frac{\hat{h}_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}_t\left[V_{t+1}^h\left(D_{t+1}, \Sigma_{t+1}\right)\right]$$

where

$$U\left(c_t^d\right) \equiv \frac{\left(c_t^d\right)^{1-\gamma^d}}{1-\gamma^d},$$

and $\gamma^d > 0$.

Consumption of the first good is subject to a cash-in-advance constraint, where i_t^d is the deposit rate and P_t the price of the consumption goods:

$$P_t c_t^d \leq \left(1 + i_t^d\right) D_t \tag{29}$$

The household consists of a continuum of workers, with a fraction π supplying labor $\hat{h}_t$, which is provided *elastically* at the *endogenous flexible* nominal wage$\hat{w}_t$. The remaining workers supply labor $\bar{h}_t$, which is provided *perfectly elastically* at the *exogenous fixed* nominal wage $\bar{w}_t$. The household's budget constraint in nominal terms is

$$P_t\left(c_t^d + c_t^h\right) + D_{t+1} + q_t \Sigma_{t+1} = \left(1 + i_t^d\right) D_t + \left(q_t + P_t r_t^h\right) \Sigma_t + \hat{\pi}\hat{w}_t \hat{h}_t + (1-\hat{\pi})\bar{w}_t \bar{h}_t + T_t^h, \tag{30}$$

where q_t is the nominal price of a firm share, r_t^h the real dividend of a firm share, and T_t^h a lump sum transfer.

The optimality condition for the flexible wage labor, denoted as $\hat{h}_t$, is expressed as follows

$$\hat{h}^{\nu} = \frac{\hat{w}_t}{P_t} \tag{31}$$

As the fixed wage labor is supplied perfectly elastically, there is no associated optimality condition for $\bar{h}_t$.

The optimality conditions for consumption goods require

$$\frac{q_t}{P_t} = \beta \mathbb{E}_t \left(\frac{q_{t+1}}{P_{t+1}} + r^h_{t+1} \right)$$
$$c^d_t = \min \left\{ R^d_t \frac{D_t}{P_{t-1}}, 1 \right\}$$

where the second element inside the min operator is the smaller one whenever the cash-in-advance constraint is not binding at t.

The optimality condition for deposits implies

$$\beta \mathbb{E}_t \left[R^d_{t+1} U'(c^d_{t+1}) \right] = 1$$

Using the solution for c^d_{t+1} and the fact that monetary policy ensures a fixed level of inflation (hence, R^d_{t+1} is known at t), we have that

$$\frac{D_{t+1}}{P_t} = \begin{cases} \beta^{\frac{1}{\gamma^d}} \left(R^d_{t+1} \right)^{\frac{1}{\gamma^d}-1} & \text{if } \beta R^d_{t+1} < 1 \\ \left[\left(R^d_{t+1} \right)^{-1}, \infty \right) & \text{if } \beta R^d_{t+1} = 1 \end{cases}$$

where the second case corresponds to the case where cash-in-advance constraint is not binding at $t+1$. The latter equation correspond to the deposit supply curve (**??**). This demand curve is increasing in the interest rate on deposits when $\gamma_d < 1$ and the deposit rate is below the subjective discount rate.

A.2 Firms

A continuum of firms, with a measure of one, produces a uniform intermediate good that can be costlessly differentiated into the two types of goods consumed by the household. The intermediate and the two final goods trade at the same price P_t. The production technology is given by $Y_{t+1} = A_{t+1}(h^d_t)^{\alpha}$, where A_{t+1} is the exogenous total factor productivity (TFP), which we assume normally distributed, and $\alpha < 1$. A fraction λ of firms utilizes flexibly priced labor, while the remainder employs sticky price labor. It is important to note that production occurs one period later, requiring the firm to pre-finance the wage bill $w_t h_t^d$ by borrowing loans L_{t+1}^d at time t. These loans have a state-dependent interest rate $i_{t+1}^{\ell}(A_{t+1})$, i.e., these loans represent a mixture of debt and equity funding.

The firm maximizes expected real profits

$$\max_{\{\ell_t^d, h_t^d\} \geq 0} \mathbb{E}_t \left[A_{t+1} \left(h_t^d\right)^{\alpha} - R_{t+1}^{\ell}(A_{t+1}) \frac{L_{t+1}^d}{P_t} \right]$$

which are paid out as dividends to share holders, subject to the working capital constraint

$$w_t h_t^d \leq L_{t+1}^d.$$

The FOC of this problem is

$$\mathbb{E}_t \left[A_{t+1}\right] \alpha \left(h_t^d\right)^{\alpha-1} P_t = w_t \mathbb{E}_t \left[R_{t+1}^{\ell}(A_{t+1})\right] \tag{32}$$

The state-contingent interest rate is assumed to be proportional to A_t. This assumption makes the bank's problem tractable. It can be motivated by a pledegability constraint: Assume that firms cannot credibly pledge more than a fraction α of their output as interest payment: $\alpha A_{t+1} \left(h_t^d\right)^{\alpha-1} P_t \geq w_t R_{t+1}^{\ell}(A_{t+1})$. Then, the only state-contingent interest rate schedule that satisfies the first order condition, while also satisfying the firm pledgabilty constraint for each state A_{t+1} is the one where $R_{t+1}^{\ell} = \alpha A_{t+1} \left(h_t^d\right)^{\alpha-1} P_t / w_t$. That is, the real rate R_{t+1}^{ℓ} becomes a normally distributed variable.

A.3 Labor markets and aggregate loan demand

Market clearing for labor markets demand

$$\hat{h}_t = \hat{h}_t^d$$
$$\bar{h}_t = \bar{h}_t^d$$

where the labor demand for each type of labor follow from (32), each facing a different nominal wage, while labor supply $\hat{h}_t$ follows from (31) and $\bar{h}_t$ is supplied perfectly elastically (meets demand). We can express equilibrium labor quantities as functions of expected return on loans and expected productivity

$$\hat{h}_t = \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{1}{1-\alpha-\nu}}$$
$$\bar{h}_t = \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{(\bar{w}_t/P_t)\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{1}{1-\alpha}}$$

Hence, the aggregate loan demand of firms can be written as

$$\frac{L_{t+1}}{P_t} = \lambda \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{1+\nu}{1-\alpha-\nu}} + (1-\lambda)\left(\frac{\bar{w}_t}{P_t}\right)^{-\frac{\alpha}{1-\alpha}} \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{1}{1-\alpha}}$$

The latter equation correspond to the loan demand curve (**??**). Assuming $\nu < 1-\alpha$, this loan demand is decreasing in the expected return on loans, i.e., $\mathbb{E}_t\left[R^{\ell}_{t+1}\right]$, and increasing in the price level P_t.

A.4 Other equilibrium outcomes

Once we have solved for the expected return on loans and the deposit rate from the model in the text, it is straightforward to recover the rest of equilibrium outcomes. In particular, aggregate output can be written as

$$y_{t+1} = \lambda A_{t+1} \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{\alpha}{1-\alpha-\nu}} + (1-\lambda)A_{t+1}\left(\frac{\bar{w}_t}{P_t}\right)^{-\frac{\alpha}{1-\alpha}} \left[\frac{\alpha \mathbb{E}_t\left[A_{t+1}\right]}{\mathbb{E}_t\left[R^{\ell}_{t+1}\right]}\right]^{\frac{\alpha}{1-\alpha}}$$

which is decreasing in the expected return on loans and increasing in the price level. Market clearing conditions for shares and goods

$$A_t\left(\lambda \bar{h}^{\alpha}_{t-1} + (1-\lambda)\hat{h}^{\alpha}_{t-1}\right) = c^d_t + c^h_t + div_t \tag{33}$$
$$\Sigma_t = 1 \tag{34}$$

pin down the price of shares q_t and the consumption of the quasi-linear good c^h_t.

B Risk and the objective function of banks

We detail the approximation of the objective function of banks. The payoff of banks in the second period is

$$\mathbb{CE}(div_1(\omega, z)) = \phi^{-1}\left(\mathbb{E}\left[\phi\left(div_1(\omega, z)\right)\right]\right)$$

Given the functional forms chosen, we have that

$$\begin{aligned}\mathbb{E}\left[\phi\left(div_1(\omega, z)\right)\right] &= \gamma^{-1} - \gamma^{-1}\mathbb{E}\left[\exp\left(-\gamma\left[R^\ell(z)\ell + R^m m - R^d d + \frac{T_1(z)}{P(1+\pi)} + \chi(s(\omega)|\theta)\right]\right)\right] \\ &= \gamma^{-1} - \gamma^{-1}\mathbb{E}\left[\exp\left(-\gamma\left[R^\ell(z)\ell + R^m m - R^d d + \frac{T_1(z)}{P(1+\pi)}\right]\right)\right]\mathbb{E}\left[\exp\left(-\gamma\left[\chi(s(\omega)|\right.\right.\right.\end{aligned}$$

where the second line follows from the independence of idiosyncratic withdrawals and aggregate uncertainty about loans' return. Applying the inverse function ϕ^{-1}, we have

$$\begin{aligned}\phi^{-1}\left(\mathbb{E}\left[\phi\left(div_1(\omega, z)\right)\right]\right) &= -\frac{1}{\gamma}\log\left(\mathbb{E}\left[\exp\left(-\gamma\left[R^\ell(z)\ell + R^m m - R^d d + \frac{T_1(z)}{P(1+\pi)}\right]\right)\right]\mathbb{E}\left[\exp\left(-\gamma\left[\chi(\right.\right.\right.\right. \\ &= \underbrace{-\frac{1}{\gamma}\log\left(\mathbb{E}\left[\exp\left(-\gamma\left[R^\ell(z)\ell + R^m m - R^d d + \frac{T_1(z)}{P(1+\pi)}\right]\right)\right]\right)}_{\equiv\mathcal{P}_1} - \frac{1}{\gamma}\log\left(\mathbb{E}\left[\mathrm{e}\right.\right.\end{aligned}$$

We can simplify the first term as follows

$$\begin{aligned}\mathcal{P}_1 &= -\frac{1}{\gamma}\log\left(\exp\left(-\gamma\left[\bar{R}^\ell\ell + R^m m - R^d d + \frac{\bar{T}_1}{P(1+\pi)}\right] + \frac{\gamma^2}{2}\mathbb{V}\left(R^\ell(z)\ell + \frac{T_1(z)}{P(1+\pi)}\right)\right)\right) \\ &= \bar{R}^\ell\ell + R^m m - R^d d + \frac{\bar{T}_1}{P(1+\pi)} - \frac{\gamma}{2}\mathbb{V}\left(R^\ell(z)\ell + \frac{T_1(z)}{P(1+\pi)}\right)\end{aligned}$$

We approximate the second term with a second order Taylor expansion around $\chi^+ = \chi^- = 0$

$$\begin{aligned}\mathcal{P}_2 &= \frac{1}{\gamma}\log\left(0.5\exp\left(-\gamma\left[\chi^-(m-\delta d)\right]\right) + 0.5\exp\left(-\gamma\left[\chi^+ m\right]\right)\right) \\ &\approx 0.5\chi^-(\delta d - m) - 0.5\chi^+ m + \frac{\gamma}{4}\left[\left(\chi^-\right)^2(m-\delta d)^2 + \left(\chi^+\right)^2 m^2 - \left(\chi^+\chi^-\right)m\left(m-\delta d\right)\right]\end{aligned}$$

Hence,

$$\mathbb{CE}(div_1(\omega, z)) = \bar{R}^\ell\ell + R^m m - R^d d + \frac{\bar{T}_1}{P(1+\pi)} - \frac{\gamma}{2}\mathbb{V}\left(R^\ell(z)\ell + \frac{T_1(z)}{P(1+\pi)}\right) - 0.5\chi^-(\delta d - m) + 0.5\chi^+ m + h\left(\chi^-,\right.$$

where $h\left(\chi^{-},\chi^{+}\right)/\sqrt{\left(\chi^{-}\right)^{2}+\left(\chi^{+}\right)^{2}}\rightarrow 0$ as $\left(\chi^{-},\chi^{+}\right)\rightarrow 0$.