Risky low-volatility environments and the stability paradox*

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August 2020

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Abstract

Can low risk be risky? In this macro-financial model, smooth business-cycle-frequency fluctuations may disguise vulnerability to panics. Self-fulfilled panics trigger collapses in asset prices and output. When moderately capitalized, the financial sector uses debt to shield allocation efficiency from real shocks, smoothing business-cycle-frequency fluctuations and supporting high asset prices. However, an indebted financial sector, coupled with high asset prices, fuels the exposure to panics: these low-volatility environments are, indeed, risky. A stability paradox emerges: more stable fundamentals lead to a less stable economy, more prone to panics. Importantly, business-cycle-frequency risk, and not the exposure to collapses, drive portfolio allocations.

Keywords: financial stability, risky low-volatility periods, stability paradox.

*A previous version of this paper circulated under the name "Risk to Control Risk" and was a chapter of my dissertation at Princeton University. I am deeply indebted to Markus Brunnermeier for his invaluable guidance and support, as well as to Nobuhiro Kiyotaki and Mark Aguiar for many helpful conversations and advice. I also thank Joseph Abadi, Oleg Itskhoki, Moritz Lenel, Chris Sims, Césaire Meh, Sebastien Merkel, Motohiro Yogo, Paul Ho, Julius Vutz, Nick Huang, Yann Koby, Brunella Bruno (discussant) and seminar participants at numerous universities and conferences for their comments and suggestions. Financial support of the Alfred P. Sloan Foundation and the Macro Financial Modeling (MFM) project is gratefully acknowledged. Giancarlo Acevedo provided outstanding research assistance.

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Conflict-of-interest disclosure statement

I have nothing to disclose.

1 Introduction

The seemingly paradoxical idea that stability breeds instability is widespread in policy and academic discussions. In particular, the hypothesis that periods of stable macroeconomic and financial outcomes disguise and even foster financial fragility became a first-order concern after the Global Financial Crisis 2007-09. However, more than a decade later, the interaction between the volatility of business-cycle-frequency (BCF) fluctuations and the vulnerability to financial collapses has not found its place within macro-financial models. Due to this gap, some key questions remain unresolved. Do prolonged periods of macroeconomic stability such as the Great Moderation make the economy more vulnerable to financial meltdowns? Can the financial system be simultaneously resilient to real shocks but highly susceptible to self-fulfilled crashes? Can macroprudential policies limit financial fragility while allowing sufficient intermediation to smooth business cycles? This paper makes a step forward towards answering these questions.

Understanding the interaction between BCF fluctuations and financial collapses involves important challenges. First, the framework must recognize the multidimensional nature of the instability generated by financial distress: it can amplify and increase the persistence of BCF fluctuations and also make the economy more prone to crashes associated with panics (e.g., systemic runs of creditors). Second, since the intensities of these endogenous instabilities are nonlinear,[1] studying the economy's dynamics away from normal times becomes critical. This paper uses continuous-time methods to deliver a tractable framework to study both instabilities jointly.

Following the standard parable in the literature, I set up a macro-financial model with two types of risk-averse agents: *unproductive* households and *productive* experts (or financial institutions[2]). The productivity gap allows experts to earn a higher return from managing capital, the only physical asset in the economy. These returns are subject to an aggregate productivity shock, whose volatility follows an exogenous stochastic process. These real

[1] Brunnermeier and Sannikov (2014) show that the amplification of real shocks is highly nonlinear: muted around the steady-state but significant when the financial sector is impaired. Gertler and Kiyotaki (2015) pointed out that the exposure to bank runs increases after adverse productivity shocks but, to the best of my knowledge, the full dynamics of this instability are unexplored and a contribution of this paper.

[2] Since any friction between financial institutions and non-financial productive firms is assumed away, experts represent both. Given the focus on the relationship between financial institutions and their creditors, I refer to experts also as financial institutions.

disturbances drive BCF fluctuations and are modeled as Brownian motions. Experts finance capital holdings beyond their net worth by issuing non-contingent debt to households. Debt contracts are the only financial asset in the economy.

A panic is a generalized loss of confidence in the solvency of experts. Panics arrive according to a Poisson process but only become self-fulfilled when experts are not able to meet their debt obligations at liquidation price for capital, which is the price when unproductive households hold the entire capital stock. Self-fulfilled panics force experts into bankruptcy and trigger substantial drops the capital price and output. Two variables summarize the state of the economy: the wealth share of experts (or the financial sector's capitalization relative to the entire economy[3]) and the volatility of real shocks.

The economy presents two instabilities: BCF fluctuations, which depend on the *exogenous* volatility of real shocks and the *endogenous* amplification of their effects, and the vulnerability to panics. The paper's main contribution is to include both instabilities in a tractable framework and study their joint dynamics. The analysis unveils two central messages. First, the endogenous emergence of *risky low-volatility* environments: periods with stable BCF fluctuations –due to a null amplification of real shocks– but exposed to economic meltdowns triggered by panics. Second, the *stability paradox*: a decline in the volatility of real shocks fuels their amplification mechanism and the vulnerability to financial panics, delivering a less stable economy.

Two insights regarding the interaction between the instabilities and the capital price are fundamental to understand the main messages. First, although both instabilities operate through financial institutions' balance sheets and intensify as these weaken, each depends on a distinct feature of the dynamics of capital price. The amplification of real shocks hinges on the sensitivity of capital price to small variations on the financial sector's capitalization, while the exposure to panics depends on the gap between current capital price and its liquidation value. As a consequence, high capital price fuels the vulnerability to panics but does not influence the amplification of real shocks. This distinction opens the door for the risky low-volatility (RLV) periods.

The second insight highlights the different effects each instability has over the capital price:

[3]This is an inverse measure of how much funds the financial sector would need to raise to provide full intermediation and allow efficient input allocation. It is different from capitalization measured relative to the assets that the financial sector holds, which I refer to as the financial sector's leverage.

while more volatile BCF fluctuations depress the capital price, greater vulnerability to panics does not influence it. The reason behind is the asymmetric effect over the capital demand of experts relative to households'. Despite reducing the capital demand of both types of agents, more extensive exposure to panics does not affect the relative capital demand. The equivalence of the two demand contractions follows from all marginal losses being ultimately borne and priced by households. The absence of feedback from the vulnerability to panics to capital price is crucial for the two central messages.

The emergence of RLV periods illustrates that at least one dimension of financial fragility is decoupled from volatile outcomes, sluggish growth, depressed asset prices, or weak financial institutions' balance sheets: crashes triggered by panics can arrive during relatively good times. In terms of the economy's states, RLV periods arise when the volatility of productivity shocks is sufficiently low and the financial sector is moderately capitalized.

During RLV periods, financial institutions have enough wealth to ensure that productive agents hold the entire capital stock and shield this efficient allocation from real shocks. After an adverse productivity shock, financial institutions increase their debt instead of contracting their balance sheets. Hence, there is no impact on allocation efficiency and no amplification. BCF fluctuations are stable, and the financial system seems robust. However, financial institutions are not sufficiently capitalized to avoid heavily relying on debt financing. The efficient capital allocation implies high capital price, which coupled with an indebted financial sector, makes the economy highly vulnerable to panics. So, the stable environment is, indeed, fragile.

Besides the novel RLV periods, the economy transits other two risk regimes: a crisis regime, when financial institutions are poorly capitalized, and a safe regime, when financial institutions are extensively capitalized. During crisis periods, the financial sector's intermediation is not sufficient to achieve the efficient capital allocation, and it is sensitive to real shocks. These features translate into volatile BCF fluctuations and a depressed capital price. Moreover, the economy can also be vulnerable to panics. During safe periods, financial institutions ensure all capital is managed by productive agents maintaining low debt levels. Therefore, output and capital price are high, and the economy is resilient to real shocks and panics.

Studying the economy's dynamics as the exogenous volatility of real shocks varies unveils the stability paradox: more stable fundamentals lead to a less stable economy. In equilibrium, lower volatility of real shocks generates financial institutions to expand their balance sheets. In terms of BCF fluctuations, the financial sector's weaker balance sheets intensify the amplification of real shocks. Brunnermeier and Sannikov (2014) named this result the volatility paradox. However, the amplification response is not sufficiently strong to overcome the decline in the volatility of real shocks, i.e., more stable fundamentals translate into smoother, or at least equally smooth, BCF fluctuations. The stability paradox follows from the response of the exposure to panics to more stable fundamentals. More substantial leverage translates into a higher capital price, so both determinants of the vulnerability to panics rise as the volatility of real shocks declines. Moreover, as fundamentals becomes more stable, the economy spends more time in the RLV regime and less in the safe regime.

The paper examines two sets of policy interventions to mitigate the economy's instabilities. The first one corresponds to leverage constraints. Leverage enables productive experts to hold more capital, so leverage caps lead to the trade-off between stability and output. The analysis unveils that a constant leverage cap has significant shortcomings: it severely constraint leverage during downturns and decelerates recoveries after deep recessions. A risk-based leverage constraints improves the output-stability trade-off because, in equilibrium, this leverage cap is counter-cyclical. A leverage constraint that limits the debt-to-output ratio in the economy lead to further improvements. In equilibrium, the latter constraint does not limit leverage during downturns, instead it constrains experts' risk-taking only when potential losses due to self-fulfilled panics would be the greatest.

The three mentioned leverage constraints limit experts' leverage when instabilities would be extensive. However, the optimal state-contingent leverage cap focuses on preventing the economy from escaping the safe regime. It limits experts' risk-taking only when the financial sector is moderately or extensively capitalized to ensure that households are the marginal buyers of capital, which accelerates experts' wealth accumulation.

Using an extension of the model, the second set of policy exercises examines the economy's response to variations in the volatility of idiosyncratic shocks. An interpretation is that policy can influence the degree of idiosyncratic risk-sharing in the economy. Idiosyncratic fluctuations influence capital allocation and its price, so allowing for idiosyncratic volatility reduces experts' leverage, limiting the exposure to panics and the amplification of real shocks: idiosyncratic risk controls aggregate risks. A non-monotonicity emerges: instability (including idiosyncratic fluctuations) is U-shaped as function of idiosyncratic volatility.

Related literature. The idea that *stability breeds instability* has been discussed at least since Minsky (1986). Existing theories embody the spirit of this idea by illustrating how economic booms can lead to financial fragility, e.g., through looser lending standards (Gorton and Ordoñez (2014, 2019), Farboodi and Kondor (2020)) or extrapolative expectations (Bordalo et al. (2018), Greenwood et al. (2019)). The present paper offers a different perspective by emphasizing the multidimensional nature of financial fragility, opening the possibility for the limited exposure to one dimension (amplification of real shocks) to coexist and even fuel the vulnerability to another one (exposure to crashes triggered by panics).

The literature has extensively studied each of the two dimensions of financial instability considered in this paper separately. First, there is lengthy literature on the role of financial frictions in the propagation of real disturbances, which follows seminal contributions by Kiyotaki and Moore (1997), and Bernanke et al. (1999).[4] Particularly relevant are the studies that highlight that the intensity of the amplification of real shocks varies in a nonlinear fashion along the business cycle (e.g., Mendoza (2010), Brunnermeier and Sannikov (2014)). The present paper shows that the amplification mechanism is invariant to the economy's vulnerability to systemic crashes.

Second, the literature includes several examples of self-fulfilled financial crises that can materialize only in certain states of the economy.[5] The present paper shares with this literature the emergence of a feedback loop between asset prices and the fundamentals behind them. The branch that studies banking panics following Diamond and Dybvig (1983) is the most relevant for this paper.[6] Unfortunately, most of it uses frameworks not suitable for macroeconomic analysis (i.e., finite-horizon partial equilibrium models). Only after the central role of runs in the last financial crisis (Bernanke (2018), Gorton (2010)), the profession has worked towards incorporating bank runs into the macroeconomic analysis.[7] The closest to our work is Gertler et al. (2019), which incorporates banking panics into a macroeconomic model and focuses on propagation mechanism of real shocks that trigger systemic runs. Complement-

[4] This literature has expanded rapidly since the financial crisis and is too vast to attempt to summarize it here. For recent surveys, see Gertler and Kiyotaki (2010), Quadrini (2011), and Brunnermeier et al. (2013). The literature focuses on productivity shocks but also explores uncertainty shocks (e.g., Christiano et al. (2014)and Di Tella (2017)).

[5] For recent examples, see Perri and Quadrini (2018), and Bocola and Lorenzoni (2020).

[6] For example, Goldstein and Pauzner (2005), and Allen and Gale (1998, 2004).

[7] Recent efforts to incorporate bank runs into macroeconomic models include Gertler and Kiyotaki (2015), Gertler et al. (2016), Martin et al. (2014) and Uhlig (2010).

ing their work, this paper focuses on the interactions between the two instabilities and their response to variations in the fundamental volatility. In contrast to their study, this paper unveils that the vulnerability to financial panics can arise not only after adverse productivity shocks but also during booms.

From a methodological point of view, this paper relates to the continuous-time macro-finance models[8] that follow He and Krishnamurthy (2011, 2013) and Brunnermeier and Sannikov (2014, 2016), while the approach to include banking panics in an infinite-horizon economy follows Gertler and Kiyotaki (2015). Technically, the closest paper is Brunnermeier and Sannikov (2015), which studies the amplification of real shocks together with a different class of discrete collapses: sudden stops of international capital flows. Relative to this literature, the methodological contribution is the analysis of anticipated aggregate jump risk.

2 Model

2.1 Environment

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space that satisfies the usual conditions, and assume all stochastic processes are adapted. The economy evolves in continuous time with $t \in [0, \infty)$ and is populated by a continuum of two types of agents: *experts* and *households*. Individual agents are indexed by j, and the set of experts and households are denoted by $\mathbb{J}_{e,t}$ and $\mathbb{J}_{h,t}$, respectively.[9] The economy features two goods: the consumption good and capital (measured in "efficiency units").

In reality, the intermediation chain that channels resources from savers to end-borrowers often has several links. For tractability, I bundle the entire chain in the two mentioned groups: *experts* that include end-borrowers and financial institutions, and *households* that represent savers. The focus is on the liability side of financial institutions' balance sheets, so I refer to experts also as financial institutions throughout the paper.

Technology. The production technology has constant returns-to-scale and takes only capital as input: an agent with $k_{j,t}$ units of capital produces $a_j k_{j,t}$ units of the consumption

[8]This growing literature includes Adrian and Boyarchenko (2012), Moreira and Savov (2017), Di Tella (2017) and He and Krishnamurthy (2019).

[9]Since decision agents' decisions will scale with net worth, the reader can think of a single expert and a single household with the entire wealth of their respective sectors.

good. Within each group, productivities are homogeneous, so abusing notation, I refer to the two productivities as a_e and a_h. Experts are more efficient than households at managing capital, i.e., $a_e > a_h$. The productivity gap summarizes the value that financial institutions create by intermediating funds.

Over a short period (from t to $t+dt$), capital evolves according to

$$dk_{j,t} = k_{j,t}\left[\left(\Phi\left(\iota_{j,t}\right) - \delta\right)dt + \sqrt{s_t}\sigma \cdot dZ_t\right], \tag{1}$$

where $\iota_{j,t}$ is investment per unit of capital, δ is the depreciation rate, Z_t is a d-dimensional Brownian motion with independent components, and $\sqrt{s_t}\sigma$ (with $s_t \in \mathbb{R}$, $\sigma \in \mathbb{R}^d$) is the sensitivity of capital efficiency units to the Brownian shocks. I interpret stochastic changes in capital efficiency units as *productivity shocks*.[10] For expositional convenience, the law of motion presented does not include capital sales or purchases, which may also occur.

The investment technology allows for transforming $\iota_{j,t}k_{j,t}$ units of consumption goods into $\Phi(\iota_{j,t})k_{j,t}$ units of capital. Function Φ represents technological illiquidity or adjustment costs and satisfies $\Phi(0) = 0$, $\Phi'(0) = 1$, $\Phi'(\cdot) > 0$, and $\Phi''(\cdot) < 0$. The sensitivity of capital to Brownian shocks is also stochastic. In particular, the time-variant component follows a mean-reverting process

$$ds_t = \lambda_s\left(\bar{s} - s_t\right)dt + \sqrt{s_t}\sigma_s \cdot dZ_t \ , \tag{2}$$

where $\bar{s}$ is the long-run mean, λ_s the mean reversion parameter, and $\sqrt{s_t}\sigma_s \in \mathbb{R}^d$ the sensitivity to the Brownian shocks. I refer to s_t as the *volatility of productivity (or real) shocks* and interpret its stochastic changes as *volatility shocks*. The framework accommodates an arbitrary number of shocks that simultaneously affect productivity and volatility. However, for pedagogical purposes, the exposition considers two "pure" shocks ($d = 2$), i.e., a *productivity shock* ($\sigma^{(1)} > 0$, $\sigma_s^{(1)} = 0$) and a *volatility shock* ($\sigma^{(2)} = 0$, $\sigma_s^{(2)} > 0$). The superscript inside parenthesis denotes the position within the vector.

Since financial panics will be rare events, the response of macroeconomic variables to real shocks (modeled by the Brownian motion) is the only source of stochastic fluctuations for prolonged periods. As standard in macroeconomic, I think about these stochastic movements

[10] The macro-finance literature uses these *capital quality shocks* (as opposed to shocks to the output per unit of capital, TFP) to capture productivity disturbances and preserve maximum tractability. A capital quality shock behaves similarly to a persistent TFP shock that induces a higher utilization rate.

as business-cycle-frequency fluctuations: the first instability studied in this paper.

Overlapping Generations. To ensure a non-degenerate stationary wealth distribution across agents' types, I assume a "perpetual youth" overlapping generations (OLG) structure. All agents perish independently at the Poisson rate λ_d, and newborns arrive at the same rate. Among them, a fraction ν are designated as experts, while $1-\nu$ are households. Dying agents' wealth is pooled and redistributed equally to newborns, regardless of their type. To ensure that these bequests are positive, I assume there are no markets to hedge these idiosyncratic death shocks.

Preferences. Preferences are symmetric for both types of agents. In particular, the utility that agents obtain from a stochastic consumption path $\{c_{j,t}\}_{t\geq 0}$ is

$$\rho\mathbb{E}_0\left[\int_0^\infty e^{-\rho t}\log(c_{j,t})dt\right], \tag{3}$$

where ρ is the preference discount rate, which includes the Poisson death rate λ_d.

Markets and financial frictions. There are markets for capital and consumption goods. Debt contracts are the only financial assets in this economy. A debt contract ensures the repayment of principal and interest as long as the borrower remains solvent, and it transforms into a claim on borrowers' assets in the case of default. Debts of different financial institutions are distinct assets because their exposures to default and recovery values may differ.

The complete absence of other financial markets[11] is an extreme assumption made for tractability, but it is not essential for results. The necessary feature is that financial institutions are not able to perfectly share the risks they take on the asset side of their balance sheets. I also assume that financial institutions that underperform relative to their peers incur in a prohibitively high cost ϕ (measured in consumption goods). This assumption allows me to focus on symmetric equilibriums among financial institutions.

Financial panics. A panic is a generalized loss of confidence in the solvency of financial institutions. Agents fear financial institutions will file for bankruptcy defaulting on their debt and severely impairing capital allocation efficiency. This fear can lead agents to bid the

[11] The absence of equity markets is a limit case of a "skin in the game" constraint, which can be microfounded by an agency problem in which financial institutions can divert a fraction of asset returns at the expense of households (see, for example, Brunnermeier and Sannikov (2014)). Due to this constraint, markets are incomplete, so agents cannot write contracts conditional on the aggregate shocks.

capital price down to a level consistent with unproductive households managing the entire capital stock, which I refer to as the liquidation price of capital. If the capital price drop to liquidation value triggers the bankruptcy of financial institutions, the concern becomes self-fulfilled. If not, the initial fear has no consequences.

In other words, the economy is vulnerable to panics if financial institutions are not able to meet their deposit obligations at liquidation price for capital, i.e., $b_{e,t} > \tilde{q}_t k_{e,t}$, where $b_{e,t}$ represents debt, $k_{e,t}$ capital holdings, and $\tilde{q}_t$ the liquidation value of capital. Abusing notation, I use subindexes e and h to refer to an arbitrary expert and household, respectively. The latter condition is equivalent to require strictly positive (percentage) losses for financial institutions' debt holders when capital drops to its liquidation value, i.e.,

$$\ell_{b,t} \equiv \left[1 - \frac{\tilde{q}_t k_{e,t}}{b_{e,t}}\right]^+ > 0 \ , \tag{4}$$

where $\ell_{b,t}$ represents the mentioned loss and $[\cdot]^+ \equiv \max\{\cdot, 0\}$. After financial institutions file for bankruptcy, their assets are distributed among creditors proportionally to their claims.

The vulnerability of the economy to panics depends on the exposure of the entire sector to bankruptcy. No individual financial institution can influence this vulnerability by itself. Formally, the economy is vulnerable to panics if the measure of experts that file for bankruptcy at liquidation price for capital is equivalent to the total measure of experts, i.e., $\mathcal{B}(j \in \mathbb{J}_{e,t} : b_{j,t} > \tilde{q}_t k_{j,t}) = \mathcal{B}(j \in \mathbb{J}_{e,t})$ where $\mathcal{B}$ is the Borel measure. A finite number of experts avoiding bankruptcy is irrelevant from an aggregate point of view. Condition (4) is equivalent to this definition when there is no within sector heterogeneity.

The Poisson process J_t with endogenous intensity p_t captures the arrival of panics that become self-fulfilled (and trigger the bankruptcy of the financial sector as well as the collapses of output and capital price). It satisfies

$$p_t = 1_{\{\ell_{b,t} > 0\}} \Gamma_t \tag{5}$$

where Γ_t is the arrival intensity of panics (the losses of confidence). Over a short period (from t to $t + dt$), the probability that a panic arrives is $\Gamma_t dt$ and it becomes self-fulfilled only if the economy is vulnerable, i.e., $1_{\{\ell_{b,t} > 0\}} = 1$. Γ_t is an exogenously given mapping from *aggregate* variables to $\mathbb{R}_+$. Analytical results will be robust to the particular function chosen, which will be adjusted to reflect that financial collapses are rare events.

Panics are sunspots that allow agents to coordinate their concerns about the viability of the financial sector. The vulnerability of the economy to this generalized fear (i.e., the situations in which it becomes self-fulfilled) is the second instability studied in this paper.

2.2 Agents' problems and equilibrium

Agents choose their consumption $c_{j,t}$, investment $\iota_{j,t}$, capital holdings $k_{j,t}$, and debt position $b_{j,t}$. Let q_t denote the capital price in terms of the consumption good (the numeraire).[12]

Households. Denote the household's holdings of debt issued by experts as $b_{j,t}$ (an asset) and his net worth as $n_{j,t} = q_t k_{j,t} + b_{j,t}$. Given initial wealth $n_{j,0}$, household j maximizes (3) subject to $k_{j,t} \geq 0$, $n_{j,t} \geq 0$, and

$$dn_{j,t} = \left[(a_h - \iota_{j,t})\, k_{j,t} + r_t b_{j,t} - c_{j,t}\right] dt + d(q_t k_{j,t})\Big|_{dJ_t=0} - \left[\ell_{b,t} b_{j,t} + (q_t - \tilde{q}_t) k_{j,t}\right] dJ_t \qquad (6)$$

where the last term represents the cost households bear in case of a self-fulfilled panic: losses on their debt holdings $\ell_{b,t} b_{j,t}$ and on capital value $(q_t - \tilde{q}_t) k_{j,t}$. The other terms are standard and correspond to production net of investment $(a_h - \iota_{j,t})\, k_t$, return on debt holdings $r_t b_{j,t}$, consumption expenditure $c_{j,t}$, and capital gains absent self-fulfilled panics $d(q_t k_{j,t})\Big|_{dJ_t=0}$. The effects of productivity and volatility shocks are embedded in the latter.

For simplicity, this formulation assumes that all experts choose the same portfolio, and therefore, there is a unique debt contract available for households, which has return r_t conditional on no default. In general, the debt of financial institutions with different portfolios render different returns, i.e., they are distinct assets, because the portfolio determines the recovery rate in case of default.

Experts. Denote expert's financing through debt as $b_{j,t}$ (a liability) and his net worth as $n_{j,t} = q_t k_{j,t} - b_{j,t}$. Given initial wealth $n_{j,0}$, expert j maximizes (3) subject to $k_{j,t} \geq 0$,

[12] Given the presence of the aggregate Poisson process, some stochastic processes might not be continuous in time, i.e., it might be that $\lim_{s \uparrow t} x_s := x_{t-} \neq x_t$ (all processes are right-continuous). I do not make that distinction to alleviate notation and use the following convention. Given the stochastic differential equation $dx_t = \mu_{x,t} dt + \sigma_{x,t} dZ_t + \ell_{x,t} dJ_t$, processes $\{\mu_{x,t}, \sigma_{x,t}, \ell_{x,t}\}$ are evaluated in their left-limit. Since (t, Z_t) are continuous, this distinction is only relevant for the value of x_t in the case of $\{\ell_{x,t}\}$.

$n_{j,t} \geq 0$, and

$$dn_{j,t} = \left[(a_e - \iota_{j,t})\, k_{j,t} - r_t(k_{j,t}, b_{j,t}) b_{j,t} - c_{j,t}\right] dt + d(q_t k_{j,t})\Big|_{dJ_t=0} \\ - (\underbrace{\mathbb{1}_{\{\ell_{b,j,t}>0\}} n_{j,t}}_{\text{risky porfolio}} + \underbrace{\mathbb{1}_{\{\ell_{b,j,t}\leq 0\}} (q_t - \tilde{q}_t) k_{j,t}}_{\text{safe portfolio}}) dJ_t - \underbrace{\mathbb{1}_{\{\ell_{b,j,t}\leq 0 \wedge \ell_{b,t}>0\}} \phi}_{\text{underperforming cost}}\, dt \tag{7}$$

where $\ell_{b,j,t} \equiv \left[1 - \frac{\tilde{q}_t k_{j,t}}{b_{j,t}}\right]^+$ represents the percentage losses for expert's creditors (i.e., households) in case of a self-fulfilled panic when the expert chooses portfolio $(k_{j,t}, b_{j,t})$. Since assets are distributed proportionally to creditors' claims after bankruptcy, they recover $\tilde{q}_t k_{j,t}/b_{j,t}$ percent of their investment. The potential losses are priced into the interest rate charged to the expert, so it depends on his portfolio choice, i.e., $r_t(k_{j,t}, b_{j,t})$. In equilibrium, the latter function needs to be consistent with households' corresponding asset-pricing condition.

When a self-fulfilled panic arrives, i.e. $dJ_t = 1$, there are two cases depending on the strength of the chosen portfolio. Either the expert had chosen a risky portfolio, i.e., $\ell_{b,j,t} > 0$, and has to file for bankruptcy ($n_{j,t} = 0$) or he had chosen a safe portfolio, i.e., $\ell_{b,j,t} \leq 0$, and bears the losses from the drop in capital value ($(q_t - \tilde{q}_t)k_{j,t}$) without going bankrupt ($n_{j,t} > 0$).

If an expert chooses a safe portfolio when (almost) all other experts decide on a risky one, i.e., $\ell_{b,j,t} \leq 0$ and $\ell_{b,t} > 0$**,** he will underperform the market. I assume that, in these situations, the expert incurs in a prohibitively high underperforming cost ϕ. Given that financial panics are rare events, choosing the safe investment strategy implies underperforming the market for a prolonged period. The underperformance cost captures the fact that no asset manager can stay in his job long if he consistently underperforms the market.[13]

This assumption allows us to focus on symmetric equilibriums among experts. Otherwise, there is the possibility that some experts choose a risky portfolio and others a safe one. In symmetric equilibriums, the portfolio chosen for every individual expert is the same, so individual potential losses coincide with potential losses for the sector, i.e., $\ell_{b,j,t} = \ell_{b,t}$. This symmetry implies that the last two terms in (7) are zero in equilibrium.

Definition 1. *(Competitive equilibrium) For any initial endowment of capital* $\{k_{j,0}\}$ *such that* $\int_{\mathbb{J}_{h,0} \cup \mathbb{J}_{e,0}} k_{j,t} dj = k_0$*, a competitive equilibrium is a set of stochastic functions on the*

[13] Recent history is replete with examples of financial institutions that have chosen to underperform the market while waiting for a large collapse but were not able to sustain the strategy long enough, such as Tiger Management funds during the dot-com bubble.

filtered probability space defined by $\{Z_t, J_t : t \geq 0\}$*: aggregate outcomes* $\{q_t, \tilde{q}_t, r_t(k,b), \ell_{b,t}, p_t\}$*, and agents' decisions* $\{c_{j,t}, \iota_{j,t}, k_{j,t}, b_{j,t}\}_{j \in \mathbb{J}_h \cup \mathbb{J}_e}$ *such that*[14]

1. *Initial net worths satisfy* $n_{j,0} = q_0 k_{j,0}$ *for all* $j \in \mathbb{J}_{h,0} \cup \mathbb{J}_{e,0}$.
2. *Agents optimize: (i) Given* $\left\{q_t, \tilde{q}_t, r_t = r_t(k_{e,t}, b_{e,t}), \ell_{b,t}, p_t\right\}$*, decisions* $\{c_{j,t}, \iota_{j,t}, k_{j,t}, b_{j,t}\}$ *solve household's problem* $\forall j \in \cup_t \mathbb{J}_{h,t}$*, (ii) Given* $\{q_t, \tilde{q}_t, r_t(k,b), p_t\}$*, decisions* $\{c_{j,t}, \iota_{j,t}, k_{j,t}, b_{j,t}\}$ *solve expert's problem* $\forall j \in \cup_t \mathbb{J}_{e,t}$.
3. *Markets clear: capital* $\int_{\mathbb{J}_{h,t} \cup \mathbb{J}_{e,t}} k_{j,t} dj = k_t$*, debt* $\int_{\mathbb{J}_{h,t}} b_{j,t} dj = \int_{\mathbb{J}_{e,t}} b_{j,t} dj$*, and consumption goods* $\int_{\mathbb{J}_{h,t} \cup \mathbb{J}_{e,t}} c_{j,t} dj + \mathbb{1}_{\{\ell_{b,t} > 0\}} \phi \int_{\mathbb{J}_{e,t}} \mathbb{1}_{\ell_{b,j,t} < 0} dj = \int_{\mathbb{J}_{h,t} \cup \mathbb{J}_{e,t}} (a_j - \iota_{j,t}) k_{j,t} dj$.
4. *Aggregate volatility* s_t *follows (2) and aggregate capital stock satisfies the following law of motion, starting with* k_0

$$dk_t = \left(\int_{\mathbb{J}_{h,t} \cup \mathbb{J}_{e,t}} \Phi(\iota_{j,t}) k_{j,t} dj - \delta k_t \right) dt + k_t \sqrt{s_t} \sigma \cdot dZ_t \tag{8}$$

5. *Consistency conditions hold: (i)* $r_t(k_{j,t}, b_{j,t})$ *is consistent with the corresponding asset-pricing condition of households, (ii)* $\{p_t\}$ *is consistent with the endogenous exposure to panics, (iii)* $\{\tilde{q}_t\}$ *is consistent with* $\{q_t\}$*, i.e.,* $\lim_{z \uparrow t} \tilde{q}_z = q_t$ *if* $dJ_t = 1$.

3 Instabilities' dynamics

This section introduces the central insights of the paper: 1) the endogenous emergence of *risky low-volatility environments*: periods of smooth BCF fluctuations extensively exposed to crashes triggered by panics, and 2) the *stability paradox*: more stable fundamentals delivering a less stable economy. First, I discuss the metrics for the amplification of real shocks and the vulnerability to panics. Then, I show that, in the complete markets and autarky benchmarks, the economy follows a stable growth path without these endogenous instabilities. Finally, I describe the instabilities' joint dynamics, which deliver the main messages. The discussion about the mechanisms behind the results is postponed until we solve the model.

[14] The definition includes the symmetry assumptions embedded in agents' problems. In particular, when defining the assets available to households, it assumes a symmetric portfolio decision among experts. This considerably simplifies notation and exposition.

3.1 Instabilities' metrics

I focus on dynamics of aggregate output to summarize the two instabilities studied. However, results are qualitative equivalent if we consider the dynamics of capital value, i.e., if we focus on financial outcomes instead of macroeconomic ones.

Macroeconomic outcomes. Given the constant returns-to-scale production function, aggregate output satisfies $y_t = a_t k_t$ where $a_t \equiv \kappa_t(a_e - a_h) + a_h$ and κ_t is the fraction of capital managed by productive experts. Aggregate capital follows $dk_t = k_t \left(\mu_{k,t} dt + \sigma_{k,t} \cdot dZ_t\right)$ where $\mu_{k,t}$ and $\sigma_{k,t}$ are defined by the equivalence to (8). Meanwhile, capital allocation follows $da_t = a_t \left(\mu_{a,t} dt + \sigma_{a,t} \cdot dZ_t - \ell_{a,t} dJ_t\right)$ where $\{\mu_{a,t}, \sigma_{a.t}, \ell_{a,t}\}$ are endogenous equilibrium outcomes. This conjecture is verified later. Then, aggregate output evolves according to

$$\frac{dy_t}{y_t} = \underbrace{\left(\mu_{k,t} + \mu_{a,t} + \sigma_{a,t} \cdot \sigma_{k,t}\right)}_{\equiv \mu_{y,t}} dt + (\underbrace{\sigma_{k,t}}_{\text{exogenous}} + \underbrace{\sigma_{a,t}}_{\text{endogenous}}) \cdot \underbrace{dZ_t}_{\text{real shocks}} - \underbrace{\ell_{a,t}}_{\text{loss}} \underbrace{dJ_t}_{\text{self-fulfilled panics}} \tag{9}$$

The last two terms correspond to the two instabilities studied. Exogenous real shocks dZ_t are meant to capture BCF disturbances, so the sensitivity of output growth to these shocks summarized by $\|\sigma_{y,t}\| \equiv \sqrt{\sigma_{y,t} \cdot \sigma_{y,t}}$, where $\sigma_{y,t} \equiv \sigma_{k,t} + \sigma_{a,t}$, measures *the volatility of BCF fluctuations*. This volatility depends on the response of capital quantity $\sigma_{k,t}$, i.e., the exogenous or fundamental component of BCF fluctuations, and on the response of capital allocation efficiency $\sigma_{a,t}$, i.e., the endogenous component of BCF fluctuations. There is endogenous *amplification of real shocks* when $\left\|\sigma_{a,t} + \sigma_{k,t}\right\| > \left\|\sigma_{k,t}\right\|$. The latter condition is equivalent to $\|\sigma_{a,t}\| > 0$ if the capital share managed by experts decreases with adverse productivity shocks, as is the case in the equilibrium studied.

In contrast to real shocks, the arrival of self-fulfilled panic, i.e., $dJ_t = 1$, is endogenous: the economy is only vulnerable if the drop of capital price to liquidation value generates losses to financial institutions' creditors, i.e., $\ell_{b,t} > 0$. This loss is the metric I emphasize for the economy's *vulnerability to panics*. A complementary metric is the loss in output generated by the self-fulfilled panic $\ell_{a,t}$.[15]

Financial outcomes. Capital price follows $dq_t = q_t \left(\mu_{q,t} dt + \sigma_{q,t} \cdot dZ_t - \ell_{q,t} dJ_t\right)$ where $\{\mu_{q,t}, \sigma_{q.t}, \ell_{q,t}\}$

[15] This measure only depends on the drop in capital allocation efficiency because panics do not affect aggregate capital. Also, the expected loss of output $p_t \ell_{y,t}$ summarizes the two metrics discussed, however, it also depends on exogenous mapping Γ.

are endogenous equilibrium outcomes. This conjecture is verified later. Then, using law of motion for aggregate capital (8), total capital value in the economy (or total wealth) follows

$$\frac{d\left(q_t k_t\right)}{q_t k_t} = \left(\mu_{k,t} + \mu_{q,t} + \sigma_{q,t} \cdot \sigma_{k,t}\right) dt + \left(\sigma_{k,t} + \sigma_{q,t}\right) \cdot dZ_t - \ell_{q,t} dJ_t \tag{10}$$

Macro-financial linkages. Appendix A.2.2 shows that the effects of real shocks over aggregate output are amplified (i.e., $\|\sigma_{a,t}\| > 0$), if and only if, there is amplification over capital value (i.e., $\|\sigma_{q,t}\| > 0$). Moreover, in equilibrium, the drop in capital value $\ell_{q,t}$ after a self-fulfilled panic is a monotone transformation of the drop in aggregate output $\ell_{a,t}$. The intuition behind is simple: in equilibrium, capital price and the share of capital managed by experts co-move.

3.2 Benchmarks: complete markets and autarky

With complete financial markets, this is a standard AK growth model that features no amplification of real shocks or exposure to financial panics, i.e., all unexpected fluctuations in aggregate output and capital value correspond to the exogenous component of BCF fluctuations. Without financial frictions, the capitalization of the financial sector is irrelevant for aggregate outcomes: productive experts manage the entire capital stock and share risks optimally through financial markets. This competitive equilibrium is the first-best outcome.

Proposition 1. *(Complete markets) With complete markets, aggregate output is given by $y_t = a_e k_t$, capital price is fixed at $q^\star$, and*

$$\frac{dy_t}{y_t} = \frac{d(q_t k_t)}{q_t k_t} = g^\star dt + \sigma_{k,t} \cdot dZ_t \tag{11}$$

where $g^\star \equiv \Phi\left(\iota(q^\star)\right) - \delta$, function $\iota(q)$ is defined by $\Phi'(\iota) = 1/q$, and $q^\star$ solves $\rho q^\star + \iota(q^\star) = a_e$.

Given this benchmark, we can attribute the emergence of endogenous instabilities and their dynamics to financial frictions. Nevertheless, the intensity of the instabilities is not monotone in the degree of market incompleteness since shutting down markets entirely –i.e., reverting to autarky– also delivers an economy without amplification of real shocks or exposure to crashes.

Proposition 2. *(Autarky) Let experts' initial share of capital satisfy $\kappa_0 = \kappa^{aut}$. In autarky, aggregate output is given by $y_t = a^{aut} k_t \equiv \left[(a_e - a_h)\, \kappa^{aut} + a_h \right] k_t$, internal value of capital for experts and households are fixed at $q^\star$ and $q^\dagger$, respectively, and*

$$\frac{dy_t}{y_t} = g^{aut} dt + \sigma_{k,t} \cdot dZ_t \tag{12}$$

where $g^{aut} < g^\star$, $a^{aut} < a_e$, $\kappa^{aut} \in (\nu, 1)$ and $q^\dagger < q^\star$ are constants. If $\kappa_0 \neq \kappa^{aut}$, then there is a deterministic transition towards κ^{aut} before reaching the stochastic balanced-growth path described.

Debt is the essential ingredient for the appearance of endogenous instabilities: in autarky, there are no debt markets, while with complete markets, agents choose to finance through state-contingent contracts instead of issuing debt.

3.3 Financial frictions and instabilities

In contrast to the benchmark cases, the model with financial frictions occasionally exhibits endogenous instabilities: amplification of real shocks and vulnerability to panics. The presence and intensity of these instabilities depends on the states of the economy: the capital stock k_t, the volatility of productivity shocks s_t, and the distribution of wealth among agents. The latter is summarized by the wealth share of experts $w_t \equiv \int_{\mathbb{J}_{e,t}} n_{j,t} dj / q_t k_t$ and allocations scale linearly with the capital stock, so the relevant state-space reduces to (w_t, s_t). I also refer to w_t as the financial sector's capitalization relative to the entire economy. From now on, I switch to recursive notation, so time subindexes are suppressed. All equilibrium objects are functions of the aggregate states of the economy (w, s), but this dependence is left implicit.[16]

First, consider the instabilities' dynamics conditional on a given volatility of productivity shocks as illustrated by Figure 1(a).[17] BCF fluctuations are amplified when the financial sector's capitalization below threshold $w_\sigma(s)$ (i.e., $w < w_\sigma(s)$), while the economy is vulnerable to panics when that capitalization lies within an intermediate range (i.e., $\underline{w}_\ell(s) < w < \bar{w}_\ell(s)$).

[16] Also, I denote by $\tilde{z}$ the value that variable z would take if a self-fulfilled panic arrives when the aggregate state is (w, s), i.e., $\tilde{z} = z(\tilde{w}(w, s), s)$, where $\tilde{w}(w, s)$ is experts' wealth share just after a self-fulfilled panic.

[17] The given volatility level s must be sufficiently low. As discussed later, for sufficiently large s, the economy is not vulnerable to panics independently of the capitalization of the financial sector.

Even though both vulnerabilities will depend on the weakness of financial institutions' balance sheets, the vulnerability regions are different. Hence, the other determinants of these instabilities must differ. We shall discuss the mechanisms in detail later.

Figure 1: Risky low-volatility environments

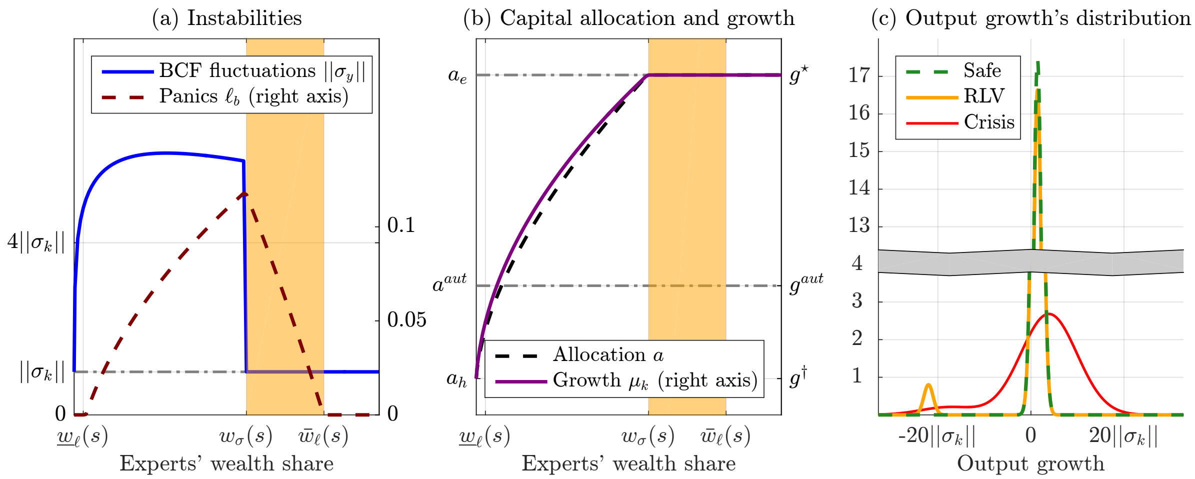

Figure 1. RLV environments correspond to the regions shaded in orange in panels (a) and (b). In panel (c), the distributions correspond to a mixtures of two normal distributions: $N(\mu_y\Delta t, \|\sigma_y\|^2\Delta t)$ with probability $(1-p\Delta t)$ and $N(\mu_y\Delta t-\ell_a, \|\sigma_y\|^2\Delta t)$ with probability $p\Delta t$. The period length is set to $\Delta t = 1$. The levels of w selected to illustrate the distributions correspond to $w^{crisis} = 0.75 w_\sigma$, $w^{RLV} = 0.5(w_\sigma + \bar{w}_\ell)$, and $w^{safe} = 1.25\bar{w}_\ell$. For all panels, the volatility of real shocks is fixed at its long run mean $s = \bar{s}$. The numerical illustration considers baseline parametrization detailed in Appendix B.

When the financial sector is moderately capitalized (i.e., $w_\sigma(s) < w < w_\ell(s)$), periods of low BCF volatility due to the absence of amplification of real shocks (i.e., $\|\sigma_a\| = 0$) that are susceptible to crashes triggered by panics (i.e., $\ell_b > 0$) endogenously emerge. See the shaded area in Figures 1(a) and 1(b). During these *risky low-volatility* (RLV) periods, the economy seems to follow a stable balanced-growth path with sound economic outcomes: the efficient capital allocation delivers high output, high capital price, and strong capital growth. In fact, output growth follows

$$\frac{dy}{y} = g^\star dt + \sigma_k \cdot dZ_t - \ell_a dJ_t \tag{13}$$

which is the complete-markets' outcome except for the vulnerability to panics.

There are two other risk regimes besides the novel *risky low-volatility* one: a *crisis* regime and a *safe* one. Each of these risk regimes presents qualitatively different equilibrium outcomes.

Crisis periods arise when the financial sector is poorly capitalized (i.e., $w < w_\sigma(s)$). These periods correspond to the typical downturn with depressed output, capital price, investment, and capital growth. Moreover, they are also highly unstable: BCF fluctuations are volatile (i.e., $\|\sigma_a\| > 0$) and the economy can be vulnerable to panics (i.e., $\ell_b \geq 0$).

On the other side of the spectrum, *safe* periods emerge when the financial sector is extensively capitalized (i.e., $w > \bar{w}_\ell(s)$). During these periods, the financial sector allows the economy to exhibit first-best outcomes in terms of output, capital price, investment and output growth without exposing the economy to endogenous instabilities: there is no amplification of real shocks or vulnerability to panics (i.e., $\|\sigma_a\| = \ell_b = 0$ and output evolves according to (11)).

Figure 1(c) illustrates the qualitative differences in the dynamics of equilibrium outcomes among risk regimes by plotting the conditional distributions of output growth. In the safe regime, capital allocation is fixed at its efficient level, so output growth is stable and inherits the distribution of capital growth: $N(g^\star dt, \|\sigma_k\| \sqrt{dt})$ for a horizon of length dt. In the RLV regime, output dynamics are identical except for one fundamental difference: with probability pdt, a panic arrives and becomes self-fulfilled, triggering an output drop of ℓ_a percent. This drop is captured by the second mode on the left tail the distribution. These two regimes are observationally equivalent in terms of output until a panic triggers a substantial collapse, sending the economy to the crisis regime.

During crisis periods, real shocks influence capital allocation efficiency, so output growth is more volatile than in the previous regimes. Moreover, the economy can be vulnerable to panics. If not exposed to panics, the conditional distribution of output growth is $N(\mu_y dt, \|\sigma_k + \sigma_a\| \sqrt{dt})$. If vulnerable to panics, a second mode on the left tail appears due to the possibility of a crash. Interestingly, the eventual crash in this crisis regime is less severe than the one that can arrive during RLV periods (i.e., loss ℓ_a is greater during RLV periods).

Now, consider the instabilities' dynamics as the volatility of productivity shocks varies. In terms of BCF fluctuations, more stable fundamentals reduce the range of financial sector's capitalizations where real shocks are amplified (i.e., $\partial w_\sigma(s)/\partial s > 0$), but it also intensifies the amplification wherever still present (i.e., $\partial \|\sigma_a\| / \partial s < 0$ for (w, s) s.t. $w < w_\sigma(s)$). Nevertheless, the response of the endogenous amplification is not sufficiently strong to overcome the exogenous decline in the volatility of real shocks, hence more stable fundamentals generate smoother, or at least equally smooth, BCF fluctuations (i.e., $\partial \left\|\sigma_k + \sigma_a\right\| / \partial s \geq 0$). These dynamics are illustrated by the heat maps in Figures 2(a) an 2(b).

Figure 2: The stability paradox (conditional on w)

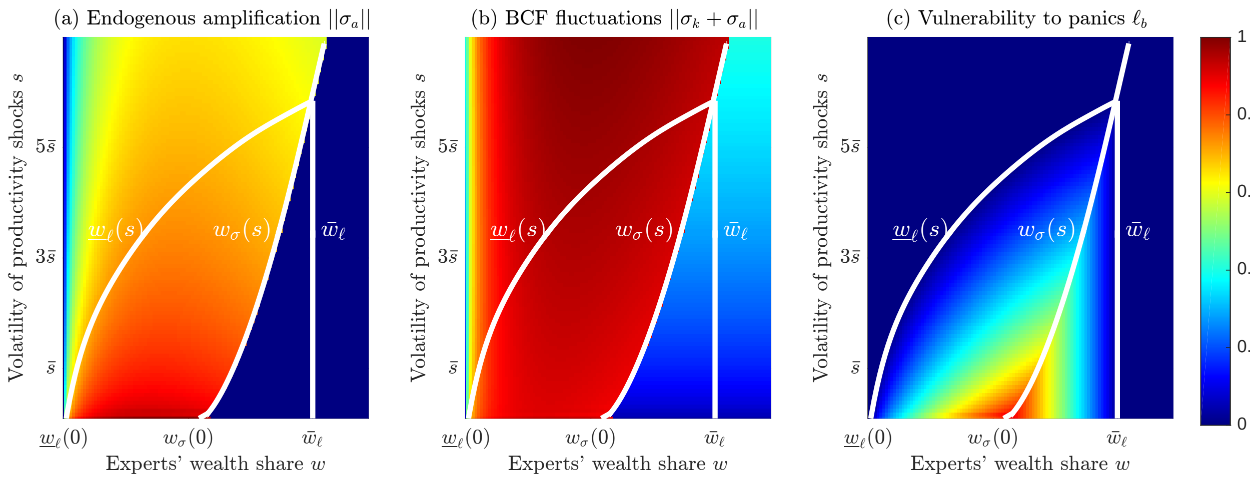

Figure 2. In each heat-map values are scaled by the maximum value of the variable. The bar at the right provides the color code for all panels. White lines in these panels correspond to thresholds for the presence of endogenous instabilities: $w_\sigma(s)$ for amplification and $\{\underline{w}_\ell(s), \bar{w}_\ell\}$ for the vulnerability to panics. The numerical illustration considers baseline parametrization detailed in Appendix B.

The stability paradox emerges when considering the effect of more stable fundamentals over the economy's exposure to panics. More stable fundamentals increase the vulnerability to panics (i.e., $\partial \ell_b / \partial s \leq 0$ and $\partial \left(\bar{w}_\ell(s) - \underline{w}_\ell(s)\right) / \partial s < 0$) as illustrated by Figure 2(c). This response, coupled with the weak reaction of the volatility of BCF fluctuations (due to the opposing responses of their exogenous and endogenous components), delivers a less stable economy when the volatility of real shocks decreases.

So far, the analysis has been conditional on the capitalization of the financial sector. However, more stable fundamentals also induce variation in the share of time the economy spends at different levels of w. Figure 3(a) illustrates that the stability paradox, if anything, strengthens after accounting for the response of conditional distribution $\mathcal{G}(w|s)$. The figure shows that overall instability is U-shaped as a function of the volatility of productivity shocks, where the instability metric is the average local variance of output growth conditional on s, i.e., $\mathbb{E}[\Sigma(w,s)|s]$ with

$$\Sigma(w,s) \equiv \mathbb{E}\left[(dy/y)^2|w,s\right] = \underbrace{\|\sigma_k\|^2}_{\text{exogenous BCF}} + \underbrace{\|\sigma_k+\sigma_a\|^2 - \|\sigma_k\|^2}_{\text{endogenous BCF}} + \underbrace{p\ell_a^2}_{\text{panics}} .$$

The stability paradox result applies when fundamentals are sufficiently stable, i.e., the s-region where the slope of the instability measure is negative.

The decomposition shown in Figure 3(a) evidences the central role of panics. As the volatility of productivity shocks declines, the contribution of the endogenous component of BCF fluctuations decreases or slightly increases at most. Hence, BCF fluctuations' contribution to instability declines. However, the contribution of crashes due to panics sharply rises, driving the increase of overall instability as s declines.

Indeed, the role of panics in the stability paradox goes beyond the contribution shown in the decomposition. Without panics, the contribution of the amplification of real shocks would collapse as s vanishes. The dashed line in Figure 3(a) shows the contribution of the endogenous component of BCF fluctuations in an economy without panics. The reason is that more substantial exposure to panics implies more frequent crashes that send the economy to a severe crisis, which counters the fact that experts need less wealth to escape the crisis regime, delivering a roughly constant share of time spent in these situations. Figure 3(b) summarizes the time the economy spends in each risk regime for different volatilities of real shocks. Interestingly, as fundamentals become more stable, the RLV regime becomes the most visited one at the expense of safe periods.

In terms of the level of outcomes, for a fixed capitalization of the financial sector, more stable fundamentals lead to a booming economy: it increases in aggregate output, capital price, investment, and capital growth. However, once it is recognized that the economy is more prone to deep crises when s is low (see dashed in Figure 3(b)), the potential boom becomes an expected recession. Figure 3(c) shows that expected aggregate output, investment, capital price, and capital growth have an inverted U-shape as functions of the volatility of real shocks.

At this point, the reader might be concerned about the lack of intuition provided about the presented results and their potential fragility. Recall that this subsection only previewed the main insights before working through the solution of the model. The next section explains the mechanisms and provides analytical characterization.

Figure 3: The stability paradox (including the effect over $\mathcal{G}(w|s)$)

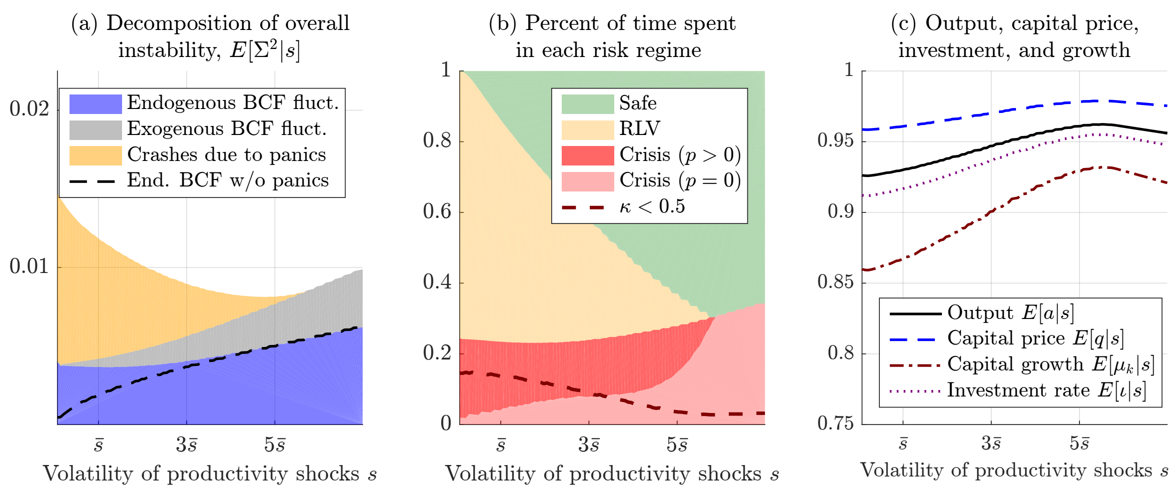

Figure 3. Panel (a) decomposes local variance into: exogenous BCF fluctuations $\mathbb{E}[\|\sigma_k\|^2|s]$, endogenous BCF fluctuations $\mathbb{E}[\|\sigma_k+\sigma_a\|^2-\|\sigma_k\|^2|s]$, and crashes due to panics $\mathbb{E}[p\ell_a^2|s]$. All variables in panel (d) are scaled by their level in the complete-markets benchmark. The numerical illustration considers baseline parametrization detailed in Appendix B.

4 Model characterization

The goal of this section is to understand the mechanisms behind the two endogenous instabilities –the amplification of real shocks and the vulnerability to panics– and provide an analytical characterization of the paper's two main insights: the emergence of risky low-volatility environment and the stability paradox. The first subsection characterizes the recursive equilibrium. The second provides two key insights about the interaction between the instabilities and the capital price. The final subsection revisits the rise of RLV environments and the stability paradox illustrated in section 3, detailing the intuition of the mechanisms behind them and providing analytical characterization.

4.1 Recursive Equilibrium

Capital returns. The return on capital managed by agent j is

$$dR_j^k \equiv \frac{(a_j - \iota_j)\,k_j}{qk_j}dt + \frac{d(qk_j)}{qk_j},$$

where the first term represents capital's dividend yield net of investment and the second term the capital gains rate, which includes price and capital quantity fluctuations. Recall that capital is measured in efficiency units, so $d(qk_j)$ captures the variation in the value of physical capital. Capital gains follow a process similar to (10) but considering individual capital evolution (1) instead of the aggregate one. Then, capital return for agent j becomes

$$dR_j^k = \mu_{R,j}dt + (\sigma_k + \sigma_q) \cdot dZ_t - \ell_q dJ_t \ ,$$

where $\mu_{R,j} \equiv (a_j - \iota_j)/q + \Phi(\iota_j) - \delta + \mu_q + \sigma_k \cdot \sigma_q$. The expected return conditional on no aggregate shocks, $\mu_{R,j}$, includes the dividend component and the deterministic capital gains. The latter part is associated with deterministic quantity gains $\mu_{k,j} \equiv \Phi(\iota_j) - \delta$, deterministic price gains μ_q, and the interaction between unexpected quantity and price gains due to real shocks (Ito's term).

Consumption. For both type of agents, optimal consumption is proportional to wealth, i.e., $\hat{c}_j = \rho$ where $\hat{c}_j \equiv c_j/n_j$ is agent j's consumption scaled by its wealth. Logarithmic preferences implies that consumption decisions are independent of asset returns.

Investment. The return on capital for both type of agents is maximized by choosing the investment rate that solves $\max_\iota \ q\Phi(\iota) - \iota$. Then, the investment rate is the same for all agents. The FOC $q\Phi'(\iota) = 1$ (marginal Tobin's Q) equates the marginal benefit of investment (extra capital $\Phi'(\iota)$ times its price q) with its marginal cost (a unit of final goods) and implicitly defines $\iota(q)$. The concavity of adjustment cost $\Phi(\cdot)$ implies that $\iota(q)$ is an increasing function of the price of capital. Therefore, anything that depresses the capital price will have a real effect through investment and capital growth.

Portfolio problems. The dynamic budget constraint for agent j, i.e. (6) or (7), can be written as

$$\frac{dn_j}{n_j} = (\mu_{n,j} - \hat{c}_j)\, dt + \sigma_{n,j} \cdot dZ_t - \ell_{n,j} dJ_t \ , \tag{14}$$

and the portfolio decision (k_j, b_j) is summarized by the share of net worth invested in capital $\hat{k}_j \equiv qk_j/n_j$ (if $\hat{k}_j > 1$, I refer to it also as leverage). Appendix A.2 shows that the portfolio problem is

$$\max_{\hat{k}_j \geq 0} \mu_{n,j} - \frac{1}{2}\left\|\sigma_{n,j}\right\|^2 + p \log\left(1 - \ell_{n,j}\right) \ . \tag{15}$$

Agents value the deterministic growth of net worth $\mu_{n,j}$ and dislike its fluctuations due to real shocks $\|\sigma_{n,j}\|$ and losses after self-fulfilled panics $\ell_{n,j}$. This problem will be independent of net worth's level, i.e., all experts will choose the same $\hat{k}_e$ and all households the same $\hat{k}_h$.

For a household, the dynamic budget constraint (6) becomes

$$\mu_{n,h} = \hat{k}_h \mu_{R,h} + \left(1 - \hat{k}_h\right) r \tag{16a}$$

$$\sigma_{n,h} = \hat{k}_h \left(\sigma_k + \sigma_q\right) \tag{16b}$$

$$\ell_{n,h} = \hat{k}_h \ell_q + \left(1 - \hat{k}_h\right) \ell_b \tag{16c}$$

Each component is a weighted average between the corresponding values for capital return and the return on debt holdings. The FOC for capital holdings is

$$\mu_{R,h} - r \leq \pi_h \cdot \left(\sigma_k + \sigma_q\right) + \alpha_h \left(\ell_q - \ell_b\right) \tag{17}$$

where $\pi_h \equiv \sigma_{n,h}$ and $\alpha_h \equiv p\left(1 - \ell_{n,h}\right)^{-1}$ are households' prices of risks, i.e., the excess return (absent shocks) that households require for additional exposure to real shocks and additional losses in case of a self-fulfilled panics, respectively. The LHS is the market excess return of capital over debt holdings (absent shocks) and the RHS is the excess return demanded by households to compensate for the additional exposure of capital to both risks. If households hold capital, i.e., $\hat{k}_h > 0$, then the latter expression must hold with equality. Lower required excess return represents an increase in capital demand. The following lemma characterizes households' capital demand.

Lemma 1. *(Households' capital demand) When $\hat{k}_h > 0$, the optimal portfolio share $\hat{k}_h$ is a strictly decreasing function of p and ℓ_q. Also, given $\mu_{R,h}$, $\hat{k}_h$ is a strictly decreasing function of $\|\sigma_k + \sigma_q\|$.*

Before moving on to experts' portfolio decisions, it is necessary to discuss how households would price the debt of experts with different portfolios. In equilibrium, households have access to a unique type of experts' debt because all choose the same portfolio $\hat{k}_e$; however, experts need to know how households would price their debt if they were to choose a different portfolio. The asset-pricing condition for an expert's portfolio with capital share $\hat{k}$ is

$$r(\hat{k}) - r^f = \alpha_h \ell_b(\hat{k}), \tag{18}$$

where $r^f \equiv -\mathbb{E}\left[dc_h^{-1}/c_h^{-1}\right]$ is the return households would require to hold a risk-free asset, and $\ell_b(\hat{k})$ is the percentage loss experienced by the creditors of an expert with share $\hat{k}$ invested in capital, in case of a self-fulfilled panic. The latter was introduced in equation (4) and can be written as a function of the share of net worth invested in capital:

$$\ell_b(\hat{k}) = \left[1-(1-\ell_q)\left(\frac{\hat{k}}{\hat{k}-1}\right)\right]^+. \tag{19}$$

The spread over the risk-free rate needs to compensate households for the potential losses and is increasing in the likelihood of a self-fulfilled panic p, the net worth drop $\ell_{n,h}$, and the loss rate on deposits $\ell_b(\hat{k})$. An intuitive derivation of this condition follows from including a risk-free asset with return r^f and a deposit with return $r(\hat{k})$ and loss $\ell_b(\hat{k})$ in households' problem.

For an expert, the dynamic budget constraint (7) becomes

$$\mu_{n,e} = \hat{k}_e\mu_{R,e} - \left(\hat{k}_e - 1\right) r(\hat{k}_e) - \phi(1-\vartheta(\hat{k}_e))\Upsilon \tag{20a}$$

$$\sigma_{n,e} = \hat{k}_e\left(\sigma_k + \sigma_q\right) \tag{20b}$$

$$\ell_{n,e} = \vartheta(\hat{k}_e) + \left(1-\vartheta(\hat{k}_e)\right)\hat{k}_e\ell_q \tag{20c}$$

where $\vartheta(\hat{k}_e) \equiv 1_{\{\ell_b(\hat{k}_e)>0\}}$ indicates if the portfolio chosen implies bankruptcy in case of a self-fulfilled panic and $\Upsilon \equiv 1_{\{\ell_b>0\}}$ indicates if the economy is exposed to financial panics, i.e., if (almost) all other experts chose such a portfolio. Υ is taken as given by any individual expert. The FOC for capital holdings is

$$\mu_{R,e} - r\left(\hat{k}_e\right) = \pi_e \cdot (\sigma_k + \sigma_q) + \left(1-\vartheta(\hat{k}_e)\right)\alpha_e\ell_q + r'\left(\hat{k}_e\right)(\hat{k}_e - 1),$$

where $\pi_e \equiv \sigma_{n,e}$ and $\alpha_e \equiv p\left(1-\ell_{n,e}\right)^{-1}$ are experts' prices of risks. The first term on the RHS is the compensation for capital return's exposure to real shocks. The second term is the compensation for capital losses in case of a self-fulfilled panic, which is only taken into account if the portfolio chosen prevents bankruptcy when capital price drops to liquidation value, i.e., $\vartheta(\hat{k}_e) = 0$. Otherwise, limited liability implies that marginal losses are irrelevant for expert's decision. The third term corresponds to the increase in financing cost, i.e., the increase in the deposit rate, $r'\left(\hat{k}_e\right) > 0$, times total debt scaled by net worth, $(\hat{k}_e - 1)$.

When the economy is vulnerable to self-fulfilled panics, the FOC presented has two solutions that correspond to two local maximums: a safe portfolio ($\vartheta(\hat{k}_e^s)=0$) that implies no failure after the panic, and a risky one ($\vartheta(\hat{k}_e)=1$) that triggers bankruptcy if the panic arrives. In the absence of the underperforming cost ϕ, there could be an asymmetric equilibrium among experts, i.e., one in which a fraction of them chooses the risky portfolio and the rest the safe one. Selecting the safe portfolio implies underperforming the market and paying the corresponding cost, which is never optimal by assumption.

Considering the risky portfolio (when the economy is exposed to panics) and using households' asset pricing condition (18), the FOC for experts becomes

$$\mu_{R,e}-r\left(\hat{k}_e\right)=\pi_e\cdot(\sigma_k+\sigma_q)+\alpha_h\ell_b'(\hat{k}_e)(\hat{k}_e-1). \tag{21}$$

It is key to notice that losses in case of a self-fulfilled panic are priced using the households' price of losses α_h. The reason is that experts only consider these losses in their marginal decision through the change in the interest rate paid on their debt, which is priced by its creditors: households. The following lemma characterizes experts' capital demand.

Lemma 2. *(Experts' capital demand) Optimal portfolio share* $\hat{k}_e=\left(\mu_{R,e}-r^f-\alpha_h\ell_q\right)/\left\|\sigma_k+\sigma_q\right\|^2$ *is a strictly decreasing function of* ℓ_q *and* p*. Also, given* $\mu_{R,e}$*, it is strictly decreasing in* $\|\sigma_k+\sigma_q\|$*.*

Market clearing. Market clearing for capital (scaled by total wealth) is

$$\hat{k}_e w+\hat{k}_h(1-w)=1, \tag{22}$$

where $\left(\hat{k}_e w\right)qk$ is total wealth invested in capital by experts, and $\left(\hat{k}_h(1-w)\right)qk$ is the corresponding quantity for households. Since capital is the only asset in positive net supply in this economy, total wealth in the economy is qk. For convenience, define $\kappa\equiv\hat{k}_e w$ as the capital share managed by experts. Market clearing for goods (scaled by total capital) is

$$\rho q+\iota(q)=\kappa a_e+(1-\kappa)a_h, \tag{23}$$

where the LHS represents aggregate demand, and the RHS is aggregate supply. The condition assumes that no financial institution pays the underperforming cost as it is the case in a symmetric equilibrium.

Evolution of aggregate states. While aggregate volatility of productivity shocks s evolves exogenously according to (2), the evolution of experts' wealth share w is endogenous with law of motion $dw/w = \mu_w dt + \sigma_w \cdot dZ - \ell_w dJ_t$ where $\{\mu_w, \sigma_w, \ell_w\}$ are equilibrium objects characterized in the following lemma.

Lemma 3. *(Experts' wealth share law of motion)*

$$w\mu_w = w(1-w)\left(\hat{k}_e - \hat{k}_h\right)\left(\left(\hat{k}_e + \hat{k}_h - 1\right)\|\sigma + \sigma_q\|^2 + \alpha_h\left(\ell_q - \ell_b\right)\right) + \lambda_d(\nu - w) \tag{24a}$$

$$w\sigma_w = w(1-w)\left(\hat{k}_e - \hat{k}_h\right)(\sigma + \sigma_q) \tag{24b}$$

$$w\ell_w = w - \tilde{w} \tag{24c}$$

where $\tilde{w}$ is the wealth share of experts after a self-fulfilled panic.

Consistency conditions. (i) Capital price dynamics is consistent with the evolution of the aggregate states, i.e.,

$$q\mu_q = q_w\left(w\mu_w\right) + q_s\left(\lambda_s\left(\bar{s} - s_t\right)\right) + \frac{1}{2}q_{ww}\|w\sigma_w\|^2 + \frac{1}{2}q_{ss}\left\|\sqrt{s}\sigma_s\right\|^2 + q_{ws}\left(w\sigma_w\right)\cdot\left(\sqrt{s}\sigma_s\right) \tag{25a}$$

$$q\sigma_q = q_w\left(w\sigma_w\right) + q_s\left(\sqrt{s}\sigma_s\right) \tag{25b}$$

$$q\ell^q = q - \tilde{q} \tag{25c}$$

These conditions follow from applying Ito's lemma to function $q(w,s)$. (ii) Losses of experts' creditors are consistent with their portfolio and capital price dynamics, i.e., (19) evaluated at $\hat{k}_e$ delivers ℓ_b. (iii) The arrival rate of self-fulfilled panics is consistent with the endogenous exposure to panics, i.e., (5) holds. (iv) The wealth share of experts after a self-fulfilled panic is consistent with bankruptcy procedures, i.e., $\tilde{w} = 0$.

Definition 2. *A Markov equilibrium in (w,s) is a set of aggregate functions $q, \mu_q, \sigma_q, \ell_q, r, \ell_b, p, \tilde{w}$, policy functions $\hat{k}_e, \hat{k}_h$ and a law of motion for the endogenous state variable μ_w, σ_w, ℓ_w such that: 1) Given aggregate functions and the evolution of w and s, $\hat{k}_e$ and $\hat{k}_h$ solve portfolio problems for experts and households, respectively:* (17) *and* (21)*. 2) Markets clear:* (22) *and* (23)*. 3) The law of motion of w satisfies* (24)*. 4) Consistency conditions (i)-(iv) hold.*

4.2 Instabilities and capital price

This subsection discusses two insights that link the capital price and instabilities. First, while capital price decreases when the amplitude of BCF fluctuations increases, it does not respond to variations in the exposure to panics. Second, while both instabilities hinge on the strength of financial institutions' balance sheet, they depend on distinct features of the dynamics of capital price.

4.2.1 Instabilities' effects over capital price

Relative capital demand. Optimal portfolio conditions (17) and (21) together with expert's balance sheet identity (28), which is described below, characterize the relative capital demand $\hat{k}_e - \hat{k}_h$:

$$\left(\hat{k}_e - \hat{k}_h\right) \|\sigma_k + \sigma_q\|^2 \leq \frac{a_e - a_h}{q} \tag{26}$$

with equality for $\hat{k}_h > 0$. The relative capital demand of experts increases with the excess capital return related to the productivity gap $(a_e - a_h)/q$ and decreases with the risk associated a BCF fluctuations $\|\sigma_k + \sigma_q\|$ but is invariant to the exposure to panics.

In equilibrium, experts choose a portfolio more biased towards capital than households (i.e., $\hat{k}_e > \hat{k}_h$), so they are more vulnerable to instabilities and require greater compensations for the exposure (i.e., $\pi_e > \pi_h$ and $\alpha_e > \alpha_h$). The latter implies that an increase in an instability should reduce the demand of experts relative to households. This logic holds for the volatility of BCF fluctuations but breaks for the exposure to panics because of bankruptcy.

The invariance result follows from the equivalence of the contractions in capital demands of experts and households (last terms in (21) and (17), respectively) when there is exposure to panics: $\alpha_h \ell_b'(\hat{k}_e)(\hat{k}_e - 1) = \alpha_h (\ell_q - \ell_b)$. Two factors are behind this equality, both related to bankruptcy. First, the increase in the interest rate paid by experts –the cost of additional losses to creditors– is priced using households' price of losses α_h. Second, losses in capital value ℓ_q and to experts' creditors ℓ_b are linked through expert's balance sheet identity (scaled by net worth)

$$\hat{k}_e \ell_q = 1 + \ell_b\left(\hat{k}_e\right)\left(\hat{k}_e - 1\right) \tag{27}$$

where the LHS represents total losses on expert's assets and the RHS, how these losses are split between the expert (drop in net worth) and his creditors (drop in value of their claims).

In particular, the marginal increase in expert's capital holdings, i.e., the derivative with respect to $\hat{k}_e$, delivers the condition needed:

$$\ell_q = \ell_b + \ell_b'(\hat{k}_e)(\hat{k}_e - 1) \tag{28}$$

Since expert's losses are capped to net worth (due to limited liability), all additional losses from increasing capital holdings $\hat{k}_e$ are ultimately born by households: through the losses on the credit extended to finance the additional capital holdings ℓ_b and the extra losses $\ell_b'(\hat{k}_e)$ imposed over the entire stock of debt $(\hat{k}_e - 1)$.

To illustrate the critical role of bankruptcy and limited liability, consider the *off-equilibrium* relative capital demand of an expert that chooses a safe portfolio ($\vartheta(\hat{k}_e^s) = \ell_b(\hat{k}_e^s) = 0$) and his creditors:

$$\left(\hat{k}_e^s - \hat{k}_h^s\right) \|\sigma_k + \sigma_q\|^2 + \left(\alpha_e \hat{k}_e^s - \alpha_h \hat{k}_h^s\right) \ell_q \leq \frac{a_e - a_h}{q} \tag{29}$$

In this case, the relative capital demand $\hat{k}_e^s - \hat{k}_h^s$ is influenced by the exposure to self-fulfilled panics. In particular, larger exposure to panics (p or ℓ_q) translates into a decrease of $\hat{k}_e^s - \hat{k}_h^s$, i.e., the agent with the larger exposure to the instability has greater incentives to reduce this exposure. This follows from experts now taking into account marginal losses on capital value and pricing them according to their marginal valuation of losses α_e (instead of households' α_h).

The described invariance rules out direct effects from the exposure to panics to capital allocation through capital demands. Also, since the invariance relies basically on limited liability and accounting identities, it will be robust to several extensions of the model as shown later.

Capital allocation and price. The previous discussion characterizes *relative capital demand,* I now illustrate the effect of instabilities over capital allocation κ and capital price q. Together with market clear condition for capital (22), relative capital demand (26) becomes a relation between κ and q conditional on the relevant instability: $\kappa^d(q; \|\sigma_k + \sigma_q\|)$. This demand-side equilibrium relationship is decreasing in capital price $\partial\kappa^d/\partial q \leq 0$ (higher capital price decreases the return gap due to experts' productivity advantage) and in the volatility of BCF fluctuations of capital returns $\partial\kappa^d/\partial\|\sigma_k + \sigma_q\| \leq 0$ (experts are more exposed to this instability). The supply-side equilibrium relationship $\kappa^s(q)$ is implicitly defined by goods market equilibrium condition (23) and it satisfies $\partial\kappa^d/\partial q > 0$: a more efficient capital allocation corresponds to a larger capital price. Instabilities have no direct influence over this condition. Together demand and supply determine κ and q conditional on instabilities (and the economy's states).

More volatile BCF fluctuations (i.e., $\|\sigma_k + \sigma_q\|$) reduce both, κ and q, while the exposure to self-fulfilled panics (i.e., p, ℓ_b or ℓ_q) has no effect over them. Of course, since instabilities are endogenous objects, this is a partial equilibrium analysis. There is still the possibility that an exogenous change in the exposure to self-fulfilled panics (e.g., an increase in the exogenous arrival rate of sunspots Γ that allows coordination of agents' fears) influences κ and q by affecting the other instability $\|\sigma_k + \sigma_q\|^2$. The following theorem rules this out: exposure to panics has no role in the determination of capital allocation and price (conditional on the economy's states).

Theorem 1. *(Capital's invariance to panics) Capital allocation κ is invariant to financial panics, i.e., equilibrium function $\kappa(w,s)$ is the same as in the model without panics ($p \equiv 0$). The same applies to capital price $q(w,s)$ and capital allocation efficiency $a(w,s)$.*

Indeed, this result is stronger than the central message of this discussion: the key instability for equilibrium capital allocation and price is the amplitude of BCF fluctuation of capital returns and not the exposure to panics.

4.2.2 Asset prices' effects over instabilities

The following discussion emphasizes that, although both instabilities operate through the balance sheets of financial intermediaries and intensify as these weaken, each one depends on a distinct feature of capital price's dynamics.

The exposure to panics is characterized by (19) evaluated at $\hat{k}_e$, i.e., $\ell_b\left(\hat{k}_e, \ell_q\right)$, while using (24b) and (25b), the volatility of BCF fluctuations can be written as

$$\sigma_k + \sigma_q\left(\hat{k}_e, \varepsilon_{q,w} | \mathcal{Q}\right) = \frac{\mathcal{Q}}{1 - \varepsilon_{q,w}\left(\hat{k}_e - 1\right)} \tag{30}$$

where $\varepsilon_{q,w} \geq 0$ is the elasticity of capital price with respect to experts' wealth share and $\mathcal{Q} = \sigma_k + q_s\sqrt{s}\sigma_s \in \mathbb{R}^d$ corresponds to the direct impacts of the real shocks over capital value.

Both instabilities intensify when experts' leverage $\hat{k}_e$ is substantial, i.e, $\partial \|\sigma+\sigma_q(\cdot|\mathcal{Q})\| / \partial \hat{k}_e \geq 0$ and $\partial \ell_b / \partial \hat{k}_e \geq 0$. However, while the amplification of real shocks increases with the sensitivity of capital price to relatively small changes in the experts' wealth share, i.e., $\partial \|\sigma+\sigma_q(\cdot|\mathcal{Q})\| / \partial \varepsilon_{q,w} \geq 0$, the exposure to self-fulfilled panics increases with the gap between current capital price and its liquidation value $\partial \ell_b / \partial \ell_q \geq 0$.

The denominator in (30) summarizes the amplification mechanism for a real shock with direct impact of $\mathcal{Q}$ percent on capital value. A one percent drop in capital value generates a $\left(\hat{k}_e - 1\right)$ percent drop in experts' wealth share w (combine equations (24b) and (22)) because experts are more exposed to capital price fluctuations, i.e., $\hat{k}_e > \hat{k}_h$. Then, the one percent initial drop in capital price generates a second-round reduction of $\zeta \equiv \varepsilon_{q,w}\left(\hat{k}_e - 1\right)$ percent. Iteration on this logic delivers a third-round effect of ζ^2 percent and a total drop of $1/(1-\zeta)$ percent, as illustrated in the expression above.

There are two necessary conditions for this amplification mechanism to operate: First, the endogenous state w needs to respond to a change in capital price $\sigma_w \neq 0$. Second, the capital price needs to be sensitive to small changes in capital price $\varepsilon_{q,w} \neq 0$. Since experts exploit their productivity advantage by taking leverage, the first condition is always satisfied (see equation (24b)). Then, absence of amplification corresponds to situations where $\varepsilon_{q,w} = \zeta = 0$.[18]

4.3 Instabilities' dynamics

Having understood the determinants of the two instabilities and their feedback effects over asset prices, we revisit the two main insights of the paper, i.e., *risky low-volatility* environments and the stability paradox, to discuss the intuition behind these results and provide the analytical characterization.

4.3.1 Dynamics for a given volatility of real shocks

First, I examine the vulnerability regions of each instability for a given volatility of real shocks. Then, I discuss the distinct risk regimes that emerge from these dynamics, focusing on the novel RLV regime. Figure 1 introduced the results discussed below.

[18] As detailed later, situations with no amplification coincide with the ones in which real shocks have no direct effects over capital price, because it is fixed at the level consistent with the efficient capital allocation (i.e., $q = q^\star$). Therefore, it is indifferent to define the absence of amplification as $\|\sigma_q\| = 0$ or $\zeta = 0$, but, conceptually, it corresponds to the latter.

BCF fluctuations. Real shocks are amplified when experts' wealth share is sufficiently low, i.e., $w < w_\sigma(s)$. As discussed above, this endogenous amplification only occurs if the capital price is sensitive to the variation in financial sector's capitalization induced by the real shock (i.e., if $\varepsilon_{q,w} \neq 0$), and changes in w affect capital price only if they generate a reallocation of capital among sectors. When experts are sufficiently capitalized, they hold the entire capital stock ($\kappa = 1$ and condition (26) holds as an strict inequality), and after a small negative shock to their wealth share, they decide to increase their debt and keep managing all capital. Therefore, capital price is not affected (i.e., $\varepsilon_{q,w} = 0$) and there is no amplification mechanism.

When experts are not sufficiently capitalized, holding the entire capital stock (i.e., $\hat{k}_e = 1/w$) implies an excessive exposure to BCF fluctuations, so experts hold only a fraction of the capital stock ($\kappa < 1$ and condition (26) holds with equality). In these cases, a small negative shock to experts' wealth share induces a reduction in their capital holdings: experts fire-sale capital to unproductive households to limit their exposure to the relevant instability. As a consequence, capital allocation efficiency decreases as well as capital price ($\varepsilon_{q,w} > 0$). The first part of Lemma 4 formalizes the economy's exposure to the amplification of real shocks, which drive BCF fluctuations.

Lemma 4. *(Amplification of real shocks) Assume the existence of an equilibrium in which* $w_\sigma(s) \equiv \min\{w : q(w,s) = q^\star\}$ *is continuous in* s*. Then, there is an equilibrium in which BCF fluctuations are amplified (i.e.,* $\|\sigma_q\| > 0$ *for a subset of the state-space with positive measure) if and only if financial sector's capitalization is sufficiently low (i.e., states* (w,s) *satisfy* $w < w_\sigma(s)$*).*

At one extreme, when the volatility of productivity shocks explodes, there amplification vanishes (i.e., $\lim_{s\to\infty} \|\sigma_q\| = 0 \ \forall w \in [0,1]$*). At the other extreme, when volatility of productivity shocks vanishes, amplification of real shocks persist (i.e.,* $w_\sigma(0) > 0$*).*

Vulnerability to panics. The economy can be vulnerable to panics only at intermediate levels of experts' capitalization, i.e., $\underline{w}_\ell(s) < w < \bar{w}_\ell(s)$. At one extreme, when experts' wealth share is large, i.e., $w > \bar{w}_\ell(s)$, the leverage they need to hold the entire capital stock is low, so although the potential fall in capital price is substantial, experts' net worth is sufficient to absorb any losses. At the other extreme, when experts hold a small share of wealth, i.e., $w < \underline{w}_\ell(s)$, capital misallocation is severe, and the capital price is depressed.

Even though experts' leverage is high, the economy is shielded from panics because the potential drop to liquidation value is not sufficiently large to wipe out experts' net worth. The first part of Lemma 5 formalizes this intuition.

Lemma 5. *(Vulnerability to panics) There exist an equilibrium in which the economy is not vulnerable to panics (i.e., $\ell_b(w,s)=0$) if the financial sector is deeply undercapitalized or extensively capitalized (i.e., states (w,s) such that $s>0$ and $w \notin [\underline{w}_\ell(s), \bar{w}_\ell]$ where $0 < \underline{w}_\ell(s) \leq \bar{w}_\ell < 1$).*

At one extreme, when the volatility of productivity shocks is sufficiently high, the economy is not exposed to panics. At the other extreme, when volatility of productivity shocks vanishes, the exposure to self-fulfilled panics is guaranteed (i.e.,$\underline{w}_\ell(s) < \bar{w}_\ell$). The latter statement considers $\Phi(\iota) \equiv \log(\varphi_\iota \iota - 1)/\varphi_\iota$.

Risk regimes and RLV periods. The amplification of real shocks translates into more volatile BCF fluctuations, while the vulnerability to panics represents a low probability of a substantial economic and financial meltdown. Therefore, the dynamics of equilibrium outcomes have distinct qualitative features depending on the presence of these instabilities as illustrated by the conditional distribution of output growth (recall Figure 1(c)). Section 3 introduced three distinct regimes: 1) the unstable *crisis* regime, 2) the *risky low-volatility* regime, and 3) the stable *safe* regime.

In the *safe* regime, experts are sufficiently capitalized (i.e., $w > \bar{w}_\ell$) to hold the entire capital stock without relying on substantial leverage and shield this allocation from real shocks by adjusting their debt financing after unexpected variations on their wealth. The limited leverage prevents vulnerability to panics (i.e., $\ell_b = 0$), and the resilience to real shocks implies no amplification (i.e., $\|\sigma_q\| = 0$). In the *crisis* regime, experts' wealth share is sufficiently low (i.e., $w < w_\sigma(s)$) to prevent them from holding the entire capital stock and make their capital holdings sensitive to real shocks, which means that they are endogenously amplified (i.e., $\|\sigma_q\| > 0$). Moreover, in these situations, the economy might be exposed to self-fulfilled panics.

These two regimes –also highlighted by previous research– deliver a one-dimensional picture of financial stability: either real shocks are not amplified, and the economy is stable, or they are amplified, and volatile BCF fluctuations evidence the economy's instability. The introduction of financial panics opens the possibility for fragile periods (i.e., $\ell_b > 0$) with smooth

BCF fluctuations (i.e., $\|\sigma_q\| = 0$): the *risky low-volatility* environments. Moreover, while unstable periods are associated with a poorly capitalized financial sector when considering only the amplification of real shocks, the introduction of panics unveils fragilities when the financial sector is moderately capitalized but still heavily relies on debt.

In the RLV regime, experts' wealth share is sufficiently large (i.e., $w > w_\sigma(s)$) to enable them to hold the entire capital stock and maintain their capital holdings after negative real shocks (i.e., $\|\sigma_q\| = 0$). However, it is not large enough (i.e., $w < \bar{w}_\ell$) to achieve this without extensive debt, which coupled with a high capital price –consistent with the efficient capital allocation– makes the economy vulnerable to panics (i.e., $\ell_b > 0$). This discussion about risk regimes assumed the presence of RLV environments for the volatility of real shocks s studied (i.e., $\underline{w}_\ell(s) < w_\sigma(s) < \bar{w}_\ell$). The first part of Theorem 2 characterizes the necessary and sufficient condition for the emergence of these seemly stable environments.

Theorem 2. *(Risky low-volatility periods) Under the assumption of Lemma 4, there exist an equilibrium that satisfies the following. For a given volatility of productivity shocks, there exists a risky low-volatility environment (i.e., $\mathcal{B}(\Psi(s)) > 0$ where $\Psi(s) \equiv \{w : \ell_b(w,s) > 0 \wedge \|\sigma_q(s,w)\| = 0\}$) if and only if the economy is vulnerable to panics at the threshold $w_\sigma(s)$ (i.e., $\ell_b(w_\sigma(s),s) > 0$). Moreover, if $\mathcal{B}(\Omega(s)) > 0$, then $\Psi(s) = (w_\sigma(s), \bar{w}_\ell)$.*

At one extreme, when the volatility of productivity shocks is sufficiently high, there are no RLV environments (i.e., $\mathcal{B}(\Psi(s)) = 0$ if $s > s_{up}$ where $s_{up} \in \mathbb{R}_+$). At the other extreme, when volatility of productivity shocks vanishes, the presence of a risky low-volatility environment is guaranteed (i.e., $\ell_b(w_\sigma(0),0) > 0$). The latter statement considers $\Phi(\iota) \equiv \log(\varphi_\iota \iota - 1)/\varphi_\iota$.

Consider the threshold situation in which experts have just accumulated sufficient wealth to ensure the efficient capital allocation and no amplification of BCF fluctuations, i.e., $w = w_\sigma(s) + \epsilon$ where $\epsilon > 0$ small. The recently achieved allocation efficiency and resilience to real shocks rely heavily on debt funding. In fact, among the circumstances in which there is no amplification of real shocks, the mentioned situation presents the largest experts' leverage. Therefore, if the economy is resilient to panics in this situation, it will be when experts own a larger share of wealth, and they need less leverage to manage the entire capital stock.

The absence of feedback from the vulnerability to panics to the capital price plays a key role in the emergence of RLV periods. If the exposure to panics depressed experts' capital demand relative to households –and therefore, capital price–, then the mere vulnerability to panics could make the capital allocation sensitive to changes in experts' wealth share ($\varepsilon_{q,w} > 0$) triggering the amplification mechanism.[19] In other words, without the invariance result, threshold $w_\sigma(s)$ increases and RLV periods might not be viable (i.e., $w_\sigma(s) > \bar{w}_\ell$).

4.3.2 Dynamics as the volatility of real shocks varies

I examine how the instabilities' dynamics vary with the exogenous volatility of real shocks, providing the intuition behind the stability paradox. Figures 2 and 3 introduced the results discussed below.

Dynamics conditional on financial sector's capitalization. More stable fundamentals encourage experts to expand their capital holdings, so they are able to hold the entire capital stock and shield this capital allocation from real shocks with less wealth (i.e., $w'_\sigma(s) > 0$). Although the range of capitalizations where real shocks are amplified shrinks, the amplification intensifies wherever still present because experts choose weaker balance sheets (i.e., larger leverage $\hat{k}_e$). These dynamics correspond to the *volatility paradox* result coined by Brunnermeier and Sannikov (2014): the increase of endogenous amplification makes the volatility of BCF fluctuations persist as the direct effect of real shocks vanishes. The second part of Lemma 4 captures a version of the volatility paradox result for this economy.

The volatility paradox, however, does not imply that more stable fundamentals lead to a less stable economy: the rise of the endogenous component of BCF fluctuations is not sufficiently strong to overcome the initial reduction of the exogenous one. The amplitude of BCF fluctuations (i.e., $\|\sigma_k + \sigma_q\|$) declines when the volatility of real shocks decreases for a given capitalization of the financial sector. Proposition 3 formalizes this result considering a static comparison exercise across economies with constant volatility of real shocks.[20]

In contrast to their effect on BCF fluctuations, more stable fundamentals do increase the economy's vulnerability to panics. As the volatility of real shocks declines, the more stable BCF fluctuations translate into a more efficient capital allocation and a higher capital price. Then, both determinants of the vulnerability to panics –experts' leverage $\hat{k}_e$ and the po-

[19] The LHS of (26) would include a difference in compensations for exposure to panics –as in (29)– that would make it hold with equality at states where it was an inequality before.

[20] The result also holds for numerical exercises with the baseline model. The simplified economy allows a tighter analytical characterization.

tential drop to liquidation price ℓ_q— increase as s declines.[21] The second part of Lemma 5 captures the negative relationship between stability of fundamentals and exposure to panics by characterizing the limiting cases. It shows that the economy is shielded against panics if s is sufficiently large, and that the exposure to panics is guaranteed when s vanishes. Proposition 3 formalizes the result beyond the limiting cases considering a static comparison exercise.

Proposition 3. *(Stability paradox illustration) Consider a continuum of economies with constant volatility of productivity shocks (i.e., $\sigma_s = \lambda_s = 0$), and index these otherwise identical economies by the mentioned volatility s. Given w, the exposure to self-fulfilled panics $\ell_b(w|s)$ increases, while the amplitude of BCF fluctuations $\|\sqrt{s}\sigma + \sigma_q(w|s)\|$ decreases, as s declines.*

In terms of the risk regimes, as real shocks become less volatile, the range of financial sector's capitalizations that corresponds to crisis regimes shrinks in favor of the corresponding range for RLV periods. The second part of Theorem 2 characterize the limiting cases: for a sufficiently large s, there are no RLV periods, while the presence of RLV periods is guaranteed for sufficiently stable fundamentals.

Impact over the distribution of financial sector's capitalization. The analytical results discussed provide an insight on the stability paradox conditional on a given capitalization of the financial sector. However, the stability of fundamentals also influences the share of time the economy spends at distinct levels of capitalization of the financial sector, i.e., the stationary distribution $\mathcal{G}(w|s)$. In order to incorporate this effect into the instability analysis, we rely on numerical illustrations (analytical result are not available). This is not a quantitative paper, and numerical exercises are meant to illustrate qualitative properties using parameters that deliver reasonable outcomes. The baseline functional forms and parameters are briefly discussed in Appendix B and they target standard moments except for the frequency of financial crashes (once every 35 years) and the maximum drop in asset prices after these events (30 percent).

As illustrated by Figure 3, taking into account the variation in $\mathcal{G}(w|s)$, if anything, intensifies the stability paradox message: more stable fundamentals deliver a less stable economy.

[21] The mechanism described does not operate when experts already manage the entire capital stock (i.e, in the RLV and safe regimes). Hence, the upper bound of the range of experts' wealth shares vulnerable to panics (i.e., $\bar{w}_\ell$) is independent of s.

Figure 4: Stability of fundamentals, the stationary distribution, and welfare

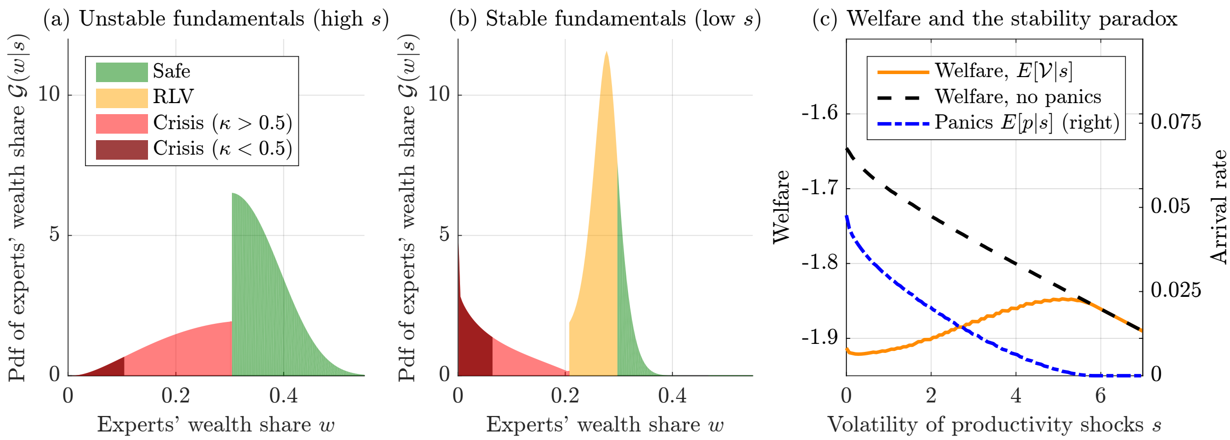

Figure 4. Panel (a) considers $s = 6\bar{s}$ and panel (b) to $s = \bar{s}$. Panel (c) considers the welfare measure associated with a fictional representative agent. See appendix A.2.1. This numerical illustration considers the baseline parametrization except for the aggregate volatility process, which dynamics are muted in the last panel (i.e., $\lambda_s = \sigma_s = 0$).

Figures 4(a) and 4(b) illustrate the distribution $\mathcal{G}(w|s)$ when fundamentals are stable (i.e., s is sufficiently low to make the economy vulnerable to panics) and when they are unstable, respectively.

In the case of unstable fundamentals, the economy is frequently knock down from the safe regime into the crisis one by the real shocks, but the system transits out of the crisis relatively fast, because agents receive compensation (i.e., higher portfolio returns) for bearing the larger exposure to real shocks that the endogenous amplification implies, and experts are more exposed than households. This explains the sharp variation in the density at threshold $w_\sigma(s)$. The economy rarely enters deep crises (i.e., cases with severe capital misallocation: $\kappa < 0.5$), which occurs only after a long sequence of negative real shocks.

In the case of unstable fundamentals, the economy spends roughly the same amount of time in the crisis regime. The economy is rarely knocked down to the crisis regime by the real shocks, i.e., the system seldom enters the crisis regime through threshold $w_\sigma(s)$. However, the economy is occasionally send directly to a deep recession by the arrival of a self-fulfilled panic, i.e., the system jumps to $w = 0$. These dynamics explain the second mode of the distribution at $w = 0$[22] and the fact that the economy spends more time at deep downturns

[22]In contrast, we can show that $\lim_{w\to 0} \mathcal{G}(w|s) = 0$ when s is sufficiently large to shield the economy from

than in mild ones (i.e., with limited misallocation of capital: $\kappa \geq 0.5$).

The economy spends prolonged periods at the RLV regime because the absence of amplification reduces the gap between portfolio returns of experts and households. During RLV periods, experts slowly expand their wealth share with little stochastic disturbances due to the stable fundamentals, so it is unlikely for the system to exit this risk regime before experts accumulate extensive wealth –which takes time– or a self-fulfilled panic arrives –which is a rare event.

Welfare. Due to the stability paradox, welfare can increase when fundamentals become more volatile despite the agents' risk aversion. Numerical exercises illustrate welfare[23] as an inverted-U shaped function of the volatility of real shocks. When fundamentals are stable (i.e., low s), an increase in the volatility of real shocks generates a decline in the economy's exposure to panics, which translates into welfare gains. When fundamentals are unstable, the economy is shielded against panics, so a rise in s reduces welfare. Figure 4(c) illustrates this non-monotonicity. In order to emphasize that the presence of financial panics drives the result, the figure also shows that welfare in a model without panics (i.e., no sunspots $\Gamma \equiv 0$) is strictly decreasing in the volatility of fundamentals.

4.4 Extensions and robustness

Robustness. The two main insights of the paper –the RLV environments and the stability paradox– are built on the idea that equilibrium portfolio allocations and capital price are mainly driven by the risk associated with BCF fluctuations and not by the exposure to substantial crashes. Theorem 1 formalizes this idea. The following discussion illustrates the robustness of this message by showing that (a version of) the latter result holds for generalized versions of the model.

The online appendix extends the model to include the following features: idiosyncratic productivity shocks, aggregate shocks to capital's expected growth rate and to the volatility of idiosyncratic shocks, a transfer from households to experts when a self-fulfilled panic is realized, and derivative markets to trade aggregate risks. This generalized model keeps the

crashes due to panics.

[23] Appendix A.2.1 suggests two alternative welfare measures to aggregate the well-being of agents in this economy. The non-monotonicity appears in numerical exercises for both measures.

financial friction that prevents agents to fully unload assets' risk on financial markets.[24] The online appendix shows that Theorem 1 holds for this generalized version of the model. Moreover, when extending the model to also consider heterogeneous Epstein-Zin preferences, the online appendix shows the following weaker version of Theorem 1 holds.

Theorem 1'. *The relative capital demand of experts over households is independent of the vulnerability to panics (i.e., taken as given aggregate outcomes, $\hat{k}_e - \hat{k}_h$ is the same as in the model without panics $p \equiv 0$).*

The latter shows that the result that BCF fluctuations, and not the exposure to panics, drive capital price and allocation does not relies on non-myopic portfolio decisions. Indeed, the difference between Theorems 1 and 1' emerges because EZ preferences imply that the consumption decision depends on investment opportunities and not because the forward-looking component of portfolio decisions introduces a gap in capital demands that depends on the exposure to self-fulfilled panics.

Runs of creditors. An interpretation of the financial panics in this model are runs of creditors on financial institutions. This interpretation requires additional assumptions[25] but delivers the same mathematical model. Hence, the analysis nests the empirically most relevant and most studied class of panics: runs of creditors. Indeed, under this interpretation, the creditors' run component of the model is the continuous-time version of the one proposed by Gertler and Kiyotaki (2015), similar to Huang and Leow (2020).

5 Policy exercises

This section examines the economy's response to leverage constraints and variations in the volatility of idiosyncratic fluctuations. An interpretation of the latter is that policy can

[24] This generalized model nests the model built by Hansen et al. (2018) to compare the implications of macroeconomic models with financial frictions. Their model does not include financial panics but nests several models of the macro-finance literature such as Basak and Cuoco (1998), He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014), Di Tella (2017), Garleanu and Panageas (2015), and Bansal and Yaron (2004).

[25] The sunspot allows households to coordinate their fear about the solvency of experts, which can trigger a massive withdraw of deposits. At this instant, experts are not able to raise further deposits, so they must liquidate capital to honor their debt. If they can do so without going bankrupt, then all agents re-optimize their portfolio, and there are no real effects. However, when the run makes experts insolvent, the economic and final collapse materializes.

influence the degree of idiosyncratic risk-sharing among agents (e.g., regulation on financial engineering or targeted open market operations in certain asset markets). The model is too stylized to deliver actionable policy recommendations, but these exercises deliver useful qualitative insights regarding the policies' effects over the two dimensions of financial fragility analyzed. For tractability, this section considers a constant volatility of productivity shocks (i.e., $\sigma_s = \lambda_s = 0$).

5.1 Leverage constraints

Debt improves capital allocation efficiency, but it also fuels both endogenous instabilities. Hence, leverage constraints introduce a trade-off between stability and aggregate output.

Constant leverage constraint. A constant leverage cap limits leverage more severely when experts are poorly capitalized and prices signal they should use substantial leverage. Moreover, while it is successful reducing the amplification of real shocks when it is binding, it can fuel this instability when it is not. Regarding the exposure to panics, there is no possibility of increasing this instability; however, in order to reduce it during RLV periods, the constraint needs to be extremely tight, which leads to substantial misallocation during downturns.[26] Figures 5(a)-5(c) illustrates the effect of the constant leverage cap on experts' leverage and the instabilities, and the following proposition formalizes the discussed effects of imposing such regulation.

Proposition 4. *(Constant leverage cap) Consider the comparison between the unregulated economy and one with a constant leverage constraint, i.e., $\hat{k} \leq \mathcal{L}$ with $\mathcal{L} \geq 1$. The implementation of a constant leverage constraint implies that:*

1. *The share of capital held by experts κ weakly decreases, i.e., $\kappa(w) \geq \kappa(w;\mathcal{L})$ $\forall w \in [0,1]$. The same applies to the capital price q.*
2. *Amplification of real shocks increases, i.e., $\sigma_q(w) \leq \sigma_q(w;\mathcal{L})$, for the range of w in which there is misallocation in both environments and the leverage cap is not binding. Moreover, the region with positive amplification risk increases, i.e., $w_\sigma \leq w_\sigma(\mathcal{L})$.*
3. *The vulnerability to panics persists, i.e., $\ell_b(w) = \ell_b(w;\mathcal{L})$, for the range of w that belongs to the RLV regime in both solutions.*

[26] Moreover, when considering a transfer from households to experts after panics or more general preferences, a constant leverage cap decreases the liquidation price of capital, increasing potential drops in asset prices and fueling the economy's exposure to systemic runs.

Relative capital demand (26) captures the intuition behind the second result. Take as given that the leverage constraint reduces the share of capital managed by experts and capital price. Then, the relative capital demand of experts $(\hat{k}_e - \hat{k}_h)$ falls, and the gap between capital returns for experts and households $(a_e - a_h)/q$ broadens. Hence, the only possibility for both agents to remain marginal buyers is that BCF fluctuations become more volatile (i.e., larger $\|\sigma_k + \sigma_q\|$).

In terms of the effects over the endogenous dynamics of w, a constant leverage cap makes it more difficult for the economy to escape the crisis regime. Inside this regime, limiting leverage reduces portfolio returns for experts directly (less investment in the asset with greater expected return) but also indirectly through general equilibrium effects (the compensation for exposure to BCF fluctuations, i.e., $\pi_e = \sigma_{n,e} = \hat{k}_e(\sigma_k + \sigma_q)$, decreases).[27] Figure 5(d) illustrates that a constant leverage cap shifts the stationary distribution towards situations where capital allocation is less efficient (i.e., aggregate output is lower).

The output-instability frontier depicted in Figure 5(e) shows the combinations of output and instability that a constant leverage constraint can achieve (each point corresponds to a different cap $\mathcal{L}$) and the metrics chosen for output and instability summarize all the discussed effects.[28] The positive slope of the output-instability frontier illustrates the trade-off between output and stability. Despite the discussed shortcomings containing the endogenous instabilities, the economy does become more stable as the constant leverage constraint tightens. Importantly, this hinges on the exposure to panics. Without panics, e.g., the when volatility of productivity shocks is substantial, a tighter leverage cap can lead to a less stable economy (this case is not shown).

Endogenous (state-contingent) leverage constraints. Given the shortcomings of a constant leverage constraint, I evaluate two intuitive state-contingent leverage constraints. First, I consider an endogenous leverage constraint that becomes tighter when the economy's instability increases, i.e., $\hat{k} \leq \mathcal{L}_\Sigma (\Sigma_q)^{-1}$ where $\mathcal{L}_\Sigma \in \mathbb{R}_+$ and $\Sigma_q^2 = \|\sigma_k + \sigma_q\|^2 + p\ell_q^2$. In this framework, this leverage cap is equivalent to risk-based capital requirement that demands

[27] When the economy is outside the crisis regime and households are not marginal buyers of capital, the leverage constraint increases portfolio return for experts as it generates a discrete drop in the interest rate paid by experts on its debt.

[28] The output measure is the expected aggregate output relative to potential: $\mathbb{E}[a]/a_e = \mathbb{E}[\kappa(a_e - a_h) + a_h]/a_e$, while the instability metric is the expected local variance of output growth, $\mathbb{E}[\Sigma^2] = \mathbb{E}[\|\sigma_k + \sigma_a\|^2 + p\ell_a^2]$. The expectations are taken with respect to the stationary distribution of w.

financial institutions to have sufficient net worth to cover a certain fraction of the standard deviation of the value of their assets: $n_j \geq (\mathcal{L}_\Sigma)^{-1}\sqrt{\mathbb{E}\left[(d(qk_j))^2/dt\right]}$. Second, I examine a leverage cap that ensures that the debt-to-output ratio in the economy remains below certain threshold, i.e., $(\hat{k}_e-1)wq/a \leq \mathcal{L}_b$ where $\mathcal{L}_b \in \mathbb{R}_+$. Figures 5(a)-5(d) illustrate the outcomes from imposing the endogenous leverage caps with their tightness adjusted to generate the same overall instability $\mathbb{E}\left[\Sigma^2\right]$ as the case with the constant leverage cap shown.

In equilibrium, the risk-adjusted leverage cap is counter-cyclical, which substantially reduces output losses during downturns compared to the case with a constant leverage cap. The relatively loose constraint during severe downturns follows from the absence of exposure to panics and the limited amplification of real shocks. The latter responds to the low sensitivity of the wealth distribution to productivity shocks: even substantial productivity shocks fail to generate significant variations in w, because the initial wealth of experts is depressed. As experts accumulate wealth, the mentioned sensitivity increases, and the economy becomes vulnerable to panics, so the constraint tightens.

The debt-to-output constraint is even looser during severe downturns and allows experts to take as much leverage as in the unregulated case. The reason the constraint is loose during severe downturns is that the absolute size of experts' balance sheet, which is proportional to wq, is deeply depressed. This constraint becomes binding when experts are moderately capitalized, because, in these situations, experts' balance sheet is large and a considerable share of it is funded with debt. Hence, the constraint only binds when the economy is most vulnerable to panics.

In terms of the effects over the endogenous dynamics of w and comparing to the unregulated equilibrium, the risk-based leverage makes the economy more prone to severe and mild downturns by decelerating the economy's recovery (i.e., reducing equilibrium portfolio returns for experts during recessions). Meanwhile, the debt-to-output constraint makes the economy less vulnerable to severe downturns but more vulnerable to mild ones. The reason is that it effectively controls the economy's exposure to panics, which limits the sudden crashes that send the economy to a severe recession. To achieve this, it limits leverage during relatively good times, turning them into mild downturns. Figure 5(d) presents the CDF of aggregate output to illustrate the effects just discussed. The strategy of limiting leverage by focusing on the exposure to panics generates, on average, fewer output losses to control the instabilities as illustrated by the output-instability frontier in Figure 5(e) .

Optimal state-contingent leverage constraint. The idea behind the endogenous constraints examined is to limit leverage in situations in which agents' decisions would otherwise lead to substantial endogenous instabilities. This static approach improves over the constant leverage cap but does not fully exploit a potentially more powerful strategy: minimize the frequency with which the economy visits the crisis and RLV regimes. Indeed, the numerical exercise described below delivers an optimal state-contingent leverage constraint that focuses on this dynamic approach.

I look for the state-contingent leverage constraint, $\mathcal{L}(w)$, that maximizes the chosen welfare measure.[29] For the optimization exercise, I consider the following family of functions: $\mathcal{L}(w) = \sum_{i=0}^{5} c_i w^{i-1}$ where $c_i \in \mathbb{R}$. The optimal constraint is $\mathcal{L}(w) \approx 0.99w^{-1}$, i.e., a leverage constraint that only marginally binds when experts hold the entire capital stock (economic booms), which requires $\hat{k}_e = w^{-1}$. Surprisingly, the latter constraint allows to virtually reach first-best outcomes in terms of aggregate output and overall instability, which implies more aggregate output and a more stable economy compared to the unregulated equilibrium: the allocation for the optimal state-contingent leverage cap lies outside the output-instability frontiers in Figure 5(e).

The optimal state-contingent leverage constraint barely influences the economy's instability or aggregate output conditional on the capitalization of the financial sector. However, it does affect the endogenous dynamics of w. When households do not hold capital, a marginally binding leverage constraint on experts discretely affects portfolio returns, because returns on assets need to adjust to make households marginal buyers. In particular, the excess market compensation that households receive for holding capital, $\mu_{R,h} - r$, needs to rise. The latter happens through a decline of the interest rate paid by experts on their debt, which fuels portfolio returns for experts, allowing the economy to spend all of the time in the safe regime. Figure 5(f) illustrates the variation in the excess return of capital over the risk-free rate and the change in the stationary distribution.

This exercise underscores the importance of considering the dynamic effects of leverage constraints. Exploiting them to ensure the financial sector remains sufficiently capitalized may imply some unintuitive results from the static point of view like constraining leverage only when capital allocation is efficient, and the economy is booming.[30]

[29] The two metrics suggested in Appendix A.2.1 deliver equivalent results.

[30] The particular form of the optimal leverage constraint presented does not generalize to alternative

5.2 Idiosyncratic fluctuations

This subsection considers the following evolution for individual capital holdings

$$dk_{j,t}/k_{j,t} = (\Phi(\iota_{j,t}) - \delta)dt + \sqrt{s_t}\sigma dZ_t + \sqrt{\varsigma}dZ_{j,t}$$

where $Z_{j,t}$ is the Brownian motion associated to agent j, which is independent from Z_t and $Z_{i,t}$ for $i \neq j$. The exercise is to compare economies with distinct levels of idiosyncratic volatility $\varsigma \geq 0$.

Allowing for more volatile idiosyncratic fluctuations disciplines the market: it reduces experts' equilibrium leverage, which reduces the amplification of real aggregate shocks and the vulnerability to panics. In this sense, idiosyncratic risk controls aggregate risk. The reason behind this response is the decline in experts' relative capital demand. In the extension considered, the characterization of experts' relative capital demand (26) becomes[31]

$$\left(\hat{k}_e - \hat{k}_h\right)\left(||\sigma_k + \sigma_q||^2 + \varsigma\right) \leq \frac{a_e - a_h}{q}$$

with equality if $\hat{k}_h > 0$. The latter implies that $\hat{k}_e - \hat{k}_h$ decreases as the volatility of idiosyncratic fluctuations rises. The critical feature is that idiosyncratic fluctuations do not trigger bankruptcy of experts, so they always bear the marginal losses that idiosyncratic fluctuations generate. In this sense, idiosyncratic volatility is akin to the volatility due to real aggregate shocks and unlike the exposure to panics. The following proposition formally characterizes the economy's response to variations in ς.

Proposition 5. *(Idiosyncratic volatility) As the exposure of the agents to idiosyncratic fluctuations increases:*

1. *The share of capital held by experts decreases, i.e., $\kappa(w|\varsigma_h) \leq \kappa(w|\varsigma_l)\ \forall w$ for any $\varsigma_l < \varsigma_h$. The same applies to the capital price q.*
2. *Overall fluctuations due to real shocks become more volatile, i.e., $||\sigma_k + \sigma_q(w|\varsigma_h)||^2 + \varsigma_h \geq ||\sigma_k + \sigma_q(w|\varsigma_l)||^2 + \varsigma_l\ \forall w$ for any $\varsigma_l < \varsigma_h$.*
3. *The vulnerability to panics decreases, i.e., $\ell_b(w|\varsigma_h) \leq \ell_b(w|\varsigma_l)\ \forall w$ for any $\varsigma_l < \varsigma_h$.*

modelling approaches of the financial sector, but the exercise emphasizes the importance of a dynamic approach to regulation.

[31] For details on the derivation, see the generalized model in the online appendix.

Figure 5: Leverage constraints and idiosyncratic volatility

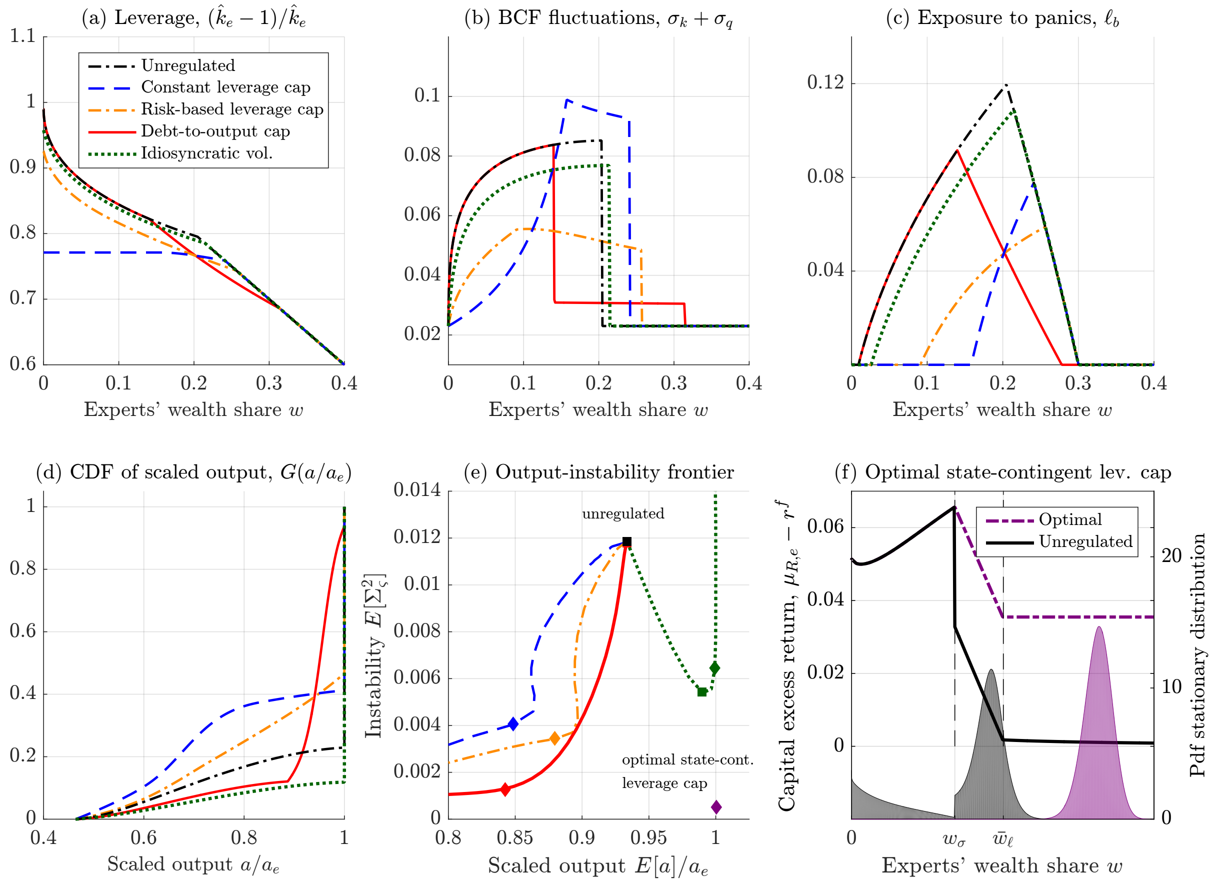

Figure 5. For panels (a)-(d), the tightness of the constraints $\{\mathcal{L}, \mathcal{L}_\Sigma, \mathcal{L}_b\}$ and the level of idiosyncratic volatility ς are adjusted such that $\mathbb{E}[\Sigma_\varsigma^2] = 0.75\%$. In panel (e), the diamonds correspond to the optimal constraint considering welfare function $\mathbb{E}[\nu\mathcal{V}_e + (1-\nu)\mathcal{V}_h]$ and squares considering $\mathbb{E}[\mathcal{V}]$. This numerical illustration considers the baseline parametrization except for the aggregate volatility process, which dynamics are muted (i.e., $\lambda_s = \sigma_s = 0$).

Despite the decline in the volatility due to real aggregate shocks (i.e., $||\sigma_k + \sigma_q||^2$) overall instability due to real shocks (i.e., $||\sigma_k + \sigma_q||^2 + \varsigma$) increases with greater ς. Again, the idea of more stable outcomes due to more volatile fundamentals is not present if the exposure to panics is not acknowledged.

The dotted lines in Figures 5(a)-5(c) illustrate the impact of idiosyncratic risk on experts' leverage and instabilities. The volatility of idiosyncratic fluctuations is adjusted to generate an overall instability, i.e., $\mathbb{E}\left[\Sigma_\varsigma^2\right] \equiv \mathbb{E}\left[\|\sigma_k + \sigma_a\|^2 + \varsigma + p\ell_a^2\right]$, equal to the previous policy exercises. Conditional on the capitalization of the financial sector, the effects of idiosyncratic volatility resembles the ones of a risk-based leverage constraint; however, the impact over the endogenous evolution of w is qualitatively different. Unlike leverage constraints, allowing for idiosyncratic fluctuations reduces the time the economy spends in both, severe and mild downturns. The reason is that idiosyncratic risk accelerates experts' wealth accumulation, because they bear most of this risk in equilibrium. Figure 5(d) illustrates this dynamic effect by plotting the CDF of aggregate output.

A non-monotonicity emerges for both aggregate output and instability: overall instability $\mathbb{E}\left[\Sigma_{\varsigma}^{2}\right]$ is U-shaped as function of the volatility of idiosyncratic shocks. Hence, for low levels of idiosyncratic volatility, there is no trade-off between instability and output: as ς rises, aggregate output increases and the economy becomes more stable. The negative slope of the output-instability frontier in Figure 5(e) illustrates the latter result. Nevertheless, for sufficiently volatile idiosyncratic fluctuations, further rises of ς depress output and make the economy less stable.

6 Conclusion

This paper presents a tractable macroeconomic framework that includes two endogenous instabilities that emerge due to financial frictions, or more precisely, due to debt contracts. These are the amplification of real shocks, which manifests as volatile outcomes at business-cycle frequency, and the exposure to self-fulfilled panics, which translates into rare economic crashes. This two-dimensional approach to financial stability unveils two critical messages connected to the widespread idea that *stability breeds instability*.

First, the endogenous emergence of *risky low-volatility environments* –i.e., periods of strong growth, high asset prices, and stable macroeconomic and financial outcomes that are extensively exposed to panics– illustrates that solid performance of the economy can conceal the threat of substantial collapses. Second, the analysis reveals a *stability paradox*: more stable fundamentals (i.e., lower volatility of the real shocks driving BCF fluctuations) lead to a less stable economy. The reason is the response of the endogenous instabilities, mainly the rise of the exposure to financial panics. These two messages build on an insight regarding the allocation and price of productive assets: they are insensitive to the exposure to crashes that trigger large-scale bankruptcies.

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Appendix

A Analytical results.

A.1 Benchmarks

Proof of Proposition 1 (Complete markets). With complete markets, the first welfare theorem applies. Optimality of production implies that experts manage the entire capital stock, i.e., $y_t = a_e k_t$. Since agents choose the same portfolio, the problem for an agent with capital k is equivalent to $\rho V(k,s) = \max_{c^k,\iota} \rho \log(c^k k) + \mathbb{E}[dV(k,s)]$ s.t. (1) and $a_e = \iota + c^k$. Guess and verify that the value function satisfies $V(k,s) = \log(k) + v(s)$ where $v(s)$ solves $\rho v(s) = \rho \log(\rho q^\star) + \Phi(\iota^\star) - \delta - \frac{1}{2} s \|\sigma\|^2 + v'(s)\lambda_s(\bar{s} - s) + \frac{1}{2} v''(s) s \|\sigma_s\|^2$ and $q^\star$ is the solution to $\rho q + \iota(q) = a_e$. Policy functions are $c^k = \rho q^\star$ and $\iota = \iota(q^\star)$ where function $\iota(\cdot)$ is implicitly defined by $\Phi'(\iota) = 1/q$. The optimal investment decision delivers law of motion (11).

Proof of Proposition 2 (Autarky). In autarky, the problem for agent j is the same as in the complete market case except fo the second constraint, w becomes $a_j = \iota_j + c_j^k$. Policy functions remain $c_j^k = \rho q_j$ and $\iota_j = \iota(q_j)$ where $q_j = q^\star$ for experts and $q_j = q^\dagger$ for households. $q^\dagger$ is the solution to $\rho q + \iota(q) = a_h$. Note that $q^\star > q^\dagger$, so $\iota_e > \iota_h$. Replacing optimal investment rates on aggregate capital evolution (8), we have $dk_t/k_t = [\kappa_t \Phi(\iota_e) + (1 - \kappa_t)\Phi(\iota_h)]\, dt + \sqrt{s_t}\sigma \cdot dZ_t$. Given the optimal investment rates and $\kappa_t = k_{e,t}/(k_{e,t} + k_{h,t})$ where $k_{e,t}$ ($k_{h,t}$) represents aggregate capital for experts (households), the law of motion for κ_t is $d\kappa = \kappa(1-\kappa)(\Phi(\iota_e) - \Phi(\iota_h))\, dt + \lambda_d(\nu - \kappa)dt \equiv F(\kappa)dt$, and κ^{aut} is defined by $F(\kappa^{aut}) = 0$. Results follow from: the uniqueness of κ^{aut}, and the fact $F(\kappa) > 0$ for $\kappa < \kappa^{aut}$ and $F(\kappa) < 0$ for $\kappa > \kappa^{aut}$.

A.2 Solving baseline with financial frictions

A.2.1 Agents' problem

Denote aggregate states as X and its law of motion as $dX = \mu_X dt + \sigma_X \cdot dZ_t - \ell_X dJ_t$. In this baseline model, $X \equiv (w,s)$. The HJB for agent j is $\rho V(n_j;X) = \max_{\hat{c}_j,\iota_j,\hat{k}_e} \rho \log(\hat{c}_j n_j) + \mathbb{E}\left[\frac{dV}{dt}\right]$ s.t. $\hat{k}_j \geq 0$, $n_j \geq 0$, dynamic budget constraint (14), and law of motion of aggregate states (2) and (24). I guess and verify the following solution for the functional equation $V(n_j,\xi(X)) = \log(n_j) + \log(\xi(X))$, where ξ is a mapping from X to $\mathbb{R}$ that summarize the investment opportunities of the agent. Given the uncertainty that drives dynamics, I conjecture the following law of motion $d\xi_j = \xi_j\left(\mu_{\xi,j}dt + \sigma_{\xi,j}\cdot dZ_t - \ell_{\xi,j}dJ_t\right)$. Then, using Ito's lemma,

$$\mathbb{E}\left[\frac{dV}{dt}\right] = \underbrace{\mu_{n,j} - \hat{c}_j - \frac{1}{2}\left\|\sigma_{n,j}\right\|^2 + p\log\left(1-\ell_{n,j}\right)}_{\text{influenced by individual decisions}} + \underbrace{\mu_{\xi,j} - \frac{1}{2}\left\|\sigma_{\xi,j}\right\|^2 + p(1-\ell_{\xi,j})}_{\text{not influenced by individual decisions}}$$

FOC for capital and investment are immediate: $\hat{c}_j = \rho$ and $\iota_j = i(q)$ which is implicitly defined by $\Phi'(\iota) = 1/q$. Both solutions are independent of the agent's type. It is also straightforward that the portfolio problem becomes (15). To verify the conjecture, it suffices to note that the portfolio solution $\hat{k}_j$ does not depend on wealth level n_j as long as $\mu_{n,j},\sigma_{n,j}$ and $\ell_{n,j}$ do not, which is the case for both agents.

Replacing these solutions in the RHS of HJB, the net worth n_j drops from the equation and it becomes

$$\rho\log(\xi_j) = \rho\log(\rho) - \rho + \mu_{n,j} + -\frac{1}{2}\left\|\sigma_{n,j}\right\|^2 + p\log\left(1-\ell_{n,j}\right) + \mu_{\xi,j} - \frac{1}{2}\left\|\sigma_{\xi,j}\right\|^2 + p(1-\ell_{\xi,j}) \quad (31)$$

which together with consistency conditions (that follow from Ito's lemma): $\mu_{\xi,j} = \mu_X \partial \ln\xi_j + \frac{1}{2}tr\left[\sigma_X'\left(\partial_{XX'}\ln\xi_j + \partial_X \ln\xi_j \partial_X \ln\xi_j'\right)\sigma_X\right]$, $\sigma_{\xi,j} = \sigma_X'\partial\ln\xi_j$, and $\ell_{\xi,j} = 1 - \tilde{\xi}_j/\xi_j$, defines a delay PDE for $\xi(X)$. This verifies the conjecture.

Welfare measures. The value function of an agent can be written as $V(n_j,\xi(X)) = \log\left(\mathcal{N}_j k\right) + \mathcal{V}_j(X)$ where $\mathcal{N}_j \equiv n_j/\left(\int_{j\in\mathbb{J}_j} n_j dj\right)$ and $\mathcal{V}_j \equiv \log(w_j q \xi_j)$ with $w_j \in \{w, 1-w\}$ and

$\mathbb{J}_j \in \{\mathbb{J}_e, \mathbb{J}_h\}$ refer to the relevant group's wealth share and interval of agents, respectively. $\mathcal{V}_j$ is the value of an agent that holds the entire wealth of its sector (i.e., $\mathcal{N}_j = 1$) normalized by the aggregate capital (i.e., considering $k = 1$). However, the latter measure is not well-defined for experts because, when they file for bankruptcy, they exit the economy and $\mathcal{V}_e \to -\infty$. An alternative is to consider the extension in which experts receive a small transfer from households after a self-fulfilled panic (see online appendix). In this case, we can consider $\mathbb{E}[\nu \mathcal{V}_e + (1-\nu)\mathcal{V}_h]$ as a measure of welfare for the economy. Another alternative is to consider a fictional representative agent with consumption path equal to the aggregate consumption (but who expects volatility that includes idiosyncratic shocks). His value function is $V(X) = \log(k) + \mathcal{V}(X)$, and we can consider $\mathbb{E}[\mathcal{V}]$ as a welfare metric.

Proof of Lemma 1 (Households' capital demand). For households, the dynamic budget constraint becomes (16), and FOC (17) follows directly. When $\hat{k}_h > 0$, solving for $\hat{k}_h$ delivers two roots but only the smaller one is a solution to the maximization problem (the other violates $n_j \geq 0$ after a self-fulfilled panic). Second-order conditions are satisfied and ensure a unique global maximum. When $\hat{k}_h > 0$, the relationship between $\hat{k}_h$ and aggregate variables follows directly from FOC (17) and the fact that $\ell_{n,h} < 1$. For the relevant solution, the RHS of (17) is strictly increasing in $\hat{k}_h$, while the LHS is constant. Then, since the RHS is strictly increasing in p and ℓ_q, the optimal $\hat{k}_h$ is strictly decreasing in those variables. Moreover, since $\|\sigma_k + \sigma_q\|$ increases the RHS, the optimal $\hat{k}_h$ decreases as this sensitivity rises for a fixed $\mu_{R,h}$ (which depends on $\sigma_k \cdot \sigma_q$).

Proof of Lemma 2 (Experts' capital demand). For experts, the dynamic budget constraint becomes (20), and FOC (21) follows directly. The FOC holds with equality when experts holds some capital, which is always the case in equilibrium. The FOC has two solutions, both local maximums with respect to $\hat{k}_e$: the safe one $\hat{k}_e^s$ that satisfies the FOC with $\vartheta(\hat{k}_e^s) = 0$ and the risky one that satisfies the FOC with $\vartheta(\hat{k}_e) = 0$.

Replacing the two local maximums into the objective function, deliver that the risky solution is optimal if

$$\left(\hat{k}_e - \hat{k}_e^s\right)(\mu_{R,e} - r^f) - (\hat{k}_e - 1)(r(\hat{k}_e) - r^f) + \phi \geq \frac{1}{2}\left((\hat{k}_e)^2 - (\hat{k}_e^s)^2\right)\|\sigma_k + \sigma_q\|^2 + p\left(\log\left(1 - \hat{k}_e^s \ell_q\right) - \log(\tau_e)\right)$$

where τ_e is the wealth of expert after bankruptcy, which is zero in the baseline model. The assumption regarding the underperforming cost ϕ is that it is sufficiently large to ensure this inequality holds at every point of the state space. In the baseline model, this implies $\phi \to \infty$ because logarithmic preferences imply that expert's utility diverges to $-\infty$ when his net worth (and therefore, his consumption) drops to zero. A positive transfer after bankruptcy, as included in the extensions, makes the underperforming cost required finite. Replacing identity (28) and pricing condition (18) into the FOC for the risky portfolio delivers the solution for $\hat{k}_e$ displayed in Lemma 2.

A.2.2 Equilibrium

Proof of Lemma 3 (Law of motion of experts' wealth share). Replacing capital FOC for $\hat{k}_h$ on households' dynamic budget constraint delivers $\mu_{n,h} = r^f + \pi_h \cdot \sigma_{n,h} + \alpha_h \ell_{n,h}$. Define risk-adjusted return on capital for experts as $\Delta_e \equiv \mu_{R,e} - r^f - \pi_h \cdot (\sigma_k + \sigma_q) - \alpha_h \ell_q$. Using FOC for $\hat{k}_e$, pricing condition for experts' debt (18), and identity $\ell_b + \ell_b'(\hat{k}_e)(\hat{k}_e - 1) = \ell_q$, we have $\Delta_e = (\pi_e - \pi_h) \cdot (\sigma_k + \sigma_q) + (1 - \vartheta)\, \alpha_e \ell_q$. Then, $\mu_{n,e} = r^f + \pi_h \cdot \sigma_{n,e} + \alpha_h \ell_{n,e} + \hat{k}_e \Delta_e - \phi(1 - \vartheta)\Upsilon + \alpha_h \left[\vartheta(\hat{k}_e \ell_q - 1) - (\hat{k}_e - 1)\ell_b\right]$. The last term is always zero if the equilibrium is symmetric among experts. Either all experts choose a safe portfolio and $\alpha_h = 0$, or they choose a risky one and identity (27) holds. Also, a symmetric equilibrium implies $(1 - \vartheta)\Upsilon = 0$. The reference to an asymmetric equilibrium is only possible for individual outcomes of experts because the households' problem was posted assuming all experts choose the same portfolio. From now on, we assume symmetric choices among experts.

The wealth share of experts satisfies $w = N_e/(N_e + N_h)$ where $N_j = \int_{\mathbb{J}_j} n_j dj$ represents the aggregate wealth of sector $j \in \{h, e\}$. The dynamics of the wealth of each sector evolves with the same (geometric) sensitivity to real shocks and (percentage) losses after self-fulfilled panics as individual net worths, i.e., $\sigma_{n,j}$ and $\ell_{n,j}$, respectively. The drift is different because of the perpetual youth structure. In particular, $\mu_{N,e} = \mu_{n,e} + \lambda_d(\nu - w)/w$ and $\mu_{N,h} = \mu_{n,h} + \lambda_d(w - \nu)/(1 - w)$. Then, using Ito's lemma, and replacing the expressions for $\mu_{n,h}$ and $\mu_{n,e}$, market clearing implication $w\sigma_{n,e} + (1 - w)\sigma_{n,h} = \sigma_k + \sigma_q$, and identity (27), delivers law of motion for w presented in Lemma 3.

Macro-financial linkages. *Endogenous component of BCF fluctuations.* Let market clearing condition for goods (23) define $a(q)$ with $\partial a/\partial q > 0$. Applying Ito's lemma delivers $\sigma_a a = (\partial a/\partial q)\, q\sigma_q$, which implies $\|\sigma_a\| = 0$ if and only if $\|\sigma_q\| = 0$. *Losses in case of a self-fulfilled panic.* Since self-fulfilled panics imply the bankruptcy of all experts, households manage the entire capital stock just after one occurs, i.e., $\tilde{a} = a_h$. From goods' market clearing condition (23), we have $\tilde{q} = q^\dagger$ where $q^\dagger$ solves $\iota(q) + \rho q = a_h$. Then, $\ell_a = 1 - \tilde{a}/a = 1 - a_h/a(q) = 1 - a_h/a(q^\dagger/(1-\ell_q)) \equiv f(\ell_q)$, and it is immediate to verify $f' > 0$.

Proof of Theorem 1 (Capital invariance). I characterize the function $q(w,s)$ and show that it is independent of the economy's vulnerability to panics. The market clearing condition for goods –equation (23)– is a one-to-one mapping between q and κ, which I denote as $\kappa(q) \equiv [\iota(q) + \rho q - a_h]/(a_e - a_h)$. Then, the invariance result also applies for κ and its linear transformation $a = (a_e - a_h)\kappa + a_h$.

First, subtract the FOC for households (17) from the FOC for expert (21), and use pricing condition (18) and identity (28) to deliver the expression for the relative capital demand: (26). The definition of κ and capital market clearing condition imply $\hat{k}_e = \kappa/w$ and $\hat{k}_h = (1-\kappa)/(1-w)$. Replacing these into relative capital demand yields

$$\left(\frac{\kappa(q) - w}{w(1-w)}\right) \|\sigma_k + \sigma_q\|^2 \leq \frac{a_e - a_h}{q} \tag{32}$$

Second, replace the solution for σ_w –equation (24b)– into the consistency condition for σ_q –equation (25b)– and express portfolios in terms of experts' capital share. This delivers

$$\sigma_q = \frac{1}{1 - \frac{q_w}{q}(\kappa(q) - w)} \left[\frac{q_w}{q}(\kappa(q) - w)\,\sigma_k + \frac{q_s}{q}\sqrt{s}\sigma_s\right] \tag{33}$$

which characterizes the amplification of real shocks and can be written as (30). Then, function $q(w,s)$ satisfies

$$\left(\frac{\kappa(q) - w}{w(1-w)}\right) s \left\|\sigma + \frac{q_s}{q}\sigma_s\right\|^2 \leq \left(1 - \frac{q_w}{q}(\kappa(q) - w)\right)^2 \frac{a_e - a_h}{q} \tag{34}$$

which holds as an equality when $q < q^\star$ (i.e., when households hold some capital $\kappa(q) < 1$). The equilibrium function must also satisfy $q(0,s) = q^\dagger$ and $q(1,s) = q^\star$, which follow from goods' market clearing condition and the fact that, without wealth, no agent can hold capital. Since these boundary conditions and (34) –or $\kappa(q) = 1$ when appropriate– fully characterize $q(w,s)$ independently of the arrival rate of self-fulfilled panics p, the proof is completed.

Proof of Lemma 4 (Amplification of real shocks). Let $\mathcal{X} \equiv [0,1] \times \mathbb{R}_+$ refer to the state space (i.e., $(w,s)\in\mathcal{X}$) and denote the capital price associated to the equilibrium which existence is assumed as $Q(w,s)$. I construct an equilibrium capital price function that satisfies $\|\sigma_q\|=0$ for $\mathcal{X}_0\equiv\{(w,s):w\geq w_\sigma(s)\}$ and $\|\sigma_q\|>0$ almost everywhere else (i.e., for a set $\mathcal{X}_\sigma\subseteq\mathcal{X}_0^c\equiv\mathcal{X}/\mathcal{X}_0$ such that $\mathcal{B}(\mathcal{X}_\sigma)=\mathcal{B}(\mathcal{X}_0^c)$).

Consider the following function: $q(w,s)=Q(w,s)$ for $(w,s)\in\mathcal{X}_0^c$ and $q(w,s)=q^\star$ for $(w,s)\in\mathcal{X}_0$. Since Q corresponds to an equilibrium capital price, $(w_\sigma(s),s)$ must satisfy

$$\left(\frac{s}{w_\sigma(s)}\right)\|\sigma\|^2\leq\frac{a_e-a_h}{q^\star}\tag{35}$$

If $(w_\sigma(s),s)$ belongs to a region in which (34) holds with equality, then $Q_w(w_\sigma(s),s)=Q_s(w_\sigma(s),s)=0$. Otherwise, $Q(w,s)>q^\star$ for some (w,s) in the neighborhood of $(w_\sigma(s),s)$ which is inconsistent with equilibrium. Given the zero partial derivatives, (34) reduces to (35). If $(w_\sigma(s),s)$ belongs to a region in which (34) holds as an inequality, then $Q=q^\star$ and partial derivatives are zero: again (34) reduces to (35). If $(w_\sigma(s),s)$ belongs to a threshold between regions and (35) does not hold, then there is a (w,s) in the neighborhood of $(w_\sigma(s),s)$ that belongs to a region in which $Q=q^\star$ and (35) does not hold. This is a contradictions because Q is an equilibrium capital function.

Condition (35) implies that any $(w,s)\in\mathcal{X}_0$ also satisfies this condition because the LHS is decreasing in w. Therefore, $q(w,s)$ satisfies equilibrium condition within subset $\mathcal{X}_0$. Since $q=Q$ for $\mathcal{X}_0^c$, it must satisfy equilibrium conditions, so all it is left to complete the proof is to show that $\|\sigma_q\|>0$ almost everywhere within $\mathcal{X}_0^c$. I prove this by contradiction.

Assume there exist a set $E\subseteq\mathcal{X}_0^c$ such that $\|\sigma_q\|=0$ for all $(w,s)\in E$ and $\mathcal{B}(E)>0$. One the one hand, since (34) holds as an equality and $\|\sigma_q\|=0$, $q(w,s)$ must satisfy $\left(\frac{\kappa(q)-w}{w(1-w)}\right)s\|\sigma\|^2=\frac{a_e-a_h}{q}$ within E. This implicitly characterize $q(w,s)$. On the other hand, $\|\sigma_q\|=0$ is equivalent to $0=q_w\left(\kappa(q)-w\right)\sigma+q_s\sigma_s$, which also implicitly characterize $q(w,s)$ within E. Both characterizations cannot be consistent with each other $\forall(w,s)$ within E. Indeed, a sufficiently rich shock structure (i.e., $\text{rank}([\sigma\ \sigma_s])\geq 2$) implies $q_w=q_s=0$.

Vanishing volatility. Denote $\hat{q}(w)\equiv q(w,0)$. Since $\hat{q}(0)=q^\dagger<q^\star$ and $\hat{q}(w)$ is continuous, it must be that $w_\sigma(0)>0$. *Exploding volatility.* As $s\to\infty$, (34) must hold as an equality and the only possibility for this to happen is that $\kappa(q)\to w\ \forall w$. This delivers $\lim_{s\to\infty}\|\sigma_q\|=0$.

Proof of Lemma 5 (Vulnerability to panics). *Upper bound.* In equilibrium, since agents cannot short-sale capital, experts' capital share satisfies $\kappa \in [0,1]$ which implies (by the goods' market clearing condition) that $q \in [q^\dagger, q^\star]$. Then, the largest percentage drop in capital price is $\bar{\ell}_q \equiv 1 - q^\dagger/q^\star$. The maximum leverage that experts can choose is $\hat{k}_e = 1/w$. Then, the maximum losses for experts' creditors are $\bar{\ell}_b(w) \equiv 1-(1-\bar{\ell}_q)/(1-w) \geq \ell_b$. Given that $\bar{\ell}_b(\bar{\ell}_q) = 0$ and $\bar{\ell}_b' < 0$, the economy is not vulnerable to panics for $w \geq \bar{w}_\ell \equiv \bar{\ell}_q$. Note that $\bar{w}_\ell < 1$.

Lower bound. Consider the function $\hat{\ell}_b(w,s) \equiv 1-(1-\ell_q)\left(\frac{\hat{k}_e}{\hat{k}_e - 1}\right)$ which is equivalent to $\ell_b(w,s)$ except that it can be negative (i.e., $\ell_b = \max\{\hat{\ell}_b, 0\}$). Since $\hat{\ell}_b(w,s)$ is continuous, if $\lim_{w\to 0}\hat{\ell}_b(w,s) < 0$, then $\hat{\ell}_b(w,s) < 0$ (i.e., $\ell_b(w,s) = 0$) for all (w,s) s.t. $w < \underline{w}_\ell(s) \equiv \min\{w : (w,s) \text{ s.t. } \hat{\ell}_b(w,s) = 0 \vee w = \bar{w}_\ell\}$ where the option of $w = \bar{w}_\ell$ is included for the volatility levels s in which $\hat{\ell}_b(w,s) < 0 \; \forall w$ so the minimum is well defined. Note that condition $\hat{\ell}_b(w,s) = 0$ is equivalent to $q(\hat{k}_e - 1) = q^\dagger \hat{k}_e$ since $\tilde{q} = q^\dagger$. Therefore, it is only left to prove that $\lim_{w\to 0}\hat{\ell}_b(w,s) < 0$.

Since $q(0,s) = q^\dagger < q^\star$, $q_s(0,s) = 0$ and (34) holds as an equality as $w \to 0$, i.e., $\left(1 - \frac{1}{q^\dagger}\lim_{w\to 0}\left(q_w\left(\kappa(q) - w\right)\right)\right)^2 \left(\frac{a_e - a_h}{q^\dagger}\right) = \left(\kappa'\left(q^\dagger\right)\left(\lim_{w\to 0} q_w\right) - 1\right) s\left\|\sigma\right\|^2$. Since the ODE is quadratic in q_w, in general, there are two solutions for it. Therefore, there are two limits that satisfy the latter equation. These are

$$\lim_{w\to 0} q_w = \frac{1}{\kappa'(q^\dagger)}\left[\left(\frac{a_e - a_h}{q^\dagger}\right)\frac{1}{s\left\|\sigma\right\|^2} + 1\right], \tag{36}$$

which is finite for $s > 0$, and $\lim_{w\to 0} q_w \to \infty$ respecting $\lim_{w\to 0} q_w\left(\kappa(q) - w\right)^2 = \kappa'(q^\dagger) s \left\|\sigma\right\|^2 \left(q^\dagger\right)^3 \left(a_e - a_h\right)^{-1}$. Each limit (potentially) correspond to a different equilibrium. I consider the former solution. Then, $\lim_{w\to 0}\hat{\ell}_b(w,s) = 1 - \left(\frac{\kappa'(q^\dagger)\lim_{w\to 0} q_w}{\kappa'(q^\dagger)\lim_{w\to 0} q_w - 1}\right) < 0$, which completes the proof for the lower bound. Note that the lower bound satisfies $\lim_{s\to 0}\underline{w}_\ell(s) = 0$ for both solutions.

Vanishing volatility. Denote $\hat{q}(w) \equiv \lim_{s\to 0} q(w,s)$, then, by continuity, $\hat{q}(w) = q(w,0)$ and $\hat{q}_w \equiv \partial \lim_{s\to 0} q(w,s)/\partial w = \lim_{s\to 0} q_w(w,s) = q_w(w,0)$. So, for the region in which $q < q^\star$, we have

$$\left(1 - \frac{\hat{q}_w}{\hat{q}}\left(\kappa(\hat{q}) - w\right)\right)^2 \left(\frac{a_e - a_h}{\hat{q}}\right) = \left(\frac{\kappa(\hat{q}) - w}{w(1-w)}\right)\lim_{s\to 0}\left(s\left\|\sigma + \frac{q_s(w,s)}{\hat{q}}\sigma_s\right\|^2\right)$$

Since the original PDE is quadratic q_s, there are two solutions for the $\hat{q}(w)$ that satisfy the previous equation. The first one in which $|\lim_{s\to 0} q_s| \to \infty$ and satisfies $\lim_{s\to 0} q_s^2 s \in \mathbb{R}_+$, and another solution in which $|\lim_{s\to 0} q_s(w,s)| < \infty$ and $\hat{q}(w)$ is characterized as follows: For $w < \hat{w}_\sigma$, $\hat{q}(w)$ solves ODE

$$\hat{q} = \hat{q}_w \left(\kappa(\hat{q}) - w\right) \tag{37}$$

with boundary conditions $\hat{q}(0) = q^\dagger$ and $\hat{q}(\hat{w}_\sigma) = q^\star$. For $w \geq \hat{w}_\sigma$, $\hat{q}(w) = q^\star$. Consider the latter solution for $\hat{q}(w)$, then we can show that creditors' losses at $s = 0$, i.e., $\hat{\ell}_b(w) \equiv 1 - \frac{q^\dagger}{q} \frac{\kappa(q)}{(\kappa(q)-w)}$, are strictly increasing for $w \leq \hat{w}_\sigma$ and strictly decreasing for $w > \hat{w}_\sigma$. Therefore, the proof is completed if $\hat{\ell}_b(\hat{w}_\sigma) > 0$.

In order to provide a close form expression for $\hat{w}_\sigma$, consider $\Phi(\iota) \equiv \log(\varphi_\iota \iota - 1)/\varphi_\iota$. Then, the investment function satisfies $\iota(q) = (q-1)/\varphi_\iota$ and there is an explicit solution for $\hat{q}(w)$: $\hat{q}(w) = \theta_1 w + \theta_0 + \sqrt{(\theta_1 w + \theta_0)^2 - \theta_0^2}$, where $\theta_1 = (a_e - a_h)/(\varphi_\iota^{-1} + \rho)$ and $\theta_0 = (\varphi_\iota^{-1} + a_h)/(\varphi_\iota^{-1} + \rho)$ for $w < \hat{w}_\sigma$ and $\hat{q}(w) = q^\star$ for $w \geq \hat{w}_\sigma$. Moreover, $\hat{w}_\sigma = \left(2\left(1 + \theta_0/\theta_1\right)\right)^{-1}$. This implies $\hat{\ell}_b(\hat{w}_\sigma) = 1 - 2\theta_0/(2\theta_0 + \theta_1) > 0$, which completes the proof.

Exploding volatility. As $s \to \infty$, we have $\kappa(q) \to w\ \forall w$. Hence, there exists a $s_{up} < \infty$ such that for all $s > s_{up}$, we have $1 - q^\dagger/q^\star < w/\kappa(q)\ \forall w$, which implies no exposure to panics.

Proof of Theorem 2 (Risky low-volatility periods). Consider the equilibrium constructed in Lemma 4 and a given level of volatility of productivity shocks s. If $\ell_b(w_\sigma(s), s) > 0$, then, by the continuity of ℓ_b on w, there exists an $\varepsilon > 0$ such that $\ell_b(w,s) > 0$ when $|w_\sigma(s) - w| < \varepsilon$. By Lemma 4 there is amplification for $w > w_\sigma(s)$, therefore there exists RLV environment for the given s, i.e., $(w_\sigma(s), w_\sigma(s) + \varepsilon) \subseteq \Psi(s)$.

If $\mathcal{B}(\Psi(s)) > 0$, then, by Lemma 4, there exists an interval $[w', w' + \epsilon]$ with $w' > w_\sigma(s)$ that is a subset of $\Psi(s)$. Then, $\ell_b(w', s) > 0$ which implies $\ell_b(w_\sigma(s), s) > 0$ because, for $w \geq w_\sigma(s)$, $\ell_b(w,s) = \max\left\{0, 1 - \frac{q^\dagger}{q^\star}\left(\frac{1}{1-w}\right)\right\}$ is decreasing in w.

Vanishing volatility. The result follows directly from the solution for $\lim_{s\to 0} q(w,s)$ that satisfies $|\lim_{s\to 0} q_s| < \infty$ presented in Lemma 5. *Exploding volatility.* The result is trivial given the absence of exposure to panics for $s > s_{up}$.

A.3 Simplified economy: constant volatility of productivity shocks.

Proof of Proposition 3 (Stability paradox illustration). I use notations $q(w|s)$ to denote the equilibrium capital price $q(w)$ for an economy with volatility of productivity shocks s, i.e., s is a parameter. The characterization of $q(w|s)$ satisfies a simplified version of (34): For $w \leq w_\sigma(s)$,

$$\left(\frac{\kappa(q)-w}{w(1-w)}\right) s\,\|\sigma\|^2 = \left(1-\frac{q_w}{q}\left(\kappa(q)-w\right)\right)^2 \frac{a_e-a_h}{q} \tag{38}$$

with boundary conditions $q(0|s) = q^\dagger$ and $q(w_\sigma|s) = q^\star$. For $w > w_\sigma(s)$, $q(w|s) = q^\star$. The ODE is quadratic in terms of q_w, so it has two solutions: one with $\sigma_k + \sigma_q = s\,\|\sigma\|^2\left(1-\frac{q_w}{q}\left(\kappa(q)-w\right)\right)^{-1} < 0$ and another one with $\sigma_k + \sigma_q > 0$ (the inequality applies to each coordinate if there is more than one shock). This proposition considers the latter solution, so the ODE is equivalent to

$$q_w = \frac{q}{\kappa(q)-w}\left[1-\sqrt{\frac{s\,\|\sigma\|^2 q}{(a_e-a_h)}\left(\frac{\kappa(q)-w}{w(1-w)}\right)}\right] \tag{39}$$

BCF fluctuations of capital returns. For $w > w_\sigma(s)$, $\|\sigma_k + \sigma_q\| = \|\sigma_k\| = s\,\|\sigma\|$ which decreases as s declines. For $w \leq w_\sigma$, the relative capital demand (26) implies

$$\|\sigma_k + \sigma_q\|^2 = \frac{a_e-a_h}{q}\left(\frac{w(1-w)}{\kappa(q)-w}\right) \tag{40}$$

Since the RHS is decreasing in q, the result is equivalent to show $q(w|s)$ increases as s declines. Note that this implies that $w'_\sigma(s) > 0$, i.e., for some w, a decrease in s implies a change from $\|\sigma_q\| > 0$ to $\|\sigma_q\| = 0$, which is consistent with a reduction of $\|\sigma_k + \sigma_q\|$ for the ODE solution considered (i.e., the one with $\sigma_k + \sigma_q > 0$) as long as the left-derivative is positive, i.e., $\lim_{w \nearrow w_\sigma} q_w > 0$. Since $q(w_\sigma(s)|s) = q^\star$ and $q \leq q^\star$ always, the latter condition must be satisfied.

Proof that $q(w|s)$ is decreasing in s. Consider $s_h > s_l$. For $w = 0$, we have $q(0|s_l) = q(0|s_h)$ and $q_w(0|s_l) > q_w(0|s_h)$. The latter follows from considering the limit as $w \to 0$ of ODE (39), which yields (36). Assume $q(w_z|s_l) < q(w_z|s_h)$ for some w_z. Since both functions are continuous in w, $\exists w_y \in (0, w_z)$ such that $q(w_y|s_l) = q(w_y|s_h)$ and $q_w(w_y|s_l) < q_w(w_y|s_h)$. This is a contradiction since q_w is decreasing in s (taken as given q and w).

Exposure to self-fulfilled panics. The result follows directly from $\partial q(w|s)/\partial s \leq 0$ since for a given w, $\ell_b(w|s)$ is a (capped from below) monotone transformation of $q(w|s)$ in equilibrium:

$\ell_b(w|s) = \left[1 - \frac{q^\dagger}{q}\left(\frac{1}{1-w/\kappa(q)}\right)\right]^+$.

Proof of Proposition 4 (Constant leverage constraint). *Equilibrium capital price.* The capital price satisfies (38) (or $\kappa(q) = 1$ when appropriate) for states in which the constraint is not binding. If the constraint is binding, then $\kappa(w;\mathcal{L}) = w\mathcal{L}$, $q(w;\mathcal{L})$ is pinned down by goods market clearing condition (23), and $\sigma_q(w;\mathcal{L})$ solves the simplified version of (33) with $\sigma_s = 0$. In these situations, we need to verify that households would prefer to hold more capital, i.e., that (26) holds. Boundary conditions at $w = 0$ and $w = 1$ remain the same.

Share of capital held by experts. Assume $\kappa(w) < \kappa(w;\mathcal{L})$ for some $w \in [0,1]$. Then, consider $w_0 \equiv \inf\{w \in [0,1] : \kappa(w) < \kappa(w;\mathcal{L})\}$. By continuity, we have that $\kappa(w_0) = \kappa(w_0;\mathcal{L})$. Note that the definition of w_0 implies that $\lim_{w\downarrow w_0} \kappa'(w) < \lim_{w\downarrow w_0} \kappa'(w;\mathcal{L})$ since κ is supposed to be greater at the economy with the leverage constraint when $w > w_0$ but remains sufficiently close. Given the characterization of $q(w;\mathcal{L})$ this is not possible. Consider the case where the constraint is not binding at $[w_0, w_0 + \epsilon]$, then in this interval the capital price solves the same ODE with the same boundary condition in both economies, so it must be that $\lim_{w\downarrow w_0} \kappa'(w) = \lim_{w\downarrow w_0} \kappa'(w;\mathcal{L})$. Now, consider the case where the constraint is binding at $[w_0, w_0 + \epsilon]$, then it must be that the solution for $q(w)$ delivers a value for $\lim_{w\downarrow w_0} \kappa'(w)$ that violates the leverage constraint. Since $\kappa'(w;\mathcal{L})$ respects such constraint and leverage in equilibrium satisfies $\hat{k}_e = \kappa/w$, we must have that $\lim_{w\downarrow w_0} \kappa'(w) \geq \lim_{w\downarrow w_0} \kappa'(w;\mathcal{L})$. This is a contradiction.

Amplification of real shocks. Consider the w-region in which there is misallocation in both environments and the leverage cap is not binding, i.e., $\{\kappa(w), \kappa(w;\mathcal{L})\} \in [0,1)$ and $\hat{k}_e(w;\mathcal{L}) < \mathcal{L}$. Then, both solutions need to satisfy relative capital demand (40) as an equality. The result follows from the fact that κ and q decrease when a leverage cap in introduced. *Vulnerability to panics.* The result is trivial since the leverage constraint cannot be binding during RLV periods since experts manage the entire capital stock in these situations.

Proof of Proposition 5 (Idiosyncratic volatility). This version of the model is an especial case of the one characterized in the online appendix. Equilibrium capital price satisfies

$$\left(\frac{\kappa(q) - w}{w(1-w)}\right)\left[||\sigma_k + \sigma_q||^2 + \varsigma\right] \leq \frac{a_e - a_h}{q} \tag{41}$$

where $||\sigma_k + \sigma_q||^2 = \|\sigma_k\|^2 / \left(1 - \frac{q_w}{q}(\kappa(q) - w)\right)^2$ and $\kappa(q)$ is a strictly increasing function defined by goods market clearing condition. The latter inequality must hold as an equality if $\kappa(q) < 1$.

Experts' capital share. Consider $\varsigma_h > \varsigma_l$ and recall that, in equilibrium, q is a monotone transformation of κ. Assume $\kappa(w|\varsigma_h) > \kappa(w|\varsigma_l)$ for some $w \in [0,1]$. Then, define $w_0 \equiv \inf\{w \in [0,1] : \kappa(w|\varsigma_h) > \kappa(w|\varsigma_l)\}$. By continuity, we have that $\kappa(w_0|\varsigma_h) = \kappa(w_0|\varsigma_l)$. Note that the definition of w_0 implies that $\kappa_w(w_0|\varsigma_h) > \kappa_w(w_0|\varsigma_l)$. Also, it must be that this crossing point satisfies $\kappa < 1$, so (41) holds as an equality and q_w is a decreasing function of ς for fixed values of q. Hence, it must be that $q_w(w_0|\varsigma_h) < q_w(w_0|\varsigma_l)$. Since $\kappa_w = \kappa'(q)q_w$, the latter implies that $\kappa_w(w_0|\varsigma_h) < \kappa_w(w_0|\varsigma_l)$. This is a contradiction.

Overall BCF fluctuations due to real shocks. If $\kappa(w|\varsigma_h) < 1$ and $\kappa(w|\varsigma_l) < 1$, then (41) holds as an equality for both ς_h and ς_l. Then, the result follows from the fact that κ and q are decreasing in ς. If $\kappa(w|\varsigma_h) = \kappa(w|\varsigma_l) = 1$, then $\sigma_q(w|\varsigma_l) = \sigma_q(w|\varsigma_h) = 0$ and the result follows trivially. If $\kappa(w|\varsigma_l) = 1$ and $\kappa(w|\varsigma_h) < 1$, we can split the variation in three: the change from ς_l to threshold $\hat{\varsigma}(w)$, the variation at threshold $\hat{\varsigma}(w)$, and the change from $\hat{\varsigma}(w)$ to ς_h, where $\hat{\varsigma}(w) \equiv \max\{\varsigma : \kappa(w|\varsigma) = 1\}$ corresponds to a threshold where $\sigma_q(w|\varsigma)$ is discontinuous as function of parameter ς. Previous results imply that it is sufficient to verify that individual fluctuations $||\sigma_k + \sigma_q||^2 + \varsigma$ increase at $\hat{\varsigma}(w)$: $\lim_{\varsigma \downarrow \hat{\varsigma}} ||\sigma_k + \sigma_q(w|\varsigma)||^2 > \lim_{\varsigma \uparrow \hat{\varsigma}} ||\sigma_k + \sigma_q(w|\varsigma)||^2$. We have that $\lim_{\varsigma \uparrow \hat{\varsigma}} \sigma_q(w|\varsigma) = 0$ because, in this region, $q = q^\star$ and there is no amplification. Then, the proof is complete if $\lim_{\varsigma \downarrow \hat{\varsigma}} \sigma_q(w|\varsigma) > 0$, which holds if $\lim_{\varsigma \downarrow \hat{\varsigma}} q_w(w|\varsigma) > 0$. The latter needs to hold because $q(w|\hat{\varsigma}) = q^\star$ which is the upper bound for the capital price in any equilibrium. *Exposure to self-fulfilled panics.* The result follows directly from $q(w|\varsigma)$ being decreasing in ς.

B Parametrization and numerical strategy

Baseline parametrization. Logarithmic adjustment cost function: $\Phi(\iota) = \log(\varphi_\iota \iota + 1)/\varphi_\iota$, and linear function for the sunspot arrival rate: $\Gamma(\ell_b) = \gamma_b \ell_b$. Time is measured in years. I use a standard value for the discount rate ($\rho = 0.02$) and a low value for the depreciation rate ($\delta = 0.03$) given that capital quality shocks are similar to persistent productivity shocks that induce pro-cyclical variation in capital utilization rate. The fraction of experts among newborns is set to 17 percent ($\nu = 0.17$).

For the set of parameters $\{a_h, a_e, \sigma^{(1)}, \varphi_\iota\}$, I match the following moments at the stochastic steady state: investment to capital ratio (8.8 percent), GDP growth volatility (2.31 percent), GDP growth (3.24 percent) and the maximum drop in asset prices (30 percent). Except to the latter, these moments correspond to U.S. data for the post-World War II period. This procedure yields $\{a_h = 0.058, a_e = 0.125, \sigma^{(1)} = 0.023, \varphi_\iota = 10.19\}$. Regarding the volatility process, the long-term mean of s is normalized to $\bar{s} = 1$, while the mean reversion parameter is set to $\lambda_s = 0.156$ and the volatility to $\sigma_s^{(2)} = 0.132$. These parameters ensure that $s > 0$ always. Recall that I consider two "pure" shocks, so $\sigma^{(2)} = \sigma_s^{(1)} = 0$. The last set of parameters $\{\lambda_d, \gamma_b\}$ are matched using the entire stationary distribution. The annual death probability for agents is set to 5.7 percent ($\lambda_d = 0.057$), targeting a conservative average leverage of 5. Finally, the parameter that governs the arrival rate of sunspots ($\gamma_b = 0.74$) ensures that self-fulfilled panics happen on average once every 35 years.

Numerical strategy. The recursive equilibrium characterize in section 4.1 defines a functional equation for $q(w, s)$. Given the presence of jumps, this functional equation is not a PDE, as is the case in almost all continuous-time macroeconomic models in the literature. Besides $\{q, q_w, q_s, q_{ww}, q_{ss}, q_{ws}\}$, the functional equation also includes $q(\tilde{w}(w), s)$, the capital price if a self-fulfilled panics were to arrive at state (w, s). There is no general approach to solving this class of functional equations (i.e., delayed PDEs).

In this model, the system of equations can be partition in two block, one of which defines a (first-order) PDE for $q(w, s)$. The capital price invariance to the exposure to panics is essential for the mentioned partition. This PDE corresponds to (34) evaluated as an equality with boundary conditions $q(0, s) = q^\dagger$, $q(w_\sigma(s), s) = q^\star$, and $q(w, 0) = \hat{q}(w)$ where $w_\sigma(s)$ is an endogenous threshold and $\hat{q}(w)$ is defined by (37). I solve this using finite differences. Once we have the solution for $q(w, s)$ the equilibrium objects $\{\sigma_q, \hat{k}_e, \hat{k}_h, \sigma_w\}$ follow directly. In order to solve for the second block of equilibrium objects, it is necessary to solve for $\tilde{w}$. In the baseline model, $\tilde{w} = 0$ is the condition consistent with bankruptcy, but this numerical approach allows for an arbitrary $0 = f(w, q, \tilde{w})$ such as $q(\tilde{w}, s)\tilde{w} = \tau q(w, s) w$ which is the corresponding equation in the extension with transfers to experts after the collapse. Once we have a solution for $\tilde{w}(w, s)$, we can retrieve solutions for $\{\mu_q, \ell_q, r, \ell_b, p, \mu_w, \ell_w\}$.

Given the solution for the recursive equilibrium, I use an iterative approach to solve for the value functions from the HJB equations (31) as described in Brunnermeier and Sannikov (2016). Also, I develop a numerical strategy to solve for the stationary distribution of the state variable without resorting to simulations.

Internet Appendix

"Risky low-volatility environments and the stability paradox"
by Fernando Mendo

This appendix presents a generalized version of the model and shows that, as long as financial frictions prevent agents from completely unload the risks of their assets into financial markets, the invariance of capital price and allocation to the exposure to panics (i.e., Theorem 1) is robust to the following features: idiosyncratic productivity shocks, aggregate shocks to the growth rate of capital and to the volatility of idiosyncratic shocks, and a derivative market to trade aggregate risks. Considering heterogeneous Epstein-Zin preferences lead to a (weaker) version of the result: the capital demand of experts relative to households is not affected by the vulnerability to panics.

1 Environment

Consider the environment described in the paper with the following generalizations.

Preferences. Heterogeneous EZ preferences with parameters $(\psi_j, \gamma_j, \rho_j)$ representing the inverse of IES, risk aversion, and discount rate (including the uninsurable death risk).

$$\begin{aligned} U_{j,t} &= \mathbb{E}\left[\int_t^\infty \varphi_j\left(c_{j,s}, U_{j,s}\right) ds\right] \\ \varphi_j(c,U) &= \frac{1}{1-\psi_j}\left\{\rho_j c^{1-\psi_j}\left[(1-\gamma_j)U\right]^{\frac{\psi_j-\gamma_j}{1-\gamma_j}} - \rho_j\left(1-\gamma_j\right)U\right\} \end{aligned} \tag{1}$$

The preferences are symmetric within each group.

Technology. Capital for agent j evolves according to

$$dk_{j,t} = k_{j,t}\left[\left(g_t + \Phi\left(\iota_{j,t}\right) - \delta\right)dt + \sqrt{s_t}\sigma \cdot dZ_t + \sqrt{\varsigma_t}dZ_{j,t}\right]$$

where $Z_{j,t} \in \mathbb{R}$ is a Brownian motion specific to agent j, which is independent of $Z_{i,t}$ $(i \neq j)$ and aggregate Brownian motion $Z_t \in \mathbb{R}^d$. Idiosyncratic volatility $\varsigma_t \in \mathbb{R}$ and expected capital growth $g_t \in \mathbb{R}$ follow

$$dg_t = \lambda_g\left(\bar{g} - g_t\right)dt + \sqrt{s_t}\sigma_g \cdot dZ_t$$
$$d\varsigma_t = \lambda_s\left(\bar{\varsigma} - \varsigma_t\right)dt + \sqrt{\varsigma_t}\sigma_\varsigma \cdot dZ_t$$

Financial markets. In addition to the financial instruments in the baseline model, we consider a derivative market to trade aggregate risk. However, agents must retain at least fraction $\underline{\chi}$ of the risks (both aggregate and idiosyncratic) associated to the capital they manage. The following markets capture this set-up.

Households have access to a set of derivatives with unitary exposure to aggregate risks. The return process for the instruments with exposure to real aggregate shocks is $\left(r_t^f + \pi_t\right)dt + dZ_t$ where $\pi \in \mathbb{R}^d$ is the market price of these risks and r_t^f represents the risk-free rate as priced by households. The corresponding return process for the instrument with exposure to panics is $\left(r_t^f + \alpha_t\right)dt - dJ_t$ where $\alpha \in \mathbb{R}$ is the price of this risk. Buying the latter instrument is effectively selling insurance against self-fulfilled panics.

Households constitute the demand in the derivative market, the supply comes a securitization process. Experts securitize fraction $(1-\chi)$ of their capital holdings, respecting the financial friction: $\chi \geq \underline{\chi}$. The return process of securitized capital (i.e., the asset experts sell to financial markets) is

$$\left(r_t^f + \pi_t \cdot \left(\sigma_{k,t} + \sigma_{q,t}\right) + \alpha_t \ell_{q,t}\right)dt + \left(\sigma_{k,t} + \sigma_{q,t}\right) \cdot dZ_t - \ell_{q,t}dJ_t$$

The latter equations simply states that securitized capital is priced according market risk prices $\{\pi_t, \alpha_t\}$. Markets are able to diversify idiosyncratic risk (i.e., its market risk price is zero).

Transfer policy. After a self-fulfilled panic, failed experts receive a transfer, which is financed by taxes on the household sector. This transfer allows them to resume operations. Policy-makers set the aggregate value of the transfer $\tau_t (q_t k_t)$. Taxes and transfers are proportional to the net worth agents had before the self-fulfilled panic materializes. Let $\tau_{e,t}$ denote the transfer per unit of net worth to experts and $\tau_{h,t}$ the tax per unit of net worth levied on households. Government runs a balance budget every period. This ingredient facilitates welfare analysis, because it bounds experts losses in terms of welfare in case of a panic.

2 Solving the model

2.1 Aggregate states

Denote aggregate states as $X \equiv (w, s, \varsigma, g)$, where w corresponds to the wealth share of experts. Let $X_{-w} \equiv (s, \varsigma, g)$ refer to the exogenous aggregate states. I switch to recursive notation from now on. Denote $dX = \mu_X dt + \sigma_X dZ - \ell_X dX$ where

$$\begin{aligned}
\mu_X &= (w\mu_w, \lambda_g (\bar{g} - g), \lambda_s (\bar{s} - s), \lambda_\varsigma (\bar{\varsigma} - \varsigma)) \\
\sigma_X &= \left(w\sigma_w, \sqrt{s}\sigma_g, \sqrt{s}\sigma_s, \sqrt{\varsigma}\sigma_\varsigma\right) \\
\ell_X &= (w\ell_w, 0, 0, 0)
\end{aligned}$$

and $\{\mu_w, \sigma_w, \ell_w\}$ are endogenous objects. Consider an analogous definition for dX_{-w}.

2.2 Agents' problems and optimal decisions

The dynamic budget constraint for agent j can be written as

$$\frac{dn_j}{n_j} = (\mu_{n,j} - \hat{c}_j)\, dt + \sigma_{n,j} \cdot dZ_t + \tilde{\sigma}_{n,j} dZ_{j,t} - \ell_{n,j} dJ_t$$

where $\tilde{\sigma}_{n,j} \in \mathbb{R}$ is the exposure to idiosyncratic risk. Abusing notation, henceforth, $j \in \{h, e\}$ denotes an arbitrary households or expert. The conjecture for capital price evolution dq_t and the expressions for capital return $dR^k_{j,t}$ are the same as in the baseline model.

Households. Choose consumption $\hat{c}_h$, investment rate ι_h, capital $\hat{k}_h$, derivatives $\theta \in \mathbb{R}^d$ and $\omega \in \mathbb{R}$, and experts' debt. The dynamic budget constraint becomes

$$\begin{aligned}
\mu_{n,h} &= \hat{k}_h \mu_{R,h} + \theta \cdot (r^f + \pi) + \omega \left(r^f + \alpha\right) + \left(1 - \hat{k}_h - \theta - \omega\right) r \\
\sigma_{n,h} &= \hat{k}_h (\sigma_k + \sigma_q) + \theta \\
\tilde{\sigma}_{n,h} &= \hat{k}_h \sqrt{\varsigma} \\
\ell_{n,h} &= \hat{k}_h \ell_q + \left(1 - \hat{k}_h - \theta - \omega\right) \ell_b + \omega + \tau_h
\end{aligned}$$

The households' problem is to maximize (1) subject to the dynamic budget constraint, $n_h \geq 0$, and $\hat{k}_h \geq 0$.

Experts. Choose consumption $\hat{c}_e$, investment rate ι_e, capital $\hat{k}_e$, fraction of capital securitized $(1-\chi)$, and debt.

$$\begin{aligned}
\mu_{n,e} &= \hat{k}_e \mu_{R,e} - (1-\chi)\hat{k}_e \left(r^f + \pi \cdot (\sigma_k + \sigma_q) + \alpha \ell_q\right) - \left(\chi \hat{k}_e - 1\right) r(\chi, \hat{k}_e) - \phi(1 - \vartheta(\hat{k}_e))\Upsilon \\
\sigma_{n,e} &= \chi \hat{k}_e (\sigma_k + \sigma_q) \\
\tilde{\sigma}_{n,e} &= \chi \hat{k}_e \sqrt{\varsigma} \\
\ell_{n,e} &= \vartheta(\chi, \hat{k}_e)(1 - \tau_e) + \left(1 - \vartheta(\chi, \hat{k}_e)\right) \chi \hat{k}_e \ell_q
\end{aligned}$$

The experts' problem is to maximize (1) subject to the dynamic budget constraint, the financial friction $\chi \geq \underline{\chi}$, $n_e \geq 0$, and $\hat{k}_e \geq 0$. The interest rate households demand to hold experts' debt given its portfolio is $r(\chi, \hat{k}_e)$.

HJB equation. The HJB for agent j is

$$0 = \max \varphi_j(c, V_j) + \mathbb{E}\left[\frac{dV_j}{dt}\right]$$

Following the same steps as in the baseline model using

$$V_j(n; \xi_j(X)) = \frac{(\xi_j n)^{1-\gamma_j}}{1-\gamma_j}$$

as a guess for the value function delivers

$$0 = \max \underbrace{\frac{\rho_j}{1-\psi_j}\left\{\left(\frac{\hat{c}_j}{\xi_j}\right)^{1-\psi_j} - 1\right\} - \hat{c}_j}_{\text{consumption decision}} + \underbrace{\mu_{\xi_j} - \frac{\gamma_j}{2}\left(\left\|\sigma_{\xi_j}\right\|^2\right) - \frac{p}{1-\gamma_j}}_{\text{not influenced by individual decisions}} \tag{2}$$
$$+ \underbrace{\mu_{n,j} - \frac{\gamma_j}{2}\left(\|\sigma_{n,j}\|^2 + \tilde{\sigma}^2_{n,j}\right) - (\gamma_j - 1)\,\sigma_{n,j}\cdot\sigma_{\xi_j} + \frac{p}{1-\gamma_j}\left((1-\ell_{\xi_j})(1-\ell_{n,j})\right)^{1-\gamma_j}}_{\text{portfolio problem}}$$

Note that ξ_j summarize the value of investment opportunities for agent j.

Consumption and investment. These decisions are symmetric for both type of agents: $\hat{c}_j = \rho_j^{1/\psi_j}\xi_j^{1-1/\psi_j}$ and $\Phi'(\iota_j) = 1/q$.

Households' portfolio. Define the (aggregate) risk-adjusted capital return Δ_h as

$$\mu_{R,h} \equiv r^f + \pi\cdot(\sigma_k + \sigma_q) + \alpha\ell_q + \Delta_h$$

Then, FOCs are

$$[\theta]: \quad \pi = \gamma_h\sigma_{n,h} + (\gamma - 1)\,\sigma_X\cdot\partial_X Ln\xi_h \equiv \pi_h \tag{3a}$$
$$[\omega]: \quad \alpha = p\left(1-\ell_{n,h}\right)^{-\gamma_h}\left(1-\ell_{\xi_h}\right)^{1-\gamma_h} \equiv \alpha_h \tag{3b}$$
$$[\hat{k}_h]: \; \Delta_h \leq \hat{k}_h\gamma_h\varsigma \tag{3c}$$

where the latter holds as an equality of $\hat{k}_h > 0$. The first two conditions imply that market price of risks $\{\pi, \alpha\}$ correspond to households' valuation of them $\{\pi_h, \alpha_h\}$.

Pricing of experts' debt. The interest rate households demand to hold the debt of an expert with portfolio $(\chi, \hat{k}_e)$ respects

$$r(\chi, \hat{k}_e) = r^f + \alpha\ell_b(\chi, \hat{k}_e)$$

where

$$\ell_b(\chi, \hat{k}_e) = \left[1 - (1-\ell_q)\left(\frac{\chi\hat{k}_e}{\chi\hat{k}_e - 1}\right)\right]^+$$

are the losses for creditors in case of a self-fulfill panic. Denote the share of wealth experts invest on direct holdings of capital as $\beta_e \equiv \chi \hat{k}_e$. This excludes the capital he manages but securitizes. The recovery rate in case of bankruptcy depends only in this fraction, i.e., $\ell_b(\chi, \hat{k}_e) \equiv \ell_b(\beta_e)$, so $r(\chi, \hat{k}_e) \equiv r(\beta_e)$ and $\vartheta(\chi, \hat{k}_e) = \vartheta(\beta_e)$.

Experts' portfolio. Define the (aggregate) risk-adjusted capital return Δ_e as

$$\mu_{R,e} \equiv r^f + \pi \cdot (\sigma_k + \sigma_q) + \alpha \ell_b + \chi \Delta_e$$

Then, FOCs are

$$[\hat{k}_e]: \quad \Delta_e = (\pi_e - \pi) \cdot (\sigma_k + \sigma_q) + \chi \hat{k}_e \gamma_e \varsigma + \underbrace{\left(1 - \vartheta(\chi, \hat{k}_e)\right) [(\alpha_e - \alpha)\, \ell_q + \alpha \ell_d]}_{\text{=0 in symmetric equilibriums}} \tag{4a}$$

$$[\chi]: \quad 0 \leq \Delta_e \tag{4b}$$

where the latter holds as an equality if $\chi > \underline{\chi}$ and

$$\alpha_e \equiv p\,(1 - \ell_{n,e})^{-\gamma_e} \left(1 - \ell_{\xi_e}\right)^{1-\gamma_e}$$
$$\pi_e \equiv \gamma_e \sigma_{n,e} + (\gamma_e - 1) \sigma_X' \partial_X Ln \xi_e$$

are the valuations of risk for experts.

2.3 Equilibrium

Market clearing. Capital

$$\hat{k}_e w + \hat{k}_h (1 - w) = 1 \tag{5}$$

for convenience denote the share of capital manage by households $\kappa \equiv \hat{k}_e w$.

Goods

$$q\,[(1 - w)\, \hat{c}_h + w \hat{c}_e] + \iota(q) = a_e \kappa + a_h (1 - \kappa) \tag{6}$$

Derivatives

$$(1 - \chi) \kappa\, (\sigma_k + \sigma_q) = (1 - w)\, \theta \tag{7a}$$

$$(1 - \chi) \kappa \ell_q = (1 - w)\, \omega \tag{7b}$$

where the LHS is the supply from securitized capital, and the RHS corresponds to the demand from households. These conditions are equivalent to

$$(1-w)\ell_{n,h} + w\ell_{n,e} = \ell_q$$
$$(1-w)\sigma_{n,h} + w\sigma_{n,e} = \sigma_k + \sigma_q$$

where the RHS is the total risk in the economy, and the LHS corresponds to the split of these risks between the two type of agents.

Government balance budget

$$\tau = (1-w)\tau_h = w\tau_e$$

Law of motion of aggregate states. Replacing optimal decision into the dynamic budget constraints delivers

$$\mu_{n,h} = r^f + \pi \cdot \sigma_{n,h} + \alpha\left(\ell_{n,h} - \tau_h\right) + \hat{k}_h \Delta_h \tag{8}$$
$$\mu_{n,e} = r^f + \pi \cdot \sigma_{n,e} + \alpha\left(\ell_{n,e} + \tau_e \vartheta\right) + \beta_e \Delta_e + \underbrace{\alpha\vartheta\left(\beta_e \ell_q - 1\right) - \alpha \ell_b \left(\beta_e - 1\right)}_{=0 \text{ in symmetric equilibriums}} \tag{9}$$

where the latter term disappears in symmetric equilibriums: either experts choose a safe portfolio $\vartheta(\beta_e) = 0$ and there is no exposure to panics $p = \alpha = 0$, or experts choose a risky portfolio $\vartheta(\beta_e) = 1$ and the compensation for losses beyond limited liability is offset by the extra interest rate payments due to the potential bankruptcy. From now on, I only consider symmetric equilibria among experts.

Replacing the dynamic budget constraints and market clearing conditions into the law of motion for experts' wealth share (derived from Ito's formula for $w \equiv N_e/(N_e + N_h)$) delivers

$$w\mu_w = w(1-w)\left(\beta_e \Delta^e - \hat{k}_h \Delta^h - \hat{c}_e + \hat{c}_h\right) + \alpha\left(\ell_q - \ell_b\right)\left(\beta_e - w\right) + \left(\pi - \left(\sigma_k + \sigma_q\right)\right) w\sigma_w + \lambda_d(\nu - w)$$
$$w\sigma_w = \left(\chi\kappa - w\right)\left(\sigma_k + \sigma_q\right)$$
$$w\ell_w = w - \tilde{w}$$

where $\tilde{w}$ solves

$$\tilde{w} = \tau(X)$$

Consistency conditions. Capital price and investment opportunities ξ_j for $j = \{h, e\}$ must be consistent with the evolution of aggregate states. These conditions follow directly from applying Ito's lemma to functions $q(X)$ and $\xi_j(X)$:

$$\mu_q = \frac{1}{q}\left[\mu_X \cdot \partial_X q + \frac{1}{2}\mathrm{tr}\left(\sigma_X' \partial_{XX'} q \sigma_X\right)\right] \tag{11a}$$

$$\sigma_q = \frac{(\chi\kappa - w)\sigma_K \partial_w \ln q + \sigma_{X_{-w}}' \partial_{X_{-w}} \ln q}{1 - (\chi\kappa - w)\partial_w \ln q} \tag{11b}$$

$$\ell_q = 1 - q(\tilde{w}, X_{-w})/q \tag{11c}$$

and

$$\mu_{\xi,j} = \mu_X \partial \ln \xi_j + \frac{1}{2} tr\left[\sigma_X' \left(\partial_{XX'} \ln \xi_j + \partial_X \ln \xi_j \partial_X \ln \xi_j'\right)\sigma_X\right] \tag{12a}$$

$$\sigma_{\xi,j} = \sigma_X' \partial \ln \xi_j \tag{12b}$$

$$\ell_{\xi,j} = 1 - \xi_j(\tilde{w}, X_{-w})/\xi_j \tag{12c}$$

HJB equations. Replace consumption and investment decision into HJB (2) delivers

$$\begin{aligned} 0 = {} & \frac{\psi_j}{1-\psi_j}\rho_j^{1/\psi_j} - \frac{\rho_j}{1-\psi_j} + \mu_{n,j} - (\gamma_j - 1)\,\sigma_{n,j} \cdot \sigma_{\xi_j} - \frac{\gamma_j}{2}\left(\|\sigma_{n,j}\|^2 + \tilde{\sigma}_{n,j}^2\right) \\ & + \frac{p}{1-\gamma_j}\left[\left(\left(1 - \ell_{\xi_j}\right)(1 - \ell_{n,j})\right)^{1-\gamma_j} - 1\right] + \mu_{\xi_j} - \frac{\gamma_j}{2}\left\|\sigma_{\xi_j}\right\|^2 \end{aligned} \tag{13}$$

where $\mu_{n,j}$ is given by (8) for households or (9) for experts, and

$$\begin{aligned} \sigma_{n,h} &= \frac{1-\chi\kappa}{1-w}\left(\sigma_k + \sigma_q\right), \quad \tilde{\sigma}_{n,h} = \frac{1-\kappa}{1-w}\sqrt{\varsigma} \\ \sigma_{n,e} &= \frac{\chi\kappa}{w}\left(\sigma_k + \sigma_q\right), \quad \tilde{\sigma}_{n,e} = \frac{\chi\kappa}{w}\sqrt{\varsigma} \\ \ell_{n,h} &= \tau_h + \left(\frac{1-\chi\kappa}{1-w}\right)\ell_q + \left(\frac{\chi\kappa - w}{1-w}\right)\ell_b \\ \ell_{n,e} &= \Upsilon(1-\tau_e) + (1-\Upsilon)\beta_e \ell_q \end{aligned}$$

which follow directly from market clearing conditions.

Definition. *A Markov equilibrium in* $X \equiv (w, s, \varsigma, g)$ *is a set of functions: aggregate outcomes and their dynamics* $r^f, \pi, \alpha, q, \mu_q, \sigma_q, \ell_q, \ell_b, p$*, portfolio decisions* $\hat{k}_e, \chi, \hat{k}_h, \theta, \omega$*, investment opportunities* $\xi_j, \mu_{\xi,j}, \sigma_{\xi,j}, \ell_{\xi,j}$ *for* $j \in \{e, h\}$*, and aggregate states' dynamics* μ_X, σ_X, ℓ_X *such that:*

1. *Given aggregate outcomes and the evolution of aggregate states,* $\hat{k}_h, \theta, \omega$ *and* $\hat{k}_e, \chi$ *solve portfolio problems for households and experts, respectively: (3) and (4). Investment opportunities* $\xi_j, \mu_{\xi,j}, \sigma_{\xi,j}, \ell_{\xi,j}$ *satisfy the HJB equation (13) for* $j = \{h, e\}$*.*
2. *Markets clear: (5), (6), (7).*
3. *The law of motion of* w *satisfies (10).*
4. *Consistency conditions hold: (12) for* $j = \{h, e\}$*, (11) and* $p = \Gamma 1_{\{\ell_b > 0\}}$*.*

3 Invariance results

Theorem 1' *(Relative capital demand's invariance to panics)*. *The relative capital demand of experts over households is independent of the vulnerability to panics (i.e., taken as given aggregate outcomes,* $\hat{k}_e - \hat{k}_h$ *is the same as in the model without panics* $p \equiv 0$*).*

Proof. Subtract the FOC for capital of experts (4a) and households (3c), then relative capital demand

$$\frac{a_e - a_h}{q} \geq \chi (\pi_e - \pi) \cdot (\sigma_k + \sigma_q) + \left(\chi \gamma_e \tilde{\sigma}_{n,e} - \gamma_h \tilde{\sigma}_{n,h} \right) \sqrt{\varsigma}$$

which holds with equality if $\kappa < 1$. Then, conditional on aggregate outcomes, the relative capital demand is independent of p.

Theorem 1 *(Capital's invariance to panics).* *Consider the generalized model with logarithmic preferences (i.e.,* $\gamma_j = \psi_j = 1$ *for* $j \in \{h, e\}$*). Then, capital allocation* κ *is invariant to financial panics, i.e., equilibrium function* $\kappa(w, s)$ *is the same as in the model without panics (*$p \equiv 0$*). The same applies to capital price* $q(w, s)$ *and capital allocation efficiency* $a(w, s)$*.*

Proof. With logarithmic preferences, the characterization of a Markov equilibrium can be divided in two blocks, where one of them is independent of investment opportunities $\{\xi_h, \xi_e\}$.

This block correspond to the following equations:

$$\Delta_e = \Big(\sigma_{n,e} - \sigma_{n,h}\Big) \cdot (\sigma_k + \sigma_q) + \beta_e \varsigma \tag{14a}$$

$$0 \leq \Delta_e \quad \text{with equality if } \underline{\chi} < \chi \tag{14b}$$

$$\Delta_h \leq \hat{k}_h \varsigma \quad \text{with equality if } 0 < \hat{k}_h \tag{14c}$$

$$\frac{a_e - a_h}{q} \geq \chi \Delta_e - \Delta_h \quad \text{with equality if } 0 < \hat{k}_h \tag{14d}$$

$$\rho q + \iota(q) = (a_e - a_h)\kappa + a_h \tag{14e}$$

$$\sigma_k + \sigma_q = \frac{\sigma_k + \sigma'_{X_{-w}} \partial_{X_{-w}} \ln q}{1 - (\chi\kappa - w)\partial_w \ln q} \tag{14f}$$

This system implicitly defines the PDE for capital $q(X)$: we can solve statically for $\{\Delta_e, \Delta_h, \kappa, \chi, \sigma_q\}$ as a function of q and its (first-order) derivatives and the boundary conditions are $q(0, X_{-w}) = q^{\dagger}$ and $q(1, X_{-w}) = q^{\star}$ solved from (14e) with $\kappa = 0$ and $\kappa = 1$, respectively. Hence, the system fully characterizes function $q(X)$ and it does not vary if we consider the case without run, i.e., $\Gamma \equiv 0$.