Heterogeneity in Macroeconomics: Methods and Applications

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Course Information

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Classes: Tuesdays 4-7pm. **Location:** Room F306, PUC Rio.
Software: Matlab and Python (sample codes).

Course Description

This course examines analytic and computational methods for working with heterogeneous agent macroeconomic models in both discrete and continuous time. The standard incomplete markets model (Bewley-Huggett-Aiyagari), which captures heterogeneity in the household sector, serves as the foundational example for introducing these techniques.

Building on this foundation, the course applies these methods to the leading framework for studying monetary policy under heterogeneity: Heterogeneous Agents New Keynesian (HANK) models, which integrate the Bewley-Huggett-Aiyagari model with the New Keynesian framework. A major focus is placed on understanding the transmission mechanism of monetary policy.

The course emphasizes mastering analytic and numerical methods, enabling students to engage with new literature on their areas of interest and to apply heterogeneous agent models in both research and policy contexts.

Evaluation

1. **Problem sets (50% - 65%)**. There will be homework assignments during the course. Some will be computational, others more theoretical. Together with the problem sets of the first part of the course, these will account for the lion share of the grade.
2. **Presentation and referee report (15% - 20%)**. Choose a paper from **Part V** of the syllabus or a different one that addresses with one of those topics or uses a similar toolbox

(e.g., you can find recent working papers at the NBER webpage, click here). First come, first serve, but the professor must approve the student's choice. Add your name and the name of the paper you want to present here by **Thursday October 17th 11:59pm**. You will prepare a 15-20' presentation and a referee report (due one week after the presentation, please also submit your slides). Presentation dates will be **November 5, 12, 26, December 3**. The professor will pick the date based on the chosen papers. Please, read: *How to Write an Effective Referee Report and Improve the Scientific Review Process*, by Berk, Harvey, and Hirshleifer, published in the *Journal of Economic Perspectives*, 2017. These are also useful resources too: Suggestion 1, Suggestions 2, Suggestions 3. If the paper is already published, the "referee report" can mainly emphasize the critical review/summary part.

3. **Research Proposal (20% - 30%)**. You can work on it by yourself or in pairs. More progress is expected from proposals worked in pairs. You will make a 20-25' presentation. Presentation will be schedule for **the week that starts Monday, December 9th**. The research proposal will consist of: a clearly defined question; motivation of why the question is economically interesting; a synthesis of relevant background literature focusing on the points most germane to the question at hand; a discussion of an appropriate model; and a discussion of next steps. Preliminary results are not required. The idea is to use (or plan to use) the toolboxes developed during the course. I suggest 5-9 pages for the proposal. Please, I will reward quality not quantity. Due date for written assignments is December 14th, please also submit your slides.

   PhD students who are **not taking macroeconomics as a main field** have the option substitute the research proposal for a **critical survey** on a selected topic (e.g., one of the sections in Part V of the syllabus). The professor must approve the selection. The critical survey should convey the current state of the literature on the selected topic (what we know as a profession) as well as the potential holes in the literature. The survey is expected to show basic understanding of multiple papers. It is not about rephrasing the abstracts of each paper but draw connections and organize their contributions. Imagine you are drafting an article for the Journal of Economic Perspectives or the Journal of Economic Literature. It also involves a 20-25' presentation and a written assignment (5-9 pages) with the same due dates as the research proposal.

Course Outline

Part 0. Preliminaries

0. **Background readings**. *Heathcote *et al.* (2009), Guvenen (2012), Quadrini and Rios-Rull (2014), Krueger *et al.* (2016), *Kaplan and Violante (2018), *Galí (2018), *Panageas *et al.* (2020).

1. **Introduction.** (a) An overview of the history of macroeconomic thought, with a focus

on the role of inequality and heterogeneity. (b) Why use heterogeneous agent models for monetary policy? Examining the monetary policy transmission mechanism in the Representative Agent New Keynesian (RANK) model and its counterfactual implications.

Part I. Standard incomplete markets model: discrete-time

2. **Partial equilibrium.** We study the consumption-saving problem of a single agent facing a stochastic income stream who can only trade a risk-free bond under a borrowing constraint. We also analyze the evolution and stationary distribution of a household sector comprising a continuum of agents, each encountering this income fluctuation problem.

   (a) **Analytic insights.** An exploration of the long-term behavior of consumption and asset holdings. The Permanent Income Hypothesis (PIH) and certainty equivalence. Challenging certainty equivalence: The role of precautionary savings driven by prudence (due to the convexity of marginal utility) and the impact of occasionally binding borrowing constraints.

   Books and lecture notes: *Ljungqvist and Sargent (2018, ch. "Self-Insurance"), *Lecture notes by Fabrizio Perri (lectures 5 and 6), available here. ***Papers***: Leland (1968), Sandmo (1970), Kimball (1990), Sibley (1975), Chamberlain and Wilson (2000), Schechtman (1976), Schechtman and Escudero (1977), Yaari (1976), Bewley (1977).

   (b) **Numerical solution for agent's problem and stationary distribution.** Numerical methods for solving the consumption and saving policy functions in the recursive formulation of the income fluctuation problem, as well as for determining the stationary distribution of the household sector.

      i. Non-uniform grids, discretization of continuous processes (Rouwenhorst).
      ii. Bellman equation *(forward looking equation, solved backward)*: Value function Iteration, Euler Equation Iteration, Endogenous Grid Points Method.
      iii. Distribution evolution *(backward looking equation, solved forward)*: Simulations, lotteries, expectation functions.

   Books and lecture notes: *Lecture notes by Auclert, Rognlie, and Straub (Standard incomplete markets model, part I, available here), Adda and Cooper (2003), Heer and Maubner (2024), Miranda and Fackler (2002), Judd (1998). ***Papers***: *Carroll (2006), *Suen and Kopecki (2010), Tauchen (1986), Lkhagvasuren (2012), Lkhagvasuren and Gospodinov (2014).

3. **General equilibrium.** We embed the household sector, composed of agents facing the income fluctuation problem, into a general equilibrium framework. We consider three alternatives for asset supply: bonds in zero net supply (Huggett model), bonds or money in positive net supply (Bewley model), and capital in positive net supply

(Aiyagari model). Our focus is on a version of the Aiyagari model that includes labor, known as the Neoclassical Growth Model with incomplete markets. We numerically solve for the steady state and discuss a strategy for solving transition dynamics using an intuitive, yet inefficient and potentially fragile, updating rule for the path of prices.

Books and lecture notes: *Ljungqvist and Sargent (2018, ch. "Incomplete Markets Models"), Heer and Maubner (2024). ***Papers***: Imrohoroglu (1989), Huggett (1993), Aiyagari (1994).

4. **Dynamics away from steady state.**

   (a) **Partial equilibrium.** We analyze the responses of a single household and the entire household sector to current or future shocks in the real interest rate or income.

   (b) **General equilibrium.** In a Bewley economy, we investigate how to determine the path of interest rates that clear the market following a MIT productivity shock. To motivate the sequence-space Jacobian (SSJ) approach, we demonstrate the fragility of ad-hoc iterative methods. We compute SSJs using a "brute force" method.

   (c) **SSJs and fake news algorithm.** An efficient approach to calculate SSJs.

   (d) **General equilibrium Jacobians.** We use partial equilibrium (PE) Jacobians (e.g., the response of aggregate asset demand to interest rate and income shocks) to compute general equilibrium (GE) Jacobians (e.g., the response of the interest rate to a productivity shock) by applying the implicit function theorem, which reduces the calculation to simple matrix multiplication. GE Jacobians capture the first-order effects of MIT shocks, eliminating the need for iterative methods to determine the price path.

   (e) **First-order certainty equivalence and stochastic economies.** We apply certainty equivalence to approximate impulse response functions (IRFs) of a stochastic economy using the first-order IRFs derived from an MIT shock (i.e., using GE Jacobians). This approach allows us to compute second moments both analytically and through simulations.

   Books and lecture notes: *Lecture notes by Auclert, Rognlie, and Straub (Standard incomplete markets model, parts II and III, available here). ***Papers***: *Auclert *et al.* (2021), Boppart *et al.* (2018).

Part II. HANK models in discrete-time

5. **A canonical HANK model.** We present a canonical Heterogeneous Agent New Keynesian (HANK) model that integrates two leading frameworks of modern macroeconomics. On the household side, it builds on the standard Aiyagari-Huggett incomplete markets model, while the remaining components follow the New Keynesian tradition, featuring sticky wages on the supply side. The model is closed with a Taylor rule for

monetary policy. We introduce three special cases: (i) Representative-agent model, (ii) Two-agent model, and (iii) Zero-liquidity model.

(a) **Solving using the Sequence-Space Jacobian toolbox.** We formalize the notion of a model block and introduce the concept of directed acyclical graphs (DAGs).

Books and lecture notes: *Lecture notes by Auclert, Rognlie, and Straub (Canonical HANK, available here), Galí (2015), Woodford (2003). ***Papers***: *Auclert *et al.* (2021), *Auclert *et al.* (2024b), Auclert *et al.* (2023), *Broer *et al.* (2020).

6. **Monetary policy in a canonical HANK model.** We review the transmission mechanism of monetary policy in the Representative Agent New Keynesian (RANK) model and highlight its counterfactual implications. We then examine monetary policy transmission in the Heterogeneous Agent New Keynesian (HANK) model, noting that while its aggregate effects are similar to those in RANK models, the transmission channels differ significantly. In RANK models, direct effects, such as intertemporal substitution of consumption, are predominant, whereas in HANK models, indirect effects play a more prominent role.

   Books and lecture notes: Lecture notes by Auclert, Rognlie, and Straub (Monetary policy, available here). ***Papers***: Auclert (2019), McKay *et al.* (2016), Werning (2015).

Part III. Standard incomplete markets model: continuous-time

7. **Continuous-time methods.**

   (a) **Deterministic optimization.** Hamiltonian and Hamilton-Jacobi-Bellman equation. ***Books and lecture notes***: Acemoglu (2009, Ch. 7, "Review of the Theory of Optimal Control"), Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lectures 1 and 3).

   (b) **Finite difference method for deterministic case.**
   Books and lecture notes: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 3). ***Papers***: Tourin (2013), Barles and Souganidis (1991).

   (c) **Stochastic calculus basics and optimization.** Standard Brownian Motion, Ito diffusion, Stochastic Differentiation: Ito formula, Generator of an Ito diffusion, Stochastic optimization: Dynamic Programming, Kolmogorov Forward Equation, basic stochastic calculus with jumps.
   References: For references on stochastic calculus and a useful discussion about them, see this syllabus by Jonathan Payne.

   (d) **Finite difference method for stochastic case.**
   Books and lecture notes: Lecture notes by Yuliy Sannikov (videos available here, see section 8), Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 4), Kushner and Dupuis (1992), Candler (1999).

8. **General Equilibrium: A Bewley-Huggett Economy.** We analyze the continuous-time version of a Bewley-Huggett economy with a two-state income process. We provide additional analytical insights into the income fluctuation problem encountered by individuals.

   (a) **Analytic insights.** We characterize consumption and saving behavior near the borrowing constraint and provide a uniqueness result for the stationary equilibrium, given that the intertemporal elasticity of substitution (IES) is equal to or greater than one.

   (b) **Numerical solution using finite differences.** Solve for steady state and transition dynamics.

   Books and lecture notes: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lectures 7 and 8). ***Papers***: Achdou *et al.* (2022), including Online Appendix available here and codes available here.

Part IV. HANK models in continuous-time

9. **Two-asset HANK model.** We extend a canonical HANK model with sticky prices to incorporate two assets/accounts with varying degrees of liquidity. This combination, along with adjustment costs for reallocating resources between accounts, creates "wealthy hand-to-mouth" agents who exhibit high marginal propensities to consume (MPCs) despite having substantial wealth. We decompose the transmission channel of monetary policy on aggregate consumption and discuss different notions of model equivalence —non-equivalence, weak equivalence, and strong equivalence— highlighting how these concepts can vary depending on the nature of the shock.

   Books and lecture notes: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 9). ***Papers***: Kaplan *et al.* (2018), Kaplan and Violante (2018), Alves *et al.* (2020).

Part V. Topics in HA macro models

(Included as references. If time permits, selected topics will be discussed. Some discussed in my other elective course next semester.)

10. **HA models and aggregate risk**

    (a) Approximations in sequence space

        i. Discrete time: Boppart *et al.* (2018), Auclert *et al.* (2021). Perfect foresight nonlinear solution: Boehl (2023).

        ii. Continuous time: Glawion (2023), Bilal (2023b).

    (b) Perturbation methods in state space

i. Discrete time: Preston and Roca (2007), Mertens and Judd (2018), Reiter (2009), Algan *et al.* (2014), Reiter (2023), Winberry (2018), Bayer and Luetticke (2020), Bayer *et al.* (2024), Bhandari *et al.* (2023), Childers (2018), Childers (2022), Childers and Dogra (2019).

ii. Continuous time: Ahn *et al.* (2018), Bilal (2023a).

(c) Projections and simulation methods: traditional approach

i. Discrete time: Early contributions. Algan *et al.* (2014), Krusell and Smith (1998), Den Haan (1996), Den Haan (1997), Algan *et al.* (2008), Den Haan and Rendahl (2010). Further contributions. Cao *et al.* (2023), Pröhl (2019).

ii. Continuous time: Schaab (2020), Schaab and Zhang (2022).

(d) Projections and simulation methods: machine learning approach

i. Discrete time: Azinovic *et al.* (2022), Kahou *et al.* (2021), Scheidegger and Bilionis (2019), Maliar *et al.* (2021).

ii. Continuous time: Fernández-Villaverde *et al.* (2023), Gopalakrishna *et al.* (2024), Gu *et al.* (2024), Huang (2023a), Huang (2023b), Han *et al.* (2021), Payne *et al.* (2024), Duarte *et al.* (2024), Gopalakrishna (2021).

11. **Estimating HA models**

DSGE estimation: Herbst and Schorfheide (2016), Fernández-Villaverde *et al.* (2016). Estimation of HA models: Fernández-Villaverde and Guerrón-Quintana (2021), Parra-Alvarez *et al.* (2023), Auclert *et al.* (2021), Auclert *et al.* (2020), Winberry (2018), Glawion (2023), Bayer *et al.* (2024), Acharya *et al.* (2023), Hagedorn *et al.* (2018), Liu and Plagborg-Møller (2023).

12. **Tractable HANK Models**

- Bilbiie (2024). Analytic characterization of core micro-heterogeneity channels: cyclical inequality and risk, self-insurance, precautionary saving, and intertemporal MPCs.
- Acharya and Dogra (2020). Incomplete markets and the resolution of New Keynesian paradoxes: the role of income risk cyclicality.
- Werning (2015). A zero liquidity HANK.
- Bilbiie (2020). A (New) Keynesian cross: a way to think graphically and analytically about the properties of HANK.
- Bilbiie *et al.* (2022). Complementarity between capital and income inequality leading to amplification of the effects of aggregate-demand shocks on consumption.
- Ravn and Sterk (2021). A tractable HANK model that integrates Search and Matching frictions in the labor market.

- Debortoli and Galí (2024). Merits and limitations of simple tractable New Keynesian models (RANK and TANK) in accounting for the aggregate predictions of HANK.
- Broer *et al.* (2020). Interaction between inequality and monetary policy.
- Bilbiie (2008). Limited asset markets participation.
- Sterk and Tenreyro (2018). Redistribution channel for the transmission of monetary policy.

13. **Monetary Policy topics in HA models**

- Auclert *et al.* (2020). A HANK model with sticky household expectations that matches microeconomic evidence on MPCs and macro evidence on the IRFs to a monetary policy shock.
- Luetticke (2021). The importance of heterogeneity in household portfolios for the transmission of monetary policy with assets with different liquidity.
- Auclert *et al.* (2024c). Endogenous portfolios and risk premia.
- Guerreiro (2023). Belief disagreement.
- Farhi and Werning (2019). Bounded rationality.
- McKay and Wieland (2021). Fixed cost model of durable consumption where monetary policy affect the future natural interest rate.
- Laibson *et al.* (2021). Present-biased time preferences and naive beliefs.
- Beraja *et al.* (2019). Time-varying regional distribution of housing equity influences the aggregate consequences of monetary policy through its effects on mortgage refinancing.
- McKay *et al.* (2016). Incomplete markets and the power of forward guidance.
- Hagedorn *et al.* (2019). Forward guidance in a liquidity trap.
- Cui and Sterk (2021). QE in a heterogeneous-agents model with liquid and partially liquid wealth, and nominal rigidities.
- Kekre and Lenel (2022). Monetary policy, redistribution between agents with different risk aversions and risk premia.
- Auclert (2019). Redistribution in the transmission mechanism of monetary policy to consumption.
- Berger *et al.* (2021) mortgage prepayment and path-dependent effects of monetary policy
- Eichenbaum *et al.* (2022). How the impact of monetary policy depends on the distribution of savings from refinancing mortgages.
- Gornemann *et al.* (2021). Welfare implications of monetary policy in a HANK model with labor market frictions.
- Greenwald (2018), Wong *et al.* (2019). The mortgage credit channel.

14. **Optimal policy in HA models**

- Aiyagari (1995). Optimal capital income taxation with incomplete markets.
- Acikgoz *et al.* (2018). The dynamic optimal Ramsey taxation problem in a model with incomplete markets.
- Chien and Wen (2024). The Ramsey steady-state conundrum in heterogeneous-agent economies.
- Auclert *et al.* (2024a). Optimal long-run policy with heterogeneous agents.
- Nuño and Moll (2018). Social optima in an Aiyagari economy with stochastic lifetimes and in a model of on-the-job search with learning.
- Hong *et al.* (2012). Constrained efficiency in the one-sector neoclassical growth model with uninsurable idiosyncratic shocks.
- Bigio *et al.* (2019). Optimal debt-maturity management problem of a government.
- Bigio and Sannikov (2021). Implementation of monetary policy through the banking system into an incomplete-markets economy with wage rigidity.
- Nuño and Thomas (2022). Ramsey-optimal monetary policy under commitment with uninsurable idiosyncratic risk, long-term nominal bonds, and costly inflation.
- Lucas Jr and Moll (2014). Planner problem in an endogenous growth model of time allocation.
- La'O and Morrison (2024). Optimal monetary policy with heterogeneous workers.

15. **Firm Heterogeneity and Stopping Time Problems.** We discuss (the continuous time version of) a workhorse model of firm dynamics: Hopenhayn (1992), and study stopping time problems in continuous time. We discuss the standard solution approach: "smooth pasting", but we focus on a more powerful approach: HJB "Variational Inequality."

   Books and lecture notes: Lecture notes by Benjamin Moll (Distributional Macroeconomics course), available here (Lecture 10). Stokey (2008), Dixit (1993), Dixit and Pindyck (1994). ***Papers*** (Theories of Firm Size Distribution): Hopenhayn (1992), Luttmer (2007), Bertola and Caballero (1994), Caballero and Engel (1999), Khan and Thomas (2008), Winberry (2016), Zwick and Mahon (2017).

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