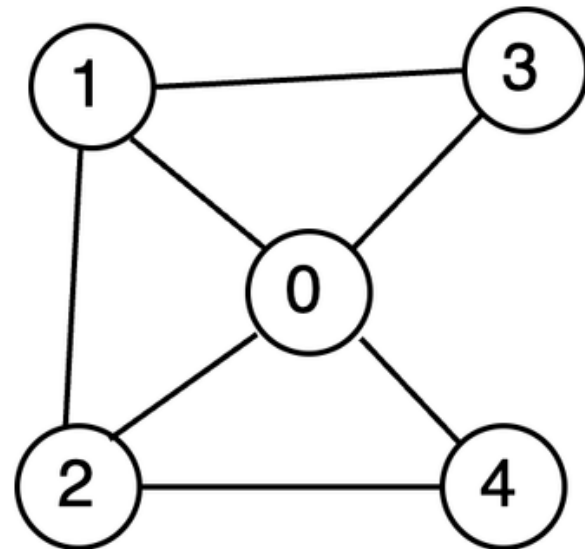


**Cenia Meeting 26-07-24**  
**Compositional Generalization:**  
**Triangles Dataset**

# Dataset Generation

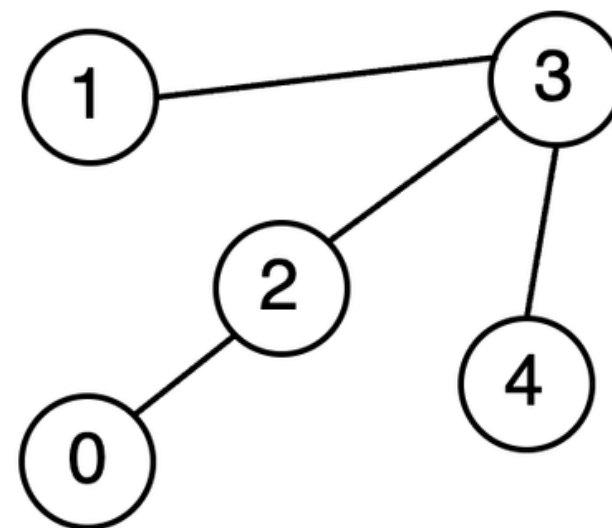
Con 

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$



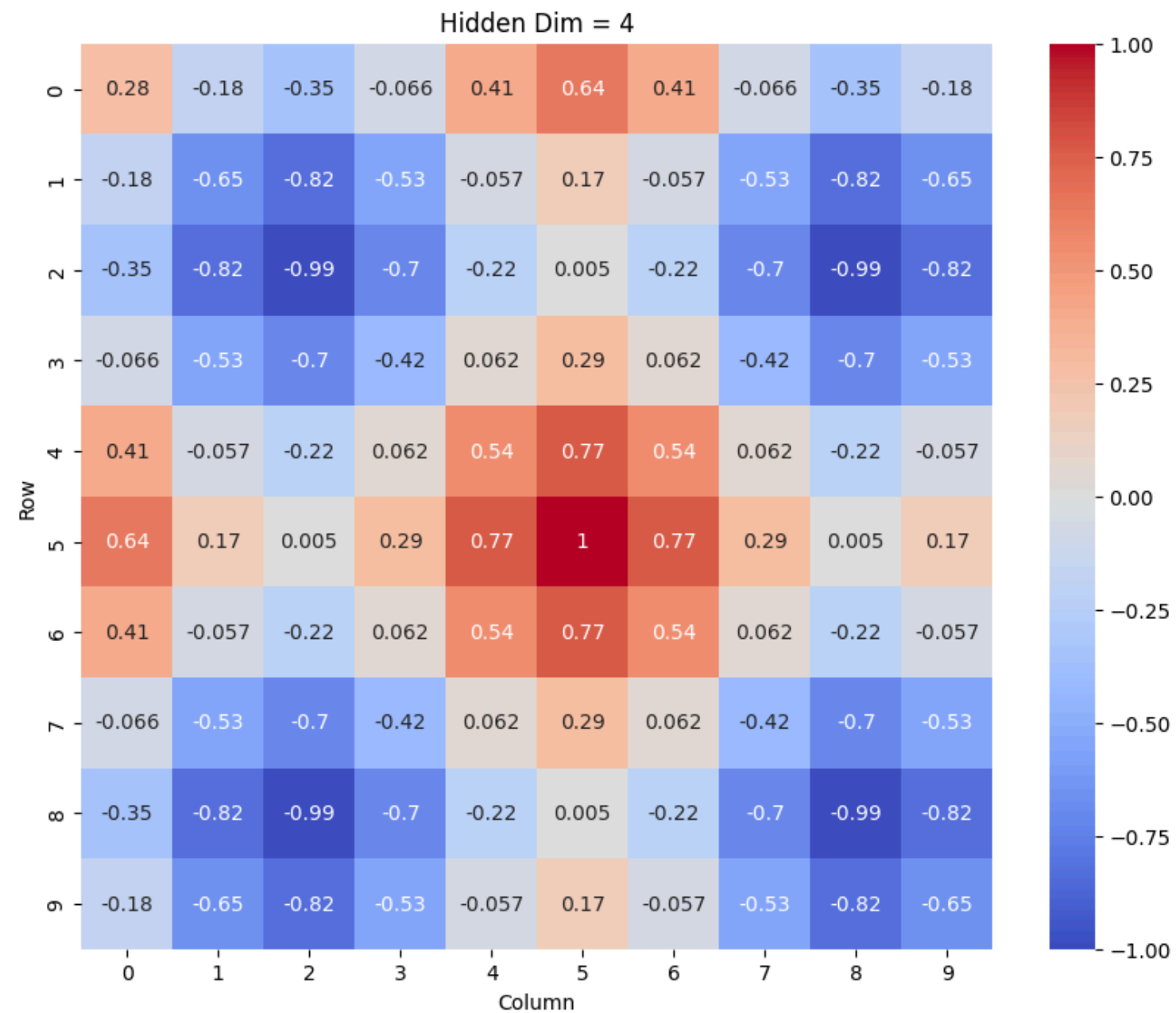
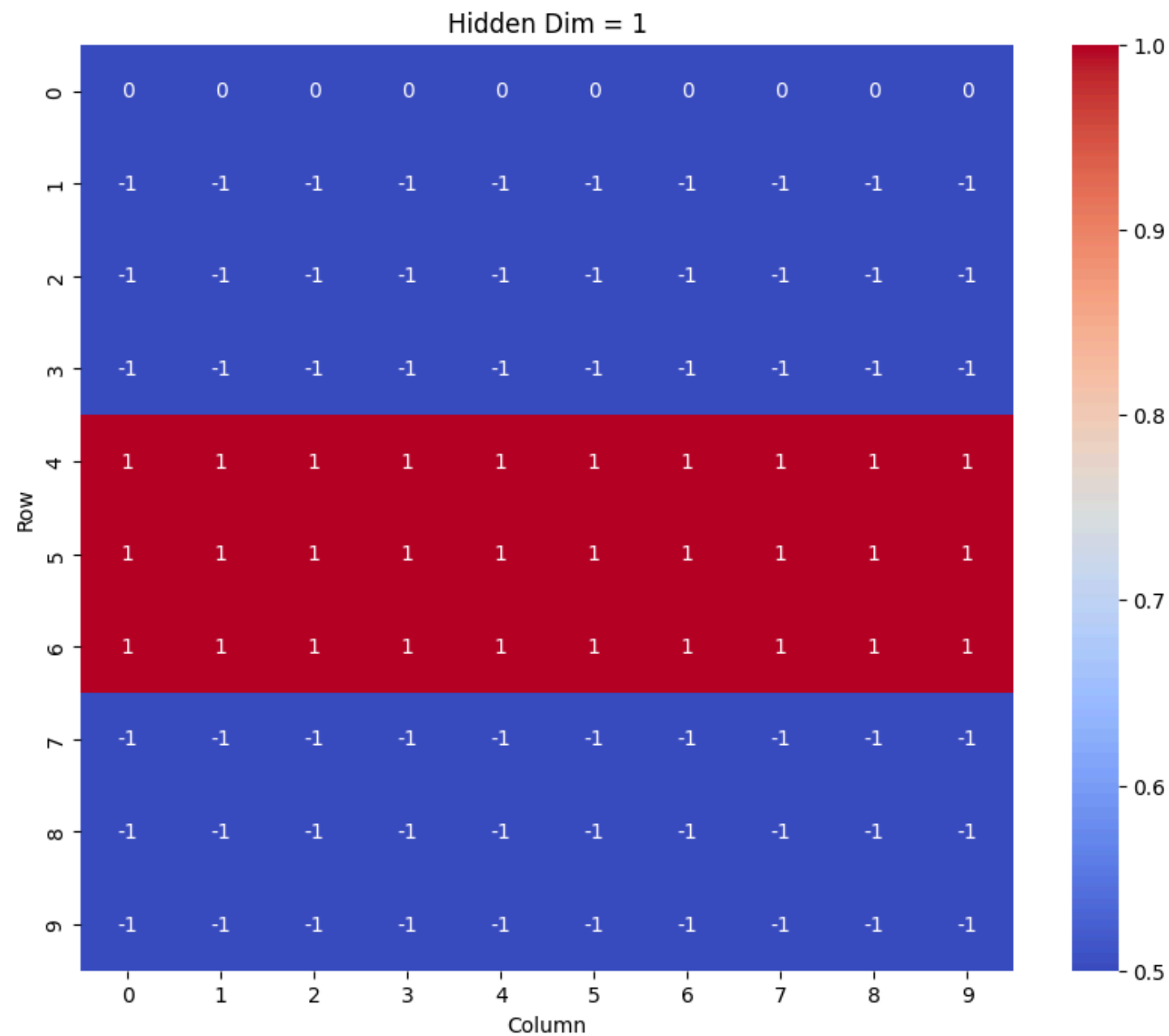
Sin 

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

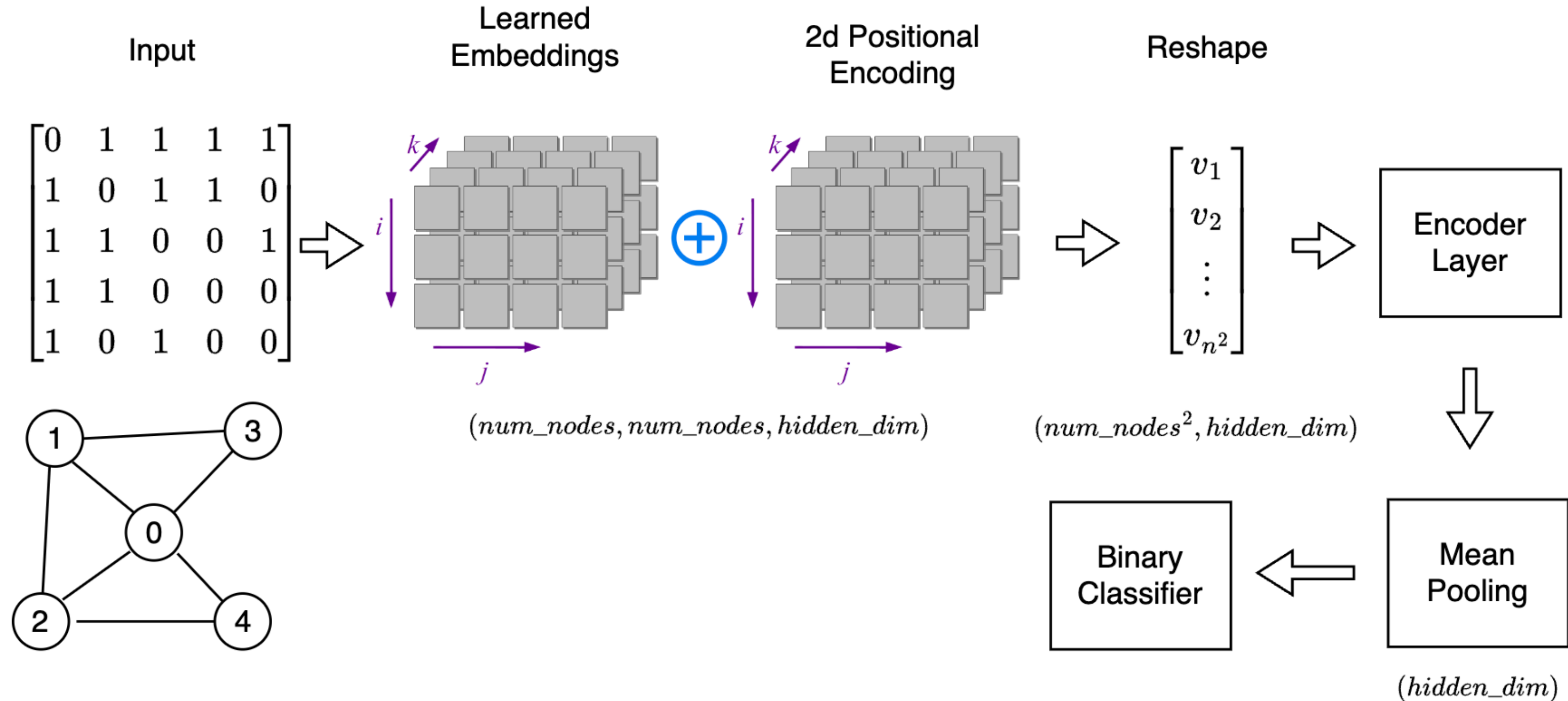


$num\_nodes \in \{5, \dots, 10\}$

# 2d Positional Encoding



# Encoder Experiment



# Encoder Experiment

## Architecture:

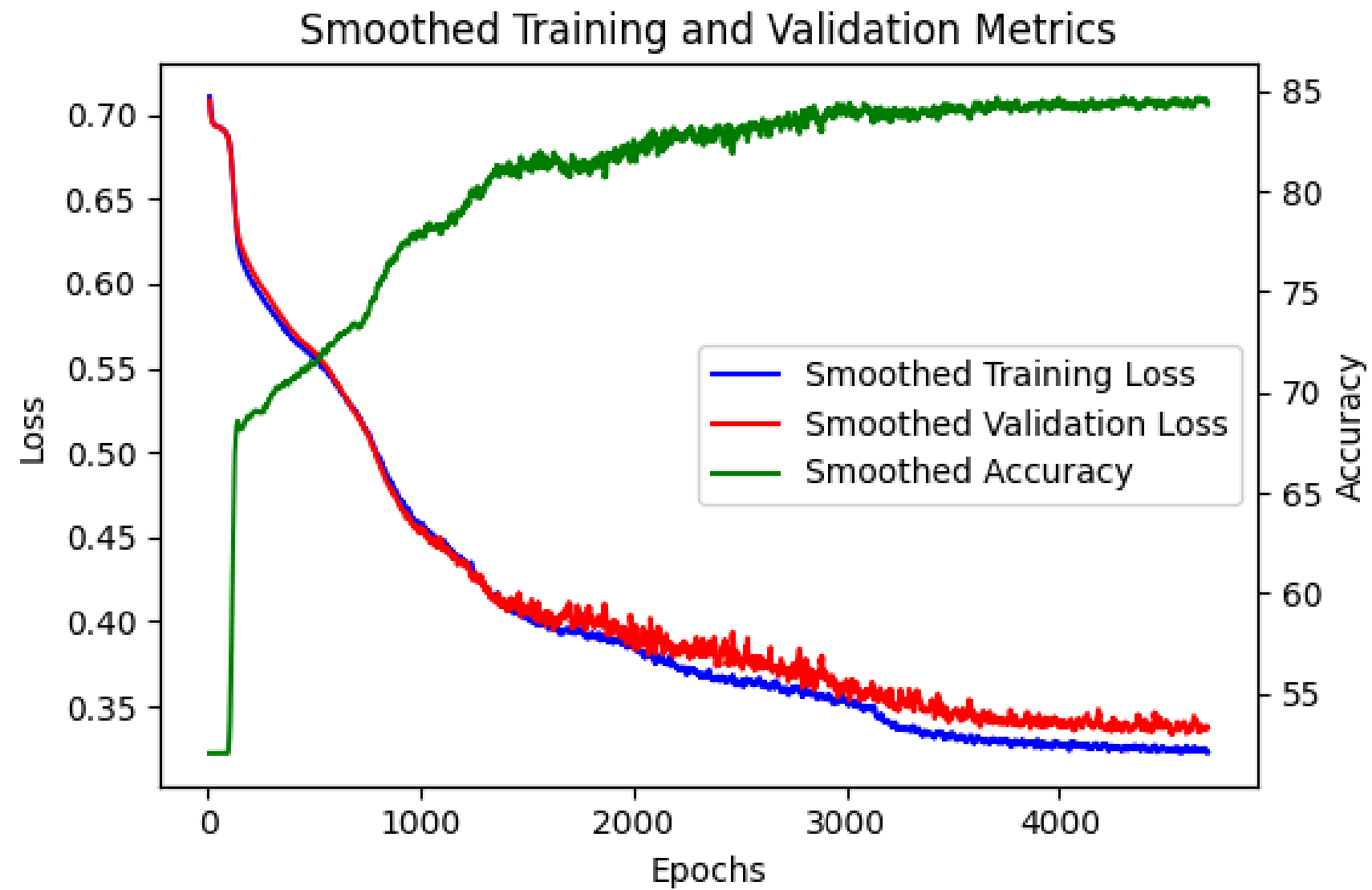
- Input\_projection -> Adj. Matrix to nn.Linear
- Positional Encoding -> Positional encoding 2D
- Standard Pytorch Attention Layer
- Binary Classifier -> Sigmoid

## Hyperparameters:

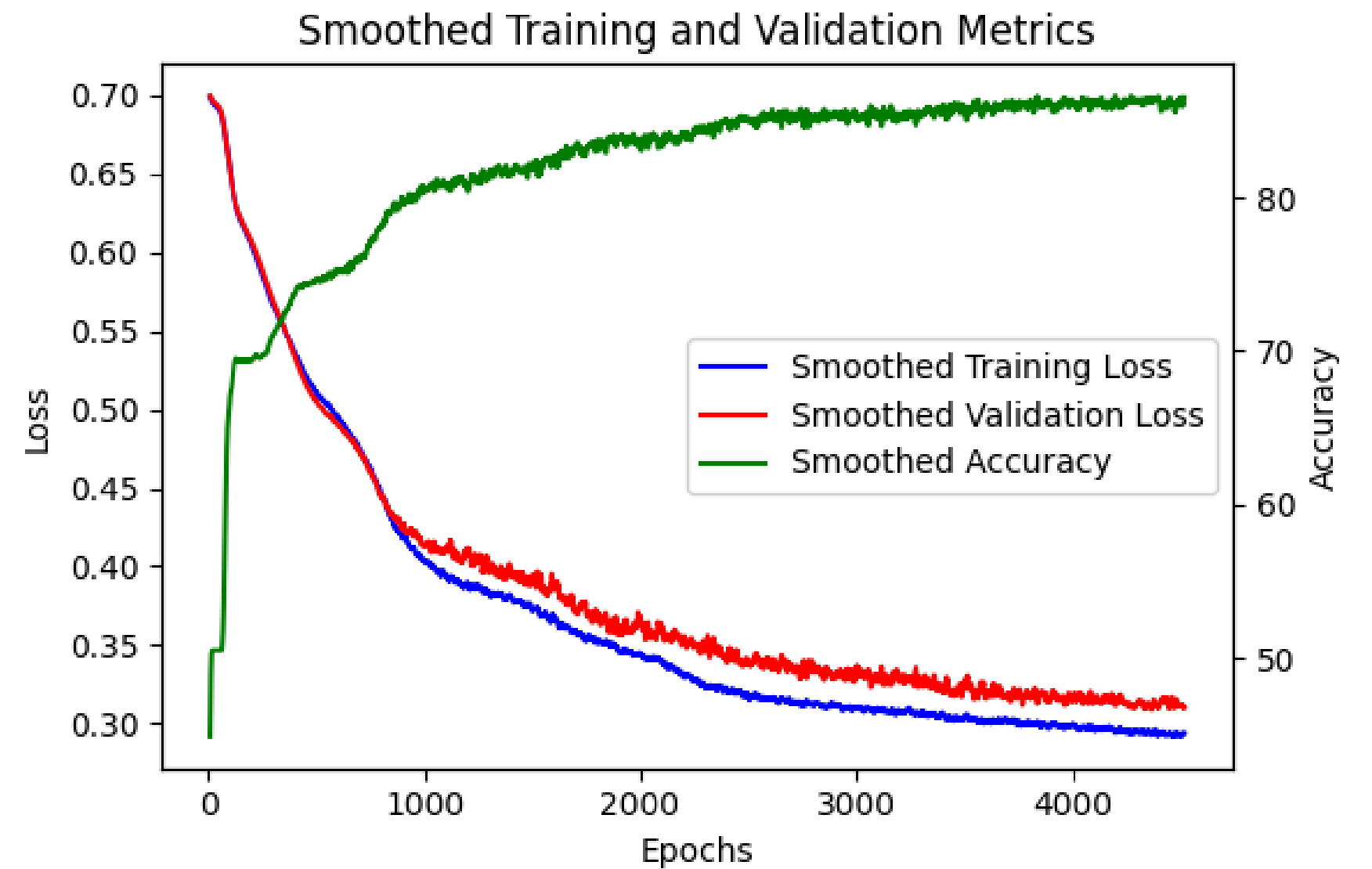
- Hidden\_dimension -> 4, 6, 32
- Attention\_heads -> 4, 6, 8
- Learning\_rate -> 1e-3
- Epochs -> 4500
- Max\_nodes -> 10

# Results

hidden\_dim = 4  
num\_heads = 4  
num\_layers = 1



hidden\_dim = 6  
num\_heads = 6  
num\_layers = 1

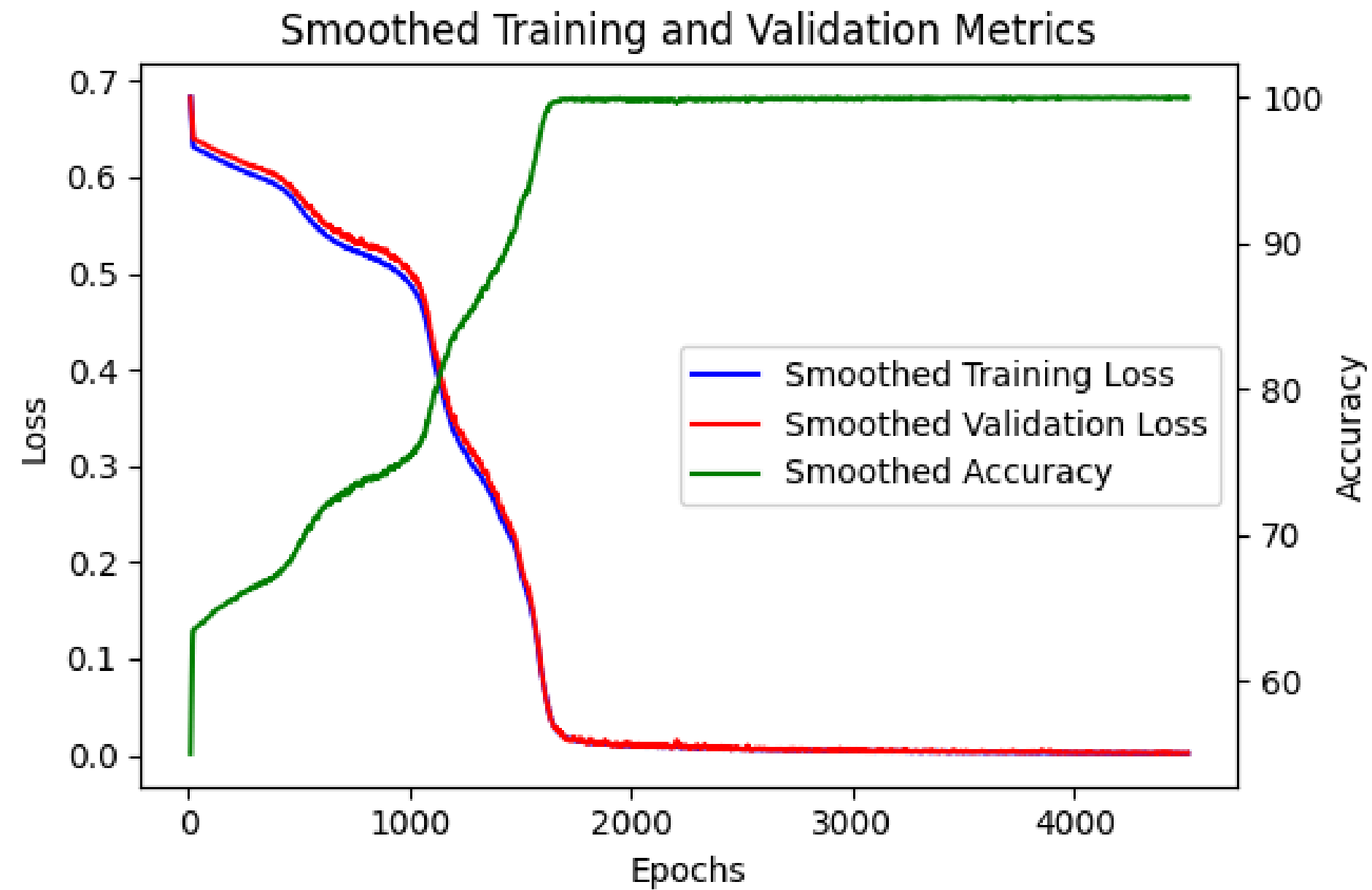


# Experiment 2 Results

hidden\_dim = 32

num\_heads = 8

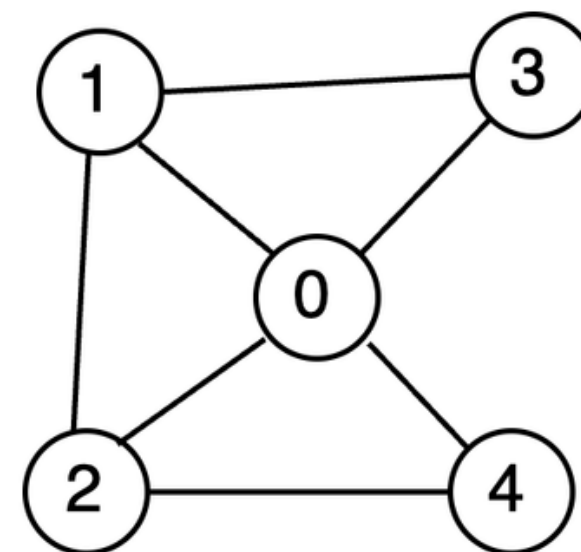
num\_layers = 1



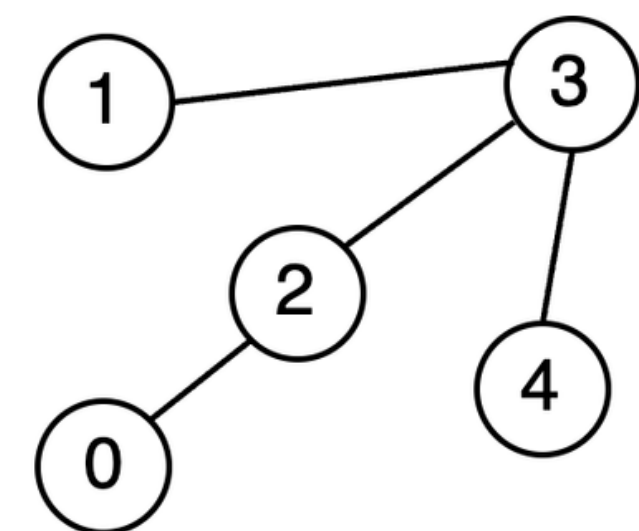
# Discussion

When tokenizing  
an adjacency  
matrix each  
token represents  
a relation  
between nodes

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

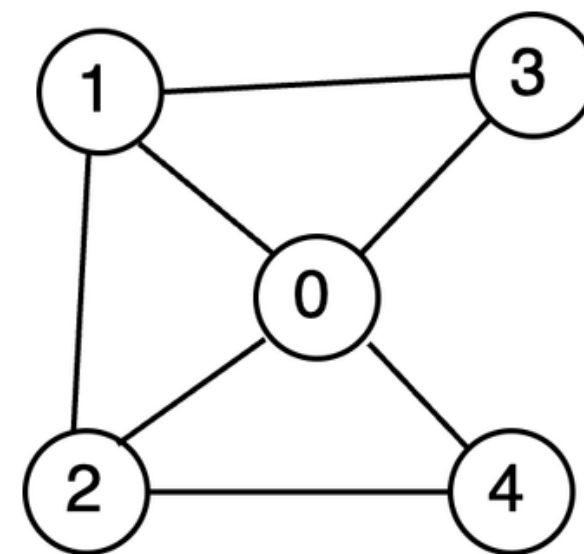




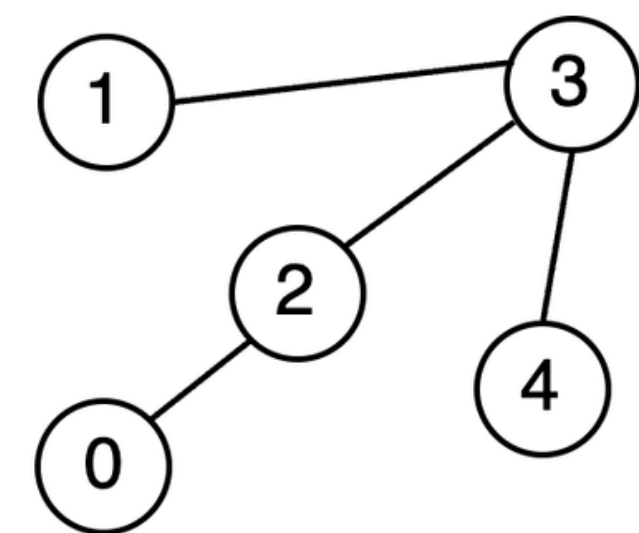
# Discussion

Then attention is  
directly learning  
“relations  
between  
relations”

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



# Discussion

On NLP transformers, each input token represents a set of characters, but not directly their relationships (the relationships are learned)

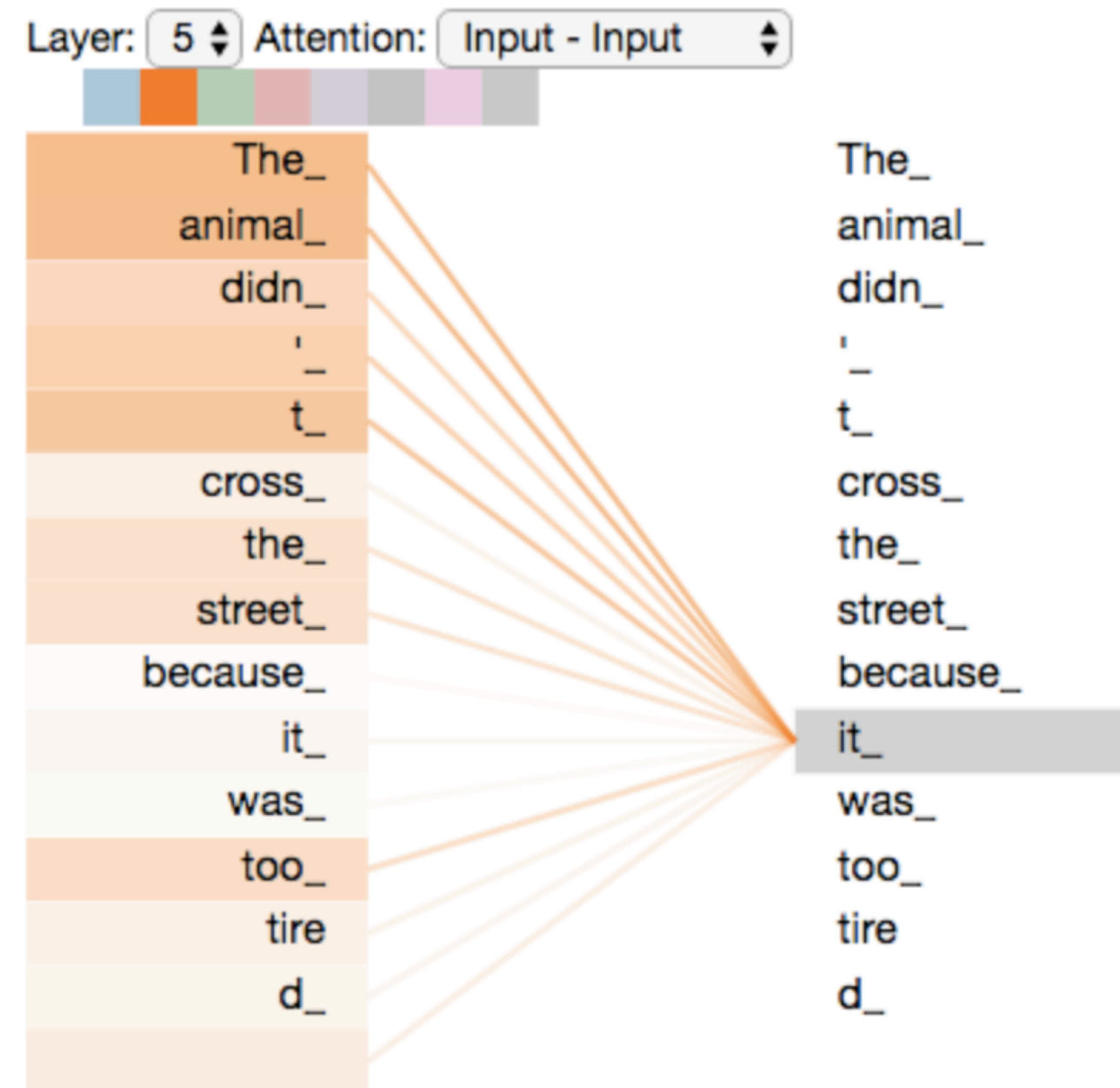
$x_1$    
Je

$x_2$    
suis

$x_3$    
étudiant

# Discussion

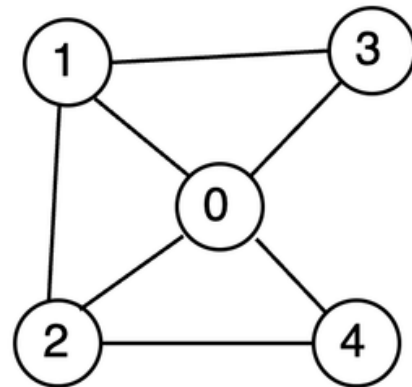
Token relationships are  
learned through attention

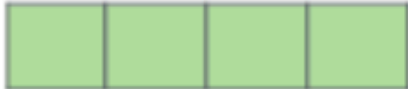


# Discussion

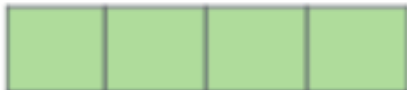
The mismatch between the tokenized information  
might imply that our test is not assessing  
composition

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

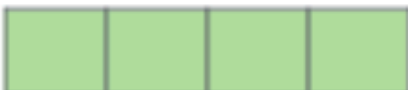


$x_1$  

Je

$x_2$  

suis

$x_3$  

étudiant

# Options?

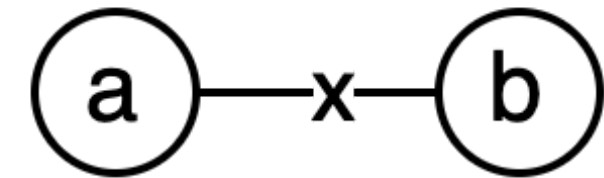
Maybe our dataset should feature both nodes and relations as tokens expressed as a set of “fact” sequences

Node tokens:  $\{a, b, c, d\}$

Relation tokens:  $\{x, y\}$

Input  
sequence:

t1   t2   t3  
”*a*   *x*   *b*”



# Options?

And also include compositions as tokens, all of this expressed as a set of “fact” sequences

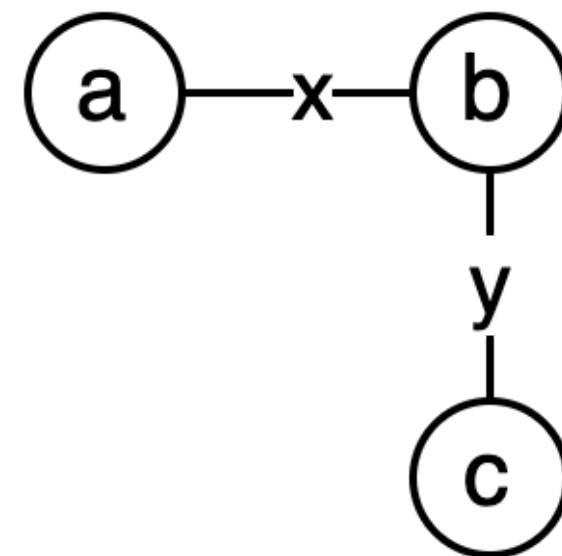
Node tokens:  $\{a, b, c, d\}$

Relation tokens:  $\{x, y\}$

Composition token:  $\{z\}$

$$z = (x \circ y)$$

sequence:  $\begin{matrix} & t1 & t2 & t3 \\ & a & z & c \end{matrix}$



$$(a, c) \in (x \circ y)$$