Quick Invariant Signature Extraction from Binary Images

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***Abstract*—It is shown how basic geometric notions can be used to extract an image signature independently of position, orientation and size. Simple primitives as lengths and slopes remain invariant by affine 2D similarity transformations. They can easily be used to define the invariant signature of an image. Contrary to the previous work in this area, images can be directly analyzed. This means that the extraction of interest points of the image is avoided. The method remains formal and no estimation or compression is needed. It is formally demonstrated that 100% of transformations are taken into consideration and that the signature of the image is totally invariant. The Quick Invariant Signature (QIS) extraction is a formal and fast method. It can be used either, only for signature extraction, or be integrated into a neural architecture for both extraction and classification. Unusual invariances such as cylindrical translation or toric translation are also defined by QIS.**

***Index Terms*—Shape matching, invariant descriptor, image signature, binary.**

I. Introduction

A complete vision system has to be able to process and analyze objects independently of their positions, orientations and scales. Many different methods for invariant pattern recognition have been developed[1][2][3][4][5]. We can classify them into three main classes: Invariant moments, invariant descriptors, and invariant neural architectures. Invariant moments[6] and Fourier-mellin descriptor[7][8] require complex calculation and high computation time. They remain quite sensible to noise and images cannot be directly analyzed. Therefore only a few of the interest points are taken into consideration. Other invariant descriptors, such as the circular descriptor, are highly sensible to noise and can cause sampling problems. Whereas the most popular neural network approach[9] with a multilayer perceptron network and the backpropagation learning algorithm requires a huge training set to learn the invariance to affine 2D transformations. Other neural networks try to integrate invariance into their architecture but this causes a problem of network size by combinatory explosion. The size of their input is thus limited. The Quick Invariant Signature (QIS) extraction is a formal method which uses simple geometric notions such as length or slopes to define an invariant signature of a binary image. The size of the input is not limited and no pre-processing is needed. The whole image is taken into account for the extraction of the signature. QIS can be used to extract a signature, invariant not only from classic affine 2D transformations such as translation, rotation and scale, but also from unusual transformations such as symmetry, cylindrical translation, toric translation and fovea size change. QIS can also be applied directly to rectangular images. We demonstrate that QIS is totally invariant to the considered transformation. QIS can also be integrated into a neural network architecture to provide a 100% invariant neural network. It is also possible to use QIS to test, in a formal approach, if two classes of forms are "invariant separable".

II. Invariances

When an image is mathematically transformed, some evident low level properties (lengths, slopes, angles) between couples of points or triplets of points remain unchanged. It can be easily demonstrated that, for example, lengths and slopes of couples of points remain unchanged when translated. Table II illustrates some of these invariants. Instead of defining a binary

Transformation	Invariant
Translation	Distances+Slopes
Translation + Rotation	Distances
Translation + Scale	Slopes
Transl + Rot. + Scale	Similar triangles

image by active or inactive pixels, we can define it in terms of active couple of points. A couple is considered as active if the two points are active. Once the image is defined as couples of active points we can use the invariants of table II to extract the invariant signature

III. Signature Extraction

The idea is quite simple. If an image is defined in terms of similar couples of points, we have an invariant signature. Similar couples are couples which have the same mathematical properties needed to define an invariance. For example the invariance to translation is defined by identical length and slope. So, similar couples in this case are all couples with the same length and the same slope. The most simple way to define an image in terms of similar couples is to count them. The signature is then obtained by counting the number of similar couples in each similarity group. The details of this process are given case by case in the following sections.

A. Translation QIS

In the case of translation invariance the similarity properties taken into account are the lengths and slopes of the active couples of points. The first step is to count the number of different similarity groups. We can easily demonstrate that in

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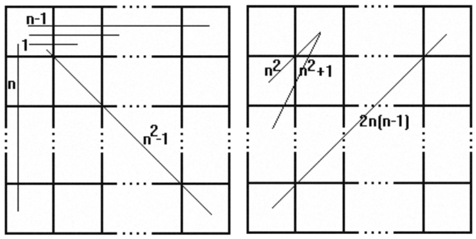


(a) Negative, nil. (b) Positive.

Fig. 1. Possible slopes.

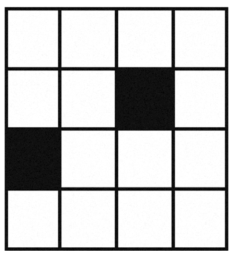


Fig. 2. One active couple in a 4×4 grid.

a $n \times n$ grid, the number of different length-slope entities is $2n(n-1)$. There are $n^2 - 1$ different lengths with a negative, nil or right angle slope and $(n-1)^2$ different lengths with a positive slope (see Figure 1). These groups are ordered as follows: -positive slope groups from n^2 to $2n(n-1)$; -other slopes from 1 to $n^2 - 1$. The invariant signature of the image is obtained by counting the number of active couples in each of these numbered groups. The result is a table of $2n(n-1)$ integers where the index is the order of the similarity group and the value is the number of active couples in this group. The size of the signature is, thus, allways less than twice the size of the image. Two examples are given in figure 2 and 3. In figure 2, only one couple of points is active in the 4×4 grid. There are 24 different similarity groups and the order of the group of the active couple is 19. The signature of this image will be:

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0

All the translated views of this couple will give the same signature since the order of the similarity group of the active couple remains 19. Figure 3 shows a 10×10 grid with 180 similarity groups. All the active couples are counted to define the invariant signature. Array III-A defines the signature of

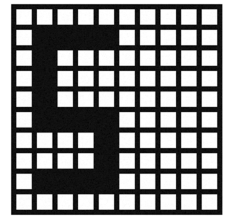


Fig. 3. Multiple active couples in a 10×10 grid.

figure 3.

9	6	3	0	0	0	0	0	0	7	2	2	2	0	0
0	0	0	0	5	2	2	3	0	0	0	0	0	0	6
4	3	4	0	0	0	0	0	0	5	4	3	4	0	0
0	0	0	0	2	2	2	3	0	0	0	0	0	0	2
2	2	2	0	0	0	0	0	0	4	3	2	1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	2	4	3	0	0
3	0	0	2	2	3	2	0	0	2	0	0	2	2	2
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

This table is totally invariant to translation. This is quite obvious since each similarity group is translation invariant and the number of couples in a similarity group does not change by translation. In fact this table can be considered as a redefinition of an image in a new 3D space having as axis the length, the slope and the number of couples. This 3D space is then reduced to a 2D space where the area covered by the slope and the length is represented by the similarity group order axis. To find to which group a couple of points belongs, we should compare exhaustively the length and the slope of the couple to the length and the slope of all the similarity groups. Considering a $n \times n$ grid, we will have at most $2n(n-1)-1$ comparisons per couple and at most $n^2! \div (2!(n^2-2)!)$ couples. One can see that the comparison process for extracting the signature is far too slow. That's why we have formalized the access to the order number of the similarity groups by a simple function that does not go through the comparison process. The problem is: "given two pixels, find the number of their similarity group". To do this we numerate the pixels of a $n \times n$ grid from 1 to n^2. Then, given two pixels p and q where the subscripts $_i$, $_r$ and $_c$ refer to the matrix index, the row, and the column, respectively, the algorithm III.1 will find the similarity group number (SGN) of a couple. Let $w = \lfloor \frac{q_i}{n} \rfloor - \lfloor \frac{p_i}{n} \rfloor$.

Algorithm III.1: GET-SGN-T(p, q, n)

if $p_c \leq q_c$
  then $s \leftarrow q_i - p_i$

  else $s \leftarrow (p_c - q_c - 1)(n-1) + (n^2 - 1) + w$
return (s)

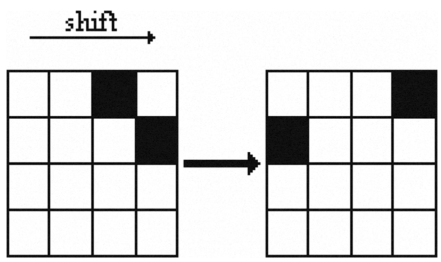


Fig. 4. Cylindrical translation.

This function will find the same number for all couples with equal slopes and equal lengths. The signature extraction is then achieved by algorithm III.2.

Algorithm III.2: $\text{EXTRACTSIGNATURE-T}(\mathcal{A}, n)$

$$\begin{array}{l}
\textbf{local } t[2n(n-1)] \\
\textbf{for } i \leftarrow 0 \textbf{ to } 2n(n-1) \\
\quad \textbf{do } t[i] \leftarrow 0 \\
\textbf{for each } (p,q) \in \mathcal{A} \\
\quad \textbf{do } \begin{cases} \textbf{local } s \leftarrow \text{GET-SGN-T}(p,q,n) \\ t[s] \leftarrow t[s] + 1 \end{cases} \\
\textbf{return } (t)
\end{array}$$

$\mathcal{A}$ is the set of all active couples (p, q). Now we can use the translation invariant signature to define many other invariances such as cylindrical or toric translation.

B. Cylindrical Translation QIS

A Cylindrical Translation (CT) invariance can seem to be useless in the pattern recognition domain, but one of its possible uses will be shown later on in this paper. Although the formal definition of a CT is quite difficult to establish, its expression in terms of similarity groups is rather simple. In figure 4, the two possible positions of a couple of points in a CT are shown. It can be seen that one couple of points in a CT can belong, at most, to two similarity groups of a simple translation. In this case the length of the couple and its slope are not unchanged by the CT any more. If the two possible simple translation similarity groups are considered as the same similarity group for the CT, we can easily express the invariance to CT. The QIS extraction remains exactly the same excepted for the SGN function. The SGN function in this case should find two similarity group order numbers, being given a couple of points. When the couple has a right angle slope, it will define only one similarity group. Therefore, the QIS extraction for a CT invariance uses the same similarity groups as the simple translation invariance. But the counting of entities belonging to each similarity group is different.

The function described by algorithm III.3 will find the order number of the similarity group (or groups) of a couple of pixels (p, q).

Algorithm III.3: $\text{GET-SGN-CT}(p, q, n)$

$$\begin{array}{l}
\textbf{if } p_r = q_r \\
\quad \textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow n - (q_i - p_i) \end{cases} \\
\quad \textbf{else if } p_c = q_c \\
\quad \textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow \emptyset \text{ (empty)} \\ \textbf{comment: } \text{Only has one similarity group} \end{cases} \\
\quad \textbf{else if } p_c < q_c \\
\quad \textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow (n - 1 - (q_c - p_c))(n-1) + (n^2 - 1) + w \end{cases} \\
\quad \textbf{else } \begin{cases} s[1] \leftarrow q_i - p_i + n \\ s[2] \leftarrow (p_c - q_c - 1)(n-1) + (n^2 - 1) + w \end{cases} \\
\textbf{return } (s)
\end{array}$$

This function will find the same numbers for all couples belonging to one of the two similarity groups. The signature extraction may be performed by algorithm III.4.

Algorithm III.4: $\text{EXTRACTSIGNATURE-CT}(\mathcal{A}, n)$

$$\begin{array}{l}
\textbf{local } t[2][2n(n-1)] \\
\textbf{for } i \leftarrow 0 \textbf{ to } 2 \\
\quad \textbf{do } \begin{cases} \textbf{for } j \leftarrow 0 \textbf{ to } 2n(n-1) \\ \quad \textbf{do } t[i][j] \leftarrow 0 \end{cases} \\
\textbf{for each } (p,q) \in \mathcal{A} \\
\quad \textbf{do } \begin{cases} \textbf{local } s \leftarrow \text{GET-SGN-CT}(p,q,n) \\ t[1][s[1]] \leftarrow t[1][s[1]] + 1 \\ t[2][s[2]] \leftarrow t[2][s[2]] + 1 \end{cases} \\
\textbf{return } (t)
\end{array}$$

As it is possible to see, the size of the signature will still be $2n(n-1)$. An example will be given later on.

C. Toric Translation QIS

A Toric Translation (TT) invariance can be used for regular texture recognition. As for the CT invariance, we will use the simple translation similarity groups to define the TT invariance. The only thing to be changed is the SGN function and the number of similarity groups involved in a TT. In figure 5, the four possible positions of a couple of points in a TT are shown. It can be seen that one couple of points in a TT can belong, at most, to four similarity groups of a simple translation. The QIS extraction in this case follows the same reasoning as for the CT invariance. The SGN function for TT QIS extraction is presented in algorithm III.5.

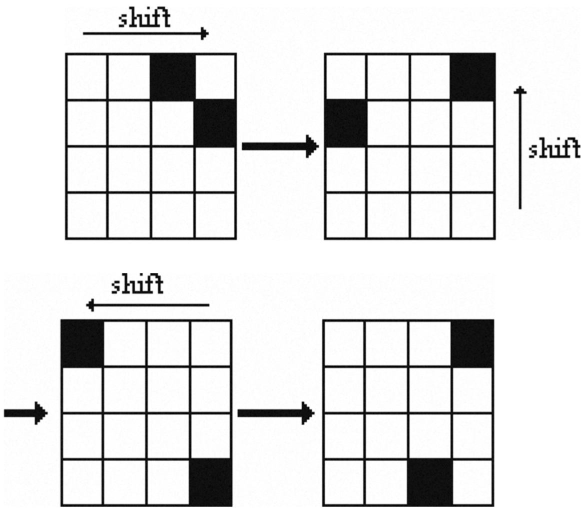


Fig. 5. Toric translation.

Algorithm III.5: GET-SGN-TT(p, q, n)

if $p_r = q_r$

$$\textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow n - (q_i - p_i) \\ s[3] \leftarrow s[4] \leftarrow \emptyset \end{cases}$$

else if $p_c = q_c$

$$\textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow n^2 - n(q_i - p_i) \\ s[3] \leftarrow s[4] \leftarrow \emptyset \end{cases}$$

else if $p_c < q_c$

$$\textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow (n - 1 - (q_c - p_c))(n - 1) + \ldots \\ \ldots (n^2 - 1) + w \\ s[3] \leftarrow 2n(n-1) - \ldots \\ \ldots (n - 1 - q_i + p_i)(n-1) - w + 1 \\ s[4] \leftarrow n^2 - 1 - n(\lfloor \frac{q_i}{n} \rfloor - \lfloor \frac{p_i}{n} \rfloor) - (q_c - p_c - 1) \end{cases}$$

$$\textbf{else } \begin{cases} s[1] \leftarrow q_i - p_i + n \\ s[2] \leftarrow (p_c - q_c - 1)(n-1) + (n^2 - 1) + w \\ s[3] \leftarrow n^2 - (q_i - p_i) \\ s[4] \leftarrow 2n(n-1) - (p_c - q_c - 1)(n-1) - w + 1 \end{cases}$$

return (s)

This function will find the same numbers for all couples belonging to each of the four similarity groups. The signature extraction is done as described by algorithm III.6.

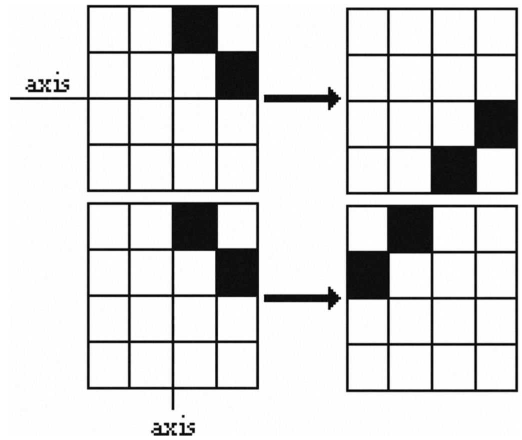


Fig. 6. Symmetry QIS.

Algorithm III.6: EXTRACTSIGNATURE-TT$(\mathcal{A}, n)$

local $t[4][2n(n-1)]$

for $i \leftarrow 0$ **to** 4

$$\textbf{do } \begin{cases} \textbf{for } j \leftarrow 0 \textbf{ to } 2n(n-1) \\ \quad \textbf{do } t[i][j] \leftarrow 0 \end{cases}$$

for each $(p, q) \in \mathcal{A}$

$$\textbf{do } \begin{cases} \textbf{local } s \leftarrow \text{GET-SGN-TT}(p, q, n) \\ t[1][s[1]] \leftarrow t[1][s[1]] + 1 \\ t[2][s[2]] \leftarrow t[2][s[2]] + 1 \\ t[3][s[3]] \leftarrow t[3][s[3]] + 1 \\ t[4][s[4]] \leftarrow t[4][s[4]] + 1 \end{cases}$$

return (t)

Again, the size of the signature remains $2n(n-1)$.

D. Symmetry QIS

A symmetry invariance can also be coded by using the simple translation similarity groups and a new SGN function. In figure 6, the two possible symmetrical positions of a couple of points are shown. It can be seen that one couple of points in a symmetry can belong, at most, to two similarity groups of a simple translation. Function presented by algorithm III.7 finds the two similarity groups involved in a symmetry invariance.

Algorithm III.7: GET-SGN-S(p, q, n)

if $p_r = q_r$ **or** $p_c = q_c$

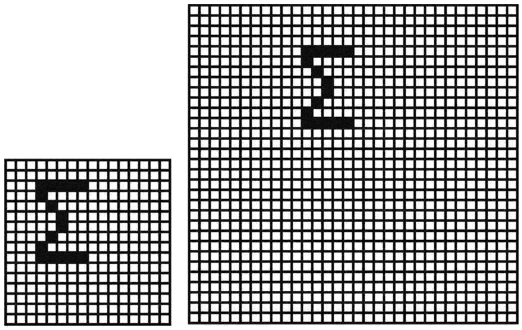
$$\textbf{then } \begin{cases} s[1] \leftarrow q_i - p_i \\ s[2] \leftarrow \emptyset \end{cases}$$

else if $p_c < q_c$

$$\textbf{then } \begin{cases} s[1] \leftarrow |q_i - p_i| \\ s[2] \leftarrow (|p_c - q_c| - 1)(n-1) + (n^2 - 1) + |w| \end{cases}$$

$$\textbf{else } \begin{cases} s[1] \leftarrow |q_i - p_i| + 2(p_c - q_c) \\ s[2] \leftarrow (|p_c - q_c| - 1)(n-1) + (n^2 - 1) + |w| \end{cases}$$

return (s)



(a) 16×16 fovea (b) 32×32 fovea

Fig. 7. Fovea size QIS.

E. Fovea Size QIS

In some cases, the size of the grid containing the reference image is not equal to the size of the grid containing the test image. The QIS extraction can be easily adjusted to allow the use of a classification independently of the fovea size. For example: A classification of the QIS of the figure 7(a) is available and we want to use it to classify the figure 7(b), or vice-versa. The idea is to extract the QIS of the test image at the size of the reference image. Let $m \times m$ be the size of the test image and $n \times n$ be the size of the reference image.

There are two cases. The **first case** happens when $m > n$. Let $\mathcal{V}$ be the order number of the similarity group of a couple in the $m \times m$ grid. $\mathcal{V}$ is found by one of the SGN functions defined earlier. Now we should find the order number of the same group in the $n \times n$ grid. The following function calculates this number for couple (p, q).

Algorithm III.8: GET-SGN-FS(p, q, m, n)

if $p_c \leq q_c$
  then $s \leftarrow \mathcal{V}(p, q, m) - (p_r - q_r)(m - n)$

  else $s \leftarrow \mathcal{V}(p, q, m) - m^2 - n^2 - (p_c - q_c - 1)(m - n)$
return (s)

The QIS extraction is then performed by algorithm III.9.

Algorithm III.9: EXTRACTSIGNATURE-FS1$(\mathcal{A}, m, n)$

local $t[2n(n-1)]$
for $i \leftarrow 0$ **to** $2n(n-1)$
  do $t[i] \leftarrow 0$
for each $(p, q) \in \mathcal{A}$
  do $\begin{cases} \textbf{local } s \leftarrow \text{GET-SGN-FS}(p, q, m, n) \\ t[s] \leftarrow t[s] + 1 \end{cases}$
return (t)

The **second case** is when $m < n$. In this case the problem is much simpler. The QIS extraction is as follows:

Algorithm III.10: EXTRACTSIGNATURE-FS2$(\mathcal{A}, m, n)$

local $t[2n(n-1)]$
for $i \leftarrow 0$ **to** $2n(n-1)$
  do $t[i] \leftarrow 0$
for each $(p, q) \in \mathcal{A}$
  do $\begin{cases} \textbf{local } a, b \\ \textbf{comment: } a,\ b \text{ are pixel structures} \\ a_i \leftarrow p_r \times m + p_c; a_r \leftarrow a_i \bmod n; a_c \leftarrow \lfloor \frac{a_i}{m} \rfloor \\ b_i \leftarrow q_r \times m + q_c; b_r \leftarrow b_i \bmod n; b_c \leftarrow \lfloor \frac{b_i}{m} \rfloor \\ \textbf{local } s \leftarrow \text{GET-SGN-FS}(a, b, m, n) \\ t[s] \leftarrow t[s] + 1 \end{cases}$
return (t)

F. Translation, Rotation and Scale QIS

We have chosen similar triangles as invariant primitives in the case of Translation, Rotation and Scale (TRS) invariance. The QIS extraction was not directly possible as it was revealed to be impossible to count down the number of different triangles in a $n \times n$ grid. The problem seems to be open and no formal answer exists. Therefore we will use a simple space change to achieve the TRS invariance. Let $P(\rho, \theta)$ be the polar coordinates of a point $p(x, y)$. In the polar imaginary space we will have $Z = \rho e^{\imath\theta}$. If P is rotated by α then $P_r(\rho, \theta + \alpha)$ and $Z_r = \rho e^{\imath(\theta+\alpha)}$. If P is scaled by a factor k then $P_s(k\rho, \theta)$ and $Z_s = k\rho e^{\imath\theta}$.

Now lets consider the Logarithm of Z:
$Z = \rho\epsilon^{\imath\epsilon} \implies \ln Z = \ln\rho + \imath\theta$.
Then
$Z_r = \rho\epsilon^{\imath(\theta+\alpha)} \implies \ln Z_r = \ln\rho + \imath\alpha$ (translation on the $\imath$ axis);
$Z_s = k\rho e^{i\theta} \implies \ln Z_s = \ln k + \ln\rho + \imath\theta$ (translation on the $\ln\rho$ axis).

A translation in the $(\ln\rho, \theta)$ space defines a rotation or a scale change in the (x, y) space. The only problem with this method is that the translation on the θ axis is periodical. This means that the translation on the θ axis in the $(\ln\rho, \theta)$ space is mod $360°$. Therefore, we deal with a cylindrical translation in this case. Our QIS extraction for cylindrical translation is specially here. The procedure to obtain the QIS for TRS is as follows :

1) Carry out the space change to obtain the logarithm-polar signature (LPS);
2) Extract the signature from the LPS matrix using the function presented in algorithm III.4.

Figure 8 illustrates three examples of images transformed into the ln-polar system.

G. Non-square images

So far we have only discussed square images and all the presented methods were based in $n \times n$ matrices. For non-square images, one quick solution would be to append zeros to make the matrix square and, then, apply the already presented methods. Nevertheless we present a method that allows the direct processing of such images. If we apply the space

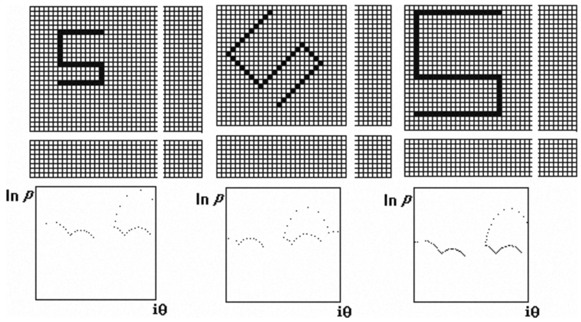
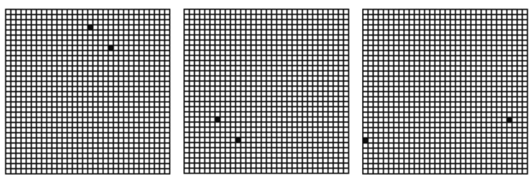


Fig. 8. ln-polar transformation.



(a) (b) (c)

Fig. 9. One active couple in each image.

transformation presented in section III-F, all we need is the translation invariance method for non-square images. Now, the same reasoning applied to square images may be applied to non-square images. The number of similarity groups in a $n \times m$ image is $(n \times m - 1) + (n-1)(m-1)$. The SGN function for the translation to invariance is expressed by algorithm III.11.

Algorithm III.11: GET-SGN-NS(p, q, m, n)

if $p_c \leq q_c$
  then $s \leftarrow q_i - p_i$

  else $s \leftarrow (p_c - q_c - 1)(n-1) + \ldots$
$\ldots (n \times m - 1) + \lfloor \frac{q_i}{m} \rfloor - \lfloor \frac{p_i}{m} \rfloor$
return (s)

Where m is the width and n the hight of the image. The possibility of direct manipulation of the non-square matrices has another advantage: -in the case of a space transformation with a non-square matrix, it is possible to set a different partitioning for each of the two axis (angle, logarithm) and get the best results according to the considered transformation.

H. Examples

Table I presents three examples of applying the several discussed SGN functions to the transforms of figures 9(a), 9(b) and 9(c).

IV. Conclusion

A very robust method to extract a signature from a binary image, in a very fast way, was presented. Nevertheless, the uniqueness of QIS seems to be partial. A few totally symmetrical shapes which give the same QIS were found, however, the uniqueness of QIS in un-symmetrical shapes is intuitively evident. Even if we considered that the uniqueness of QIS would never be formally demonstrated, the probability of finding the same QIS for two different images is nearly zero.

Figure	9(a)		9(b)		9(c)	
Pairs	p	q	p	q	p	q
r	4	8	22	26	1	29
c	17	21	7	11	26	22
i	113	245	679	811	801	701
T	132		132		1864	
CT	[132, 1864]		[132, 1864]		[1864, 132]	
TT	[132, 1864, 1144, 924]		[132, 1864, 1144, 924]		[1864, 132, 1144, 924]	
S	[132, 1120]		[132, 1120]		[1864, 156]	

TABLE I
SGN OF THE TRANSFORMATIONS

Another aspect to consider is related to the presence of noise. As QIS extraction uses couples of points, the introduction of one parasite pixel in an image with n pixels, will introduce n parasite couples. But as the image is defined in terms of couples there will be $n! \div 2!(n-1)!$ couples. Therefore when the ratio of noise of an image is $1/n$ pixels, the number of couples will be $2 \div (n-1)$. It can be considered, then, that the perturbation has as factor of two. This is theoretically a disadvantage of QIS. But in practice it seems that QIS is much more robust in the presence of noise than other invariant recognition methods such as invariant moments[10]. This is because QIS preserves the signature of the original image in a noisy grid. Consider an image of n pixels and one parasite pixel. The QIS extraction will have as result the QIS of the image incremented by n parasite couples. Therefore, the original signature is preserved but lightly incremented.

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