CSCI 2011 HW 1

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1 1.2 Problem 18

Let P, Q and R be statements. Determine whether the following is true.

$$P \oplus (Q \oplus R) \equiv (P \oplus Q) \oplus R$$

P	Q	R	$P \oplus Q$	$Q \oplus R$	$P \oplus (Q \oplus R)$	$(P \oplus Q) \oplus R$
T	T	T	F	F	T	T
T	T	F	F	T	F	F
T	F	T	T	T	F	F
T	F	F	T	F	T	T
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Since both $P \oplus (Q \oplus R)$ and $(P \oplus Q) \oplus R$ have the same truth values, the two are logically equivalent.

2 1.3 Problem 18

The inverse of the implication of $P \Rightarrow Q$ is the implication $(\sim P) \Rightarrow (\sim Q)$.

(a) **Use a truth table to verify that $P \Rightarrow Q \not\equiv (\sim P) \Rightarrow (\sim Q)$.**

P	Q	$\sim P$	$\sim Q$	$P \Rightarrow Q$	$(\sim P) \Rightarrow (\sim Q)$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

$P \Rightarrow Q$ and $(\sim P) \Rightarrow (\sim Q)$ don't contain the same truth values in row two and three, so they are not logically equivalent.

(b) **Find another implication that is logically equivalent to $(\sim P) \Rightarrow (\sim Q)$ and verify your answer.**

$$\begin{aligned}
(\sim P) \Rightarrow (\sim Q) &\equiv \sim(\sim P) \vee (\sim Q) && \text{(Theorem 1.48 from textbook and 3a of HW)}\\
&\equiv P \vee (\sim Q) && \text{(Double negation)}\\
&\equiv (\sim Q) \vee P && \text{(Commutative Law)}\\
&\equiv Q \Rightarrow P && \text{(Theorem 1.48 from textbook again)}
\end{aligned}$$

$(\sim P) \Rightarrow (\sim Q) \equiv Q \Rightarrow P$ as proven by the above logical equivalencies.

3 1.3 Problem 25

For two statements P and Q, use logical equivalencies to verify the following.

(a) $P \vee Q \equiv (\sim P) \Rightarrow Q$.

P	Q	$P \Rightarrow Q$	$(\sim P) \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

This shows that $P \Rightarrow Q \equiv (\sim P) \vee Q$, It's also theorem 1.48 in the textbook. Now we can use logical equivalencies to prove that $P \vee Q \equiv (\sim P) \Rightarrow Q$.

$$\begin{aligned}(\sim P) \Rightarrow Q &\equiv \sim (\sim P) \vee Q && \text{(Theorem 1.48)}\\ &\equiv P \vee Q && \text{(Double negation)}\end{aligned}$$

(b) $P \wedge Q \equiv \sim (P \Rightarrow (\sim Q))$.

$$\begin{aligned}P \wedge Q &\equiv \sim (\sim (P \wedge Q)) && \text{(Double negation)}\\ &\equiv \sim ((\sim P) \vee (\sim Q)) && \text{(De Morgan's Law)}\\ &\equiv \sim (P \Rightarrow (\sim Q)) && \text{(Theorem 1.48)}\end{aligned}$$

(c) $\sim (P \Rightarrow Q) \equiv P \wedge (\sim Q)$.

$$\begin{aligned}\sim (P \Rightarrow Q) &\equiv \sim ((\sim P) \vee Q) && \text{(Theorem 1.48)}\\ &\equiv \sim (\sim P) \wedge (\sim Q) && \text{(De Morgan's Law)}\\ &\equiv P \wedge (\sim Q) && \text{(Double negation)}\end{aligned}$$

4 1.4 Problem 12

For every two statements P and Q, use logical equivalencies to verify the following.

(a) $P \Leftrightarrow Q \equiv (\sim P) \Leftrightarrow (\sim Q)$.

$$\begin{aligned}P \Leftrightarrow Q &\equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P) && \text{(Definition of a biconditional)}\\ &\equiv ((\sim P) \vee Q) \wedge ((\sim Q) \vee P) && \text{(Theorem 1.48)}\\ &\equiv (Q \vee (\sim P)) \wedge (P \vee (\sim Q)) && \text{(Commutative property)}\\ &\equiv ((\sim Q) \Rightarrow (\sim P)) \wedge ((\sim P) \Rightarrow (\sim Q)) && \text{(Theorem 1.48)}\\ &\equiv (\sim P) \Leftrightarrow (\sim Q) && \text{(Definition of a biconditional)}\end{aligned}$$

(b) $P \Leftrightarrow Q \equiv (P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$.

$$
\begin{aligned}
P \Leftrightarrow Q &\equiv (P \Rightarrow Q) \wedge (Q \Rightarrow P) && \text{(Definition of a biconditional)}\\
&\equiv ((\sim P) \vee Q) \wedge ((\sim Q) \vee P) && \text{(Theorem 1.48)}\\
&\equiv ((\sim P) \wedge ((\sim Q) \vee P)) \vee (Q \wedge ((\sim Q) \vee P)) && \text{(Distributive property)}\\
&\equiv (((\sim P) \wedge (\sim Q)) \vee ((\sim P) \wedge P)) \vee ((Q \wedge (\sim Q)) \vee (Q \wedge P)) && \text{(Distributive property)}\\
&\equiv ((\sim P) \wedge (\sim Q)) \vee (Q \wedge P) && ((\sim X) \wedge X = F)\\
&\equiv (P \wedge Q) \vee ((\sim P) \wedge (\sim Q)) && \text{(Commutative property)}
\end{aligned}
$$

(c) $\sim (P \Leftrightarrow Q) \equiv P \Leftrightarrow (\sim Q)$.

$$
\begin{aligned}
\sim (P \Leftrightarrow Q) &\equiv \sim ((P \Rightarrow Q) \wedge (Q \Rightarrow P)) && \text{(Definition of a biconditional)}\\
&\equiv \sim (((\sim P) \vee Q) \wedge ((\sim Q) \vee P)) && \text{(Theorem 1.48)}\\
&\equiv (\sim ((\sim P) \vee Q)) \vee (\sim ((\sim Q) \vee P)) && \text{(De Morgan's Law)}\\
&\equiv (P \wedge (\sim Q)) \vee (Q \wedge (\sim P)) && \text{(De Morgan's Law)}\\
&\equiv (P \vee (Q \wedge (\sim P))) \wedge ((\sim Q) \vee (Q \wedge (\sim P))) && \text{(Distributive property)}\\
&\equiv ((P \vee Q) \wedge (P \vee (\sim P))) \wedge (((\sim Q) \vee Q) \wedge ((\sim Q) \vee (\sim P))) && \text{(Distributive property)}\\
&\equiv (P \vee Q) \wedge ((\sim Q) \vee (\sim P)) && (\sim X \vee X = T)\\
&\equiv (Q \vee P) \wedge ((\sim P) \vee (\sim Q)) && \text{(Commutative property)}\\
&\equiv ((\sim Q) \Rightarrow P) \wedge (P \Rightarrow (\sim Q)) && \text{(Theorem 1.48)}\\
&\equiv P \Leftrightarrow (\sim Q) && \text{(Definition of a biconditional)}
\end{aligned}
$$

5 1.5 Problem 10

Let S and R be two compound statements with the same component statements. If S is a tautology and R is a contradiction, then what is the truth value of the following?

(a) $S \vee R$
True (Always True $\vee$ Always False $\equiv$ Always True)

(b) $S \wedge R$
False (Always True $\wedge$ Always False $\equiv$ Always False)

(c) $S \Rightarrow R$
False (Always True $\Rightarrow$ Always False $\equiv$ Always False)

(d) $R \Rightarrow S$
True (Always False $\Rightarrow$ Always True $\equiv$ Always True)

(e) $S \Leftrightarrow R$
$S \Leftrightarrow R \equiv (S \Rightarrow R) \wedge (R \Rightarrow S)$
(Always False) $\wedge$ (Always True) $\equiv$ Always False
False