

CSCI 2011 HW 3

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1 3.1 Problem 14

Consider the following quantified statement: For every even integer a and every odd integer b , $a + b$ is odd.

- (a) Express this quantified statement in symbols.
- (b) Express the negation of this quantified statement in symbols.
- (c) Express the negation of this quantified statement in words.

2 3.1 Problem 18

State the negation of the quantified statement below.

For every integer a , there exists an integer b such that $|\frac{a+1}{2} - b| \leq 1$.

3 3.2 Problem 16

Prove that if a and b are positive integers, then $\frac{a}{b} + \frac{b}{a} \geq 2$.

4 3.2 Problem 18

Prove the following:

- (a) If a and b are even integers, then $a + b$ is even.
- (b) If c and d are even integers, do we know that $c + d$ is even?
- (c) For integers x and y , if we know $x + y$ is even, do we know that x and y are even?
- (d) If a and b are integers that are not both even, do we know that $a + b$ is not even?

5 3.3 Problem 8

Give a proof of

Let $n \in \mathbb{Z}$. Then $n - 3$ is even if and only if $n + 4$ is odd.

using

- (a) two direct proofs.
- (b) one direct proof and one proof by contrapositive
- (c) two proofs by contrapositive.

6 3.3 Problem 12

Let a, b and m be integers. Prove that if $2a + 3b \geq 12m + 1$, then $a \geq 3m + 1$ or $b \geq 2m + 1$.