CSCI 2011 HW 2

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1 2.1 Problem 26

For $n \in \mathbb{N}$, two sets A and C have the property that $A \subset C$, $|A| = n$, and $|C| = n + 3$. How many sets B are there such that $A \subset B \subset C$?

$|C| - |A| = 3$, so C has three more elements than A. $B \supset A$, so B must have more elements than A, B can not have the same number of elements as A because $B \not\supseteq A$. $B \subset C$, so B must have less elements than C. Again, they cannot be equal because $B \not\subseteq C$. This means that B must have one or two more elements than A that are also in C.

ex: $A = \{a\}, C = \{a, b, c, d\}$
possible sets for B: $\{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}$
Number of possible sets for B: 6

$C \cap \bar{A} = \{b, c, d\}$. So B includes any combination of those three values that sums up to one or two total values, giving 6 possible combinations... $\big\{\{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}\big\}$

2 2.2 Problem 22

For sets A and B of integers, define $A + B = \{a + b : a \in A, b \in B\}$. If $|A| = |B| = 5$, how small and how large can $|A + B|$ be?

The largest possible value for $|A + B|$ comes when each element of A and B add up to a different sum.

ex: $A = \{a, b, c, d, e\}, B = \{v, w, x, y, z\}$
$A + B = \{a + v, a + w, a + x, a + y, a + z, b + v, b + w, b + x, b + y, b + z, c + v, c + w, c + x, c + y, c + z, d + v, d + w, d + x, d + y, d + z, e + v, e + w, e + x, e + y, e + z\}$
$|A + B| = 25$

The smallest possible value for $|A + B|$ comes when almost each element of A and B add up to the same sum.

ex: $A = \{0, 1, 2, 3, 4\}, B = \{0, 1, 2, 3, 4\}$
$A + B = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
$|A + B| = 9$

3 2.3 Problem 12

Verify, for sets A, B, and C, that $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

$$
\begin{aligned}
A \times (B \cap C) &= \{(x, y) : x \in A, y \in (B \cap C)\} && \text{(definition of the product between sets)}\\
&= \{(x, y) : (x \in A) \wedge (y \in (B \wedge C))\} && \text{(comma and intercept = “and”)}\\
&= \{(x, y) : (x \in A) \wedge (y \in B) \wedge (y \in C)\} && \text{(distributive property)}\\
&= \{(x, y) : ((x \in A) \wedge (y \in B)) \wedge ((x \in A) \wedge (y \in C))\} && (X \wedge X \equiv X)\\
&= \{(x, y) : ((x, y) \in A \times B), ((x, y) \in A \times C)\} && \text{(“and” = comma)}\\
&= (A \times B) \cap (A \times C) && \text{(definition of the product between sets)}
\end{aligned}
$$

4 2.3 Problem 14

For two sets A and B of real numbers, the set $A \cdot B$ is defined by

$$A \cdot B = \{ab : a \in A, b \in B\}.$$

Determine each of the following sets.

(a) $A \cdot B$ **for** $A = \{\frac{1}{2}, 1, \sqrt{2}\}$ **and** $B = \{\sqrt{2}, 2, 4\}$**.**

$\{\frac{\sqrt{2}}{2}, 1, 2, \sqrt{2}, 2, 4, 2, 2\sqrt{2}, 4\sqrt{2}\} \Rightarrow \{\frac{\sqrt{2}}{2}, 1, \sqrt{2}, 2, 2\sqrt{2}, 4\sqrt{2}\}$

(b) $\mathbb{R} \cdot \mathbb{R}$**.**

any real number multiplyed by another real number is just a real number, so $\mathbb{R} \cdot \mathbb{R} = \mathbb{R}$

(c) $\mathbb{R} \cdot C$ **where** $C \subseteq \mathbb{R}$ **with** $|C| = 2$**.**

C contains two elements that are both real numbers. Assuming that one of it's elements is zero, the other can be any non-zero number. For any non-zero real number, you can multiply it by another real number in the $\mathbb{R}$ set to produce any possible number in the $\mathbb{R}$ set. This means that...

$\mathbb{R} \cdot C = \mathbb{R}$

5 2.4 Problem 10

Let $A = \{1, 2, 3, 4\}$. Partition the power set $\mathcal{P}(A)$ of A into as many subsets as possible such that no two subsets have the same number of elements.

$\mathcal{P}(A) = \big\{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\big\}$

P = Partition of $\mathcal{P}(A)$

$P = \Big\{\{\emptyset\}, \{\{1\}, \{2\}\}, \{\{3\}, \{4\}, \{1, 2\}\}, \{\{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}\}, \{\{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}\Big\}$

5 possible subsets.