

3.10 Problem 10: Let $\mathbf{A} = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}$. Prove that $\mathbf{e}^{\mathbf{A}t} = \mathbf{e}^{\lambda t} \begin{bmatrix} 1 & t & \frac{1}{2}t^2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$

3.11 Problem 10: Determine whether $e^t \begin{bmatrix} 1 & t+1 & t^2+1 \\ 1 & 2(t+1) & 4t^2 \\ 1 & t+2 & 3 \end{bmatrix}$ is a fundamental matrix solution of $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ for some \mathbf{A} ; if yes, find \mathbf{A} .

3.12 Problem 1: Use the method of variation of parameters to solve the given initial-value problem.

$$\dot{\mathbf{x}} = \begin{pmatrix} 4 & 5 \\ -2 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4e^t \cos t \\ 0 \end{pmatrix}, \quad \mathbf{x}(\mathbf{0}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

3.13 Problem 1: Find the solution to the initial-value problem.

$$\dot{\mathbf{x}} = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix}, \quad \mathbf{x}(\mathbf{0}) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$