CSCI 2011 HW 5

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1 4.1 Problem 10

Let $r \geq 2$ be an integer. Prove that $1 + r + r^2 + ... + r^n = \frac{r^{n+1}-1}{r-1}$ for every positive integer n.

Base Case: Since this claim must hold true for every positive integer n, we can use $n = 1$ as our base case. Therefore $1 + \cdots r^n = 1 + r^1 = 1 + r$. Since $\frac{r^{n+1}-1}{r-1} = \frac{r^2-1}{r-1} = \frac{(r+1)(r-1)}{r-1} = r + 1$, our base case holds.

Inductive Step: Now let's assume for some $k \geq 1$, that $1 + r + r^2 + ... + r^k = \frac{r^{k+1}-1}{r-1}$. We show that $1 + r + r^2 + ... + r^k + r^{k+1} = \frac{r^{k+2}-1}{r-1}$.

$$
\begin{aligned}
1 + r + r^2 + ... + r^k + r^{k+1} &= \frac{r^{k+1}-1}{r-1} + r^{k+1} && \text{(by the inductive hypothesis)} \\
&= \frac{r^{k+1} - 1 + r^{k+1}(r-1)}{r-1} \\
&= \frac{r^{k+1}(1 + r - 1)}{r-1} - \frac{1}{r-1} \\
&= \frac{r \cdot r^{k+1} - 1}{r-1} \\
&= \frac{r^{k+2} - 1}{r-1}
\end{aligned}
$$

Therefore the claim holds for the inductive step as well. Hence, by the principle of mathematical induction, the claim $1 + r + r^2 + ... + r^n = \frac{r^{n+1}-1}{r-1}$ is true for all integers $n \geq 1$.

2 4.2 Problem 14

Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{n}} > \sqrt{n+1}$ for ever integer $n \geq 3$.

3 4.2 Problem 16

Prove for every positive integer n that $2! \cdot 4! \cdot 6! \cdots (2n)! \geq ((n+1)!)^n$.

4 4.3 Problem 14

A sequence $a_1, a_2, a_3...$ is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1} + 1$ for $n \geq 2$.

(a) **Determine a_2, a_3, a_4 and a_5.**

(b) **Based on the variables obtained in (a), make a guess for a formula for a_n for every positive integer n and use induction to verify that your guess is correct.**

5 4.3 Problem 22

Use induction to show the following for Fibonacci numbers: $F_2 + F_4 + \cdots + F_{2n} = F_{2n+1} - 1$ for every positive integer n.