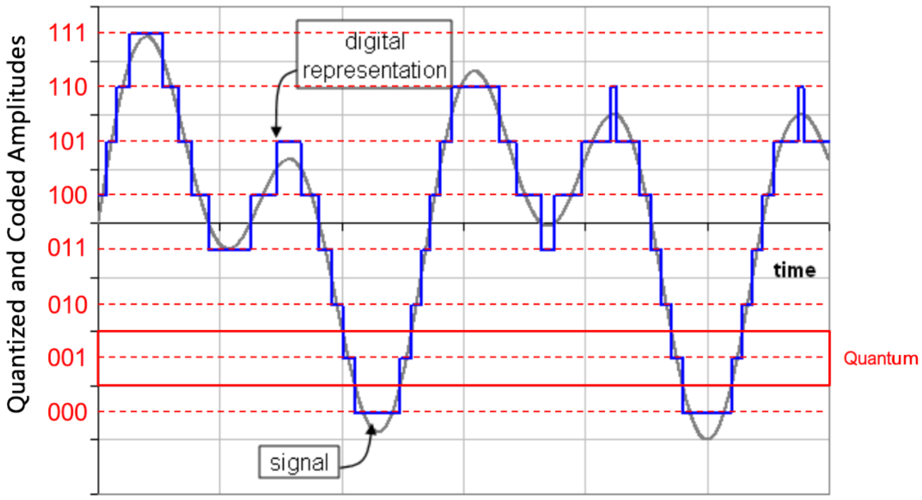
Quantization

The transformation of a continuous value signal into a discrete value signal is called quantization.

Original value will be associated with a new value in a discrete set of levels. This will introduce a certain error, since originally different values can collapse on the same level, becoming indistinguishable. The quantization precision is linked to the minimum variation in the original quantity which induces a passage from one level to another in the quantized domain. The smaller the minimum variation required, the more precise the quantization will be.



- **Uniform or linear**

  The quantization is uniform when equal intervals of quantization levels correspond to intervals of equal amplitude in the original domain.

- **Non-uniform or non-linear**

  The quantization is non-uniform when at different intervals of equal amplitude in the original domain, a different number of quantization levels correspond. In other words, one is more precise in quantifying certain intervals and less precise for others.

- **Dynamic range (Full Scale value)**

  It is the size of the range to be represented.

  For example, if you want to represent a voltage between 5 and 20 volts:

  $VFS = 20 - 5 = 15$ or $VFS = Max - Min$

Uniform Quantization-Error

Quantization is a process of approximation which as such introduces a loss and therefore a distortion. As a rule, the middle value of that range is associated with the values that fall in the same range. Therefore, in the case of uniform quantization, the maximum quantization error E*max* holds:

$$E_{max} = \frac{VFS}{2Q}$$ where **Q** equals the number of quantization levels

In the case of digital audio, the maximum number of levels (uniform or not) will depend on the number of bits that we will decide to use to represent the levels of amplitude (bit depth). So, if N is the number of bits used:

$$Q = 2^N$$

E.g.: if N = 3 → Q = 8

Distortion SQNR

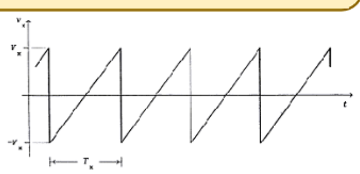
The SQNR reflects the relationship between the maximum nominal signal strength and the quantization error (also known as quantization noise) introduced in the analogue-to-digital conversion.

As in the case of analogue audio, it is possible to estimate the distortion introduced by the treatment of the original signal. An index that considers the distortion introduced by quantization is the Signal to Quantization Noise Ratio.

The average SQNR in the case of uniform quantization can be calculated as follows:

$$SQNR = 2^N \times \sqrt{\frac{3}{2}}$$ Where **N** is the bit depth

In this case the average quantization error follows a sawtooth trend between the various quantities



In decibels:

$$SQNR_{dB} = 10\log_{10} SQNR^2 = 6{,}02N + 1{,}7609$$

It is clear that by using a larger number of bits, it is possible to quantify more precisely and introduce less distortion. In fact, a high SQNR is an indication of higher quality.

Example with average SQNR in the case of uniform quantization:

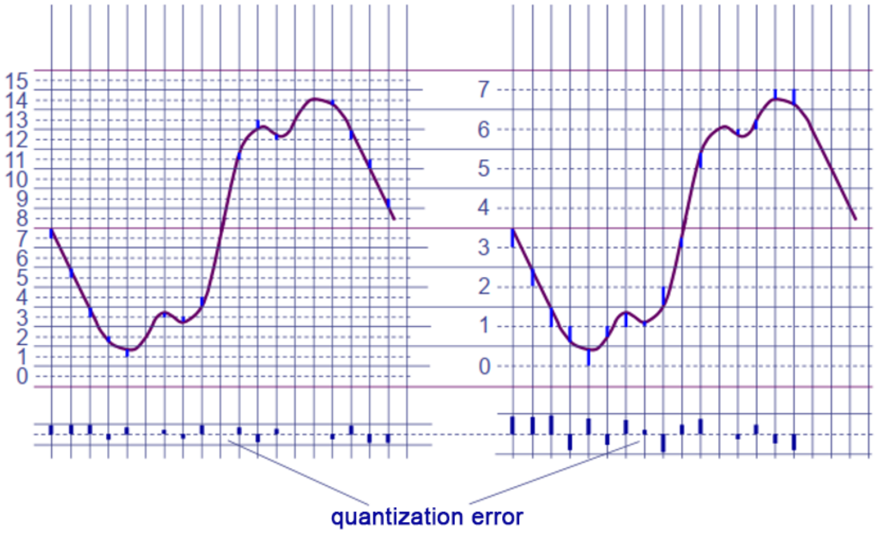
$$SQNR_{dB} = 10\log_{10} SQNR^2 = 6{,}02N + 1{,}7609$$

This relation establishes that each bit contributes about 6 dB to the SQNR.

E.g.: Given N = 8, the SQNR will be about 6 * 8 = 48 dB

Given an SQNR equal to about 120 dB, then N = 120/6 = 20

Quantization is precisely the operation of assigning discrete values to describe a continuous signal; the more bits are used, the more accurate the quantization is. This is because in quantization the signal / noise ratio is proportional to the number of bits used with a reduction of the "quantization noise" of 6 dB for each bit used. The more the steps, the lower the quantization error (or noise).



Example of Difference between 4-bit and 3-bit quantization

Uniform quantization

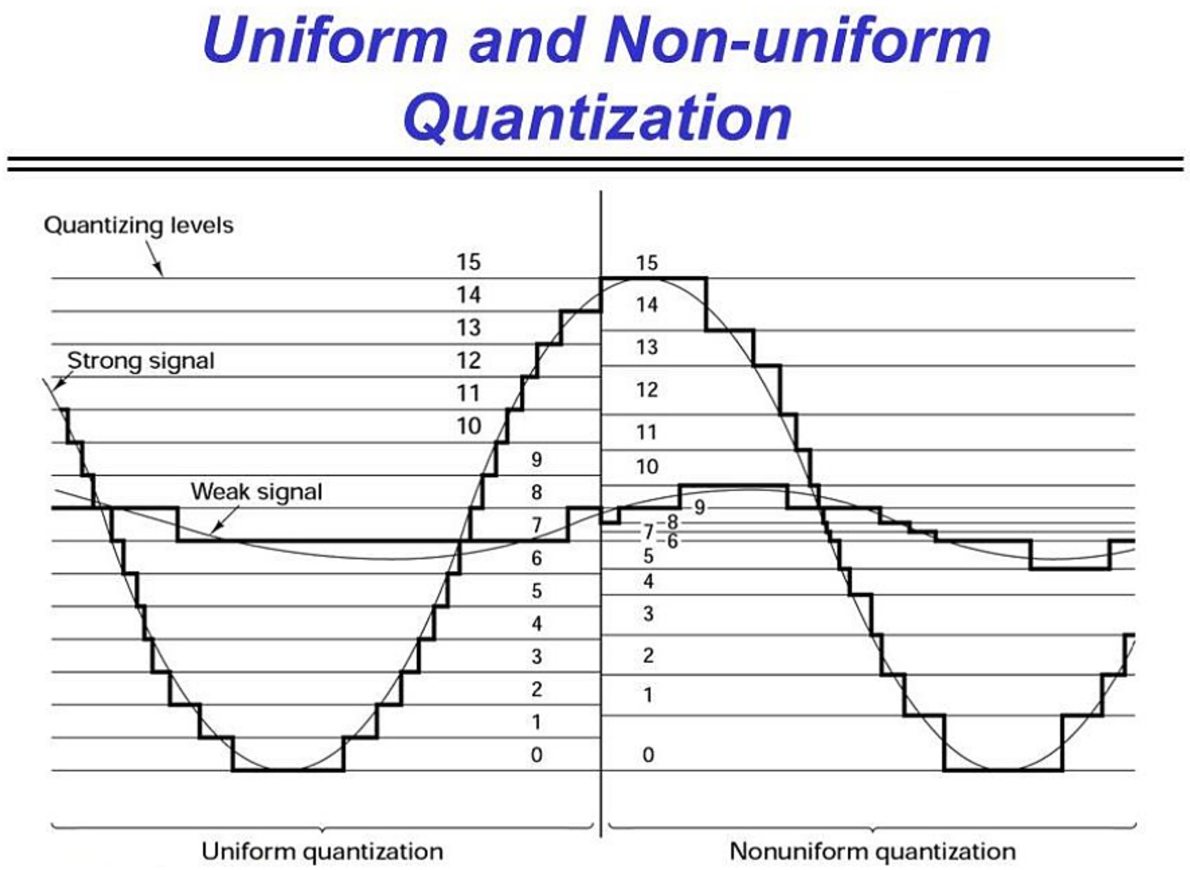
Explanation of average SQNR in the case of uniform quantization:

- With the same quantization error, the smaller amplitudes in absolute value are more distorted (in proportion), compared to the larger amplitudes.
  E.g.: Very small amplitudes could all fall within the same range and become indistinguishable.
- To be precise, the distortion should be calculated for each possible amplitude value.
- The SQNR index is calculated considering the mean square amplitude of a signal (RMS)
- SQNR > 60dB are acceptable values
- 16-bit CD standards have SQNR = 96dB

Uneven quantization

The factors of the idea of Non-uniform quantization: Humans perceive volume variations more clearly, which affect the low amplitudes, and the quantization error is more significant for small amplitudes.

Uneven quantization can help improve overall quality for the same number of bits used.



The essential idea is to reduce noise at weak amplitudes, admitting instead that it increases at strong amplitudes, where it is however masked. This result is achieved by uneven spacing of the quantization regions: regions near zero amplitudes are quantified much finer than regions and high amplitudes.

Digital audio

The quantities needed to characterize a digital audio signal:

- Number of channels
- Sampling frequency (number of samples per second)
- Depth in bits per sample (quantization bits by value)

Memory space

Number of bits required in memory to represent a digital audio signal:

Let *f* be the sampling rate, N the bit depth, D the duration of the audio stream and C the number of channels, then the number of bits needed to represent the signal (without compression) is calculated:

$$Size = f_c \times N \times D \times C$$

The number of bits flowing in the unit of time (one second) is called the bit rate. It is measured in bps (bits per second):

$$bitrate = f_c \times N \times C$$

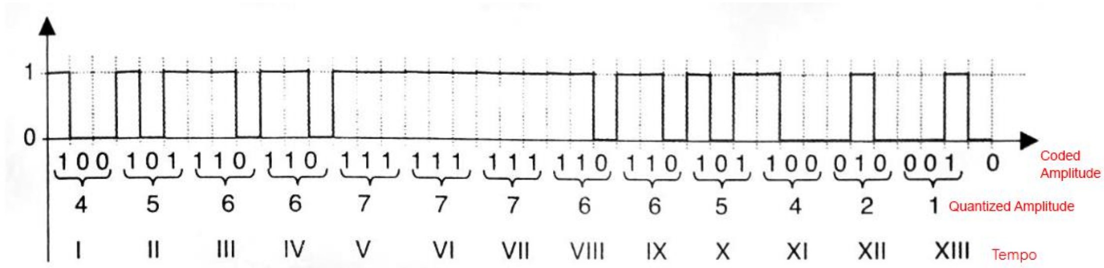
The goal is to guarantee good quality using the minimum amount of memory. The compression methods represent a next step that allows you to lower the bit rate while maintaining quality.

Some Standards:

Support	Sampling rate (Hz)	Bit per sample	SQNR (dB)	Channels	Bitrate (KBps)
Telefon	8000	8	48	1	8.00
Radio AM	11025	8	48	1	11.05
Radio FM	22050	16	96	2	88.20
CD Audio	44100	16	96	2	176.40
Digital Audio Tape (DAT)	48000	16	96	2	192.00
DVD Audio	192000	24	144	6	1200.00

Pulse-Code Modulation (**PCM**)

It is a method used to digitally represent sampled analogue signals. This is possibly the simplest encoding technique for digital audio. We consider each sample as an impulse and combine it with a binary word that represents its amplitude. The length of the binary words depends on the (linear) quantization bits used.



The example shows the 3-bit PCM coding of an audio signal. The 13 samples take values between 0 and 7.

Signal encodings

The fundamental difference from the point of view of binary representation is whether the coding is signed or unsigned.

ID Quantum	Offset binary	Sample at 2	Sign and Magnitude
7	111_2 (7_{10})	011_2 ($+3_{10}$)	011_2 ($+3_{10}$)
6	110_2 (6_{10})	010_2 ($+2_{10}$)	010_2 ($+2_{10}$)
5	101_2 (5_{10})	001_2 ($+1_{10}$)	001_2 ($+1_{10}$)
4	100_2 (4_{10})	**000_2 ($+0_{10}$)**	**000_2 ($+0_{10}$)**
3	011_2 (3_{10})	111_2 (-1_{10})	**100_2 (-0_{10})**
2	010_2 (2_{10})	110_2 (-2_{10})	101_2 (-1_{10})
1	001_2 (1_{10})	101_2 (-3_{10})	110_2 (-2_{10})
0	**000_2 (0_{10})**	100_2 (-4_{10})	111_2 (-3_{10})

Error-Correcting Code

ECC is used to detect errors in the storage and transmission of data and to correct them if possible.

Error detection procedures are limited to determining whether there is an error.

Parity bit

A **parity bit**, or **check bit**, is a bit added to a string of binary code to ensure that the total number of 1-bits in the string is even or odd. Parity bits are used as the simplest form of **error detecting code**.

The meaning of the additional bit can be as follows: If the number of bits in the sequence is equal to 1, the bit is set to 0. If the number of bits in the sequence is odd at 1, the bit is reset to 1.

The parity bit enables you to find out whether a bit accidentally flips (varies from 0 to 1 or the other way around).

Assume the sequence contains 5 bits of 1. The parity bit is set to 1. If one of the bits changes polarization (both from 1 to 0 and from 0 to 1), there is an even number of 1 and there is an inconsistency between the bit sequence and the parity bit.

It should be noted that it is also possible that the error is in the parity bit.