heuristic_analysis

April 16, 2017

1 Introduction

This document is an analysis of the search algorithms and heuristics that were used to solve the three Air cargo problems described in the project.

The experimental settings were as follows: * **search limit**: up to the memory limit (16 GiB), no timeout * **elements tested**: all problems (3) and searches (10) were tested * **benchmark sript**: a modified run_search.py that output json files * **hardware**: Processor 8 x Intel Core i7-4710Hq CPU 2.50 GHz, 16 GiB of RAM

In the first part of this analysis, we will prepare the data prior to its exploration.

Then, we will review the main four characteristics associated to AI search algorithms: * optimality * completeness * time complexity * space complexity

Finally, we will provide an answer to suggest which algorithm should be used in the general case.

2 Preparation

```
In [1]: from itertools import product
        from glob import iglob
        import pandas as pd
        import numpy as np
        import json
        import os
        import re

        %matplotlib inline
        import matplotlib
        matplotlib.rcParams['figure.figsize'] = (12.0, 4.0)
```

```
In [4]: # files and formats
        DIR = '../data/run/'
        FILES = iglob(os.path.join(DIR, '*.json'))
        RES_PAT = re.compile('air_cargo_problem_(\d+)-(\w+)-(\w+)')
        COLUMNS = ['problem', 'search', 'algorithm', 'heuristic', 'uninformed', 'runtime',
                   'plan_length', 'expansions', 'goal_tests', 'new_nodes', 'plan']

        # problems and searches
```

```python
PROBLEMS = ['1', '2', '3']

SEARCHES = [
    ["breadth_first_search", "uninformed"],
    ['breadth_first_tree_search', "uninformed"],
    ['depth_first_graph_search', "uninformed"],
    ['depth_limited_search', "uninformed"],
    ['uniform_cost_search', "uninformed"],
    ['recursive_best_first_search', 'h_1'],
    ['greedy_best_first_graph_search', 'h_1'],
    ['astar_search', 'h_1'],
    ['astar_search', 'h_ignore_preconditions'],
    ['astar_search', 'h_pg_levelsum'],
]

def sname(algo, heur):
    """Format an algorithm and heuristic to a search name."""
    return "{} - {}".format(algo, heur).replace('_', ' ')


RESULTS = list()

# import results
for f in FILES:
    file, ext = os.path.splitext(f)
    filename = os.path.basename(file)
    match = RES_PAT.match(filename)

    if match is None:
        raise ValueError("Match failed for: {}".format(filename))

    prob, algo, heur = match.groups()
    uninformed = heur == 'uninformed'

    with open(f, 'r') as r:
        res = json.load(r)

    res['problem'] = prob
    res['algorithm'] = algo
    res['heuristic'] = heur
    res['uninformed'] = uninformed
    res['search'] = sname(algo, heur)

    RESULTS.append(res)

# create a dataframe from results
df = pd.DataFrame(RESULTS, columns=COLUMNS).sort_values(['problem', 'heuristic'])
```

3 Visualization

3.1 Overview

In [5]:
```
df.drop(['search', 'plan'], axis=1)
```

Out[5]:

	problem	algorithm	heuristic	uninformed
10	1	greedy_best_first_graph_search	h_1	False
12	1	astar_search	h_1	False
22	1	recursive_best_first_search	h_1	False
17	1	astar_search	h_ignore_preconditions	False
7	1	astar_search	h_pg_levelsum	False
1	1	depth_first_graph_search	uninformed	True
2	1	breadth_first_tree_search	uninformed	True
9	1	depth_limited_search	uninformed	True
13	1	uniform_cost_search	uninformed	True
15	1	breadth_first_search	uninformed	True
8	2	greedy_best_first_graph_search	h_1	False
21	2	astar_search	h_1	False
6	2	astar_search	h_ignore_preconditions	False
11	2	depth_limited_search	uninformed	True
14	2	breadth_first_search	uninformed	True
18	2	depth_first_graph_search	uninformed	True
20	2	uniform_cost_search	uninformed	True
0	3	greedy_best_first_graph_search	h_1	False
19	3	astar_search	h_1	False
16	3	astar_search	h_ignore_preconditions	False
3	3	uniform_cost_search	uninformed	True
4	3	breadth_first_search	uninformed	True
5	3	depth_first_graph_search	uninformed	True

	runtime	plan_length	expansions	goal_tests	new_nodes
10	0.010160	6	7	9	28
12	0.118958	6	55	57	224
22	4.504784	6	4229	4230	17023
17	0.076575	6	41	43	170
7	10.033305	6	18	20	77
1	0.021924	20	21	22	84
2	1.592876	6	1458	1459	5960
9	0.146723	50	101	271	414
13	0.049170	6	55	57	224
15	0.024268	6	43	56	180
8	0.130309	15	35	37	299
21	86.061412	9	4853	4855	44041
6	26.639975	9	1506	1508	13820
11	1656.404348	50	222719	2053741	2054119
14	21.990256	9	3343	4609	30509
18	5.494803	619	624	625	5602
20	84.186726	9	4853	4855	44041

```
0    258.498690          24        4587        4589       40379
19   939.075264          12       18236       18238      159726
16   218.743101          12        5118        5120       45650
3    966.414331          12       18236       18238      159726
4    213.483141          12       14663       18098      129631
5      2.490629         392         408         409        3364
```

3.2 Completeness

In [14]:
```python
def is_complete(df, p, a, h):
    return ((df['problem'] == p) & (df['algorithm'] == a) & (df['heuristic'] == h)).any


COMPLETES = list()

for p, s in product(PROBLEMS, SEARCHES):
    a, h = s

    doc = {
        'problem': p,
        'search': sname(a, h),
        'complete': 1 if is_complete(df, p, a, h) else 0
    }

    COMPLETES.append(doc)

complete = pd.DataFrame(COMPLETES, columns=['problem', 'search', 'complete']) \
            .sort_values('problem')
```

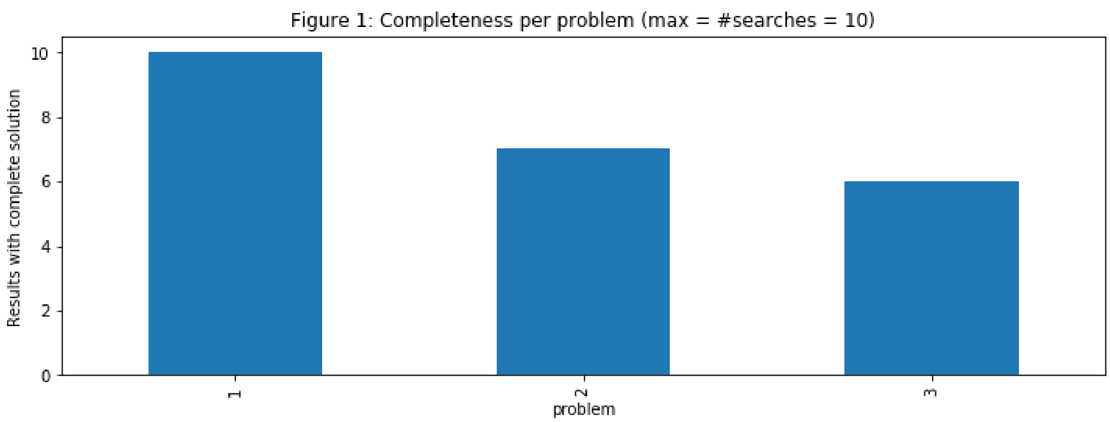
In [16]:
```python
complete
```

Out[16]:

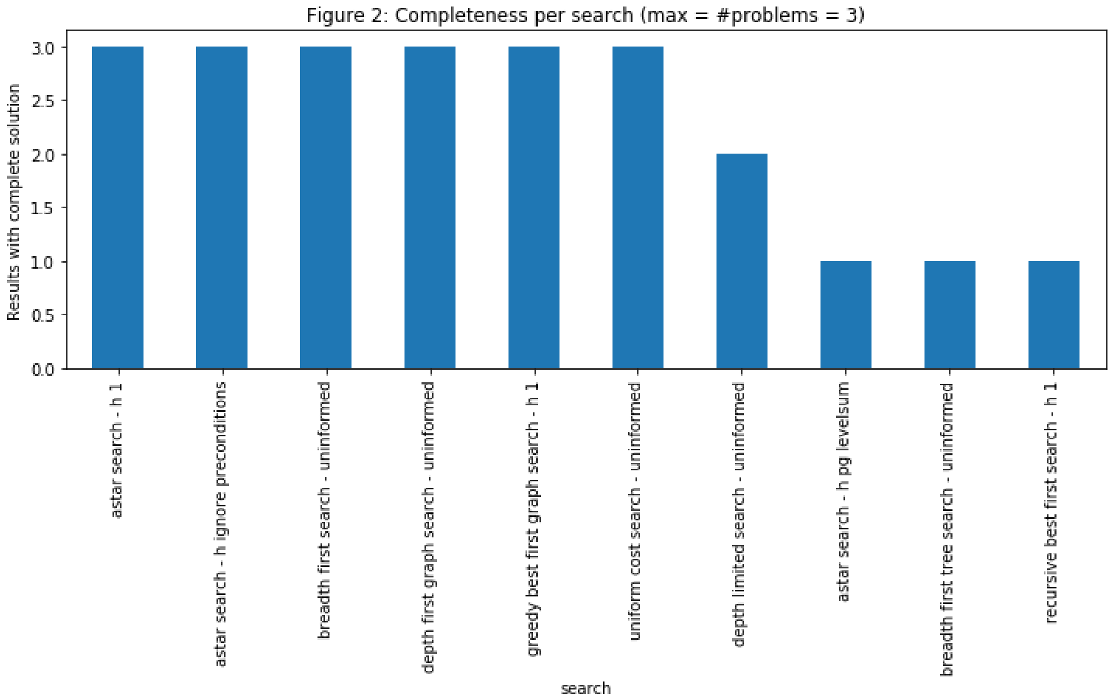
	problem	search	complete
0	1	breadth first search - uninformed	1
1	1	breadth first tree search - uninformed	1
2	1	depth first graph search - uninformed	1
3	1	depth limited search - uninformed	1
4	1	uniform cost search - uninformed	1
5	1	recursive best first search - h 1	1
6	1	greedy best first graph search - h 1	1
7	1	astar search - h 1	1
8	1	astar search - h ignore preconditions	1
9	1	astar search - h pg levelsum	1
19	2	astar search - h pg levelsum	0
18	2	astar search - h ignore preconditions	1
17	2	astar search - h 1	1
16	2	greedy best first graph search - h 1	1
15	2	recursive best first search - h 1	0
14	2	uniform cost search - uninformed	1

13	2	depth limited search - uninformed	1
12	2	depth first graph search - uninformed	1
11	2	breadth first tree search - uninformed	0
10	2	breadth first search - uninformed	1
20	3	breadth first search - uninformed	1
21	3	breadth first tree search - uninformed	0
22	3	depth first graph search - uninformed	1
23	3	depth limited search - uninformed	0
24	3	uniform cost search - uninformed	1
25	3	recursive best first search - h 1	0
26	3	greedy best first graph search - h 1	1
27	3	astar search - h 1	1
28	3	astar search - h ignore preconditions	1
29	3	astar search - h pg levelsum	0

In [18]:
```
ax = complete.groupby('problem').sum().sort_values('complete', ascending=False)['comple
ax.set_title('Figure 1: Completeness per problem (max = #searches = {})'.format(len(SEA
ax.set_ylabel('Results with complete solution')
None
```



In [19]:
```
ax = complete.groupby('search').sum().sort_values('complete', ascending=False)['complet
ax.set_title('Figure 2: Completeness per search (max = #problems = {})'.format(len(PROB
ax.set_ylabel('Results with complete solution')
None
```



Out of 30 cases, we found that 23 cases have a complete solution in our experimental settings.

We can see from Figure 1 that it is easier to find a complete solution for smaller problems (e.g. Problem 1).

We can see from Figure 2 that some searches that were supposed to had complete solution, such as breadth first tree and A* search, were not complete in our settings. The reason might be the lack of memory on the machine.

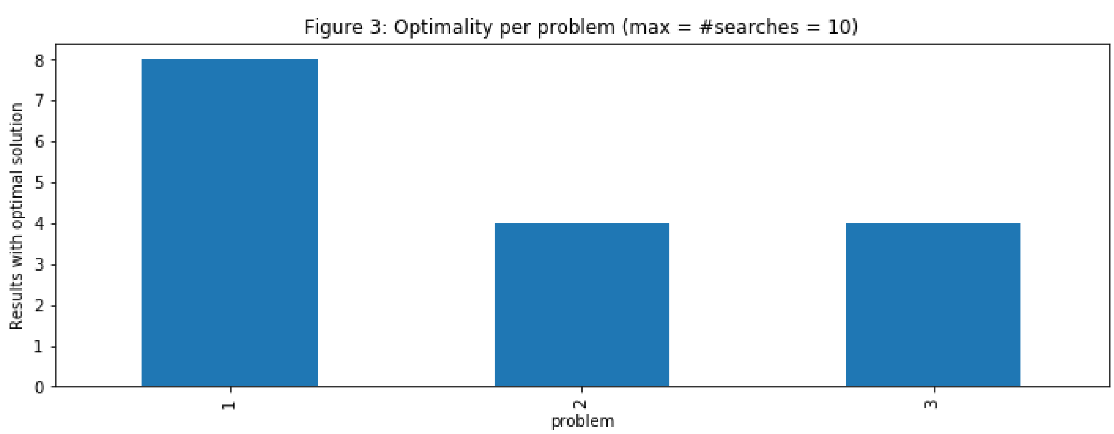
On the contrary, some searches which do not guarantee completeness, such as greedy best first graph search and depth first graph search, had a complete solution. As mentioned, these algorithms can find a solution in some cases, and the result do not contradict that we learned in the session.

3.3 Optimality

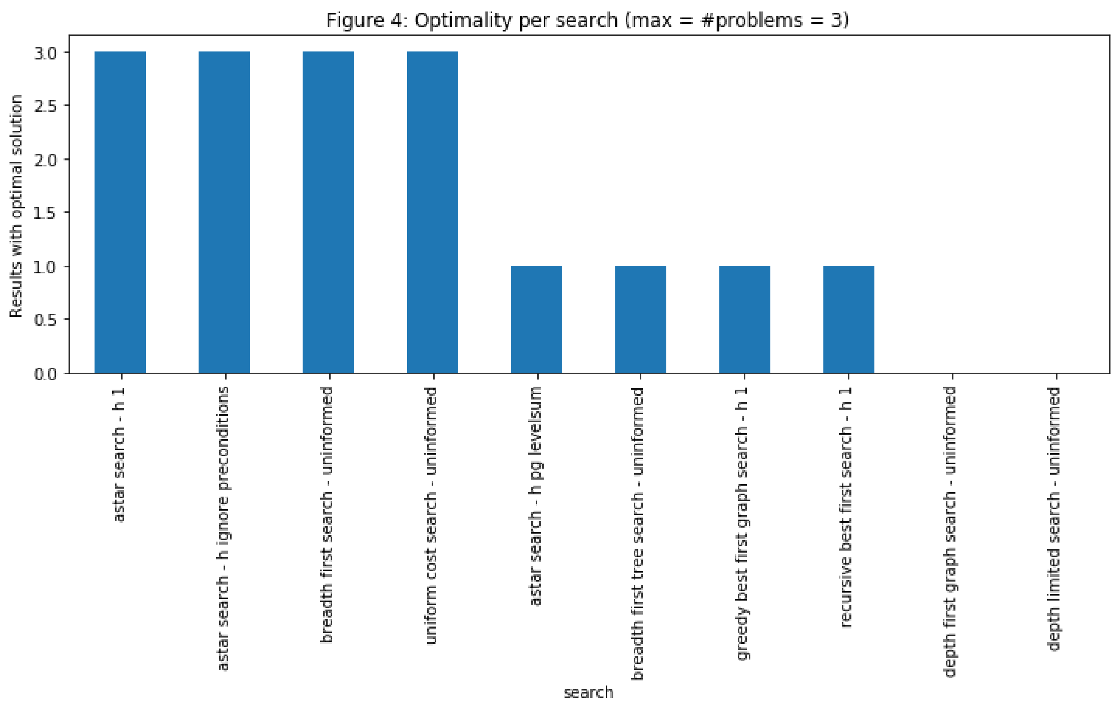
```
In [17]: BEST = {'1': 6, '2': 9, '3': 12}

         def is_optimal(row):
             return row['plan_length'] == BEST[row['problem']]


         optimal = df[['problem', 'search', 'plan_length', 'plan']].copy()
         optimal['optimal'] = optimal.apply(lambda r: 1 if is_optimal(r) else 0, axis=1)

In [19]: optimal.drop('plan', axis=1)

Out[19]:    problem                                  search  plan_length  optimal
         10       1    greedy best first graph search - h 1            6        1
```

12	1	astar search - h 1	6	1
22	1	recursive best first search - h 1	6	1
17	1	astar search - h ignore preconditions	6	1
7	1	astar search - h pg levelsum	6	1
1	1	depth first graph search - uninformed	20	0
2	1	breadth first tree search - uninformed	6	1
9	1	depth limited search - uninformed	50	0
13	1	uniform cost search - uninformed	6	1
15	1	breadth first search - uninformed	6	1
8	2	greedy best first graph search - h 1	15	0
21	2	astar search - h 1	9	1
6	2	astar search - h ignore preconditions	9	1
11	2	depth limited search - uninformed	50	0
14	2	breadth first search - uninformed	9	1
18	2	depth first graph search - uninformed	619	0
20	2	uniform cost search - uninformed	9	1
0	3	greedy best first graph search - h 1	24	0
19	3	astar search - h 1	12	1
16	3	astar search - h ignore preconditions	12	1
3	3	uniform cost search - uninformed	12	1
4	3	breadth first search - uninformed	12	1
5	3	depth first graph search - uninformed	392	0

In [22]:
```
ax = optimal.groupby('problem').sum().sort_values('optimal', ascending=False)['optimal'
ax.set_title('Figure 3: Optimality per problem (max = #searches = {})'.format(len(SEARC
ax.set_ylabel('Results with optimal solution')
None
```



In [23]:
```
ax = optimal.groupby('search').sum().sort_values('optimal', ascending=False)['optimal']
ax.set_title('Figure 4: Optimality per search (max = #problems = {})'.format(len(PROBLE
ax.set_ylabel('Results with optimal solution')
None
```



Out of 30 cases, we found that 16 cases have an optimal solution.

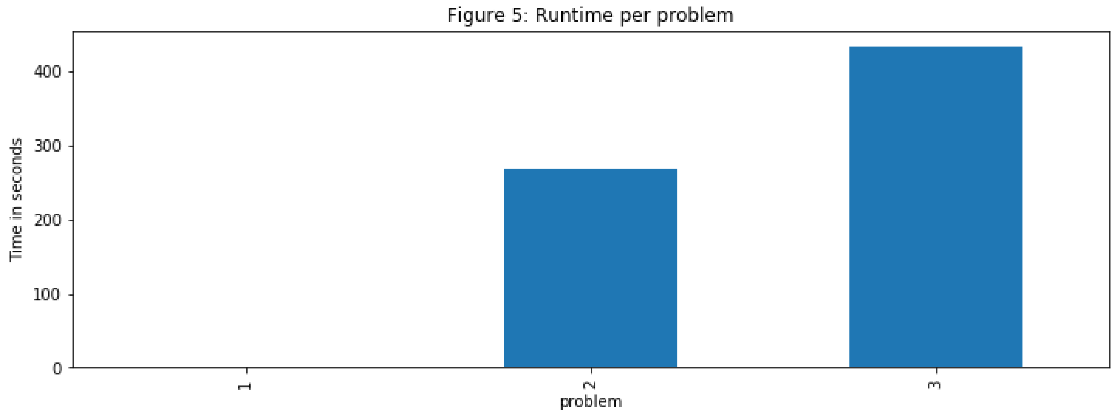
In Figure 3, we see that an optimal solution was easier to found for smaller problems. At this scale, finding an optimal solution for small problem could be influenced by a factor of chance.

In Figure 4, we see that 4 search algorithms consistently found an optimal solution. This confirm what we learned in the textbook about the optimality of certain algorithms, such as A*, breadth-first search and uniform cost search algorithm. On the other hand, other search algorithms such as greedy search and depth first search do not have this guarantee, which is shown in the results as well.

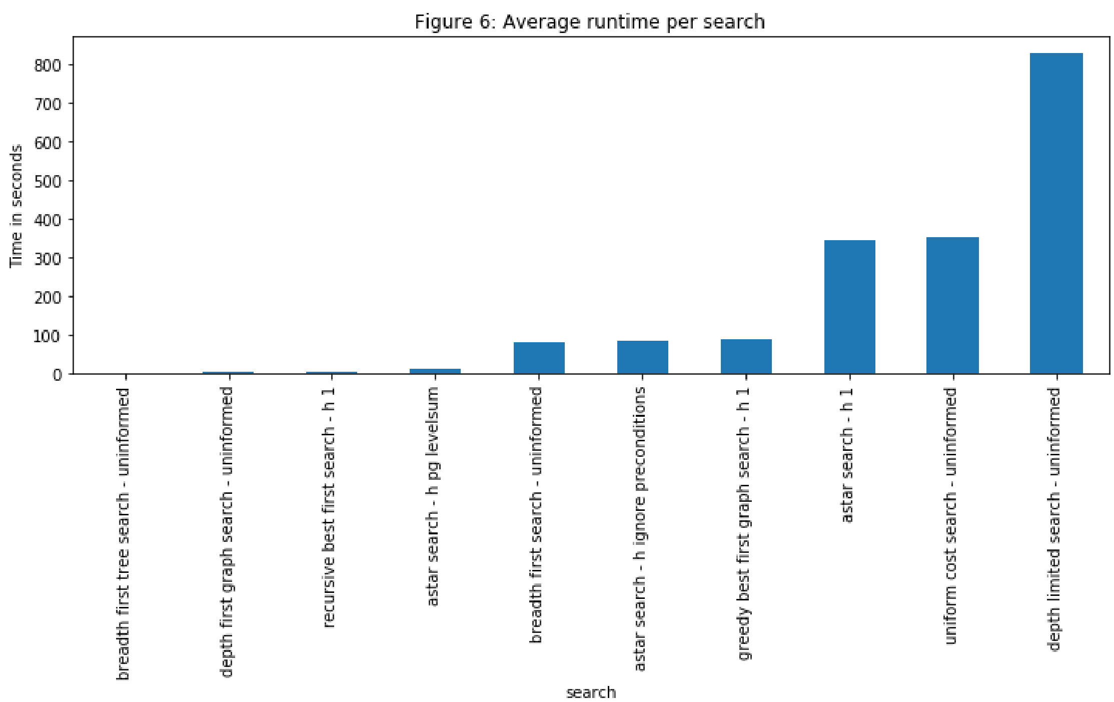
We also found some search algorithm supposed to be optimal but without optimal solutions in our experiment, such as breadth first tree search. However, we we observed prior to this analysis that these searches were incomplete.

3.4 Time complexity

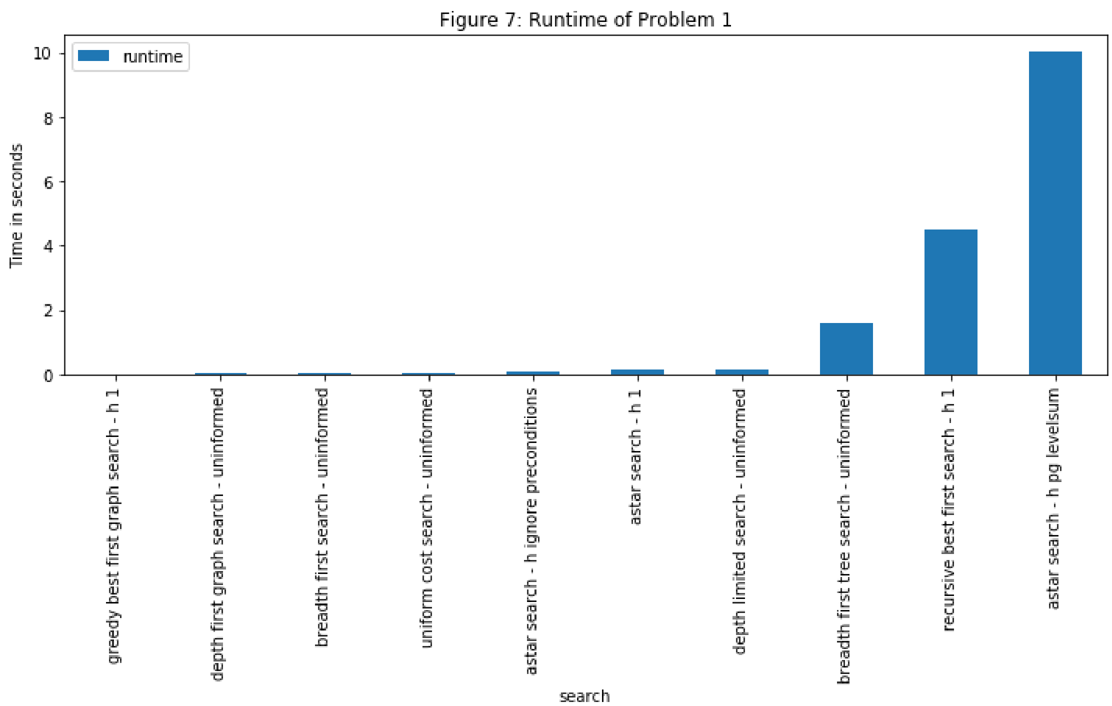
```
In [25]: ax = df.groupby('problem').mean()['runtime'].sort_values().plot(kind='bar')
         ax.set_title('Figure 5: Average runtime per problem')
         ax.set_ylabel('Time in seconds')
         None
```



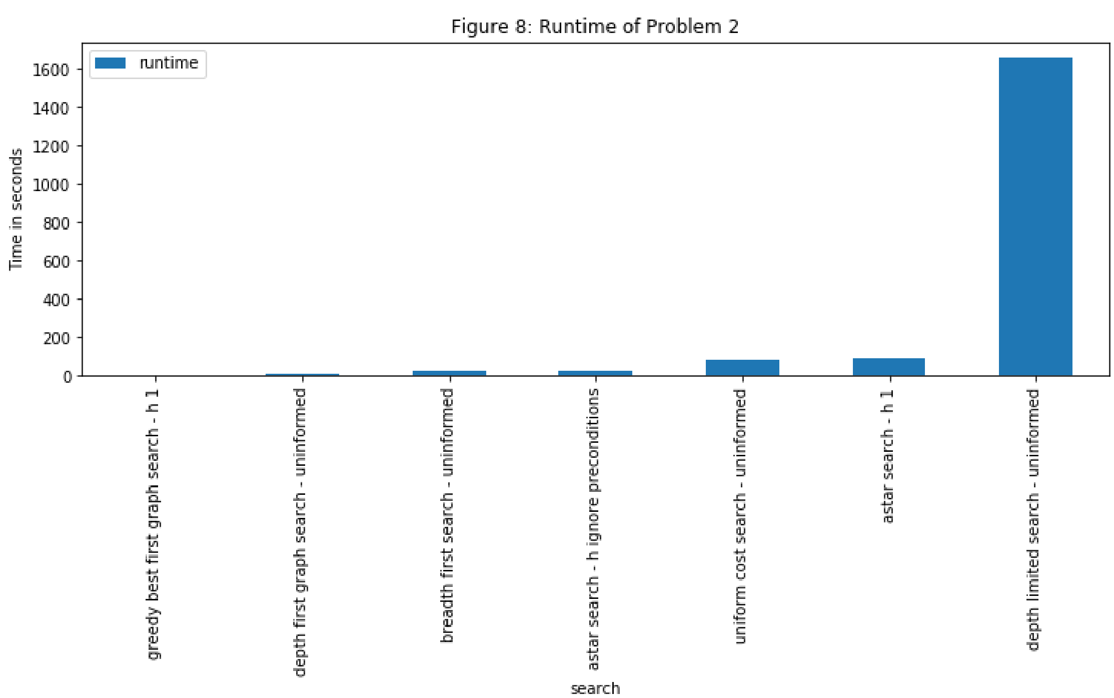

```
In [9]: ax = df.groupby('search').mean()['runtime'].sort_values().plot(kind='bar')
        ax.set_title('Figure 6: Average runtime per search')
        ax.set_ylabel('Time in seconds')
        None
```



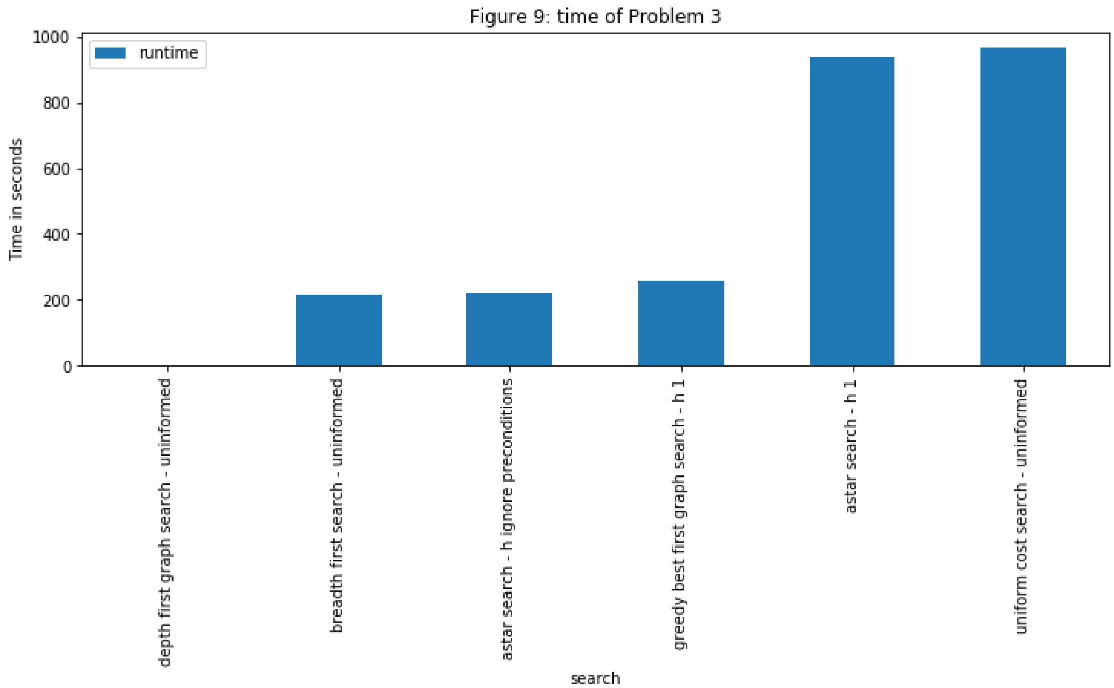

```
In [27]: ax = df[df['problem'] == '1'].sort_values('runtime').plot(x='search', y='runtime', kind
         ax.set_title('Figure 7: Runtime per search for Problem 1')
         ax.set_ylabel('Time in seconds')
         None
```



In [28]:
```
ax = df[df['problem'] == '2'].sort_values('runtime').plot(x='search', y='runtime', kind
ax.set_title('Figure 8: Runtime per search for Problem 2')
ax.set_ylabel('Time in seconds')
None
```



```
In [29]: ax = df[df['problem'] == '3'].sort_values('runtime').plot(x='search', y='runtime', kind
         ax.set_title('Figure 9: Runtime per search for Problem 3')
         ax.set_ylabel('Time in seconds')
         None
```



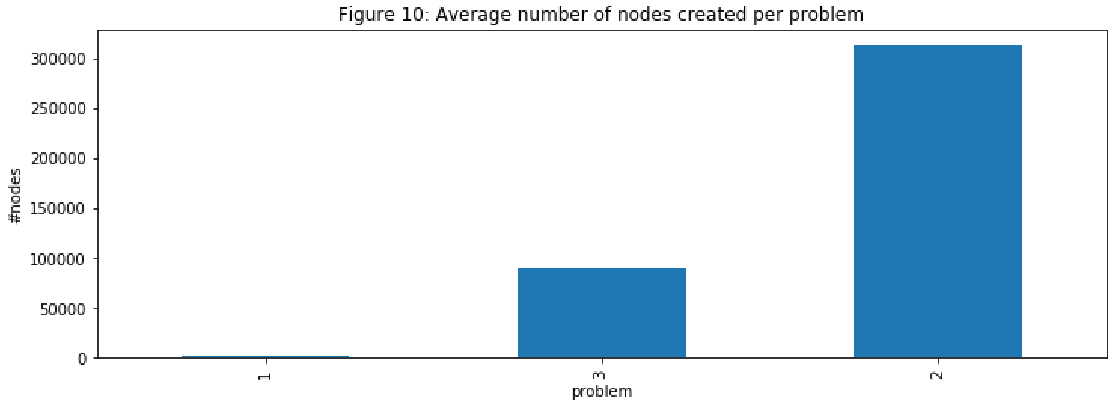
In figure 5, we can see that the time complexity varies greatly depending on the complexity of the problem, from less than 1 second for Problem 1 to more than 400 seconds on average for problem 3. This is due to the algorithmic complexity associated to search algorithms, which are exponential in most cases.

In figure 6, we see that the average runtime per search depends on the algorithm. Greedy and depth-first searches tend to have smaller runtime than breadth-first search and A* algorithms. On the other hand, we solution they found were not optimal.

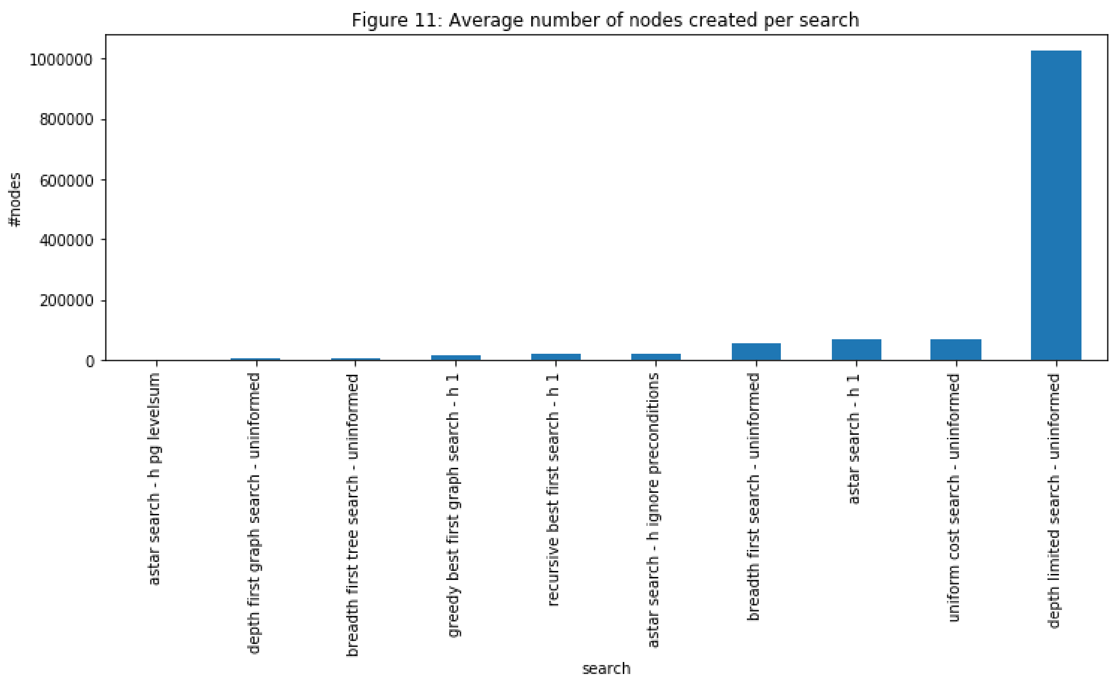
In the context of the Air Cargo Problem, finding an optimal solution is much important than the runtime of the search algorithm. A company would be more inclined to spend more computer resources and avoid the cost of flying more planes than necessary. The cost of the latter is much greater than the former. The only constraint in this case is to find search algorithms with tractable executions.

3.5 Space Complexity

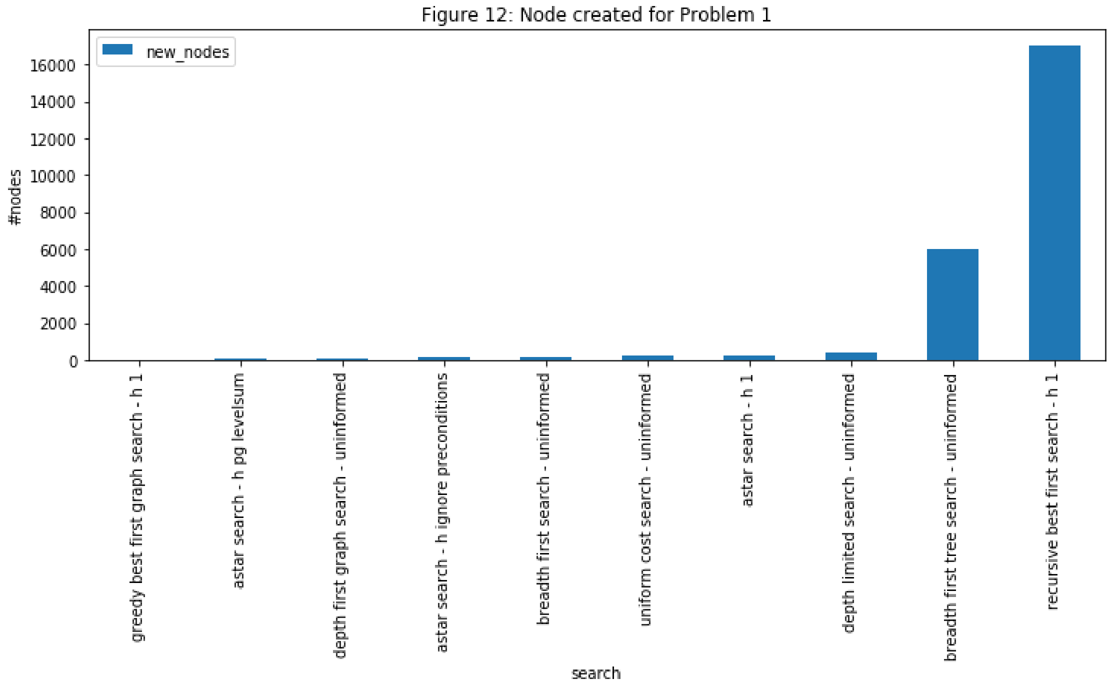
```
In [11]: ax = df.groupby('problem').mean()['new_nodes'].sort_values().plot(kind='bar')
         ax.set_title('Figure 10: Average number of nodes created per problem')
         ax.set_ylabel('#nodes')
         None
```



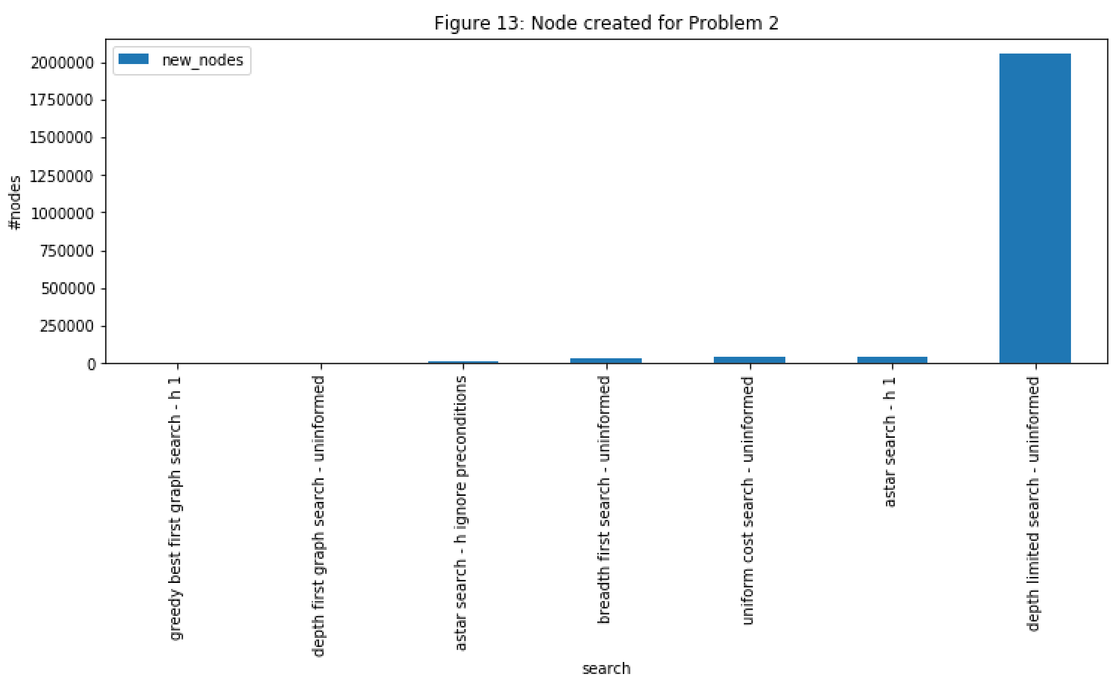

```
In [10]: ax = df.groupby('search').mean()['new_nodes'].sort_values().plot(x='problem', y='new_no
         ax.set_title('Figure 11: Average number of nodes created per search')
         ax.set_ylabel('#nodes')
         None
```



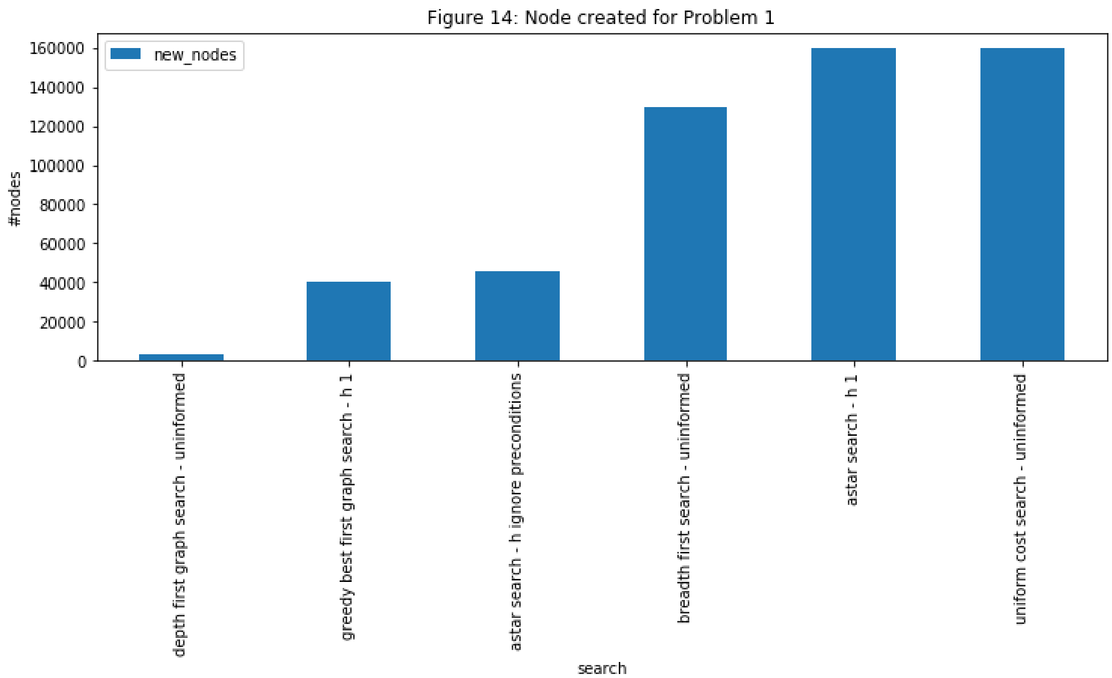

```
In [32]: ax = df[df['problem'] == '1'].sort_values('new_nodes').plot(x='search', y='new_nodes',
         ax.set_title('Figure 12: Nodes created per search for Problem 1')
         ax.set_ylabel('#nodes')
         None
```




```
In [33]: ax = df[df['problem'] == '2'].sort_values('new_nodes').plot(x='search', y='new_nodes',
         ax.set_title('Figure 13: Node created per search for Problem 2')
         ax.set_ylabel('#nodes')
         None
```



```
In [34]: ax = df[df['problem'] == '3'].sort_values('new_nodes').plot(x='search', y='new_nodes',
         ax.set_title('Figure 14: Node created per search for Problem 1')
         ax.set_ylabel('#nodes')
         None
```



We may be surprised to find in Figure 10 than more nodes were created for Problem 2 than Problem 3. This is because most searches could not be completed for Problem 3, while they could for Problem 2, at the cost of more expansions.

In figure 11, it is striking to see the high number of node created for depth-limited search compare to the other algorithms. In this particular case, the depth-limited search must do more work when the depth limit increases, causing more expansions than the other algorithms.

Overall, the number of node created is not a problem with the resources we have on our machine today, as long as the search can be completed at some point.

3.6 Summary

The following searches had incomplete in our settings: * astar search - h pg levelsum * depth limited search - uninformed * recursive best first search - h 1 * breadth first tree search - uninformed

The following searches had non optimal results: * depth limited search - uninformed * greedy best first graph search - h 1 * depth first graph search - uninformed

The following searches had a runtime greater than 400 seconds: * astar search - h 1 * uniform cost search - uninformed * depth limited search - uninformed

The following searches created more than 100 000 new nodes: * astar search - h 1 * uniform cost search - uninformed
* depth limited search - uninformed * breadth first search - uninformed

The following searches are not included in the previous listings: * astar_search - h_ignore_preconditions

4 Conclusion

Through this report, we analyzed the main characteristics of search algorithms and confirmed the theory learned in this session on planning and search. We noticed that even for small problems, some algorithms were not tractable or did not provided sufficient solutions.

In particular, the capacity to find optimal and complete solution is critical in the case of flying planes. The high cost of this activity could greatly benefit from the solutions that some of these algorithms can provide. The main constraint is for optimal algorithms to be tractable, both in term of time and space complexity.

The only algorithm which had this property is the A* search with the "ignore preconditions" heuristic. This search was always optimal and complete in our settings. Moreover, it had acceptable time and space complexity. The heuristic exposed one of the critical insight that I learned through this session: an heuristic could be created automatically by relaxing some of the problem constraints (such as removing preconditions). I think this is a powerful idea that could improve the automation of AI techniques for new problem domain.

You can find in the appendix below the optimal solutions that were found for each three problems.

5 Appendix

5.1 Optimal plans

```
In [12]: for problem, group in optimal[optimal['optimal']].groupby('problem'):

             plans = {'\n'.join(plan) for plan in group['plan']}
             print("### Problem: {} (#plans = {}) ###".format(problem, len(plans)))
             print()

             for i, plan in enumerate(plans, 1):
                 print("- Plan: {}".format(i))
                 print(plan)
                 print()
```

```
### Problem: 1 (#plans = 4) ###

- Plan: 1
Load(C1, P1, SFO)
Fly(P1, SFO, JFK)
Unload(C1, P1, JFK)
Load(C2, P2, JFK)
Fly(P2, JFK, SFO)
```

```
Unload(C2, P2, SFO)

- Plan: 2
Load(C2, P2, JFK)
Load(C1, P1, SFO)
Fly(P2, JFK, SFO)
Unload(C2, P2, SFO)
Fly(P1, SFO, JFK)
Unload(C1, P1, JFK)

- Plan: 3
Load(C1, P1, SFO)
Load(C2, P2, JFK)
Fly(P1, SFO, JFK)
Fly(P2, JFK, SFO)
Unload(C1, P1, JFK)
Unload(C2, P2, SFO)

- Plan: 4
Load(C1, P1, SFO)
Load(C2, P2, JFK)
Fly(P2, JFK, SFO)
Unload(C2, P2, SFO)
Fly(P1, SFO, JFK)
Unload(C1, P1, JFK)

### Problem: 2 (#plans = 3) ###

- Plan: 1
Load(C3, P3, ATL)
Fly(P3, ATL, SFO)
Unload(C3, P3, SFO)
Load(C2, P2, JFK)
Fly(P2, JFK, SFO)
Unload(C2, P2, SFO)
Load(C1, P1, SFO)
Fly(P1, SFO, JFK)
Unload(C1, P1, JFK)

- Plan: 2
Load(C1, P1, SFO)
Fly(P1, SFO, JFK)
Load(C2, P2, JFK)
Fly(P2, JFK, SFO)
Load(C3, P3, ATL)
Fly(P3, ATL, SFO)
Unload(C3, P3, SFO)
Unload(C2, P2, SFO)
```

```
Unload(C1, P1, JFK)

- Plan: 3
Load(C1, P1, SFO)
Load(C2, P2, JFK)
Load(C3, P3, ATL)
Fly(P2, JFK, SFO)
Unload(C2, P2, SFO)
Fly(P1, SFO, JFK)
Unload(C1, P1, JFK)
Fly(P3, ATL, SFO)
Unload(C3, P3, SFO)

### Problem: 3 (#plans = 3) ###

- Plan: 1
Load(C1, P1, SFO)
Load(C2, P2, JFK)
Fly(P2, JFK, ORD)
Load(C4, P2, ORD)
Fly(P1, SFO, ATL)
Load(C3, P1, ATL)
Fly(P1, ATL, JFK)
Unload(C1, P1, JFK)
Unload(C3, P1, JFK)
Fly(P2, ORD, SFO)
Unload(C2, P2, SFO)
Unload(C4, P2, SFO)

- Plan: 2
Load(C2, P2, JFK)
Fly(P2, JFK, ORD)
Load(C4, P2, ORD)
Fly(P2, ORD, SFO)
Unload(C4, P2, SFO)
Load(C1, P1, SFO)
Fly(P1, SFO, ATL)
Load(C3, P1, ATL)
Fly(P1, ATL, JFK)
Unload(C3, P1, JFK)
Unload(C2, P2, SFO)
Unload(C1, P1, JFK)

- Plan: 3
Load(C1, P1, SFO)
Fly(P1, SFO, ATL)
Load(C2, P2, JFK)
Fly(P2, JFK, ORD)
```

```
Load(C3, P1, ATL)
Load(C4, P2, ORD)
Fly(P2, ORD, SFO)
Unload(C4, P2, SFO)
Fly(P1, ATL, JFK)
Unload(C3, P1, JFK)
Unload(C2, P2, SFO)
Unload(C1, P1, JFK)
```