Confidence Weighted Mean Reversion Strategy for On-Line Portfolio Selection – Online Appendix

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In the main paper, we have demonstrated the efficacy of the proposed CWMR algorithm. Due to the similarity of some results, we cannot include all the details of our extensive experiments. In this online appendix, we provide more details on experimental setup and metrics, and more experimental results to support the efficacy of the proposed CWMR algorithms.

1. PERFORMANCE CRITERIA

One of the standard criteria to evaluate the performance of a on-line portfolio selection strategy is the *portfolio cumulative wealth* achieved by the strategy until the end of the whole trading period. In our study, we simply set the initial wealth $\mathbf{S}_0 = 1$ and thus the notation $\mathbf{S}_n$ also denotes the *portfolio cumulative return* at the end of the n^{th} trading day, which is the ratio of the portfolio cumulative wealth divided by the initial wealth. Another equivalent criterion is *annualized percentage yield* (APY) that takes the compounding effect into account, that is, $\text{APY} = \sqrt[y]{\mathbf{S}_n} - 1$, where y is the number of years corresponding to n trading days. APY measures the average wealth increment that a strategy could achieve compounded in a year. Typically, the higher the value of *portfolio cumulative wealth* or *annualized percentage yield*, the more performance preferable the trading strategy.

For some process-dependent investors [Moody et al. 1998], it is also important to evaluate the *risk* and *risk-adjusted return* of portfolios [Sharpe 1963; Sharpe 1994]. Volatility risk of the portfolio is typically measured using the *annualized standard deviation* of daily returns. We compute this by multiplying the standard deviation of daily returns with $\sqrt{252}$ (here 252 is the average number of annual trading days). On the other hand, the *annualized Sharpe Ratio* (SR) is typically used to evaluate the risk-adjusted return. We obtain this as $\text{SR} = \frac{\text{APY} - R_f}{\sigma_p}$, where R_f is the risk-free return (typically the return of Treasury bills, fixed at 4% in this work), and σ_p is the annualized standard deviation of daily returns. Basically, the higher the *annualized Sharpe Ratio*, the more performance efficient the trading strategy is concerning the volatility risk.

For the portfolio management in finance community, *drawdown* (DD) analysis [Magdon-Ismail and Atiya 2004] is used to measure the decline from a historical peak in the cumulative wealth achieved by a financial trading strategy. Formally, let $\mathbf{S}(\cdot)$ denote the process of cumulative wealth achieved by a trading strategy, that is, $\{\mathbf{S}_1, \ldots, \mathbf{S}_t, \ldots, \mathbf{S}_n\}$. The *drawdown* at any time t, is defined as $\text{DD}(t) = \max\left[0, \max_{j\in(0,t)} \mathbf{S}(j) - \mathbf{S}(t)\right]$. The *maximum drawdown* for the horizon n, MDD(n) is defined as, $\text{MDD}(n) = \max_{t\in(0,n)}\left[\text{DD}(t)\right]$, which is an excellent way to measure the downside risk of different strategies. Moreover, we also adopt the *Calmar Ratio* (CR) to measure the return relative of the drawdown risk of a portfolio, calculated as $\text{CR} = \frac{\text{APY}}{\text{MDD}}$. Generally speaking, the smaller the *maximum drawdown*, the more downside risk tolerable the financial trading strategy. The higher the *Calmar Ratio*, the higher the performance efficient concerning the drawdown risk.

To test whether simple luck can generate the return of the proposed strategy, we can also conduct a statistical test to measure the probability of this situation, as is popularly done in the fund management industry [Grinold and Kahn 1999]. First, we separate the portfolio daily returns into two components: one benchmark-related and the other non-benchmark-related by regressing the portfolio excess returns[1] against the benchmark excess returns. Formally, $s_t - s_t(\text{F}) = \alpha + \beta\left(s_t(\text{B}) - s_t(\text{F})\right) + \epsilon(t)$, where s_t stands for the portfolio daily returns, $s_t(\text{B})$ denotes the daily returns of the benchmark (market index) and $s_t(\text{F})$ is the daily return of the risk-free as-

[1] excess return is daily return less risk-free return.

sets (here we simply choose Treasury bill and set it to 1.000156, or equivalently, annual interest of 4%). This regression estimates the portfolio's alpha (α), which indicates the performance of the investment after accounting for the risk it involves. Then we conduct the *t*-statistical test to evaluate whether alpha is significantly different from zero, by using the *t*-statistic $\frac{\alpha}{\mathrm{SE}(\alpha)}$, where $\mathrm{SE}(\alpha)$ is the standard error for the estimated alpha. Thus, by assuming the alpha is normally distributed, we can obtain the probability that the returns of the proposed strategy are generated by simple luck. Generally speaking, the smaller the probability, the higher confidence the trading strategy.

2. PRACTICAL ISSUES

Our model described preceding is concise and omits some practical issues faced in portfolio management, in order to allow ease of understanding. We shall now relax some constraints in our model to address these issues.

In reality, an important and unavoidable issue is the *transaction cost*. Generally, there are two ways to handle the transaction costs. The first, commonly adopted by existing online portfolio selection strategies, is that the transaction costs is suffered when the portfolio selection doesn't take into account the transaction cost for the choice of next portfolio. The second is that the transaction cost is directly involved in the portfolio selection process [Györfi and Vajda 2008]. In this work, we adopt the *proportional transaction costs* model proposed in Blum and Kalai [1999] and Borodin et al. [2004]. To be specific, rebalancing the portfolio incurs a transaction cost on every buy and sell operation, based upon a transaction cost rate $\gamma \in (0,1)$. At the beginning of the t^{th} trading day, the portfolio manager rebalances the portfolio from the previous closing price adjusted portfolio $\hat{\mathbf{b}}_{t-1}$ to a new portfolio $\mathbf{b}_t$, incurring a transaction cost of $\frac{\gamma}{2} \times \sum_i \left| b_{(t,i)} - \hat{b}_{(t-1,i)} \right|$, where the initial portfolio is $(0,\ldots,0)$. Thus, the cumulative wealth at the end of the n^{th} trading day can be expressed:

$$\mathbf{S}_n^{c(\gamma)} = \mathbf{S}_0 \prod_{t=1}^{n} \left[(\mathbf{b}_t \cdot \mathbf{x}_t) \times \left(1 - \frac{\gamma}{2} \times \sum_i \left| b_{(t,i)} - \hat{b}_{(t-1,i)} \right| \right) \right].$$

Another practical issue in portfolio selection is *margin buying*, which allows the portfolio managers to buy securities with cash borrowed from security brokers. Following previous studies [Cover 1991; Helmbold et al. 1998; Agarwal et al. 2006], we relax this constraint in the model and evaluate it empirically in Section 3.3. In this research, the margin setting is assumed to be 50% down and 50% loan, at an annual interest rate of 6%, so the interest rate of the borrowed money, c is set to 0.000238. Thus, for each security in the asset pool, a new asset named "Margin Component" is generated. Following the down and loan percentage, the price relative for the "Margin Component" of asset i would be $2 * x_{ti} - 1 - c$, where x_{ti} is the price relative of the i^{th} asset for the t^{th} trading day. By adding this "Margin Component", we magnify both the potential profit and loss of the trading strategy on the i^{th} asset.

3. ADDITIONAL EXPERIMENTAL RESULTS

3.1. Experiment 1: Evaluation of Cumulative Wealth

Please refer to Figure 1 for the curves on the high frequency data and weekly data.

3.2. Experiment 3: Evaluation of Parameter Sensitivity

Please refer to Figure 2 for experimental results.

3.3. Experiment 4: Evaluation of Practical Issues

Please refer to Figure 3 for experimental results.

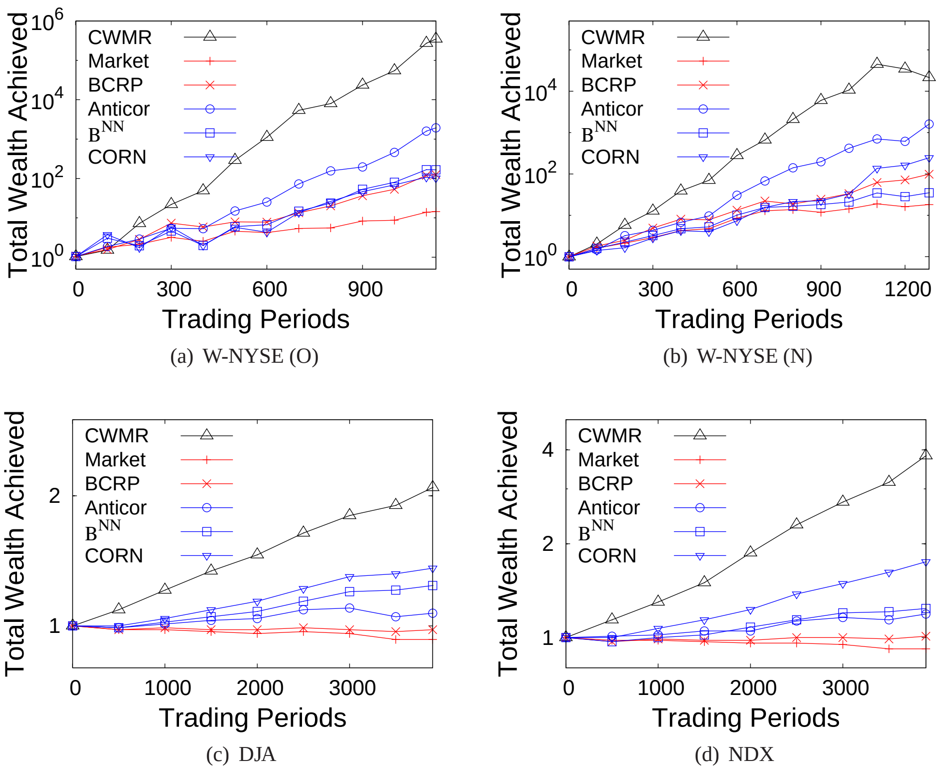

Fig. 1. Trend of cumulative wealth achieved by various strategies during the entire period on the stock datasets.

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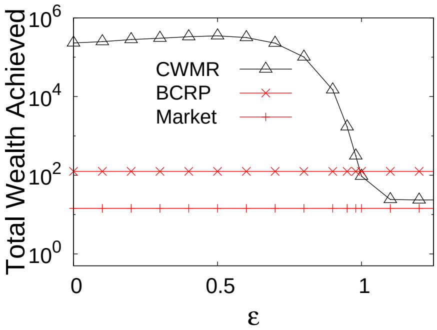
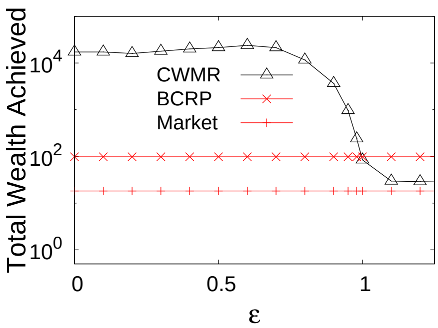
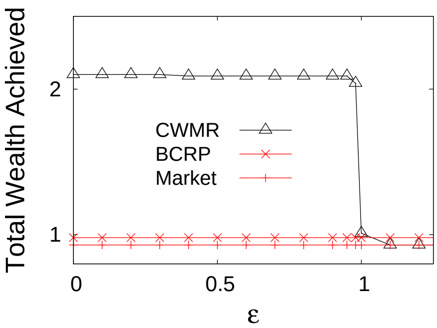


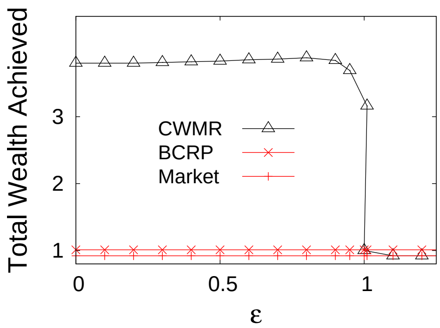
(a) W-NYSE(O)



(b) W-NYSE(N)

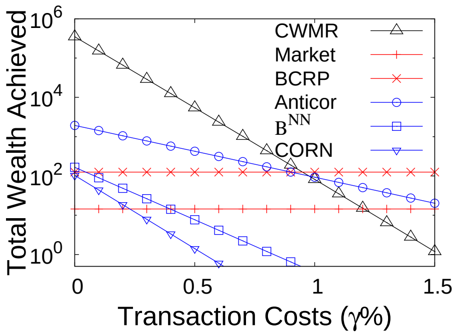


(c) DJA

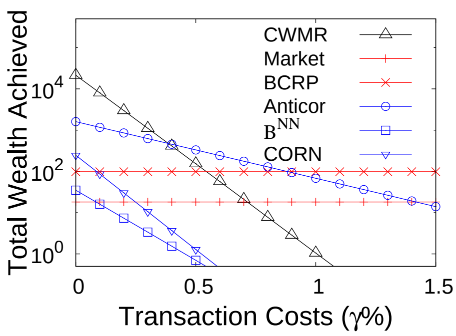


(d) NDX

Fig. 2. Parameter Sensitivity of the total wealth achieved by CWMR-Stdev with respect to the mean reversion sensitivity parameter ϵ on four daily datasets.



(a) W-NYSE (O)



(b) W-NYSE (N)

Fig. 3. Scalability of the total wealth achieved by CWMR with respect to transaction cost rate (γ).